Hydrodynamic models
(specific to RHIC BES/FXT)

”life beyond net proton/baryon cumulants”

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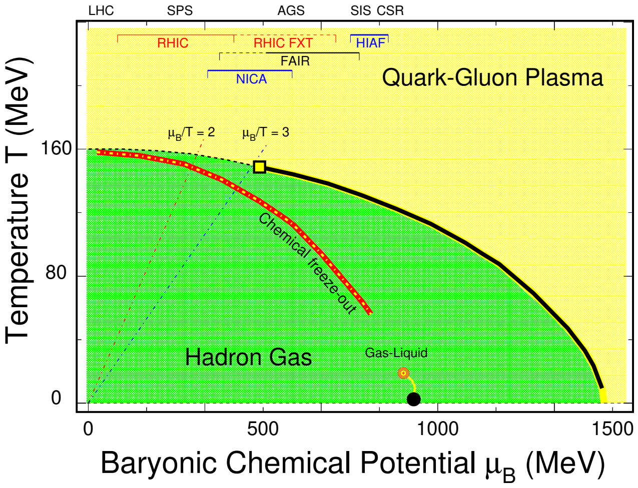
QCD Critical Point and Hydrodynamic Evolution, Yukawa Institute for Theoretical Physics, 1-4 June 2026

What do we want to achieve with fluid-dynamic modelling ?

Goal #1

Heavy-ion collisions <==> simulations of medium dynamics



<==>

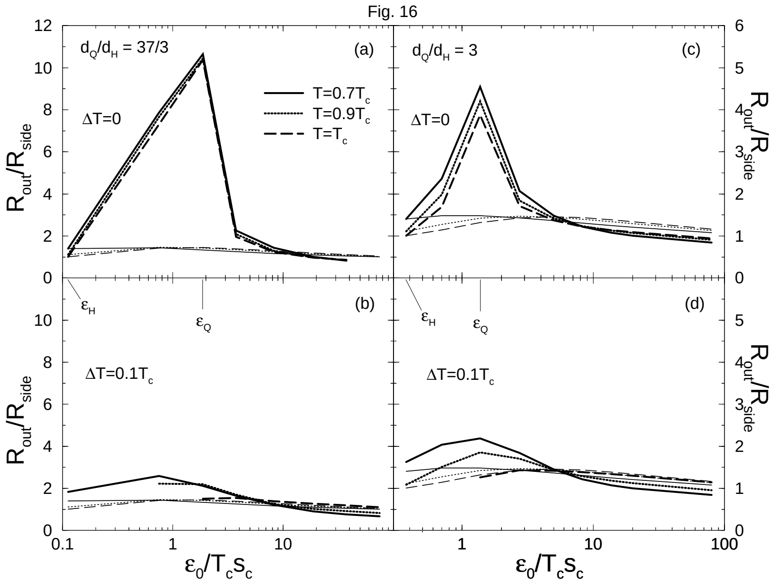
Look at the observables **beyond net proton cumulants**:

- Shape of net proton distribution
- Flow
- HBT
- …

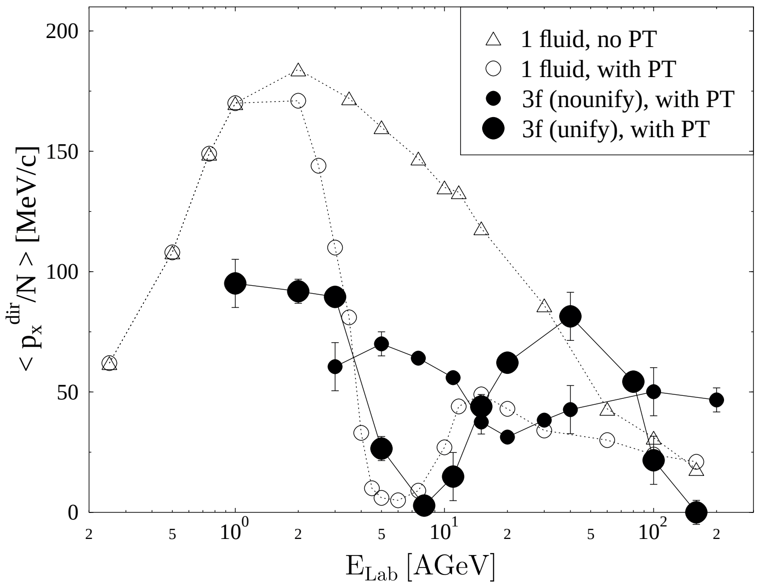
Old phase transition signatures

Not criticality but softening of the EoS

HBT (time-delay signature of QGP formation)
Nucl.Phys.A 608, 479 (1996)

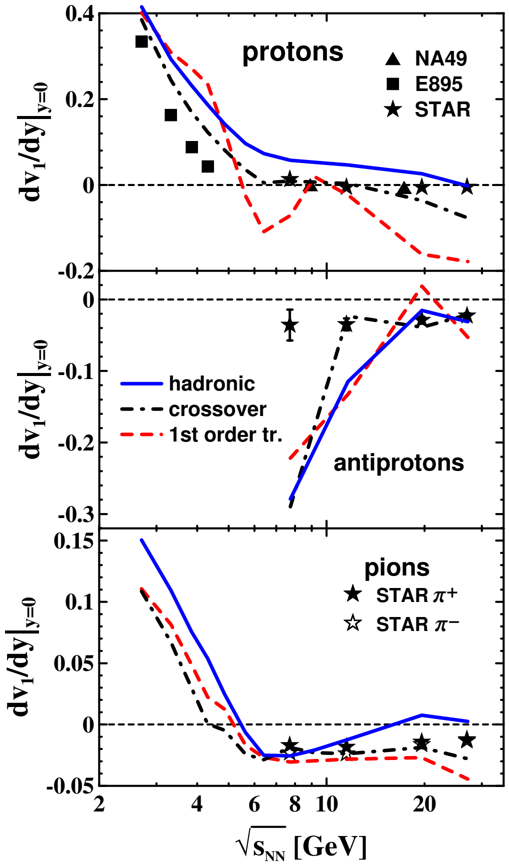


Directed Flow: Brachmann, et al,
Phys.Rev. C61 (2000) 024909



… do not hold water anymore, but the idea of a smoking gun signature is still tempting

A more recent motivation for directed flow observable



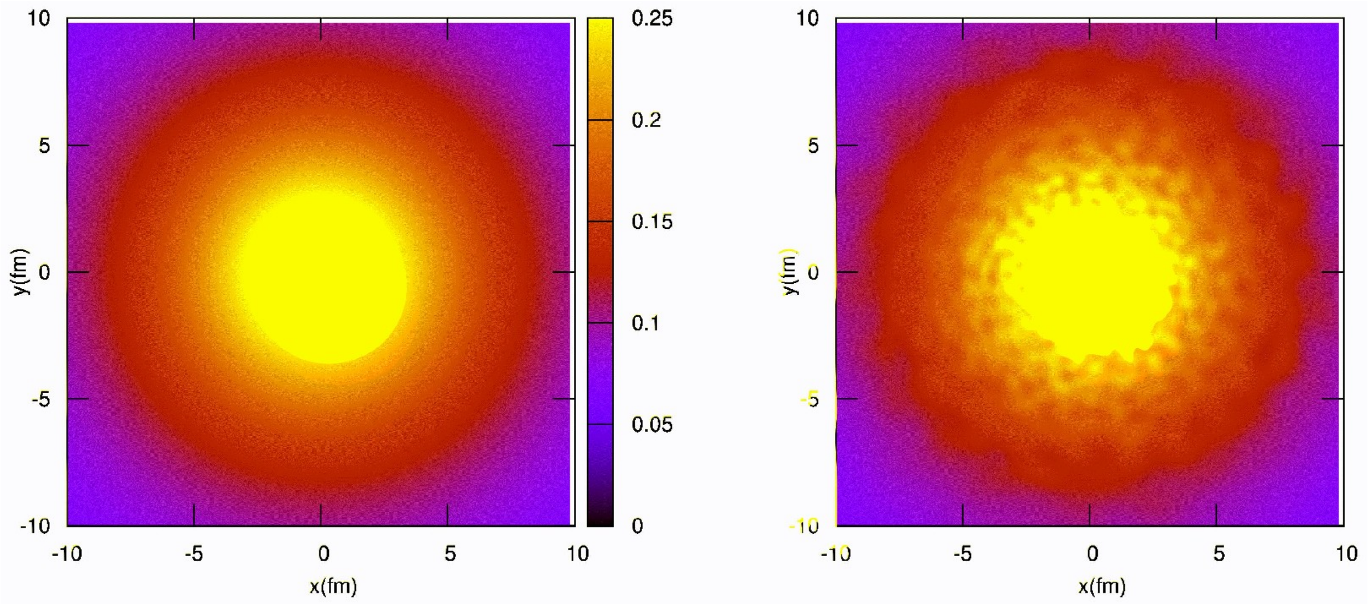
Ivanov, Soldatov, Phys. Rev. C 91, 024915 (2015)

Simulations in 3-fluid dynamics

Strong EoS dependence,
Crossover EoS preferred

Goal #2

Embed stochastic noise, or non-hydro modes, into realistic 3D expansion



And hopefully see the effects of enhanced critical fluctuations

Model construction

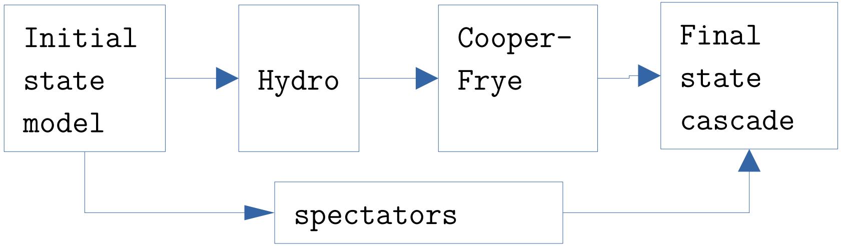
Non-critical, vanilla hydro

The recipe is seemingly simple:

- 3D initial state: from transport or parametrized
  Start hydro later (nuclei are not thin pancakes)
- 3D hydro with finite viscosity (not a challenge)
  and conserved charges (not a challenge either)
- EoS at finite μ_B – an input to the modelling
- Final state (particlization):
  same or slightly adapted Cooper-Frye
- final-state hadronic cascade (SMASH, UrQMD)

Modelling is complex

- We end up having a multi-step simulation:



- 3 to 4 different codes stacked one after another to simulate different stages
- Need to take care of every interface

Successful examples:

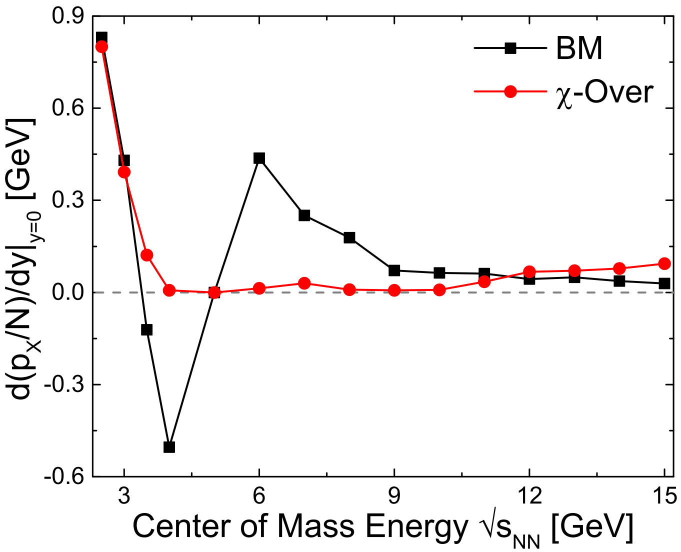
[SMASH] + [vHLLE] + [smash-hadron-sampler] + [SMASH]

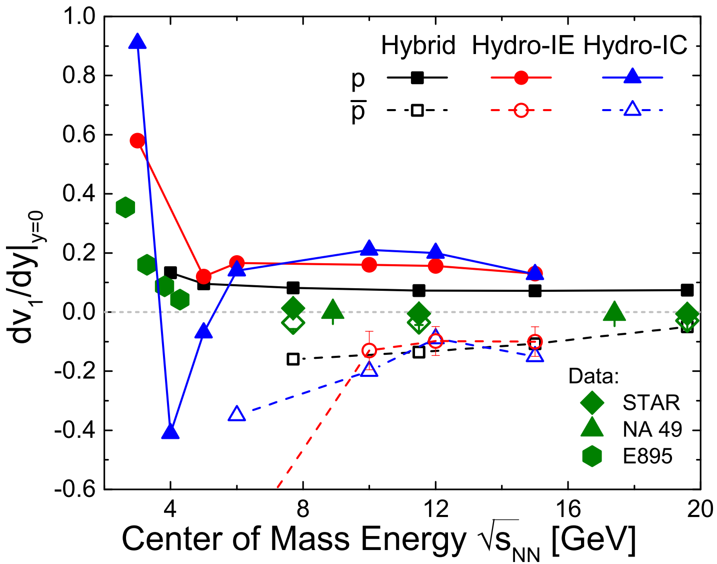
[superMC] + [MUSIC] + [ISS] + [UrQMD]

nuclear conficionery

Why the need for multi-step modeling?

J. Steinheimer, J. Auvinen, H. Petersen, M. Bleicher, H. Stöcker, Phys. Rev. C 89 (2014) 054913

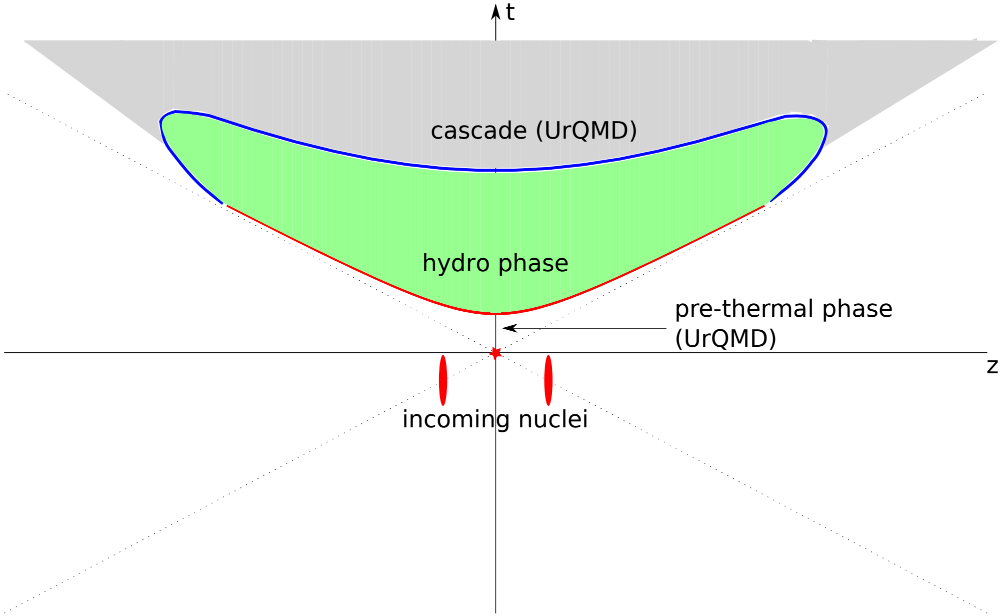




An oversimplified simulation where v1 is sensitive to PT in the EoS

State-of-the-art simulation where v1 is sensitive to freezeout and also the EoS

The first interface: initial state to hydro



The easy choice:

- start fluid-dynamic picture at $\tau=\tau_0$
- assume some transverse profile (MC Glauber, TrENTO, IP-Glasma)
- superimpose longitudinal profile (or not)

Motivated at **high** collision energies:

- Longitudinal boost invariance

  $\longrightarrow$ keep calm and run 2+1D hydro

- Physics changes as a function of τ

Strategies to construct initial state for lower energies

1. Parametrize the initial state

2. Run transport and melt products at fixed $\tau = \tau_0$

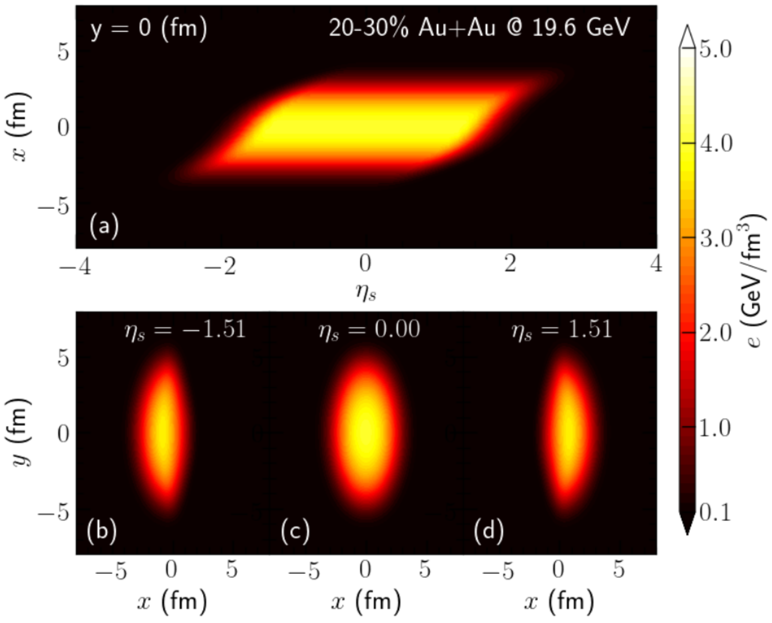
3. Run transport and melt dynamically (dynamic fluidization)

4. Assume everything is a fluid from the start

1. Parametrized 3D initial state at τ=τ$_0$

superMC code (?)

$$e(x,y,\eta_s;y_{\rm CM}) = \mathcal{N}_e(x,y)\times\exp\left[-\frac{(|\eta_s-y_{\rm CM}|-\eta_0)^2}{2\sigma_\eta^2}\theta(|\eta_s-y_{\rm CM}|-\eta_0)\right]$$



$\mathcal{N}_e(x,y)$ determined from the full energy count

Parameter tuning (sqrt(s) - dependent):

$\sqrt{s_{\rm NN}}$ (GeV)	τ_0 (fm/c)	η_0	σ_η	$\eta_{B,0}$	$\sigma_{B,\rm in}$	$\sigma_{B,\rm ou}$
AuAu & dAu @ 200	1.0	2.5	0.6	3.5	2.0	0.1
AuAu & dAu @ 62.4	1.0	2.25	0.3	2.7	1.9	0.2
AuAu & dAu @ 39	1.3	1.9	0.3	2.2	1.6	0.2
AuAu@27	1.4	1.6	0.3	1.8	1.5	0.2
AuAu & dAu @ 19.6	1.8	1.3	0.3	1.5	1.2	0.2
AuAu@14.5	2.2	1.15	0.3	1.4	1.15	0.2
AuAu@7.7	3.6	0.9	0.2	1.05	1.0	0.1
PbPb@17.3	1.8	1.25	0.3	1.6	1.2	0.2
PbPb@8.77	3.5	0.95	0.2	1.2	1.0	0.1

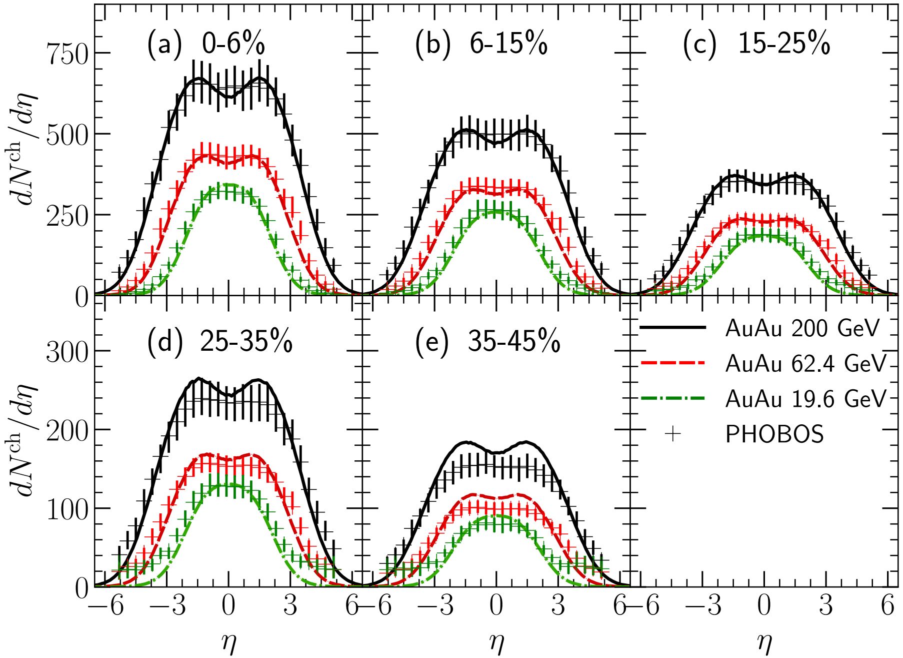
Shen, Alzhrani, Phys.Rev.C 102 (2020) 1, 014909

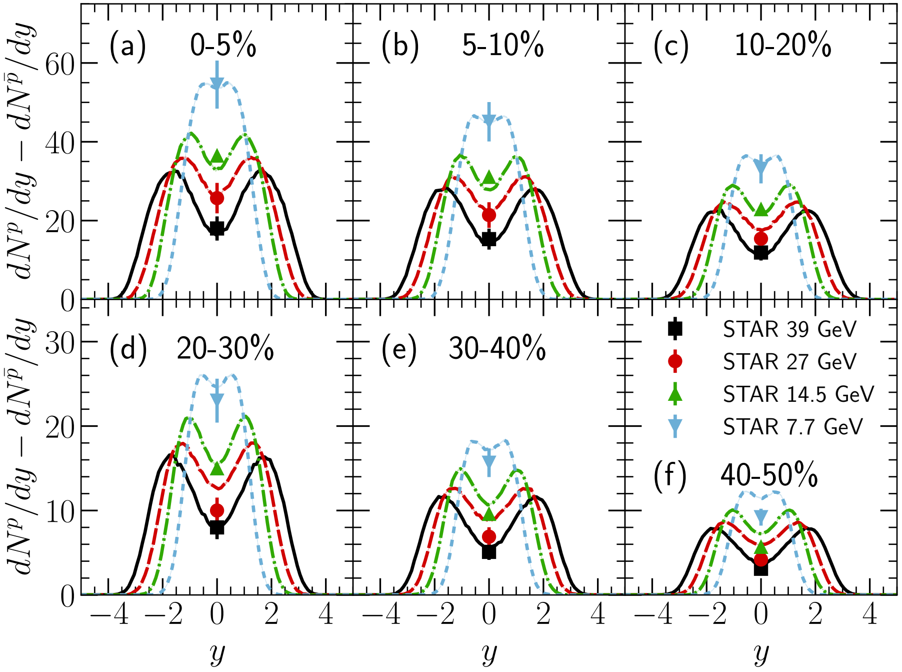
+ follow-up by Lipei du et al

One can reproduce the data, at the expense of highly parametrized initial state

Data reproduction with parametrized initial state

For more plots: Shen, Alzhrani, [Phys.Rev.C 102 (2020) 1, 014909]

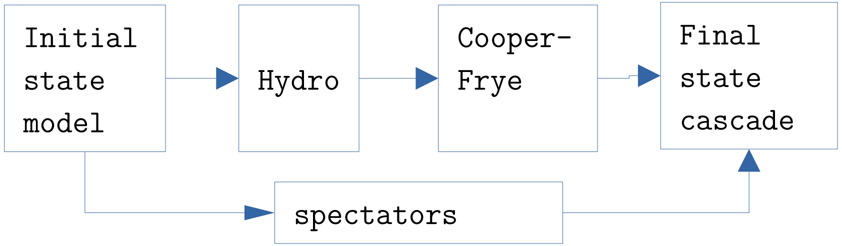


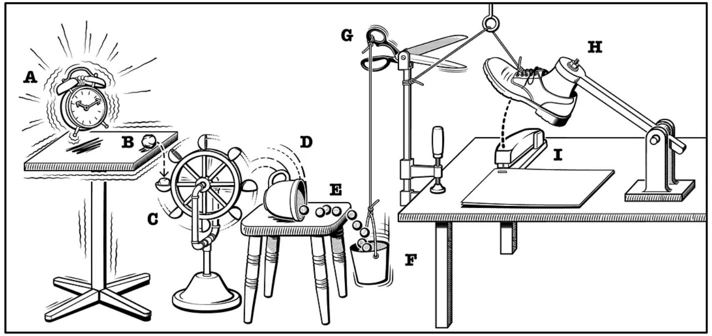


3D initial state at $\tau=\tau_0$ from transport

- hopefully fewer parameters as initial state dynamics dictates the shape
- there is dynamics before fluid phase starts => better physics

(UrQMD, SMASH, EPOS, JAM)





Karpenko, Huovinen, Petersen, Bleicher, Phys.Rev.C 91 (2015) 6, 064901

- Patch a transport code to stop evolution or disable interactions after after $\tau=\tau_0$
- Collect hadrons at $\tau=\tau_0$
- Assume that hadrons melt into fluid:

$$\Delta P^{\alpha}_{ijk} = P^{\alpha} \cdot C \cdot \exp\left(-(\Delta x_i^2 + \Delta y_j^2)/R_\perp^2 - \Delta\eta_k^2 \gamma_\eta^2 \tau_0^2 / R_\eta^2\right)$$
$$\Delta N^{0}_{ijk} = N^{0} \cdot C \cdot \exp\left(-(\Delta x_i^2 + \Delta y_j^2)/R_\perp^2 - \Delta\eta_k^2 \gamma_\eta^2 \tau_0^2 / R_\eta^2\right)$$

$$E/\Delta V = (\epsilon + p)(u^0)^2 - p, \quad P^i/\Delta V = (\epsilon + p) u^0 u^i$$

- 3D initial state with finite n_B, n_Q, n_S

Does the medium look like a fluid when it is fluidized?

Inghirami, Elfner, Eur.Phys.J. C 82 (2022) 9, 796

A study from Elab=1.23 GeV to sqrt(s)=7.7 GeV

Energy-momentum tensor from transport (SMASH):

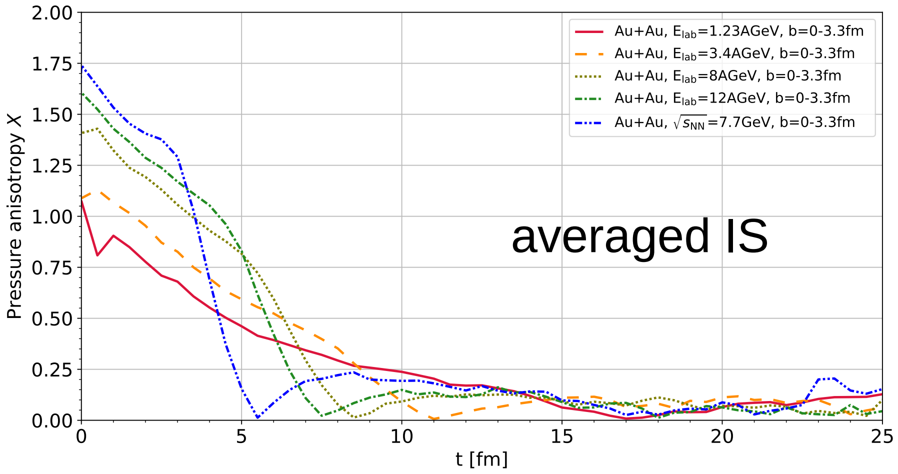
$$T^{\mu\nu} = \sum_i \frac{p_i^\mu p_i^\nu}{p_i^0} K(\mathbf{r} - \mathbf{r}_i, \mathbf{p_i})$$

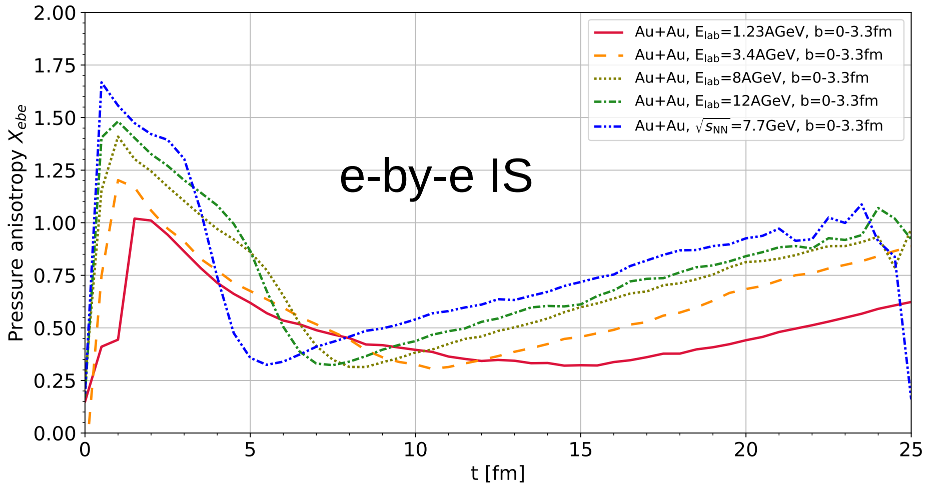
Anizotropy measures:

$$X \equiv \frac{|\langle T_L^{11}\rangle - \langle T_L^{22}\rangle| + |\langle T_L^{22}\rangle - \langle T_L^{33}\rangle| + |\langle T_L^{33}\rangle - \langle T_L^{11}\rangle|}{\langle T_L^{11}\rangle + \langle T_L^{22}\rangle + \langle T_L^{33}\rangle}$$

$$Y \equiv \frac{3(|\langle T_L^{12}\rangle| + |\langle T_L^{23}\rangle| + |\langle T_L^{13}\rangle|)}{\langle T_L^{11}\rangle + \langle T_L^{22}\rangle + \langle T_L^{33}\rangle}$$

"OK to switch to viscous hydro when X~0.3"

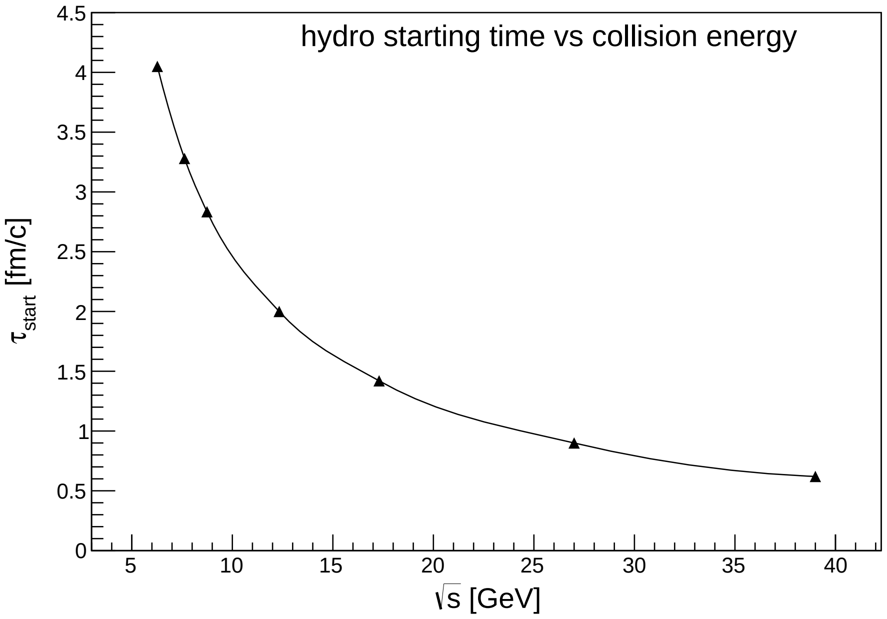




At low energies [sqrt(s)<10 GeV], it takes time for the nuclei to pass through each other.

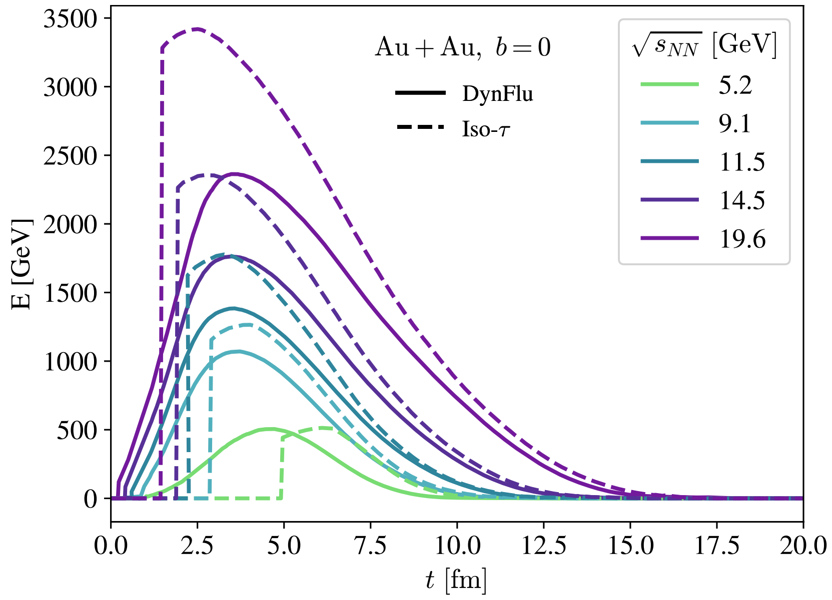
Initial state dynamics can take almost as much time as the fluid phase.

$$\tau_0 = \frac{2R}{\gamma v_z}$$



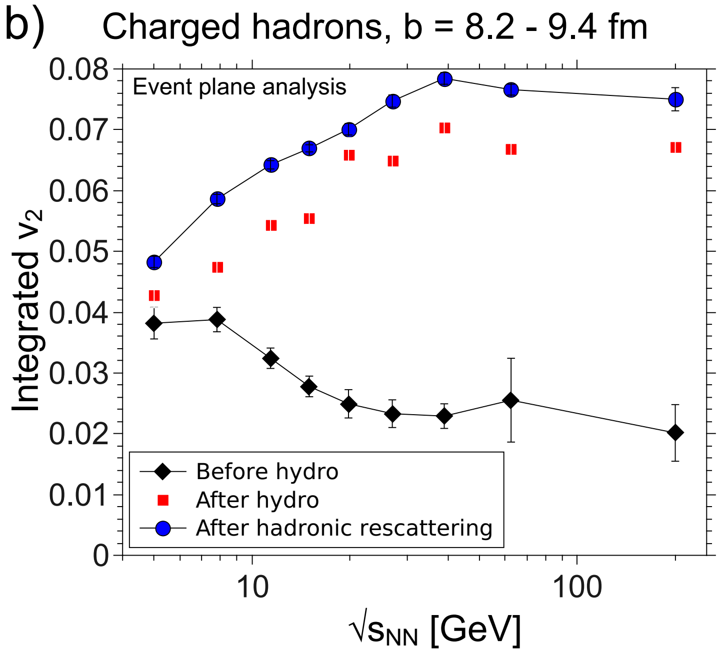
At low energies [sqrt(s)<10 GeV], one has to start fluid description early

Góes-Hirayama, Egger, Paulínyová, IK, Elfner, [2507.19389](2507.19389)



Different time-evolution of energy density if it starts early

Auvinen, Petersen(Elfner), PRC 88, 064908 (2013)



A lot of flow is produced by initial stage if hydro description starts late

A better physics-motivated switching: dynamical fluidization

The idea: once the energy density in LRF is high enough **locally**, fluidize this part.

Initial state SMASH

$$j^\mu(\mathbf{r}) = \sum_i p^\mu K\left(\mathbf{r} - \mathbf{r}_i, \frac{\mathbf{p}_i}{m_i}\right)$$

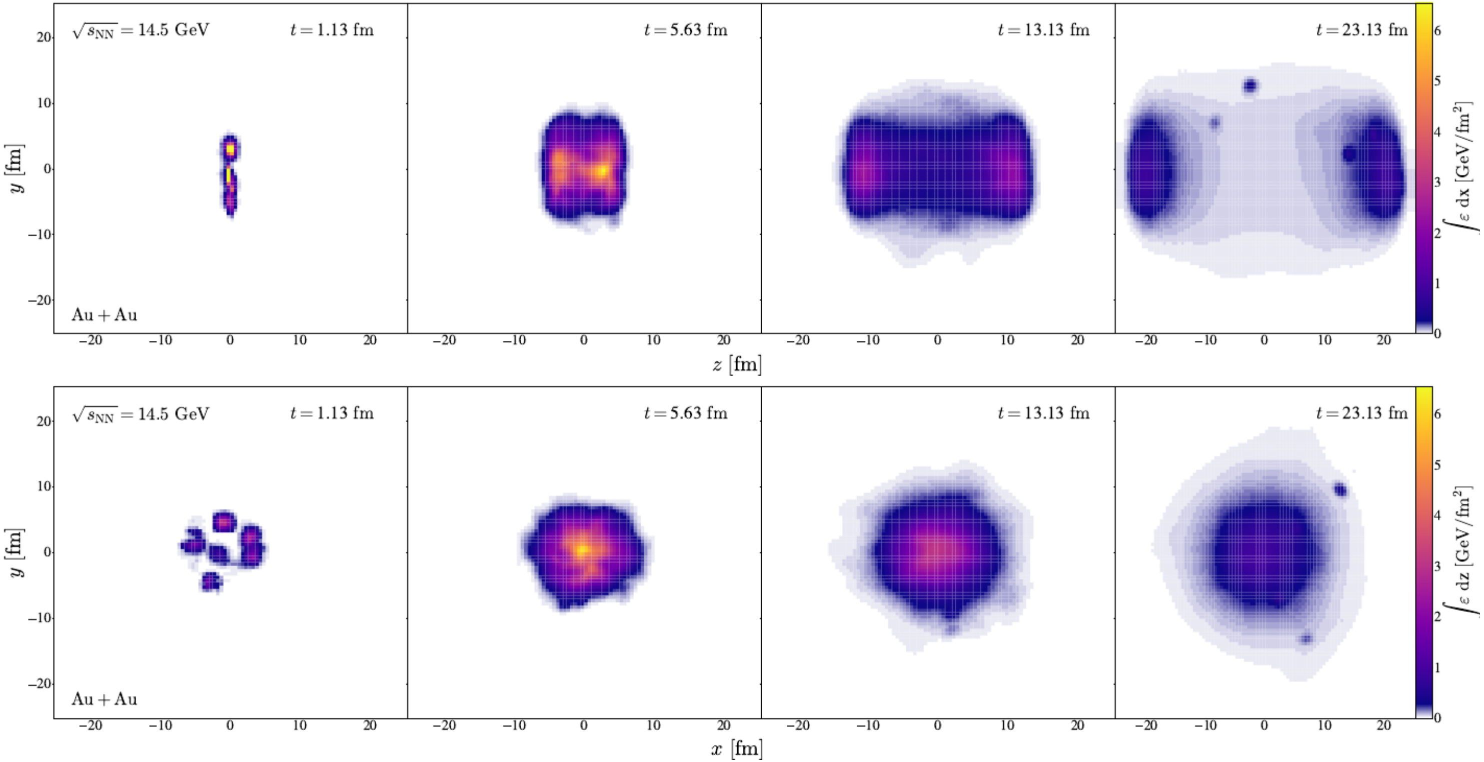
$$j_C(\mathbf{r}) = \sum_i C_i K\left(\mathbf{r} - \mathbf{r}_i, \frac{\mathbf{p}_i}{m_i}\right)$$

Hydro with source feeding energy-momentum and charges:

$$\partial_\nu T^{\mu\nu} = j^\mu(\mathbf{r}),$$

$$\partial_\nu N^\nu_C = j_C(\mathbf{r}),$$

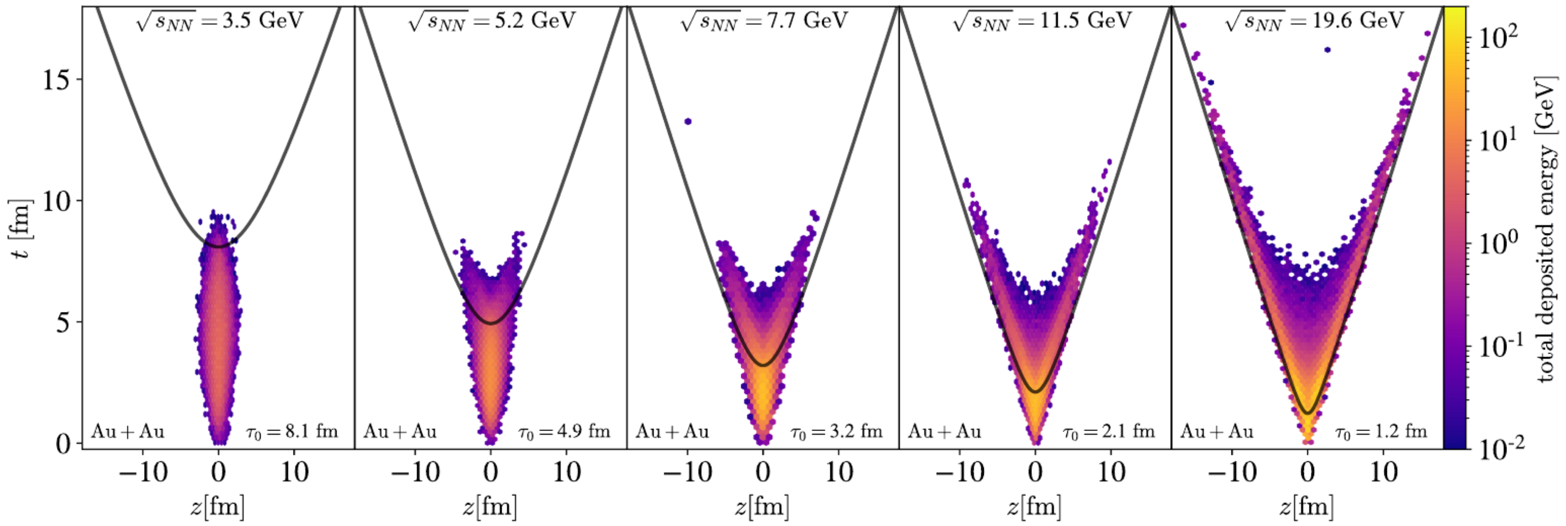
+remove the particles from cascade



Góes-Hirayama, Egger, Paulínyová, Karpenko, Elfner, 2507.19389

As a result, the simulation does not wait for the two nuclei to completely pass through each other, which is a long time at low √s

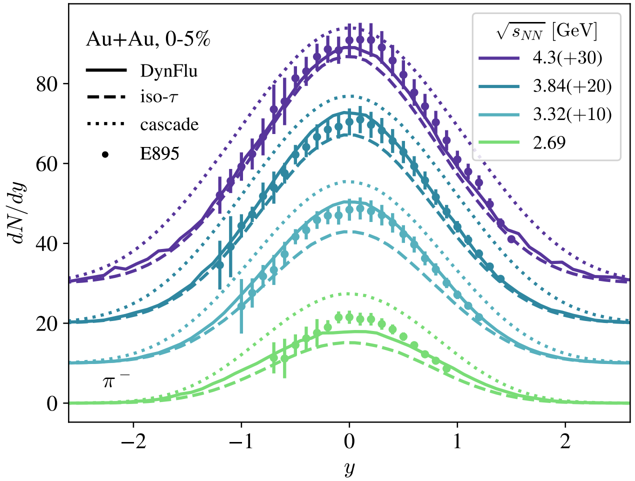
Fluid approximation still applies for the most dense part of the evolution

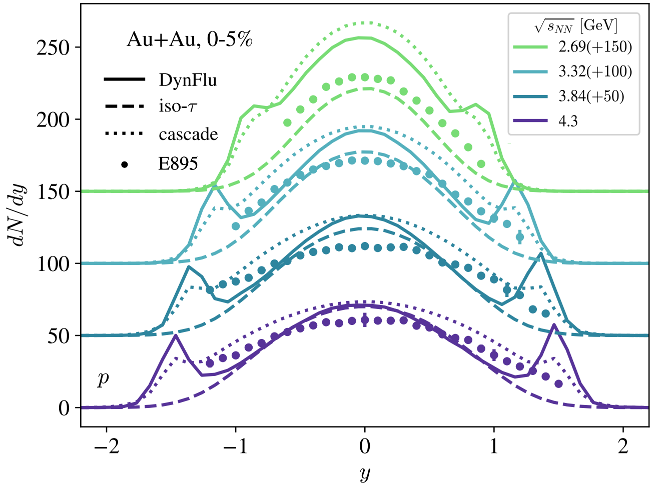


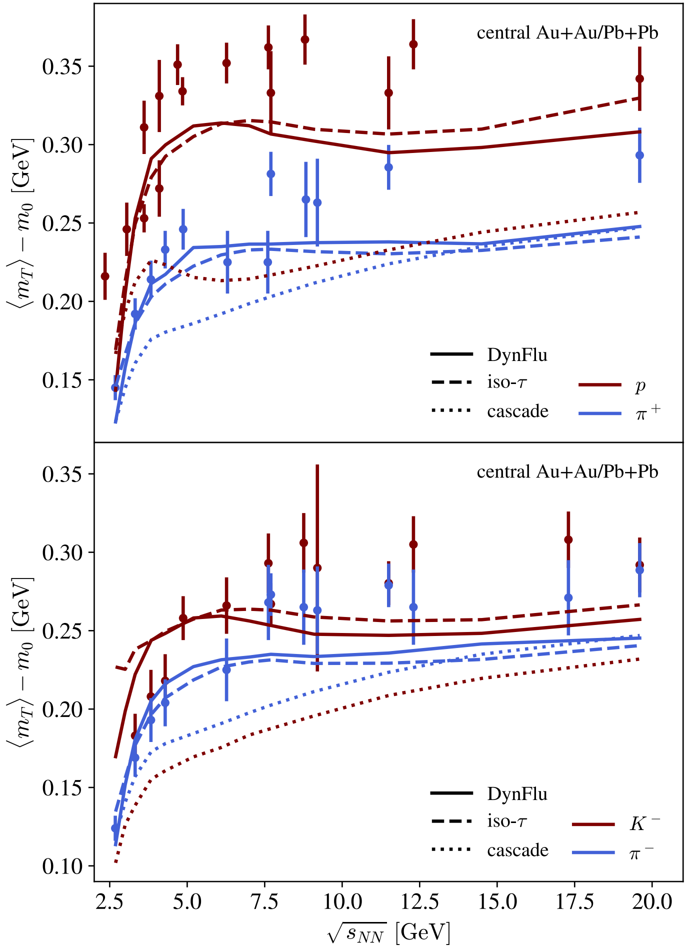
Góes-Hirayama, Egger, Paulínyová, Karpenko, Elfner, 2507.19389

dynamical fluidization vs. experimental data

Góes-Hirayama, Egger, Paulínyová, Karpenko, Elfner,
[2507.19389](2507.19389)







JAM + dynamical fluidisation + hydro + JAM

Y. Akamatsu, M. Asakawa, T. Hirano, M. Kitazawa, K. Morita, K. Murase, Y. Nara, C. Nonaka, A. Ohnishi, Phys. Rev. C 98, 024909 (2018)

JAM IS: HIJING string excitation + PYTHIA6 fragmentation + rescatterings of produced hadrons.

Hadrons are converted to fluid if the local energy density $e > e_f = 0.5$ GeV/fm^3.

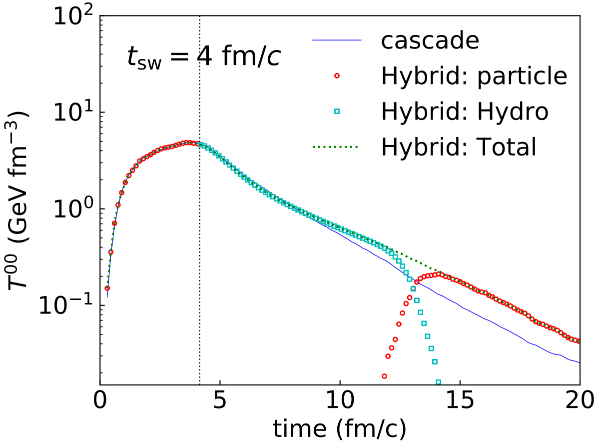
$$\partial_\mu T_f^{\mu\nu} = J^\nu, \quad \partial_\mu N_f^\mu = \rho$$

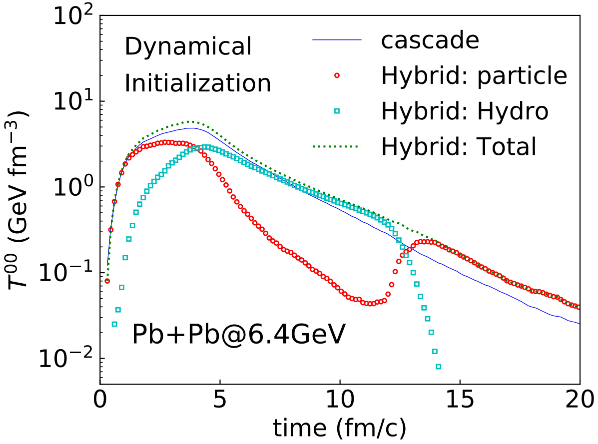
$$J^\mu(r) = \frac{1}{\Delta t}\sum_i p_i^\mu G(r - r_i(t))$$

$$\rho(r) = \frac{1}{\Delta t}\sum_i B_i G(r - r_i(t))$$

$G(r)$ is a Gaussian smearing profile.

EoS: EoS Q, a.k.a. BM+HRG EoS



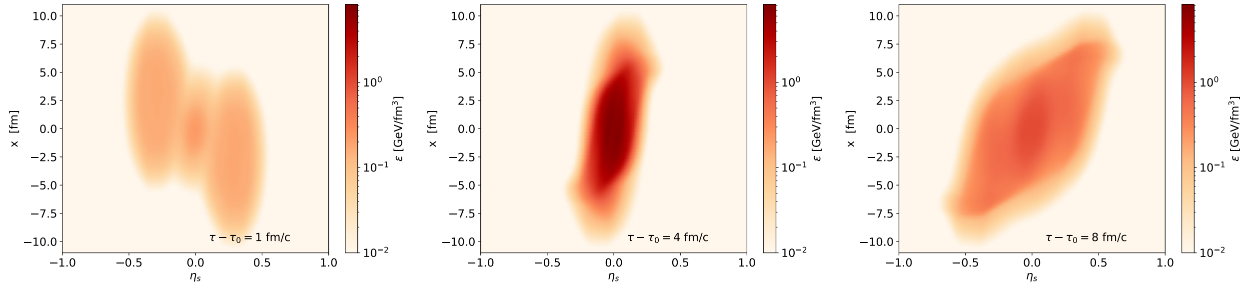


4. Skipping the first switching/interface altogether: *multi-fluid approach*

- Incoming nuclei = two fluids labelled as projectile and target
- Interaction of the fluids (slowing down) via "friction" terms
- Friction transports energy and momentum into the third fluid labelled as fireball
- It is a minimal setup to reproduce baryon stopping at low $\sqrt{s}$ and baryon transparency at high $\sqrt{s}$.

$$\partial_\mu T_{\rm p}^{\mu\nu}(x) = -F_{\rm p}^{\nu}(x) + F_{\rm fp}^{\nu}(x),$$
$$\partial_\mu T_{\rm t}^{\mu\nu}(x) = -F_{\rm t}^{\nu}(x) + F_{\rm ft}^{\nu}(x),$$
$$\partial_\mu T_{\rm f}^{\mu\nu}(x) = F_{\rm p}^{\nu}(x) + F_{\rm t}^{\nu}(x) - F_{\rm fp}^{\nu}(x) - F_{\rm ft}^{\nu}(x),$$

Cimerman, IK, Tomasik, Huovinen, Phys.Rev.C 107 (2023) 4, 4



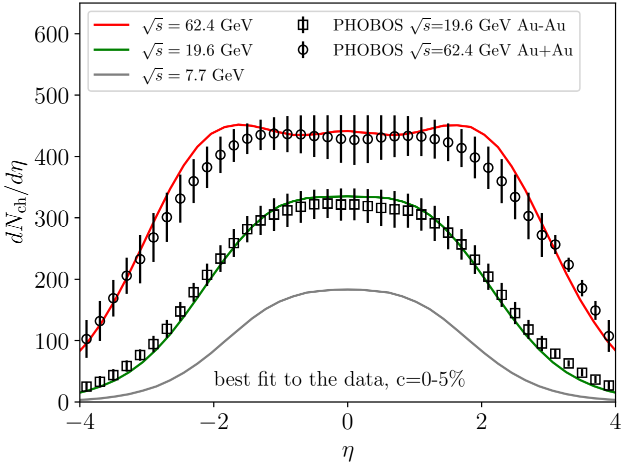
Werthmann, IK, Huovinen, Phys.Rev.C 113 (2026), 034908

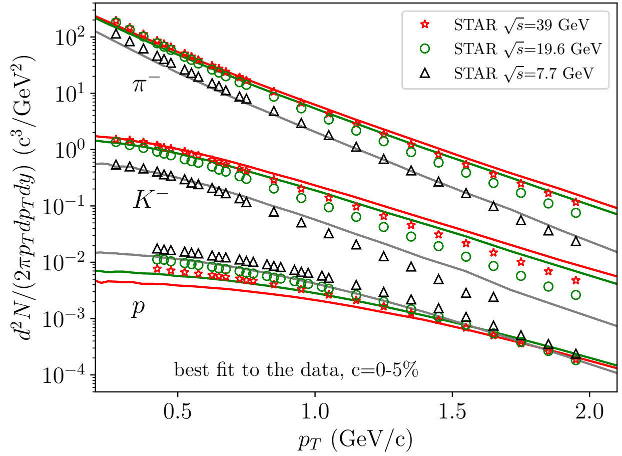
Reichert, Spieles, Bleicher

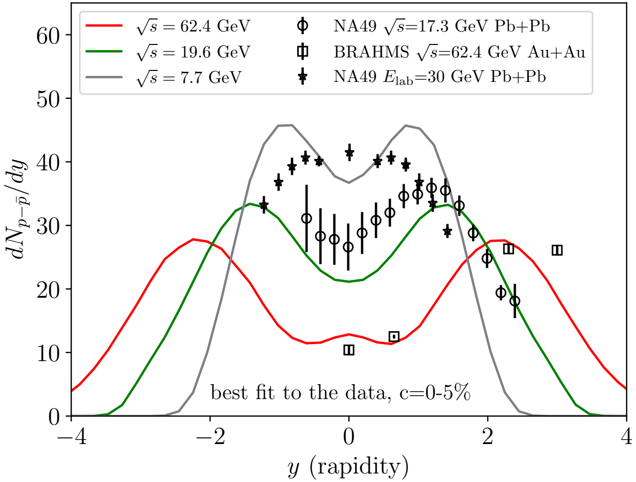
Snapshots of multi-fluid evolution in x-η_s plane, Au-Au collision at $\sqrt{s_{\rm NN}} = 7.7$ GeV

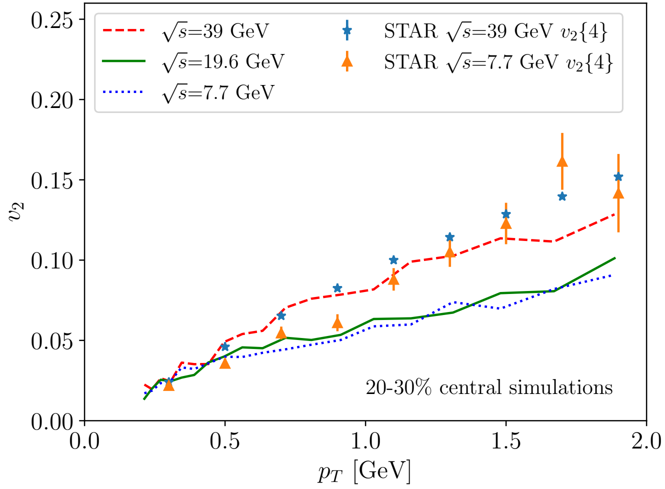
Multi-fluid hydro (MUFFIN) vs. experimental data

Werthmann, IK, Huovinen, Phys.Rev.C 113 (2026), 034908









Why not to run transport?

✅ natively 3D

✅ EoS can be emulated with potentials

✅ Exact energy-momentum, charge conservation

✅ no interfaces (to/from hydro)

Caveats:

⚠️ No change in the degrees of freedom (UrQMD, SMASH, HSD, JAM)

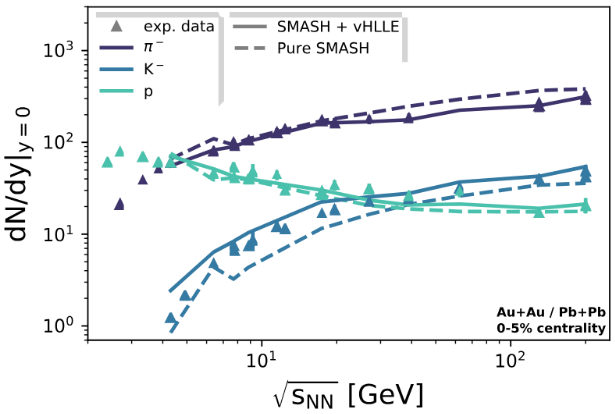
Unless: partonic phase + coalescence to hadrons (AMPT, PHSD, PHQMD(?))

⚠️ No good agreement with the data above sqrt(s)>10 GeV (except for JAM!)

(see one of next slides)

What does the modelling tell us

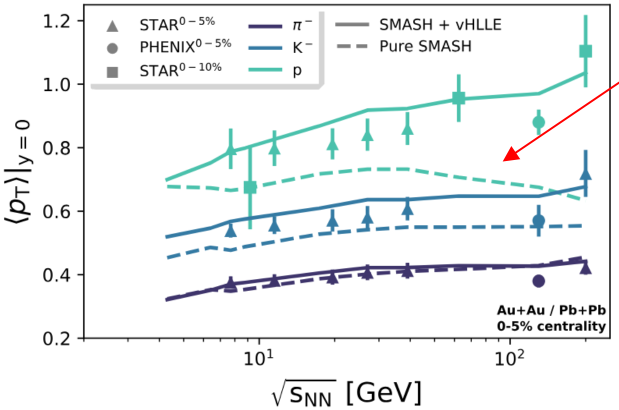
Aggregate conclusions from the hydro modelling

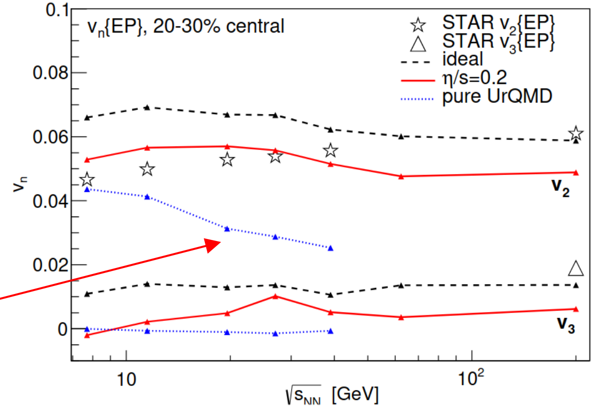


❗ Fluid phase improves agreement with the data; pure cascade fails to describe flow and mean pT above √s~7 GeV

Schäfer, IK, Wu, Hammelmann, Elfner,
Eur.Phys.J.A 58 (2022) 11, 230

IK, Huovinen, Petersen, Bleicher,
Phys.Rev.C 91 (2015) 6, 064901

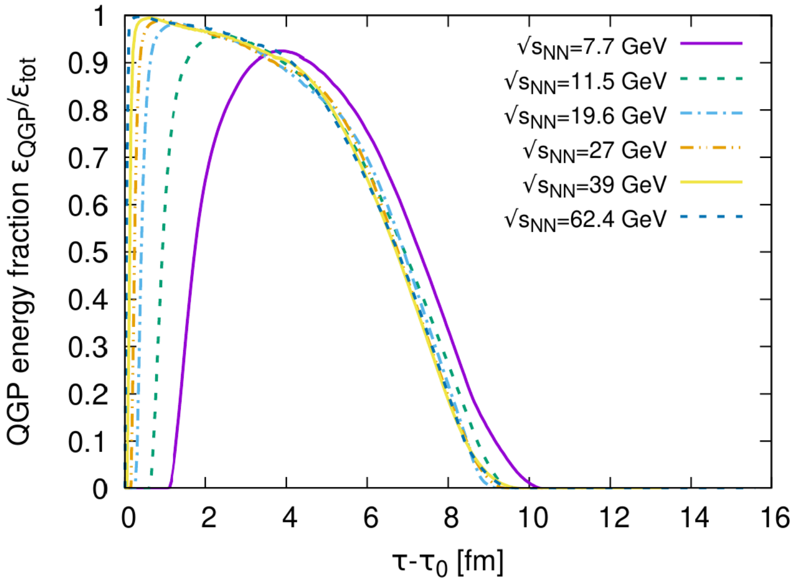




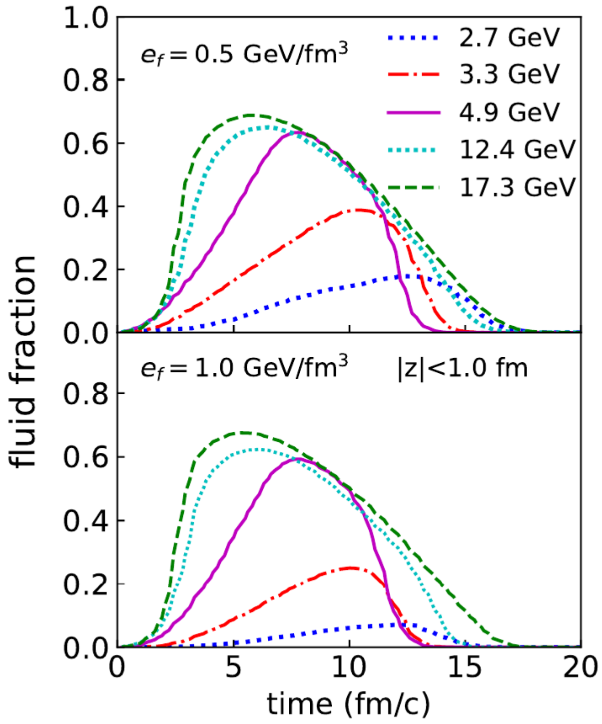
Rule of thumb:
√s>10 GeV
=> include hydro

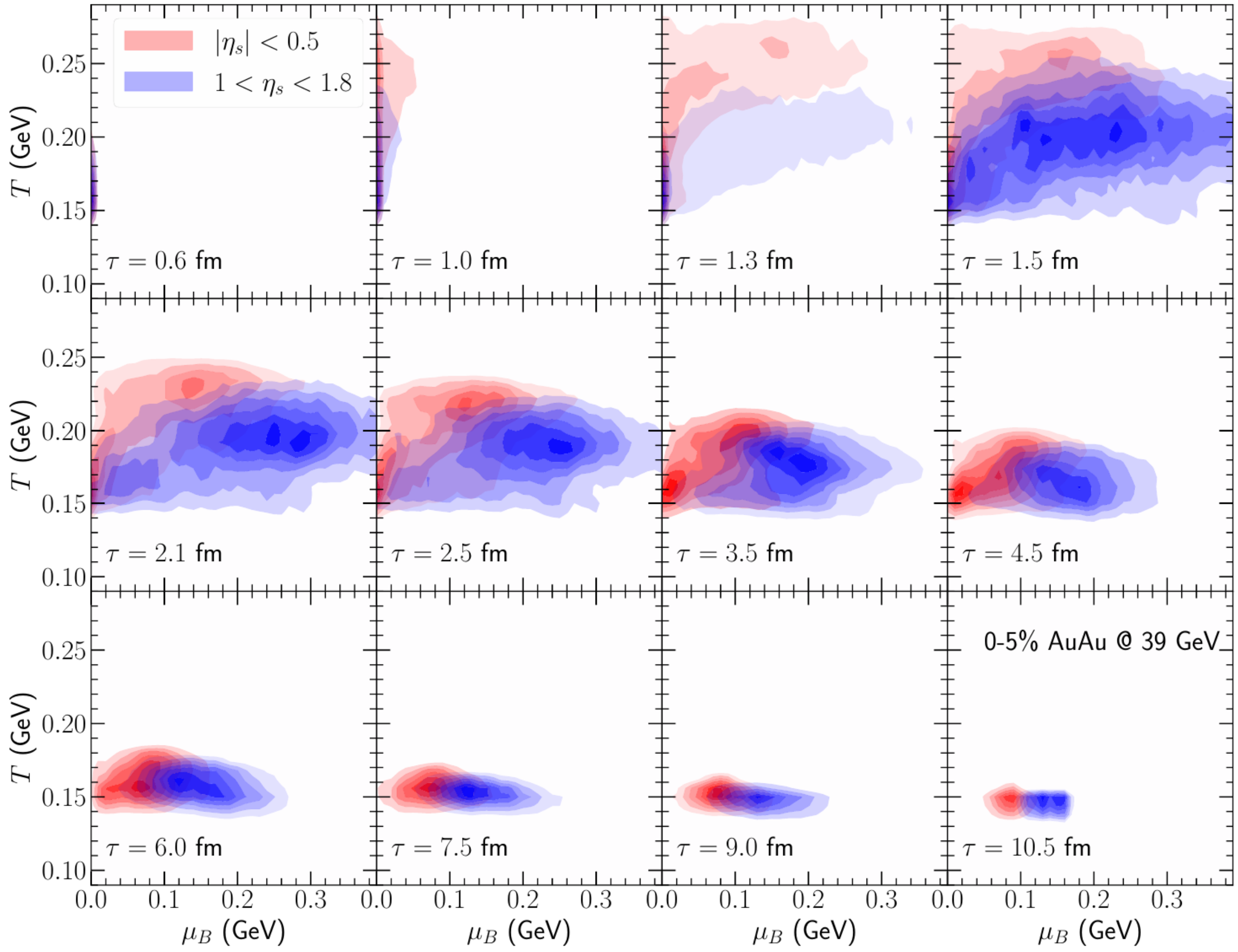
❗ High densities are reached at low collision energies

MUFFIN 1.0: Cimerman, IK, Tomasik, Huovinen, [Phys.Rev.C 107 (2023) 4]



JAM+hydro: Yasushi Nara et al





Evolution movies: https://smash-transport.github.io/movies-hybrid.html

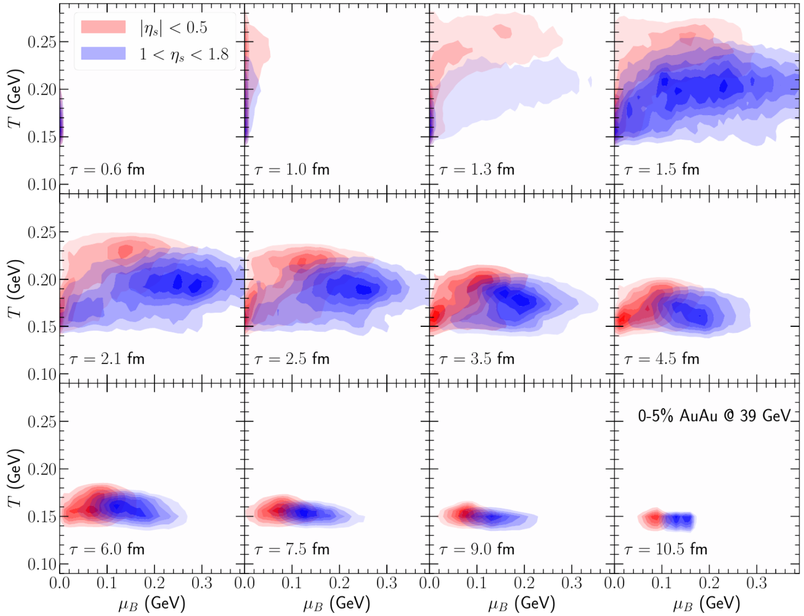
❗ The fireball is quite inhomogeneous

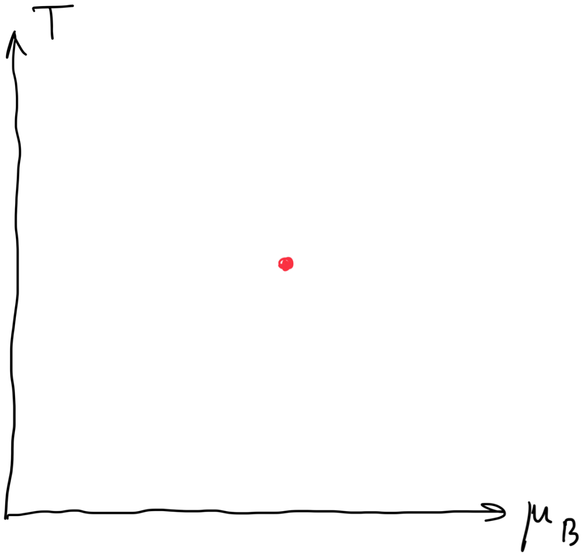
A √s point does not map to a (T, μ_B) point.

This plot is taken from Chun Shen, 2108.04987

Other hybrids show a similar picture

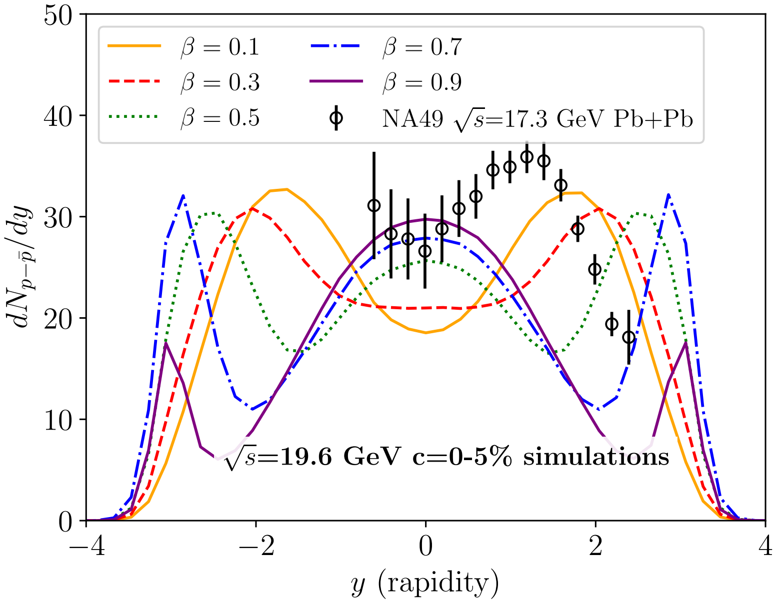
For particle number ratios, the whole system is integrated over, which results in one point at the phase diagram





plot is taken from: Chun Shen, 2108.04987

Baryon stopping/transparency is driven by initial state



MUFFIN (3-fluid hydro) simulations:

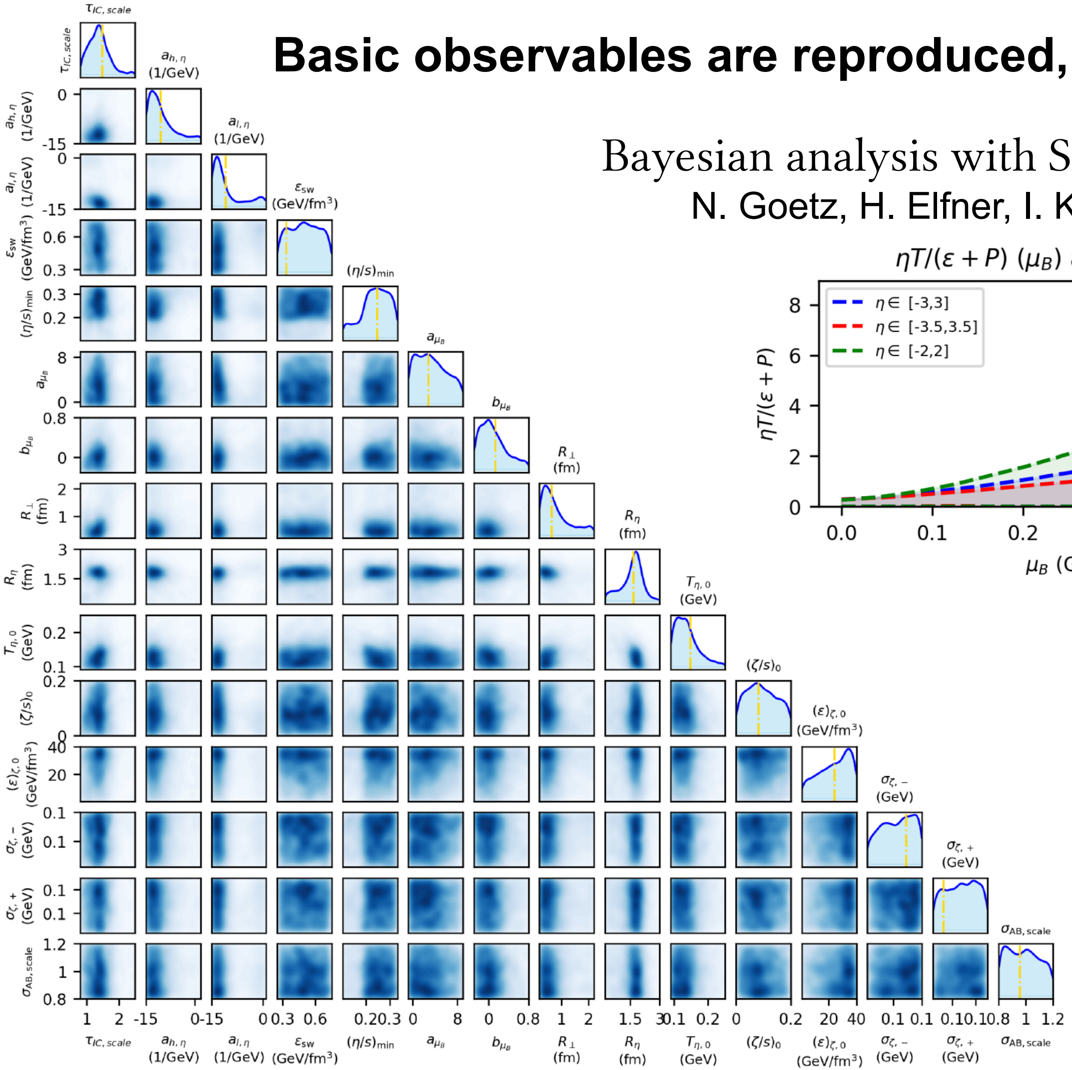
Werthmann, IK, Huovinen,
Phys.Rev.C 113 (2026), 034908

Can result in very different shapes of net proton rapidity distribution

β => relative rapidity cut for initial state nucleons to melt into fireball or p/t fluids

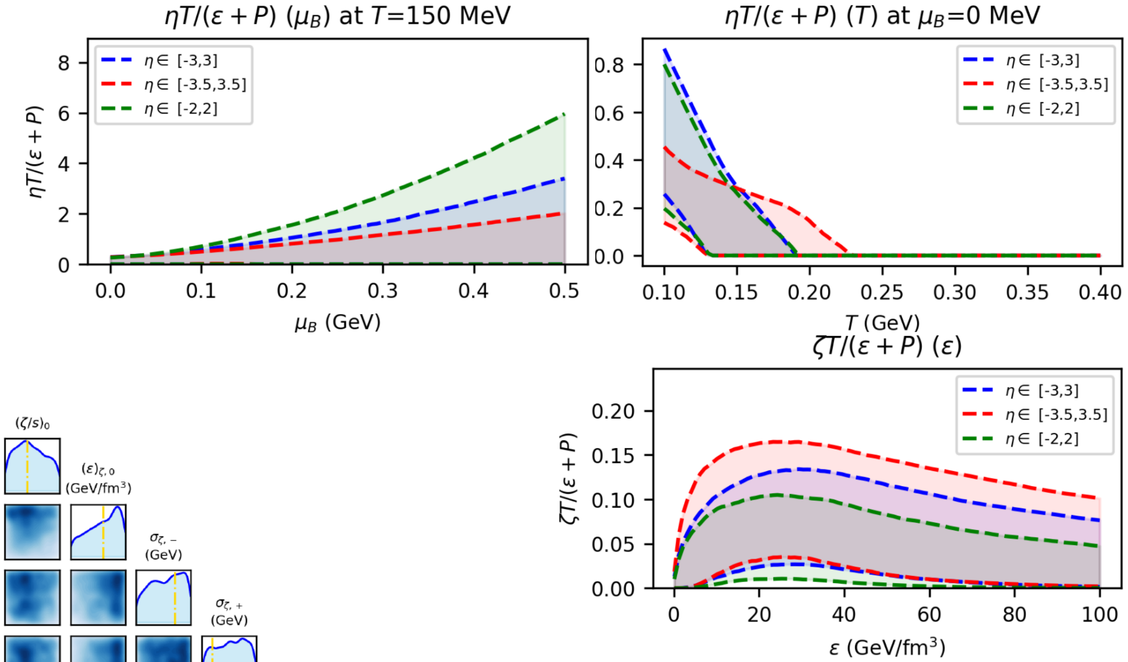
Bayesian analyses

Basic observables are reproduced, models constrained via BA



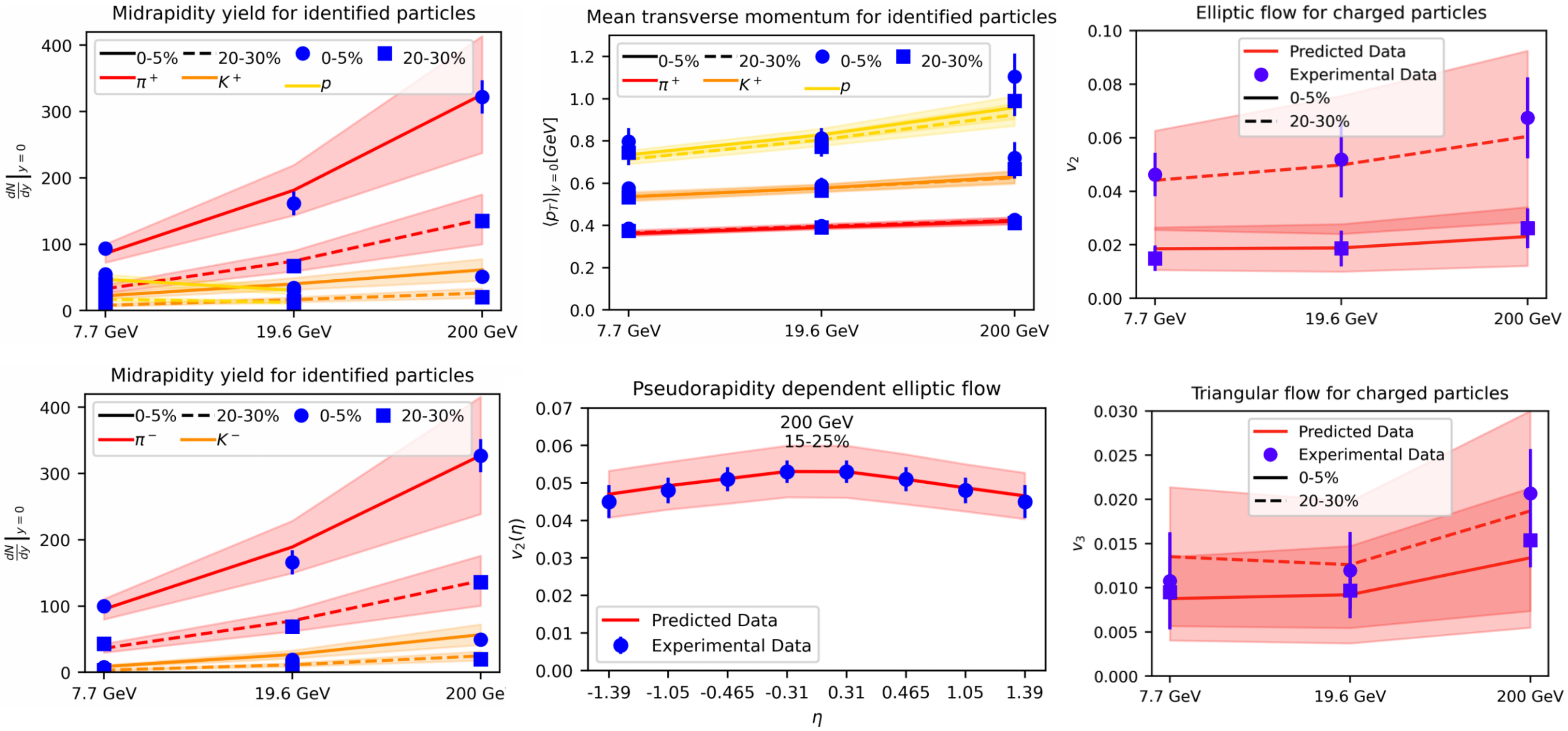
Bayesian analysis with SMASH+vHLLE, sqrt(s)=7.7…200 GeV

N. Goetz, H. Elfner, I. Karpenko, [Phys.Rev.C 112 (2025) 1, 014910]



Parameter space is large,
Bayesian analysis is challenging,
<u>exp. data is not always high quality</u>

What experimental data is reproduced in (being used for) the Bayesian analysis:

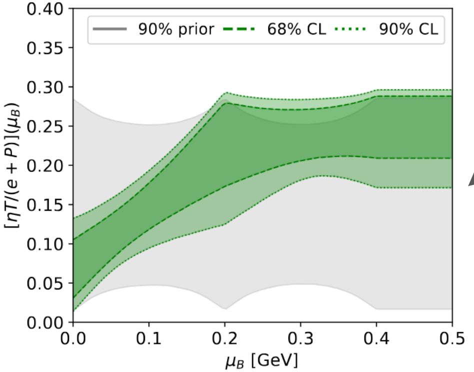


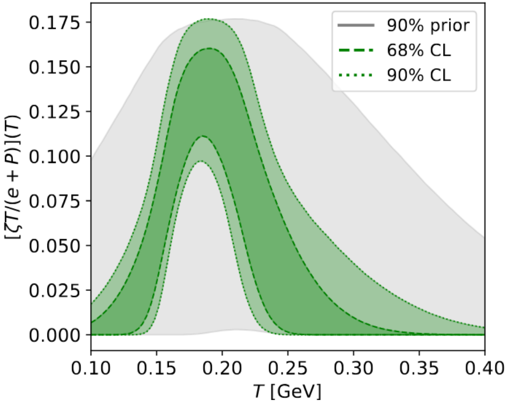
N. Goetz, H. Elfner, I. Karpenko, Phys.Rev.C 112 (2025) 1, 014910

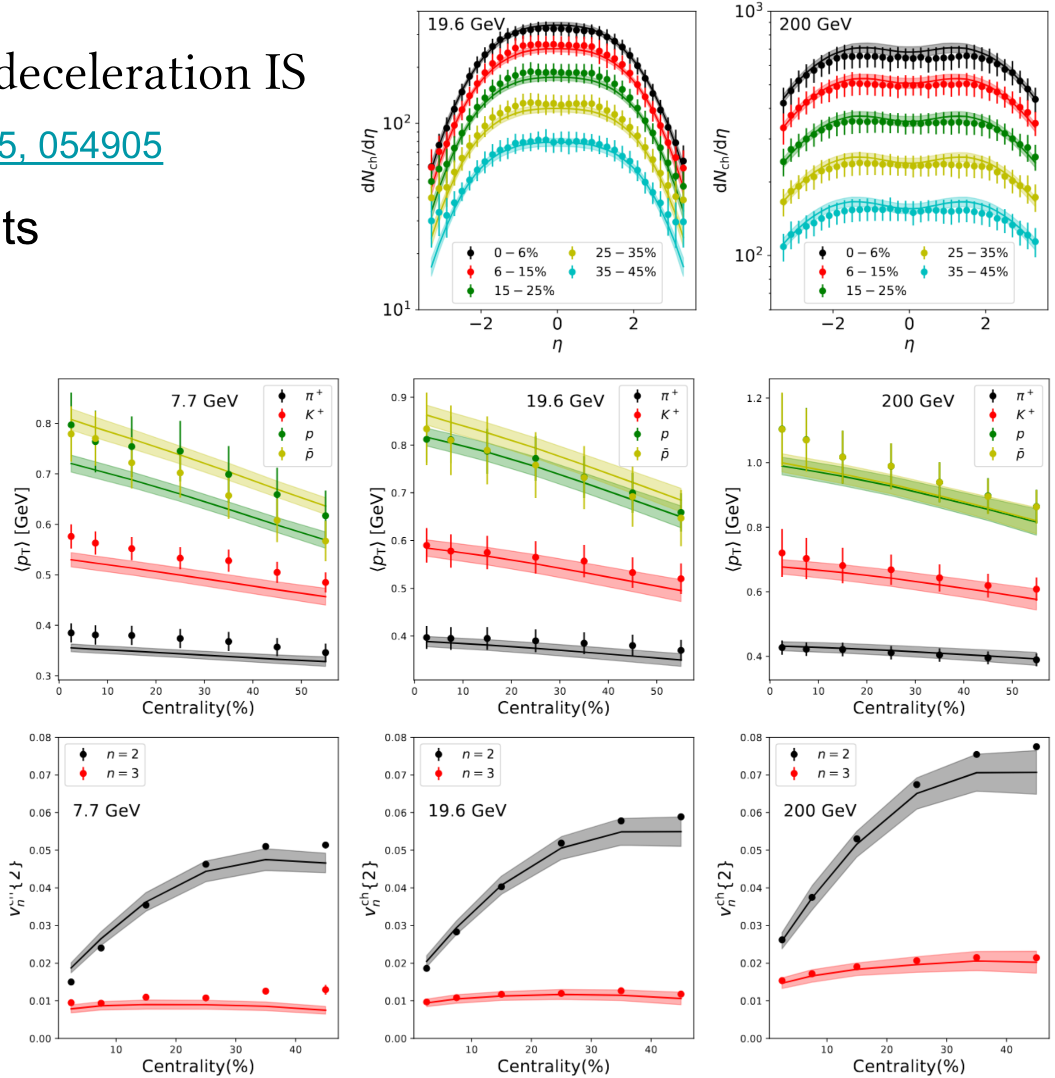
Another Bayesian analysis: string deceleration IS

Jahan, Roch, Shen, [Phys.Rev.C 110 (2024) 5, 054905]

Constraints







Caveats for the BAs above:

- no agreement so far in the extracted viscosities
- tied to particular initial state(s)
- **Experimental data situation is not boring**

Try to figure out what the centrality classes are 😎

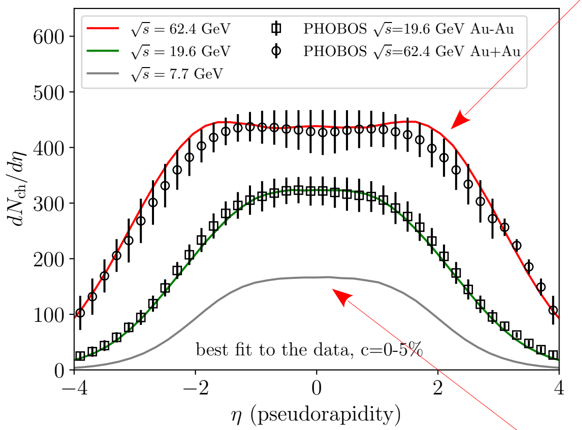
Not available in public (servers down); Luckily, asked a colleague for a zip file with data points. Got the file!!

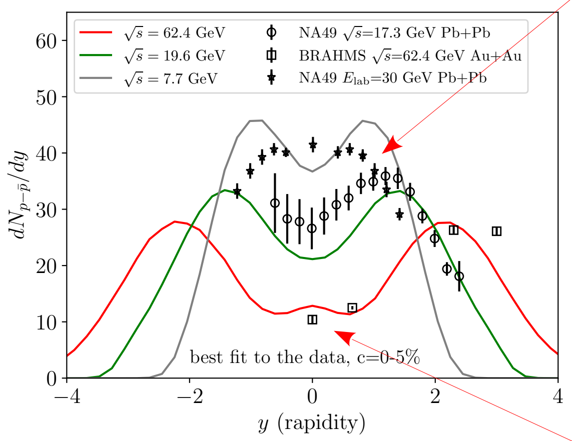
PHOBOS (RHIC)

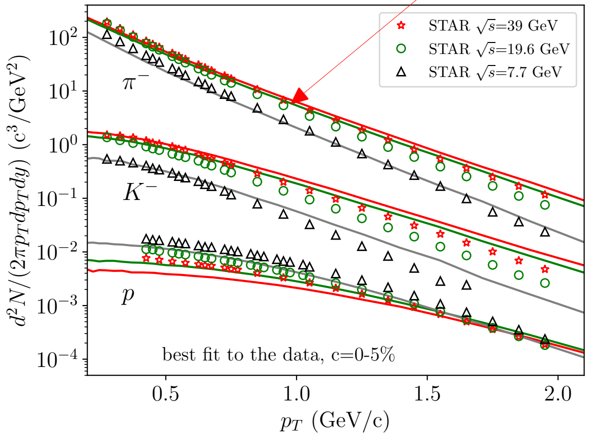
NA49 (SPS)

good quality!

STAR (RHIC)







No data

BRAHMS (RHIC)

Either don't do BA at 7.7 GeV or ask for preliminary STAR data (??)

ALT: use E895 data from AGS maintained by Dan Cebra

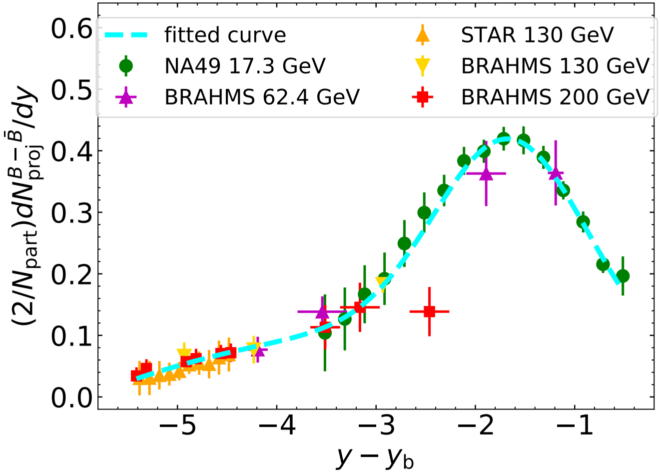
Werthmann, IK, Huovinen, Phys.Rev.C 113 (2026) 3, 034908

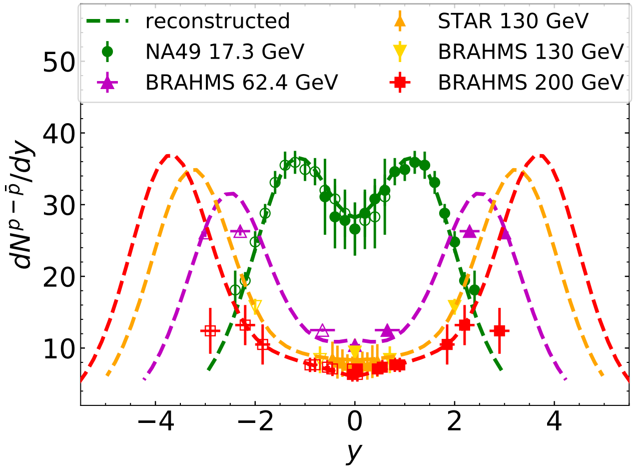
…reconstruction of experimental rapidity distributions

Lipei Du, Phys. Rev. C 110, 014904 (2024)

B. Reconstructing rapidity distributions

Rapidity-dependent measurements play a crucial role in constraining theoretical models for collisions at the beam energy scan. Notably, the charged particle multiplicity in pseudo-rapidity, $dN^{\mathrm{ch}}/d\eta$, holds significance in probing longitudinal distributions of entropy and energy densities, while the rapidity-dependent net proton yields, $dN^{p-\bar{p}}/dy$, aid in constraining the distribution of net baryon density. Regrettably, contemporary measurements have predominantly focused on the midrapidity region, resulting in limited rapidity-dependent data, largely obtained from early experiments. In this section,





Reconstructed rapidity distributions of net protons, easy to compare to models

EoS sensitivity

EoS sensitivity / constraints?

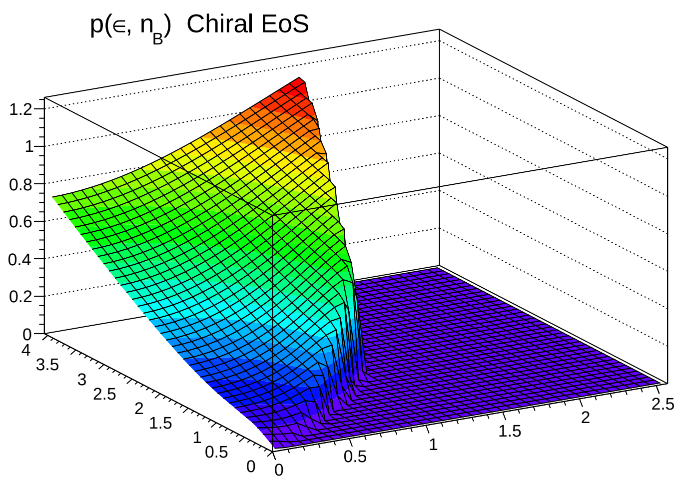
There is no simple answer for that yet, but we are slowly getting there.

- EoS sensitivity in the models can be obscured by other sensitivities (just like at high energies); parameter space is complex but there is hope.
- E.g. EoS constraining via directed flow or HBT does not seem to be a straightforward procedure.
- State-of-the-art EoS with CP location as a free parameter are started to be applied in *some* fluid-dynamic models (MUSIC code, NEOS).
- But there is no BA constraining the EoS at non-zero baryon density yet.

New EoS at finite n_B (optionally n_Q, n_S) are coming

EoS can include first-order PT but only Maxwell construction

EoS has to be known/tabulated down to very low T and high mub → limits the candidate EoS to use.



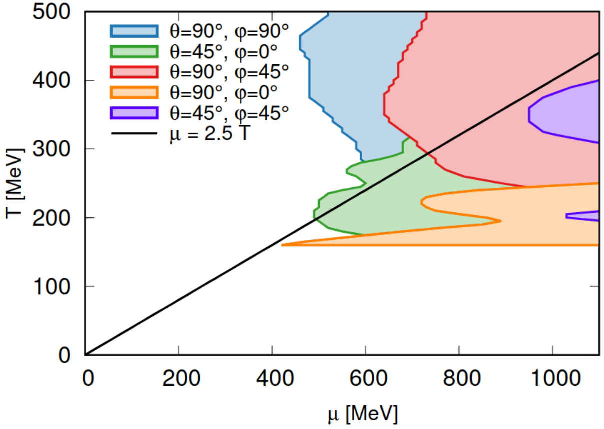
Tabulated in (ε, n_B) !!

Hydro evolution runs in (ε, n_B)

Steinheimer, Schramm, J.Phys.G 38 (2011) 035001

- Energy and baryon density (2D)
- Compatible with latticeQCD at μ_B=0
- Reproduces ground state of nucleus (?!)
- Crossover transition at all μ_B

State-of-the-art: **MUSES**



White region is where the used expansion works

Ahmed Abuali et al (MUSES), Phys.Rev.D 112 (2025) 5, 054502

- 4D (T, μ_B, μ_Q, μ_S)
- latticeQCD at μ_B=0
- CP location parameterized

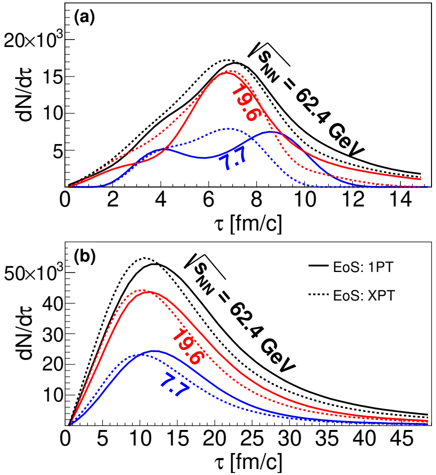
EoS/PT sensitivity via HBT/femtoscopy

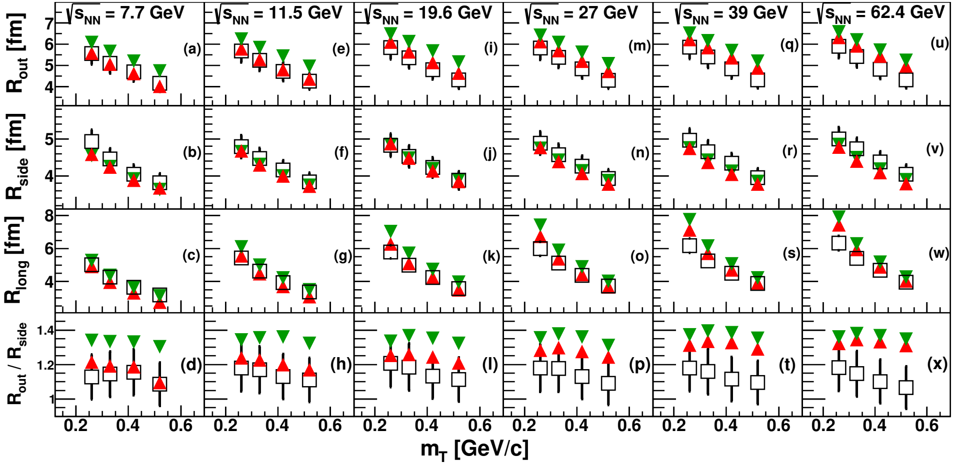
Initial idea: enhancement of $R^2_{\text{out}} - R^2_{\text{side}}$ would signal a passage through the mixed phase.

"Reality check":

`UrQMD` (initial state) + fluidisation at fixed τ_0 + `vHLLE` (hydro) + `UrQMD`

Batyuk, IK, Lednicky, Malinina, Mikhaylov, Rogachevsky, Wielanek, Phys. Rev. C 96, 024911 (2017)





1PT = 1st order PT, XPT = crossover; crossover EoS is red, 1PT EoS is green

There is weak EoS sensitivity, crossover EoS is preferred.

Directed flow indicates a cross-over deconfinement transition in relativistic nuclear collisions

Yu. B. Ivanov[1,2,*] and A. A. Soldatov[2,†]

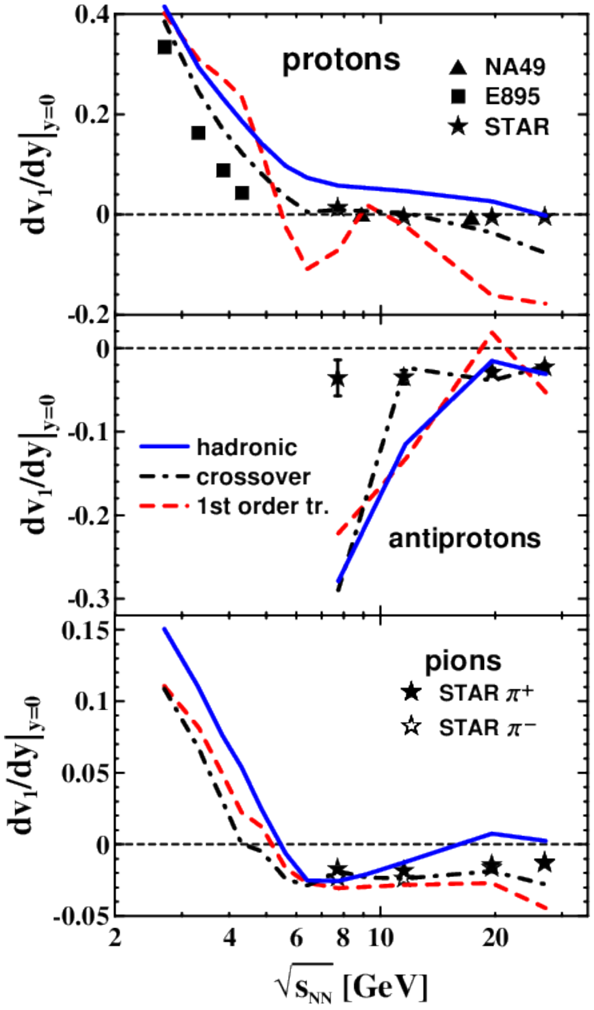
Show more

Phys. Rev. C **91**, 024915 – **Published 26 February, 2015**

DOI: https://doi.org/10.1103/PhysRevC.91.024915

Crossover-type EoS is preferred,

but there is no globally good description of the data



A path forward

- Figure out discrepancies between the models (wherever possible)
  E.g. parametrized initial state vs. initial state from transport (at fixed τ_0)
  vs. dynamical fluidization from transport vs. multi-fluid vs. hadron(-parton) cascade

- Bayesian analysis to provide constraints on the EoS
  For that, one needs an EoS with parametrized location of CP

- Get more consistent experimental data (depends on experimental collaborations)

Summary

- There is a zoo of models for RHIC BES energies, covering a region of $\sqrt{s}$=5-200 GeV.
- There is WIP to extend the picture further down to a few GeV, with either dynamical fluidisation or multi-fluid dynamics.
- Most of models cover most of basic observables: rapidity distributions, net protons, radial flow, elliptic flow, (triangular flow). HBT seems to be not much off from the data.
- There has been a few Bayesian analyses that constrain shear/bulk viscosities not not EoS yet.
- The studies above are done mostly with crossover-type EoS. There has been no constraints on the EoS yet.
- Hadronic cascades allow to emulate EoS softening (~1PT EoS) but fundamentally their applicability stops (gradually) at around sqrt(s)=10 GeV.