Conditions and Phenomenology for Symmetric Mass Generation

许岑珂

Cenke Xu

Conditions and Phenomenology for Symmetric Mass Generation

Outline:

1. Definition of SMG, ‘t Hooft anomaly

2. Example of 0+1d SMG, Green’s function zero

3. Example of 2+1d SMG, connection to topo-insulator, Green’s function zero, conformal defect

4. Examples of 1+1d SMG, relation to Lieb-Shultz-Mattis

5. Conditions and possible phenomenology of SMG in 3+1d

Collaborators: Kevin Slagle, Yi-Zhuang You, Yuan-Yao He, Zi-Hong Liu, Zi-Yang Meng, Fakher Assaad, Anna Hasenfratz …….

Trivially gapped = symmetric, gapped and nondegenerate (unique GS/vacuum);

Symmetric mass generation: the system has a trivially gapped phase; but there is also a point or phase with (weakly interacting) massless Dirac fermions, and **there is a key symmetry $\mathcal{G}$ under which all fermion-bilinear mass operators transform nontrivially**.

$$\mathcal{G}\,(\bar{\psi}M\psi)\,\mathcal{G}^{-1} = -\bar{\psi}M\psi$$

Massless Dirac fermion, "Semi-metal",

SMG, trivially gapped, $\mathcal{G}$ symmetric

Symmetry $\mathcal{G}$ prohibits all fermion mass terms

Nature of the transition?

$\langle\bar{\psi}M\psi\rangle = 0$

The gapless phase often has some large emergent and also anomalous symmetry G_{IR} in the IR. In SMG phase, this G_{IR} symmetry must be broken. " S " in SMG refers to " $\mathcal{G}$ ", not G_{IR}.

Example of SMG in 2+1d

Key symmetry is a nonunitary $\mathcal{G} = CT$. One can verify that, with CT, all Dirac fermion mass operators change sign:

$$\mathcal{G}\,(\bar{\psi}M\psi)\,\mathcal{G}^{-1} = -\bar{\psi}M\psi$$

The self 't Hooft anomaly of CT is cancelled for multiple of 8 Dirac fermions, or 16 Majorana fermions (Z_{16} of ^{3}He-B).

Relation to interacting topological insulator/superconductor.

Many entries reduces from Z to Z4, Z8 ... Under interaction. Meaning, for 8 copies of AIII topological insulator (or 16 copies of He3-B), its 2+1d boundary **cannot be trivially gapped without interaction**, but **can be trivially gapped with interaction, precisely SMG.**

Class	$d=1$	$d=2$	$d=3$	$d=4$
A	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	$\mathbb{Z} \to \mathbb{Z}_4$	0	$\boxed{\mathbb{Z} \to \mathbb{Z}_8}$	0
AI	0 F&K, 2009	0	0	$\mathbb{Z}$
BDI	$\boxed{\mathbb{Z} \to \mathbb{Z}_8,}$ $\mathbb{Z} \to \mathbb{Z}_4$	0	0	0
D	$\mathbb{Z}_2$	$\mathbb{Z}$	0 ^{3}He-B	0
DIII	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\boxed{\mathbb{Z} \to \mathbb{Z}_{16}}$	0
AII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

Trivially gapped = symmetric, gapped and nondegenerate (unique GS/vacuum);

Common wisdom is that, this is only possible when the system has no ‘t Hooft anomaly.

For SMG we need to ensure:

(1) there is a symmetry $\mathcal{G}$ under which all fermion mass operators transform nontrivially;

(2) There is no self ‘t Hooft anomaly for $\mathcal{G}$, and there is no mixed anomaly between $\mathcal{G}$ and other exact symmetries.

Remark 1: Anomaly cancellation is only the **necessary** but **not sufficient** condition for SMG.
Anomaly cancellation means that SMG in principle can happen, but it does not answer whether a specific model (action, Hamiltonian) realizes SMG or not.

Remark 2: With ‘t Hooft anomaly, the ground state degeneracy can come from either spontaneous symmetry breaking, or topological order.

Remark 3: three different "things" called anomaly:

1, Gauge anomaly for a dynamical gauge field: the worst kind of anomaly, the theory should not exist (no way to realize this theory).

2, ABJ-anomaly for a symmetry G means that the symmetry is explicitly broken (by instanton).

3, **'t Hooft anomaly** is much more "benign": the symmetry is still valid, and the theory is still consistent.

But 't Hooft anomaly of G means three things:

1, Constraint on how the symmetry is realized on the lattice;

2, Obstruction to gauging: once we gauge the symmetry G, there will be gauge anomaly (most systematic diagnosis of 't Hooft anomaly);

3, the spectrum above the ground state cannot be trivially gapped.

Remark 4: Choice of the key symmetry $\mathcal{G}$ (**minimal symmetry** under which **all** fermion-bilinear mass operators transform nontrivially): In 0+1d and 2+1d, most natural choice of $\mathcal{G}$, is CT.
Example: in 0+1d, consider arbitrary numbers of complex fermions.

$$CT \;:\; c_a \;\rightarrow\; c_a^\dagger, \quad \mathrm{i} \rightarrow -\mathrm{i},$$

$$c^\dagger M c \;\rightarrow\; c M^* c^\dagger = -c^\dagger M^\dagger c = -c^\dagger M c.$$

For 1+1d and 3+1d, the most natural choice of $\mathcal{G}$ is Spin-Z4.

$$\psi \rightarrow e^{\mathrm{i}k\frac{\pi}{2}\gamma_5}\psi, \quad k = 0, 1, 2, 3. \qquad\qquad \bar{\psi}_a M_{ab} \psi_b, \quad \mathrm{i}\bar{\psi}_a \gamma_5 M_{ab} \psi_b$$

Is $\mathcal{G}$ a symmetry at all?

In many lattice system, $\mathcal{G}$ is often realized as an **exact non-onsite** symmetry. For example, spin-Z4 is often realized as translation (shown later).

Self ‘t Hooft anomaly of the key symmetry.

dimension	(0+1)	(1+1)	(2+1)	(3+1)
$\mathcal{G}$	$\mathcal{T},\ \mathcal{T}^2=1$	Spin-Z_4,	$\mathcal{T},\ \mathcal{T}^2=-1$	Spin-Z_4
Anomaly class	$\mathbb{Z}_4$	$\mathbb{Z}_4$,	$\mathbb{Z}_8$	$\mathbb{Z}_8$

Remark: in all cases, the smallest element is a single complex Dirac fermion. The classification will be different if the smallest element is taken to be Majorana, or Weyl fermion.

Example of SMG in 0+1d:

In 0+1d, most natural choice of $\mathcal{G}$, is *CT*.

$$CT \;:\; c_a \;\to\; c_a^\dagger, \quad \mathrm{i} \to -\mathrm{i},$$

$$c^\dagger M c \;\to\; c M^* c^\dagger = -c^\dagger M^\dagger c = -c^\dagger M c.$$

Based on the classification table, with total number of 2 complex fermions, there is self 't Hooft anomaly of *CT*, cannot have SMG. Luckily, we can show this exactly.

The only *CT* invariant Hamiltonian:

$$H = U\left(c_1^\dagger c_1 - \frac{1}{2}\right)\left(c_2^\dagger c_2 - \frac{1}{2}\right)$$

GS is two-fold degenerate,
spontaneously breaks CT:

$$|\mathrm{GS}\rangle = |10\rangle \;\;\text{and}\;\; |01\rangle, \;\;\text{for}\;\; U > 0$$

Example of SMG in 0+1d:

In 0+1d, most natural choice of $\mathcal{G}$, is CT.

$$CT \;:\; c_a \;\rightarrow\; c_a^\dagger, \quad \mathrm{i} \rightarrow -\mathrm{i},$$

$$c^\dagger M c \;\rightarrow\; c M^* c^\dagger = -c^\dagger M^\dagger c = -c^\dagger M c.$$

with total number of $4k$ complex fermions, no self 't Hooft anomaly of CT, can have SMG. Again we can show this exactly.

$$H = -U\left(c_1 c_2 c_3 c_4 + h.c.\right), \quad \rightarrow \quad |\mathrm{GS}\rangle = \frac{1}{\sqrt{2}}\left(|0000\rangle + |1111\rangle\right) \qquad \langle \mathrm{GS}|c^\dagger M c|\mathrm{GS}\rangle = 0$$

The GS is trivially gapped and symmetric with this Hamiltonian.

Example of SMG in 0+1d:

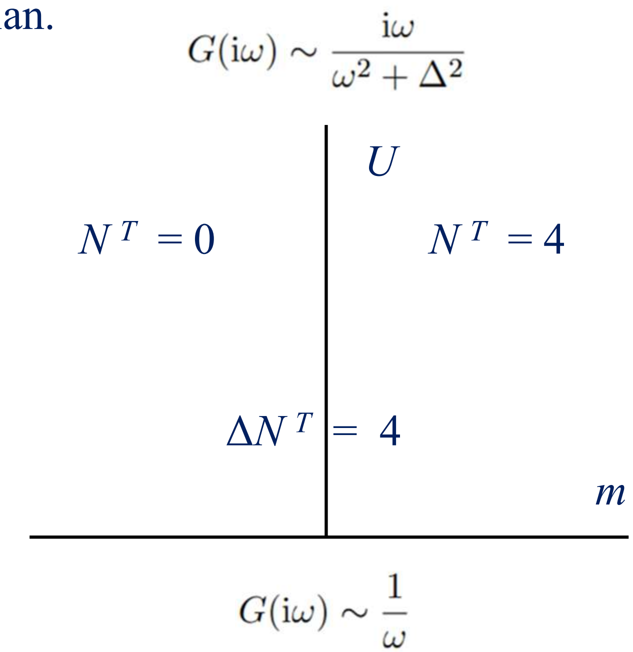
Now let us include a *CT* breaking mass term in the Hamiltonian.

$$H = \sum_{i=1}^{4} m c_i^\dagger c_i - U\left(c_1 c_2 c_3 c_4 + h.c.\right)$$

Consider the fermion Green's function, and the following function

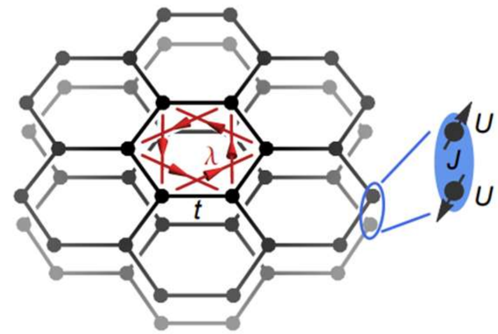
$$N^T = \frac{1}{2\pi \mathrm{i}} \int_{-\infty}^{+\infty} d\omega \ \mathrm{tr}\left(G^{-1}(\mathrm{i}\omega)\partial_\omega G(\mathrm{i}\omega)\right)$$

Guaranteed to be an integer if well defined, thanks to $\pi_1[\mathrm{GL}(N, C)]$. It can only change discontinuously when Green's functions become singular. But it can change through either "poles", or "zeros" of the Green's function.



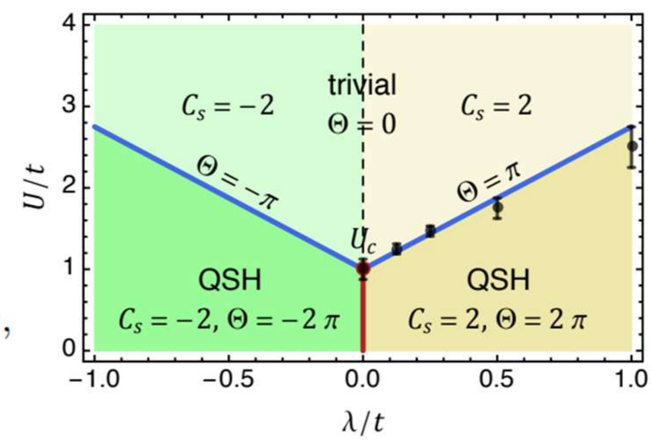
Example of SMG in 2+1d

Slagle et.al. 1409.7401
Consider a bilayer spin-1/2 quantum Hubbard model in 2d, *CT* is realized as a non-onsite symmetry.



The Hamiltonian includes nearest neighbor hopping t, a "topological" hopping λ, and interaction U, J.

$$H = H_t + H_\lambda + H_J \qquad H_J = \sum_i J S_{i,1} \cdot S_{i,2}$$



There are 2 x 2 x 2 = 8 Dirac fermions in the IR at weak coupling, ensuring anomaly cancellation of $\mathcal{G}$.

In the strong coupling limit, the state is quite obviously a trivially gapped phase:

$$|\Psi\rangle = \prod_i (c^\dagger_{i1\uparrow} c^\dagger_{i2\downarrow} - c^\dagger_{i1\downarrow} c^\dagger_{i2\uparrow})|0\rangle,$$

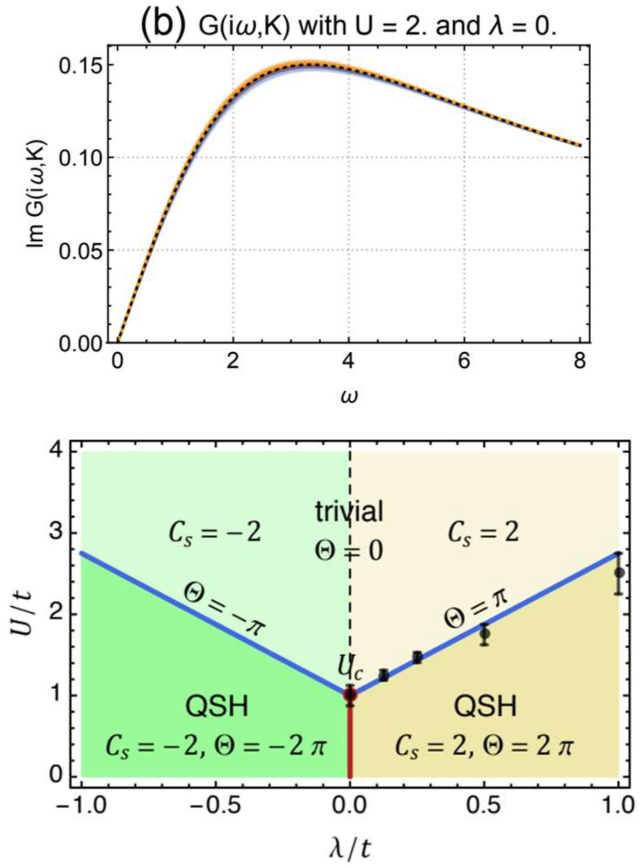
Example of SMG in 2+1d

Another key signature of SMG:
Green's function zero.

Topological argument: with nonzero parameter λ, the system has a nonzero topological number defined with the Green's function.

$$C_s = \frac{1}{48\pi^2}\int \mathrm{d}^3k\epsilon^{\mu\nu\lambda}\mathrm{Tr}[-\sigma^z G\partial_\mu G^{-1}G\partial_\nu G^{-1}G\partial_\lambda G^{-1}]$$

Guaranteed to be an integer if well defined, thanks to $\pi_3[\mathrm{GL}(N, C)]$. It can only change discontinuously when Green's functions become singular. But it can change through either "poles", or "zeros" of the Green's function. This means that **the Green's function must have zeros in the SMG phase**. Gurarie, 2011, Slagle et.al. 2014

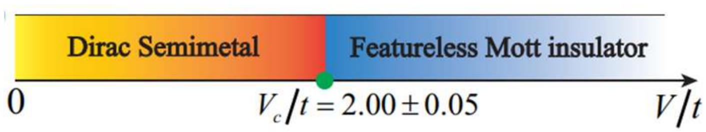


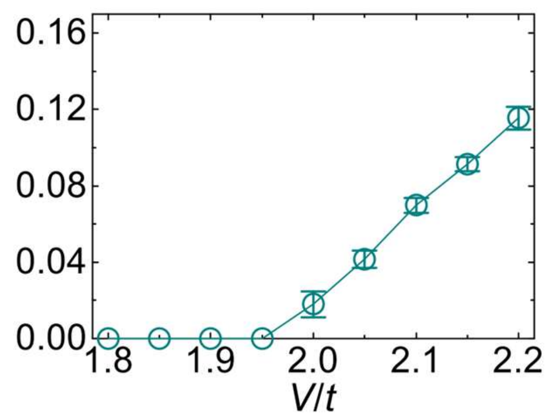
Example of SMG in 2+1d

Nature of the SMG transition: discussed more carefully in another very similar model on honeycomb lattice:

$$\hat{H} = H_{\text{band}} + H_{\text{int}}$$
$$\hat{H}_{\text{band}} = -t \sum_{\langle l,r\rangle \alpha} (-1)^\alpha (c^\dagger_{l\alpha} c_{r\alpha} + c^\dagger_{r\alpha} c_{l\alpha})$$
$$\hat{H}_{\text{int}} = V \sum_r (c^\dagger_{r1} c_{r2} c^\dagger_{r3} c_{r4} + c^\dagger_{r4} c_{r3} c^\dagger_{r2} c_{r1}),$$

Phase diagram





Numerically, all the gap (fermion and "mesons") opens up continuously at V_c. (He, et.al. 1603.08376). But need a better diagnosis for 2+1d CFT.

Example of SMG in 2+1d

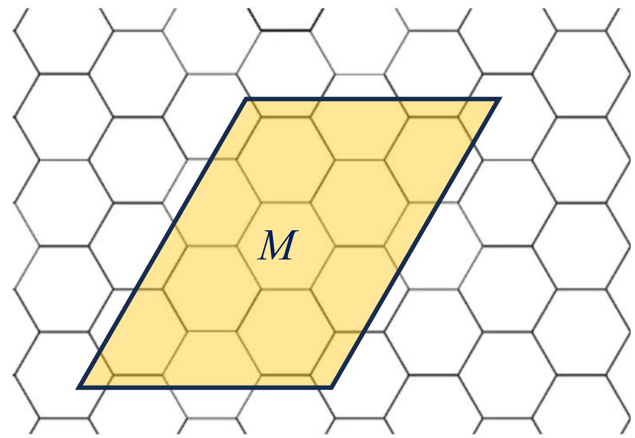
Better diagnosis: conformal defect.

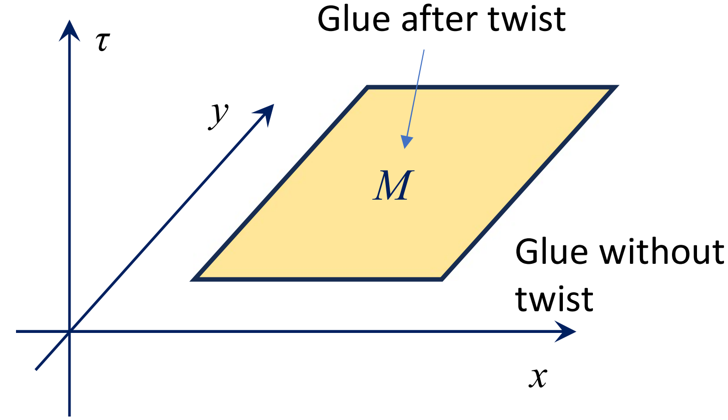
$$\hat{X}_M(\theta) = \prod_{i \in M} \exp(\mathrm{i}\theta \hat{n}_i),$$

Also called the disorder operator.

In the space-time path-integral, the expectation value of this operator maps to a path integral of the system action with a "twist" at temporal plane $\tau = 0$:
glueing $\tau > 0$ and $\tau < 0$ with a twist in M.

This is like inserting flux along the boundary of M in spacetime.

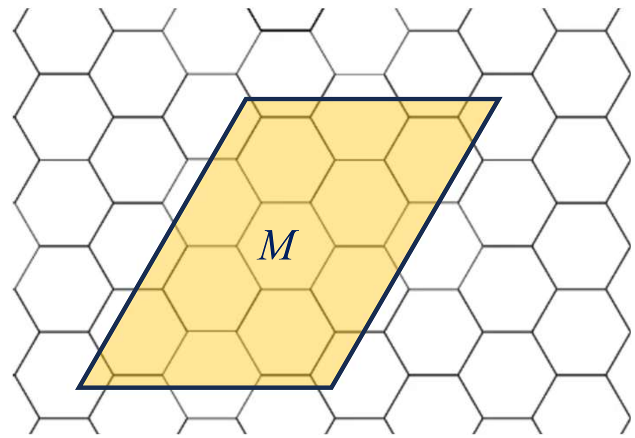




Example of SMG in 2+1d

Better diagnosis: conformal defect.

$$\hat{X}_M(\theta) = \prod_{i \in M} \exp(\mathrm{i}\theta \hat{n}_i),$$



Also called the disorder operator.

If the system is a trivially gapped state, then $\ln |\langle \hat{X}_M(\theta) \rangle| \sim -al,$

If the system spontaneously breaks the (continuous symmetry) $\ln |\langle \hat{X}_M(\theta) \rangle| \sim -al \ln l,$

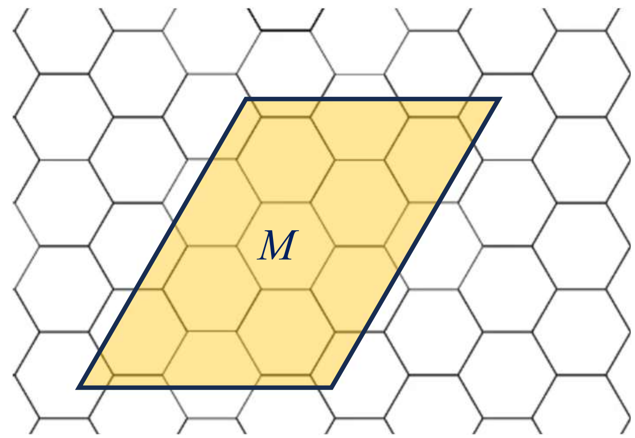
If the system is a 2+1d CFT: $\ln |\langle \hat{X}_M(\theta) \rangle| \sim -al + \beta(\theta)$ Smooth curve

$s(\theta)$ related to current central charge. $\ln |\langle \hat{X}_M(\theta) \rangle| \sim -al + s(\theta) \ln l$ With corners

Example of SMG in 2+1d

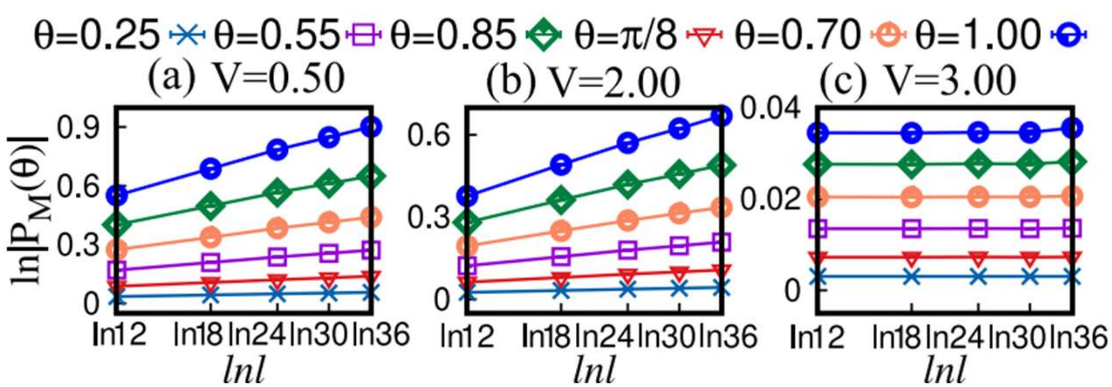
Better diagnosis: conformal defect.

$$\hat{X}_M(\theta) = \prod_{i \in M} \exp(\mathrm{i}\theta \hat{n}_i),$$



For the gapless Dirac fermion phase, and at the SMG transition, the CFT behaviors were observed Liu et.al. 2308.07380:

$$P_M(\theta) = \frac{|\langle \hat{X}_M(\theta) \rangle|}{|\langle \hat{X}_M(\theta)|}$$



Example of SMG in 2+1d

Key symmetry is a nonunitary $\mathcal{G} = CT$. One can verify that, with CT, all Dirac fermion mass operators change sign:

$$\bar{\psi}M\psi \;\rightarrow\; -\bar{\psi}M\psi.$$

The self 't Hooft anomaly of CT is cancelled for multiple of 8 Dirac fermions, or 16 Majorana fermions (famous Z_{16} classification of ^{3}He-B).

But there could still be mixed anomaly between CT and flavor symmetry. For example, for 8 flavors of Dirac fermions, if there is a SU(8) flavor symmetry, there is a mixed anomaly between SU(8) and CT.

If we gauge SU(8), there will be a parity anomaly.

$$CT + \text{Gauge SU(8)} \rightarrow 1/2\ \text{CS(A)}$$

Parity anomaly is cancelled, when the flavor symmetry is smaller, say SU(4): the eight Dirac fermions form two fundamentals of the same SU(4).

Examples of SMG in 1+1d

Lieb-Shultz-Mattis theorem:

Simplest version: for a spin system with SU(2) symmetry and translation symmetry, with odd number of spin-1/2s per unit cell, the system must be either gapless, or gapped but degenerate, as long as the Hamiltonian is local.

This is a statement at the level of UV lattice model.

LSM theorem in the UV implies SMG cannot happen in the IR analysis (must be some uncancelled ‘t Hooft anomalies of exact symmetries in the IR).

Examples of SMG in 1+1d

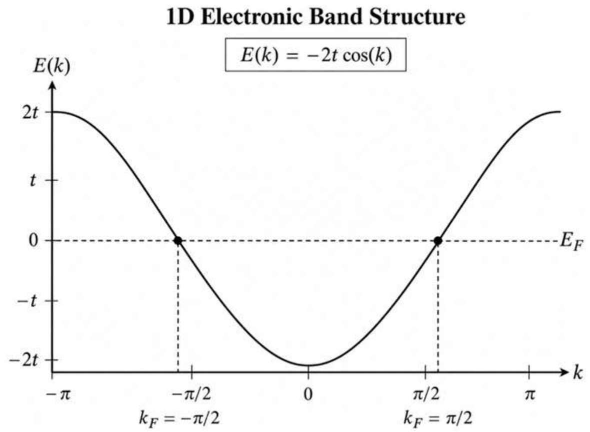
Example 1: 1+1d QED with $N_f = 2$ Dirac fermions, and $\mathrm{SU}(2)_\mathrm{V}$ flavor symmetry. It is the $\mathrm{SU}(2)_1$ CFT, the CFT describing the quantum spin-1/2 chain. Based on well-known result from Haldane, and also the **LSM theorem**, this system **cannot be trivially gapped (no SMG)**.

Spin-Z4: $x \to x+1, \quad \psi_R \to \mathrm{i}\psi_R, \quad \psi_L \to -\mathrm{i}\psi_L$

Spin-Z4 anomaly **not canceled**.

$$S = \int dx d\tau \sum_{a=1,2} \bar{\psi}_a \gamma_\mu (\partial_\mu - \mathrm{i}A_\mu)\psi_a$$

$$H = \sum_i J_1 \vec{S}_i \cdot \vec{S}_{i+1} + \sum_i J_2 \vec{S}_i \cdot \vec{S}_{i+2}$$



Indeed, LSM theorem in the UV, consistent with the fact that some anomalies are not cancelled in the IR theory.

Examples of SMG in 1+1d

Example 2: 1+1d QCD with U(2) gauge field, and $N_f = 2$ Dirac fermions, again with $SU(2)_V$. It is the $SU(2)_2$ CFT, the CFT describing a critical point of the quantum spin-1 chain. Based on well-known result from Haldane, this system **can be trivially gapped**.

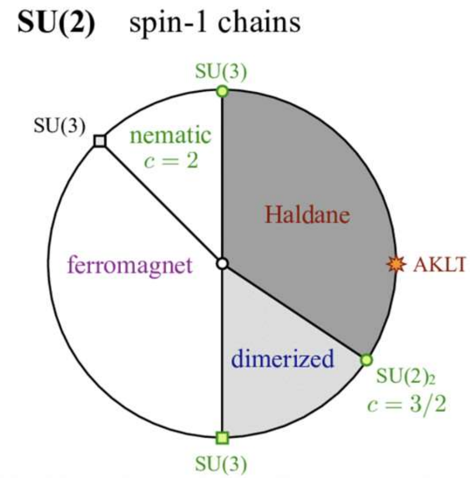
Spin-Z4: $x \to x+1, \quad \psi_R \to \mathrm{i}\psi_R, \quad \psi_L \to -\mathrm{i}\psi_L$

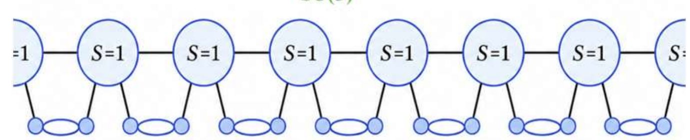
Spin-Z4 anomaly **canceled**.

$$S = \int dx d\tau \sum_{a=1,2} \bar{\psi}_a \gamma_\mu (\partial_\mu - \mathrm{i}A_\mu - \mathrm{i}\vec{a}_\mu \cdot \vec{\sigma}/2)\psi_a$$

$$H = \sum_i J_1 \vec{S}_i \cdot \vec{S}_{i+1} + \sum_i J_2 (\vec{S}_i \cdot \vec{S}_{i+1})^2$$

The $SU(2)_2$ CFT has an emergent (enlarged) chiral flavor symmetry $G_{IR} = SU(2)_L \times SU(2)_R$. In SMG phase, this G_{IR} symmetry must be broken.





Examples of SMG in 1+1d

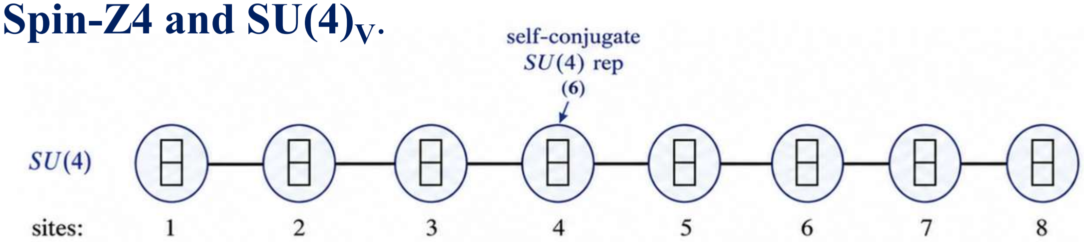
Not only need to cancel self-anomaly of Spin-Z4, but also need to cancel the mixed anomaly between Spin-Z4 and other (vector) flavor symmetries.

Example 3: 1+1d QED with $N_f = 4$ Dirac fermions, with $SU(4)_V$. It is the $SU(4)_1$ CFT, the CFT describing the quantum SU(4) chain. If we also (only) assume the Spin-Z4, this theory describes the SU(4) chain with self-conjugate rep. This model **cannot be trivially gapped, due to LSM.**

Spin-Z4 anomaly is indeed **canceled**.

$$S = \int dx d\tau \sum_{a=1\cdots 4} \bar{\psi}_a \gamma_\mu (\partial_\mu - \mathrm{i} A_\mu) \psi_a$$

But some anomalies must remain uncancelled. Indeed, in field theory, there is another **mixed anomaly between Spin-Z4 and $SU(4)_V$.**

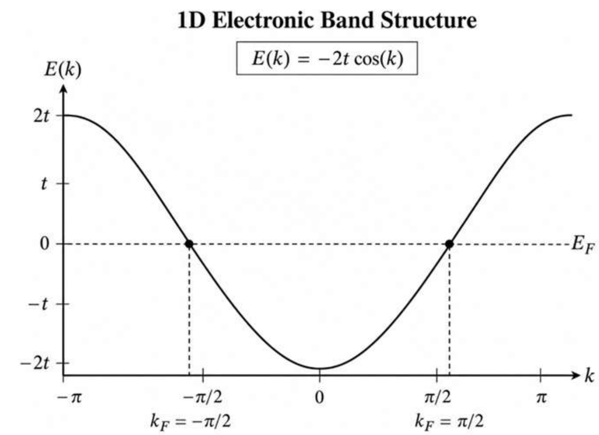


Examples of SMG in 1+1d (without gauge fields)

Now consider a bilayer (two-orbital) spin-1/2 quantum Hubbard model in 1d.

$$H = \sum_{i,j} \sum_{a=1,2} \sum_{l=1,2} -t c^\dagger_{i,a,l} c_{j,a,l} + h.c. + H_{\text{int}}$$

The noninteracting part of the system maps to Dirac fermion in the same way as before, as Spin-Z4 implemented as translation. But now there are $N_f = 4$ Dirac fermions. The Spin-Z4 anomaly is cancelled out, and there is no LSM obstruction. Indeed, **the system can be drive into a trivially gapped (SMG) state (Slagle et.al. 1409.7401)**.



Important note: the free (or weakly interacting) fermion has an **emergent** (enlarged) chiral flavor symmetry G_{IR}= U(4) $_L$ x U(4)$_R$ with **clear 't Hooft anomaly**. In SMG phase, this $\tilde{G}$ symmetry must be broken. " S " in SMG refers to " $\mathcal{G}$ " (in this case Spin-Z4), not G_{IR}.

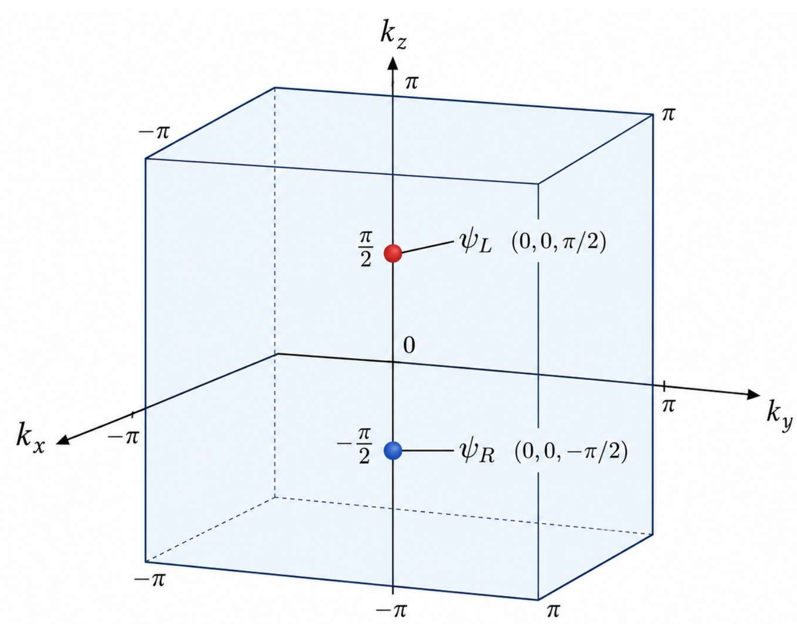
A 3+1d system: Weyl semimetal

Weyl semimetal is a condensed matter system (realized in experiment). Just like the 1+1d examples, the left and right handed fermions are separated in momentum space.

Is there a symmetry $\mathcal{G}$ = Spin-Z4?

Translation T_z generate Spin-Z4:

$$z \to z+1, \quad \psi_R \to \mathrm{i}\psi_R, \quad \psi_L \to -\mathrm{i}\psi_L$$



Conditions for SMG in 3+1d (4d) lattice-QCD

Let's take staggered fermion as a potential platform for SMG. One staggered fermion is $N_f = 4$ QCD in the IR.

Does staggered fermion have the key symmetry $\mathcal{G}$ = Spin-Z4?

Exact symmetries of staggered fermion, includes the $\mathrm{U}(1)_\varepsilon$, and "shift" symmetries (translation by on lattice constant along each direction)

$$\mathrm{Spin}{-}Z_4 : \psi \rightarrow e^{\mathrm{i}k\frac{\pi}{2}\gamma_5}\psi,$$

is a subgroup of

$$\mathrm{U}(1)_\epsilon : \psi \rightarrow e^{\mathrm{i}\theta\gamma_5\xi_5}\psi.$$

$$Z_2^\xi : \psi \rightarrow \xi_5\psi.$$

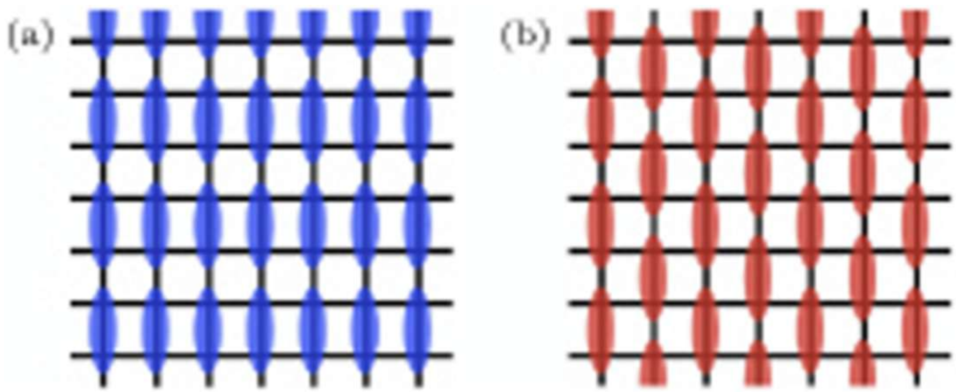
Z_2^{ξ} is the Z2 subgroup of shift symmetry, which translates along all four directions by one lattice constant.

Therefore it is important to keep Z_2^{ξ} (not necessarily the entire shift symmetry) for SMG,

Conditions for SMG in 3+1d (4d) lattice-QCD

Let's take staggered fermion as a potential platform for SMG. One staggered fermion is $N_f = 4$ QCD in the IR.

Does staggered fermion have the key symmetry $\mathcal{G}$ = Spin-Z4?



Break $Z_2{}^{\xi}$ and Spin-Z4, A fermion mass term

Preserve $Z_2{}^{\xi}$ and Spin-Z4, not a mass term

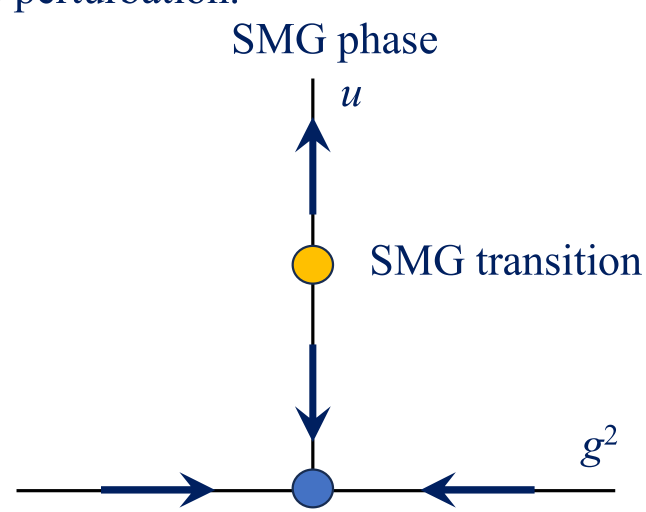
" If " SMG happens in a 3+1d (or 4d) lattice-QCD model

Let's assume $N_f N_c$ is multiple of 8, so Spin-Z4 anomaly is cancelled, and (N_f, N_c) is in the conformal window.

Motivated from the 2+1d phase diagram, the ideal situation is that there is a continuous transition from the conformal fixed point to SMG phase, driven by some perturbation.

The conformal fixed point has an IR emergent (enlarged) chiral flavor symmetry $G_{IR} = SU(N_f)_L \times SU(N_f)_R$. This symmetry has 't Hooft anomaly, it should be broken.

It should be broken either explicitly in the UV action (like the high-dimension terms for staggered fermion in Lee&Sharpe 1999);
or broken spontaneously in the SMG phase through a **four-fermion condensate**, which still preserves Spin-Z4.



" If " SMG happens in a 3+1d (or 4d) lattice-QCD model

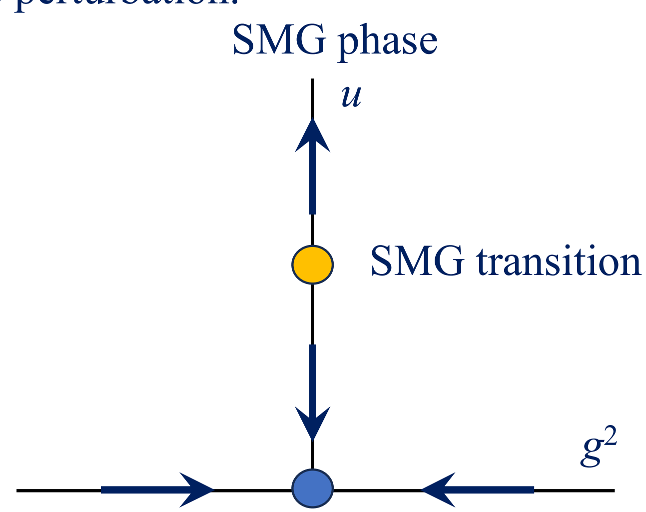
Let's assume $N_f N_c$ is multiple of 8, so Spin-Z4 anomaly is cancelled, and (N_f, N_c) is in the conformal window.

Motivated from the 2+1d phase diagram, the ideal situation is that there is a continuous transition from the conformal fixed point to SMG phase, driven by some perturbation.

If $G_{IR} = SU(N_f)_L \times SU(N_f)_R$ is spontaneously broken, then some chiral flavor current $\bar{\psi} T \gamma_5 \gamma_\mu \psi$ correlation can't be short-ranged, then according to the **Weingarten inequality**, pion $i\bar{\psi} T \gamma_5 \psi$ correlation can't be short ranged, inconsistent with the definition of SMG.

How do we invalidate the Weingarten inequality? The conditions for Weingarten inequality can be violated by including a proper Yukawa field.

$$\gamma_5 \not{D}_A \gamma_5 = \not{D}_A^\dagger .$$



Current status of the effort of searching for SMG

1+1d, reliable analytical methods, also reliable numerics, matching anomaly cancellation.

2+1d, no exactly soluble models, numerics from different groups agree, showing SMG and continuous transition between conformal and SMG phase; matching anomaly cancellation.

3+1d (4d) lattice-QCD: a strongly coupled phase with $U(1)_\varepsilon$ symmetry was reported in staggered fermion for example in $N_f = 4$ SU(2) QCD, and $N_f = 8$ SU(3) lattice-QCD. Mesons connected by $U(1)_\varepsilon$ have the same mass. Therefore this strongly coupled phase seems different from the ordinary confined phase of QCD (condensate of $\bar{\psi}\psi$).

But, latest data shows that in both 4d models there is likely shift symmetry and the Z^{ξ}_2 breaking, and hence Spin-Z4 breaking, therefore the strongly coupled phase of these models is not exactly SMG phase. More efforts needed.

Procedures of looking for SMG:

1. If the system has a gapless point/phase described by massless fermions, identify the key symmetry $\mathcal{G}$ under which all fermion mass terms transform nontrivially, this symmetry must be exact, but could realize non-onsite.

2. Ensure that $\mathcal{G}$ has no ‘t Hooft anomaly, including its self ‘t Hooft anomaly, as well as mixed anomaly with other symmetries.

3. In the putative trivially gapped (SMG) phase, verify that the key symmetry $\mathcal{G}$ is preserved.

4. It would be best if there is a continuous transition between SMG and the gapless phase.

5. Check the Green’s function zero.

Funding: DOE (Anna Hasenfratz), Simons Foundation (Cenke Xu)