TANGENT FERMIONS

Chiral fermions on a lattice without doubling

and what they can do:
Symmetric mass generation in a single Dirac cone

Vladimir Zakharov

Lorentz Institute, Leiden University

collaborators: A. Ueda, F. Verstraete, C. Beenakker, M. J. Pacholski, J. Tworzydło, J. Sánchez Fernán, S. Polla, A. Donís Vela, P. Emonts, G. Lemut

Review arXiv:2302.12793 (free particle)

QMC arXiv:2401.10828

DMRG arXiv:2407.06713

SSB arXiv:2601.09563

SMG arXiv:2606.24713

Frontiers of Lattice Fermions — Yukawa Institute for Theoretical Physics, Kyoto. With gratitude to the organizers.

One chiral cone on a lattice — and the obstacle

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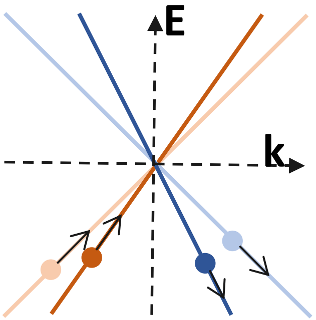

different QFTs

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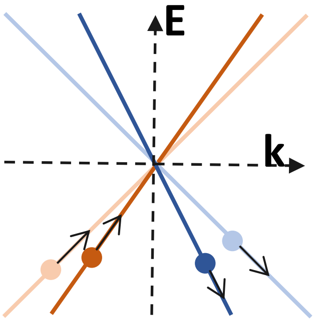

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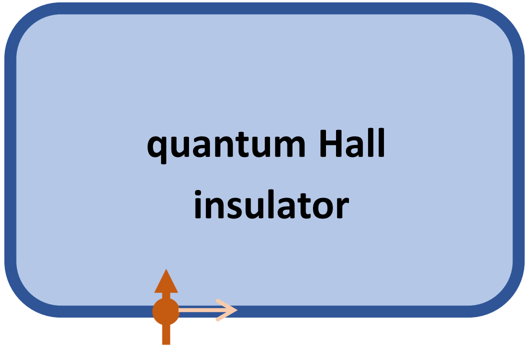

chiral Tomonaga-Luttinger liquid

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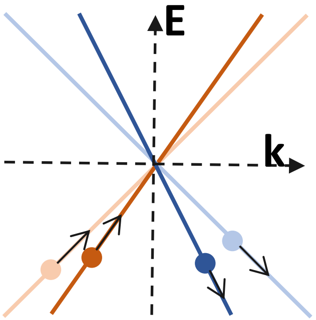

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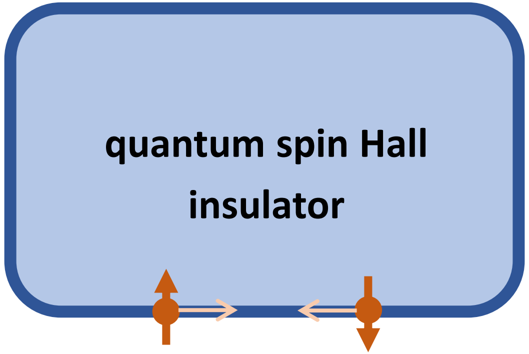

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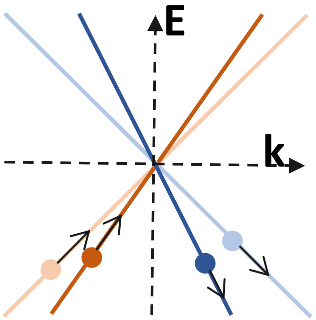

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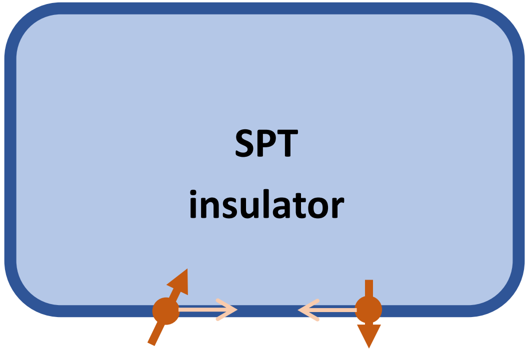

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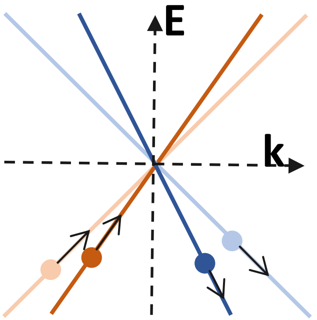

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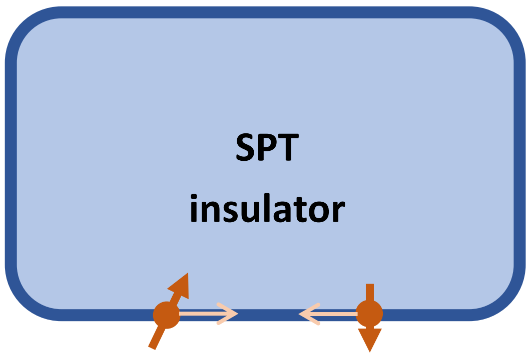

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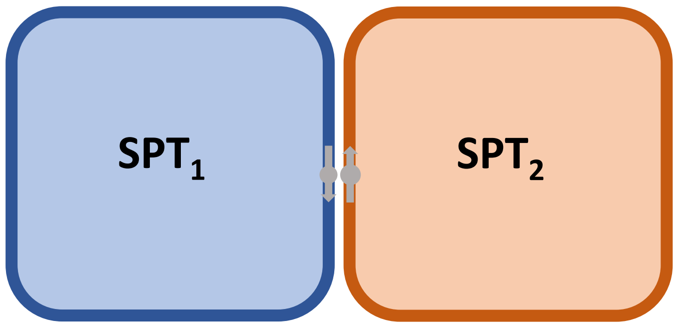

and their fusion

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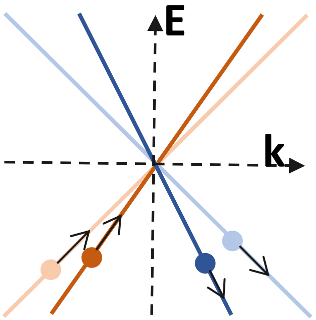

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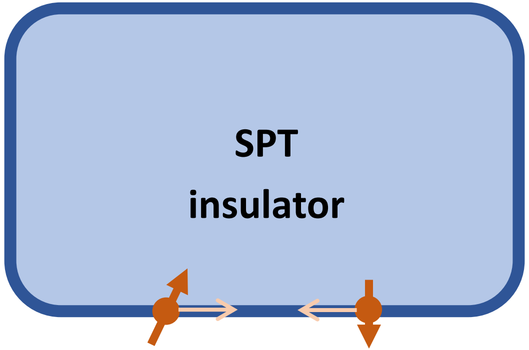

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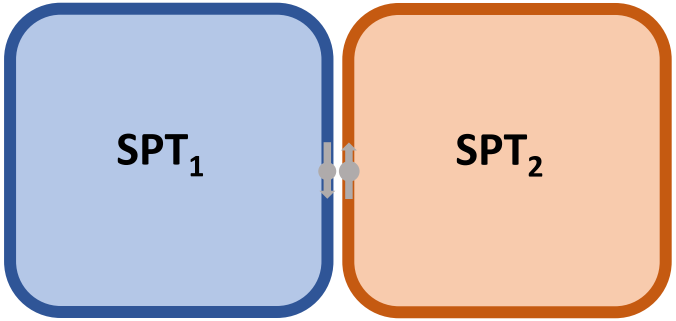

and their fusion

The obstacle on a lattice — you know it — Nielsen–Ninomiya (no-go) theorem

If you do not want to have a doubler, you must sacrifice something

What does each scheme give up?

Wilson

The Wilson term **breaks chiral symmetry** explicitly;
mass can be generated and must be tuned back.

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Staggered

The doublers survive as tastes;
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Domain-wall

Requires **extra dimension;**
residual mass decays exponentially in its extent.

Sen, Yasunaga

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Chiral symmetry is exact but **is not on-site;**
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SLAC

Exactly linear dispersion with a cusp at the zone edge (a sawtooth).

$\partial_x \leftrightarrow (-1)^{n-m} / (n-m)$ *(SLAC derivative)*

Nonlocal hoppings;
known to give the wrong continuum when interacting

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Alternative approaches (not single-particle Dirac equation discretizations):

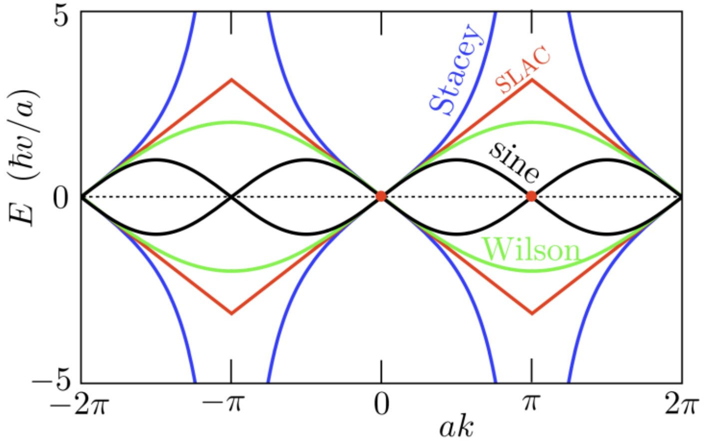
Boson-based *(Shao, Morikawa, Aoki)*; symmetry disentanglers *(Thorngren)*; Kahler-Dirac fermion *(Vaibhav)*;
SMG-based routes that lift the doublers *(Shamir, Xu, Araki)*; and non-Hermitian routes.

DEFINITION

Tangent fermions = Stacey's derivative, revived

$$H = \sum_{n \neq m} 2it_0(-1)^{n-m}\, c_n^\dagger c_m$$

$$E(k) = 2t_0 \tan(k/2)$$



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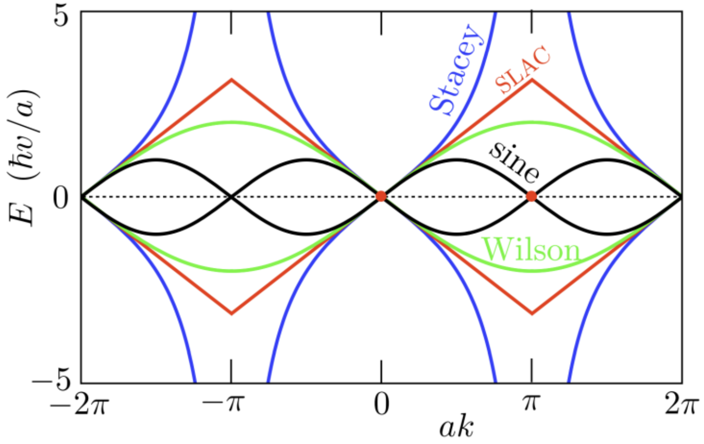

Eliminating lattice fermion doubling

Richard Stacey

*Department of Physics and Astronomy, University of College London, Gower Street, London WC1, England**

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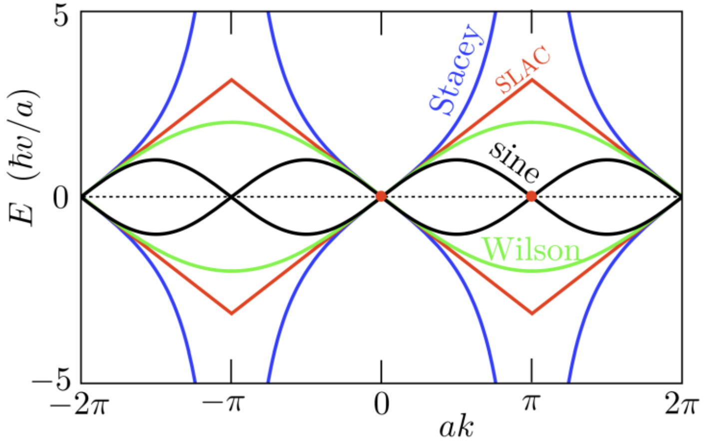

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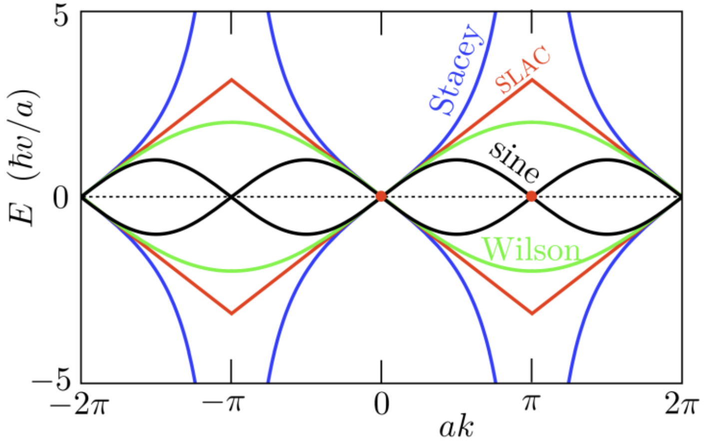

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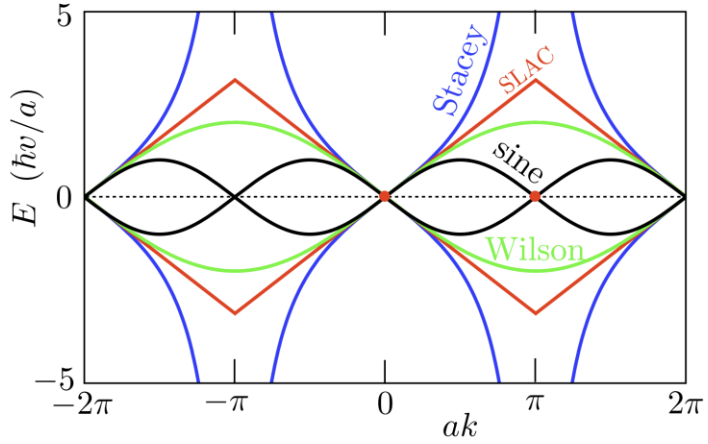

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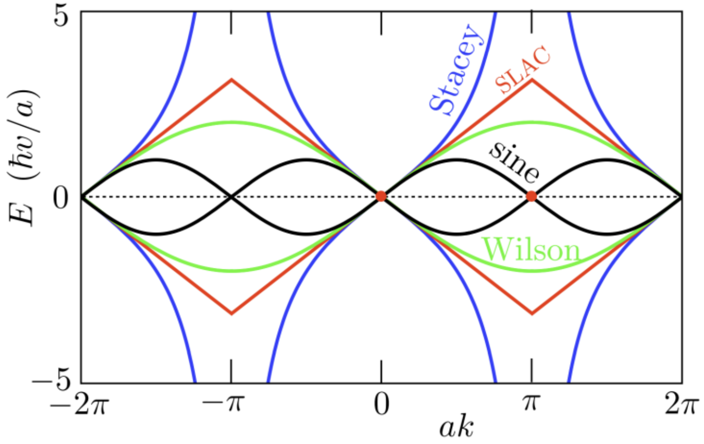

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- Chiral symmetry is **ordinary and on-site**: $\{\sigma_z, H\} = 0$
- The hopping is nonlocal, but the physics is **local** *(next slide)*

Only the hopping is nonlocal — nothing else is

Local generalized eigenvalue problem (single particle)

$\hat{H}_{\tan}|\psi\rangle = E|\psi\rangle, \;\; |\psi\rangle = \hat{D}|\phi\rangle, \;\; \hat{D}^\dagger \hat{H}_{tan}\hat{D}|\phi\rangle = E\hat{D}^\dagger\hat{D}|\phi\rangle$ with $\hat{D}^\dagger\hat{H}_{\tan}\hat{D}, \; \hat{D}$ — local transformation $\hat{\psi}(x) = \hat{D}\hat{\phi}(x) = \frac{1}{2}[\hat{\phi}(x) + \hat{\phi}(x-1)]$ gives $\frac{1+e^{i\hat{k}a}}{2} 2a^{-1}\tan(\frac{\hat{k}a}{2})\frac{1+e^{-i\hat{k}a}}{2} = a^{-1}\sin(\hat{k}a)$

- enables free particle simulations — *arXiv:2302.12793*

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$\mathcal{S}_{\tan} = \overline{\boldsymbol{\Psi}} M_{\tan}\boldsymbol{\Psi}, \ \ \boldsymbol{\Psi} = D\boldsymbol{\Phi}, \ \ \mathcal{S}_{\tan} = \overline{\boldsymbol{\Phi}} D^\dagger M_{\tan} D\boldsymbol{\Phi},$ $M_{\tan}$ — non-local $D^\dagger M_{\tan} D, \ D$ — local

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Efficient MPO representation

$$\hat{H}_{\tan} = \prod_i W_{\tan}^{(i)} \qquad W^{(n)} = \begin{pmatrix} 1 & \hat{c}_n & \hat{c}_n^\dagger & 0 \\ 0 & -1 & 0 & \hat{c}_n^\dagger \\ 0 & 0 & -1 & \hat{c}_n \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

— matrix of local operators with fixed rank

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$S_{\tan} = \overline{\mathbf{\Psi}} M_{\tan} \mathbf{\Psi}$ … $D, \; D$ — local

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These facts are connected:
Local generalized eigenvalue formulation
of a Schrödinger equation allows for
a matrix product operator representation of the Hamiltonian with
fixed bond dimension.
Extra conditions of being ***non-local,*** crossing E = 0 ***only*** at
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Two more representations

- MPO representation of D operator — *arXiv:2405.10285* (F. Verstraete group, with A. Ueda)
- Quantum circuit decomposition of the tangent-fermion Dirac operator — *arXiv:2606.19020* (compare to Singh)

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Local physics

locally conserved charge & current · correct **entanglement scaling** · **correct physics** demonstrations (one on the next slide)

Helical Tomonaga–Luttinger liquid on a lattice

$$H = \int_{-L/2}^{L/2} dx \left[\psi_\uparrow^\dagger(-iv_F\partial_x)\psi_\uparrow - \psi_\downarrow^\dagger(-iv_F\partial_x)\psi_\downarrow + 2\pi v_F\, \kappa_{inter}\, \hat{\rho}_\uparrow(x)\hat{\rho}_\downarrow(x) + \pi v_F\, \kappa_{intra} \left(\hat{\rho}_\uparrow^2(x) + \hat{\rho}_\downarrow^2(x) \right) \right]$$

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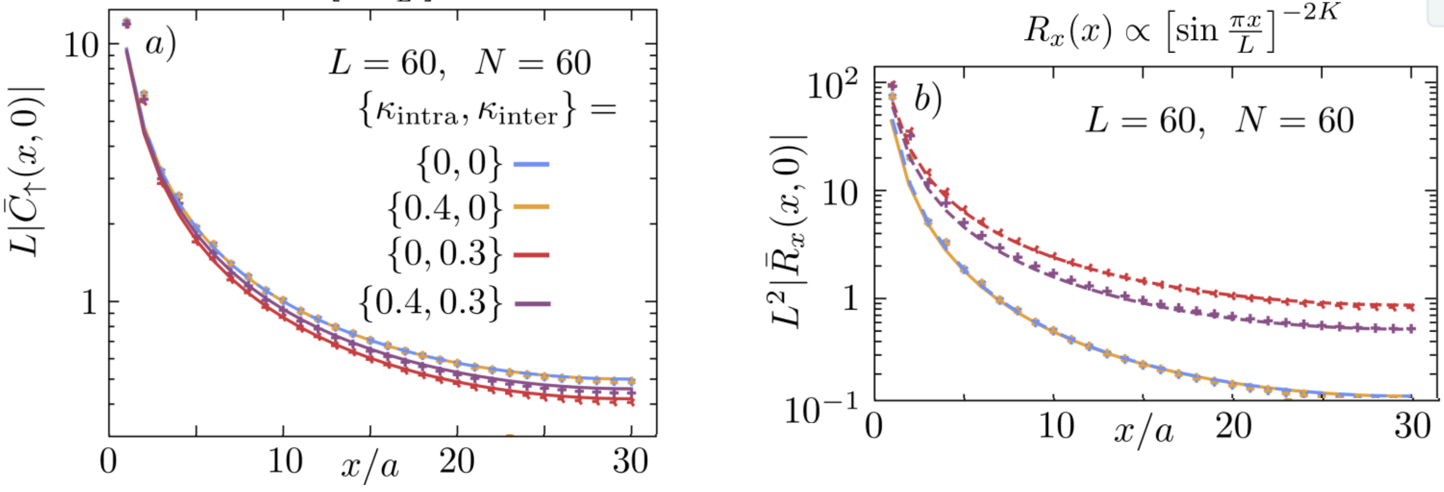
Propagator

$C_\sigma(x) \propto \left[\sin\frac{\pi x}{L}\right]^{-\frac{1}{2}\left(K+\frac{1}{K}\right)}$

Transverse spin correlator

$R_x(x) \propto \left[\sin\frac{\pi x}{L}\right]^{-2K}$



dots: tangent-fermion tensor network · curves: continuum bosonization (no fit parameter)

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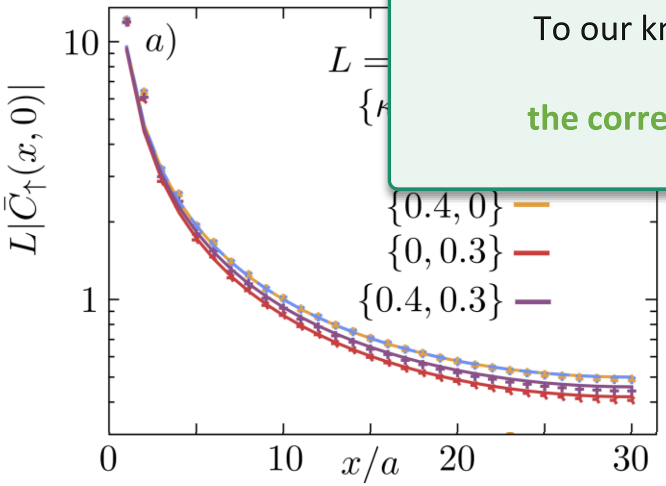
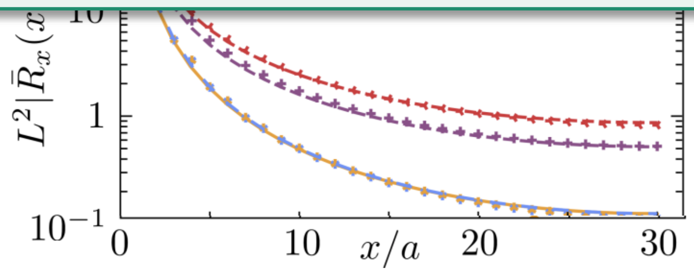
$C_\sigma(x) \propto [\sin$

$\sqrt{1 + \kappa_{intra} - \kappa_{inter}}$ ... $\kappa_{intra} + \kappa_{inter}$





To our knowledge tangent fermion is **the only** lattice-fermion representation that reproduces **the correct Tomonaga–Luttinger-liquid physics directly in 1D**.

dots: tangent-fermion tensor network · curves: continuum bosonization (no fit parameter)

WHERE TANGENT SITS

The comparison

	exact chiral symmetry	no doublers	on-site chiral symmetry	local hoppings	local action	efficient MPO
Wilson	✗	✓	✓	✓	✓	✓
staggered	remnant	✗	✓	✓	✓	✓
overlap / domain-wall	✓	✓	✗	✓	✓	✗
SLAC	✓	✓	✗	✗	✗	✗
tangent	✓	✓	✓	✗	✓	✓

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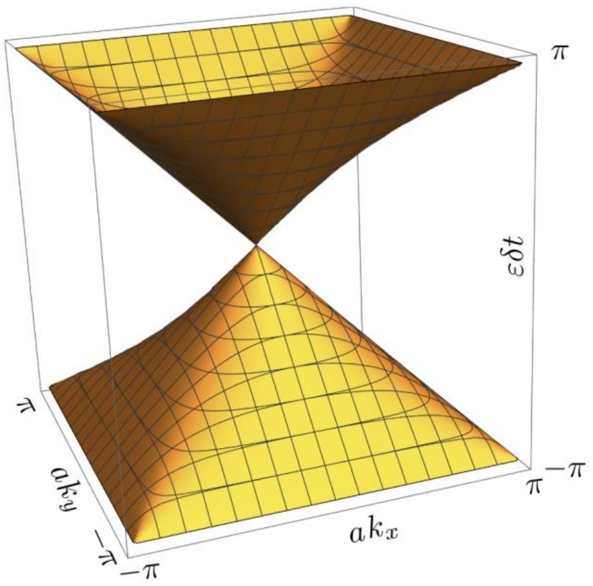
overlap / domain-wall	✓	✓	✗	✓	✓	✗
SLAC	✓	✓	✗	✗	✗	✗
tangent	✓	✓	✓	✗ (✓- GEV)	✓	✓

Why tangent fermions

- **The chiral physics of the continuum is protected** — and the representation is easy and natural.



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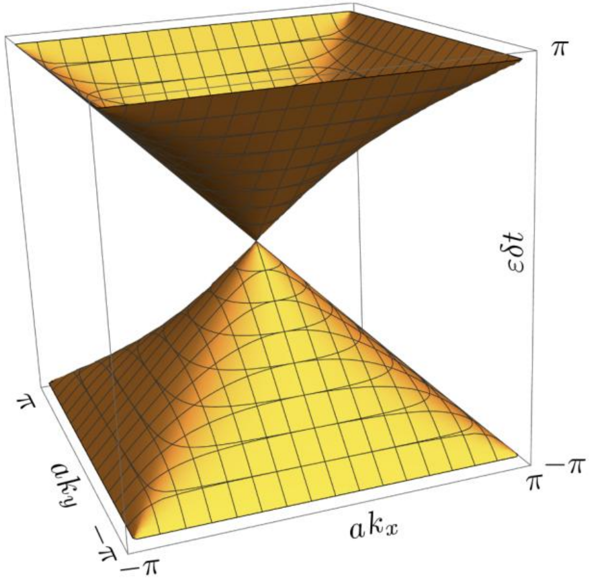

PART 1 — TAKEAWAY

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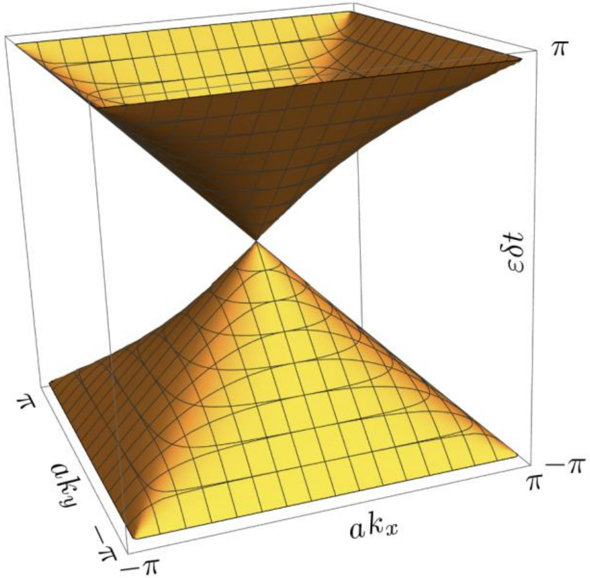

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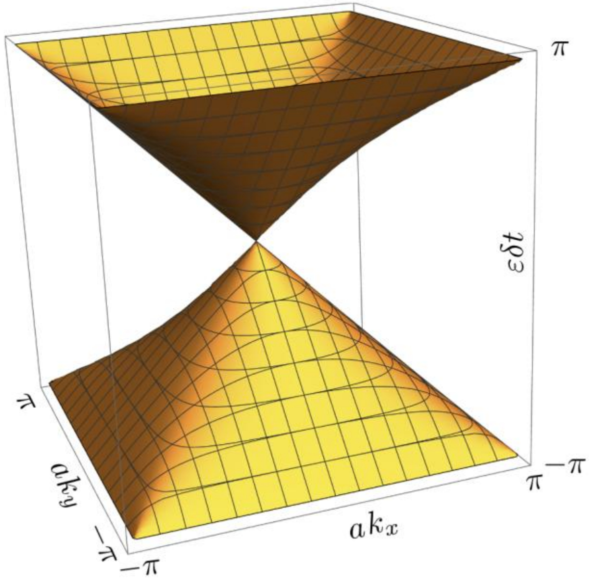

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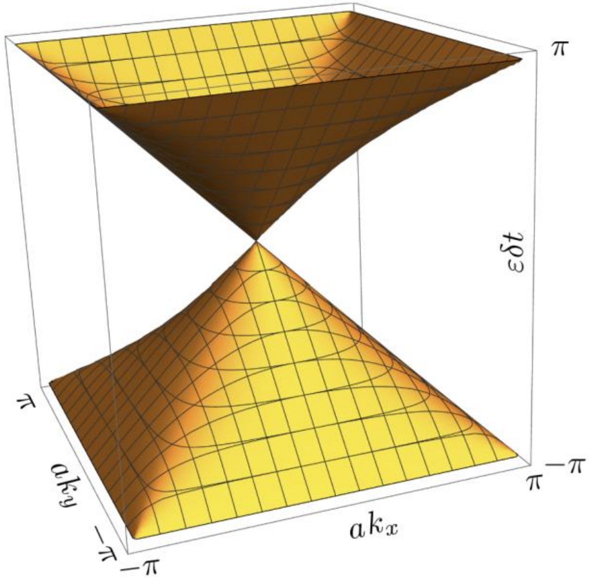

Not just a construction — a working method for real problems.

Here is one: symmetric mass generation in a single Dirac cone.

SHOWCASE

Symmetric mass generation in a single Dirac cone

3-4-5-0 model with U(1) anomaly cancelling and SMG interactions tuned to be RG relevant

Zakharov, Ueda, Verstraete, Beenakker — arXiv:2606.24713

Quantum spin Hall example

In absence of backscattering — helical TLL

$$H = \int_{-L/2}^{L/2} dx \left[\psi_\uparrow^\dagger(-iv_F\partial_x)\psi_\uparrow - \psi_\downarrow^\dagger(-iv_F\partial_x)\psi_\downarrow + 2\pi v_F\,\kappa_{inter}\,\hat{\rho}_\uparrow(x)\hat{\rho}_\downarrow(x) + \pi v_F\,\kappa_{intra}\left(\hat{\rho}_\uparrow^2(x) + \hat{\rho}_\downarrow^2(x)\right)\right]$$

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QSH is protected by time-reversal symmetry

$$\mathcal{T}: \quad \psi_\uparrow \mapsto \psi_\downarrow, \quad \psi_\downarrow \mapsto -\psi_\uparrow, \qquad \mathcal{T}^2 = -1$$

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Two-particle backscattering

$$H_{\mathrm{U2}} = g_{\mathrm{U2}} \int dx \left[\left(\psi^\dagger_\uparrow\psi_\downarrow\right)^2 + \left(\psi^\dagger_\downarrow\psi_\uparrow\right)^2\right]$$

double spin-flip: $\Delta S_z = \pm 2$
S_z not conserved (N is)

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Relevance

$$H_{\mathrm{U2}} \sim \int dx\, \cos(4\phi(x)), \qquad \Delta = 4K \qquad \frac{dg_{\mathrm{U2}}}{d\ell} = (2-4K)\, g_{\mathrm{U2}}$$

relevant for **$K < K_c = ½$**

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Gap opening ⇒ spontaneous time-reversal breaking

$$\cos(4\phi(x)) = \hat M_x^2 - \hat M_y^2$$

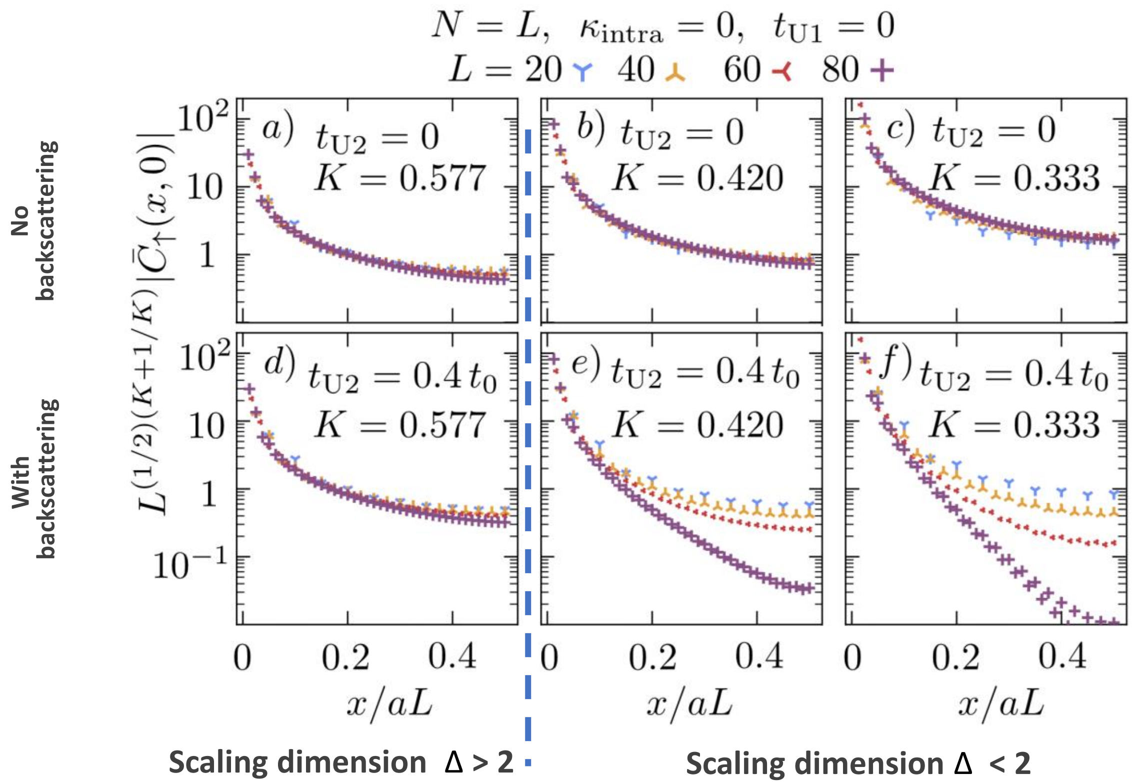
cosine term is pinned to a minimum

M_y if $g_{U2} > 0$
M_x if $g_{U2} < 0$

$$M_\alpha(x) = \tfrac{1}{2}\langle\psi^\dagger(x)\,\sigma_\alpha\,\psi(x)\rangle = \pm\sqrt{\cos(4\phi)}$$

two degenerate ground states $|\pm\rangle$ exchanged by T

Mass generation observed in the propagators

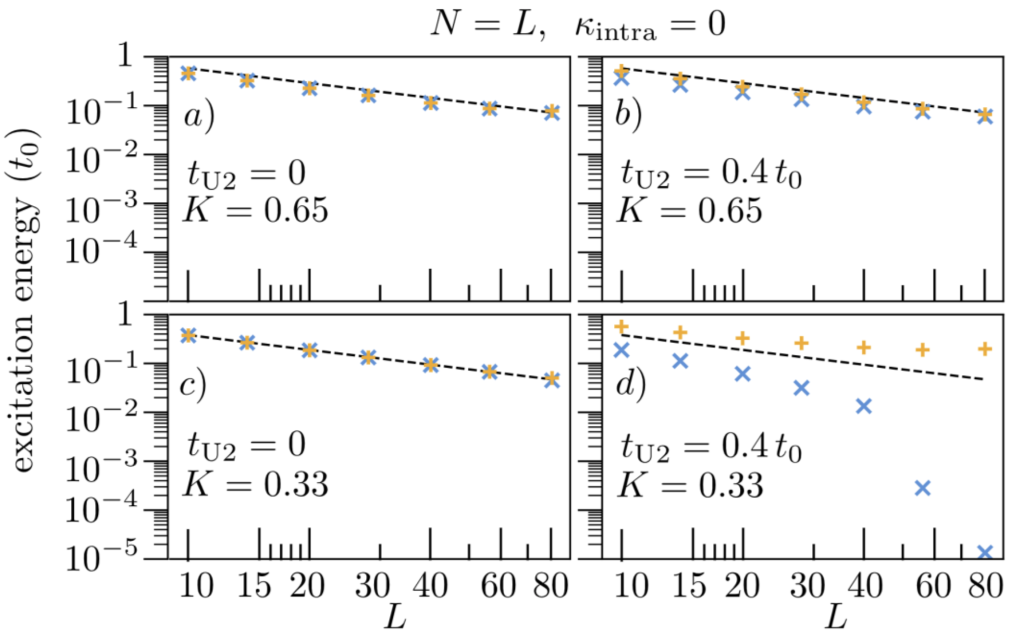


→ propagator **decays exponentially**: the signature of gap opening

SSB and gap opening observed in the excitation spectrum

No backscattering

With backscattering



Scaling dimension Δ > 2

Scaling dimension Δ < 2

→ in a gapless system the excitation energy decays as **ΔE = 2πℏv/L** (dashed line)

Anomaly-free chiral fermions on a 1D lattice

Model — free part + six-fermion SMG interaction

$$H_0 = \sum_{n>m} t_{nm}\left(c^\dagger_{n,3}c_{m,3} + c^\dagger_{n,4}c_{m,4} - c^\dagger_{n,5}c_{m,5} - c^\dagger_{n,0}c_{m,0}\right) + \text{H.c.}$$

$$t_{nm} = 2it_0(-1)^{n-m}, \qquad E(k) = 2t_0\tan(k/2)$$ *tangent hopping, no doubler*

$$H_{3450} = \sum_n \left(g_1 c_{n,3}c^\dagger_{n,4}c^\dagger_{n+1,4}c_{n,5}c_{n,0}c_{n+1,0} + g_2 c_{n,3}c_{n+1,3}c_{n,4}c^\dagger_{n,5}c^\dagger_{n+1,5}c_{n,0}\right) + \text{H.c.}$$

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U(1) symmetry and the 't Hooft anomaly

$$\sum_\alpha q_\alpha \ell_\alpha = 0 \quad \sum_\alpha \text{sign}(v_\alpha) l_\alpha^2 = 0 \Longleftrightarrow \sum_\alpha sign(v_\alpha) q_\alpha^2 = 0 \rightarrow 3^2 + 4^2 - 5^2 - 0^2 = 0$$

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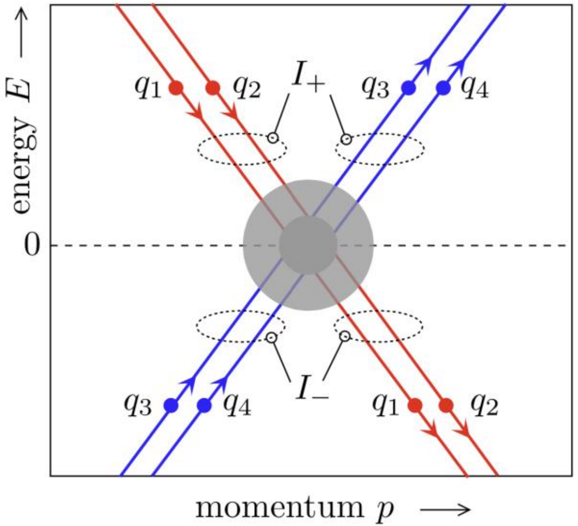

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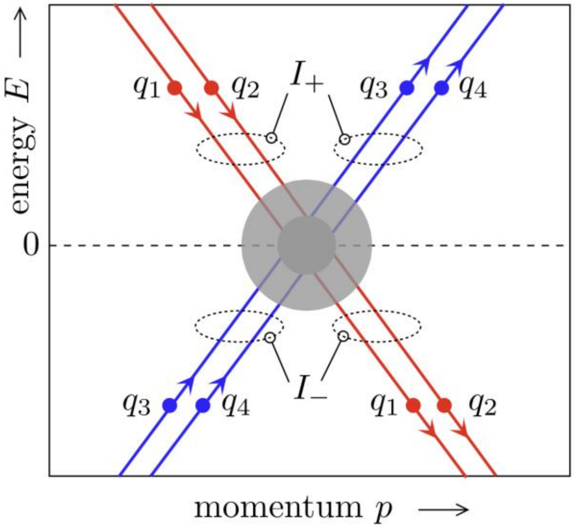

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We do not use SMG to remove doublers, we have a **different paradigm:**
Our cone is single from the start — we do SMG for the SMG physics itself.

A rotation turns the interaction into cosines

Rotate the chiral bosons — two decoupled Luttinger liquids (insight from Atsushi Ueda)

$$\widetilde{\Phi}_R = Q_R \Phi_R, \quad \widetilde{\Phi}_L = Q_L \Phi_L, \qquad Q_R = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}, \quad Q_L = -\frac{1}{\sqrt{5}}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$$

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The 3-4-5-0 interaction = cosines of the neutral fields

$$H_{\text{free}} = \sum_{p=1}^{2} \frac{v}{2\pi} \int dx \left[K_p (\partial_x \theta_p)^2 + \frac{1}{K_p} (\partial_x \phi_p)^2 \right] \qquad V_p = \cos(\boldsymbol{\ell}^{(p)} \Phi) = \cos(2\sqrt{5}\, \phi_p), \;\; p = 1, 2$$

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As in the QSH warm-up: for clean, explainable Luttinger-liquid physics the scaling dimension must drop below 2 — that calls for a density–density interaction.

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Hubbard term with weights = ℓ-vectors: K becomes a knob

$$\begin{aligned} U_n = & -U_{\mathrm{H}}^{(1)} (\delta\rho_{n,3} - 2\delta\rho_{n,4})(\delta\rho_{n,5} + 2\delta\rho_{n,0}) \\ & -U_{\mathrm{H}}^{(2)} (2\delta\rho_{n,3} + \delta\rho_{n,4})(-2\delta\rho_{n,5} + \delta\rho_{n,0}) \end{aligned}$$

$$D_{3450}^{(\alpha)} = 5K_\alpha, \quad K_\alpha = \sqrt{\frac{2\pi t_0 - C U_{\mathrm{H}}^{(\alpha)}}{2\pi t_0 + C U_{\mathrm{H}}^{(\alpha)}}}, \quad C = \sqrt{\nu_3^2 + \nu_4^2}\sqrt{\nu_5^2 + \nu_0^2} = 5$$

→ the 3-4-5-0 interaction becomes relevant for $K < 2/5 \equiv K_c$

Why the gapped ground state is unique

We want to minimize two cosine terms simultaneously

$$V_p = \cos(\boldsymbol{\ell}^{(p)}\Phi) = \cos(2\sqrt{5}\,\phi_p), \quad p = 1, 2$$

$$\Longrightarrow \quad \boldsymbol{\ell}^{(p)}\Phi = 0 \bmod 2\pi, \; p = 1, 2$$

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Mapping from original bosonic fields

$$\mathcal{A}\Phi = A\Phi \bmod 2\pi, \qquad A = \begin{pmatrix} 1 & -2 & 1 & 2 \\ 2 & 1 & -2 & 1 \end{pmatrix}$$

Is its kernel single-connected?

$$\mathcal{M} = \{x \in \mathbb{T}^4 : Ax = 0 \bmod 2\pi\}$$

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Let’s choose another l-vectors: $\boldsymbol{\ell}^{(1)} = (1,3,-3,1), \quad \boldsymbol{\ell}^{(2)} = (-3,1,1,3)$

they also satisfy Haldane rule and compatible with 3,4,5,0 charges

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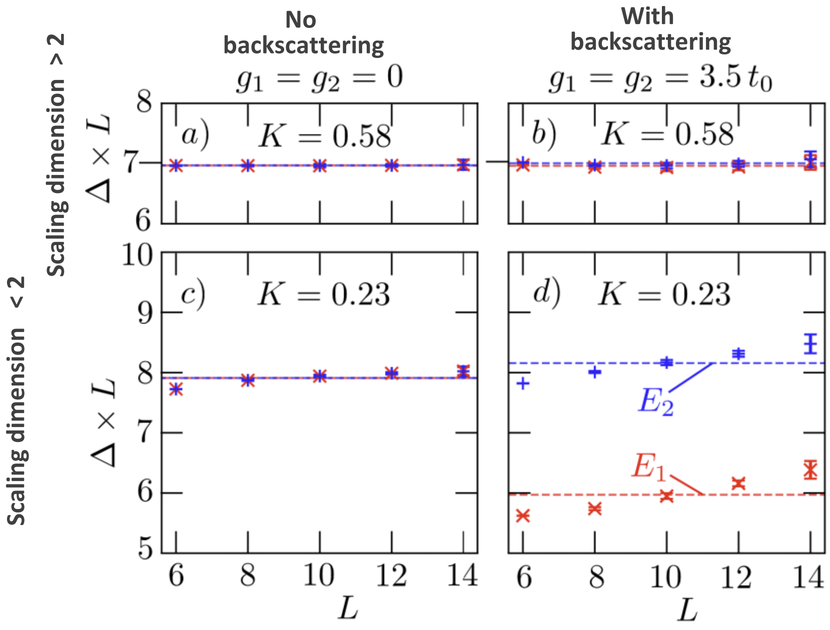
Is it still SMG then? *Cenke Xu says yes – degeneracy is accidental and can be lifted*

Gap opens, ground state stays nondegenerate

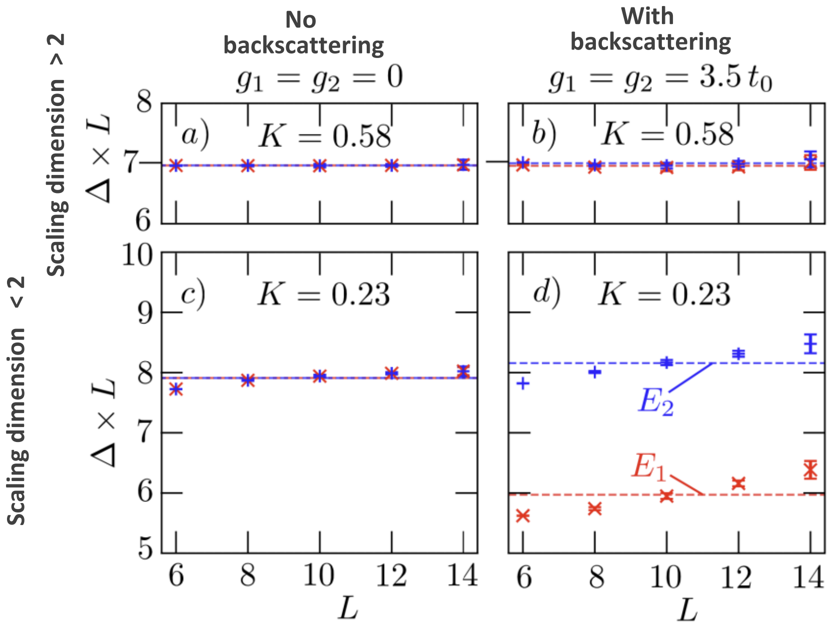


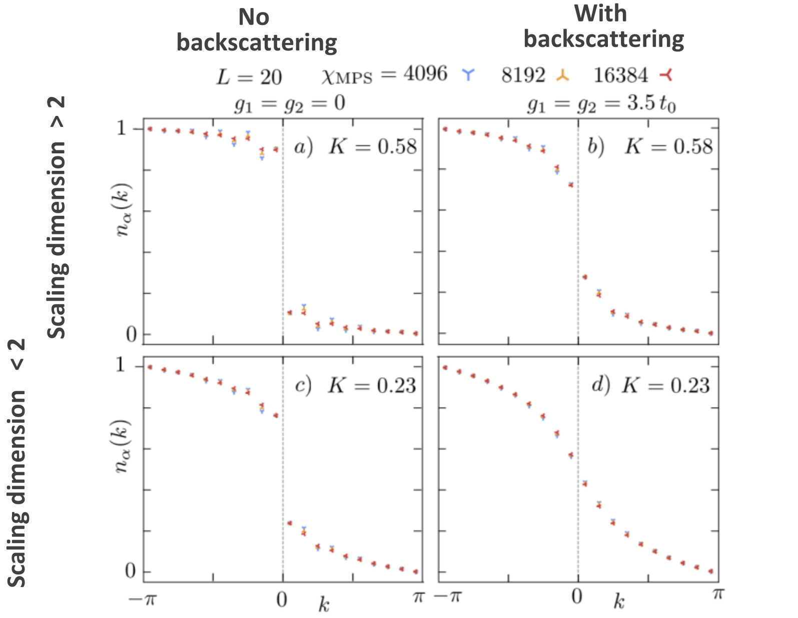
The gap appears only when both are present: the 3-4-5-0 interaction and $K < K_c$

E_1 does not collapse onto the ground state: **SMG, not SBB.**

IT WORKS

Gap opens, ground state stays nondegenerate





gapless Luttinger-liquid singularity: $n_\alpha(k) \propto |k|^{[K+K^{-1}]/2-1}$

The gap appears only when both are present: the 3-4-5-0 interaction and $K < K_c$

E_1 does not collapse onto the ground state: **SMG, not SBB.**

Conclusions

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All of this chiral fermion physics with a tunable K is accessible for one reason:
tangent fermion is a simple construction — and it works.

CONCLUSIONS

- **A single chiral cone on a strictly 1D lattice** — tangent fermions: only the hopping is nonlocal; the chiral symmetry stays ordinary and on-site.
- **Efficiency is not sacrificed** — the local generalized eigenvalue problem enables QMC and native tensor networks.
- **Symmetric mass generation in the 3-4-5-0 model** — Hubbard-tuned $K < 2/5$ makes the interaction relevant: the gap opens with no ground-state degeneracy.

Thank you