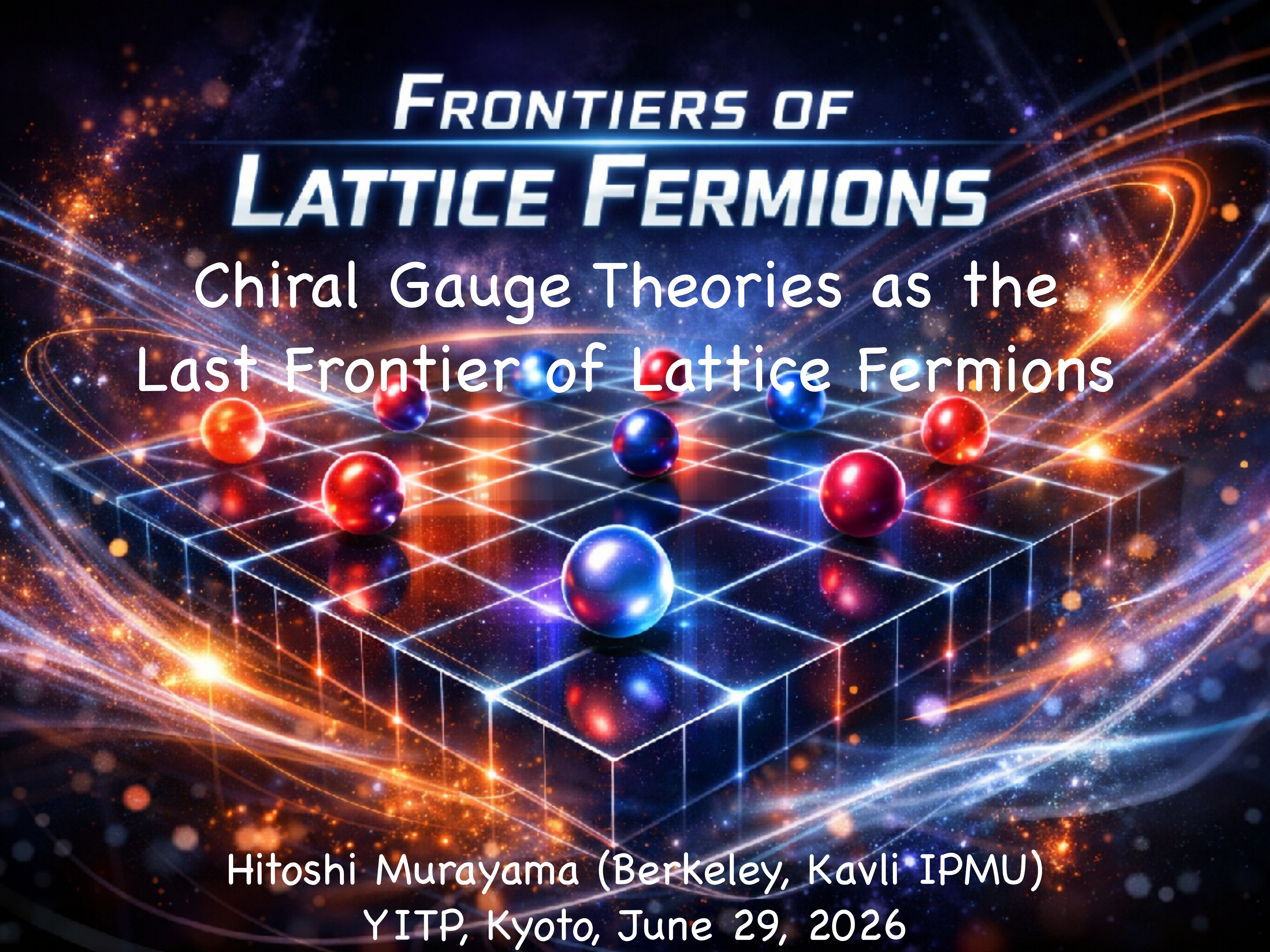


Chiral Gauge Theories as the Last Frontier of Lattice Fermions

Hitoshi Murayama (Berkeley, Kavli IPMU)

YITP, Kyoto, June 29, 2026

Lattice Gauge Theories

- I don't work on lattice fermions
- I've been awed by enormous progress in working QCD physics on lattices
- only practical tool for precision non-perturbative calculations in QFT
- lot of potential for non-QCD theories
- I'm looking forward to learn a lot during this week about the progress

Why Chiral Gauge Theories

- I don’t work on lattice fermions
  - All I know is chiral fermions are difficult
  - vector-like chiral symmetry
  - chiral gauge theories: last frontier
- Chiral Gauge Theories are difficult
  - difficult to formulate with lattice fermions
  - difficult to “guess” the universality class
- conjecture has been tumbling hypothesis
- new conjecture based on supersymmetry
- Which one is right?

Nature is chiral

- Standard model is a chiral gauge theory
- Nature taught us
  - abelian theory (QED)
  - non-abelian confining theory (QCD)
  - non-abelian broken theory (EW)
- There can be more
  - dark matter, axion, dark energy, inflation
  - likely that some of them are chiral
- need to know what chiral gauge theories do!

Outline

- Why chiral gauge theories
- Tumbling and 't Hooft anomaly matching
- near-SUSY QCD-like theories
- near-SUSY chiral gauge theories
- 2D Yang-Mills Theories

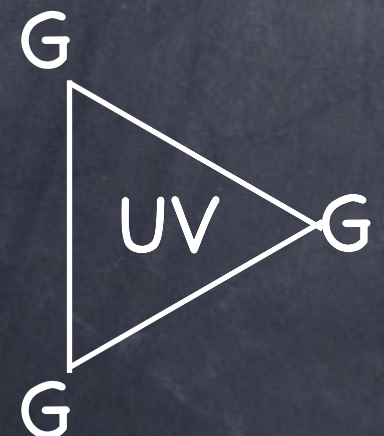
Tumbling and ’t Hooft anomaly matching

QCD-like theories

- We think they break global symmetries
- SU(N) with N_f F + F*
  - $SU(N_f)_L \times SU(N_f)_R$ to $SU(N_f)_V$ $\langle Q_R^i \bar{Q}_L^j \rangle \propto \delta^{ij}$
- SO(N) with N_f V
  - $SU(N_f)$ to $SO(N_f)$ $\langle Q^i Q^j \rangle \propto \delta^{ij}$
- Sp(N) with $2N_f$ F
  - $SU(2N_f)$ to $Sp(N_f)$ $\langle Q^i Q^j \rangle \propto J^{ij}$
- Not that we can prove it analytically
- we think this is the case at least for small N_f
- massless NGBs, no massless composite fermion

't Hooft anomaly matching

- Consider gauging global symmetry G
  - often it is anomalous



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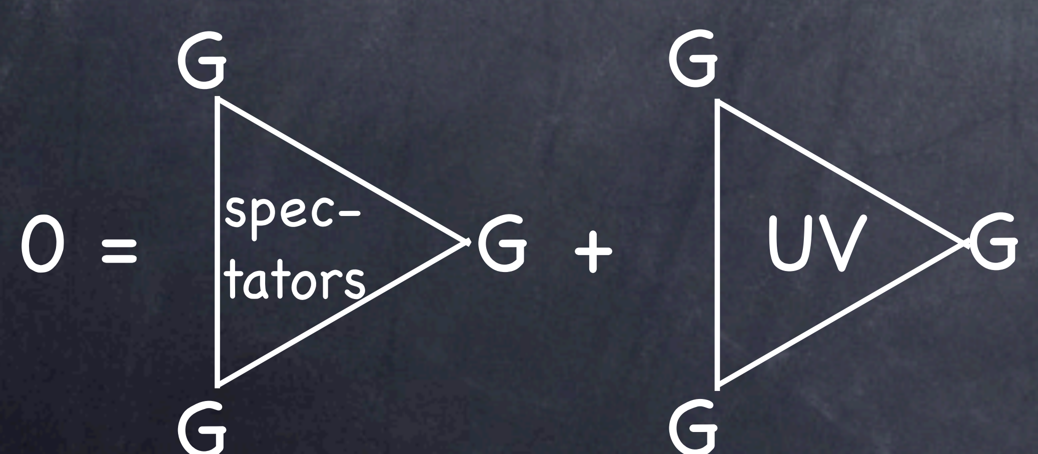

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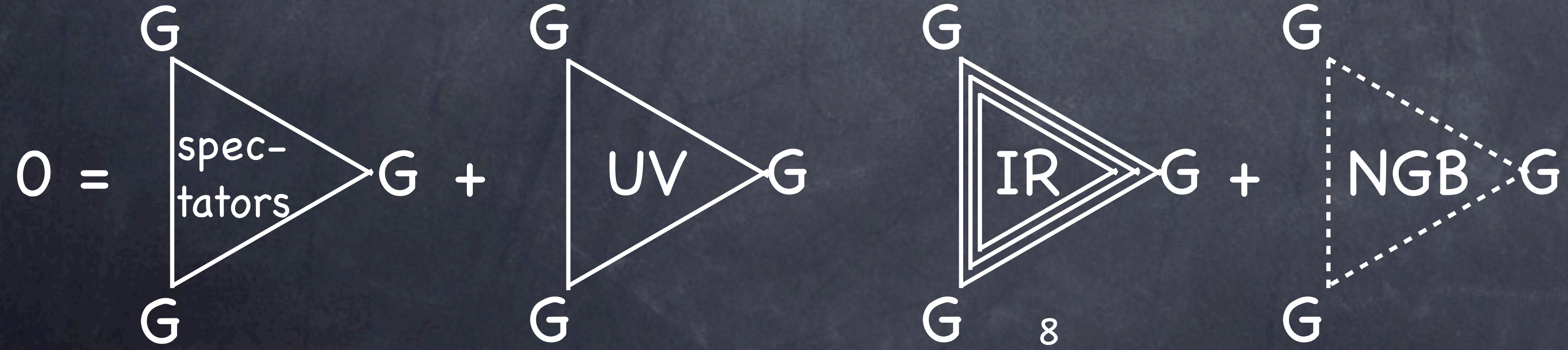
- In the IR, G may be broken to H
  - Wess-Zumino Witten term if $\pi_5(G/H)\neq 0$
  - massless composite fermions



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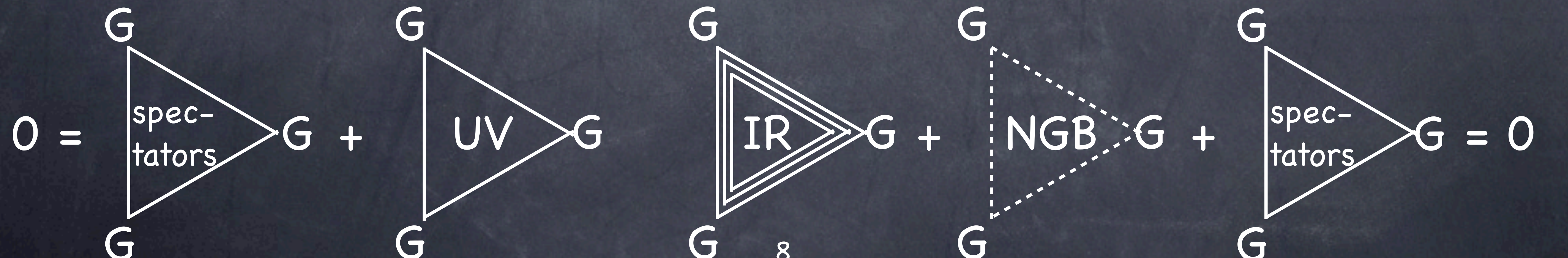

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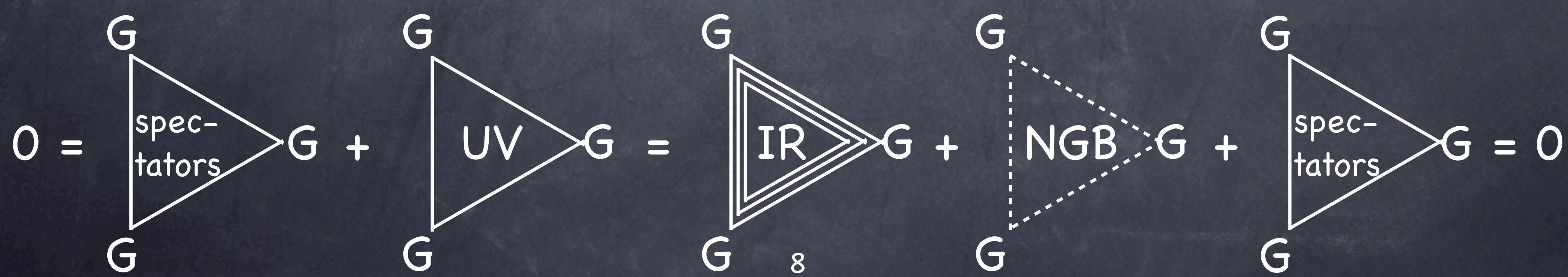

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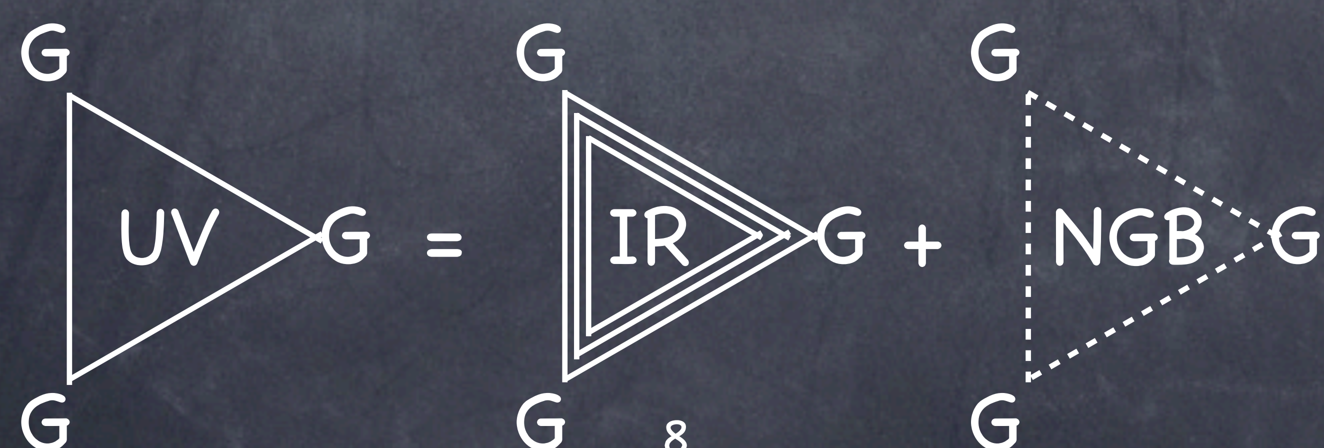

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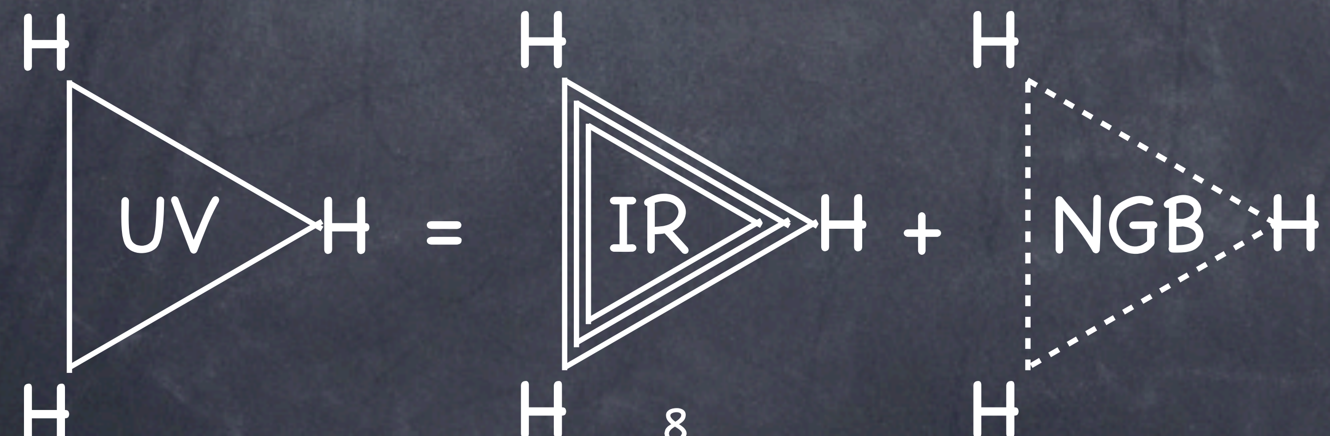

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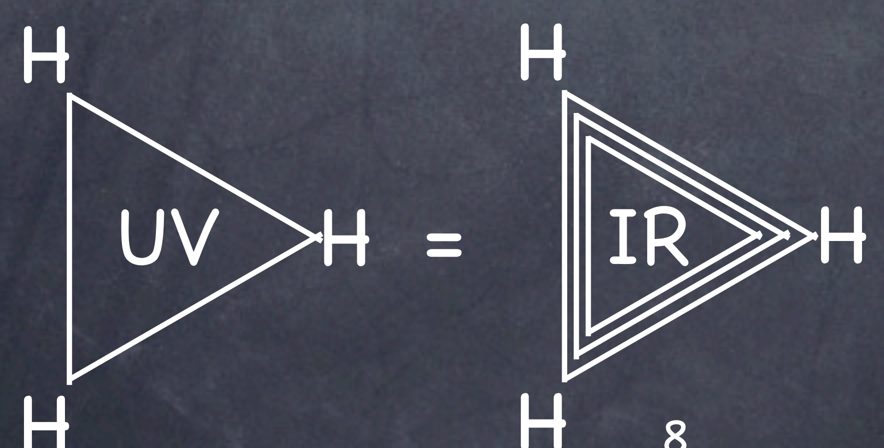

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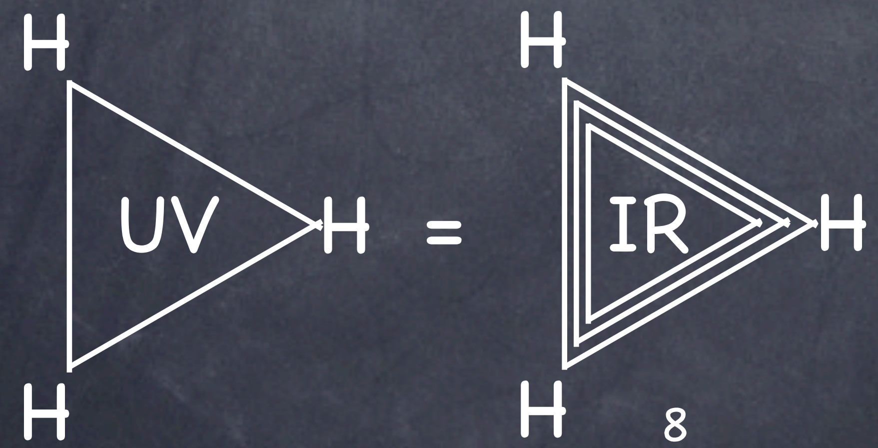
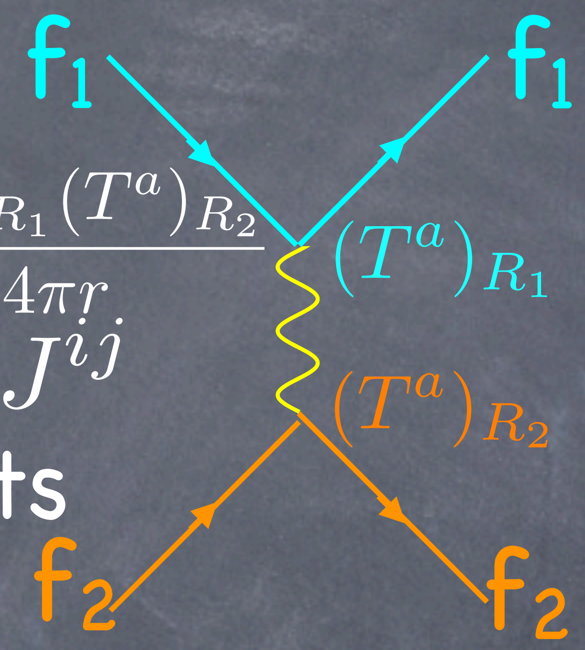
- no changes to spectator fermions
- anomalies in the UV = anomalies in the IR
- also discrete anomalies (Csáki, HM 1997)



Chiral theories



$$V = g^2 \frac{(T^a)_{R_1}(T^a)_{R_2}}{4\pi r}$$

$$\langle Q_R^i \bar{Q}_L^j \rangle \propto \delta^{ij} \quad \langle Q^i Q^j \rangle \propto \delta^{ij} \quad \langle Q^i Q^j \rangle \propto J^{ij}$$

- by definition, no bilinear gauge invariants
  - assume $\langle f_1 f_2 \rangle_R$ do form nonetheless
  - Pick MAC (Most Attractive Channel) $R \subset R_1 \otimes R_2$

$$(T^a)_{R_1}(T^a)_{R_2} = \frac{1}{2}\left[((T^a)_{R_1} + (T^a)_{R_2})^2 - (T^a)^2_{R_1} - (T^a)^2_{R_2}\right] = \frac{1}{2}\left[C_2(R) - C_2(R_1) - C_2(R_2)\right]$$

- breaks its own gauge invariance
  - gauge group Higgsed to smaller ones
- let it repeat
  - until the theory becomes vector-like so that the last step is QCD-like
- Georgi (1979), Dimopoulos-Raby-Susskind ('80)

SU(N) A_{N-4} + (N-4)F^*_{-N+2}

- SU(N) with A + (N-4) F*, global SU(N−4)xU(1)
- MAC $\langle A^{\alpha\beta} F^*_{i\beta} \rangle = \Lambda^3 \delta^\alpha_i$: SU(N)→SU(4)xSU(N−4)xU(1)
- unbroken global SU(N-4)xU(1), gauged SU(4)

A

F*

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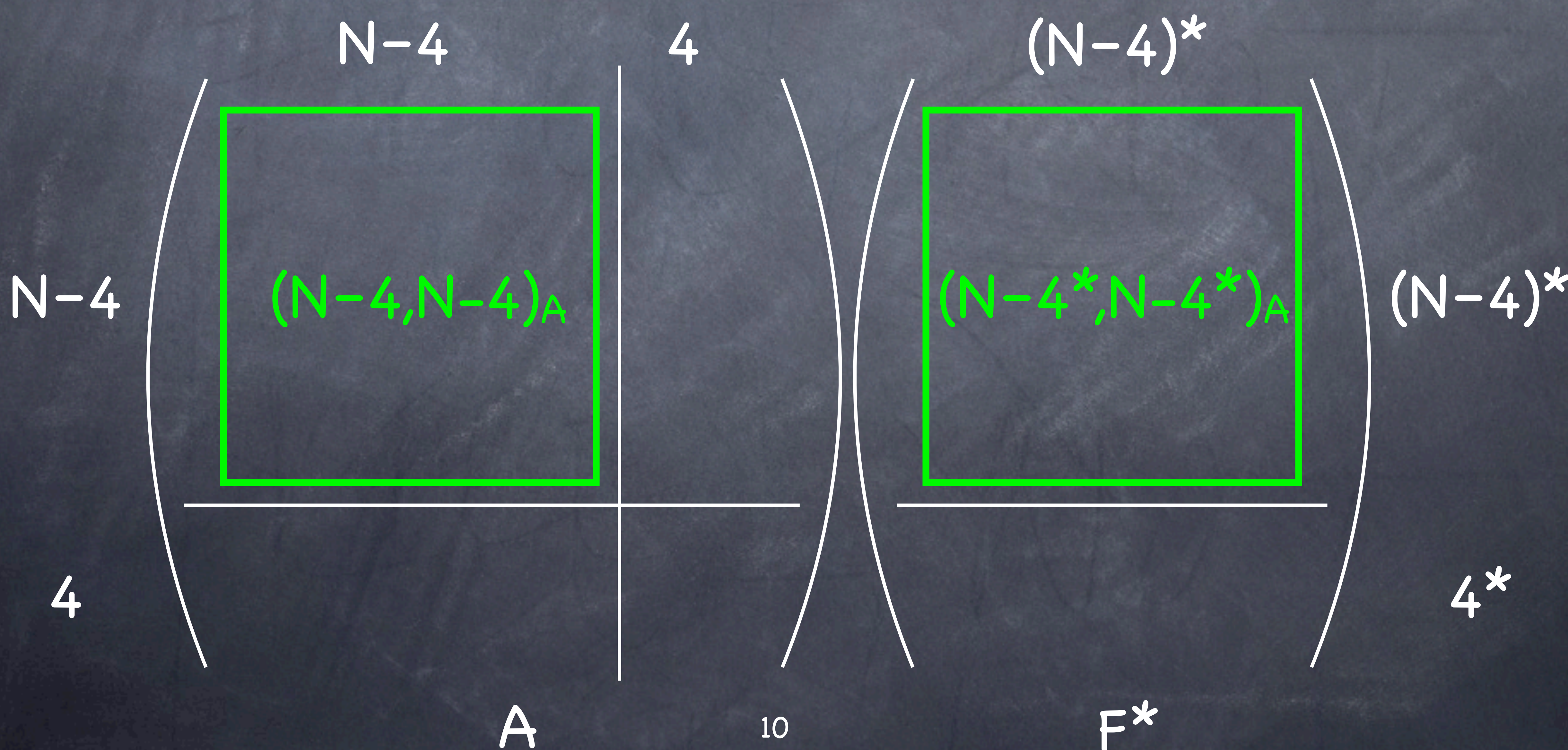

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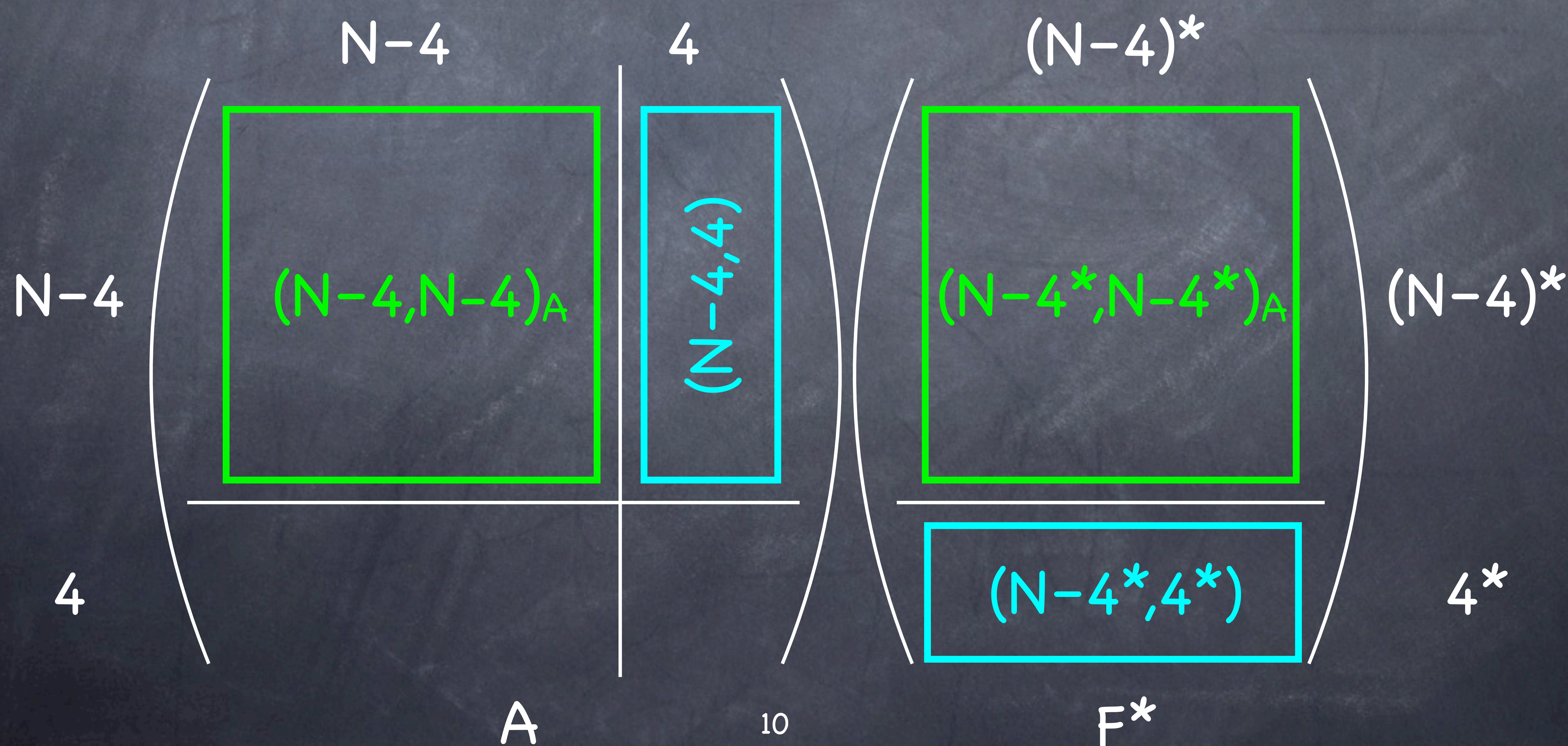

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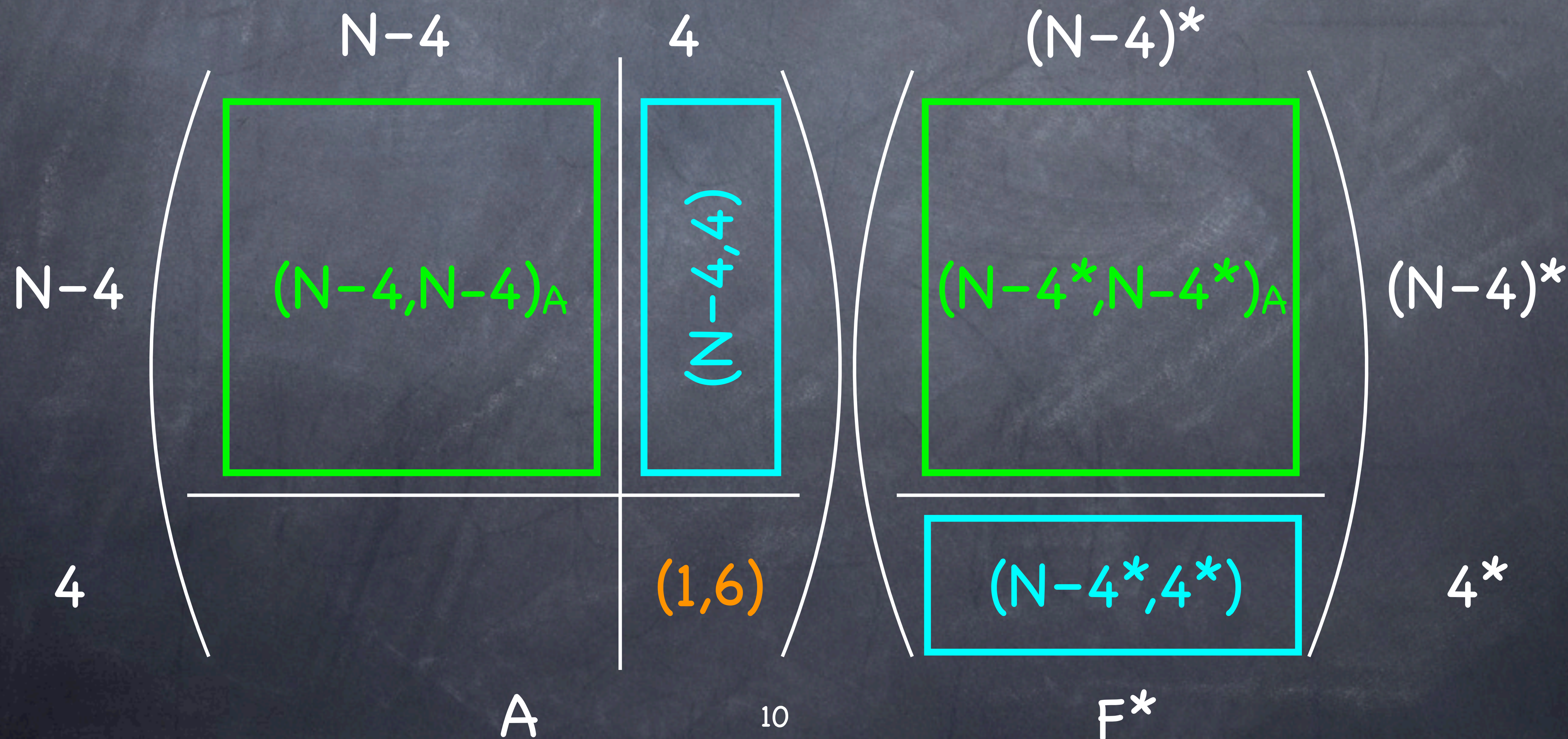
- unbroken global SU(N-4)xU(1), gauged SU(4)
- SU(4)+6 = SO(6)+V



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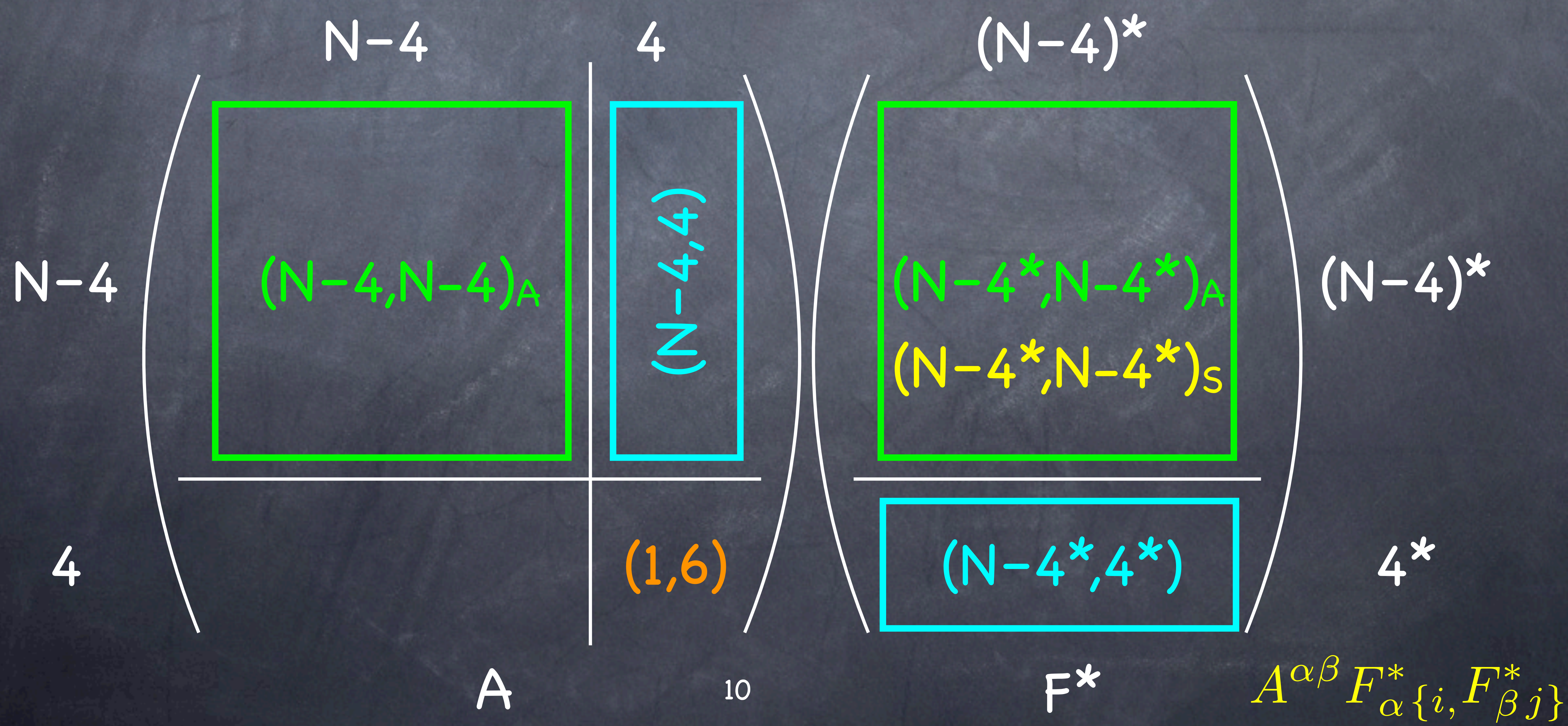

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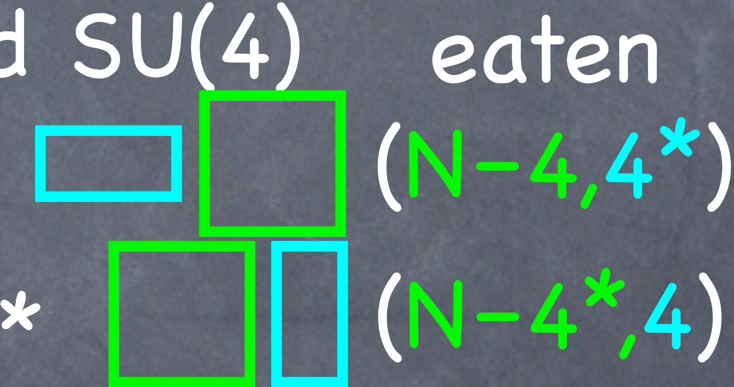
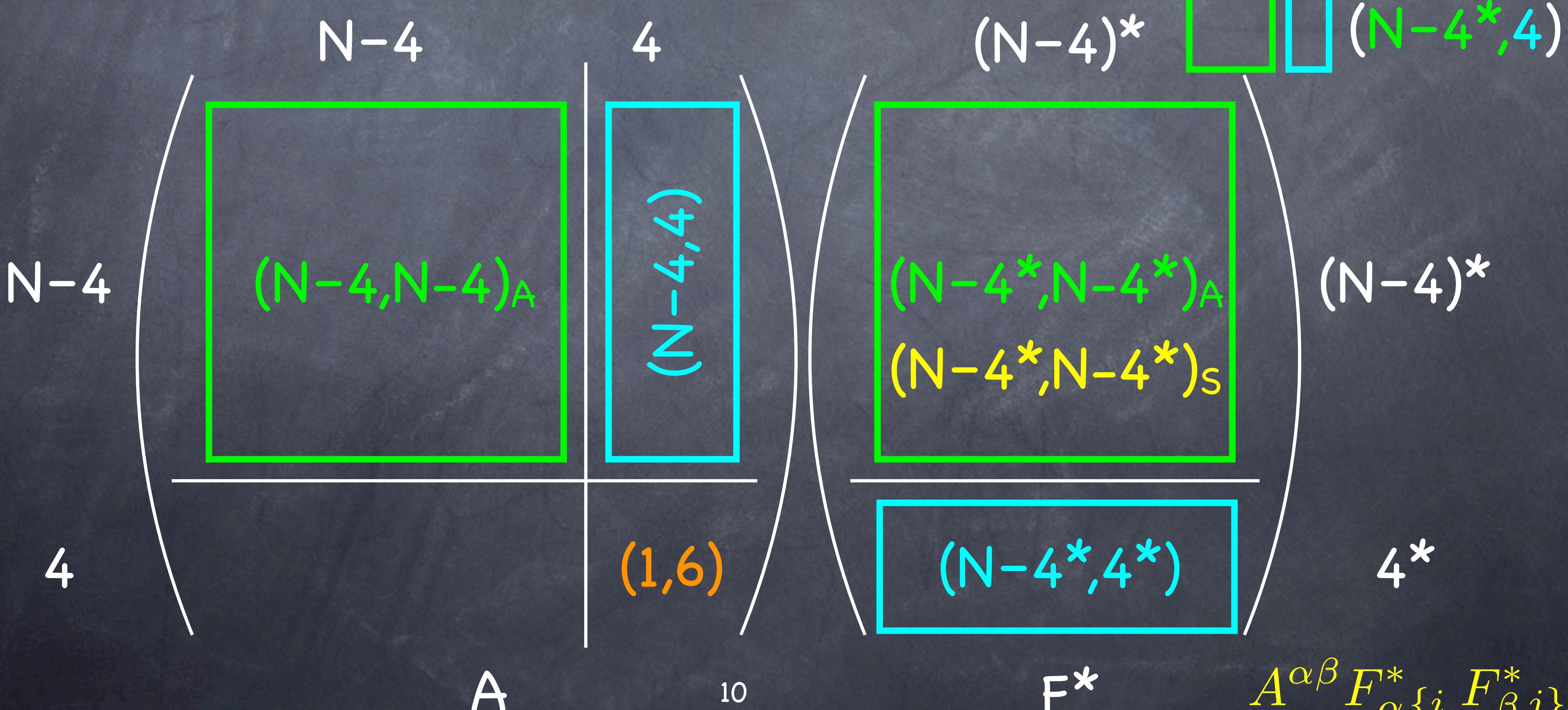




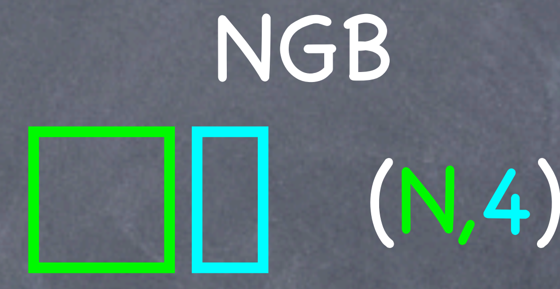
$A^{\alpha\beta} F^*_{\alpha\{i,} F^*_{\beta j\}}$

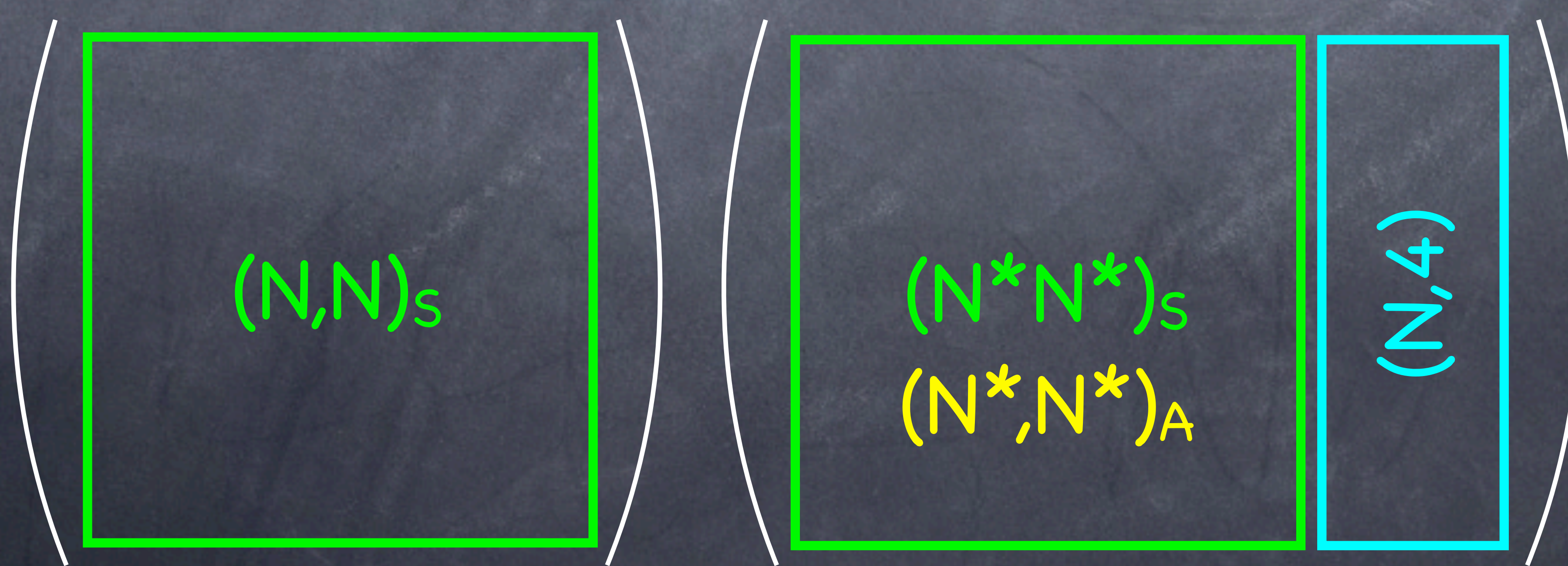
SU(N) A_{N-4} + (N-4)F^*_{-N+2}

- Unbroken SU(N-4)xU(1) global symmetry
- massless composite fermion $A^{\alpha\beta}F^*_{\alpha\{i,}F^*_{\beta j\}}$
  - SU(N-4) symmetric, charge -N
- Non-trivial anomaly matching
- $SU(N-4)^3$: -N = -N
- $SU(N-4)^2U(1)$: (1/2)xNx(-N+2) = (-N+2)/2xN
- U(1): N(N-4)x(-N+2)+N(N-1)/2x(N-4) = (N-4)(N-3)/2x(-N)
- $U(1)^3$: $N(N-4)x(-N+2)^3+N(N-1)/2x(N-4)^3 = (N-4)(N-3)/2x(-N)^3$

SU(N) S_{N+4} + (N+4)F^*_{-N-2}

- SU(N) with S + (N+4) F*, global G=SU(N+4)xU(1)
- MAC $\langle S^{\alpha\beta} F^*_{i\beta} \rangle = \Lambda^3 \delta^\alpha_i$: SU(N)→0
- unbroken global H=SU(N)xSU(4)xU(1)
- no WZW term π_5(G/H)=0
- anomalies of H need to be reproduced
- massless composite fermions $S^{\alpha\beta} F^*_{\alpha[i,} F^*_{\beta j]}$
- but there is an alternative possibility





S F*

Goity, Peccei, Zeppenfeld (1985)

SU(N) S_{N+4} + (N+4)F^*_{-N-2}

- SU(N) with S + (N+4) F*, global G=SU(N+4)xU(1)
- anomalies of G can to be reproduced by massless composite fermions $S^{\alpha\beta} F^*_{\alpha[i,} F^*_{\beta j]}$
- $SU(N+4)^3$: -N = -N
- $SU(N+4)^2U(1)$: (1/2)xNx(-N-2) = (-N-2)/2xN
- U(1): N(N+4)x(-N-2)+N(N+1)/2x(N+4) = (N+4)(N+3)/2x(-N)
- $U(1)^3$: $N(N+4)x(-N-2)^3+N(N+1)/2x(N+4)^3 = (N+4)(N+3)/2x(-N)^3$

S

F*

Eichten, Peccei, Preskill, Zeppenfeld (1985)

near-SUSY QCD-like theories

with Csaba Csáki, Andrew Gomes, Riku Ishikawa, Dan Kondo, Zhiyao Lu, Bea Noether, Shota Saito, Ofri Telem, Digvijay Roy Varier, Ruoyao Zhang, Yijun Lin

Main point

- very few methods to study strongly coupled systems, e.g., QCD
- non-perturbative effects are essential $e^{-8\pi^2/g^2\hbar}$
- supersymmetry makes exact analytic studies of non-perturbative effects possible
- small anomaly-mediated SUSY breaking still allows for exact solutions

HM, 2104.01179, *PRL* 126 (2021) 251601

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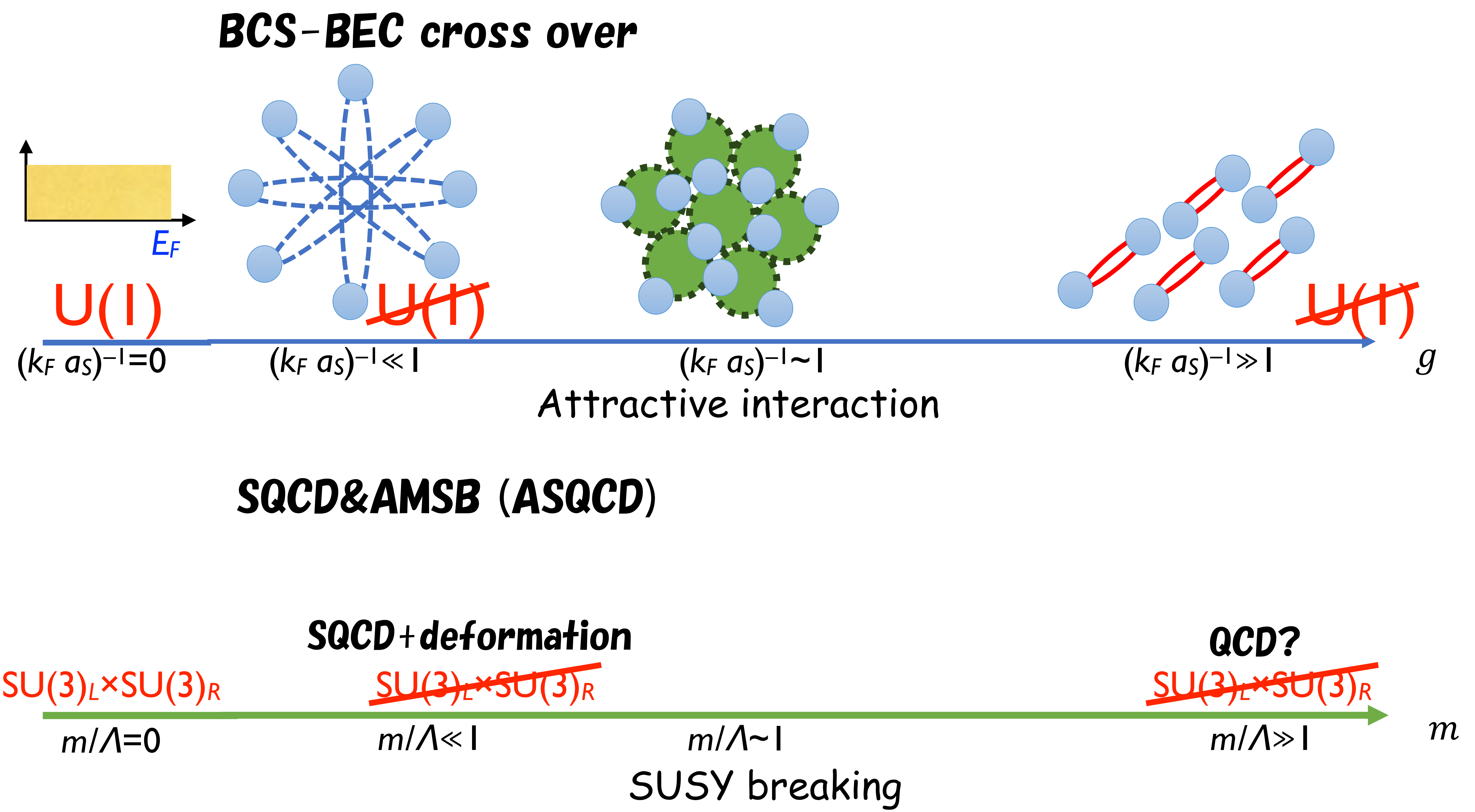


BCS-BEC cross over
E_F
U(1)
U(1)
U(1)
$(k_F a_S)^{-1}=0$
$(k_F a_S)^{-1}\ll 1$
$(k_F a_S)^{-1}\sim 1$
$(k_F a_S)^{-1}\gg 1$
g
Attractive interaction
SQCD&AMSB (ASQCD)
SQCD+deformation
QCD?
$SU(3)_L\times SU(3)_R$
$SU(3)_L\times SU(3)_R$
$SU(3)_L\times SU(3)_R$
$m/\Lambda=0$
$m/\Lambda\ll 1$
$m/\Lambda\sim 1$
$m/\Lambda\gg 1$
m
SUSY breaking

Concrete results

- Chiral Symmetry Breaking
- Confinement (when appropriate)
- U(1) problem $m_{\eta'}^2 > 0$
- Chiral Lagrangian (Hidden Local Symmetry) with Wess-Zumino-Witten term
- Large N_c Scaling $f_\pi^2 \propto O(N_c)$ etc
- quark condensates $\langle \bar{q} q \rangle$, gluon condensates $\langle G_{\mu\nu} G^{\mu\nu} \rangle$
- Witten's formula on topological susceptibility $\chi = \frac{\partial^2 E}{\partial \theta^2}$
- 0^+ hadron mass spectrum
- finite electromagnetic corrections
- symmetry breaking in chiral gauge theories

colored: challenging on lattice

Anomaly Mediation of SUSY Breaking (AMSB)

- Tree-level piece on dimensionful parameters

$$V_{\mathrm{AMSB}} = -m\left(\phi\frac{\partial W}{\partial\phi} - 3W\right)$$

- loop-level piece from running

$$M_i = -\frac{\beta_i(g^2)}{2g_i^2}m_{3/2}, \quad m_i^2 = -\frac{\dot{\gamma}_i}{4}m_{3/2}^2, \quad A_{ijk} = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k)m_{3/2}$$

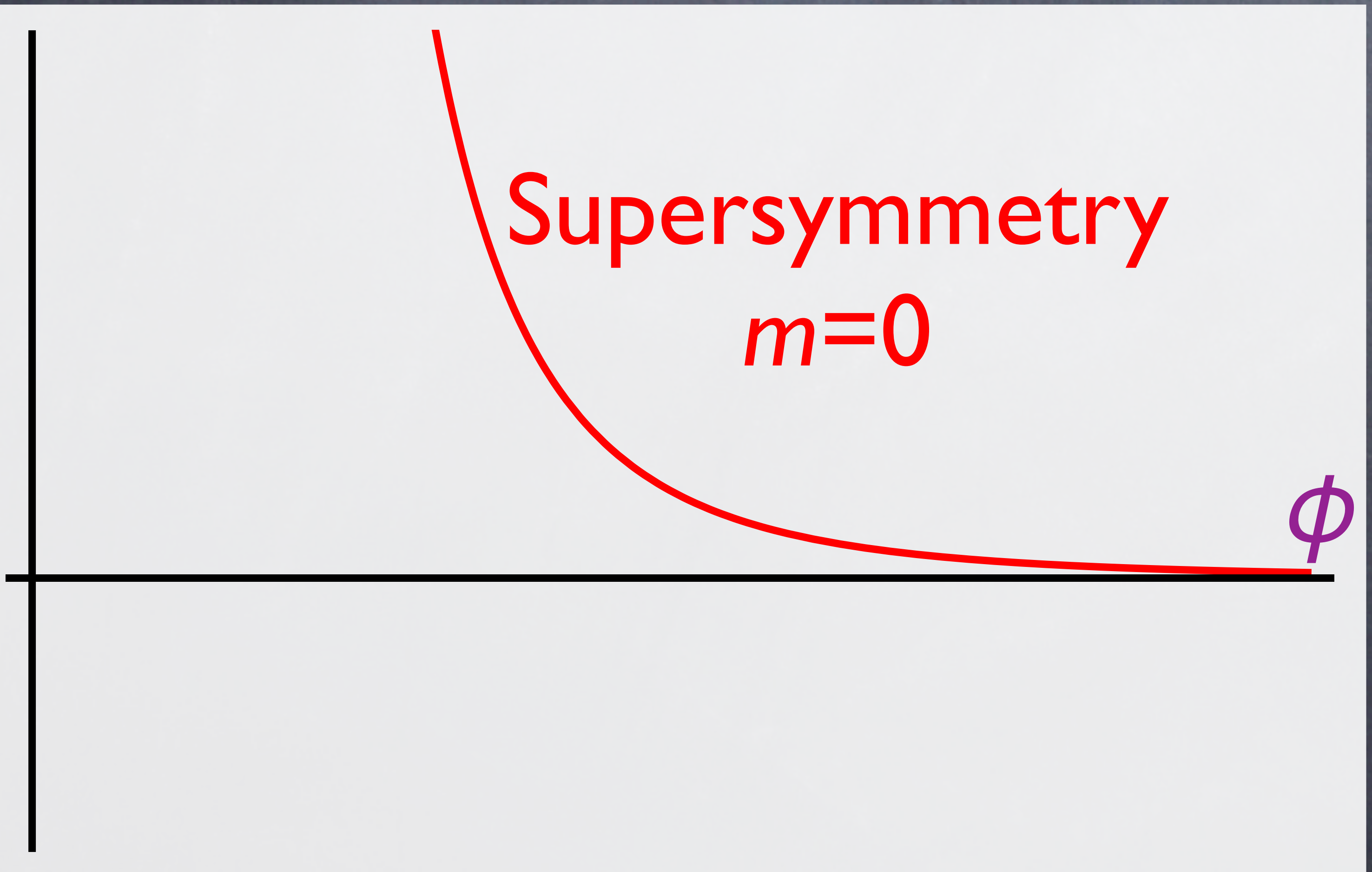
- determined only by physics at the energy scale of interest
- UV insensitivity!

Randall, Sundrum (1998)
Giudice, Luty, HM, Rattazzi (1998)

$N_f < N_c$

- run-away superpotential for $M^{ij} = \tilde{Q}^i Q^j$

$$W = (N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{1/(N_c - N_f)} \qquad M^{ij} = \delta^{ij}\phi^2$$



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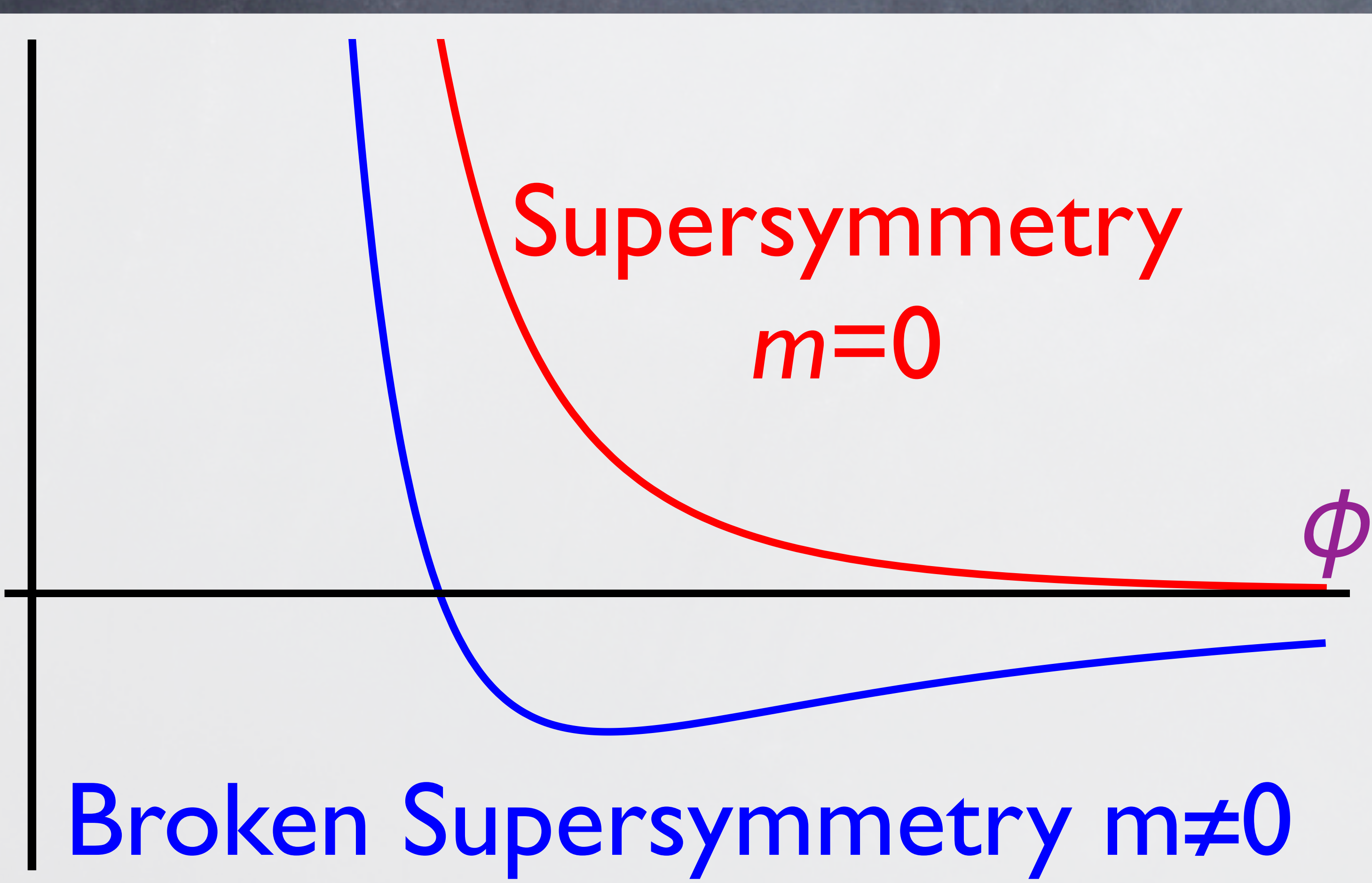
$$V = 2N_f\left|\frac{\Lambda^{3N_c-N_f}}{\phi^{N_c+N_f}}\right|^{\frac{2}{N_c-N_f}} - m(3N_c - N_f)\left(\frac{\Lambda^{3N_c-N_f}}{\phi^{2N_f}}\right)^{\frac{1}{N_c-N_f}} + c.c.$$



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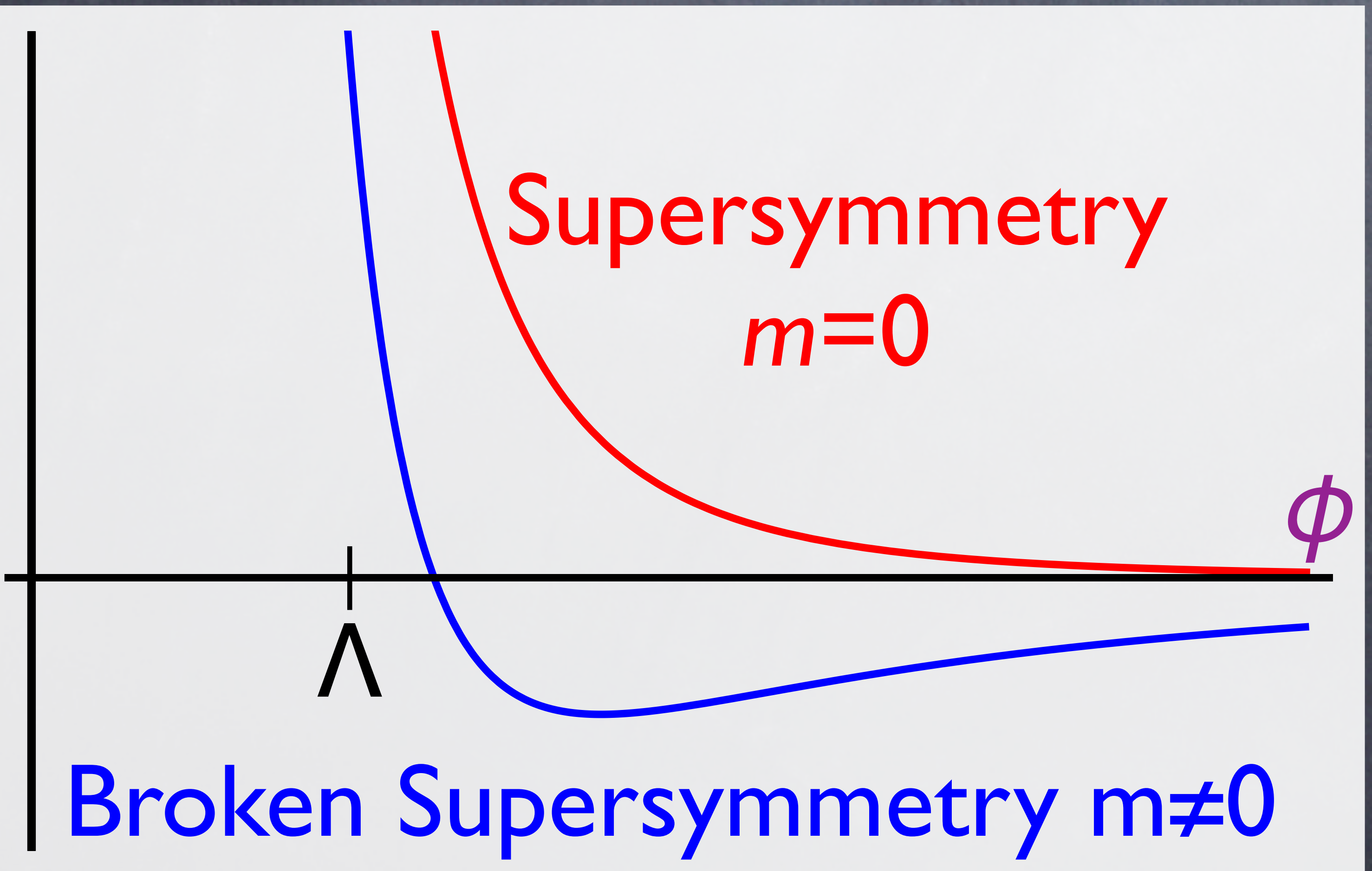
$$V = 2N_f \left| \frac{\Lambda^{3N_c - N_f}}{\phi^{N_c + N_f}} \right|^{\frac{2}{N_c - N_f}} - m(3N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\phi^{2N_f}} \right)^{\frac{1}{N_c - N_f}} + c.c.$$



$$M_{ij} = \delta_{ij} \left(\left(\frac{N_c + N_f}{3N_c - N_f} \right)^{N_c - N_f} \frac{\Lambda^{3N_c - N_f}}{m^{N_c - N_f}} \right)^{1/N_c}$$

$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$

Proving χSB!

mesino loop→WZW term

$N_f = 1$ special

no NGB, gapped

Deriving Chiral Lagrangian

$$Q = \begin{pmatrix} v\xi^T \\ 0 \end{pmatrix}, \quad \tilde{Q} = \begin{pmatrix} v\xi \\ 0 \end{pmatrix} \quad \xi^T \to h\xi^T g_L^T, \quad \xi \to h^* \xi g_R^T$$

$$U = \xi\xi \to g_L U g_R^T$$

$$|D_\mu Q|^2 + \left|D_\mu \tilde{Q}\right|^2 = \frac{v^2}{2} \mathrm{Tr} \partial_\mu U^\dagger \partial^\mu U$$

$$f_\pi^2 = 2v^2 = 2\left(\left(\frac{N_c + N_f}{3N_c - N_f} \right)^{N_c - N_f} \frac{\Lambda^{3N_c - N_f}}{m^{N_c - N_f}} \right)^{1/N_c}$$

$$\Lambda^{3N_c - N_f} = M^{3N_c - N_f} e^{-8\pi^2/g_h^2(M)}$$

$$= M^{3N_c - N_f} e^{-8\pi^2/g_c^2(M)} Z^{-N_f} (g_c^2)^{-N_c}$$

$$= M^{3N_c - N_f} e^{-8\pi^2 N_c/g_{\mathrm{tH}}^2(M)} Z^{-N_f} N_c^{N_c} (g_{\mathrm{tH}}^2)^{-N_c}$$

$$f_\pi^2 \propto N_c$$

Nima Arkani-Hamed and HM, arXiv:hep-th/9707133

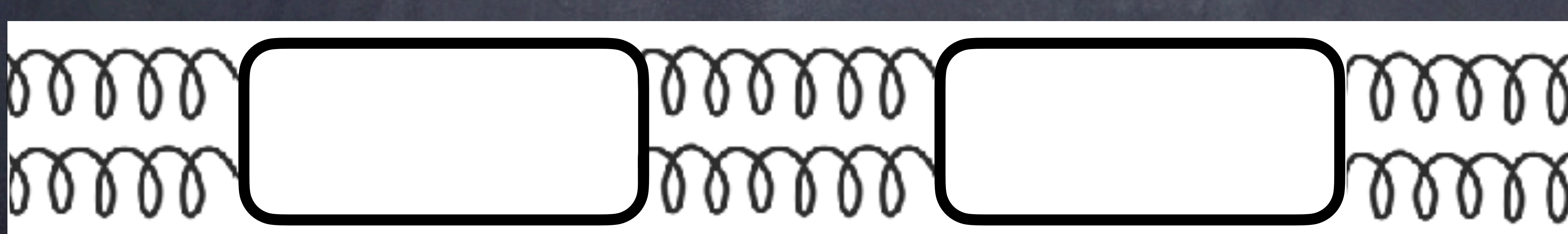
U(1) Problem

- Classically $U(N_f) \times U(N_f)$ global symmetry
- We don't see U(1) NGB in data
- U(1) broken by anomaly $\partial_\mu j_1^\mu = \frac{N_f}{8\pi^2} \mathrm{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu} \neq 0$
- Does it lead to a finite mass?

$$m_{\eta'}^2 = \frac{2(3-x)^2 x}{(1+x)(1-x)^2} m^2 = 18 m^2 x + \mathcal{O}(x^2) \qquad x = \frac{N_f}{N_c}$$

- Witten's formula $f_\pi^2 m_{\eta'}^2 = 2N_f \left(\frac{\partial^2 \mathcal{E}}{\partial \theta^2}\right)_{\theta=0}^{\text{no quarks}}$

$$f_\pi^2 m_{\eta'}^2 = 4 \frac{x(3-x)}{(1-x)^2} \left(\frac{3-x}{1+x} \frac{m}{\Lambda}\right)^x m\Lambda^3 \xrightarrow{x \to 0} 12 x m \Lambda^3 \qquad \left(\frac{\partial^2 \mathcal{E}}{\partial \theta^2}\right)_{\theta=0}^{\text{no quarks}} = \frac{6m\Lambda^3}{N_c}$$



Non-perturbative Condensates

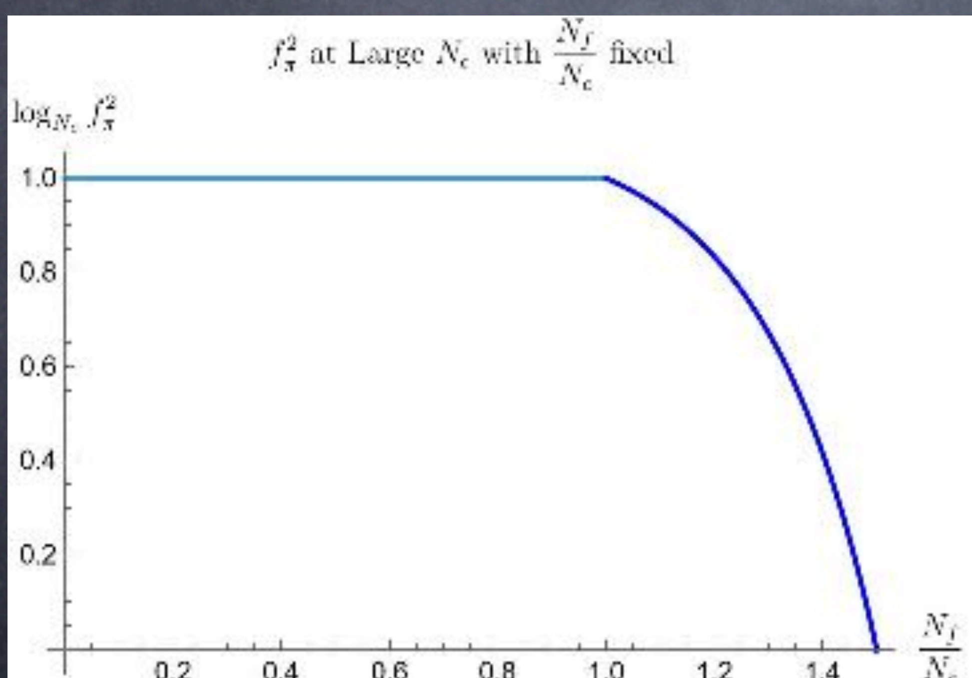
$$M_{ij} = \delta_{ij}\left(\left(\frac{N_c+N_f}{3N_c-N_f}\right)^{N_c-N_f}\frac{\Lambda^{3N_c-N_f}}{m^{N_c-N_f}}\right)^{1/N_c}$$

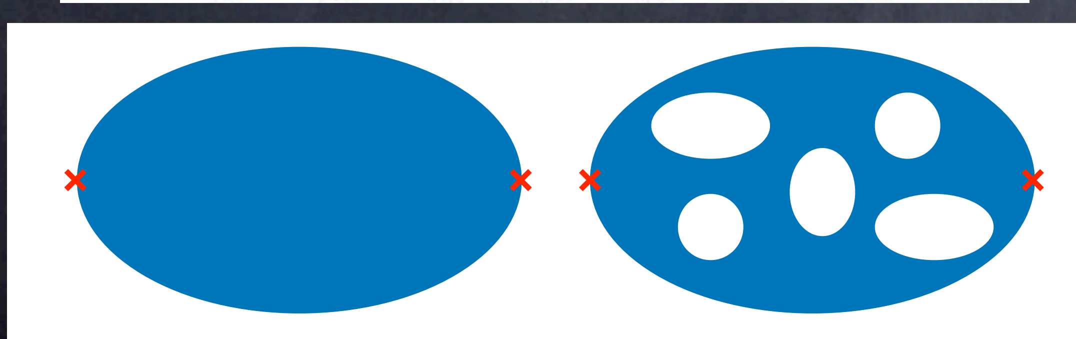
$$\langle \bar{q}_i q_j\rangle = 4m\delta_{ij}\left(\left(\frac{N_c+N_f}{3N_c-N_f}\right)^{-N_f}\frac{\Lambda^{3N_c-N_f}}{m^{N_c-N_f}}\right)^{1/N_c}$$

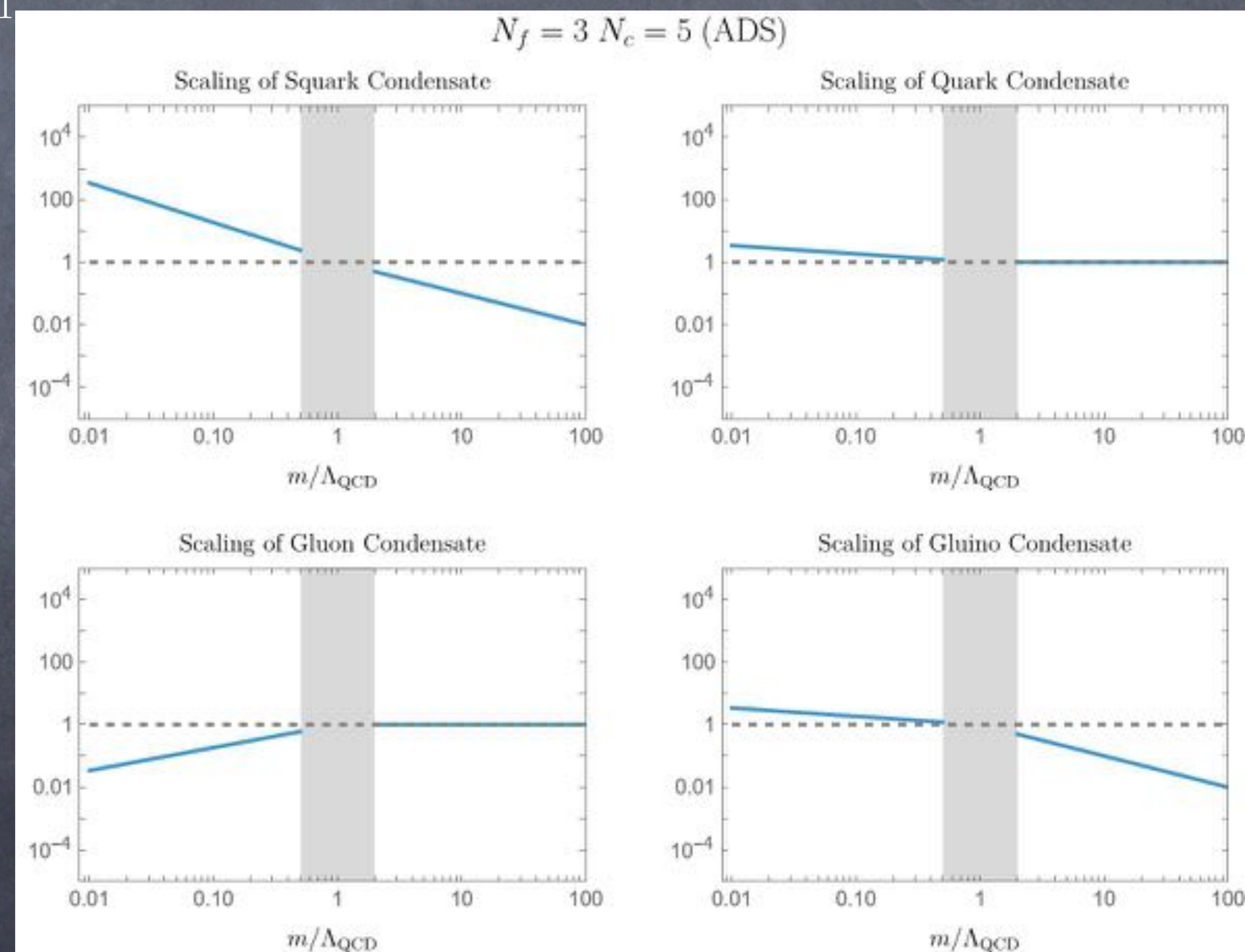
$$\langle \mathrm{Tr}\lambda\lambda\rangle = 32\pi^2\left(\left(\frac{N_c+N_f}{3N_c-N_f}\right)^{-N_f}\Lambda^{3N_c-N_f}m^{N_f}\right)^{1/N_c}$$

$$\langle \mathrm{Tr}G_{\mu\nu}G^{\mu\nu}\rangle = 32\pi^2 m\left(\left(\frac{N_c+N_f}{3N_c-N_f}\right)^{-N_c-N_f}\Lambda^{3N_c-N_f}m^{N_f}\right)^{1/N_c}$$

$$m^2_{\eta'} = 2m^2\frac{N_f(3N_c-N_f)^2}{(N_c-N_f)^2(N_c+N_f)} \propto N_c^{-1}$$

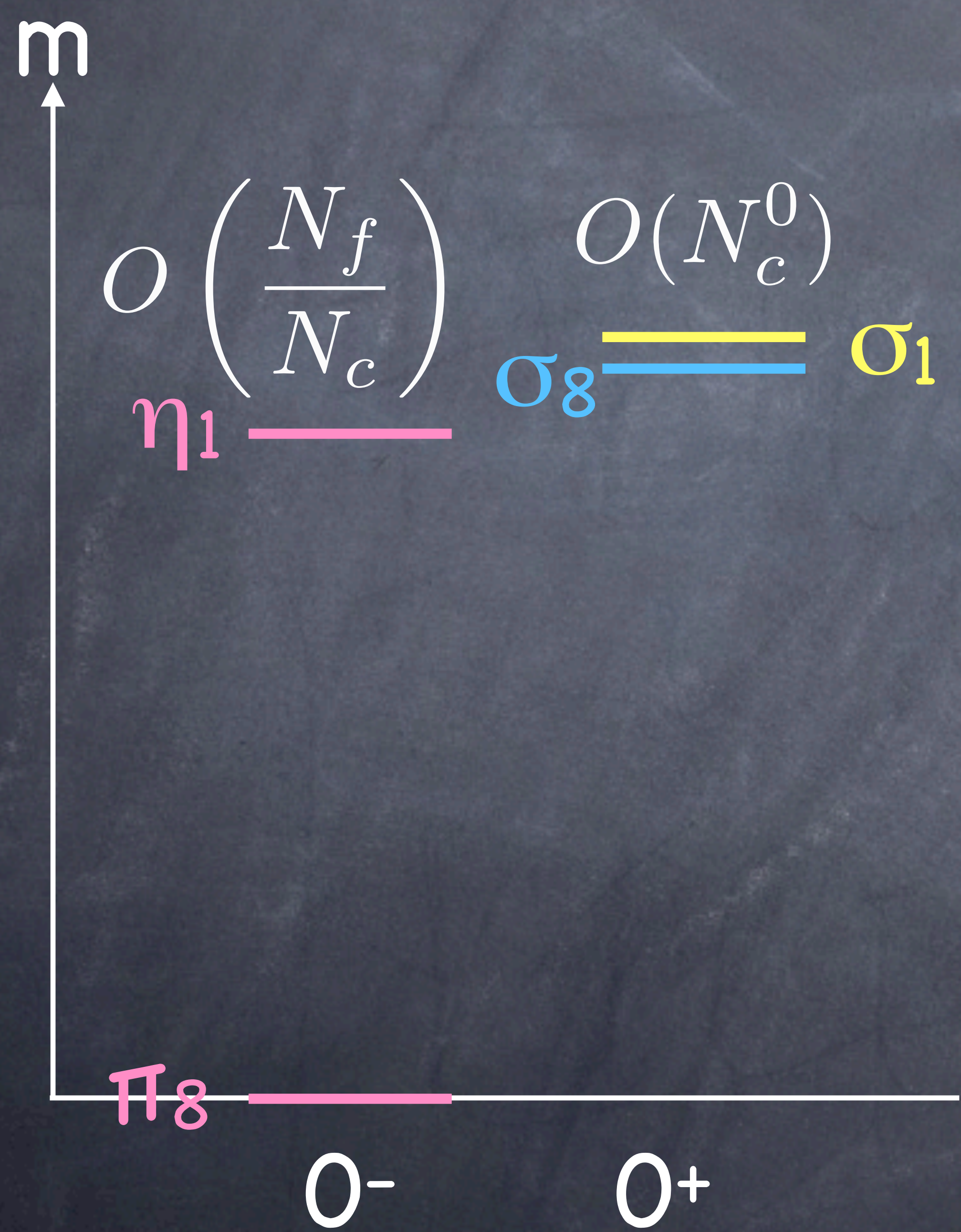




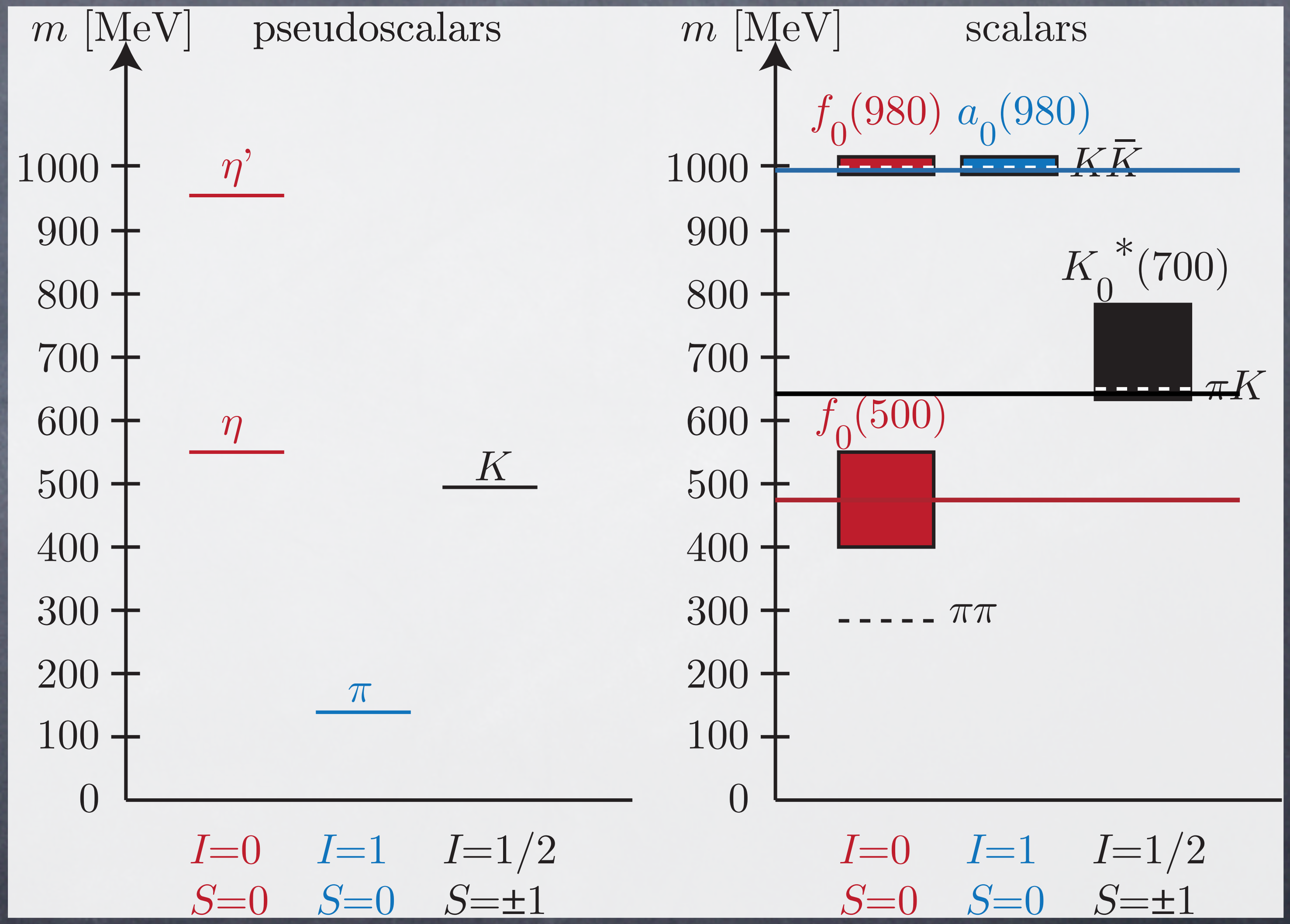


Scalar spectrum

Found c'<0



$$m_\pi^2 = c(m_{q_i} + m_{q_j}) \qquad m_\sigma^2 = m_8^2 + c'(m_{q_i} + m_{q_j})$$

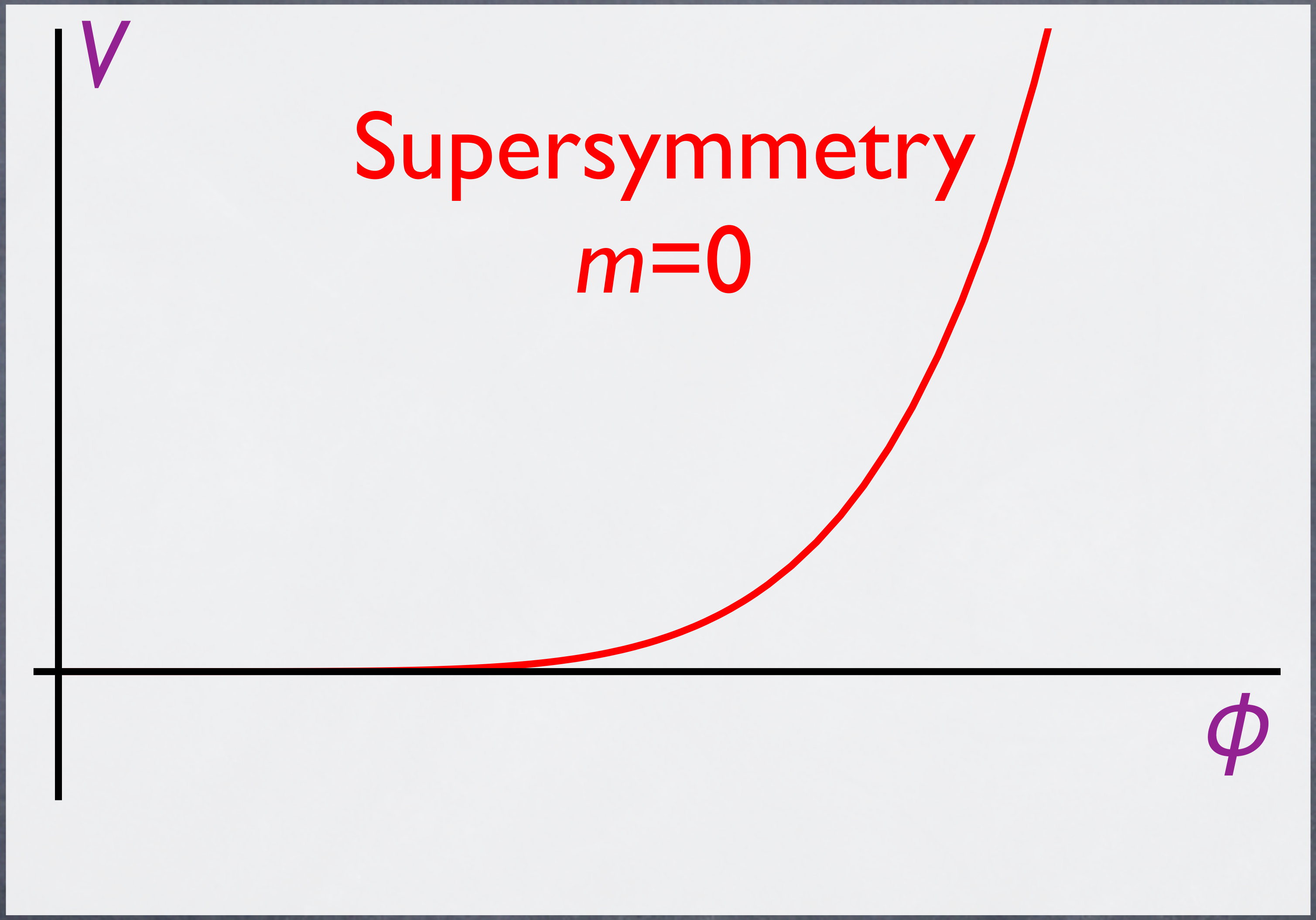


c'=−2.2c

Dan Kondo, HM, Bea Noether

SU(N_c) w/ $N_c<N_f<3N_c/2$

- "Confinement without χSB" (Seiberg)



$SU(N_c)$ w/ $N_c<N_f<3N_c/2$

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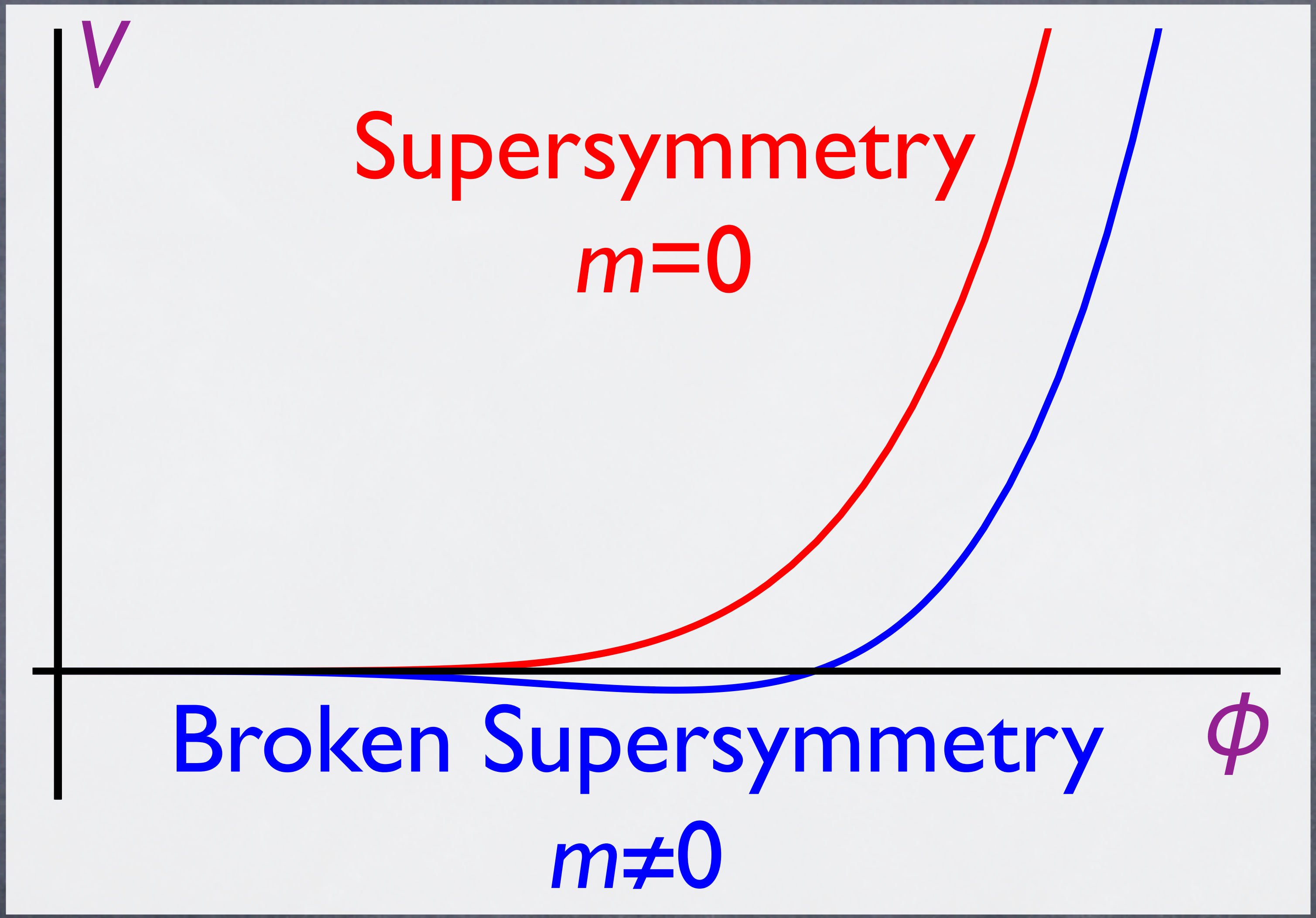
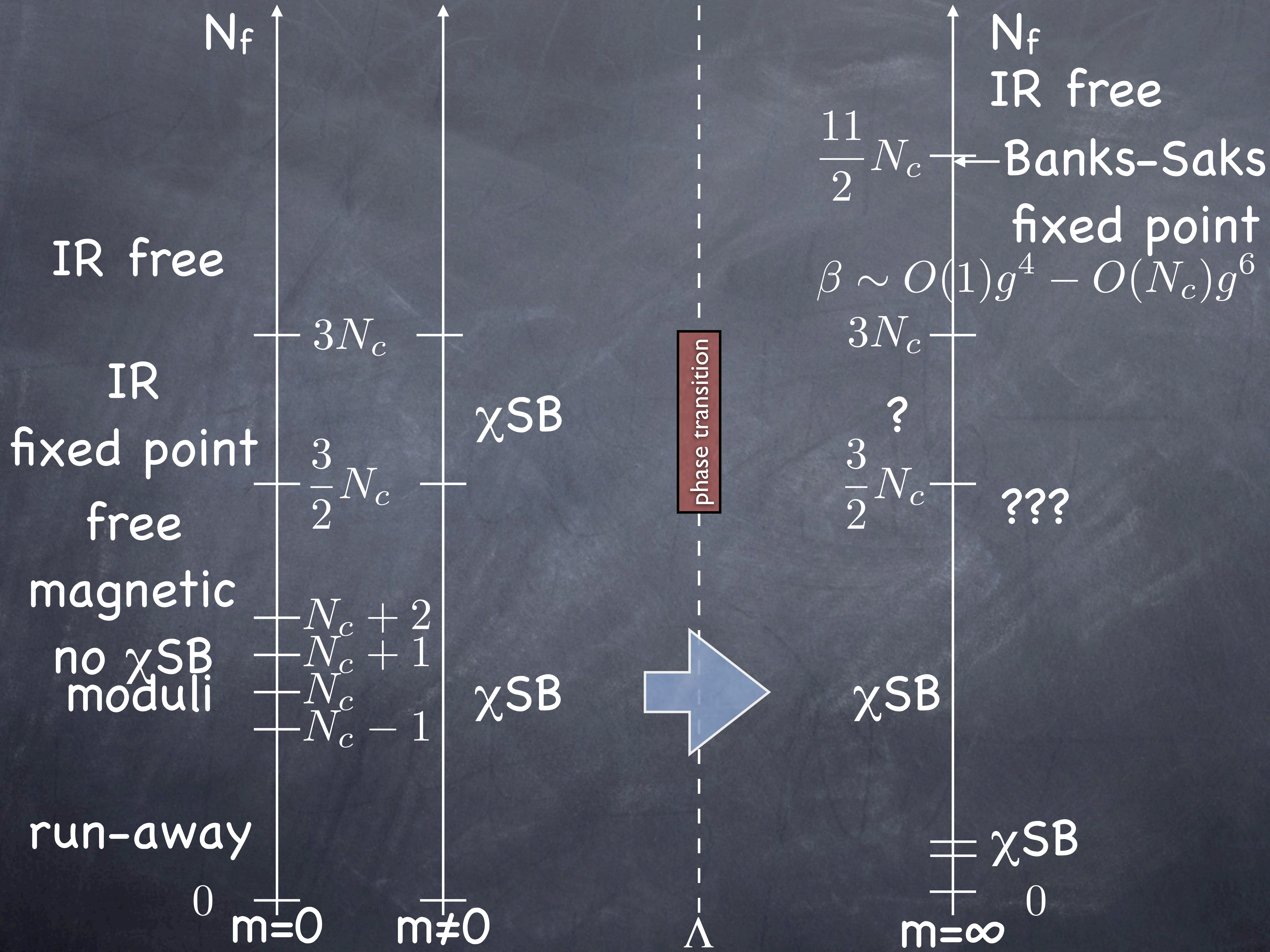


$$\bar{Q}_i Q_j = \phi \delta_{ij} \neq 0$$

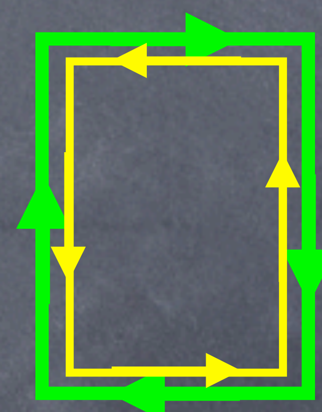
$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$

- χSB!
- massless pions


N_f
IR free
$3N_c$
IR
fixed point
$\frac{3}{2}N_c$
free
magnetic
N_c+2
no χSB
N_c+1
moduli
N_c
N_c-1
run-away
0
m=0
χSB
χSB
m≠0
phase transition
Λ
N_f
IR free
$\frac{11}{2}N_c$
Banks-Saks
fixed point
$\beta \sim O(1)g^4 - O(N_c)g^6$
$3N_c$
?
$\frac{3}{2}N_c$
???
χSB
χSB
0
m=∞

confinement vs screening

- We've derived χSB in SU(N_c) QCD
  - strictly speaking, it has no confinement
  - massless quarks in the fundamental rep can screen any color charges
  - Wilson loop is perimeter law $\langle \mathrm{Tr} e^{i \oint A dx} \rangle \propto e^{-L}$



- SO(N_c) QCD with quarks in vector rep
  - cannot screen Z_2 center (e.g. spinor rep)
  - rigorous definition of confinement $\langle \mathrm{Tr} e^{i \oint A dx} \rangle \propto e^{-A}$
  - can we see an interplay with χSB?
- Complete classification of t-confining theories

Csaba Csáki, HM, Ofri Telem
Riku Ishikawa, HM, Shota Saito

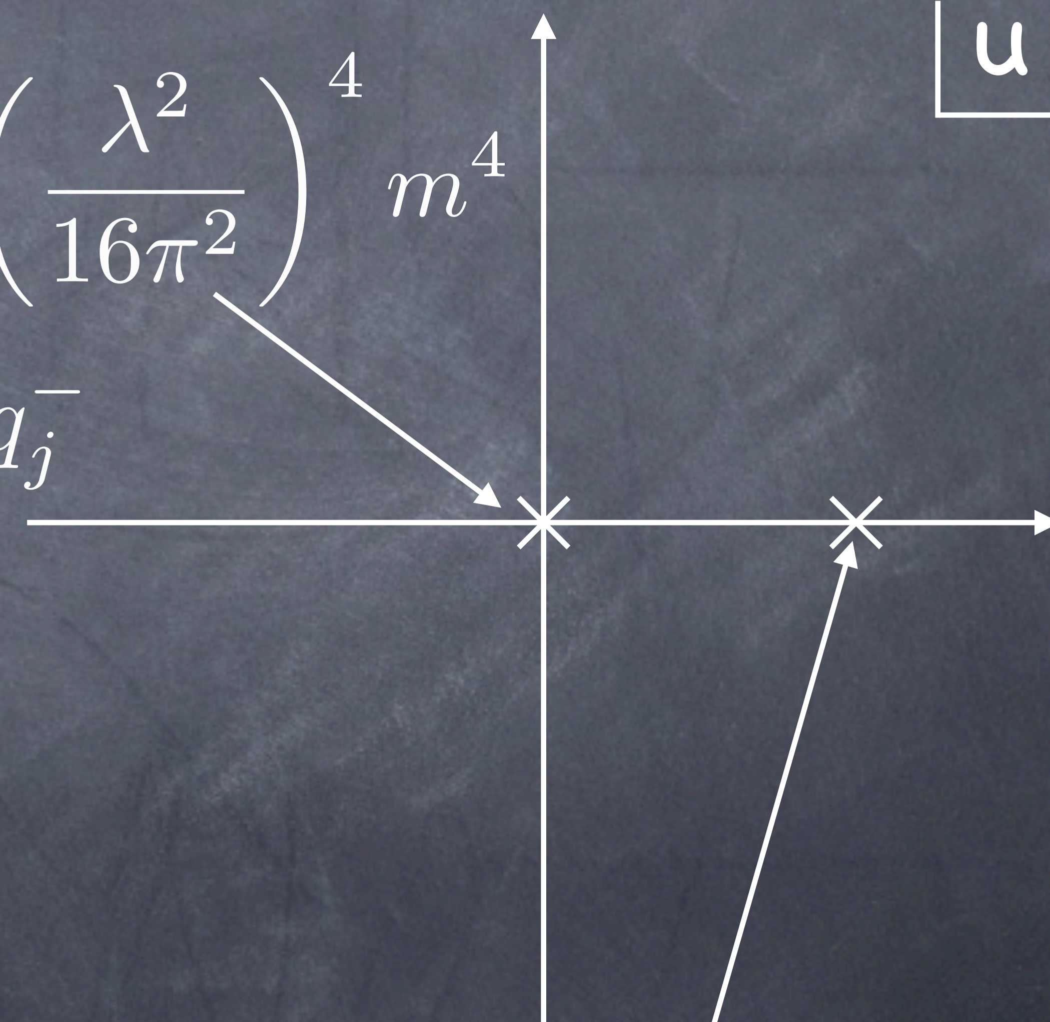
$N_f = N_c - 2$

- for $M^{ij} = Q^i Q^j \neq 0$ with rank $M = N_f$, $SO(N_c)$ is broken to $SO(2)$
- Coulomb branch $u = \det M$
- two singularities
- $u = \det M = 0$
  - dyons: $q_i^{\pm}$ $W = \frac{1}{\mu} M^{ij} q_i^+ q_j^-$
- $u = \det M = \Lambda^{2N_f}$
  - monopoles:

    $W = (u - \Lambda^{2N_f}) E^+ E^-$

    $|E^{\pm}| = (m\Lambda)^{1/2}$
  - both monopoles and meson condense!

$$V \approx -\left(\frac{\lambda^2}{16\pi^2}\right)^4 m^4$$



$$V = -N_f m^2 \Lambda^2$$

Near-SUSY Chiral gauge theories

with Csaba Csáki, Dan Kondo, Andrew Goh, Jacob Leedom, Zhiyao Lu, Pablo Quilez, Shota Saito, Gurpreet Singh, Bethany Suter, Cameron Sylber, Ofri Telem, Jason Wong

SUSY + AMSB, N_c=2k

- non-perturbative run-away superpotential

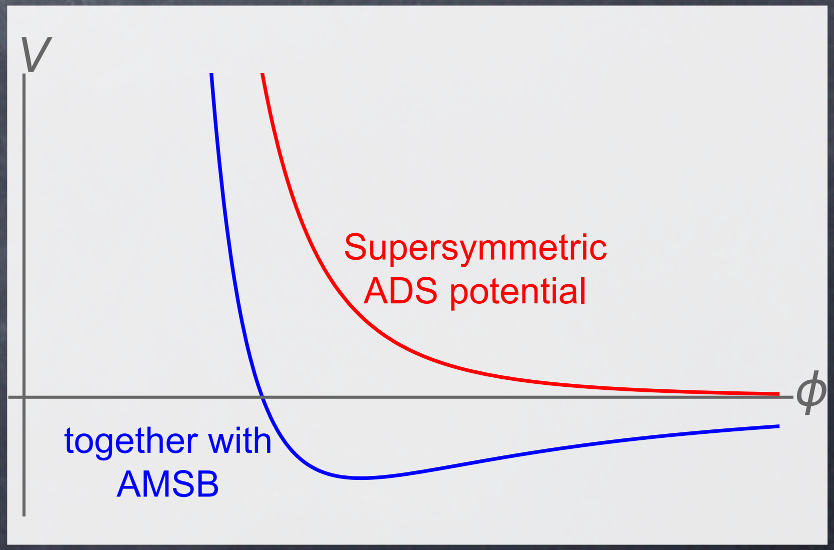
$$A = \left(\begin{array}{c|c} aJ_{(N_c-4)} & 0 \\ \hline 0 & bJ_4 \end{array}\right) \qquad \bar{F} = \left(\begin{array}{c} cJ_{(N_c-4)} \\ \hline 0 \end{array}\right)$$

$$|a|^2 - |c|^2 = |b|^2$$

$$W = \left(\frac{\Lambda^{2N+3}}{(\mathrm{Pf}A\bar{F}\bar{F})(\mathrm{Pf}A)}\right)^{1/3}$$

Pouliot (1995)

- SU(N_c-4)xU(1) broken to Sp(k-2)
- no massless fermions



SUSY + AMSB, N_c=2k+1

- non-perturbative run-away superpotential

$$A = \frac{\varphi}{\sqrt{2}} \left(\begin{array}{c|c} J_{(N-5)} & 0 \\ \hline 0 & 0_{5\times 5} \end{array} \right), \qquad \bar{F} = \varphi \left(\begin{array}{c|c} I_{(N-5)} & 0 \\ \hline 0 & 0_{5\times 1} \end{array} \right)$$

$$W = \left(\frac{\Lambda_N^{2N+3}}{(\mathrm{Pf}' A \bar{F} \bar{F})(\mathrm{Pf}' A)} \right)^{3/13}$$

Pouliot (1995)

$$\varphi \approx \Lambda \left(\frac{\Lambda}{m} \right)^{13/(4N-7)} \gg \Lambda$$

- SU(2k-3)xU(1) broken to Sp(k-2)xU(1)
- massless fermions (2k–4,N_c)

SO(10) with N_f 16

- $SU(N_f)$ global symmetry
- Tumbling: MAC is $\langle 16_i\ 16_j \rangle = \langle 10_{ij} \rangle = \langle 10_{ji} \rangle$
  - breaks gauge group SO(10) to SO(9)
  - breaks global symmetry $SU(N_f)$ to $SO(N_f)$
  - all 16 acquire "constituent quark mass"
- reduces to pure SO(9) YM
- Overall, $SU(N_f)/SO(N_f)$
  - $SO(N_f)$ has no anomalies
  - no massless fermions

SO(10) with two 16

- SU(2) global symmetry
- $S_{ij;kl} = A_{ij}^{\mu\nu\rho} A_{kl}^{\mu\nu\rho}$

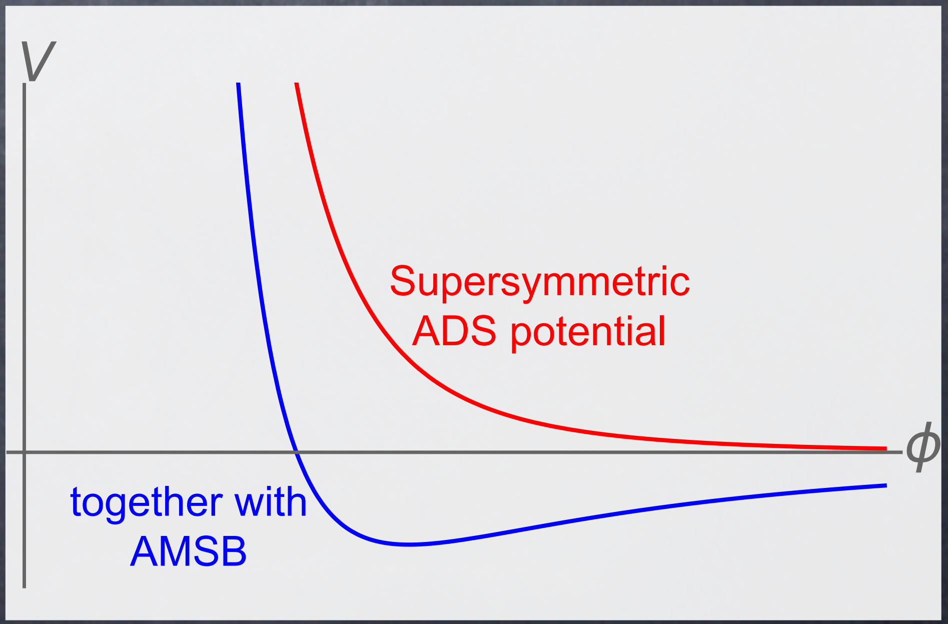
1	1
2	2

is SU(2) singlet

- D-flat direction breaks SO(10) to G_2
- gaugino condensate generates superpotential

$$W = \left(\frac{\Lambda^{20}}{S^2}\right)^{1/4}$$

- gapped, no symmetry breaking
- SU(2) non-anomalous 10=even
- testable predictions!



SO(10) with three 16

- SU(3) global symmetry
- $S_{ij;kl} = A^{\mu\nu\rho}_{ij} A^{\mu\nu\rho}_{kl}$ is SU(3) symmetric tensor

$$\begin{array}{|c|c|}\hline i & k \\ \hline j & l \\ \hline \end{array} \quad 3^* \times 3^* = 6^*$$

- D-flat direction breaks SO(10) to SU(2)
- gaugino condensate generates superpotential

$$W = \left(\frac{\Lambda^{18}}{\det S} \right)^{1/2}$$

- breaks SU(3) to SO(3)
- low-energy: SU(3)/SO(3) chiral L with WZW
- testable predictions!

<table>
<tr><th colspan="2">UV theory</th><th colspan="2">SUSY + AMSB</th><th colspan="2">Tumbling</th></tr>
<tr><th>N_F</th><th>G_{global}</th><th>H_{global}</th><th>$m_f = 0$</th><th>H_{global}</th><th>$m_f = 0$</th></tr>
<tr><td colspan="6">$\mathrm{SU}(N_C) + A(\begin{smallmatrix}\Box\\\Box\end{smallmatrix}) + (N_C - 4 + N_F)\tilde{F}(\bar{\Box}) + N_F F(\Box)$, N_C =Odd [15, 20]</td></tr>
<tr><td>0</td><td>$\mathrm{SU}(N_C - 4) \times \mathrm{U}(1)$</td><td>$\mathrm{Sp}(N_C - 5) \times \mathrm{U}(1)$</td><td>$\Box + 1$</td><td>$\mathrm{SU}(N_C - 4) \times \mathrm{U}(1)$</td><td>$\Box\Box$</td></tr>
<tr><td>1</td><td>$\mathrm{SU}(N_C - 3) \times \mathrm{U}(1)^2$</td><td>$\mathrm{Sp}(N_C - 3) \times \mathrm{U}(1)$</td><td>$\Box$</td><td>$\mathrm{SU}(N_C - 4) \times \mathrm{U}(1)$</td><td>$\Box\Box$</td></tr>
<tr><td>2</td><td>$\mathrm{SU}(N_C - 2) \times \mathrm{SU}(2) \times \mathrm{U}(1)^2$</td><td>$\mathrm{Sp}(N_C - 3) \times \mathrm{U}(1)^2$</td><td>$\Box$</td><td>$\mathrm{SU}(N_C - 4) \times \mathrm{SU}(2) \times \mathrm{U}(1)$</td><td>$(\Box\Box, 1)$</td></tr>
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<tr><td colspan="6">$\mathrm{SU}(N_C) + S(\Box\Box) + (N_C + 4)\tilde{F}(\bar{\Box})$ [18]</td></tr>
<tr><td>0</td><td>$\mathrm{SU}(N_C + 4) \times \mathrm{U}(1)$</td><td>$\mathrm{SO}(N_C + 4)$</td><td>none</td><td>$\mathrm{SU}(N_C + 4) \times \mathrm{U}(1)$</td><td>$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$</td></tr>
<tr><td colspan="6">$\mathrm{SO}(10) + N_F\psi(\mathbf{16})$ [19]</td></tr>
<tr><td>1</td><td>none</td><td>none</td><td>none</td><td>none</td><td>none</td></tr>
<tr><td>2</td><td>SU(2)</td><td>SU(2)</td><td>none</td><td>SO(2)</td><td>none</td></tr>
<tr><td>3</td><td>SU(3)</td><td>SO(3)</td><td>none</td><td>SO(3)</td><td>none</td></tr>
<tr><td colspan="6">$E_6 + N_F\psi(\mathbf{27})$ [This Work]</td></tr>
<tr><td>1</td><td>none</td><td>none</td><td>none</td><td>none</td><td>none</td></tr>
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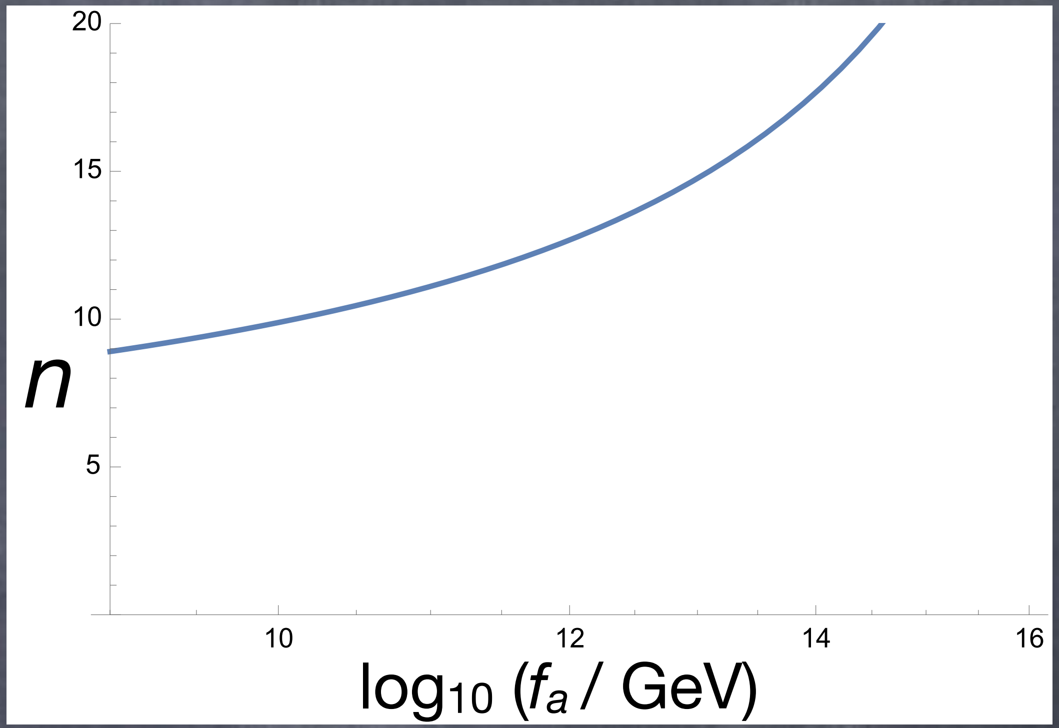
<tr><td>$\mathrm{U}(1) \times \mathrm{U}(1)$</td><td>$S_{iii} (i = 1, 2, 3)$</td></tr>
<tr><td>none</td><td>none</td></tr>
</table>

New BSM model building tools

- SUSY + AMSB appears useful to gain insight into strong gauge dynamics
- new possibilities:
  - massless composite fermions
  - light composite scalars
  - new symmetry breaking patterns
- also many (in principle) testable predictions on dynamics
  - look forward to future developments on lattice

composite axion
Gherghetta, HM, Quilez

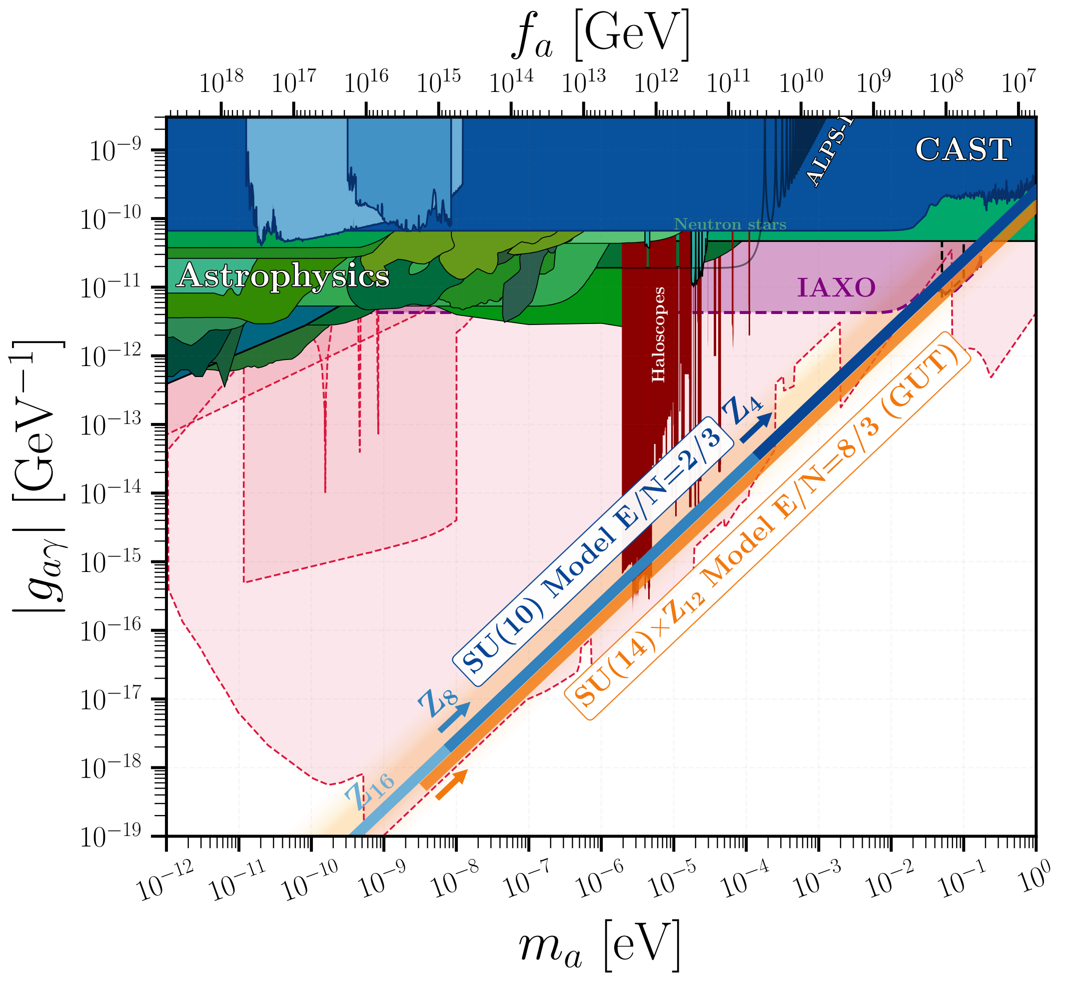
Axion Quality Problem



- there have been arguments that quantum gravity does't respect global symmetries
- string theory: no global symmetries $\phi = f_a e^{ia/f_a}$
- Is $U(1)_{PQ}$ broken at Planck scale? $V_{\rm PQV} = \frac{1}{M_{\rm Pl}^{n-4}}\phi^n$
- can shift the axion minimum away from $\theta_{QCD}=0$
- $\theta_{QCD} < 10^{-10}$, $f_a > 10^9$ GeV would

  require $n \geq 9$

Axion Quality Problem



- there have been arguments that quantum gravity doesn't respect global symmetries
- string theory: no global symmetries $\phi = f_a e^{ia/f_a}$
- Is $U(1)_{PQ}$ broken at Planck scale? W
- can shift the axion minimum away from θ_{QCD}=0
- $\theta_{QCD} < 10^{-10}$, $f_a > 10^9$ GeV would require n≥9

- can write models of composite axion using the exact SUSY dynamics to avoid the quality problem (Gherghetta, HM, Noether, Quilez)

2D Yang-Mills Theories

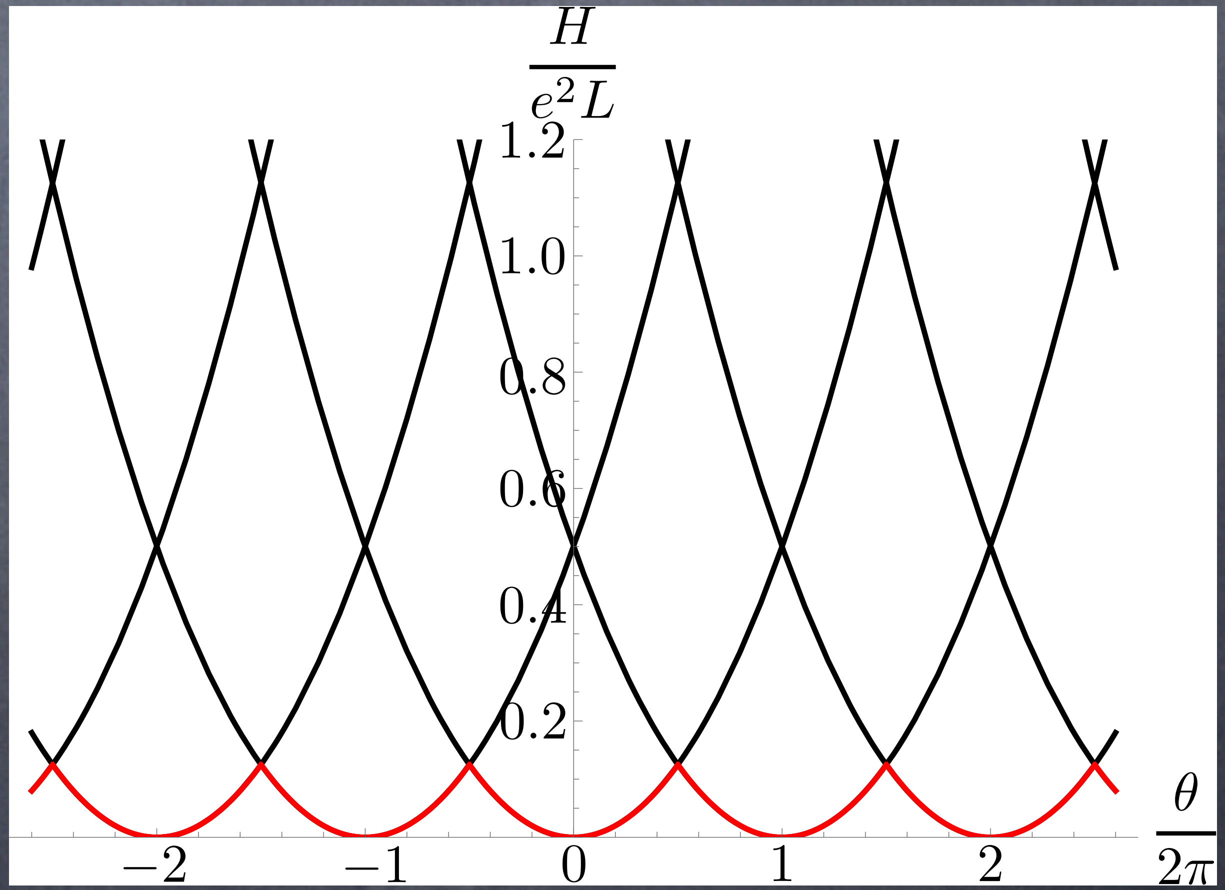
HM, Bea Noether, Digvijay Varier, Ruoyao Zheng

in preparation

Compact U(1) in 1+1D

- Many similarities with pure Yang-Mills in 4D
  - theta term
  - confinement
- Can be solved exactly
  - Electric dipole moment
  - partition function with full θ-dependence
  - deconfinement at $\theta=\pi$

$$H = \frac{e^2}{2L}\left(-i\frac{d}{da} - L\frac{\theta}{2\pi}\right)^2 = \frac{e^2 L}{2}\left(n - \frac{\theta}{2\pi}\right)^2$$



Path Integral

$$z = \frac{1}{2\pi}(-ig^2 LT)\frac{\theta}{2\pi}$$

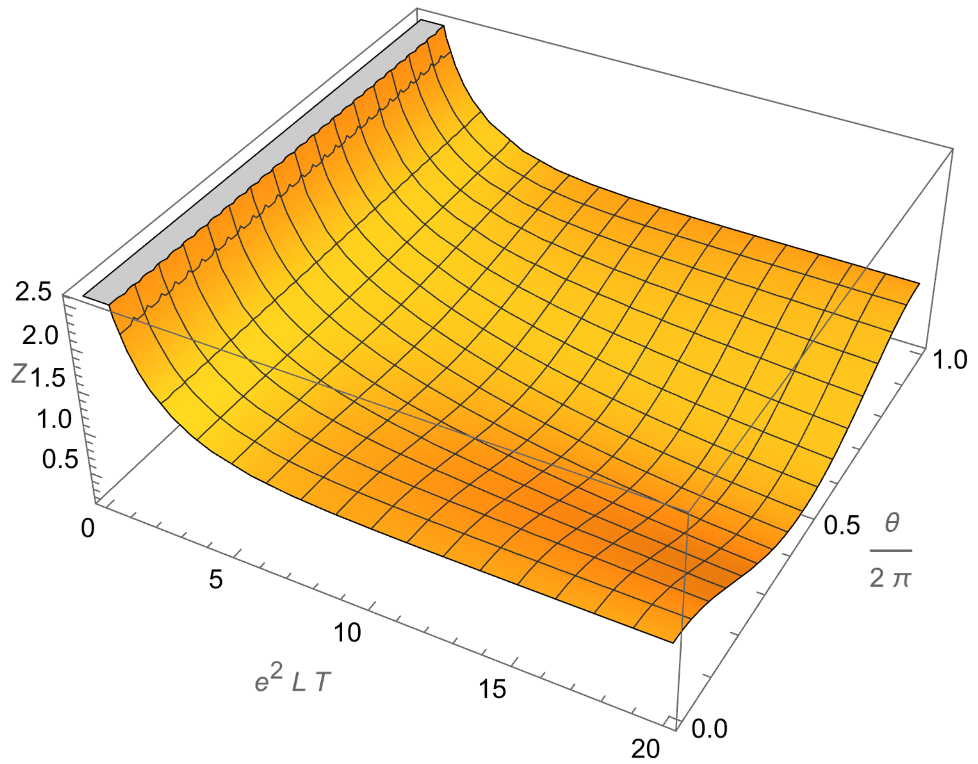
$$\tau = \frac{1}{2\pi}(ig^2 LT)$$

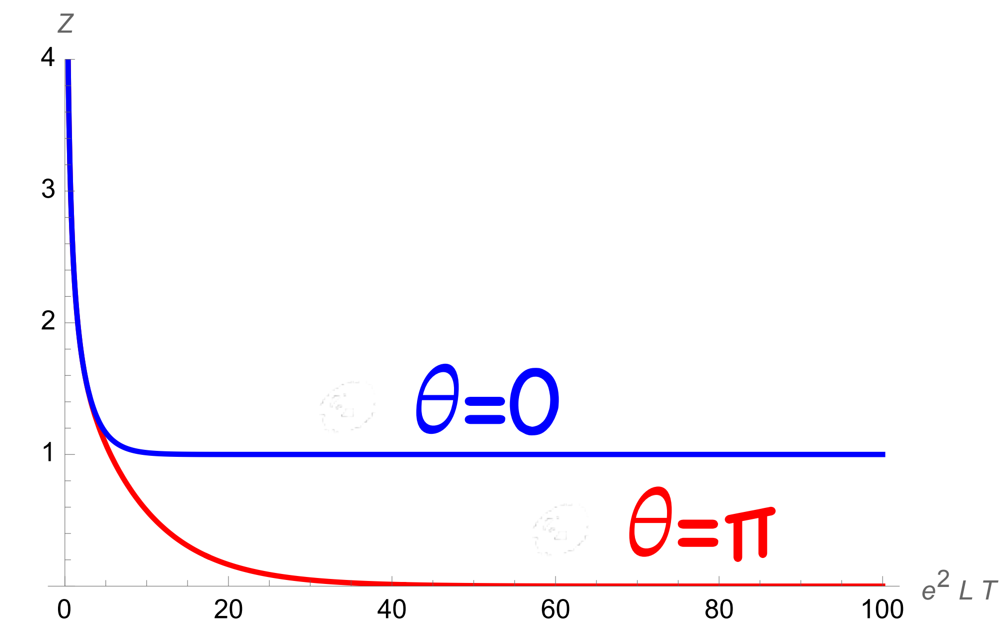
$$Z = \sum_n e^{-\beta \frac{e^2 L}{2}\left(n - \frac{\theta}{2\pi}\right)^2} = e^{-\frac{1}{2} g^2 LT \left(\frac{\theta}{2\pi}\right)^2} \vartheta(z;\tau)$$

- modular transformation of Jacobi elliptic theta functions (Poisson summation)

$$Z = \left(\frac{2\pi}{e^2 \beta L}\right)^{1/2} \sum_n e^{-\frac{1}{2e^2 L\beta}(2\pi n)^2 + in\theta}$$

- equivalent to summation over instantons

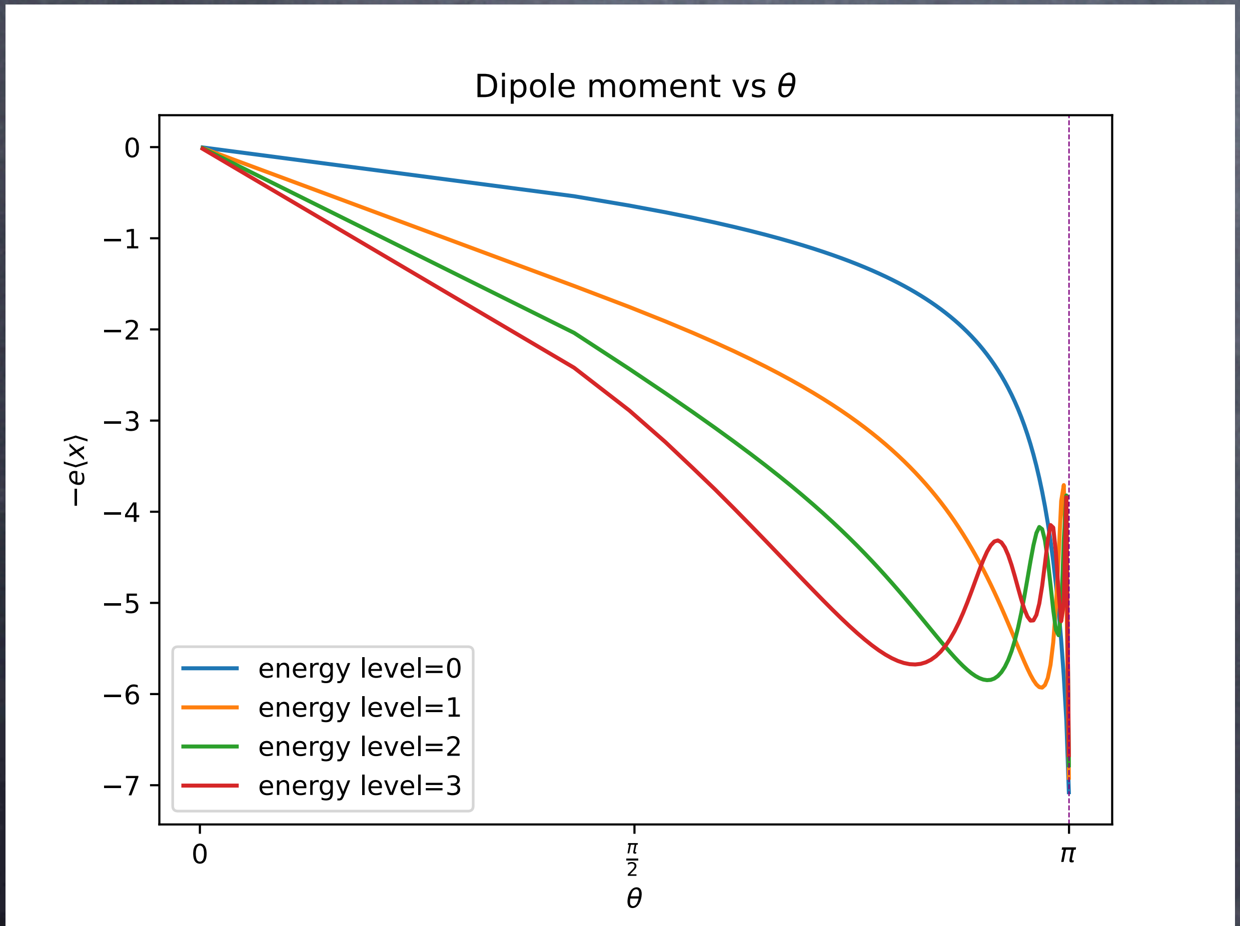


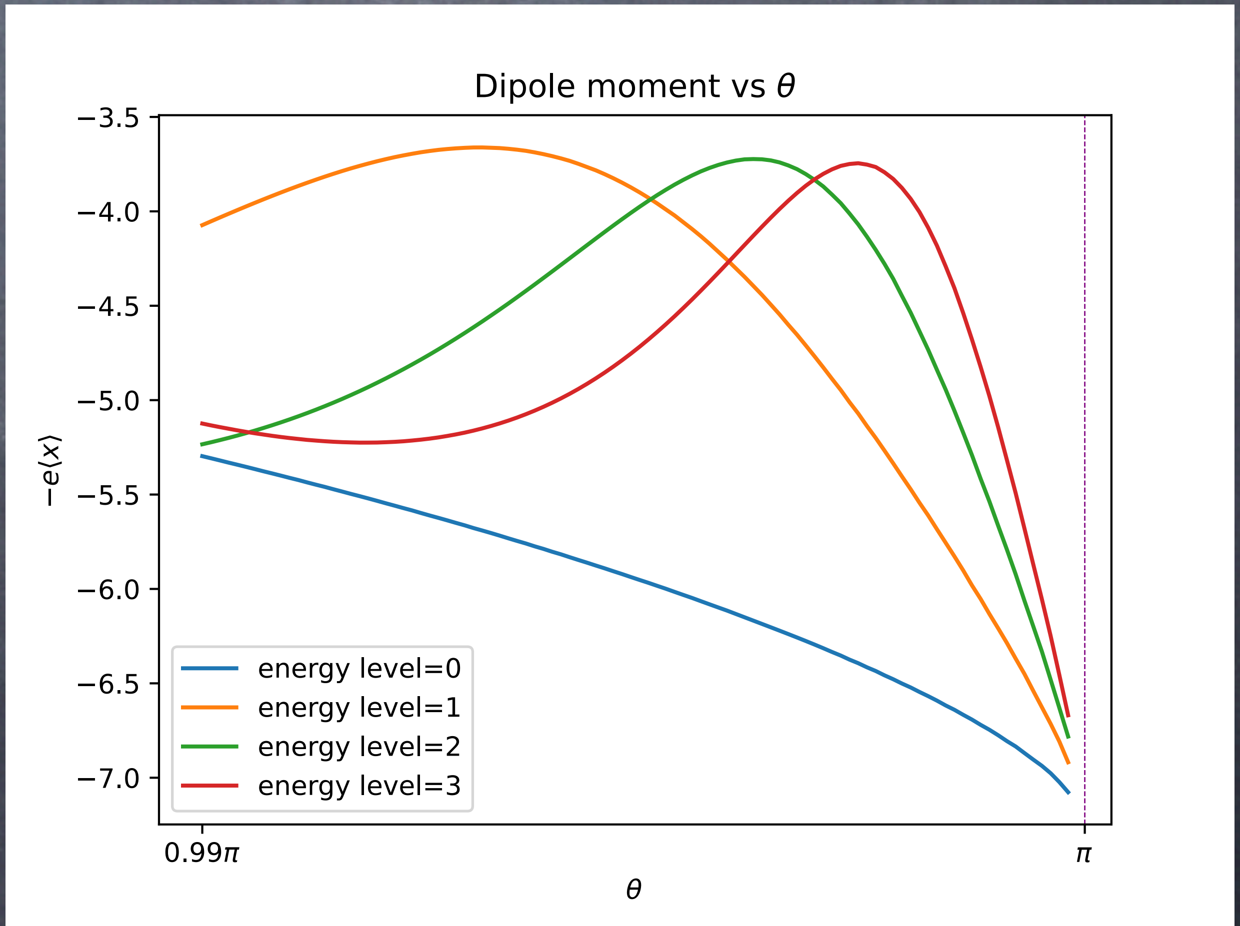


Electric Dipole Moments

$$H = \frac{1}{2\mu}p^2 + \frac{1}{2}e^2|x| + e^2\frac{\theta}{2\pi}x$$

- θ violates P and C but not CP
- EDM is induced by θ
- deconfinement at $\theta=\pi$, EDM diverges





Pure YM in 1+1D

- Exact solutions go back to Migdal (1975):
  - wave functions are characters
  - energy eigenvalues are 2nd order Casimirs
- Consider $S^1 = [0,L]$
- wave functions are characters for irreps

$$H = \frac{1}{2} g^2 L T^a T^a$$

$$E_R = \frac{1}{2} g^2 L C_2(R)$$

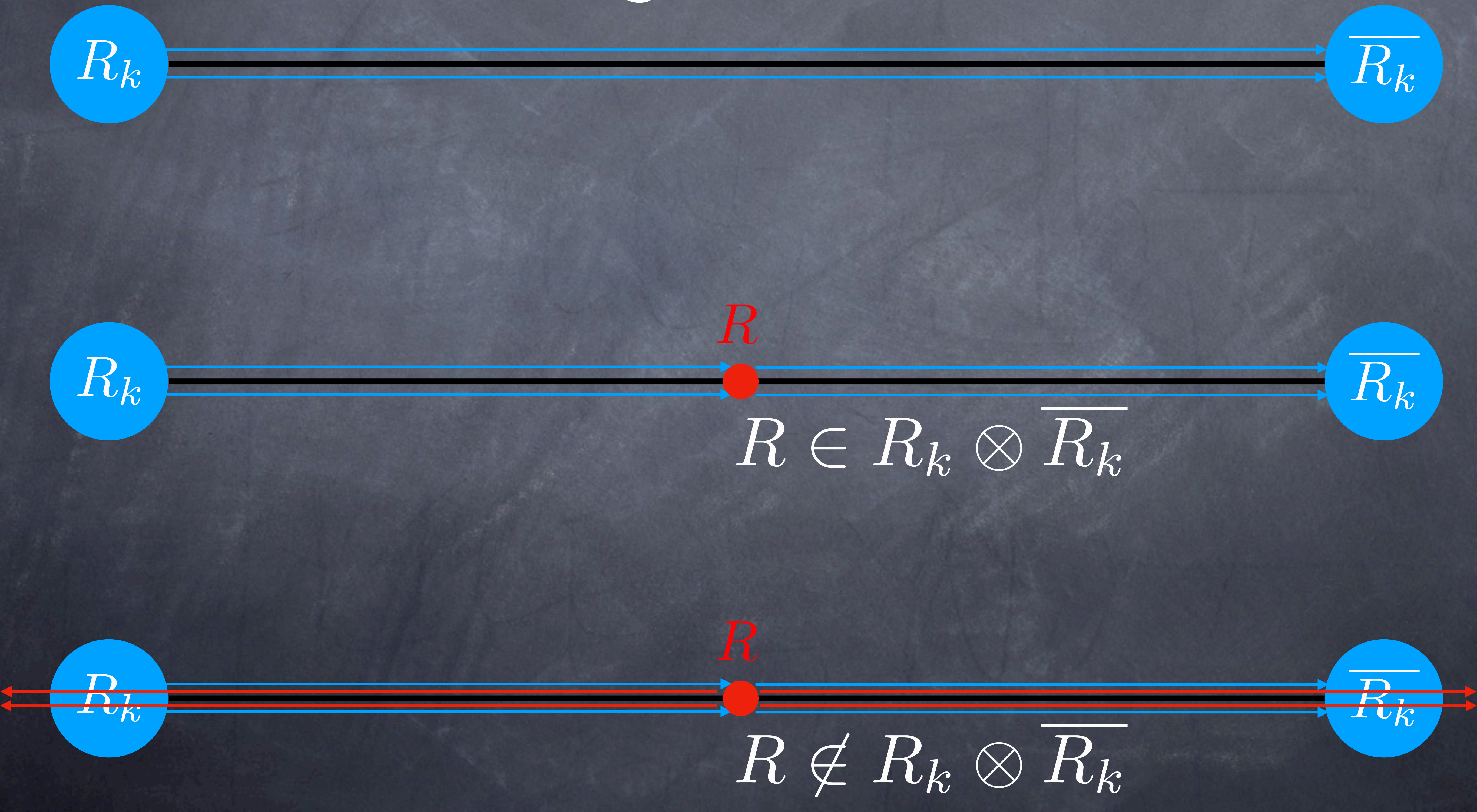
- Ground state is singlet
- all non-trivial representations confined

Pure YM in 1+1D

- $SU(N)/Z_N$: N different definitions due to Z_N-valued Stiefel-Whitney classes $w_2(SU(N)/Z_N)$ Ahanory, Seiberg, Tachikawa (2015)
- canonical: subspace of Hilbert space with k mod N boxes of Young tableaux (N-ality)
- cast to path integral using Poisson summation
- corresponding path integrals are weighted by Stiefel-Whitney class w_2: $e^{2\pi i k w_2/N}$ (odd N) or $e^{2\pi i (k+m) w_2/N}$ (even N=2m)
- without weighting by w_2, adjoint confined for odd N, not for even N

Sloppily, $\theta=\pi$ and 0 switched

Interpretation of the ground state



Conclusions

- Mysteries about chiral gauge theories
  - we don't know what they do
- conjectures to universality class, e.g. tumbling
- new addition: exact results near SUSY
  - near-SUSY to non-SUSY cross over
  - results do not agree with past conjectures
- nature may use them! e.g. dark matter, axion
- curious behavior of 2D YM
- Looking forward to the rest of the meeting!