Is there any critical behavior in heavy quark dynamics?

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QCD Critical Point and Hydrodynamic Evolution

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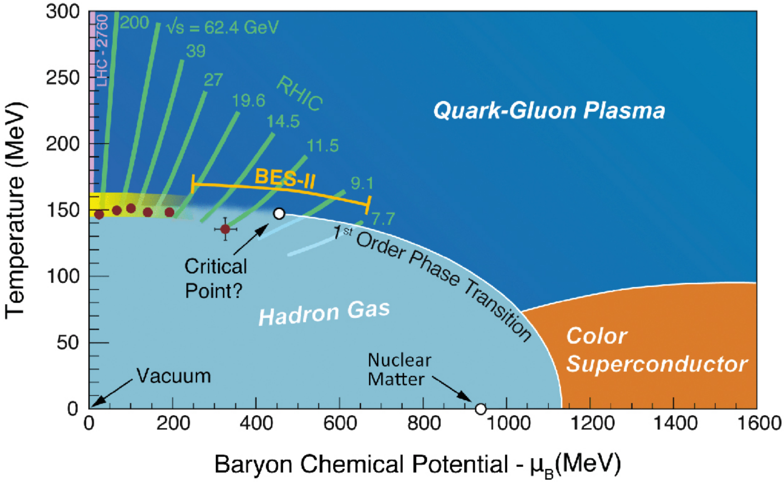
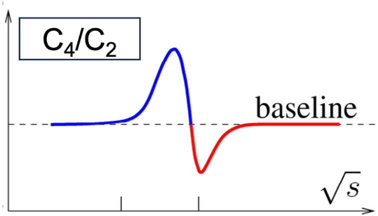
Introduction

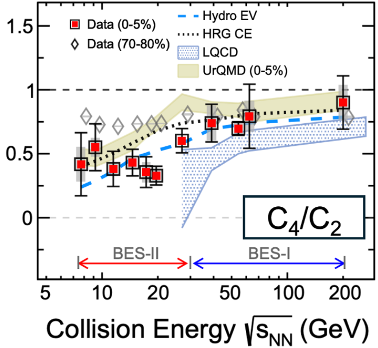
QCD phase diagram and critical point



Universality (Z_2 Ising & model H) argument provides clues to the critical point

Baryon number fluctuations [STAR (25)]





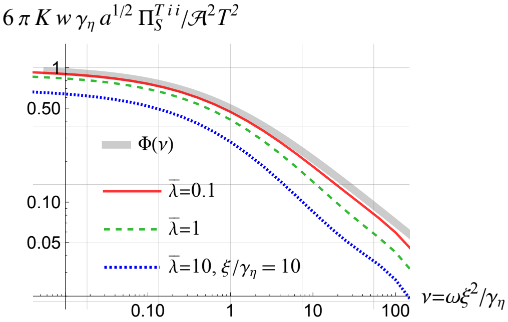
Baseline needs to be evaluated for interpretation, which is a nontrivial task

Alternative signals?

Electromagnetic probes

Critical enhancement in low energy

- Sensitive to electric conductivity $\sigma \propto \xi$ (Model H = critical modes + shear modes)
  - Dileptons in NJL model [Nishimura-Kitazawa-Kunihiro (23)]
  - Photons in model H [Akamatsu-Asakawa-Hongo-Stephanov-Yee (25)]
- Too low energy??
  - To see photon enhancement, typically $\omega \sim \gamma_\eta/\xi^2 \sim 2\mathrm{MeV}$ for $\xi \sim 10\mathrm{fm}$



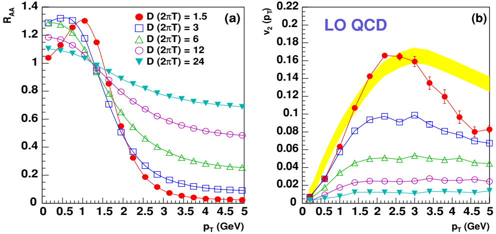
Heavy quarks?

Langevin equation for Brownian motion

$$\dot{\boldsymbol{p}} = -\gamma \boldsymbol{p} + \boldsymbol{\xi}, \quad \langle \xi_i(t)\xi_j(0)\rangle = \kappa \delta_{ij}\delta(t)$$

$$\gamma = \frac{\kappa}{2MT}, \quad D = \frac{2T^2}{\kappa} \quad \text{(diffusion constant)}$$

- Observables are sensitive to heavy quark diffusion constant ($D \propto \kappa^{-1}$)



[Moore-Teaney (05)]

Is there any critical effects in κ?

Force correlator near the critical point

[Akamatsu-Asakawa, PRC109 (2024) L031901]

Coupling to critical fluctuations

Slow modes at the critical point

- Model B: critical modes $\delta\sigma \sim \delta n_B \sim \delta e$
- Model H: critical modes $\delta\sigma \sim \delta n_B \sim \delta e$ coupled with shear modes $\boldsymbol{g}_T$
- Heavy quark is also a slow d.o.f. due to its large mass

Coupling between a heavy quark and critical modes

- At finite μ_B, coupling $\delta\sigma\delta n_B$ is allowed
  Similarly, we model a heavy quark effective Lagrangian

$$\mathcal{L}_{\mathrm{kin}} = Q^\dagger \left(i\partial_t + \frac{\boldsymbol{\nabla}^2}{2M} \right) Q, \quad \mathcal{L}_{\mathrm{int}} = -\lambda \delta\sigma n_Q = -\lambda \delta\sigma Q^\dagger Q$$

- Force acting on a heavy quark

$$\boldsymbol{P}_Q = \int_x Q^\dagger \frac{\boldsymbol{\nabla}}{i} Q, \quad \frac{d\boldsymbol{P}_Q}{dt} = -\int_x \boldsymbol{\nabla}\delta\sigma \cdot (\lambda Q^\dagger Q)$$

Heavy quark momentum diffusion constant by force correlator

Definition of κ by matching Langevin equation to QCD

$$\text{(Langevin)} \quad \kappa\delta_{ij} = \int dt \langle \xi_i(t)\xi_j(0)\rangle \xrightarrow{M\to\infty} \int dt \underbrace{\left\langle \frac{dp_i}{dt}(t)\frac{dp_j}{dt}(0)\right\rangle}_{\text{force correlator at } M\to\infty}$$

$$\text{(QCD)} \quad \kappa\delta_{ij} = \int dt \underbrace{\langle \mathrm{Tr_c}\, W(-\infty,t)E_i(t)\, W(t,0)E_j(0)\, W(0,-\infty)\rangle}_{\text{at } M\to\infty\text{, Lorentz force = electric force}}$$

- Near the QCD critical point

$$\text{(CP)} \quad \kappa\delta_{ij} = \int dt \left\langle \int_x \partial_i\delta\sigma\cdot(\lambda Q^\dagger Q)(t) \int_x \partial_j\delta\sigma\cdot(\lambda Q^\dagger Q)(0)\right\rangle_{N_Q=1}$$

Force correlator near the critical point

Weak coupling case, at order $\mathcal{O}(\lambda^2)$ (= neglecting the medium back reaction)

$$\kappa = -\frac{1}{3}\boldsymbol{\nabla}^2\lambda^2 \int dt\, \langle \delta\sigma(t,x)\delta\sigma(0,0)\rangle\Big|_{x=0} + \mathcal{O}(\lambda^3)$$
$$= \frac{1}{3}\lambda^2 \int d^d k k^2 \int dt\, \underbrace{\langle \delta\tilde{\sigma}(t,k)\delta\tilde{\sigma}(0,-k)\rangle}_{\text{decay with } \tau_\sigma(k)} \sim \lambda^2 \int d^d k k^2 \underbrace{\chi_\sigma(k)}_{\text{amplitude}} \tau_\sigma(k)$$

- In the scaling regime $1/\xi \ll k \ll 1/\ell_o$:

$$\chi_\sigma(k) \sim k^{-2+\eta}, \quad \tau_\sigma(k) \sim k^{-z} \quad \to \quad \kappa \sim \lambda^2 \int_{1/\xi}^{1/\ell_o} dk k^{d-1+\eta-z} \sim \lambda^2 \xi^{z-d-\eta}$$

- Model B ($z \approx 4$): $\kappa \sim \lambda^2 \xi$ → critical enhancement!
- Model H ($z \approx 3$): $\kappa \sim \lambda^2 \xi^0$ → difficult to observe, how about higher order in λ?

Heavy quark width near the critical point

Width from the heavy quark self-energy at weak coupling

$$\frac{\Gamma}{2} = \mathrm{Im}\left[\lambda^2 \int dt\, \langle \delta\sigma(t,0)\delta\sigma(0,0)\rangle\right] \sim \lambda^2 \xi^{z+2-d-\eta}$$

- Model B ($z \approx 4$): $\Gamma \sim \lambda^2 \xi^3 \rightarrow$ critical enhancement!
- Model H ($z \approx 3$): $\Gamma \sim \lambda^2 \xi^2 \rightarrow$ critical enhancement!?

Heavy quarks are strongly coupled with critical fluctuations $\rightarrow$ consistent with the result of κ?

Summary and outlook

Is there any critical behavior in heavy quark dynamics?

Critical scaling of heavy quark momentum diffusion constant

$$\kappa \propto \lambda^2 \xi^{z-d-\eta} + \mathcal{O}(\lambda^3)$$

- Depends on the critical dynamics:
  model B ($z \approx 4 \to \kappa \propto \lambda^2 \xi$) and model H ($z \approx 3 \to \kappa \propto \lambda^2 \xi^0$)
- Width $\Gamma \propto \lambda^2 \xi^{z+2-d-\eta} \to$ suggests strong coupling in model H

Outlook

- Higher-order expansion in λ needs to be analyzed? [Akamatsu-Asakawa-Sogabe, in progress]
- Schwinger-Keldysh EFT with heavy quarks and soft modes?
- Experimental test for impurity-critical coupling
  $\to$ Polaron in cold atoms at the superfluid transition [Lau et al, in progress]