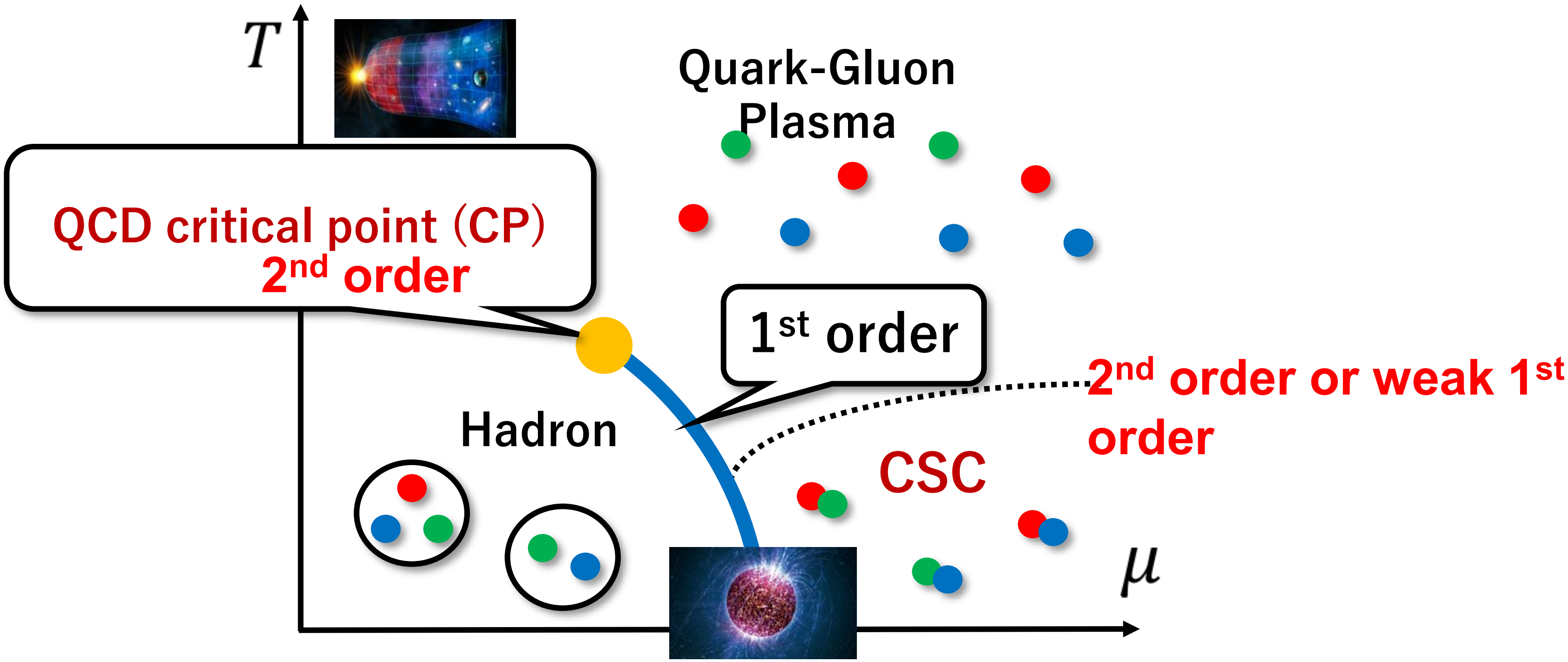
Nature of the soft mode of QCD critical point and Its effects on dilepton production rate and electric conductivity

Teiji Kunihiro (YITP, Kyoto)

Based on joint works with M. Kitazawa (YITP) and **T. Nishimura** (Osaka)

YITPWS `QCD Critical Point and Hydrodynamic Evolution',
June-1-4, @Maskawa Hall, YITP, Kyoto U.

QCD phase diagram



How can we reveal this structure of the phase diagram, in particular the existence of QCD CP, experimentally?

The notion of the soft modes of 2nd-order ph.tr. and its detectability

The soft mode:

Quantum amplitude fluctuations of the order parameters due to 2nd-order phase transitions

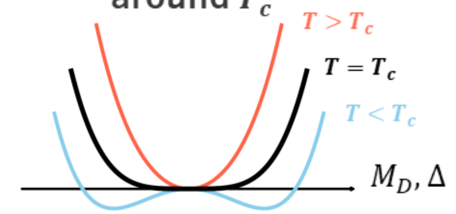
- CSC : fluctuation of diquark condensates (preformed pairs)
  Kitazawa, Koide, Kunihiro, Nemoto (2005)
- QCD CP : fluctuation of chiral condensate(` σ ')
  Fujii, Ohtani (2004); Dam T. Son and M. Stephanov (2004)

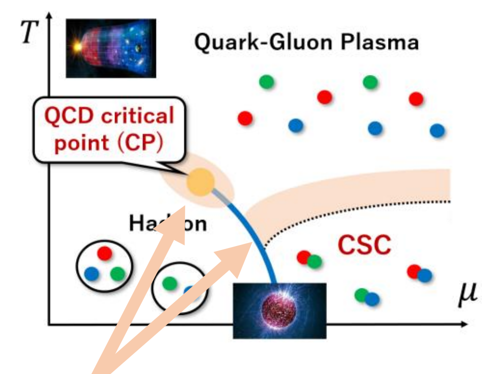
✓ They develop prominently around the critical point or line.
✓ They are excited with low energy & momentum (< 300 MeV).

These soft modes can contribute significantly to observables.

If the contributions are observed in HIC, they will provide experimental signals of QCD CP as well as CSC.

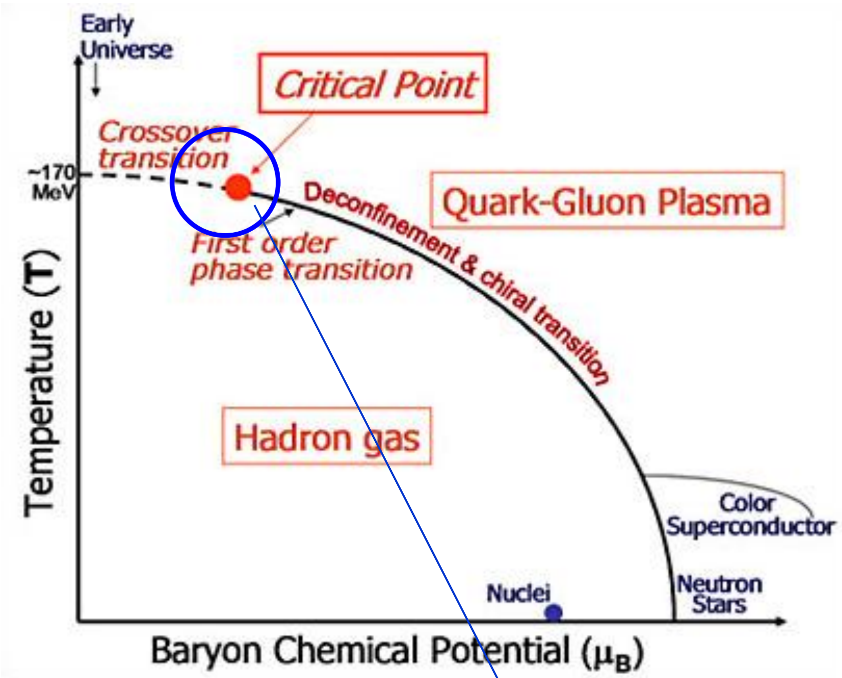
Thermodynamic potential around T_c

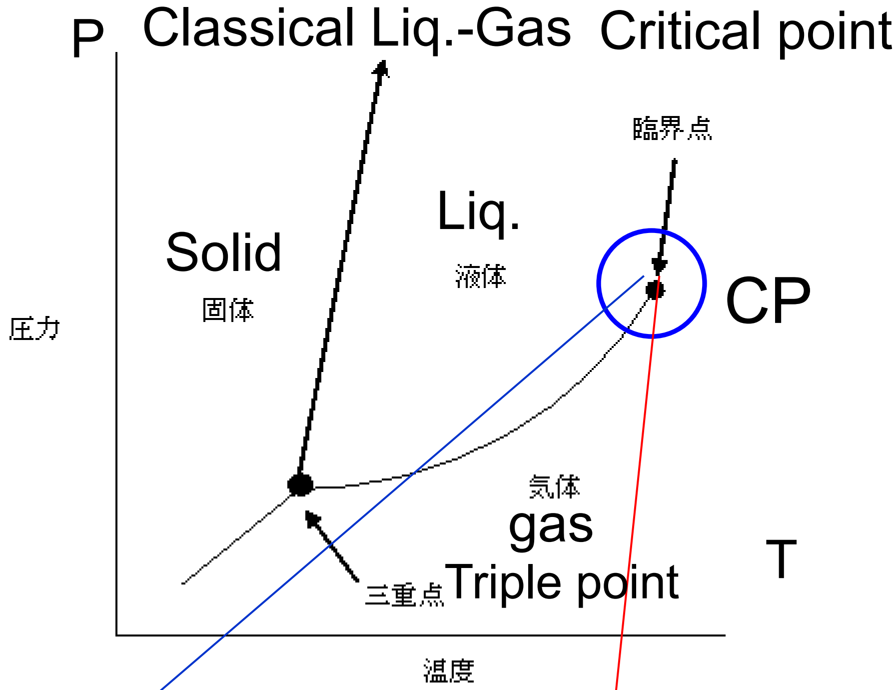




Colored areas indicate the regions that the soft modes develop.

The case of QCD critical point





The same universality class; Z2

H. Fujii, PRD 67 (03) 094018;H. Fujii and M. Ohtani, Phys.Rev.D70(2004)
Dam. T. Son and M. A. Stephanov, PRD70 ('04) 056001

Collective p-h excitations is the soft mode of QCD critical point!

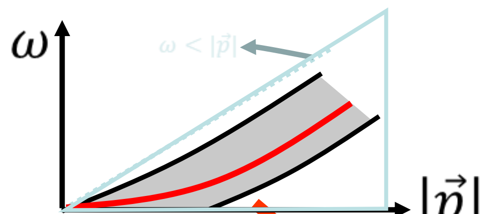
In classical liquid, a random light scattering due to density fluctuations (critical opalescence)

What is the soft mode at CP?

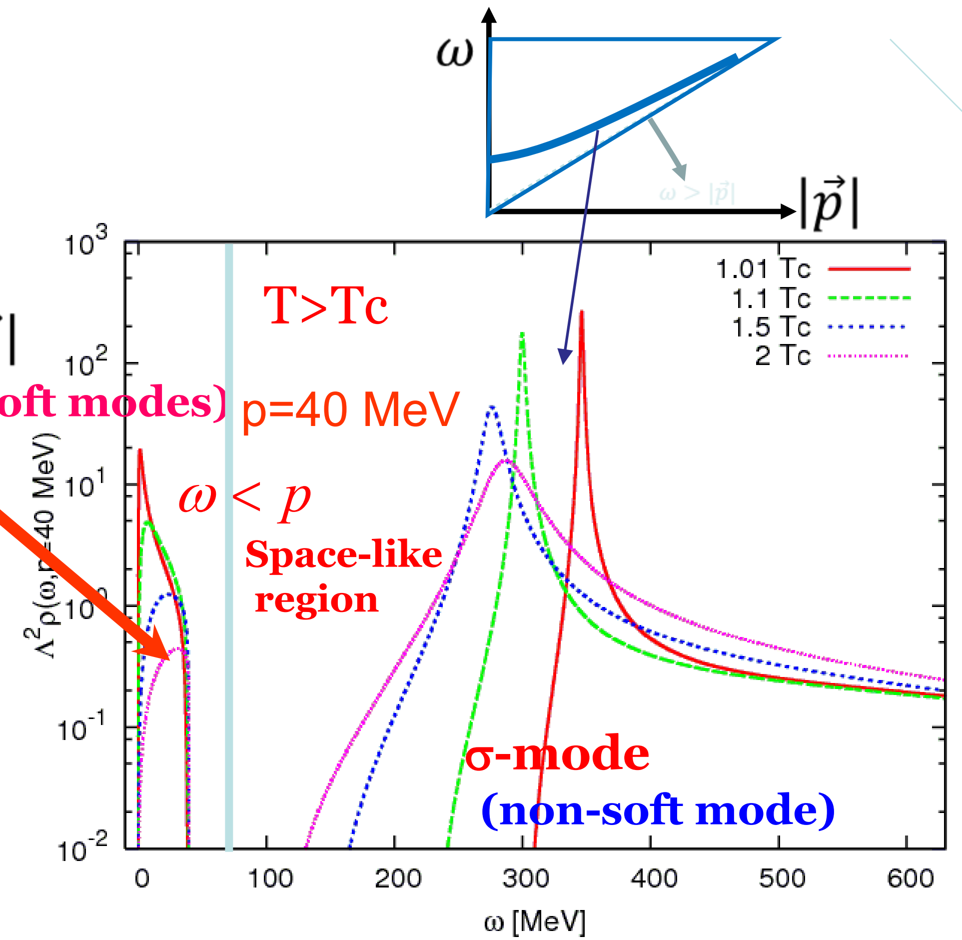
nonzero m_q and charge conjugation symm. is broken by finite chemical potential

Spectral function in the scalar(-vector) channel

H. Fujii (2003)
H. Fujii and M.Ohtani(2004)



Phonon mode in the space-like region softens at CP.



couples to hydrodynamical modes,

leading to interesting dynamical critical phenomena.

See also, D. T. Son and M. Stephanov (2004)

The σ meson has still a non-zero mass at CP, and is not the relevant soft mode.

Possible observables of the precursory soft modes of QCD-CP in HI-collisions: Dilepton production rate (DPR)

T.Nishimura, M. Kitazawa, TK,PTEP 2023 (2023) 5, 053D01 ;
Annals Phys. 469 (2024) 169768

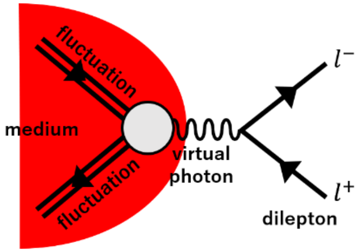
✓ **Good observable in HIC**
✓ **Computable theoretically**

$$\frac{d^4\Gamma}{dk^4}(\boldsymbol{k},\omega) = -\frac{\alpha}{12\pi^4}\frac{1}{\omega^2-\boldsymbol{k}^2}\frac{1}{e^{\beta\omega}-1}\rho(\boldsymbol{k},\omega)$$

$$\rho(\boldsymbol{k},\omega) = g_{\mu\nu}\mathrm{Im}\Pi^{R\mu\nu}(\boldsymbol{k},\omega)$$

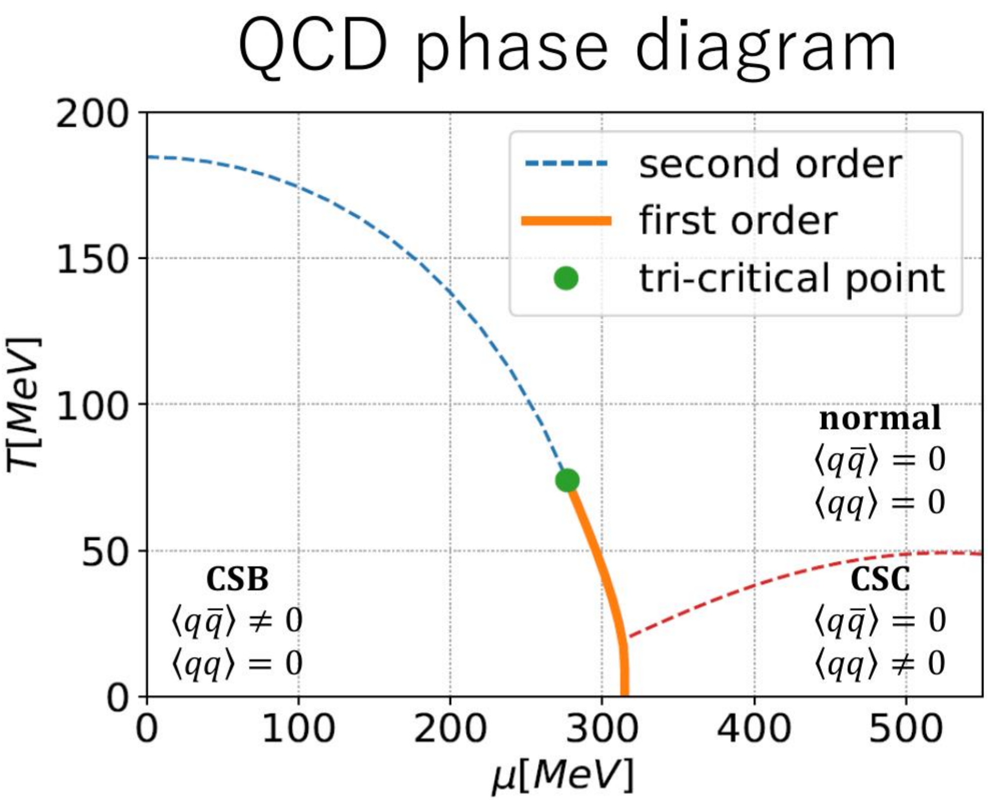
Spectral function in the vector channel

The soft modes associated to the QCD-CP modify the photon self-energy through the mechanism established to account for the para-conductivity above T_c in metal superconductivity.



Lesson from the case of 2SC color superconductivity and condensed matter physics

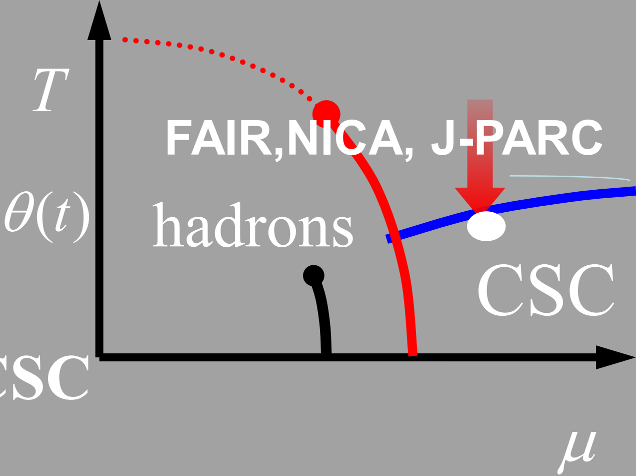
■Color superconductivity (CSC)



* Induced by **diquark** condensation
* **Low-temperature** and **high-density**
* **It is difficult to observe CSC in experiment.**
* We focus on **2SC**.

Review : Alford, Schmitt, Rajagopal, Schäfer(2008)

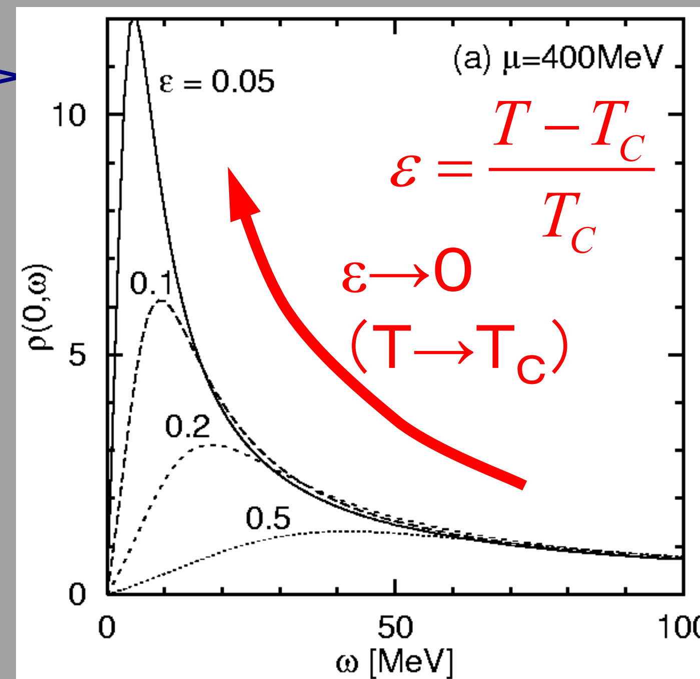
Diquark pair fluctuations prior to CSC



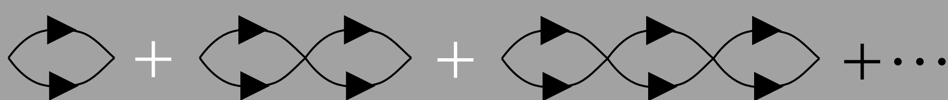
$$D^R(\mathbf{x},t) = -2G_C \left\langle \left[\bar{\psi}(x) i\gamma_5 \tau_2 \lambda_2 \psi^C(x), \bar{\psi}(0) i\gamma_5 \tau_2 \lambda_2 \psi^C(0) \right] \right\rangle \theta(t)$$

The diquark excitations as the soft mode of CSC

$$\rho(\mathbf{k},\omega) = -\frac{1}{\pi} \mathrm{Im}\, D^R(\mathbf{k},\omega)$$



$D^R(\mathbf{k},\omega) =$



The strong coupling nature of CSC leads to a sharp peak even at ε ~ 0.2

electric SC: ε ~ 0.005
even in 2d-SC

Existence of large pair fluctuations

M.Kitazawa, T. Koide, T. K., Y. Nemoto, PRD **65**, 091504 (2002)

It may affect various observables even well above T_c.

We explore the possibility that the diquark fluctuations as the soft modes of CSC anomalously affect dilepton-pair production rate in H-I collisions.

How do they ?

A hint is given by the para conductivity (i.e., anomalous enhancement of electric conductivity) through the Aslamazov-Larkin as well as the Maki-Thompson terms and DOS.

A.Aslamasov and A. Larkin,Sov. Phys. SS **10**,875('68)

K. Maki, Prog. Theor. Phys. 39, 897(1968);

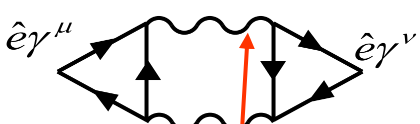
R. S. Thompson, Phys. Rev. B1, 327 (1970)

Solid line

Paraconductivity in metallic SC's near but above T_c

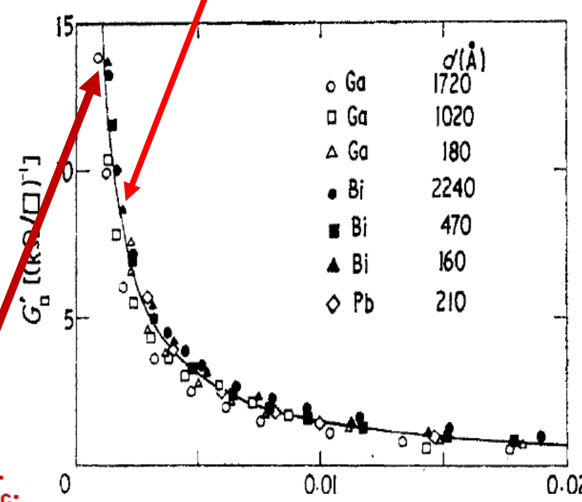
vertical axis : Electric conductivity

horizontal axis : $\epsilon = (T - T_c)/T_c$

$\Pi_{AL}^{\mu\nu} =$ 

preformed pairing oscillation

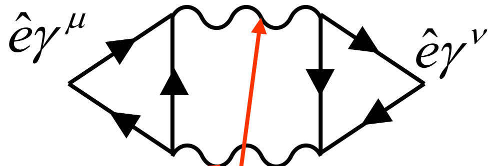
The conductivity shows a diverging behavior at $T = T_c$ due to the fluctuations of the electron pair fields above T_c.



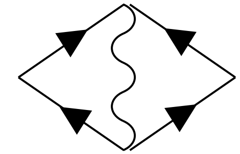
W.J.Skopal and M.Tinkham, Rep. Prog. Phys.38 (1975),1049

The relevant diagrams

$$\Pi^{\mu\nu}_{AL} =$$

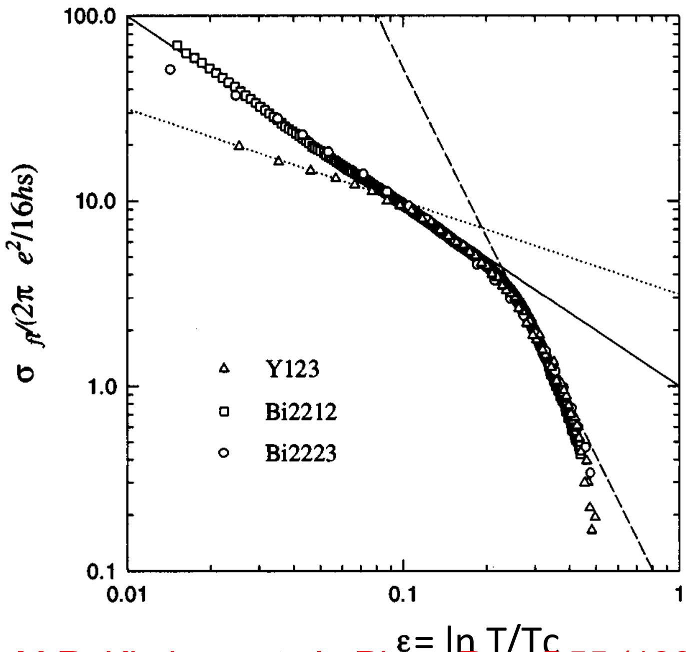


Maki-Thompson term



electron pairing oscillations → **Diquark oscillations**

@ finite q and ω.



the solid line is ε^-1 , the dashed line 0.055/ε^(-3), and the dotted line is 3.2/ε^(-1/2) .

M.R. Kimbere et al, Phys. Rev. B55 (1997): see also
A. Larkin and A. Varlamov, cond-matt/0109177

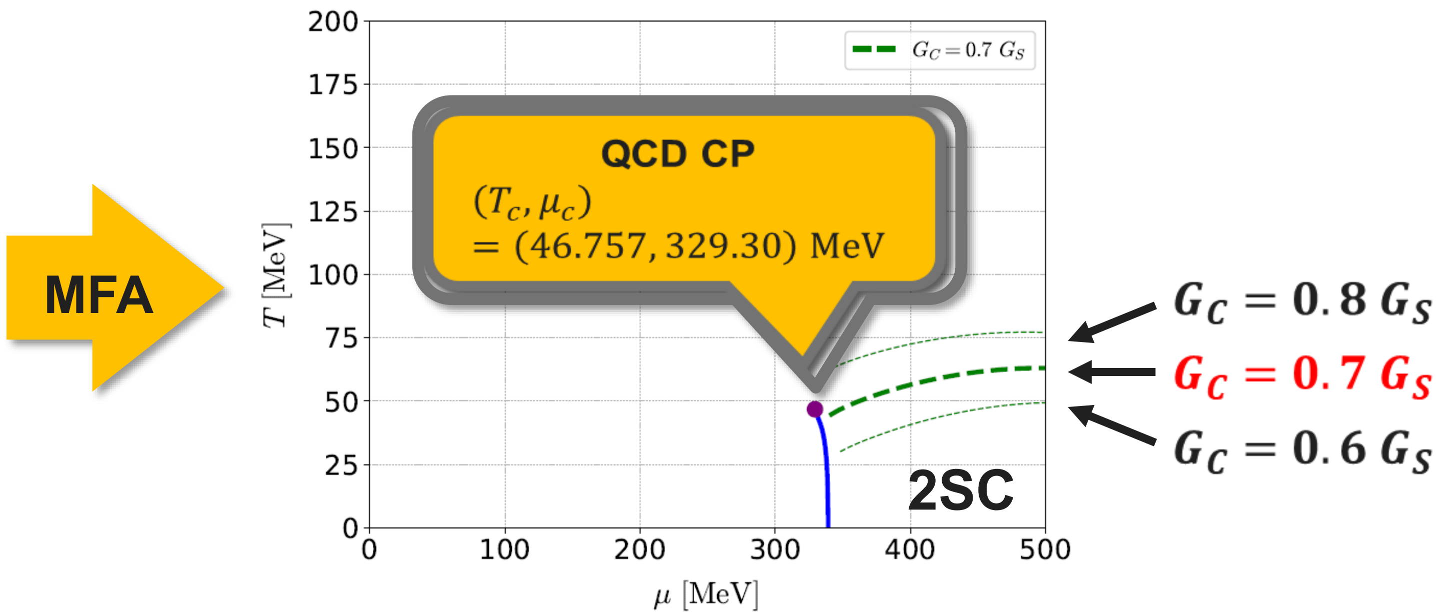
2-flavor NJL model

$$\mathcal{L} = \bar{\psi} i(\gamma^\mu \partial_\mu - m)\psi + \mathcal{L}_S + \mathcal{L}_C$$

$$\begin{cases} \mathcal{L}_S = G_S[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \boldsymbol{\tau}\psi)^2] \\ \mathcal{L}_C = G_C(\bar{\psi} i\gamma_5 \tau_2 \lambda_A \psi^C)(\bar{\psi}^C i\gamma_5 \tau_2 \lambda_A \psi) \end{cases}$$

($\boldsymbol{m = 5.5}$ **MeV**, $\boldsymbol{G_S = 5.50}$ **MeV**, $\boldsymbol{\Lambda = 631}$**MeV**)

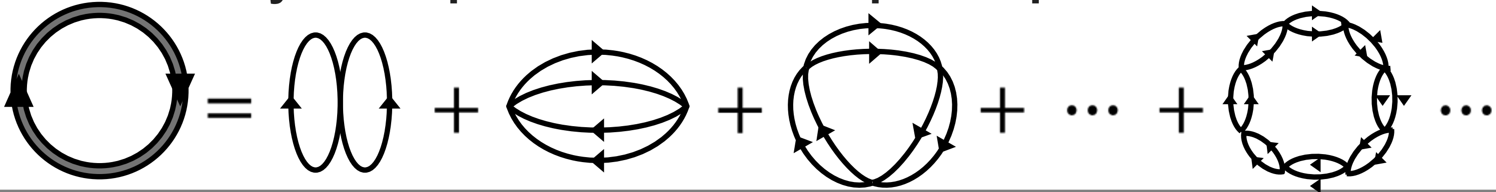
Kitazawa, Koide, Kunihiro, Nemoto (2002)

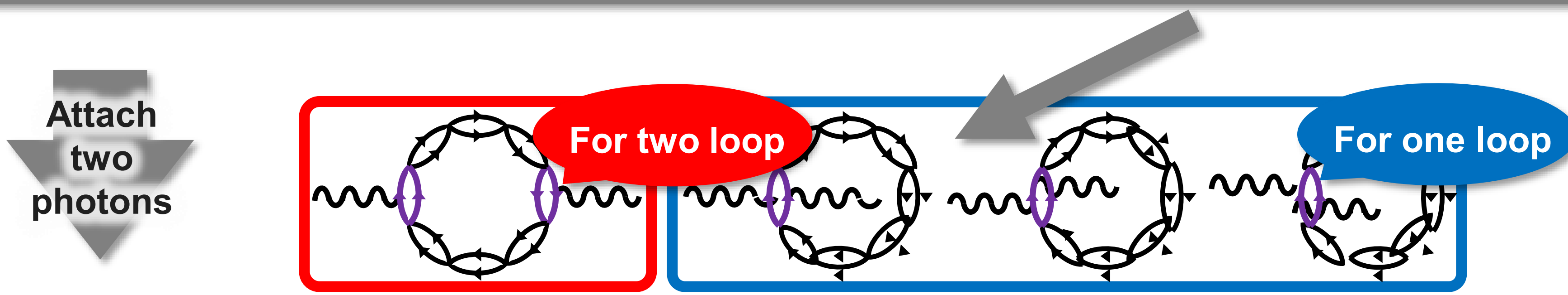


Systematic Computation of the effects of the soft mode on the photon self-energy with the Ward-Takahashi relation being satisfied

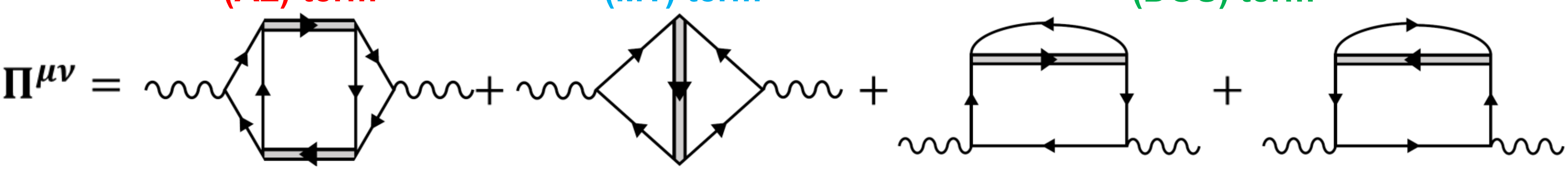
T.Nishimura, M. Kitazawa, TK, PTEP 2022 (2022) 9, 093D02

Thermodynamic potential : One loop of diquark fluctuations





Aslamazov-Larkin (AL) term | **Maki-Thompson (MT) term** | **Density of states (DOS) term**



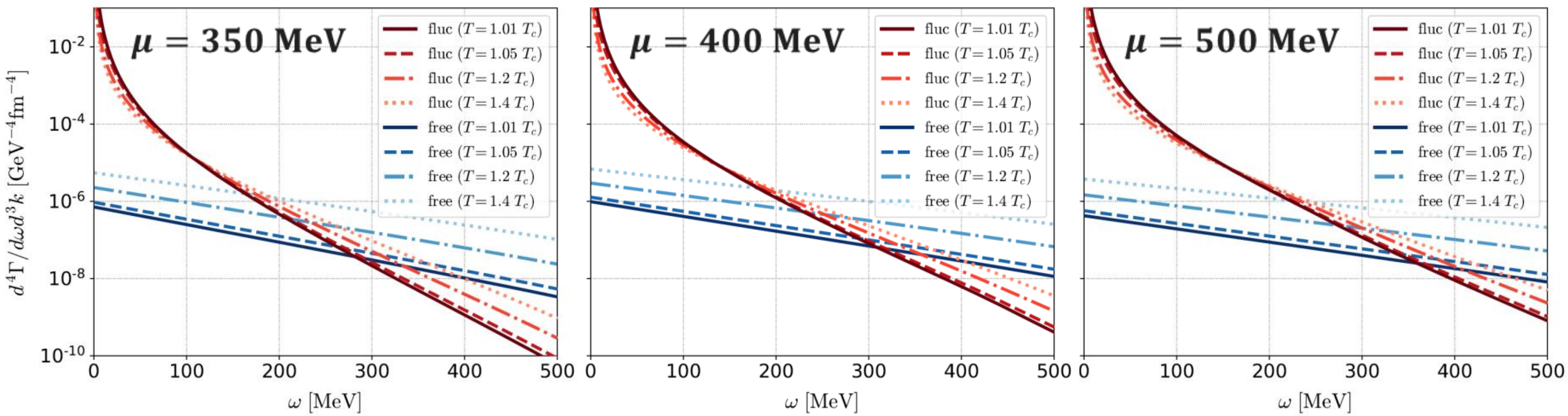
Ward identity $k_\mu \Pi^{\mu\nu} = 0$ is satisfied.

T.Nishimura, M. Kitazawa, TK, PTEP 2022 (2022) 9, 093D02

Contribution of diquark soft mode at $k = 0$ $(T > T_c)$

Red lines : Contribution of the soft modes

Blue lines : Contribution of the free quark gases



- ✓ **The dilepton production rate is enhanced by the soft mode.**
- ✓ **As $T \to T_c$, the rate becomes bigger.**

··· **This behavior is expected from the property of soft modes.**

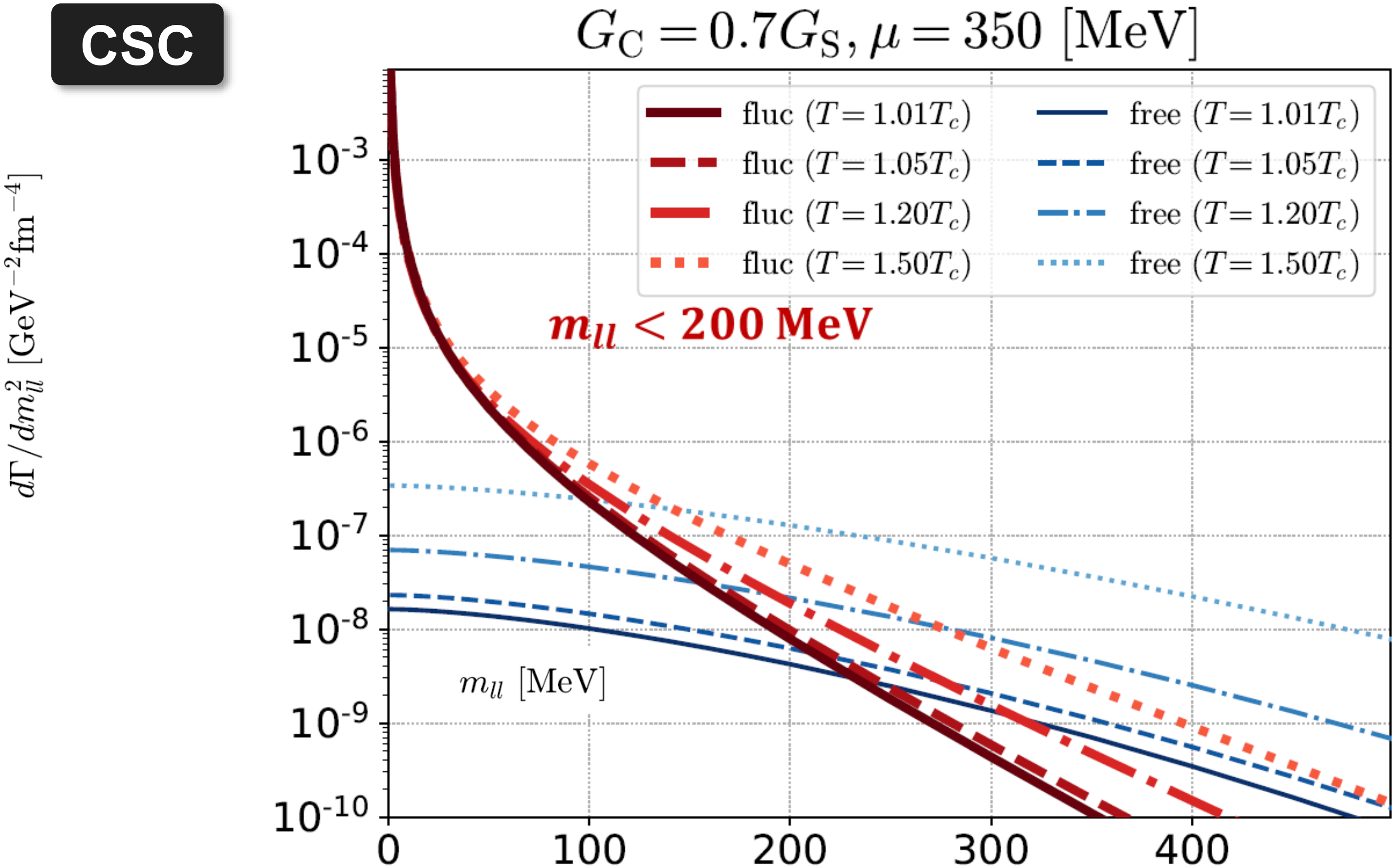
T.Nishimura, M. Kitazawa, TK, PTEP 2022 (2022) 9, 093D02

Dielectron Invariant mass spectra

$$\frac{d\Gamma}{dM^2} = \int d^3k \frac{1}{2\omega} \frac{d^4\Gamma}{d^4k}\Big|_{\omega=\sqrt{k^2+M^2}}$$

If the enhancement is confirmed, it may possibly give an experimental evidence of the phase transition to CSC !

CSC



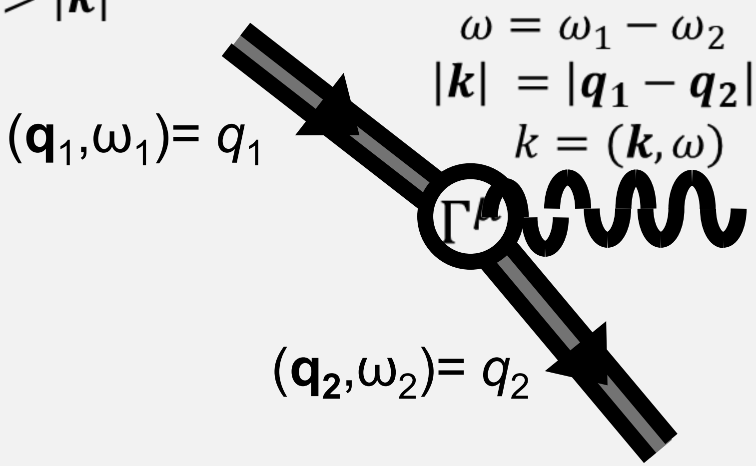
Mechanism of lepton-pair production by two space-like modes

Virtual photon creation in the time-like region by the diquark soft modes

$|\boldsymbol{q}_1| > \omega_1$

$|\boldsymbol{q}_2| > \omega_2$

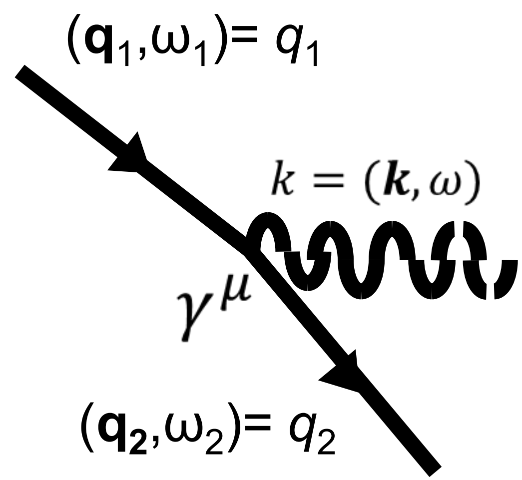
$\omega > |\boldsymbol{k}|$



Although the diquark soft modes have spectral support in the space-like region, a combined excitation of the two modes can have spectral support in the time-like region.

∗ MT&DOS cancel. → Scattering in AL.

Scattering process with free quarks



The scattering process only produces virtual photon in the space-like region, while the $q\bar{q}$ annihilation contributes dilepton production in time-like.

Transport coefficients: electric conductivity

Photon self-energy $\rho(\boldsymbol{k},\omega) = g_{\mu\nu}\mathrm{Im}\Pi^{R\mu\nu}(\boldsymbol{k},\omega)$

Dilepton production rate

$$\frac{d^4\Gamma}{dk^4}(\boldsymbol{k},\omega) = -\frac{\alpha}{12\pi^4}\frac{1}{\omega^2-\boldsymbol{k}^2}\frac{1}{e^{\beta\omega}-1}\rho(\boldsymbol{k},\omega)$$

If the hydrodynamic picture is valid in the low energy-momentum region…

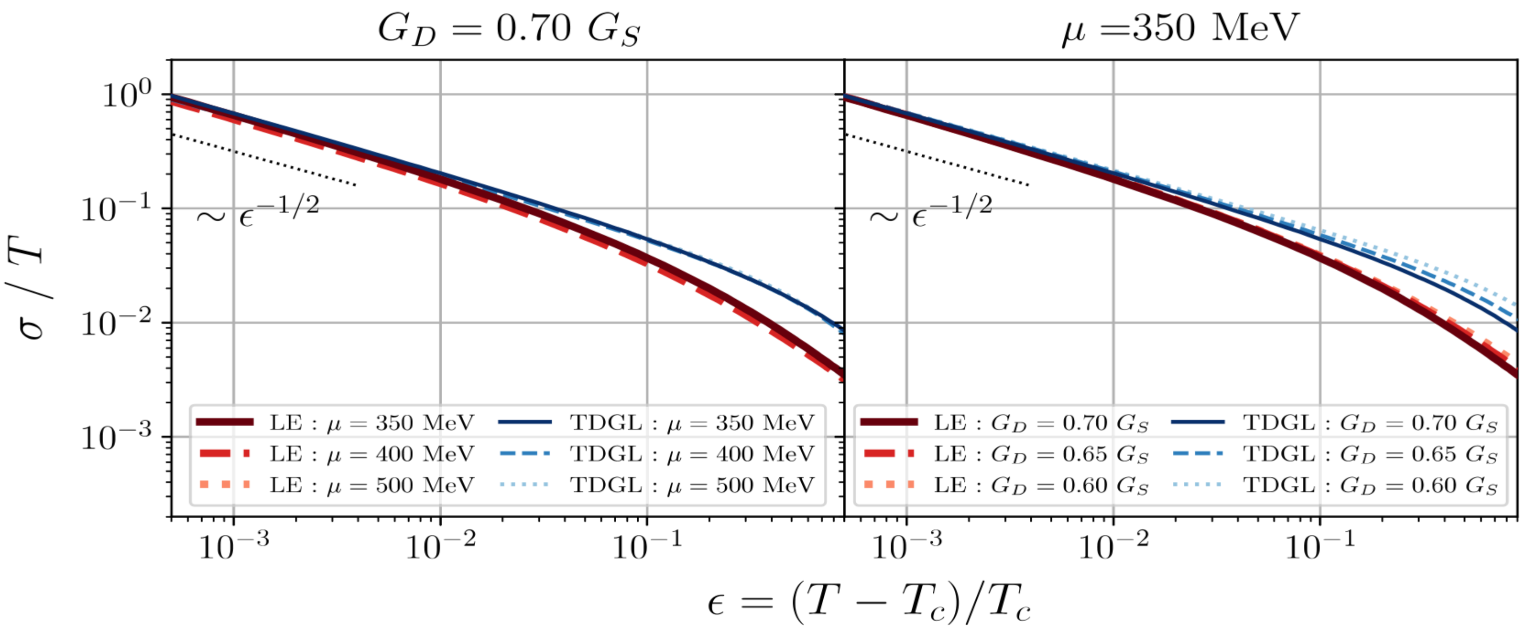
$$\rho(\boldsymbol{k},\omega) = g_{\mu\nu}\mathrm{Im}\Pi^{R\mu\nu}(\boldsymbol{k},\omega) \simeq \frac{\sigma\omega(\omega^2-\boldsymbol{k}^2)}{(\tau\omega^2-D\boldsymbol{k}^2)^2+\omega^2} + 2\frac{\sigma\omega}{\tau^2\omega^2+1}$$

Kadanoff, Martin (1963)

$$\Rightarrow \quad \rho(\mathbf{0},\omega) = -\sum_{i=1,2,3}\mathrm{Im}\Pi^{Rii}(\mathbf{0},\omega) = 3\frac{\sigma\omega}{\tau^2\omega^2+1}$$

Electric conductivity	$\sigma = \frac{1}{3}\frac{\partial\rho(\mathbf{0},\omega)}{\partial\omega}\Big\vert_{\omega=0}$	**Relaxation time**	$\tau = \sqrt{-\frac{1}{3\,!}\frac{\frac{\partial^3}{\partial^3\omega}\rho(\mathbf{0},\omega)\vert_{\omega=0}}{\frac{\partial}{\partial\omega}\rho(\mathbf{0},\omega)\vert_{\omega=0}}}$

Para-conductivity associated to 2SC

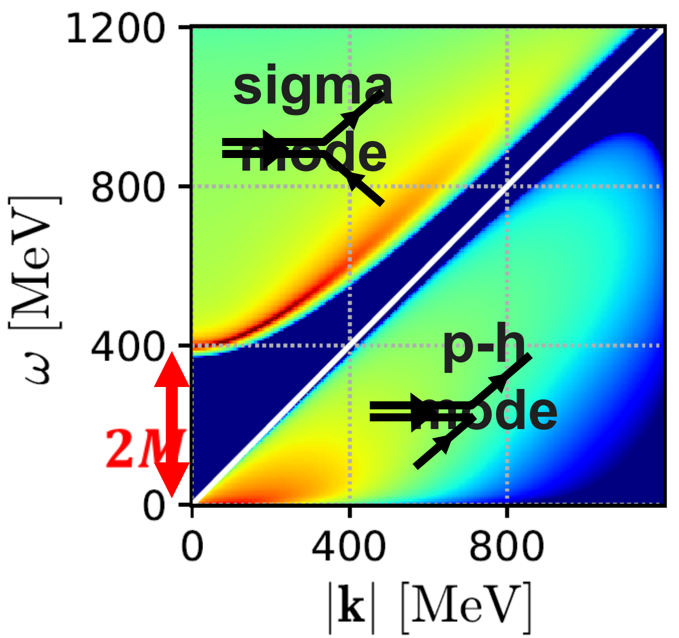


The soft modes at the QCD critical point

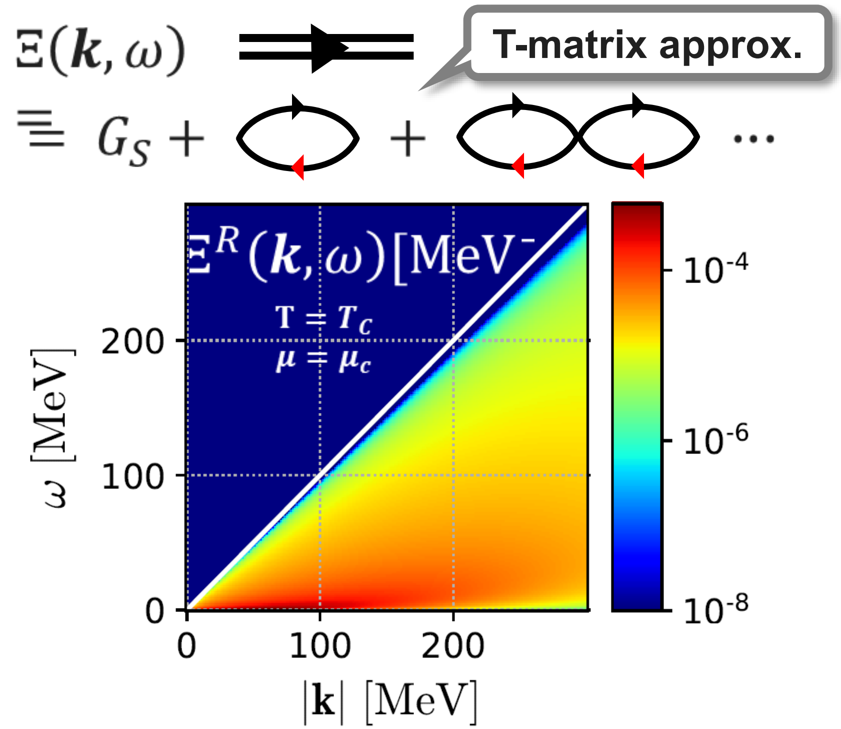
T.Nishimura, M. Kitazawa, TK,PTEP 2023 (2023) 5, 053D01

Soft modes for QCD^CP

In our work, the soft mode of QCD CP is the particle-hole (p-h) mode.



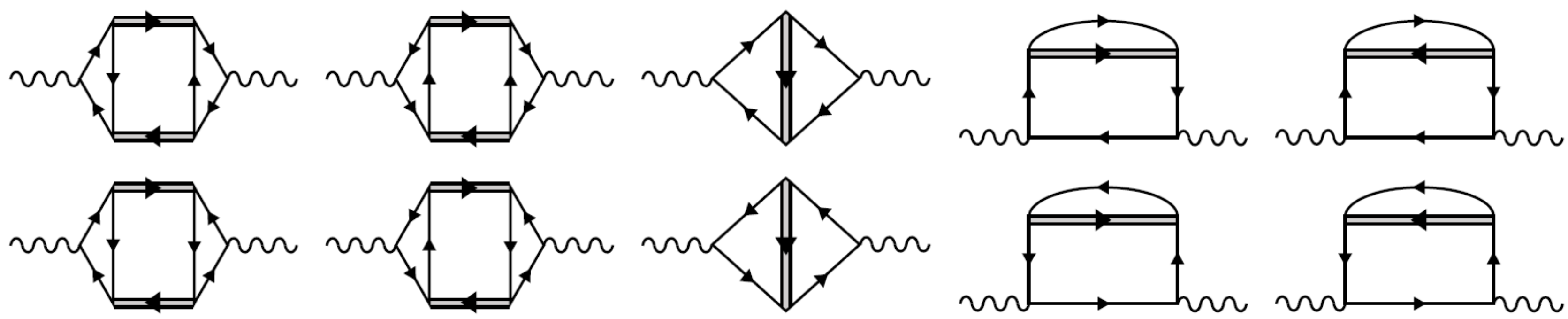
Particle-hole soft modes



Fujii, Ohtani (2004)

T.Nishimura, M. Kitazawa, TK,PTEP 2023 (2023) 5, 053D01

Photon self-energy $\Pi^{\mu\nu}(k,\omega)$ in case of p-h soft mode



CS

QCD CP

Aslamazov-Larkin (AL) term

Maki-Thompson (MT) term

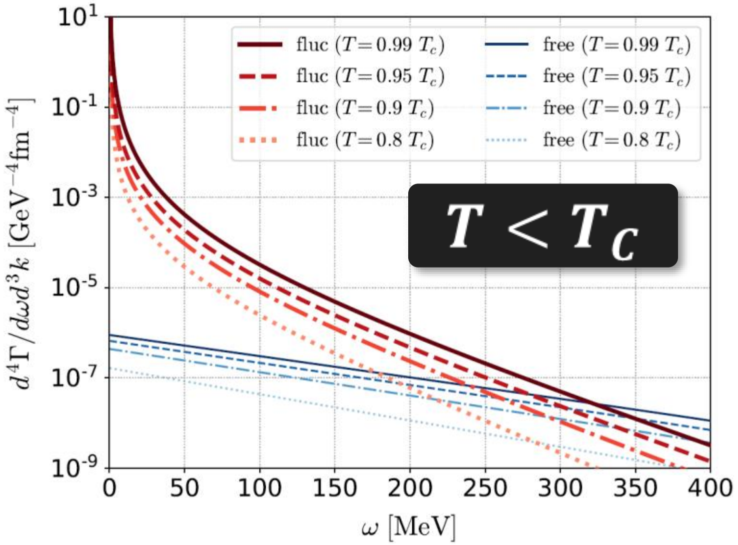
Density of states (DOS) term

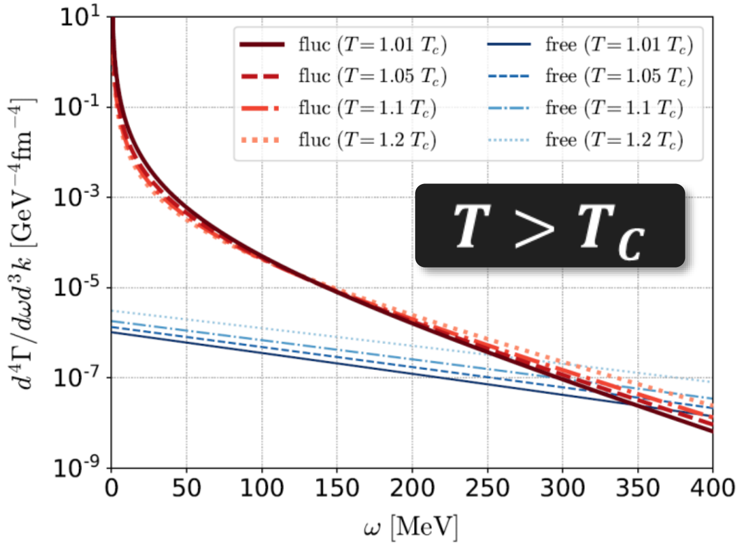
$\Pi^{\mu\nu} =$

Ward identity of $\Pi^{\mu\nu}$ is satisfied.

T.Nishimura, M. Kitazawa, TK,PTEP 2023 (2023) 5, 053D01

Contribution of p-h (QCD CP) soft mode at $k = 0$ ($\mu = \mu_c$)





Bigger as $T \to T_c$

Bigger as T is bigger

Competition between contributions of soft modes and kinematical (temperature) effects

$$\frac{d^4\Gamma}{dk^4}(\boldsymbol{k},\omega) = -\frac{\alpha}{12\pi^4}\frac{1}{\omega^2-\boldsymbol{k}^2}\frac{1}{e^{\beta\omega}-1}\rho(\boldsymbol{k},\omega),$$

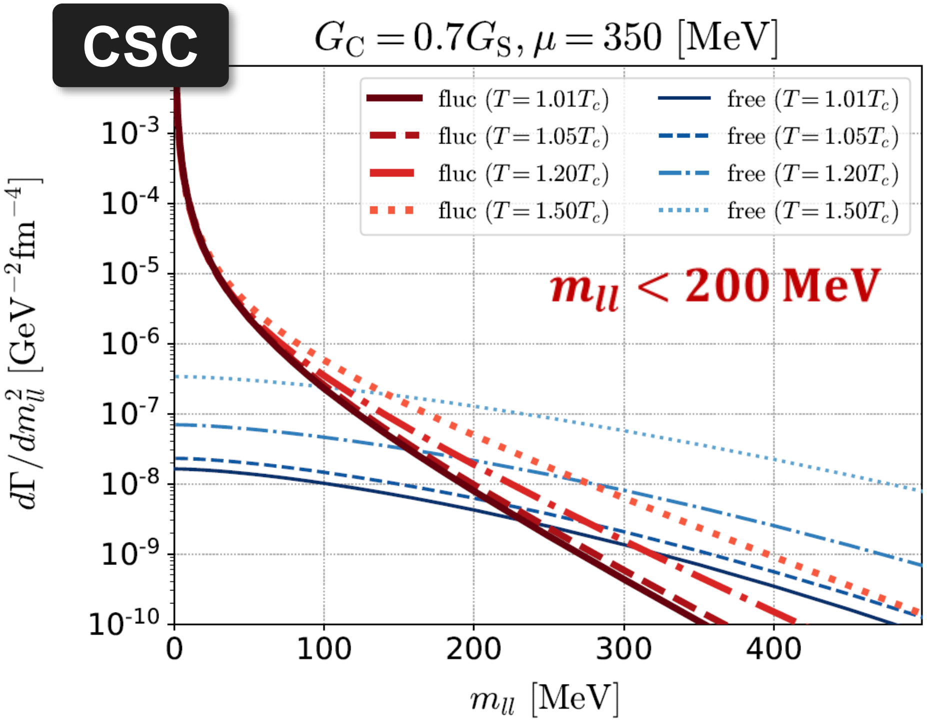
$$\rho(\boldsymbol{k},\omega) = g_{\mu\nu}\mathrm{Im}\Pi^{\mathrm{R}\mu\nu}_{\mathrm{AL}}(\boldsymbol{k},\omega)$$

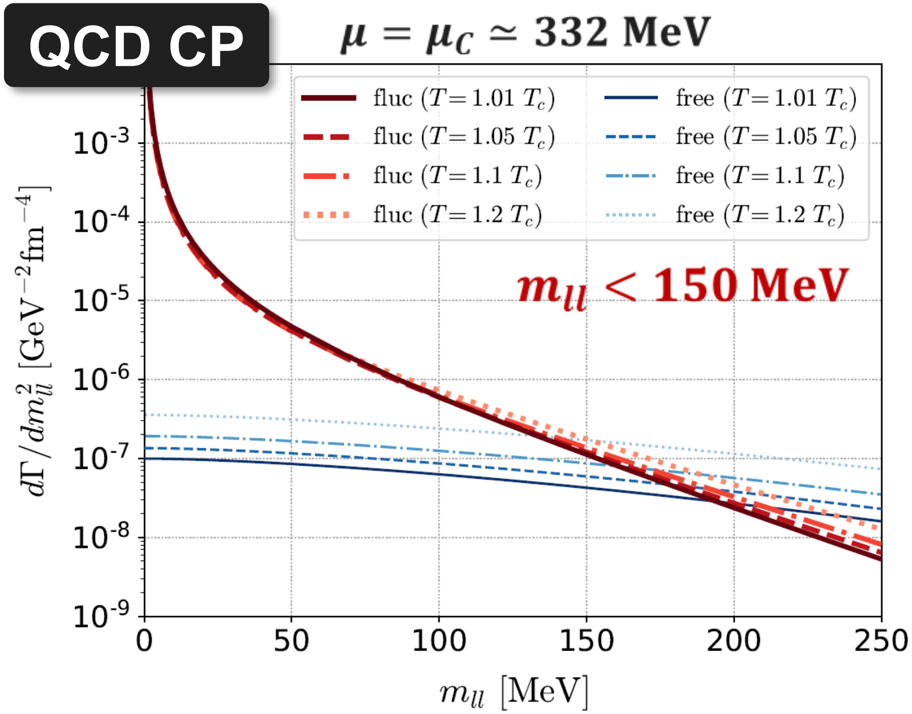
T.Nishimura, M. Kitazawa, TK,PTEP 2023 (2023) 5, 053D01

Invariant mass spectra

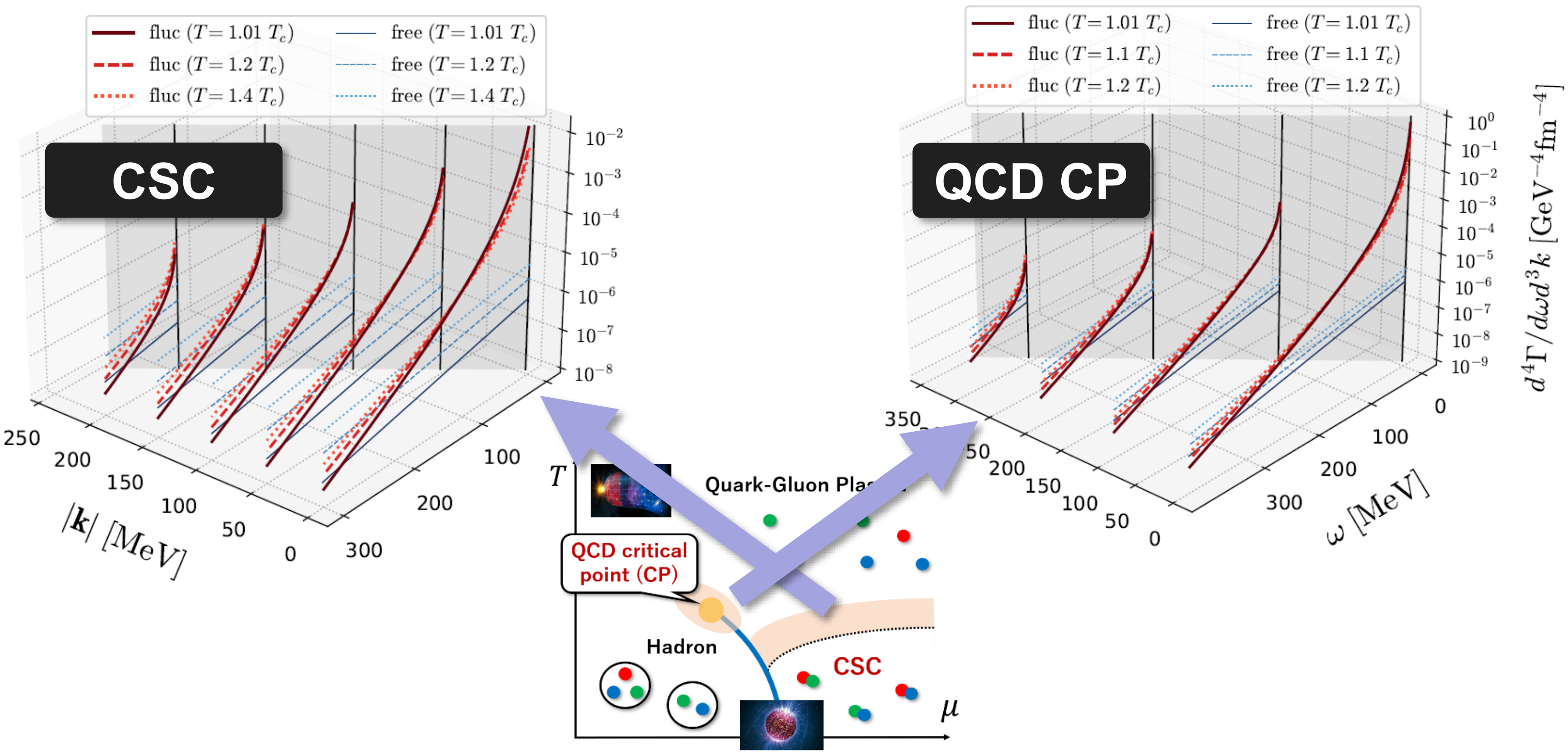
$$\frac{d\Gamma}{dM^2} = \int d^3k \frac{1}{2\omega} \frac{d^4\Gamma}{d^4k}\Big|_{\omega=\sqrt{\boldsymbol{k}^2+M^2}}$$

If the enhancement is confirmed, it may possibly give an experimental evidence of the phase transition to CSC & QCD CP!



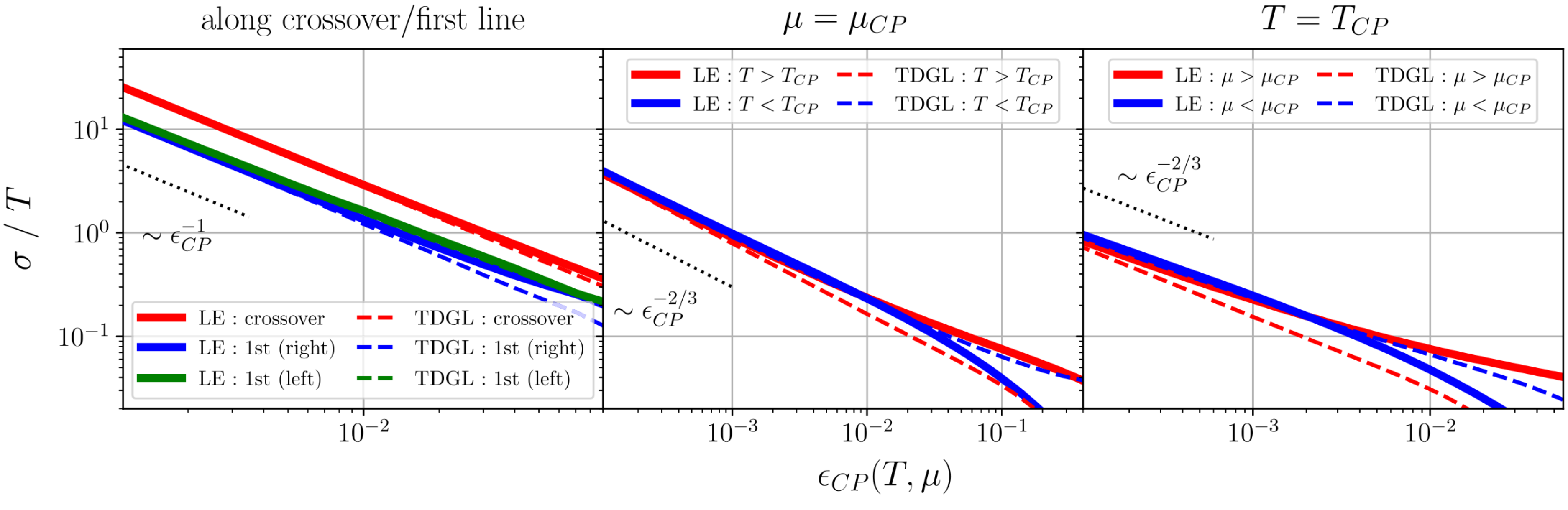


T.Nishimura, M. Kitazawa, TK,PTEP 2023 (2023) 5, 053D01

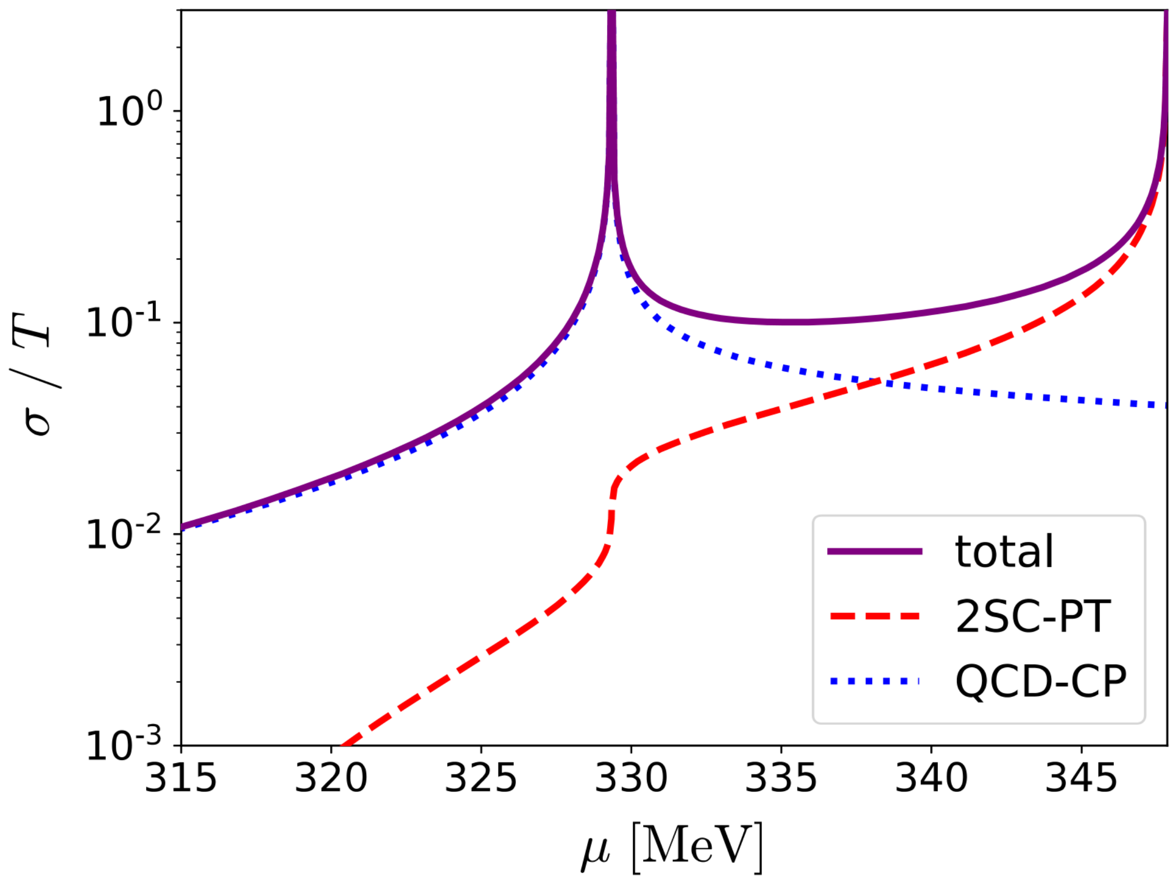


What is the fundamental difference between the two cases?
→ See the critical behavior of transport coefficients!

Electric conductivity around QCD-CP



Chemical potential dependence of conductivity around QCD-CP



Brief summary

1. We have shown how the dilepton production rate is affected by the soft modes of the QCD CP as well as 2SC, which is very specific to QCD,
2. and the soft modes also cause an enhancement of electric conductivity around the critical region, as the para-conductivity in metal superconductors.
3. The anomalous enhancement of the di-electron production in the very low energy region. We admit that it would be an experimental challenge to detect them,
 but it should be a quite useful to identify the existence of these phase transitions in the QCD phase diagram.

Remark:

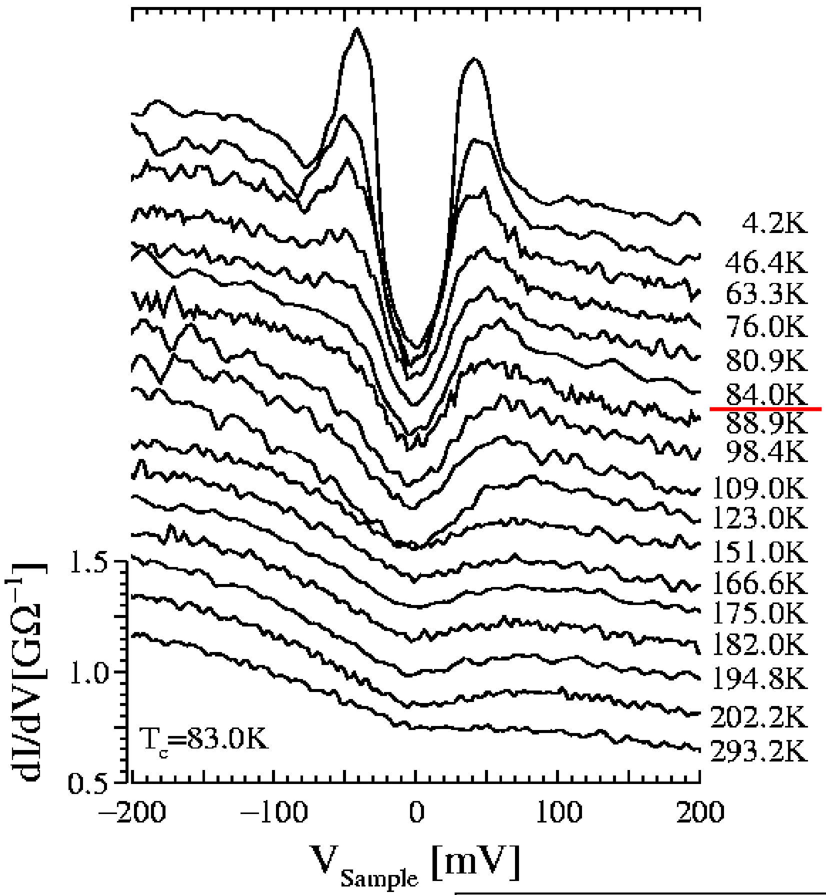
Such an experiment may provide a part of a multi-messenger HI science, analogously to multi-messenger astronomy consisting of gravitational Waves, photons, neutrinos, and other cosmic rays. >> Dynamical model cal.

Further development: an enhancement of relaxation times,
T. Nishimura, M. Kitazawa and TK,
Annals Phys. 469 (2024) 169768

BACK UPS

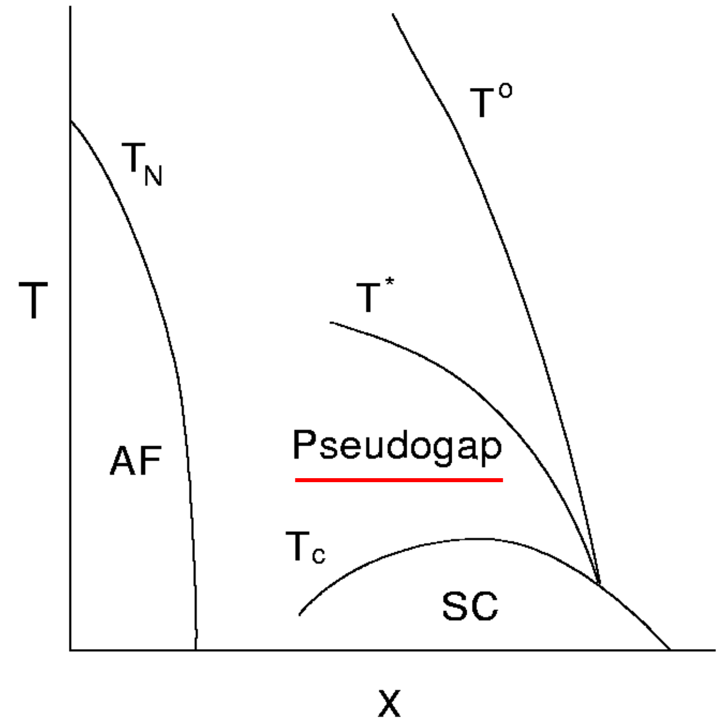
Pseudogap in High T_c Superconductors

:Anomalous depression of the density of state near the Fermi surface in the normal phase.



Renner et al.('96)

Conceptual phase diagram of HTSC cuprates



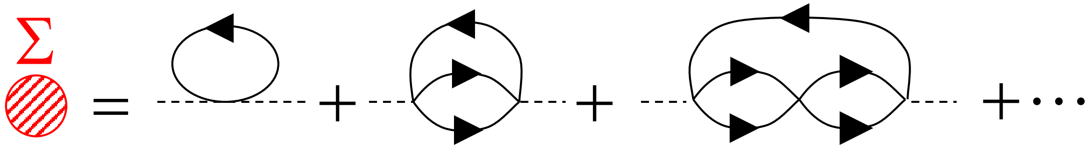
The origin of the pseudogap in HTSC is **still controversial**.

The pseudo gap in the quark spectra in the quark matter prior to CSC

M. Kitazawa, T. Koide, Y. Nemoto and T.K., Phys. Rev. D70 (2004)

$$G(\mathbf{k},\omega_n) = \frac{1}{G^0(\mathbf{k},i\omega_n) - \Sigma(\mathbf{k},i\omega_n)} \qquad G^0(\mathbf{k},i\omega_n) = \left[(i\omega_n+\mu)\gamma^0 - \mathbf{k}\cdot\vec{\gamma}\right]^{-1} =$$

$\Sigma(\mathbf{k},\omega_n) =$



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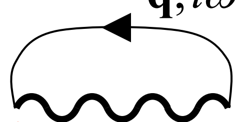
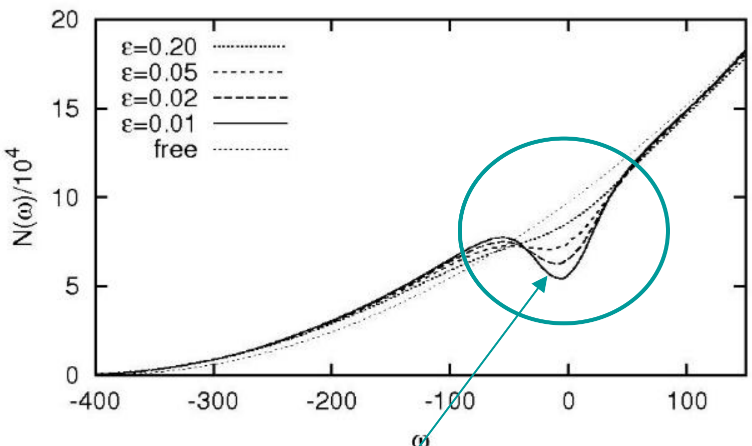


$$= T\sum_m \int \frac{d^3q}{(2\pi)^3} \Xi(\mathbf{k}+\mathbf{q},\omega_n+\omega_m) G^0(\mathbf{q},\omega_m)$$

Soft mode



Pseudo gap !

A considerable depression in the quark level density; an incomplete gap, i.e., **the pseudogap!**

Quite interesting!, but may be difficult to experimentally confirm it, unfortunately.

Is there any chance to probe the enhanced diquark soft modes of CSC in an experiment?

Functional RG approach with meson-quark model

Takeru Yokota, Kenji Morita and TK, PTEP (2016) 073D01

$$S_\Lambda\left[\overline{\psi},\psi,\phi=(\sigma,\vec{\pi})\right]=\int_0^{\frac{1}{T}}d\tau\int d^3\vec{x}\left\{\overline{\psi}\left(\partial\!\!\!/+g_s(\sigma+i\vec{\tau}\cdot\vec{\pi}\gamma_5)-\mu\gamma_0\right)\psi+\frac{1}{2}(\partial_\mu\phi)^2+V_\Lambda(\phi^2)-c\sigma\right\}$$

$$V_\Lambda(\phi^2)=\frac{1}{2}m_\Lambda^2\phi^2+\frac{1}{4!}\lambda(\phi^2)^2$$

Λ	m_Λ/Λ	λ_Λ	c/Λ^3	g_s
1000MeV	0.794	2.00	0.00175	3.2

R. Tripolt, L. Smekal, J. Wambach, PRD90 (2014)

Vacuum value

M_q	M_π	M_σ	σ_0
286MeV	137MeV	496MeV	93MeV

M_q is the constituent quark mass.

$$M_q=g_s\sigma_0$$

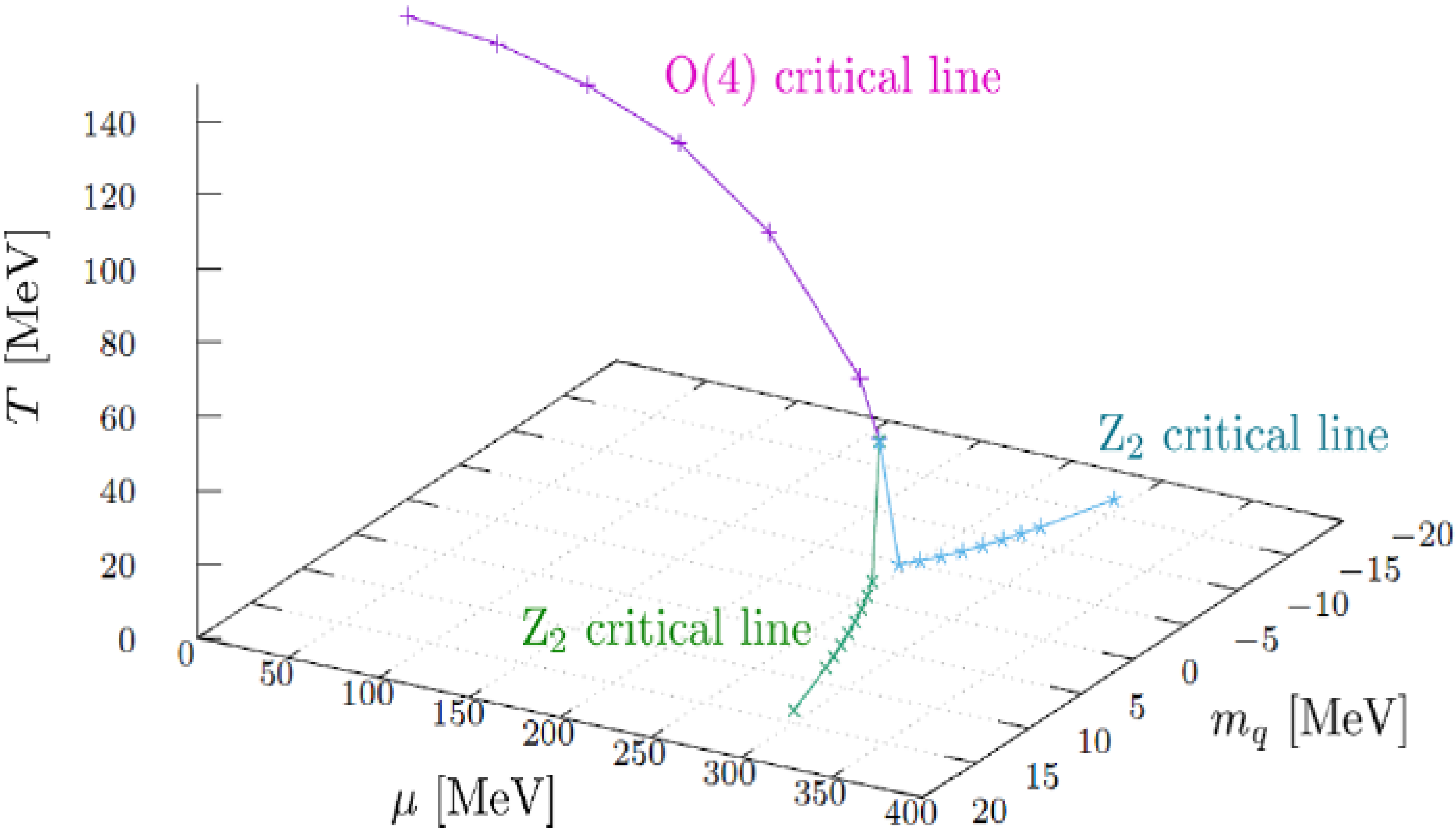
M_π and M_σ are the screening masses.

$$M_\pi=\frac{1}{\sigma_0}\frac{\partial U_0}{\partial\sigma}(\sigma_0)$$

$$M_\sigma=\frac{\partial^2 U_0}{\partial\sigma^2}(\sigma_0)$$

Quark-mass dependence of the phase transition by FRG analysis with meson-quark model

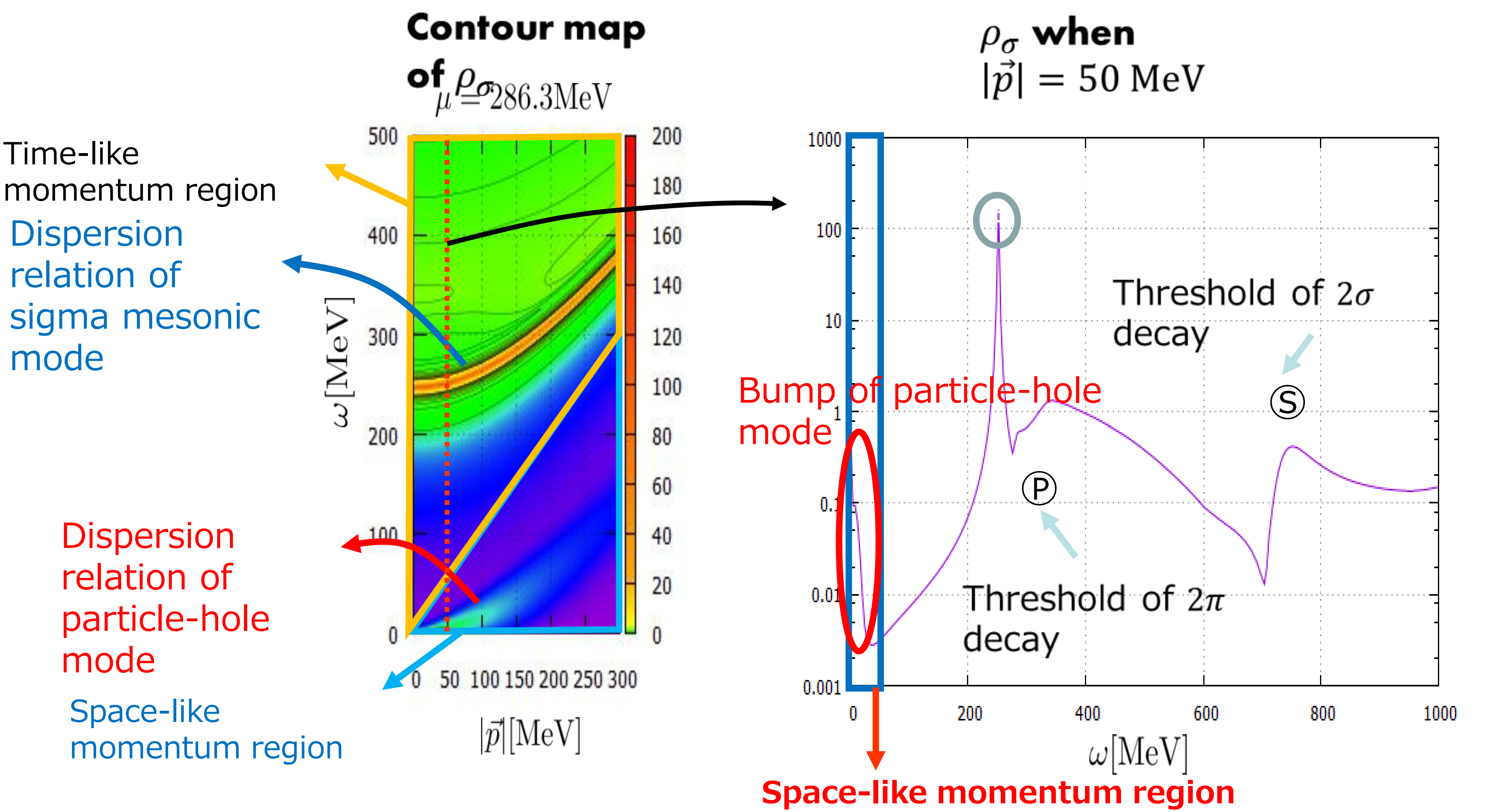
Takeru Yokota, Kenji Morita and TK, PTEP (2016) 073D01



Spectral function of scalar-density collective mode around QCDCP

Takeru Yokota, Kenji Morita and TK, PTEP (2016) 073D01

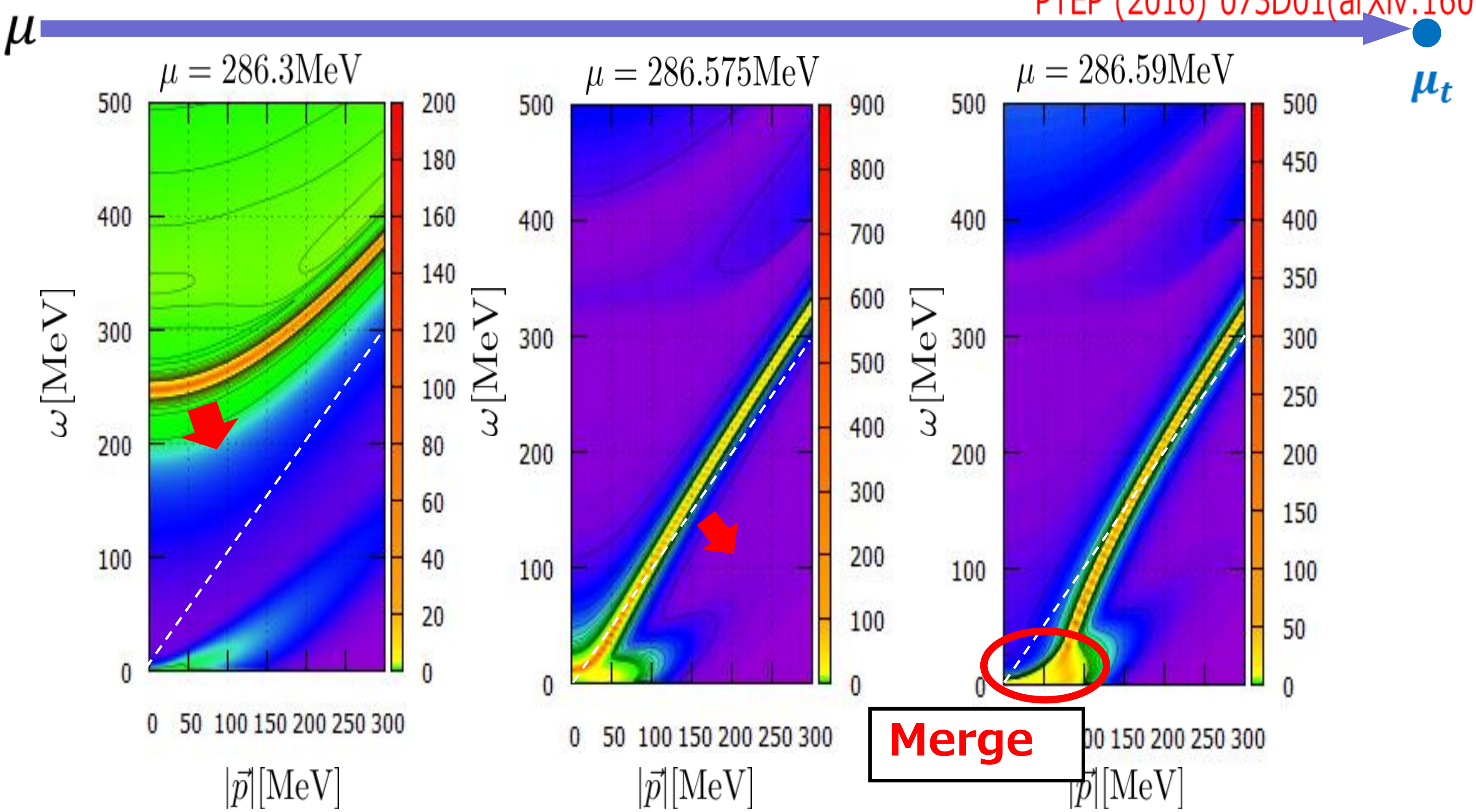
We fix $\boldsymbol{T} = \mathbf{5.1\ MeV}$ below. (At this temperature, the transition chemical potential is $\boldsymbol{\mu_t} = \mathbf{286.69\ MeV}$)



Anomalous softening of the sigma once located in the time-like region to merge into the phonon mode

We fix $\boldsymbol{T} = \mathbf{5.1}\ \mathbf{MeV}$. (At this temperature, the transition chemical potential is $\boldsymbol{\mu_t} = \mathbf{286.69}\ \mathbf{MeV}$)

Takeru Yokota, Kenji Morita and TK,
PTEP (2016) 073D01(arXiv:1603.02147)



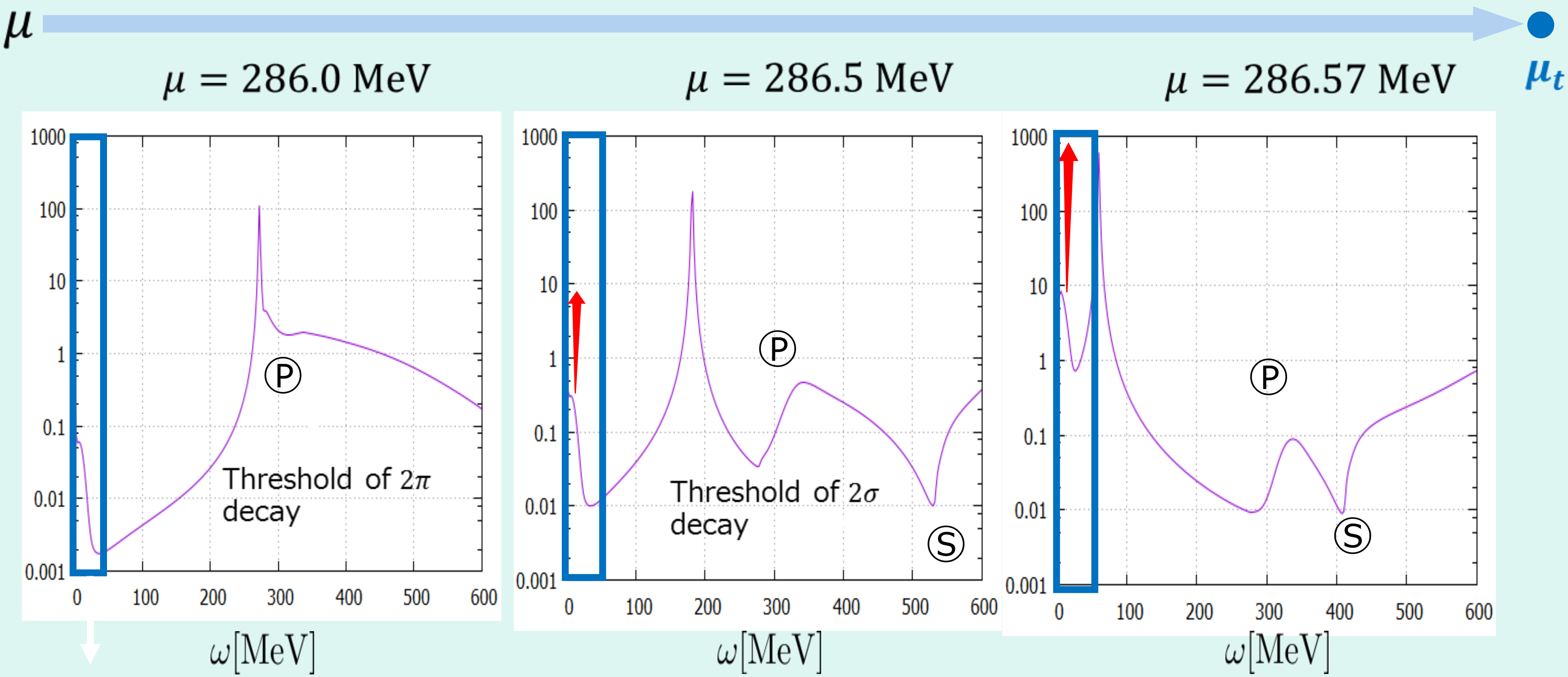
As the system approaches the QCD CP, the dispersion relation of sigma mesonic mode shifts

to low-energy region and merges with the bump of the particle-hole modes in the space-like momentum region.

Development of the phonon mode near the QCD CP

Takeru Yokota, Kenji Morita and TK,
PTEP (2016) 073D01(arXiv:1603.02147)

- We fix $\boldsymbol{T} = \mathbf{5.1}$ MeV and $|\vec{\boldsymbol{p}}| = \mathbf{50}$ MeV. (At this temperature, the transition chemical potential is $\boldsymbol{\mu_t} = \mathbf{286.69}$ MeV)



- The particle-hole mode becomes soft.

A remark to be omitted.

The merge of the sigma and phonon and appearance of the tachyonic mode @ m_q=8.88 MeV, which implies a larger scalar-density coupling.

Smaller current quark mass with smaller scalar-density coupling shows no such anomalous behavior.

Appearance of an acausal tachyonic mode at finite momenta!

↓

Suggesting an instability of the system, to an inhogeneous phase,with modurated sigma field as well as the density, no pion field, i.e., of the real kink type.

For the detailed analysis, see
T. Yokota, K. Morita and T.K., Phys. Rev. D96, 074028 (2017).

c.f. Real Kink type: D.Nickel, M. Buballa (2010).
DCDW: Nakano, Tatsumi (2005); chiral limit.

★ Importance of Vector-type Interaction in CSC

- Vector interaction naturally appears in the effective theories.
  - Instanton-anti-instanton molecule model Shaefer,Shuryak ('98)

$$L = G\left\{\frac{2}{N_C^2}\left[(\bar{\psi}\tau^a\psi)^2 + (\bar{\psi}\tau^a i\gamma_5\psi)^2\right] - \frac{1}{2N_C^2}\left[(\bar{\psi}\tau^a\gamma^\mu\psi)^2 + (\bar{\psi}\tau^a\gamma^\mu\gamma_5\psi)^2\right]\right\} + L_8$$

$\Rightarrow$ $G_V / G_S = 1/4$

  - Renormalization-group analysis N.Evans et al. ('99)

$$L_{LL}^0 = G_{ll}\left\{(\bar{\psi}_L\gamma^0\psi_L)^2 - (\bar{\psi}_L\gamma^\mu\psi_L)^2\right\}$$

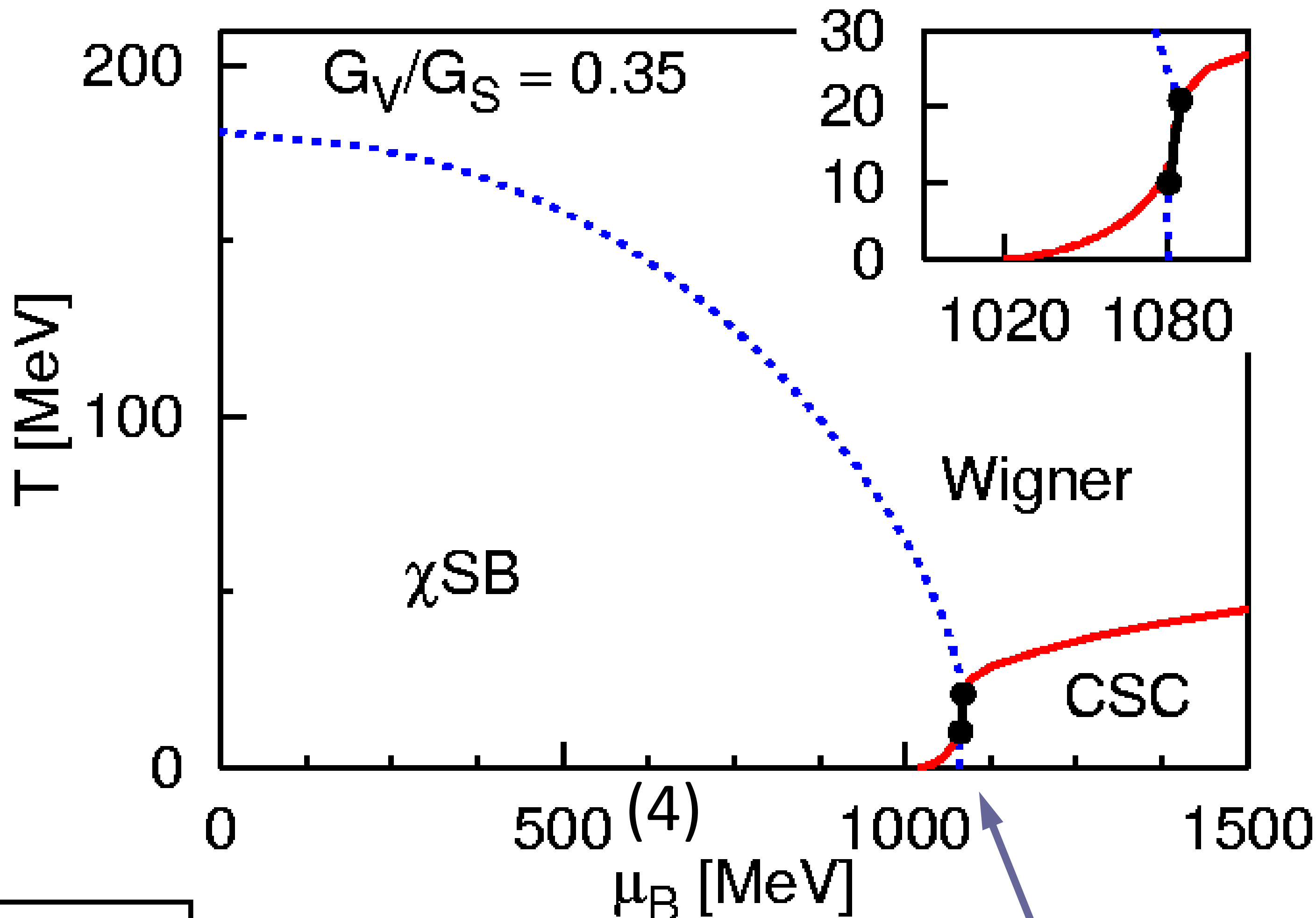
$-G_V\left(\bar{\psi}\gamma^\mu\psi\right)^2$ $\Leftrightarrow$ density-density correlation

- $-G_V\left(\bar{\psi}\gamma^0\psi\right)^2 \rightarrow -G_V\left\langle\bar{\psi}\gamma^0\psi\right\rangle^2 = -G_V\rho^2 \quad \rho = \left\langle\bar{\psi}\gamma^0\psi\right\rangle$

Chiral restoration is punished by the vector interaction!

With color superconductivity transition incorporated: Two critical end point!

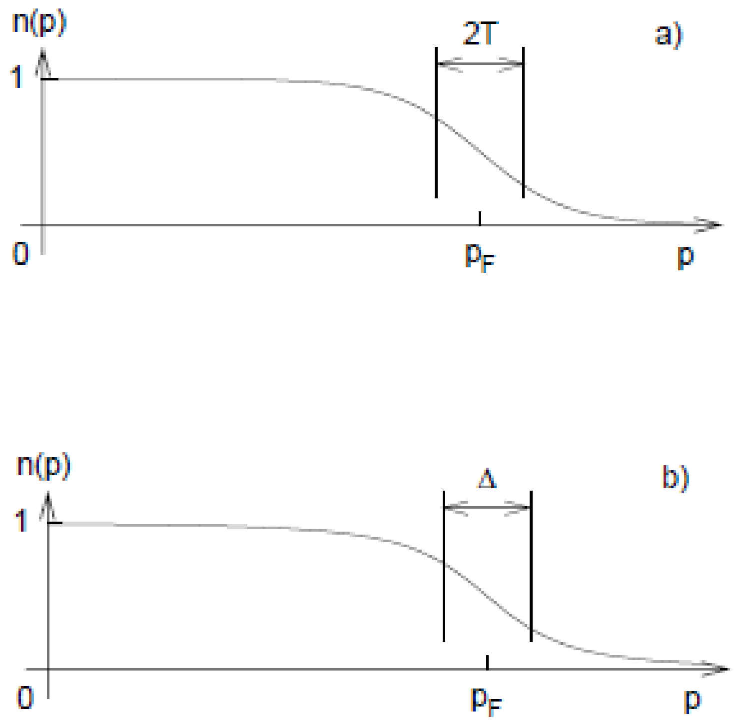
M. Kitazawa, T. Koide, Y. Nemoto and T.K., PTP ('02)



$G_V / G_S = 0.35$

Another end point appears from lower temperature, and hence **there can exist <u>two end points</u>** in some range of G_V!

Similarity of the effect of temperature and pairing gap on the chiral condensate.



M. Kitazawa et al.
PTP, 110 (2003), 185:
arXiv:hep-ph/0307278

T

↕

Δ

An enhancement of dilepton-pair production

$$\frac{dR_{ee}}{d^4q} = -\frac{\alpha}{12\pi^4 Q^2}\,\mathrm{Im}\,\Pi^{R\mu}_{\mu}\,\frac{1}{e^{q^0/T}-1}$$

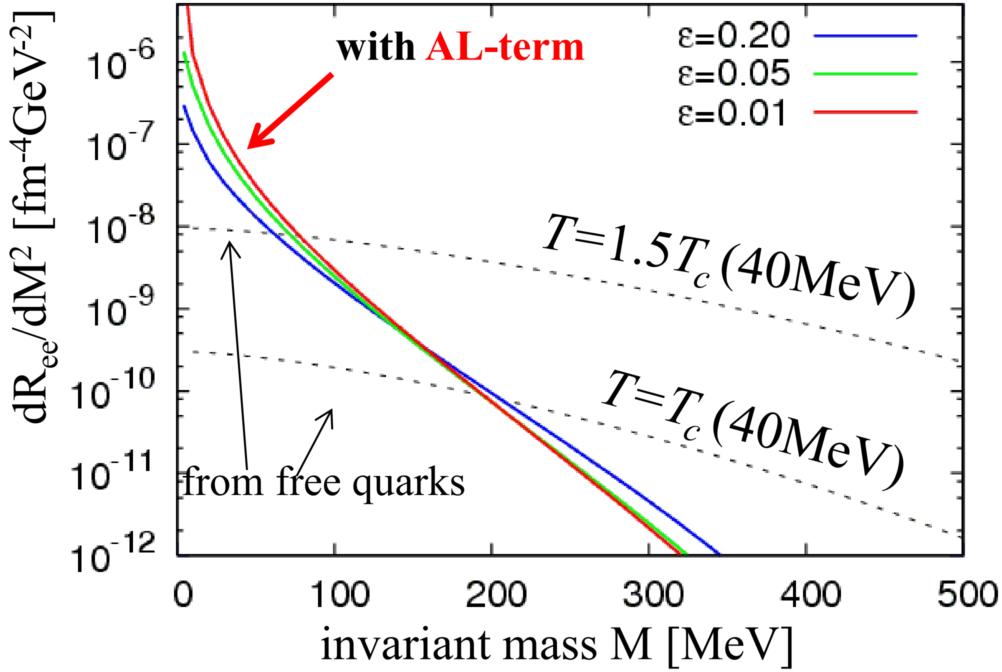
-per invariant mass

$$\frac{dR_{ee}}{dM^2} = \int \frac{d^3q}{2q^0}\frac{dR_{ee}}{d^4q}$$

Prominent enhancement at M<150MeV.

The peak becomes sharp as $\varepsilon \to 0$.

M. Kitazawa, Y. Nemoto and T.K., PoS CPOD07 (2007) 041;arXiv.0711.4429



c.f. other channel: **anomalous absorption of sound;**

B.O. Kerbikov and M.S. Lukashov, arXiv:1607.00125

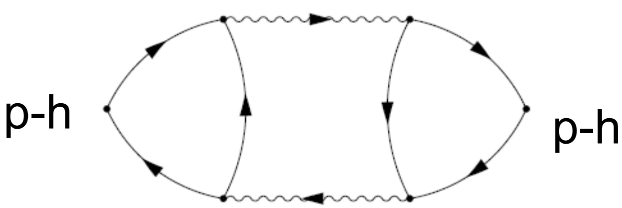


FIG. 2. Feynman diagram for the AL polarization operator for the sound absorption.

$$\mathrm{Im}\,\Pi = -\,\omega\, g^2\,\frac{m^4}{2^5 p_0^4 \varkappa^3}\,(v_0^2+1)^2\,\ln^2\frac{\omega_D}{2\pi T_c}\left(\frac{T_c}{T-T_c}\right)^{3/2}$$

→ enhanced near CP!

Unfortunately,
a weak coupling calculation a la BCS theory.

Spectral function of density fluctuations

The density fluctuation depends on the transport as well as thermodynamic quantities which show an anomalous behavior around the critical point.

Especially, the existence of the density-temperature coupling.

Missing in the previous analyses.

For non-relativistic case with use of Navier-Stokes eq.
L.D. Landau and G.Placzek(1934),
L. P. Kadanoff and P.C. Martin(1963),
R. D. Mountain, Rev. Mod. Phys. 38 (1966), 38
H.E. Stanley, `Intro. To Phase transitions and critical phenomena'
(Clarendon, 1971)

We apply for the first time relativistic hydrodynamic equations to analyze the spectral properties of density fluctuations, and examine possible critical phenomena.

Notice: The 1^{st}-order can be valid for describing the hydrodynamic modes with a long wave-length without encountering the causality problem.

Relativistic Hydrodynamics

Y.Minami, TK, Prog.Theor.Phys. 122 (2010) 881-910 • e-Print: 0904.2270 [hep-th]

$$\begin{cases} \partial_\mu T^{\mu\nu} = 0 \\ \partial_\mu N^\mu = 0 \end{cases}$$

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu} + \tau^{\mu\nu}$$

$$N^\mu = n u^\mu + \nu^\mu$$

$\tau^{\mu\nu},\ \nu^\mu$ dissipative terms

(1) Energy-frame

$$\tau^{\mu\nu} = \eta[\,\partial_\perp^\mu u^\nu + \partial_\perp^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}(\partial_\perp \cdot u)\,] + \zeta\Delta^{\mu\nu}(\partial_\perp \cdot u)$$

$$\nu^\mu = \kappa(\frac{nT}{w})^2 \partial_\perp^\mu (\frac{\mu}{T})$$

$u_\mu \nu^\mu = 0 \qquad u_\mu \tau^{\mu\nu} = 0$

(2) Particle frame ; Eckart(1940), unstable $u_\mu \tau^{\mu\nu} u_\nu = 0$

Tsumura-Kunihiro-Ohnishi, Phys.Lett.**B646**(2007);

Tsumura & T.K., PL**B668(2008)**.

$$\nu^\mu = 0$$

$$\tau^{\mu\nu} = \eta[\,\partial_\perp^\mu u^\nu + \partial_\perp^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}(\partial_\perp \cdot u)\,] - \zeta'(3u^\mu u^\nu - \Delta^{\mu\nu})(\partial_\perp \cdot u) + \kappa(u^\mu \partial_\perp^\nu + u^\nu \partial_\perp^\mu)T$$

$\tau^\mu_\mu = 0$

(3) Israel-Stewart

Linear approximation around the thermal equilibrium;

$$u^\mu(\vec{r},t) = u_0^\mu + \delta u^\mu(\vec{r},t) \qquad n(\vec{r},t) = n_0 + \delta n(\vec{r},t)$$ etc

In the rest frame of the fluid,

$$u_0^\mu = (1,0) \qquad \delta u^\mu(\vec{r},t) = (0,\vec{v}(\vec{r},t))$$

Inserting them into $T^{\mu\nu}, \ N^\mu$, and taking the linear approx.

- Linearized Landau equation (Lin. Hydro in the energy frame);

$$\frac{\partial \delta n}{\partial t} + n_0 \nabla \cdot \vec{v} - \kappa \frac{n_0}{w_0}\left[\frac{T_0}{w_0}\nabla^2(\delta P) - \nabla^2(\delta T)\right] = 0$$

$$w_0 \frac{\partial \vec{v}}{\partial t} - \eta \nabla^2 \vec{v} - (\frac{1}{3}\eta + \zeta)\nabla(\nabla \cdot \vec{v}) + \nabla(\delta P) = 0$$

$$n_0 \frac{\partial \delta s}{\partial t} - \frac{\kappa}{T_0}\nabla^2(\delta T) + \frac{\kappa}{w_0}\nabla^2(\delta P) = 0$$

Rel. effects

with

$$\delta P(x) = \frac{w_0 c_s^2}{n_0 \gamma}\delta n(x) + \frac{w_0 c_s^2 \alpha_P}{\gamma}\delta T(x) \qquad \delta s(x) = -\frac{w_0 c_s^2 \alpha_P}{n_0^2 \gamma}\delta n(x) + \frac{\tilde{c}_n}{T_0}\delta T(x)$$

Solving δn as an initial value problem using Laplace transformation, we obtain $S_{nn}(\vec{k},\omega) = <\delta n(\vec{k},\omega)\delta n(\vec{k},t=0)>$, in terms of the initial correlation.

Spectral function of density fluctuations in the Landau frame

Y.Minami, TK, Prog.Theor.Phys. 122 (2010) 881-910 • e-Print: 0904.2270 [hep-th]

In the long-wave length limit, k→0

thermal mode

sound modes

$$\frac{S_{nn}(\vec{k},\omega)}{< (\delta n(\vec{k},t=0))^2 >} = (1-\frac{1}{\gamma})\frac{2\Gamma_R k^2}{\omega^2+\Gamma_R^2 k^4} + \frac{1}{\gamma}\left(\frac{\Gamma_B k^2}{(\omega+c_s k)^2+\Gamma_B^2 k^4} + \frac{\Gamma_B k^2}{(\omega-c_s k)^2+\Gamma_B^2 k^4}\right)$$

Rel. effects appear only in the width of the peaks.

Rel. effects appear only in the sound mode.

$$\Gamma_R = \chi \qquad \Gamma_B = \Gamma + \frac{1}{2} c_s^2 T_0 (\kappa / w_0 - 2\chi\alpha_P)$$

$$\Gamma = \frac{1}{2}[\chi(\gamma-1)+\nu_l]$$

α_P rate of isothermal exp.

thermal expansion rate: $\chi = \frac{\kappa}{n_0 C_P}$

c_s : sound velocity γ : specific heat ratio Long. Dynamical : $\nu_l = (\zeta + \frac{4}{3}\eta)/w_0$

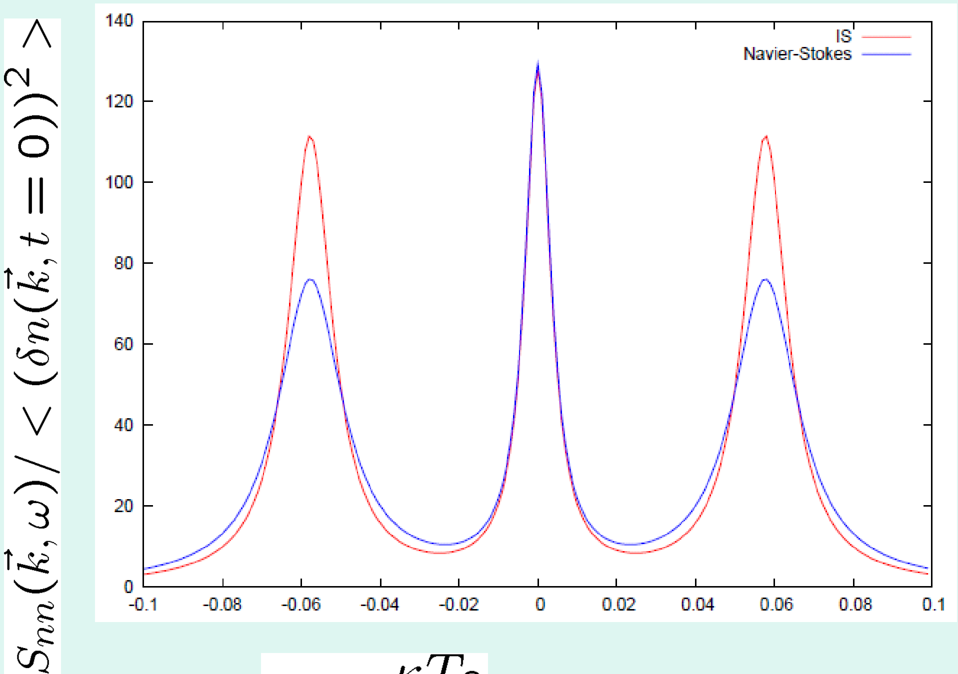
enthalpy

Notice: $\gamma = c_p / c_n = t^{-\gamma+\alpha} \rightarrow \infty$

As approaching the critical point, the ratio of specific heats diverges!

The strength of the sound modes vanishes out at the critical point.

Spectral function from I-S eq.



I-S

"Non-relativistic"

ω [1/fm]

$\Gamma_B = \Gamma - c_s^2 T_0 (2\chi\alpha_P - \kappa/w_0)$

For $\tau_\kappa > \dfrac{\kappa T_0}{w_0}$

$$\frac{S_{nn}(\vec{k},\omega)}{<(\delta n(\vec{k},t=0))^2>} = (1-\frac{1}{\gamma})\frac{2\chi k^2}{\omega^2+\chi^2 k^4} + \frac{1}{\gamma}(\frac{\Gamma_B k^2}{(w+c_s k)^2+\Gamma_B^2 k^4} + \frac{\Gamma_B k^2}{(w-c_s k)^2+\Gamma_B^2 k^4})$$

$$+O(k^2)\times[\frac{1/\tau_\zeta}{\omega^2+(1/\tau_\zeta)^2} + \frac{1/\tau_\eta}{\omega^2+(1/\tau_\eta)^2} + \frac{w_0/(\tau_\kappa w_0 - \kappa T_0)}{\omega^2+[w_0/(\tau_\kappa w_0-\kappa T_0)]^2}]$$

↓

No contribution in the long-wave length limit k→0.

Conversely speaking, the first-order hydro. Equations have no problem to describe the hydrodynamic modes with long wave length, as it should.

Critical behavior of the density-fluctuation spectral functions

Y.Minami, TK, Prog.Theor.Phys. 122 (2010) 881-910 • e-Print: 0904.2270 [hep-th]

$$S_{nn}(\vec{k},\omega) = <(n(\vec{k},t=0))^2> [\ (1-\frac{1}{\gamma})\frac{2\Gamma_R k^2}{\omega^2+\Gamma_R^2 k^4} + \frac{1}{\gamma}\{\frac{\Gamma_B k^2}{(\omega - c_s k)^2 + \Gamma_B^2 k^4} + \frac{\Gamma_B k^2}{(\omega + c_s k)^2 + \Gamma_B^2 k^4}\}\ ]$$

$$\gamma = c_p / c_n = t^{-\gamma+\alpha} \to \infty$$

$$<(n(\vec{k},t=0))^2> \frac{2\Gamma_R k^2}{\omega^2+\Gamma_R^2 k^4}$$

$$<(n(\vec{k},t=0))^2> \sim (|T-T_c| + k^2)^{-1}$$

Critical opalescence

In the vicinity of CP, only the Rayleigh peak stay out, while the sound modes (Brillouin peaks) die out.

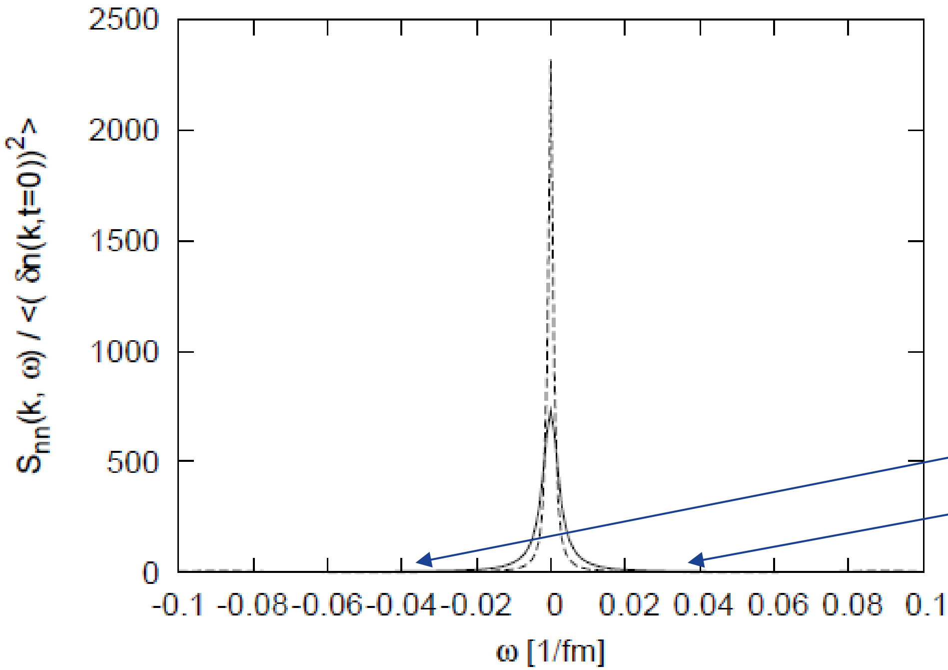
c.f. The dynamical critical exponent.

$$z = 2 + (\gamma - a)/\nu\ □\ 3$$

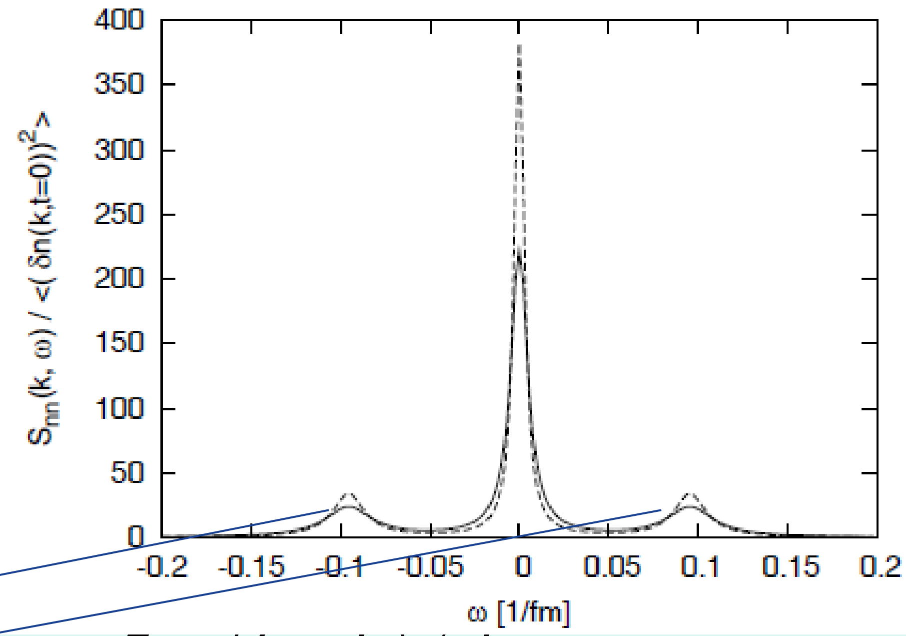
So, the divergence of Γ_B and the viscosities therein can not be observed, unfortunately.

Spectral function of density fluctuation at CP

$t = 0.1$



0.4



$t \equiv (T - T_c)/T_c =$

The sound mode (Brillouin) disappears
Only an enhanced thermal mode remains.
Furthermore, the Rayleigh peak is enhanced, meaning the large energy dissipation.

Spectral function at CP

The soft mode around QCD CP is thermally induced density fluctuations, but not the usual sound mode.

Suggesting interesting critical phenomena related to sound mode.

Y.Minami, TK, Prog.Theor.Phys. 122 (2010) 881-910 • e-Print: 0904.2270 [hep-th]

Why at all do sound modes die out at the Critical Point ?

The correlation length

$$\xi = \xi_0 t^{-\nu}$$

The wave length of sound mode

$$\lambda_s$$

The hydrodynamic regime!

$$\xi \ll \lambda_s$$

However, around the critical point $\xi \longrightarrow \infty$ as $t \longrightarrow 0$

$$\lambda_s \ll \xi$$

The system behaves as an aggregate of clumps of matter with a diameter ξ .

So the hydrodynamic sound modes can not be developed around CP!

Y.Minami, TK, Prog.Theor.Phys. 122 (2010) 881-910 • e-Print: 0904.2270 [hep-th]