Symmetric mass generation of 8 Majorana domain-wall fermions

Sho Araki (The University of Osaka)

Based on: On-going work

collaborators: Hidenori Fukaya, Tetsuya Onogi, Satoshi Yamaguchi

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Domain-wall fermion

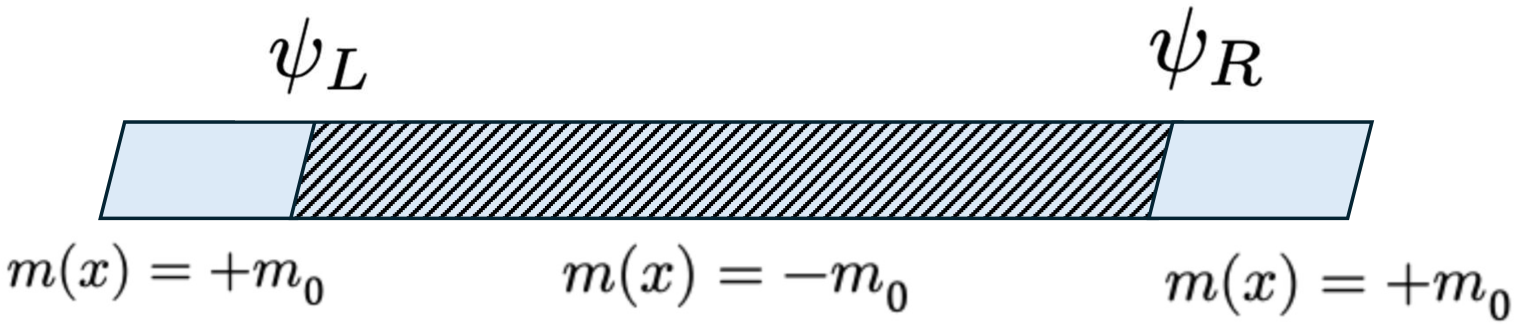
[Kaplan 1992]

A method to realize lattice Dirac fermions with precise chirality.

ψ_L (localized on a domain-wall)

$+ \ \psi_R$ (localized on the other wall)

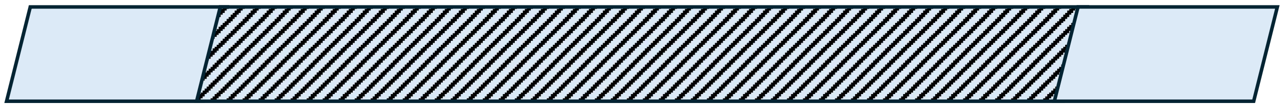
- Isolating single mode (i.e., formulating Weyl fermion) is a difficult problem. [Nielsen Ninomiya theorem]



Symmetric mass generation+ Domain-wall fermion

An approach to get only one-wall localized mode at low-energy by gapping out the mirror fermions.

Keep massless ψ_L ψ_R Gap out by SMG



Symmetric mass generation (SMG):
A symmetry-preserving mass gap generated by interactions, without a fermion bilinear mass term.

SMG is possible if the symmetry is anomaly-free.

Target model: 8 Majorana fermions in 1D

It provides a suitable testing ground for this domain-wall–SMG approach.

- **Instead of chiral symmetry, the model satisfies** $\mathbb{Z}_2^F \times \mathbb{Z}_2^T$ **(fermion parity)×(time reversal) symmetry**

→It exhibits non-perturbative $\mathbb{Z}_8$ anomaly

Our previous work:
Numerical verification of the $\mathbb{Z}_8$ structure of the bulk SPT and anomaly inflow with Wilson/domain-wall fermions.

[Araki, Fukaya, Onogi, Yamaguch 2025]

Motivation

Our work is the first numerical study of only-one-wall SMG using domain-wall fermions in the Euclidean path-integral formalism.

- Spatial separation of the two edge modes allows the SMG interaction to be localized on only one wall.
- Euclidean path integral enable non-perturbative HMC simulations.

To our knowledge, this is first numerical study of this combination.

The SMG Hamiltonian is exactly solved. [Fidkowski-Kitaev 2008]
→It provides a controlled benchmark for testing our approach.

Fidkovski-Kitaev model in the Hamiltonian formalism (0+1D model)

Exactly solvable in quantum mechanics.
(finite dimensional Hilbert space)

8 Majorana fermions

$$c = (c^1, \cdots, c^8) \qquad \{c^i, c^j\} = 2\delta^{ij} \qquad c^{i\dagger} = c^i$$

Time reversal $\mathcal{T}$

$$c^i \to c^i \qquad i \to -i$$

Mass terms (quadratic terms) are prohibited.

$$ic^i c^j \to -ic^i c^j$$

- When the number of flavors are 8 (mod 8), SMG interaction terms can exist.

A 4-fermi interaction by Fidkowski-Kitaev

$$H_{FK} = -\frac{1}{24}\sum_{\alpha=1}^{7}\left(\sum_{i,j=1}^{8} c_i c_j \Gamma_{ij}^{\alpha}\right)^2$$

This simple from manifestly satisfies global Spin(7)※ symmetry [You and Xu 2015]

$$\Gamma = \left\{\sigma^{123}, \sigma^{203}, \sigma^{323}, \sigma^{211}, \sigma^{021}, \sigma^{231}, \sigma^{002}\right\}$$

$$\sigma^{ijk} = \sigma^i \otimes \sigma^j \otimes \sigma^k$$

$$\sigma^0 = 1_{2\times 2}$$

The ground state of the H_{FK} is unique (= no symmetry breaking) and the finite energy gap (= 14 energy unit) exist.

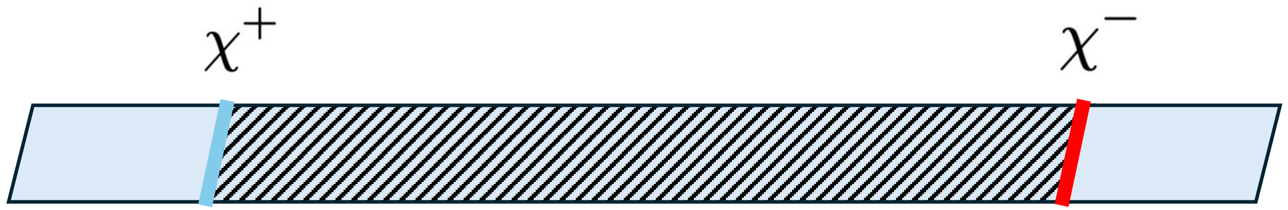
→The simplest SMG example.

※Spin(7) symmetry is not essential to SMG but it provide the direction of the model constructions.

2D domain-wall model

(We now switch to Euclidean path integral)

$$S_E = \frac{1}{2}\int dx^2 \chi^T(x,t_E) C\left(\gamma^\mu \partial_\mu + m(x)\right)\chi(x,t_E)$$



$m(x) = +m_0$ $\quad$ $m(x) = -m_0$ $\quad$ $m(x) = +m_0$

Reflection (Euclidean counterpart of time reversal) $\mathcal{R}_{t_E}$

$$\chi(x,t_E) \to i\sigma^1 \chi(x,-t_E)$$

$$\chi^+(x,t_E) \to +i\chi^+(x,-t_E)$$

$$\chi^-(x,t_E) \to -i\chi^-(x,-t_E)$$

Gamma matrices convention

$\gamma^x = \sigma^1$ $\quad$ $\gamma^{t_E} = \sigma^2$ $\quad$ $\gamma^3 = \sigma^3$ $\quad$ $C = i\sigma^2$ **(Charge conjugate)**

4-fermi interaction

$$S_{E,\mathrm{int}} = \int dx^2 \left\{ \frac{1}{2}\chi^T C \left(\gamma^\mu \partial_\mu + m\right)\chi - \frac{g_P^2}{2} \sum_{\alpha=1}^{7} \left(\frac{1}{2}\chi^T C i \gamma^3 \Gamma^\alpha \chi \right)^2 \right\}$$

Reflection invariant 4-fermi term
(pseudo-scaler)^2

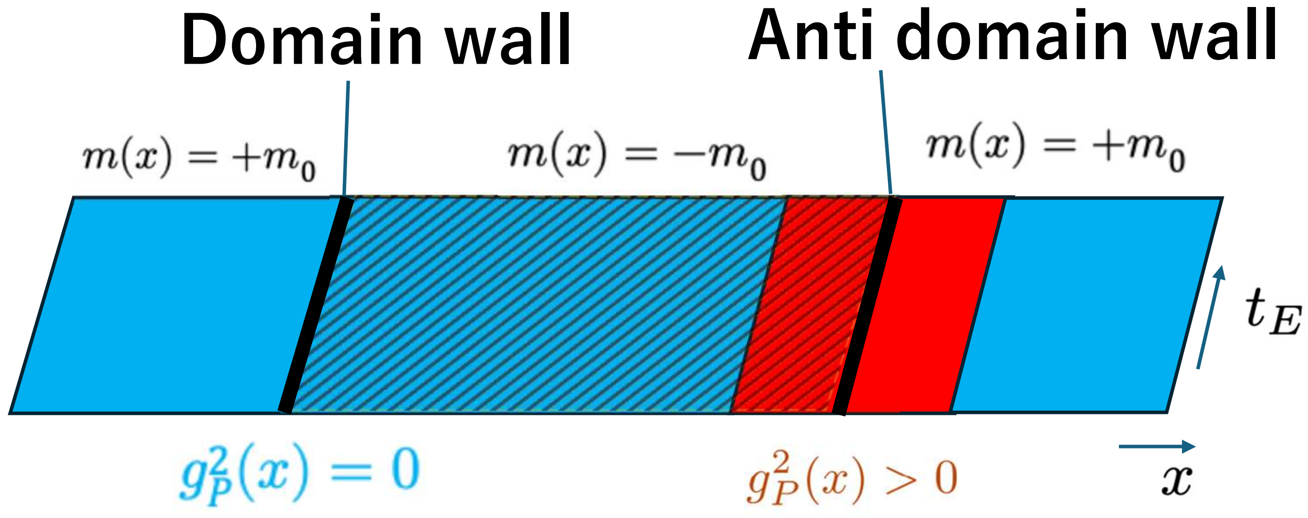
$$\Gamma = \left\{ \sigma^{123}, \sigma^{203}, \sigma^{323}, \sigma^{211}, \sigma^{021}, \sigma^{231}, \sigma^{002} \right\}$$

$$\sigma^{ijk} = \sigma^i \otimes \sigma^j \otimes \sigma^k \qquad \sigma^0 = 1_{2\times 2}$$

Turning on the coupling constant $g_P^2 > 0$ induce the SMG.

What we want to do:

① While keeping the modes on one domain wall massless,
② introduce the SMG interaction near the other (anti) wall to open a mass gap.



χ^+ :massless $\quad$ χ^- : interacting, gapped

Auxiliary-field technique (Hubberd-Stratnovich transformation)

$$Z = \int D\chi \exp(-S_{E,\mathrm{int}}[\chi])$$

$$= \int D\chi D\pi \exp(-S_{E,HS}[\chi,\pi])$$

Insert the "identity"

(Gaussian path-integral over the auxiliary field $\pi^a(x,t_E)$)

$$1 = \frac{1}{\mathcal{N}} \int D\pi \exp\left[-\int dx^2 \frac{1}{2} \sum_a \left\{ \pi^a(x,t_E) + \frac{\sqrt{g_P^2}}{2} \chi^T(x,t_E) C i\gamma^3 \Gamma^\alpha \chi(x,t_E) \right\}^2 \right]$$

$$S_{E,HS} = \int dx^2 \left\{ \frac{1}{2} \chi^T(x,t_E) C \left(\gamma^\mu \partial_\mu + m + \sqrt{g_P^2} \sum_{\alpha=1}^{7} \pi^\alpha(x,t_E) i\gamma^3 \Gamma^\alpha \right) \chi(x,t_E) + \frac{1}{2} \sum_{\alpha=1}^{7} (\pi^\alpha(x,t_E))^2 \right\}$$

4-fermi terms → bilinear Yukawa terms + boson term

2D lattice simulation

$$Z=\int D\pi \ \ \mathrm{Pf}(CD[\pi])\exp\left(-\sum_{x,t_E} a^2 \frac{1}{2}\sum_{\alpha=1}^{7}(\pi^\alpha(x,t_E))^2\right)$$

Stochastically evaluate the four-fermion interaction by sampling the auxiliary fields $\pi^a(x,t_E)$ with Hybrid Monte Carlo (HMC) method.

$$D[\pi]=\left(D_{W,\mathrm{free}}+m(x)+\sum_{\alpha=1}^{7}\pi^\alpha(x,t_E)i\gamma^3\Gamma^\alpha\right)$$

We employed the Wilson-Dirac operator.

- **It dose not break any symmetry.**

$$D_{W,\mathrm{free}}=\sum_\mu\left\{\gamma^\mu\frac{\nabla_\mu^f+\nabla_\mu^b}{2}-\frac{a}{2}\nabla_\mu^f\nabla_\mu^b\right\}$$

$\nabla_\mu^{f/b}$ **Forward/backward difference**

Parameters

・**lattice size**

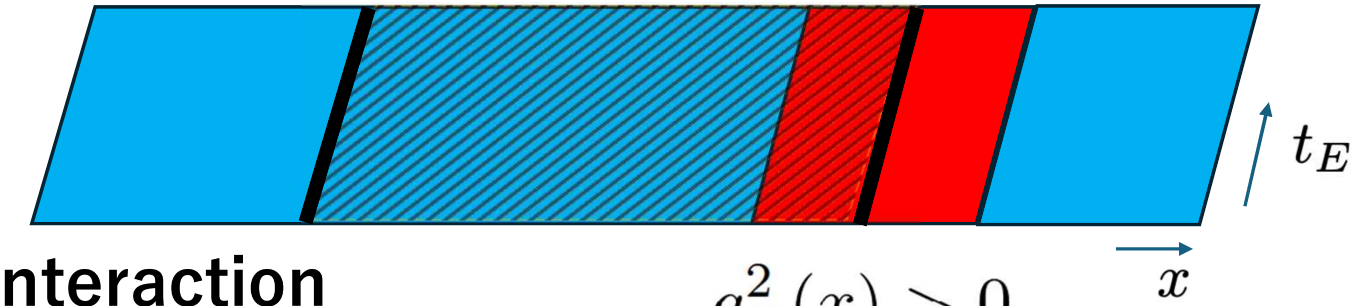
$N_x = 12$ $N_{t_E} = 12, 24$ $a = 1$

With periodic boundary conditions (for the both directions)

- **Domain-wall mass term**

$m(x) = +1$ for $x = 1, 2, 3,$ $10, 11, 12$

$m(x) = -1$ for $x =$ $4, 5, 6, 7, 8, 9$



- **DW-selective interaction**

$g_P^2(x) > 0$

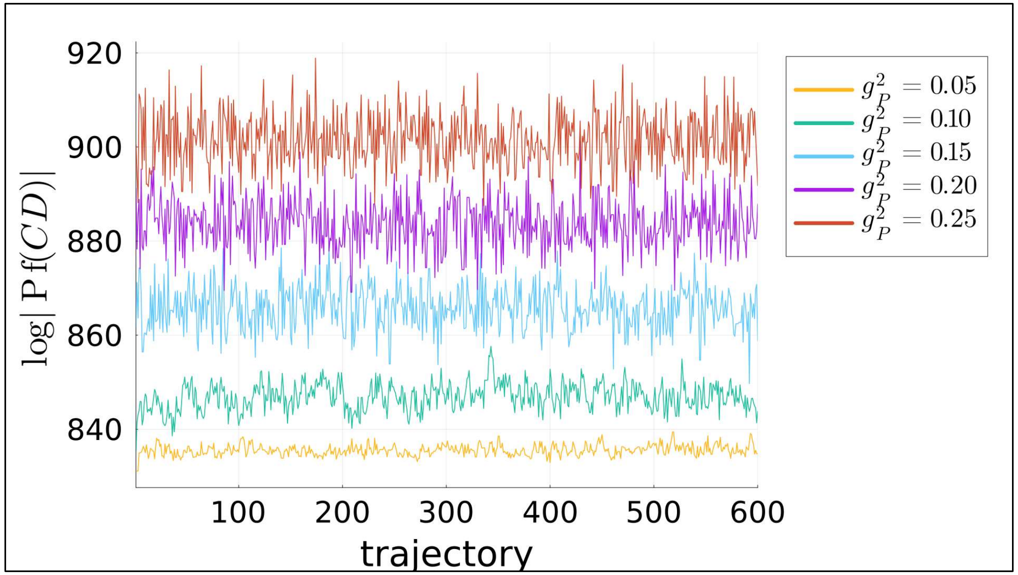
for $x = 8, 9, 10, 11$

HMC w/ <u>phase quenched</u> + reweighting $\langle O\rangle = \frac{\langle O e^{i\theta}\rangle_{pq}}{\langle e^{i\theta}\rangle_{pq}}$

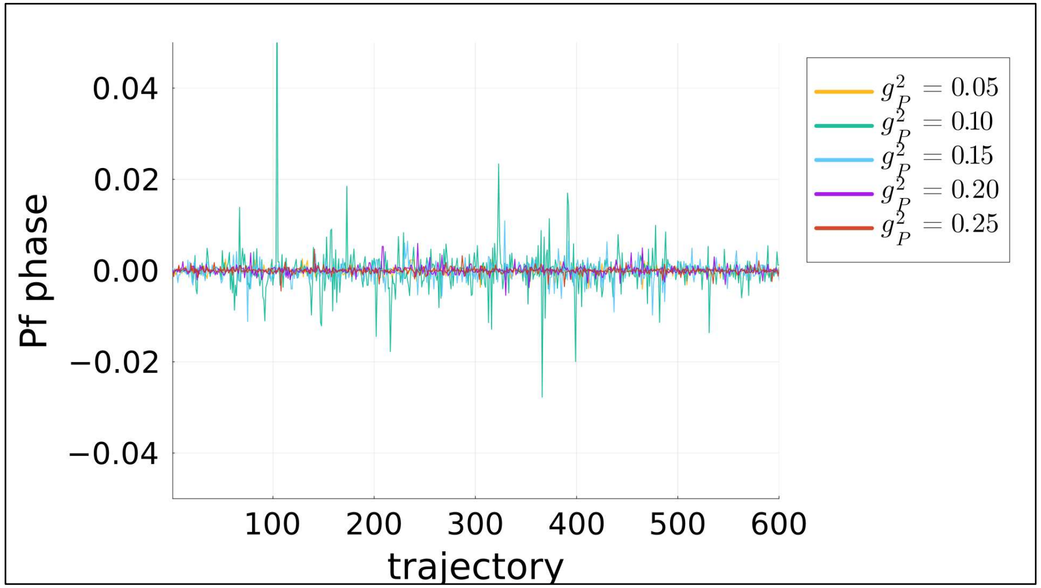
$\mathrm{Pf}(CD[\pi])$ is complex

12×12 lattice

log|Pfaffian| history



argument(Pfaffian) history



Complex phases of Pfaffian are small. =<u>Sign problem is negligible</u>

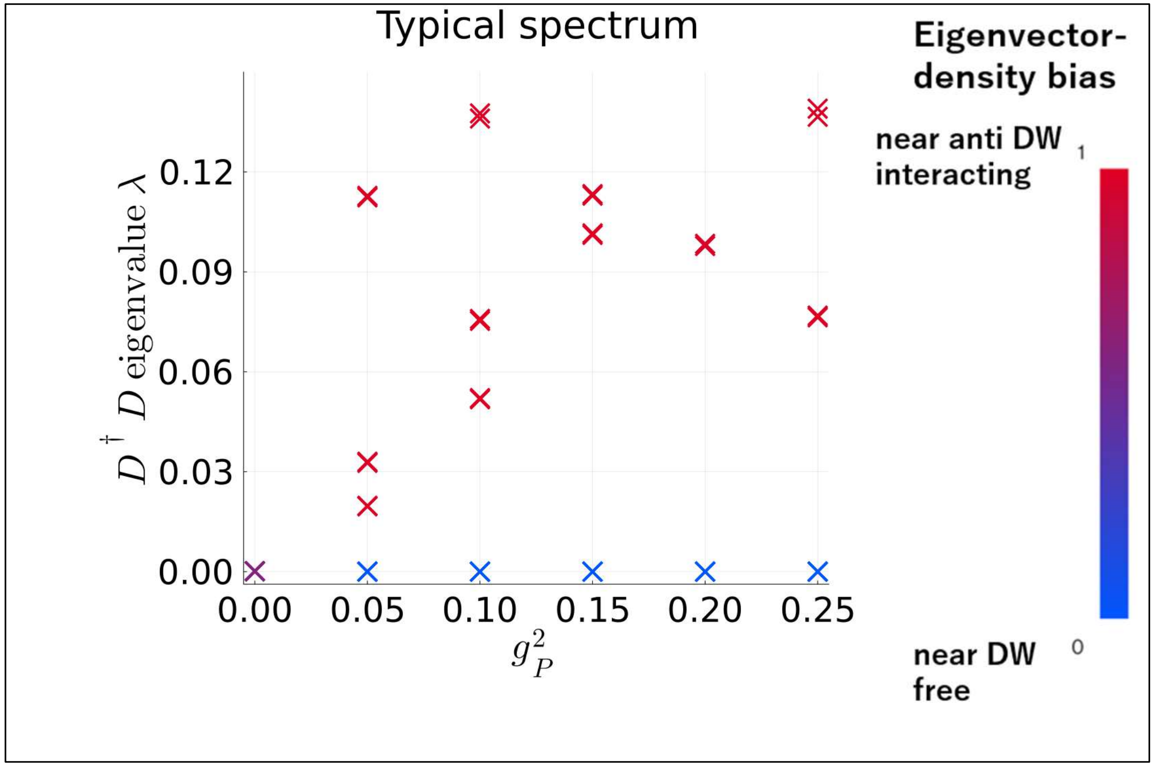
(total effect < 1 %)

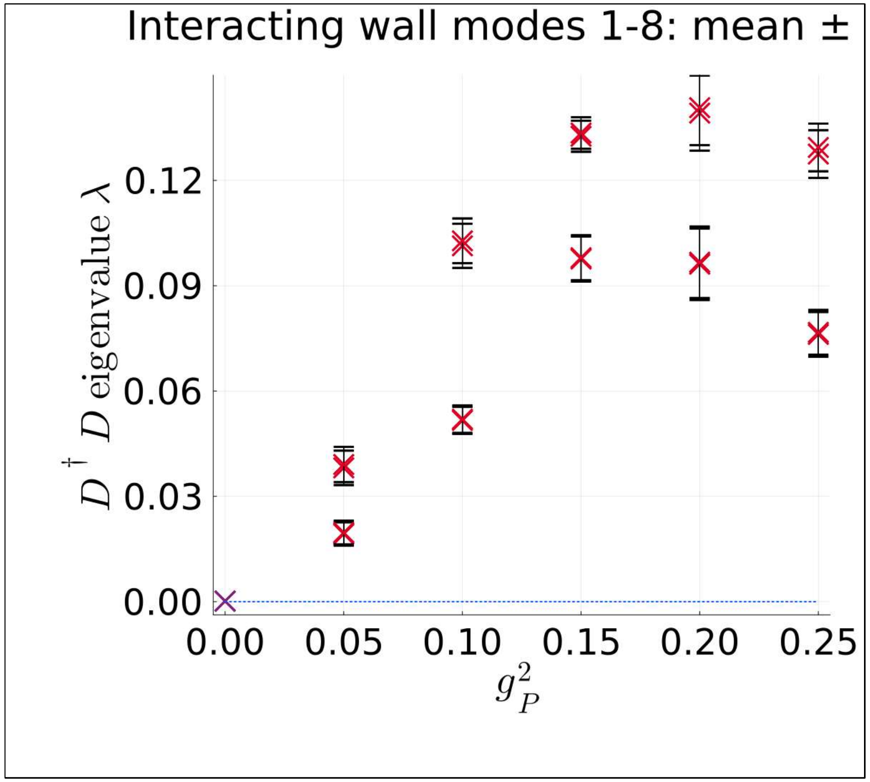
Spectrum of $D^\dagger D$

$$D = \left(D_{W,\text{free}} + m(x) + \sqrt{g_P^2} \sum_{\alpha=1}^{7} \pi^\alpha(x, t_E) i\gamma^3 \Gamma^\alpha \right)$$

preliminary

12×12 lattice





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preliminary

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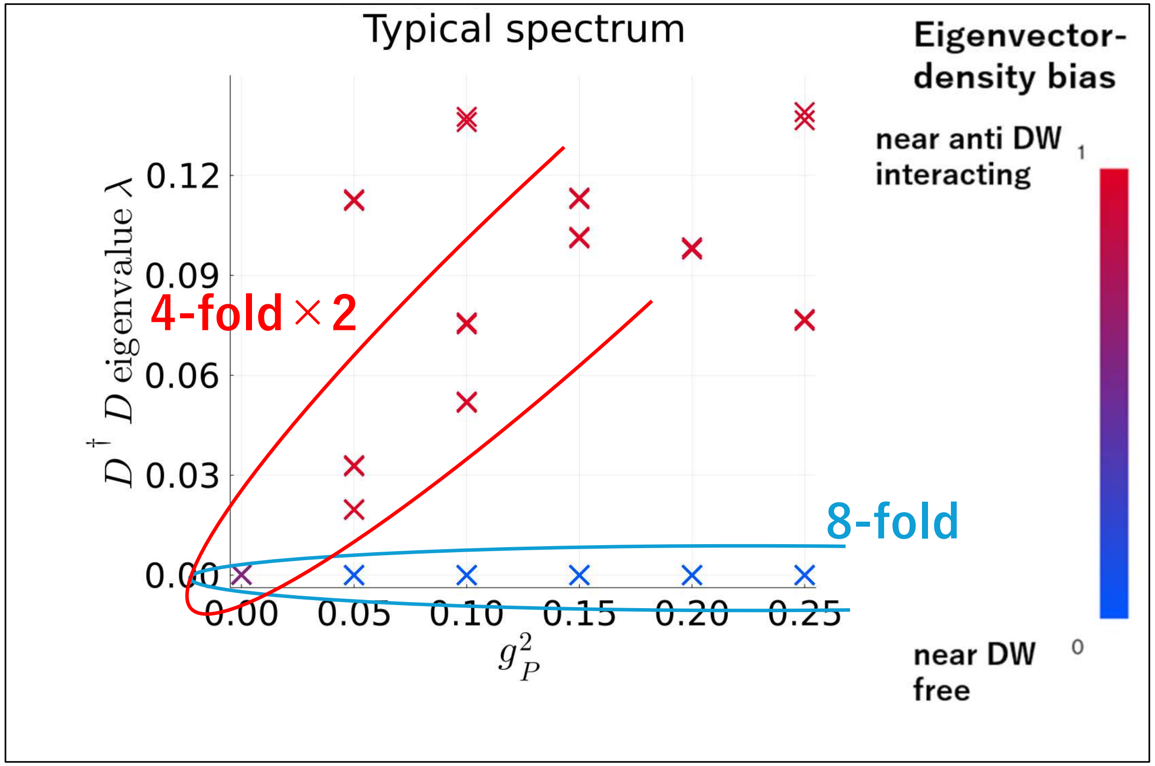
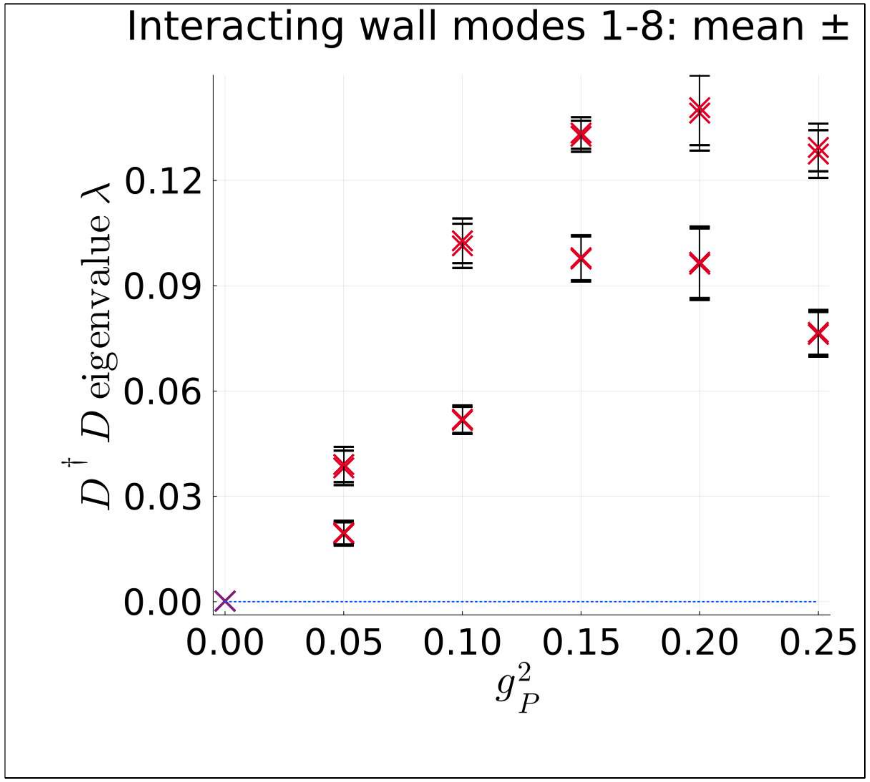
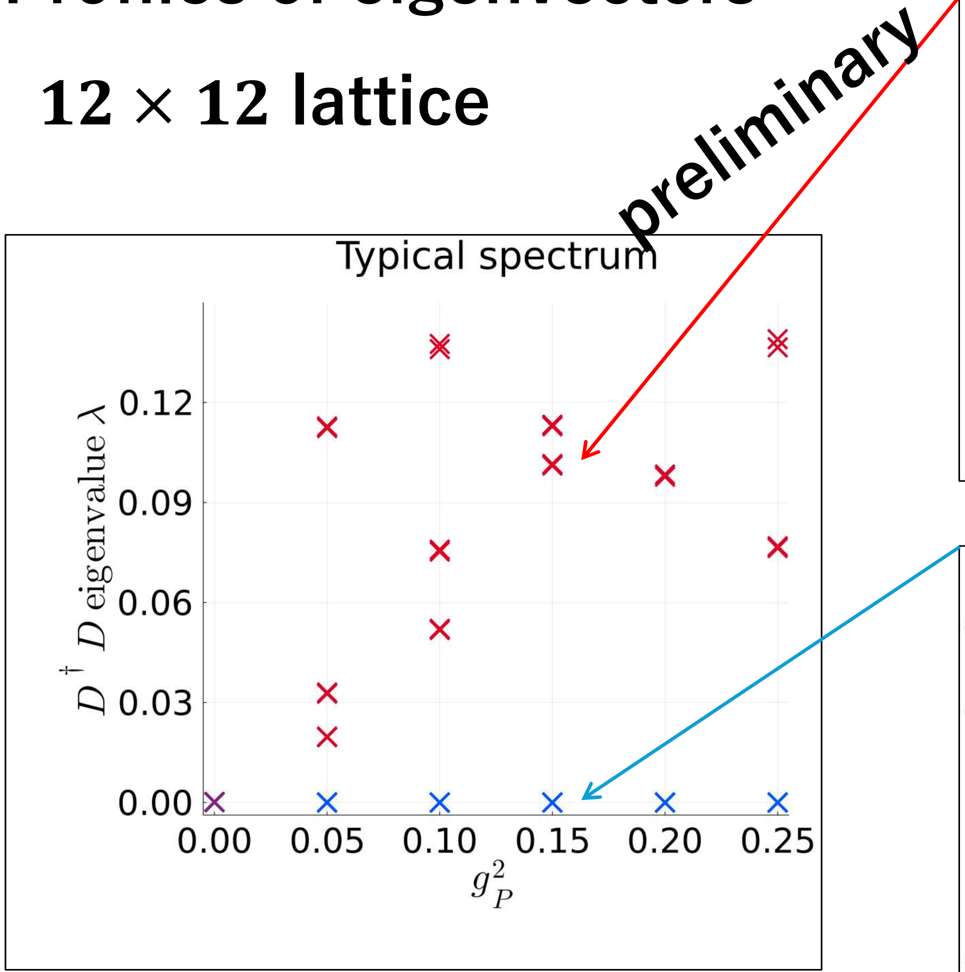
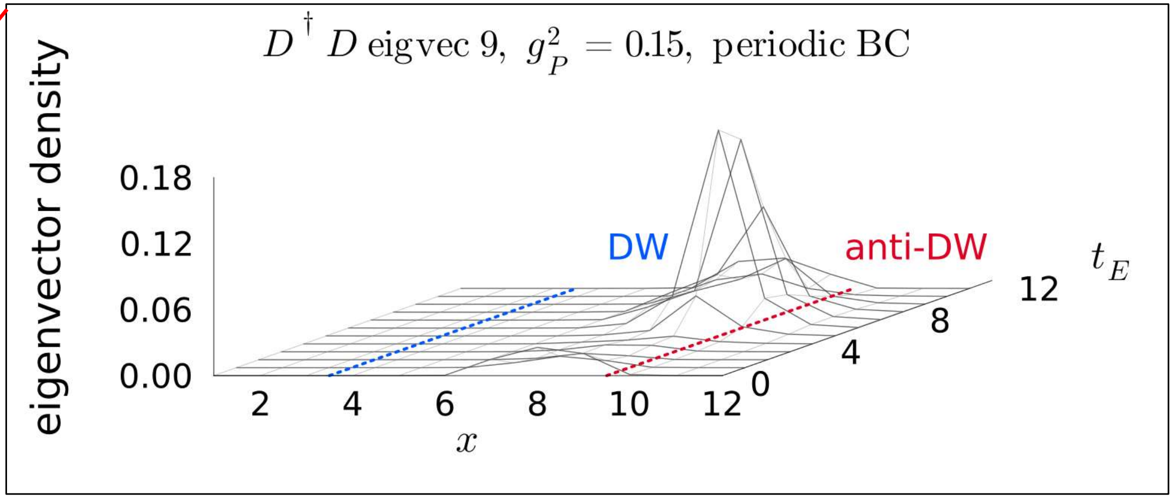


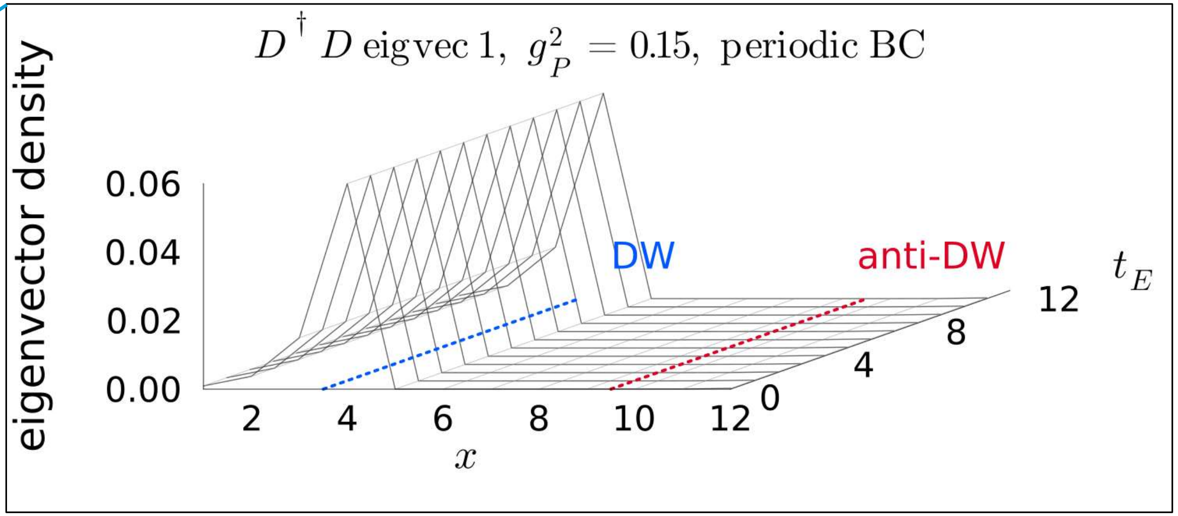


Profiles of eigenvectors

12 × 12 lattice



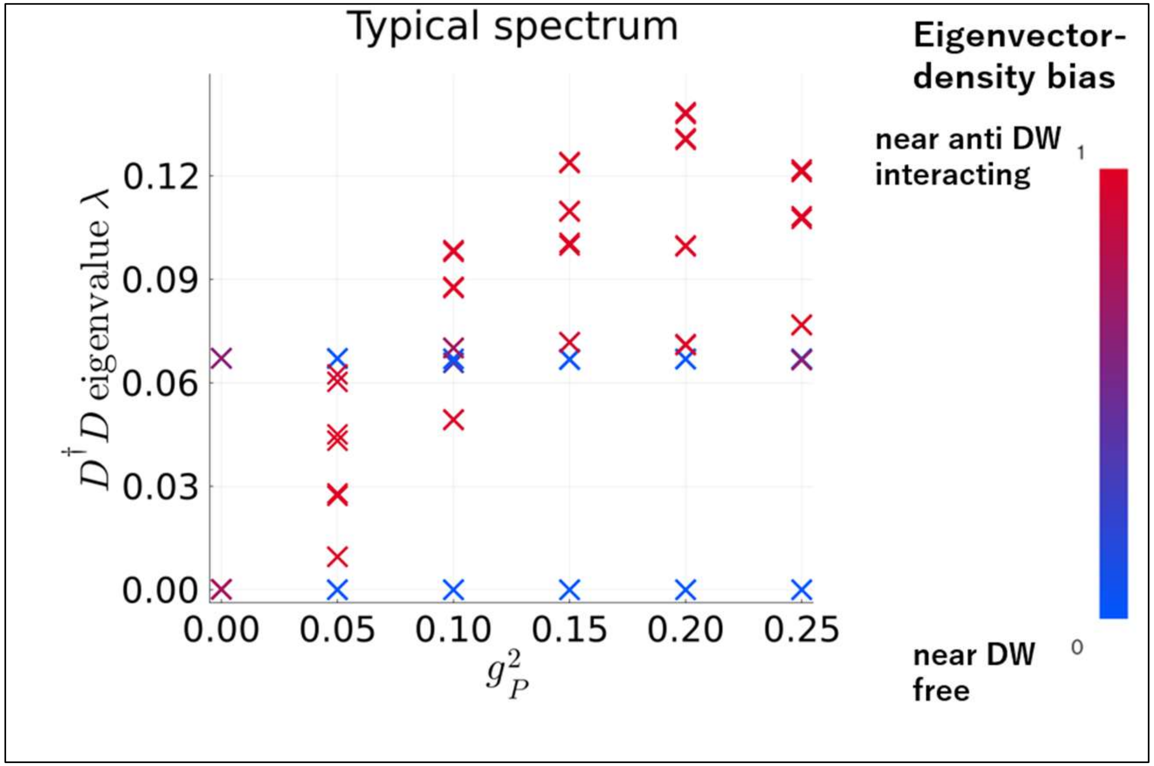


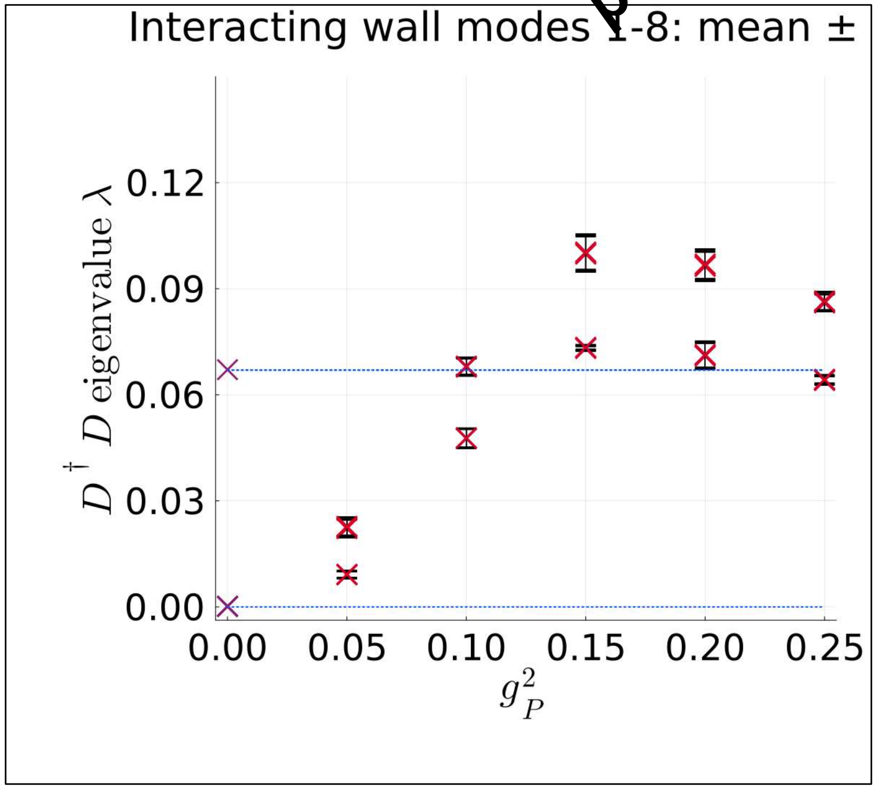


Spectrum of $D^\dagger D$

12×24 lattice

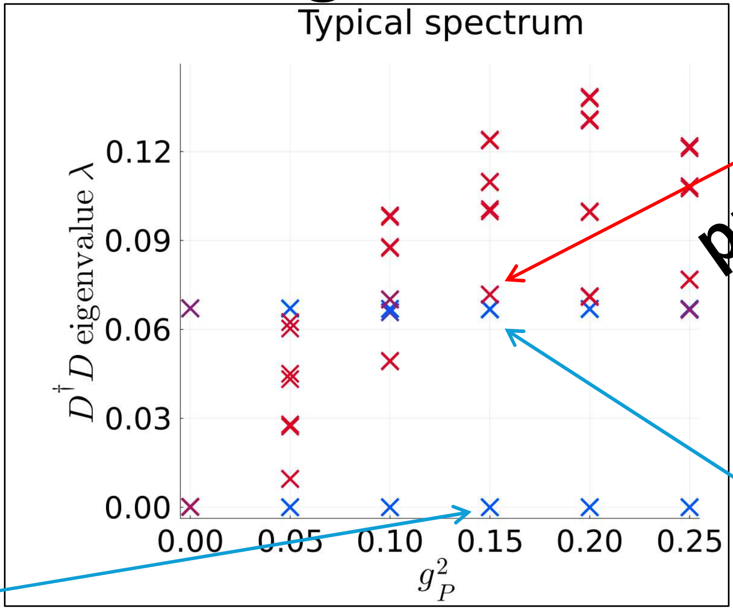
preliminary

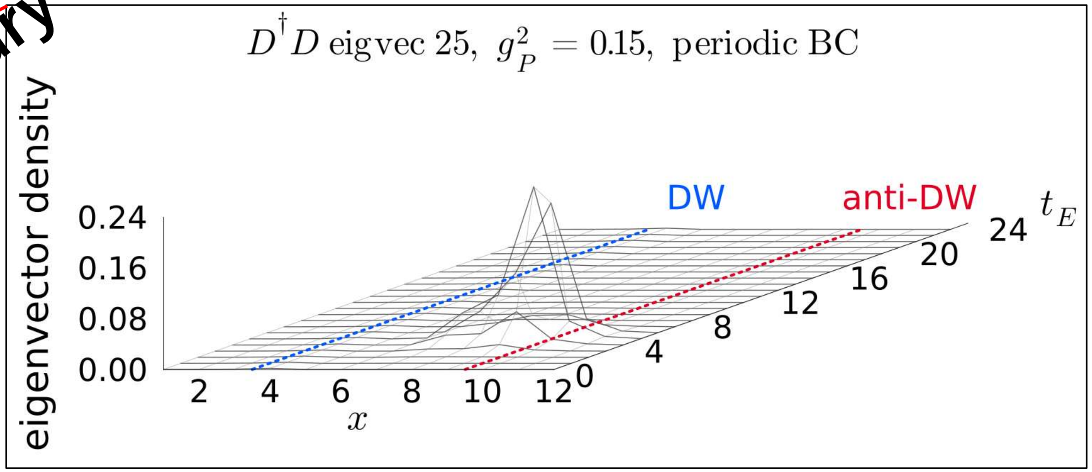




$p_{t_E} = \pm \frac{2\pi}{N_{t_E}}$ **modes (16 folded) are observed on the free DW side.**

Profiles of eigenvectors 12×24 lattice





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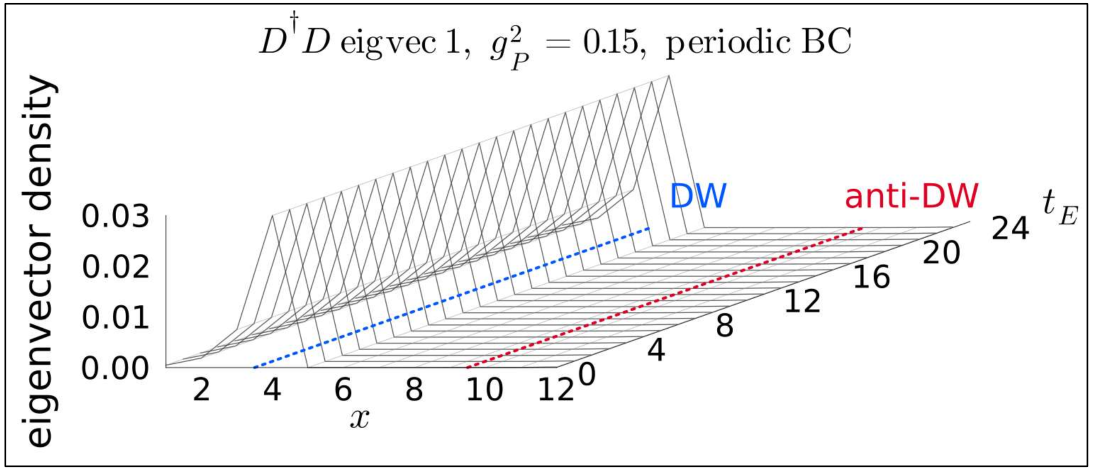

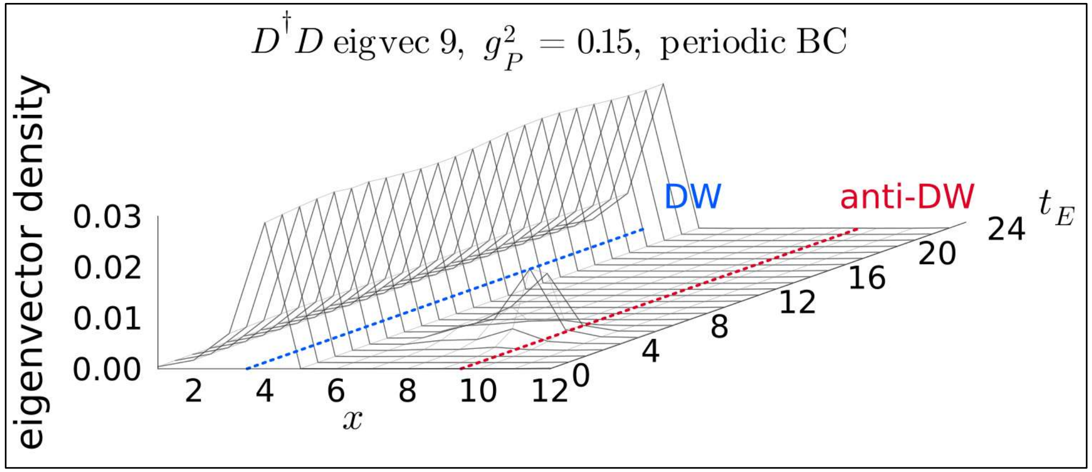

Remaining issue

Establish stronger evidence for SMG

- Determine whether the correlation functions exhibit power-law or exponential decay; the present analysis provides only a preliminary estimate.

More detailed analysis of gap generation

- Perform a scaling analysis to examine the continuum and large-gap limits, including whether both can be taken simultaneously.

Summary

We have simulated the path-integral version of Fidkowski-Kitaev model with domain-wall fermion.

- We have estimated gap by the Dirac-Yukawa operator with Higgs fields generated by HMC.

- We have introduced the SMG interaction only around one domain-wall.

→Eigenvalue spectrum exhibit that interacting modes obtain mass gap and non-interacting modes stay massless.