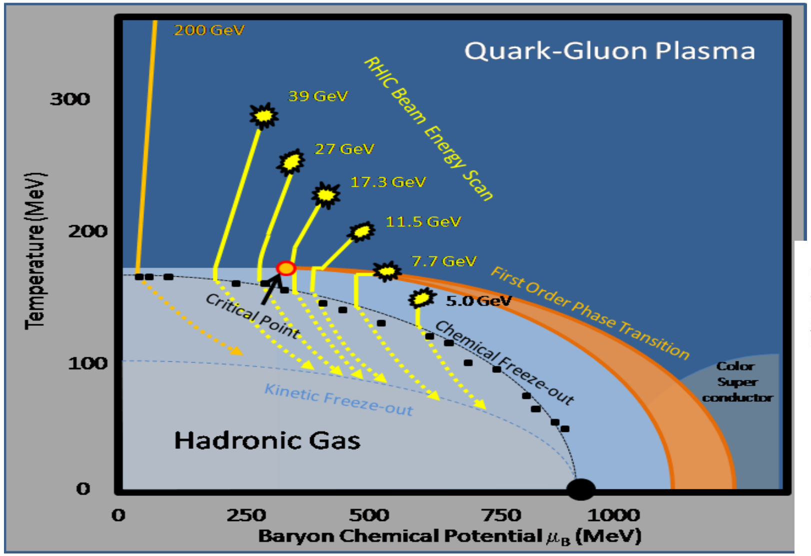
A new relativistic quantum molecular dynamics for heavy-ion collisions

Yasushi Nara (Akita International Univ.)

- Introduction
- Relativistic quantum molecular dynamics (RQMD) in JAM2
- Interaction models in JAM2

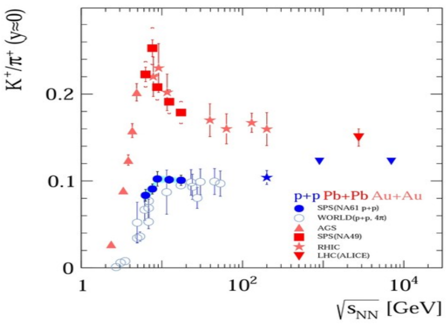
QCD critical point and hydrodynamic evolution 2026/6/1-4

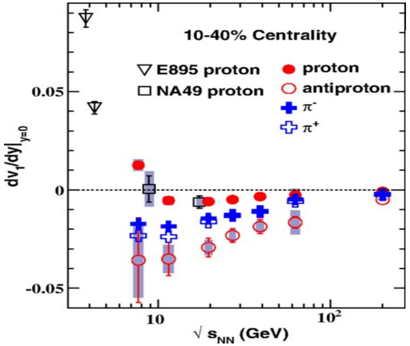
Search for the QCD equation of state (EoS) by the beam energy scan

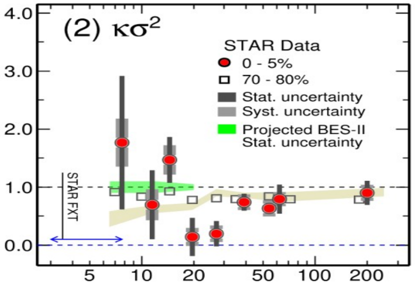


Lattice QCD has not covered the FAIR, NICA J-PARC energy regions.

On going RHIC BES II, FXT and NA61/SHINE explore the phase structure of QCD matter. New experiments FAIR, NICA, J-PARC-HI,HIAF are planned.







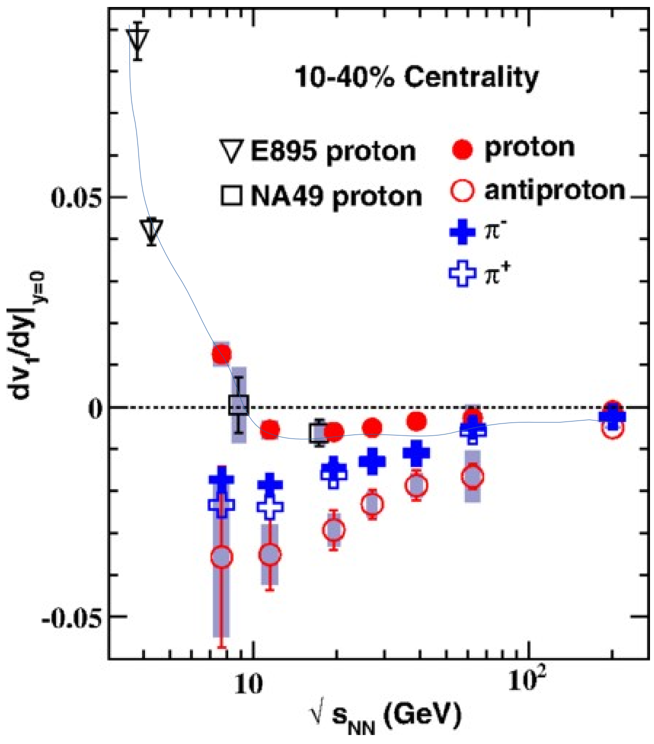
Discovery of non-monotonic behavior of beam energy dependence in K+/pi+,v1,kurtosis,$N_t N_p/N_d^2$.

Are they signals of a phase transition?

To extract the information on the EoS, transport model is needed.

Beam energy dependence of v1: discovery of negative flow

L. Adamczyk et al. (STAR Collaboration) Phys. Rev. Lett. 112, 162301 – 23 April 2014



$$v_1(y) = Fy + F_3 y^3, \quad dv_1/dy|_{y=0} = F$$

Proton slope changes sign from positive to negative at 11.5GeV. Signal of a 1st-order phase transition?

But no model with 1st-order phase transition reproduces beam energy dependence of the proton v1 data.

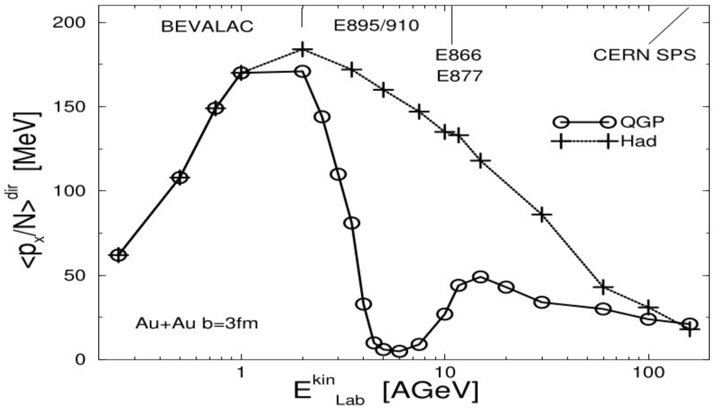
1FD: D.H.Rischke, et.al,Heavy Ion Phys.1, 309 (1995)
ART: B-A. Li and C.M. Ko,PRC58(1998)R1382
3FD: Ivanov, Heavy Ion Phys.15:117-130,2002

Effects of Momentum-dependent potential on the 1st-order PT?

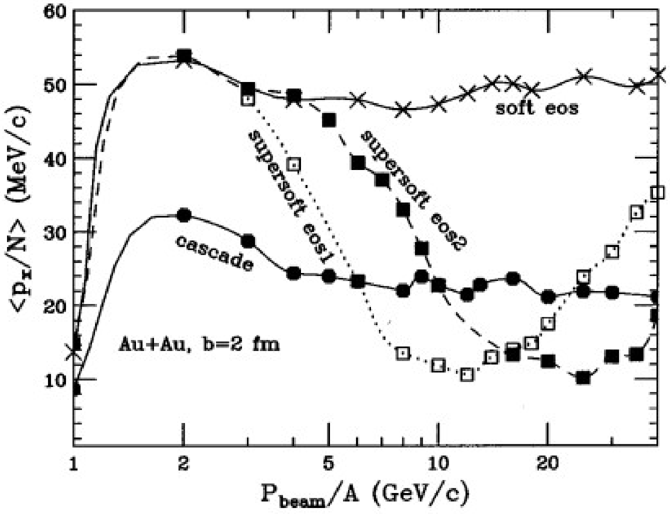
The softest point in the EoS

$$\langle p_x/N\rangle^{dir} = \frac{1}{N}\int_{-y_{CM}}^{y_{CM}} \mathrm{d}y\ \langle p_x/N\rangle(y)\ \frac{\mathrm{d}N}{\mathrm{d}y}\ \mathrm{sgn}(y)$$

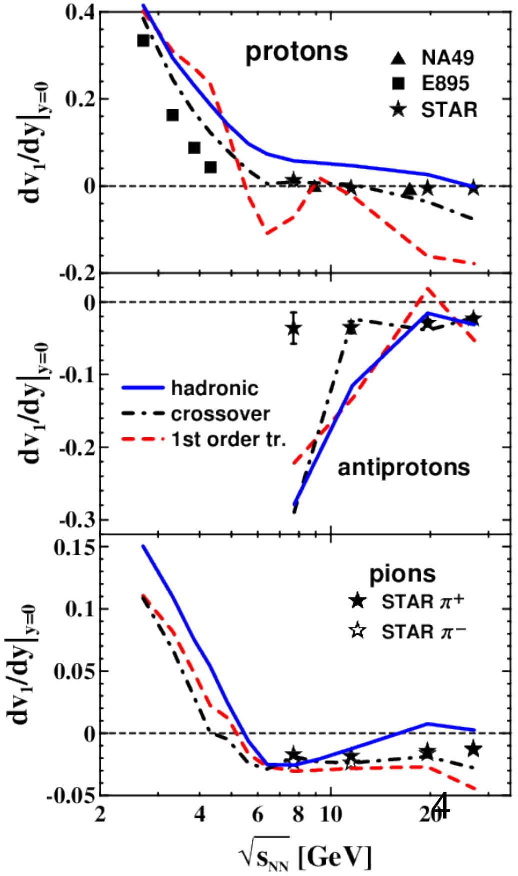
3FD:Ivanov Heavy Ion Phys.15:117-130,2002



1FD: D.H.Rischke, et.al
Heavy Ion Phys.1, 309 (1995)



ART:B-A. Li and C.M. Ko,
PRC58(1998)R1382



A minimum is predicted in the excitation function of the directed flow.

QGP signal: formation of tilted ellipsoid

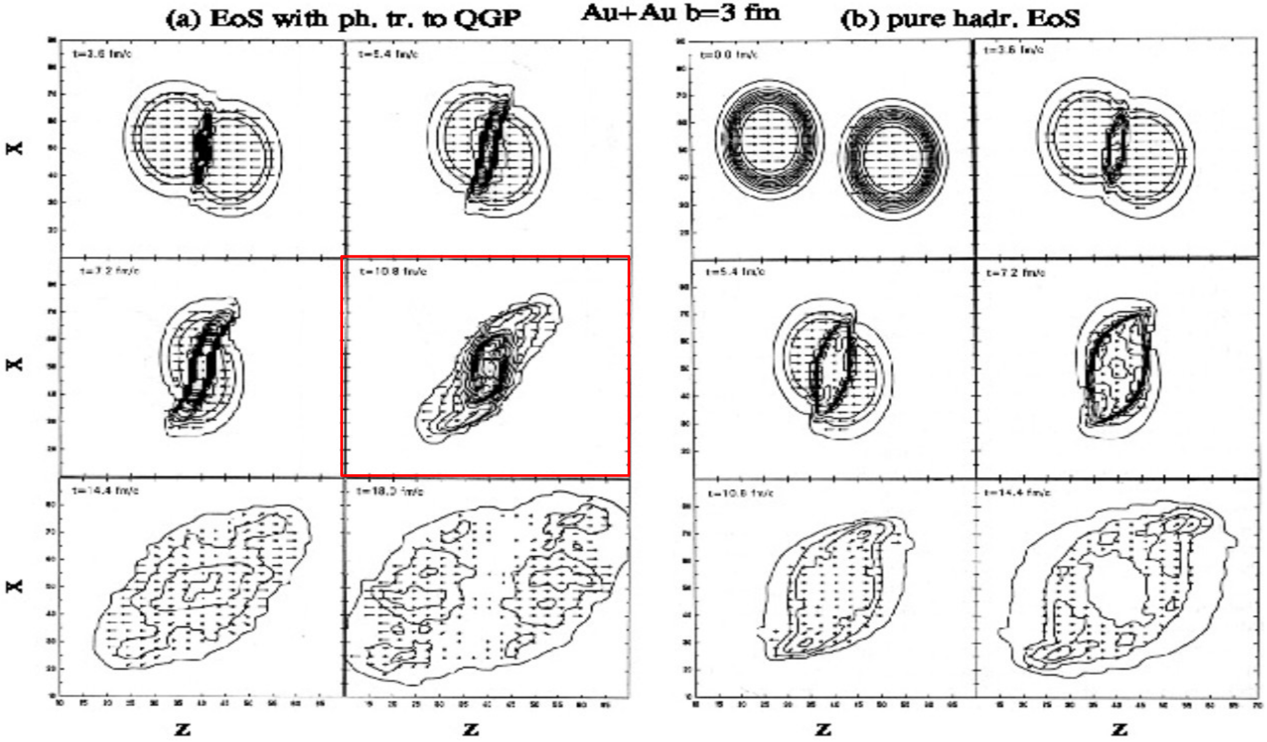
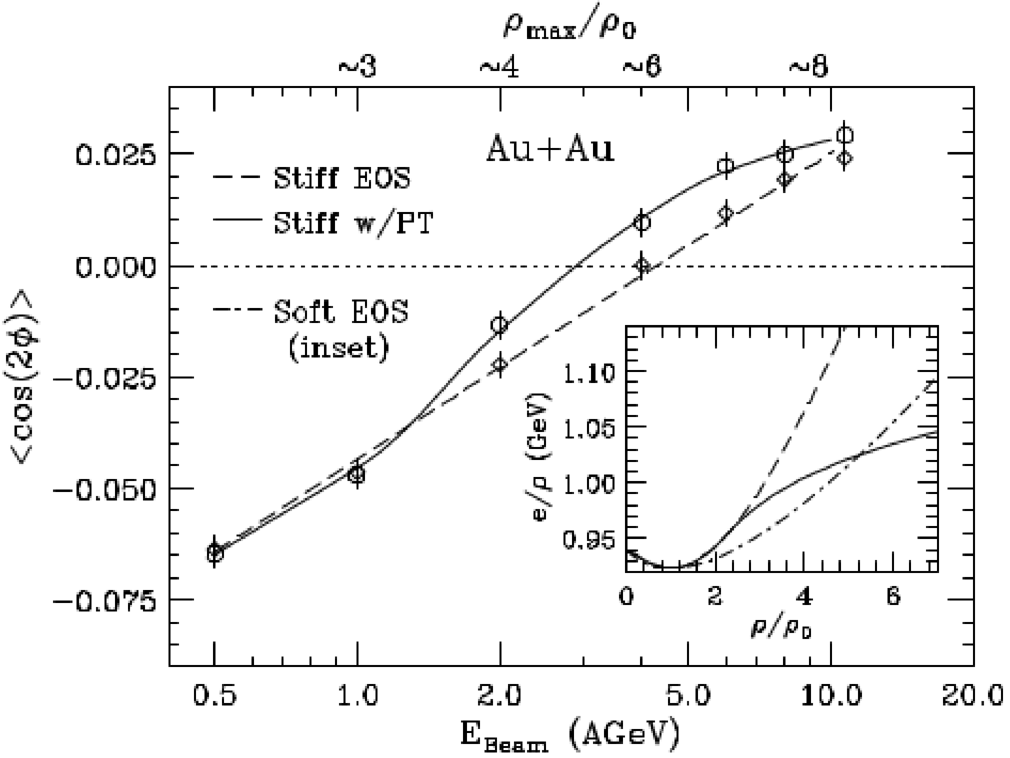


Fig. 5

D.H.Rischke, Y.Pursun,J.A.Maruhn,H.Stoeckeer,W.Greiner, Heavy Ion Phys. 1, 309 (1995)

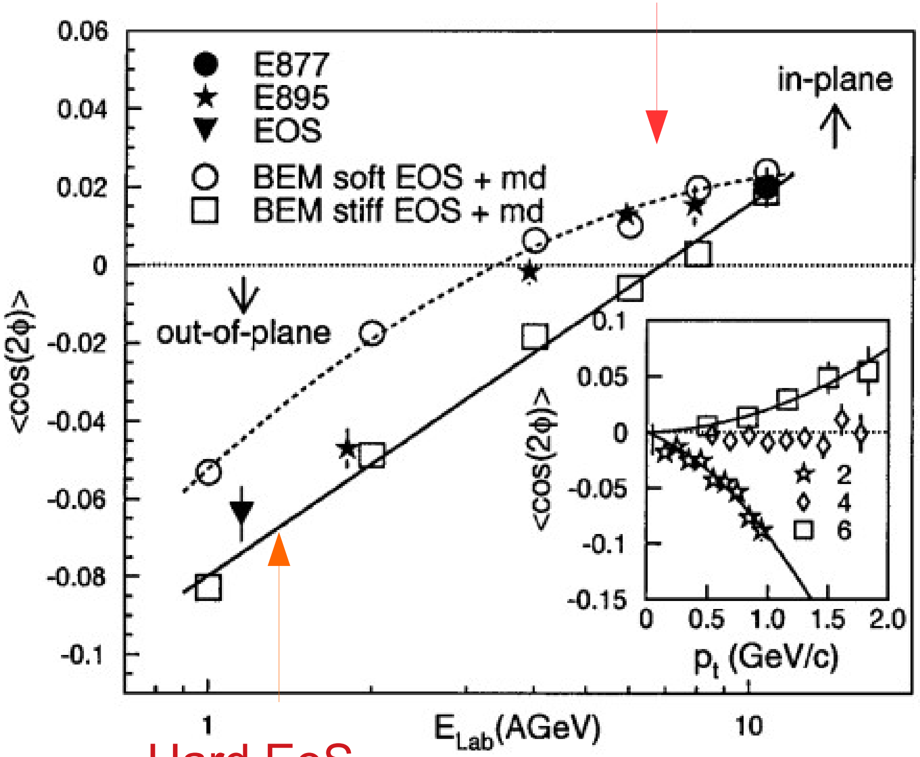
BUU with 2nd order phase transition

P.Danielewicz, PRL81(1998) 2438



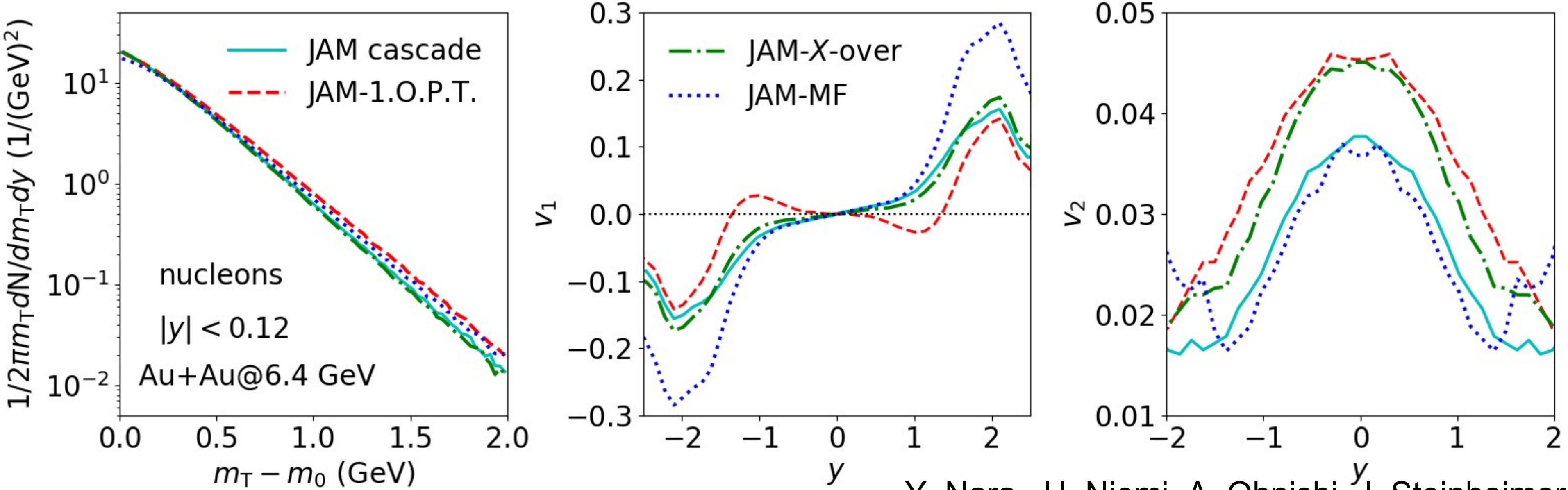
2nd order phase transition

E895 collaboration, PRL83 (1999) 1295



v0,v1, v2 at 6.4 GeV

Enhancement of v2, v4 by 1st-order phase transition



	Mt	v1	v2
Cascade			
Hadronic mean-field	enhanced	positive	reduced
First-order P.T.	enhanced	negative	enhanced
Crossover	same	positive	enhanced

Y. Nara., H. Niemi, A. Ohnishi, J. Steinheimer, X. Luo, H. Stoecker, Eur. Phys. J. A54 (2018) 18

Y.Nara, J. Steinheimer, H. Stoecker, Eur.Phys.J.A54 (2018)188

Combined analysis of v0, v1, and v2 should be very useful.

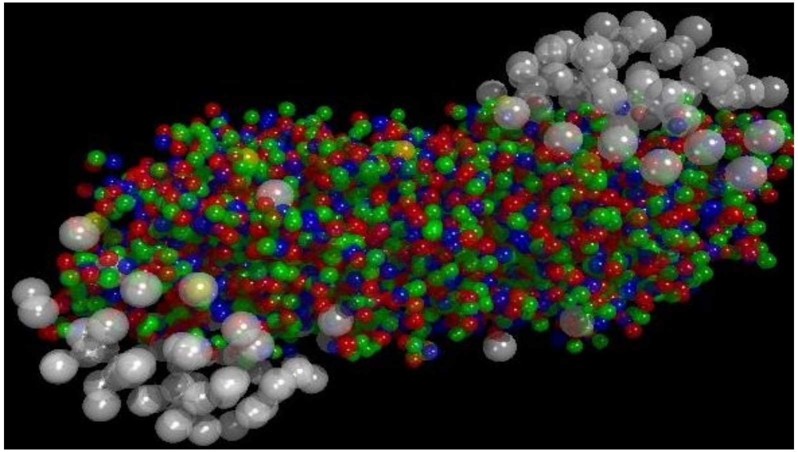
Microscopic transport models (微視的輸送模型)

- Boltzmann-Uehling-Uhlenbeck (BUU)

$$\frac{\partial f}{\partial t} + \frac{\boldsymbol{p}}{m} \cdot \frac{\partial f}{\partial \boldsymbol{r}} - \frac{\partial U}{\partial \boldsymbol{x}} \cdot \frac{\partial f}{\partial \boldsymbol{p}} = I_{\mathrm{coll}}$$

- Quantum molecular dynamics (QMD)

UrQMD simulation



- Boltzmann type collision term
  - ✓ Particle production
- Mean field propagation
  - ✓ Nuclear saturation properties
  - ✓ Equation of state (phase transition)

One can extract EoS even if a system created in heavy-ion collision does not achieve an equilibrium state.

A new relativistic quantum molecular dynamics (RQMD2)

- Covariant propagation of a system of interacting wave packets, which can be numerically solved with the numerical cost compatible to QMD.
- Simulate correct density dependence in potentials with the same computational cost to simulate accurately EoS.
- Use the mass shell condition with a single-particle potential in RQMD2.

$$\text{single-particle potential: } U(n), \quad \text{one-particle potential:} V(n) = \frac{1}{n}\int U(n)dn$$

$$\text{RQMD1: } p^{*\mu} = p^{\mu} - V^{\mu}, \quad m^{*} = m + V_s$$

$$\text{RQMD2: } p^{*\mu} = p^{\mu} - U^{\mu}, \quad m^{*} = m + U_s$$

EoM for new RQMD

$$S_{part} = \sum_{i=1}^{N} \int p_i \cdot dx_i - \int d^4x d^4p \, W(x,p) f(x,p) \quad W(x,p) = \frac{p^{*2}(x,p) - m^{*2}(x,p)}{2}$$

$$\frac{dx_i^\mu}{ds} = \frac{\bar{\Pi}_i^\mu}{\bar{\Pi}_i \cdot \hat{a}}, \quad \frac{dp_i^\mu}{ds} = -\frac{\bar{Q}_i^\mu}{\bar{\Pi}_i \cdot \hat{a}}$$

$$p^{*\mu} = p^\mu - U^\mu, \quad m^* = m + U_s$$

$$\bar{\Pi}_i^\mu = \int d^4x d^4p \, \partial_\mu^p W(x, p_i) g(x - x_i(s)) g(p - p_i(s))$$

$$\bar{Q}_i^\mu = \int d^4x d^4p \, \partial_\mu^x W(x, p_i) g(x - x_i(s)) g(p - p_i(s))$$

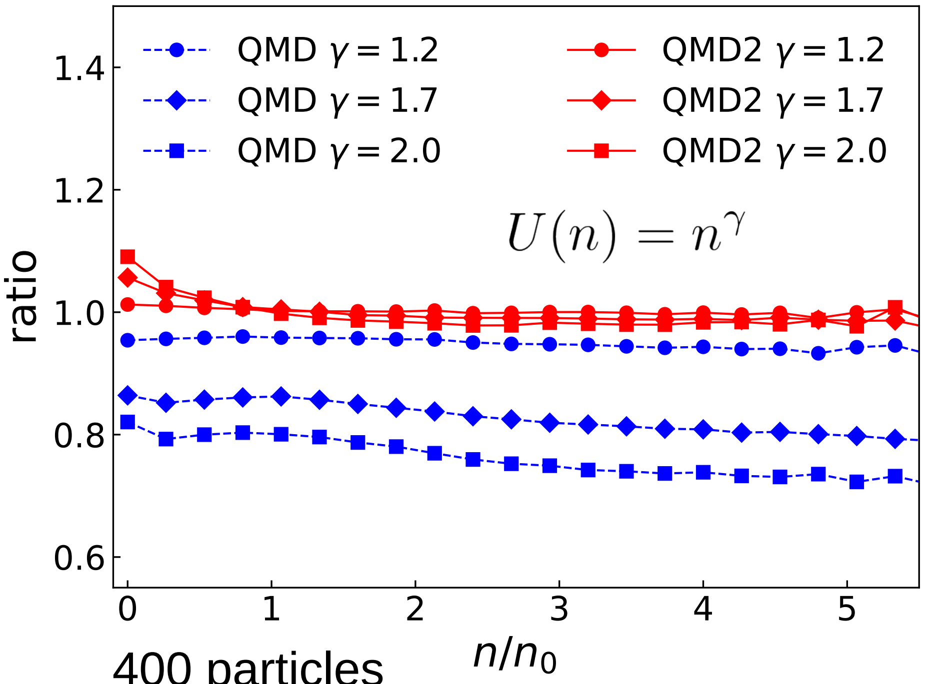
We apply a new approximation to the spatial integral to get the efficient RQMD EoM.

$$g(x - x_i) = \frac{u_i \cdot \hat{a}}{(2\pi L)^{3/2}} e^{\frac{(x-x_i)^2 - [(x-x_i)\cdot u_i]^2}{2L}} \delta((x - x_i(s)) \cdot \hat{a}).$$

Y.N., A. Jinno and K. Murase, [arXiv:2507.23294 [hep-ph]].

New QMD (QMD2) results for U=n$^\gamma$

Ratio = QMD/(numerical integration)



400 particles

Numerical: Monte-Carlo integration of the exact formula:

$$\frac{d\boldsymbol{p}_i}{dt} = -\int d^3x\, U(n) \frac{\partial G(x-x_i)}{\partial \boldsymbol{x}_i}$$

$$\text{QMD:} \quad \frac{d\boldsymbol{p}_i}{dt} = -\sum_{j=1}^{N} \left[\frac{dV(\langle n_i \rangle)}{dn} + \frac{dV(\langle n_j \rangle)}{dn} \right] \frac{\partial n_{ij}}{\partial \boldsymbol{x}_i}$$

$$\text{QMD2:} \quad \frac{d\boldsymbol{p}_i}{dt} = -\sum_{j=1}^{N} \left[\frac{dV(n(x_i))}{dn} + \frac{dV(n(x_j))}{dn} \right] \frac{\partial n_{ij}}{\partial \boldsymbol{x}_i}$$

$$V(n) = \frac{1}{n}\int U(n)dn \quad n_{ij} = \int d^3x G(x-x_i)G(x-x_j)$$

$$\text{interaction density } \langle n_i \rangle = \sum_{j \neq i}^{N} n_{ij} \quad \text{density } n = \sum_{i=1}^{N} \int d^3x G(x-x_i)$$

Covariant cascade method Phys. Rev. C 108, 2 (2023)

We consider 8N-dimentional phase space: $\{x_i^\mu(s), p_i^\mu(s)\}, i = 1, \ldots, N$

Hamiltonian is given by the sum of mass-shell constraints

$$H = \sum_{i=1}^{N} \lambda_i \frac{p_i^2 - m_i^2}{2}, \quad \lambda_i : \text{Lagrange multiplier}$$

Equation of motion:

$$\frac{dx_i}{ds} = \lambda_i p_i$$

which is equivalent to $\frac{dx_i}{dx_i^0} = \frac{p_i}{p_i^0}$, or $\frac{dx_i}{d\tau_i} = \frac{p_i}{m_i}$

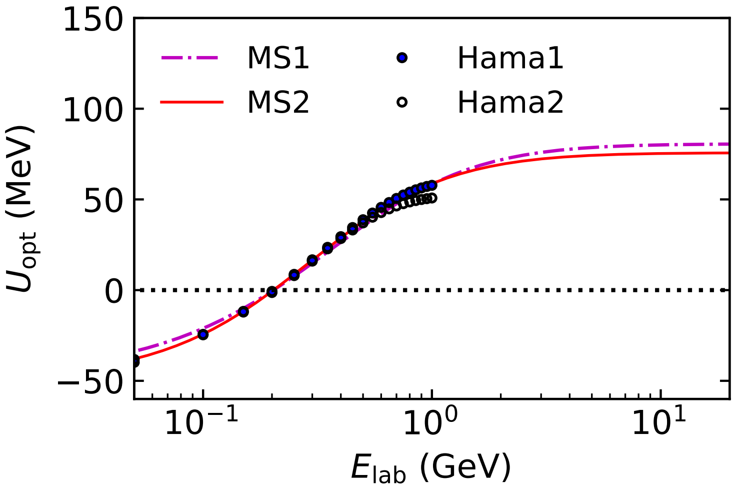
If we assume that time of all particle is the same at a Lorentz frame $\hat{a}$

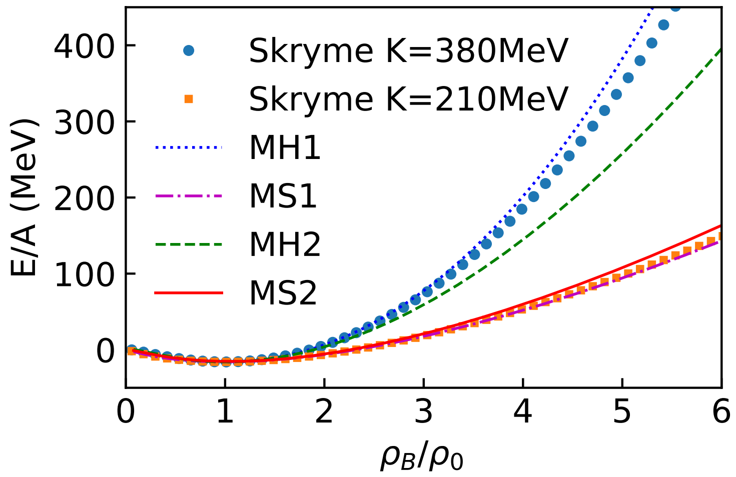
$$\hat{a} \cdot x_i = s, \quad i = 1, \ldots, N \quad \rightarrow \hat{a} \cdot \frac{dx_i}{ds} = 1 \quad \rightarrow \lambda_i = \frac{1}{\hat{a} \cdot p_i}$$

$$\text{EoM:} \quad \frac{dx_i}{ds} = \frac{p_i}{\hat{a} \cdot p_i}$$

Based on this EoM, covariant cascade method can be obtained (MC simulation of Boltzmann type collision term.)

EoS in the JAM2/RQMDv mode





Y.N. and A. Ohnishi, PRC(2022)

Skyrme type Loretnz vector potential:

$$\mathrm{p}^{*\mu} = p^\mu - U^\mu_{\mathrm{sk}}(\rho) - U^\mu_m(p).$$

$$U^\mu_{sk} = U_{sk}(n_B)\frac{J^\mu_B}{n_B}, \quad n_B = \sqrt{J^2_B}$$

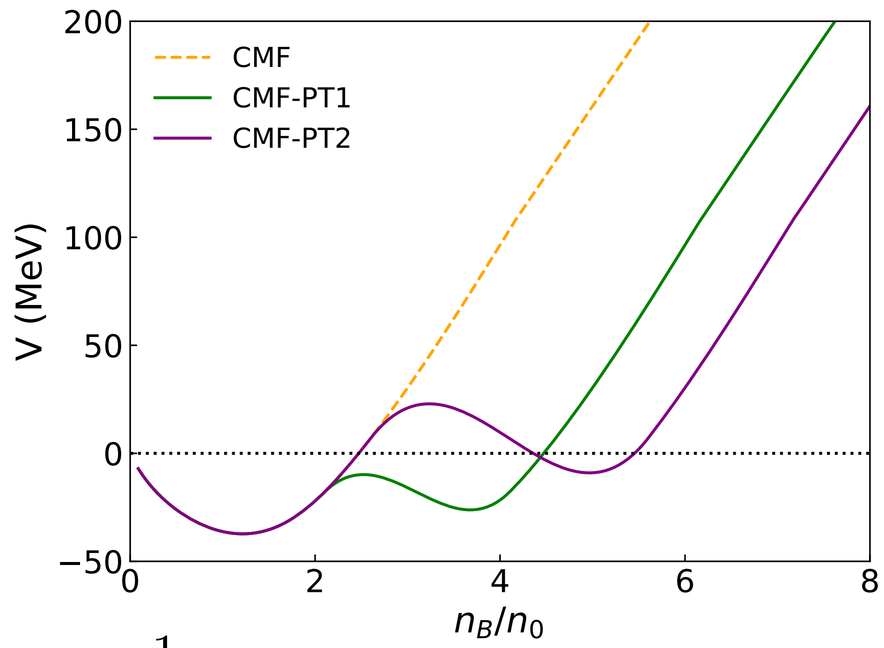
$$U_{\mathrm{sk}}(\rho) = \alpha\left(\frac{\rho}{\rho_0}\right) + \beta\left(\frac{\rho}{\rho_0}\right)^\gamma,$$

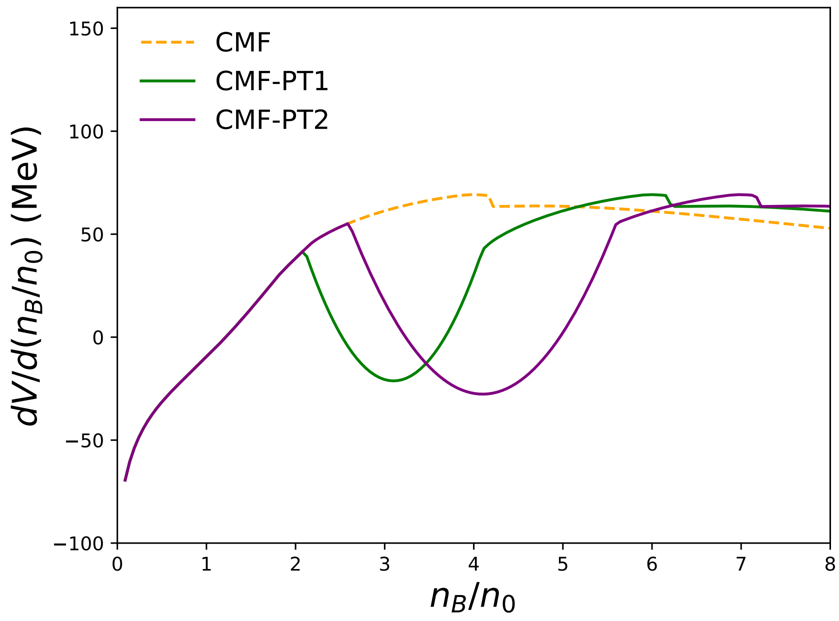
$$U^\mu_m(p) = \frac{C}{\rho_0}\int d^3p' \frac{p^{*\prime\mu}}{e'^*}\frac{f(x,p')}{1+[(\boldsymbol{p}-\boldsymbol{p}')/\mu_k]^2},$$

Chiral mean field (CMF) + 1st OPT

The chiral SU(3)-flavor parity-doublet Polyakov-loop quark-hadron mean-field model

A. Motornenko,et.al PRC103,054908(2021), J.Steinheimer,et.al, EPJC82,911(2022)





$$V = \frac{1}{n}\left[\epsilon - \epsilon_{\mathrm{free}}\right]$$

In JAM2, one can change the location of 1st-order phase transition.

relativistic mean-field

Single particle energy

$$e = \sqrt{m^{*2} + (\boldsymbol{p} - \boldsymbol{U})^2} + U^0 \qquad U^\mu = g_\omega \omega^\mu$$

σ-ω model $\quad m^* = m_N + g_\sigma \sigma$

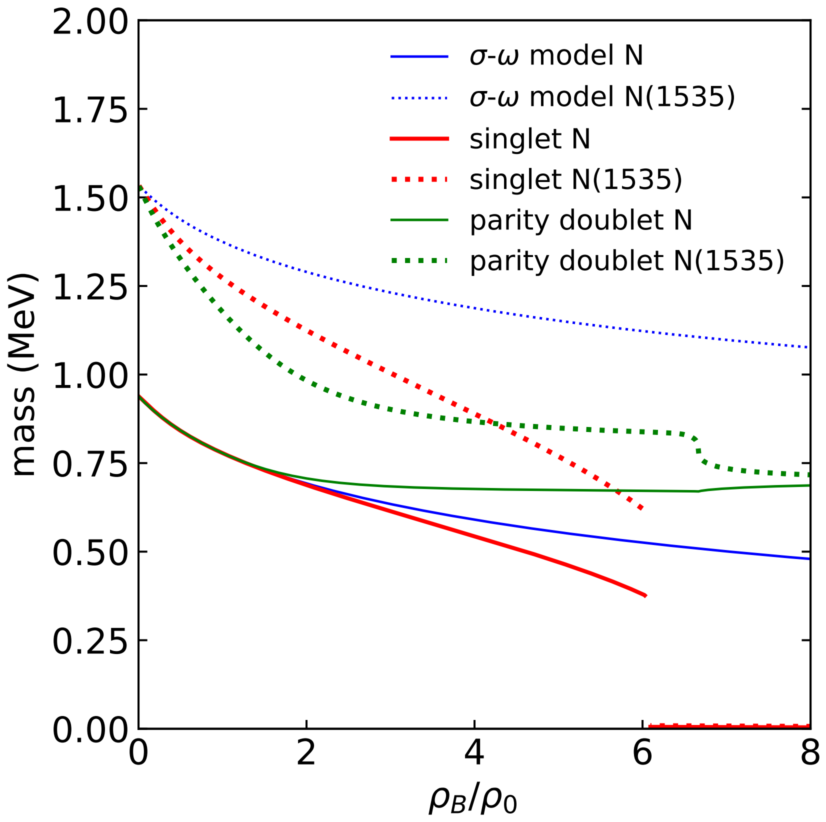
Chiral Singlet model $\quad m^* = g_\sigma \sigma$

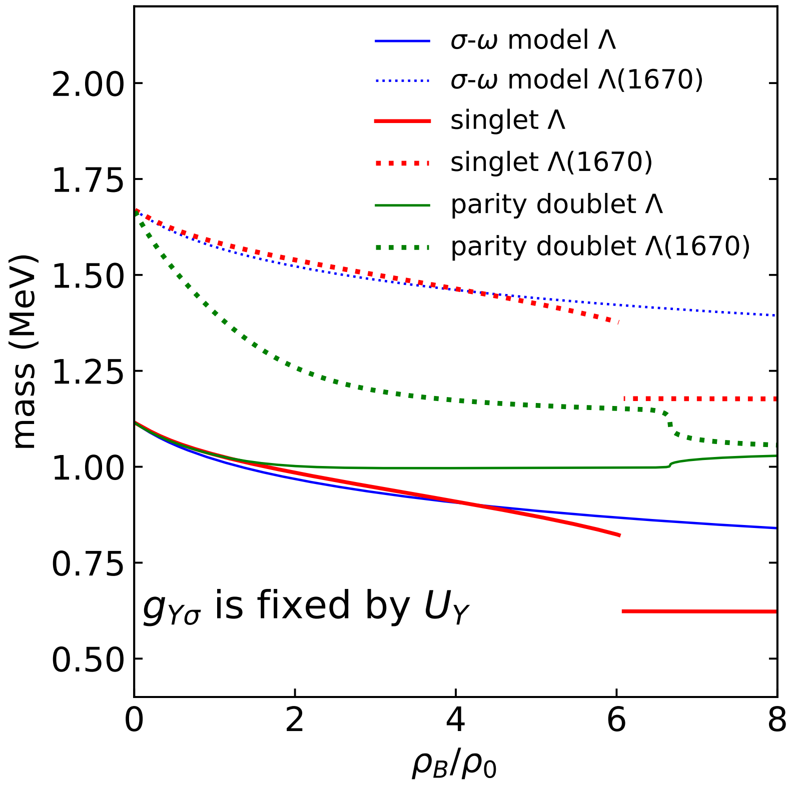
Parity doublet model $\quad m^*_\pm = \sqrt{a^2\sigma^2 + m_0^2} \pm b\sigma \qquad m_0 = 0$ chiral invariant mass

$$U(\sigma) = \sum_{i=1}^{4} \frac{a_n}{2^n n!}(\sigma^2 - f_\pi^2)^n - \epsilon(\sigma - f_\pi) \qquad V(\omega) = \frac{m_\omega^2}{2}\omega^2$$

E.S.Fraga,et.al.
PRD108(2023)

Effective mass of N and N(1535)





Mass of the Chiral partner decreases faster in the parity doublet model

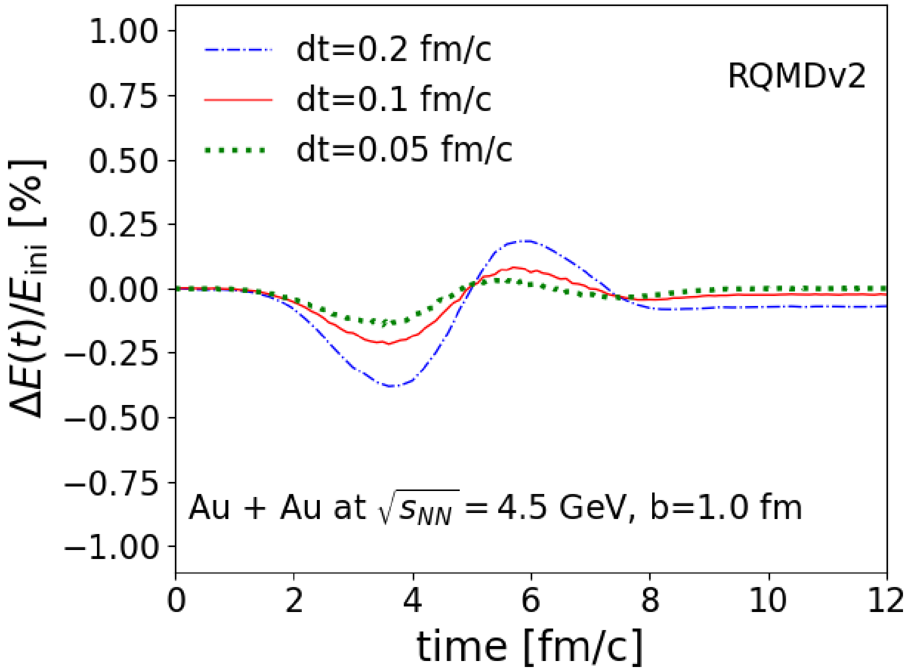
JAM2:microscopic transport model

- C++
- Pythia8
- Update of collision term: include new pp data.
  - ✓ New total hadronic cross section at high energies (PDG2016)
  - ✓ New resonance cross section (Ecm< 4GeV)
  - ✓ New string excitation low (4 < Ecm < 20 GeV)
  - ✓ New multiple-parton scattering (Pythia8) (Ecm > 20GeV)

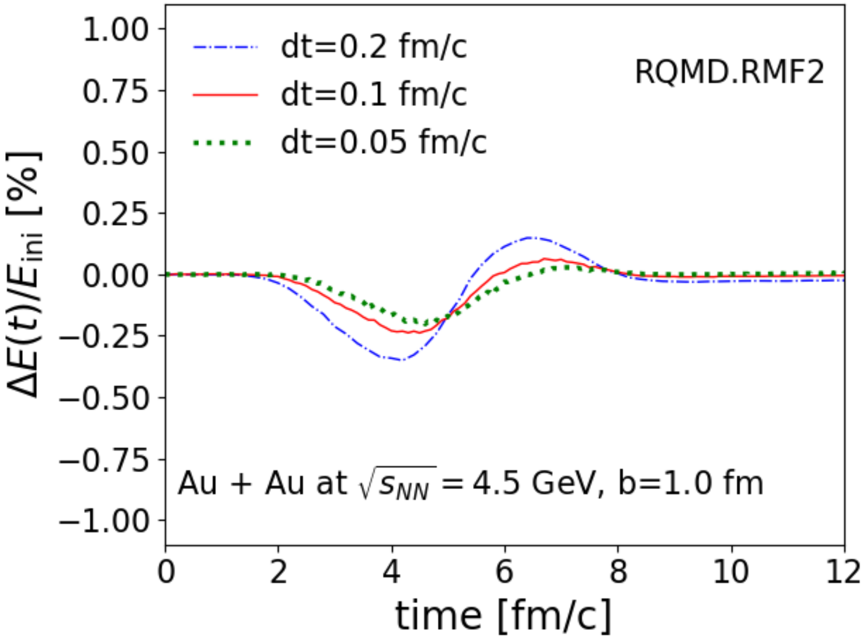
- RQMD with Skyrme force (Lorentz scalar and vector)
- RQMD.RMF with momentum-dependent potential
- RQMD.PDM parity doublet model
- Speeding up computational time by introducing expanding box for both collision term and potential evaluation

Validation of the code

Energy conservation



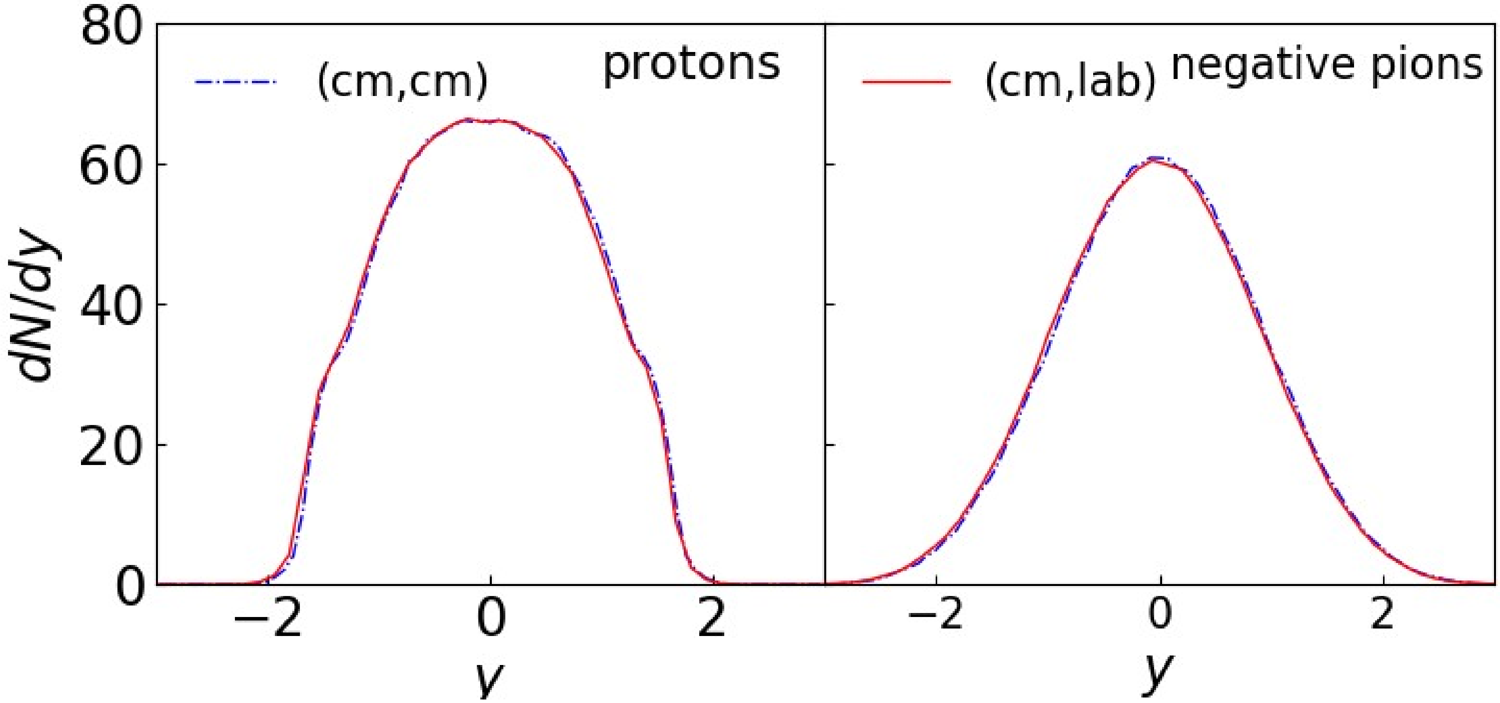
RQMDv2: Loretnz vector Skyrme force



RQMD.RMF: relativistic mean-field

Lorentz covariance

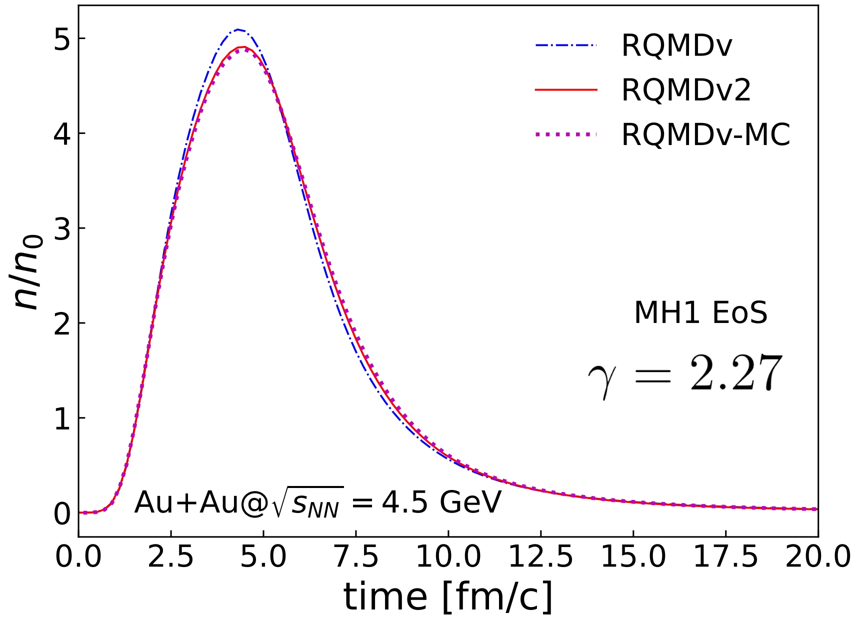
Central Au + Au collisions at 4.5 GeV

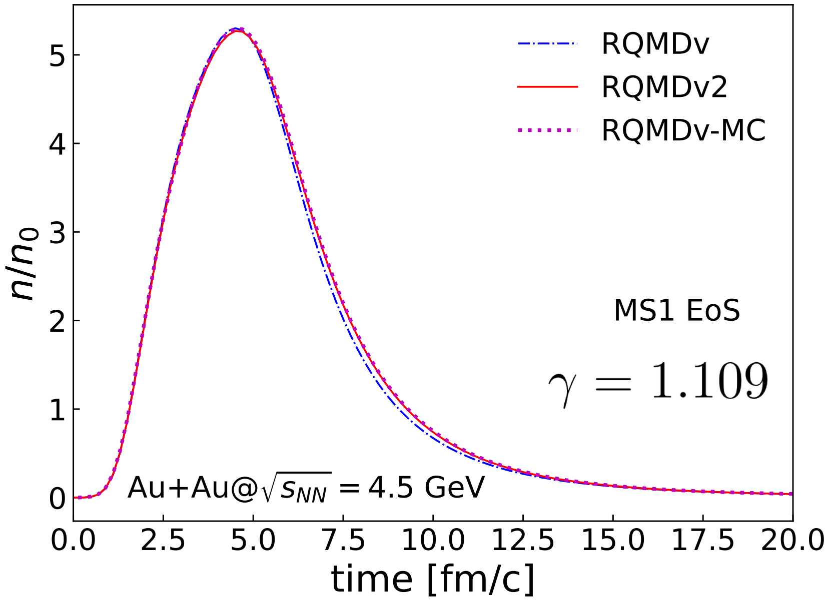


$$(\mathrm{cm}, \mathrm{cm}) = (\hat{a} = (1,0,0,0) \text{ at CM}, \text{simulation at CM frame})$$

Frame-independence: dN/dy at CM frame = dN/dy at Laboratory frame

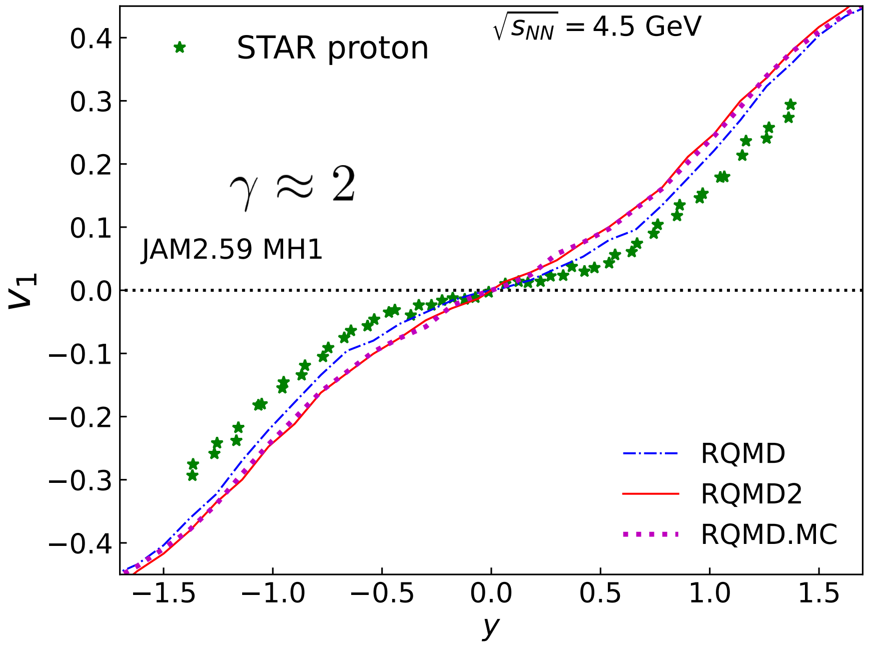
Time evolution of density at 4.5GeV

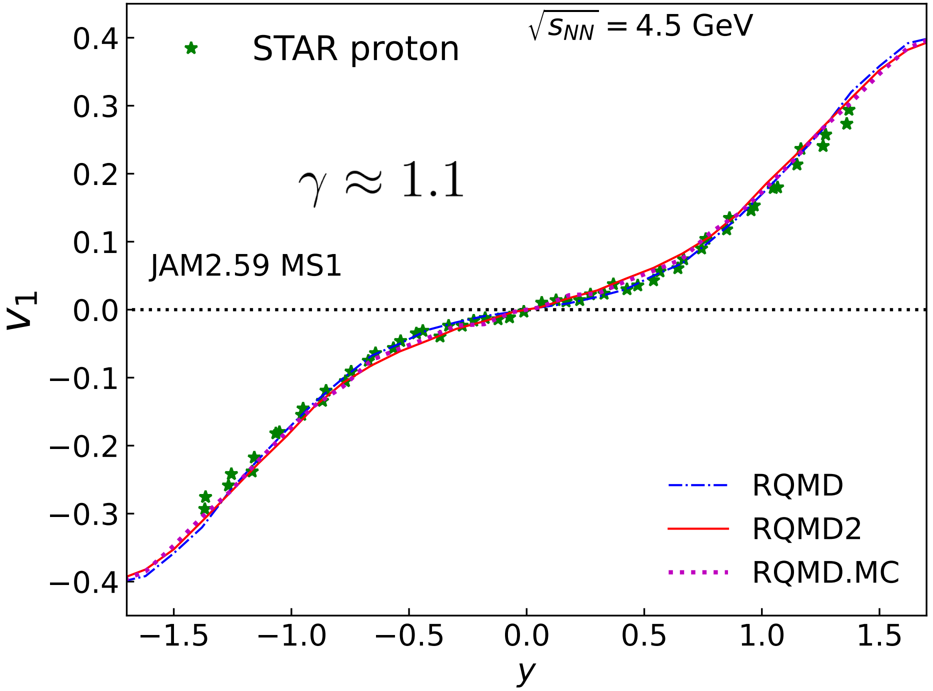




RQMDv (Lorentz vector Skyrme potential)
RQMDv2: new RQMDv
RQMDv-MC: exact integration

Directed flow in Au + Au collisions



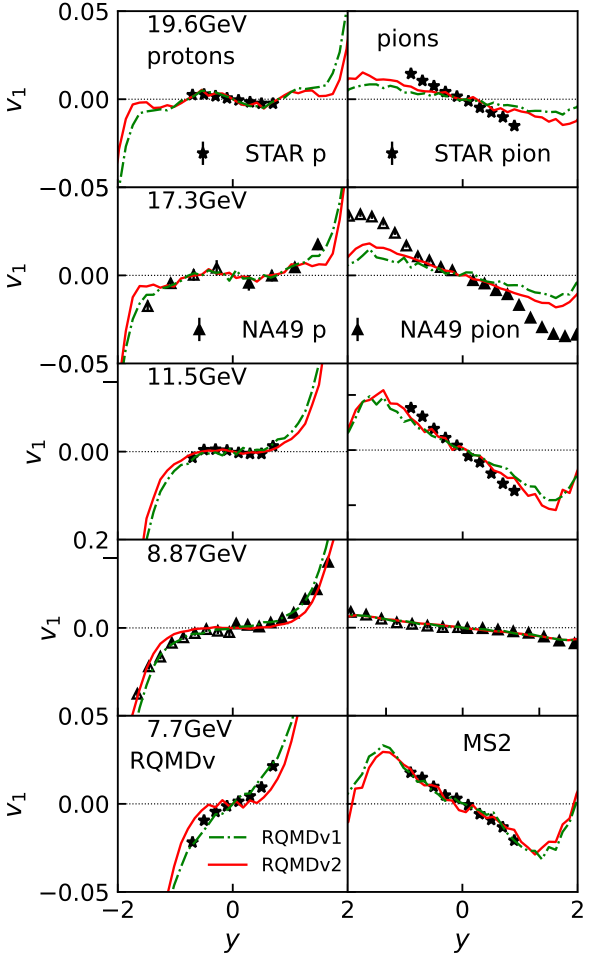


RQMDv (Lorentz vector Skyrme potential)
RQMDv2: new RQMDv
RQMDv-MC: exact integration

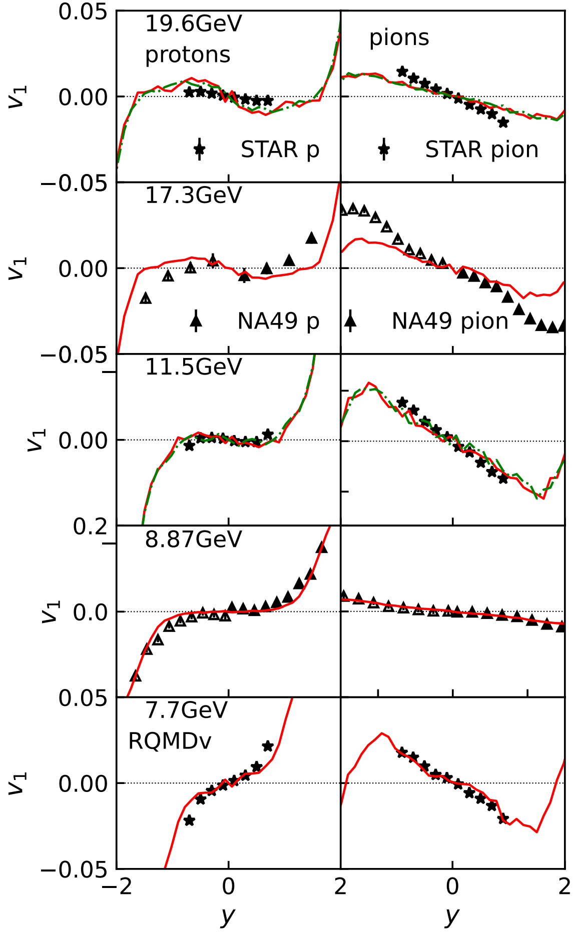
$$v_1 = \langle \cos\phi \rangle = \left\langle \frac{p_x}{p_T} \right\rangle$$

Beam energy dependence of v1 from RQMD

RQMDv



RQMD.RMF

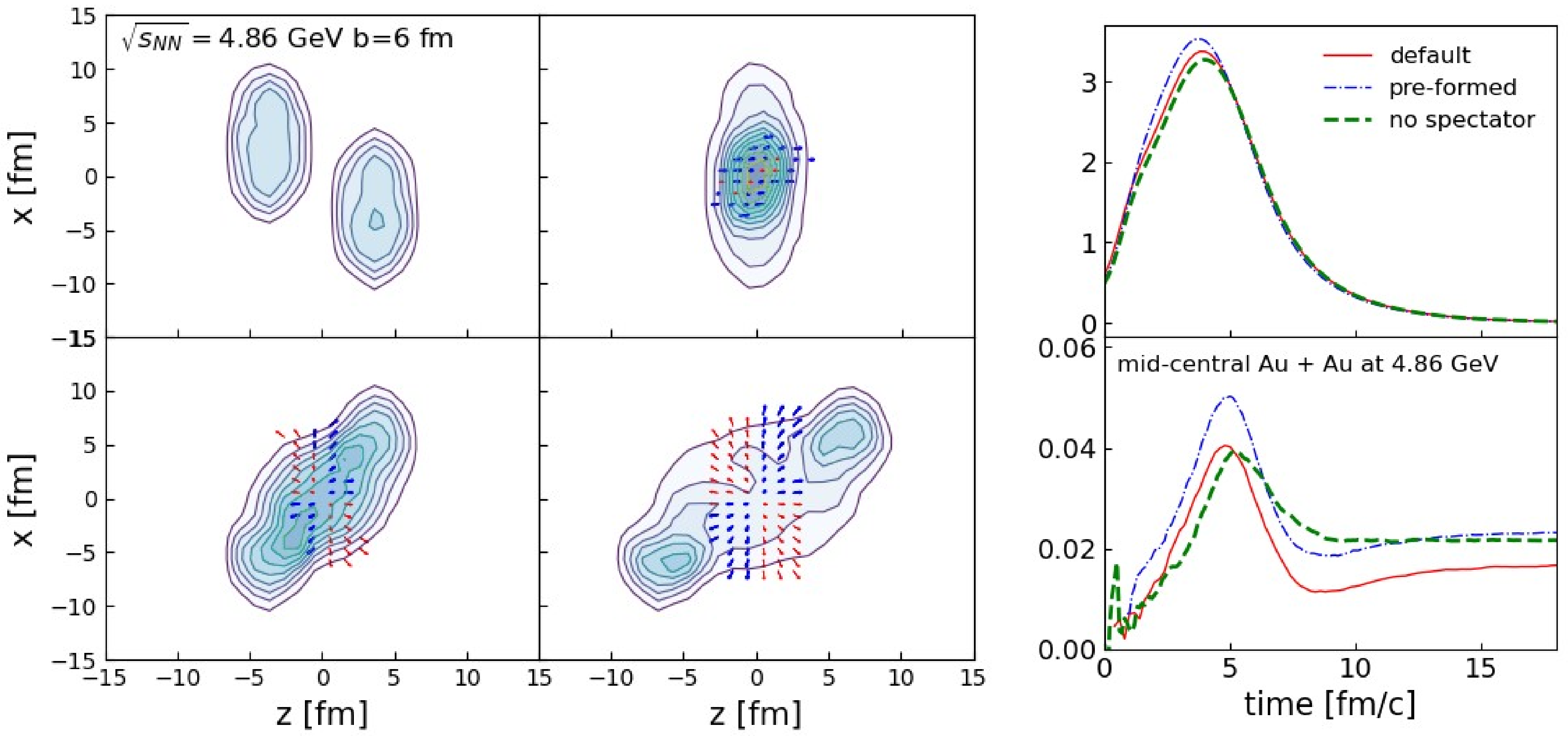


Beam energy dependence of v1 is explained by a mean-field both Skyrme type and sigma-omega without phase transition.

Y.N, A. Ohnishi, PRC (2022)

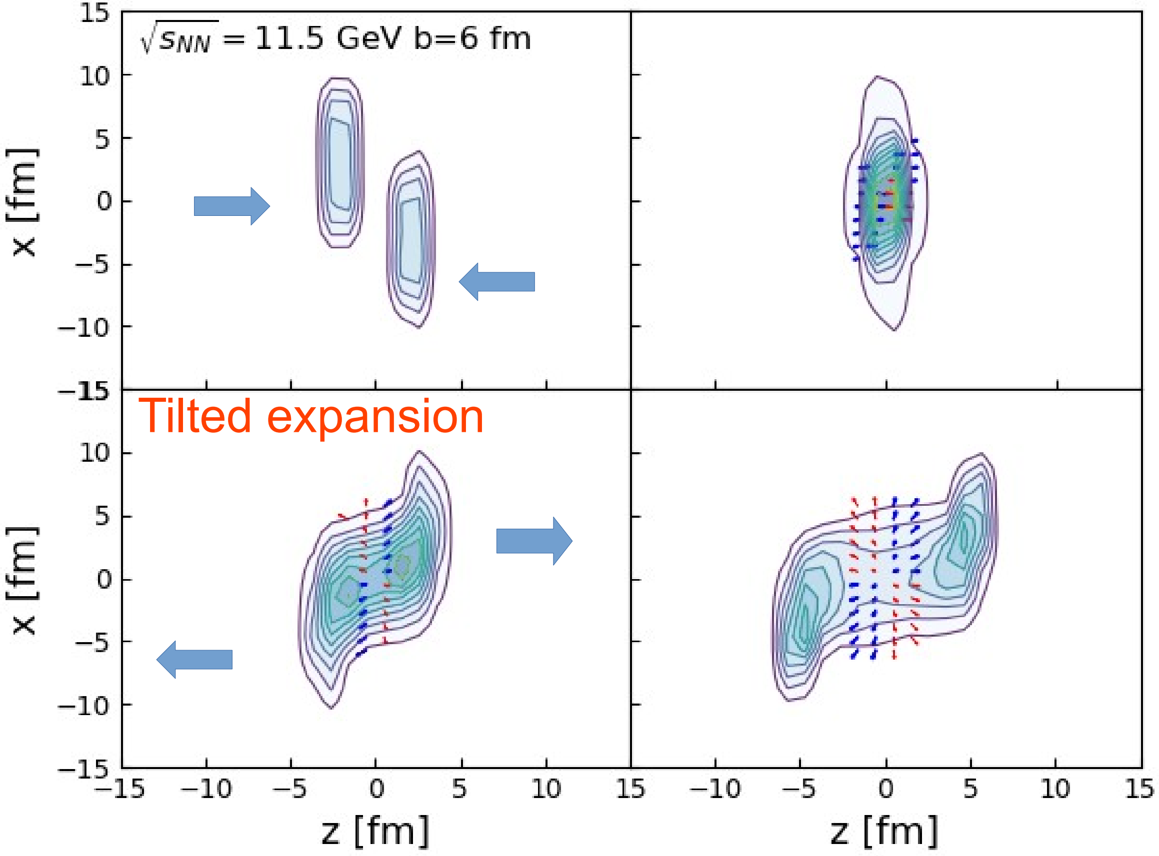
Development of v1 at 4.86 GeV

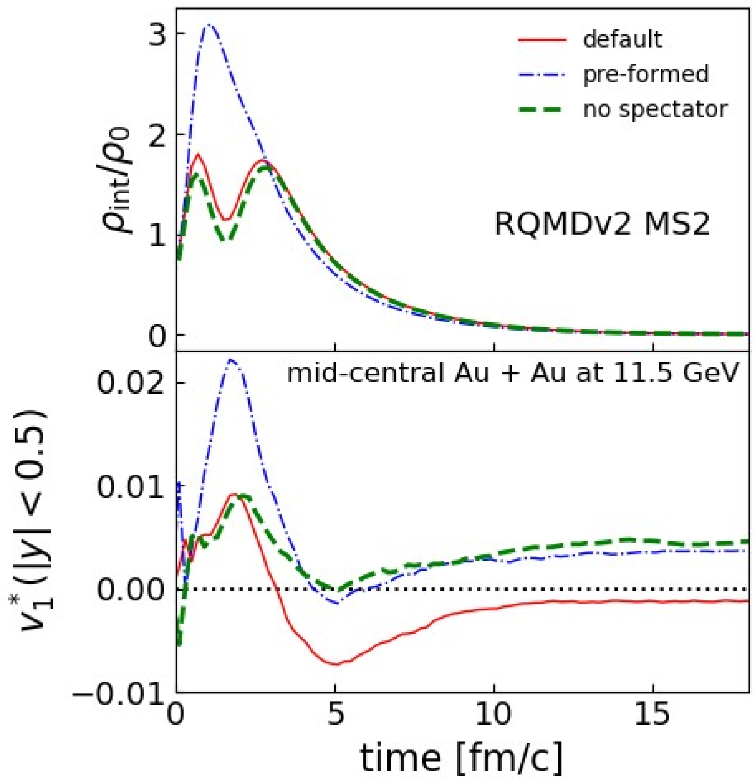
Time evolution of the baryon density in Au + Au mid-central collision (b=6fm)



Time evolution of v1 at 11.5GeV

Time evolution of the baryon density in Au + Au mid-central collision (b=6fm)

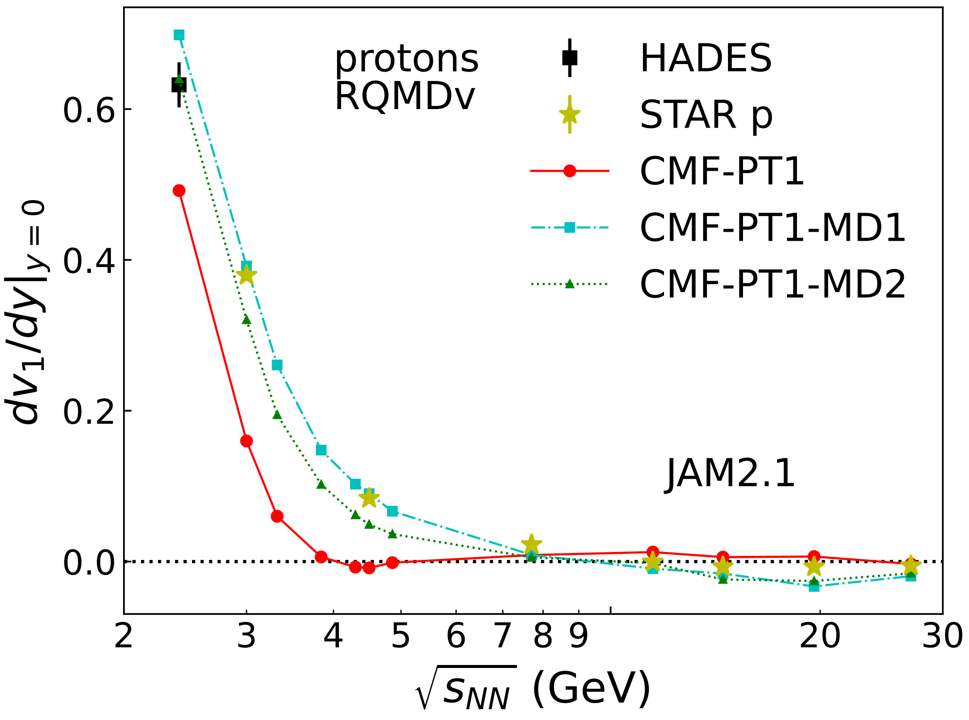


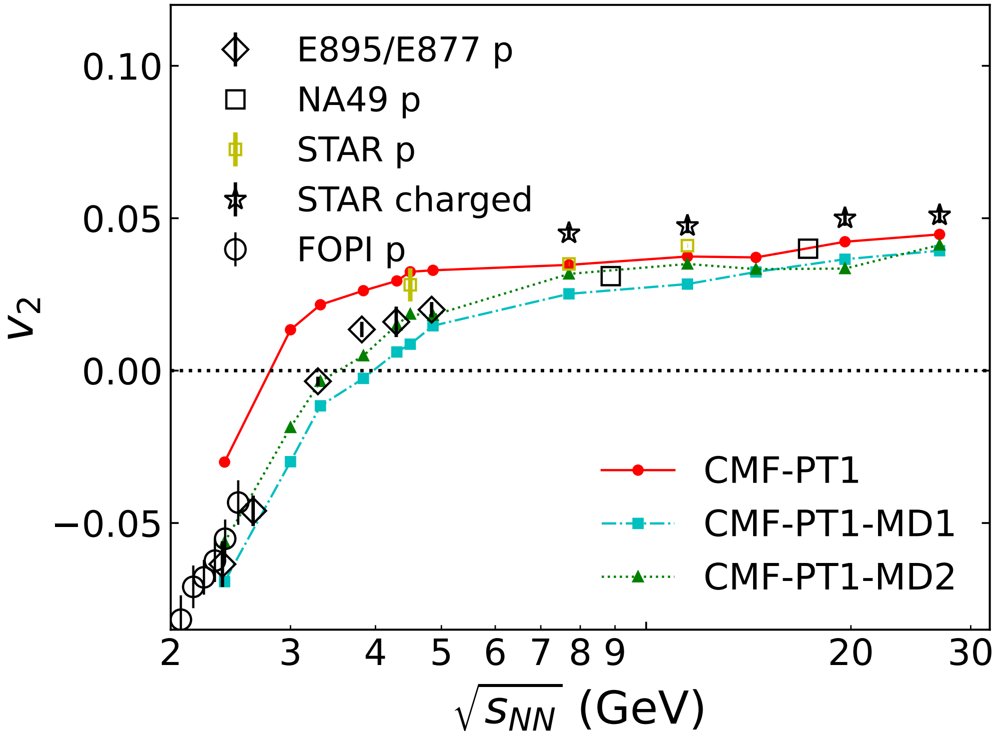


Positive v1 at compression, while negative v1 at expansion.

$$v_1^* = \int_{-0.5}^{0.5} dy v_1(y)\,\mathrm{sgn}(y)$$

V1 and v2 from CMF + 1st OPT

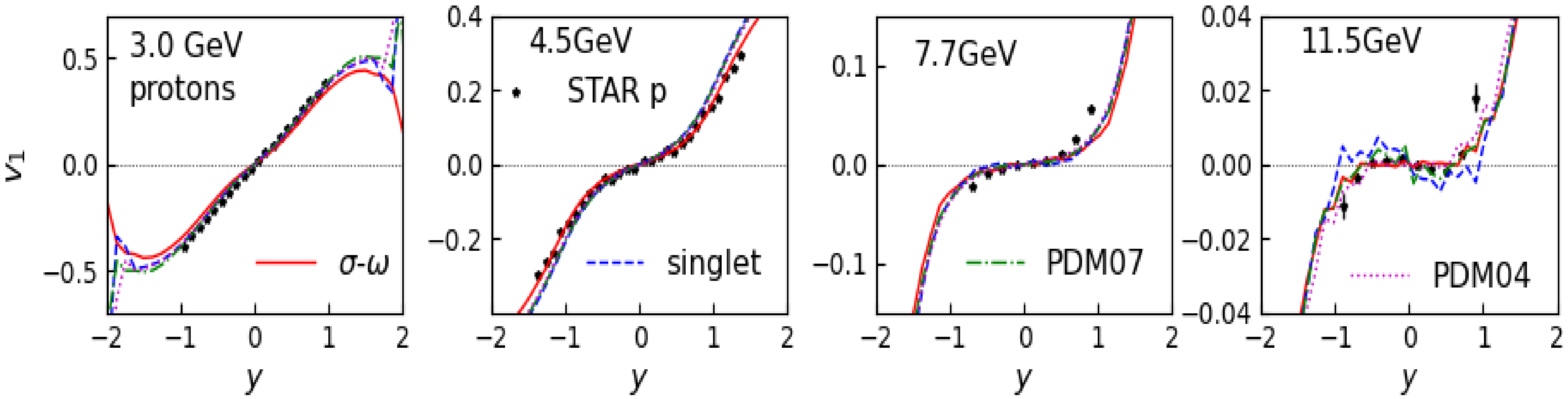




Collapse of v1 slope disappears by the momentum-dependent interaction.

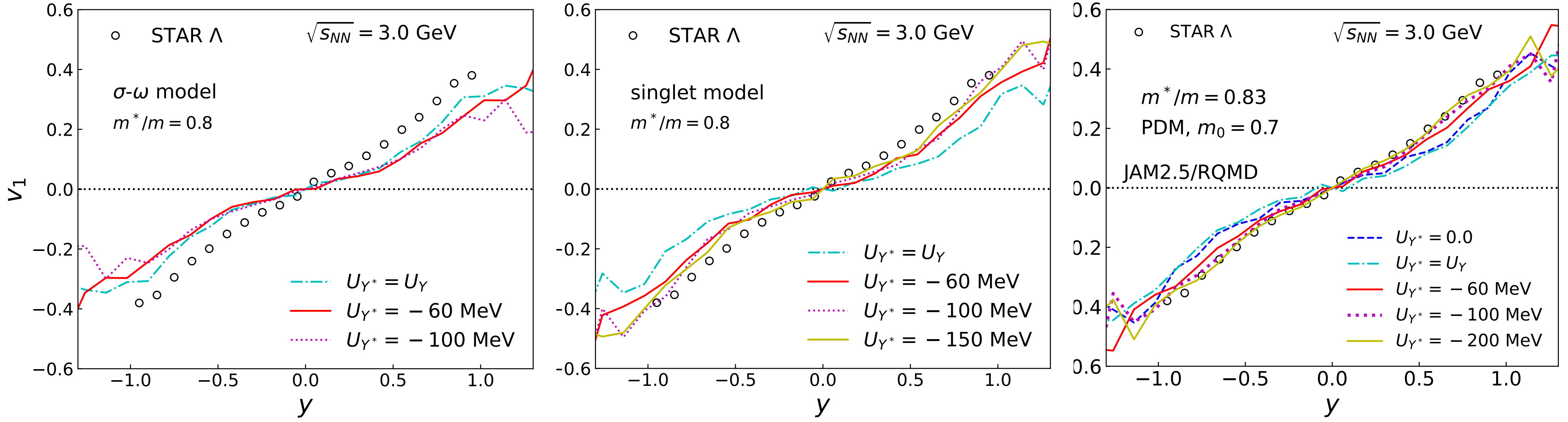
What is the prediction from the relativistic mean-field theory?

Proton v1



σ-ω, singlet, and parity doublet model can reproduce proton directed flow.

Lambda v1 at 3.0GeV



σ-ω model cannot fit the lambda v1 at 3 GeV.

Singlet model: Very deep hyperon resonance potential is required to fit the lambda v1.

Parity doublet model: good description of v1 data for $U_{Y^*} < -60$ MeV.

Summary

- A new RQMD formulation (RQMD2) is presented based on a covariant cascade method and the variational principle.
- RQMD2 is implemented in the JAM2 code, which is publicly available https://gitlab.com/transportmodel/jam2.
- RQMD2 accurately simulates the equation of state (EoS) for
  - ✓ Skyrme type potential
  - ✓ Relativistic mean field (singlet model, party doublet model)

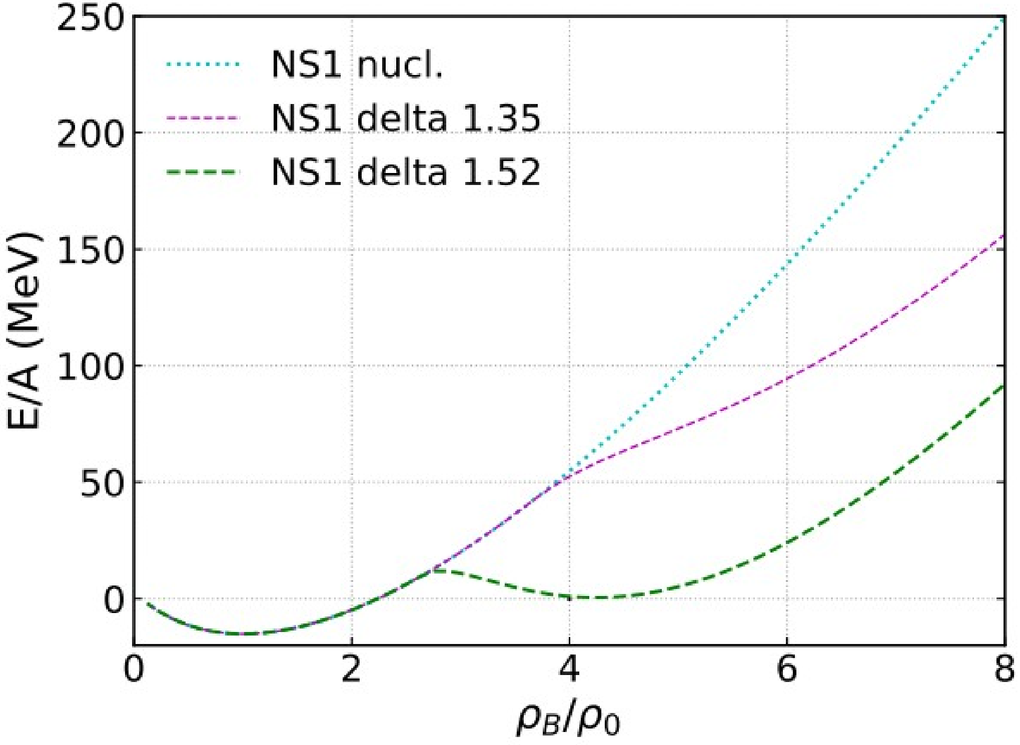
Delta-isomer state in RMF

J. Boguta, Phys. Lett.B109 (1982)251,
B. M.Waldhauser,et. al, PRC36(1987) 1019

$$g_{\Delta\omega} = g_{N\omega}$$

$$g_{\Delta\sigma} = \alpha g_{N\sigma}$$

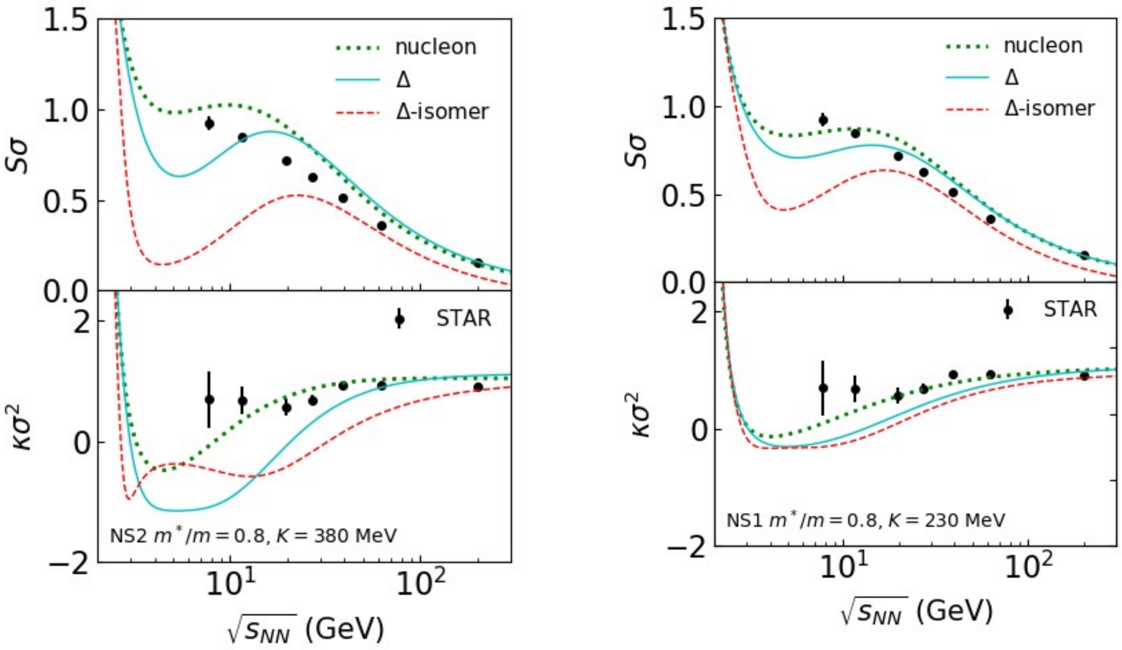
$$\alpha = 1.35, 1.52$$



Effects of Delta-isomer state on kurtosis

Compute baryon number fluctuations according to
K. Fukushima, PRC91 (2015) 044910

large EoS dependence !

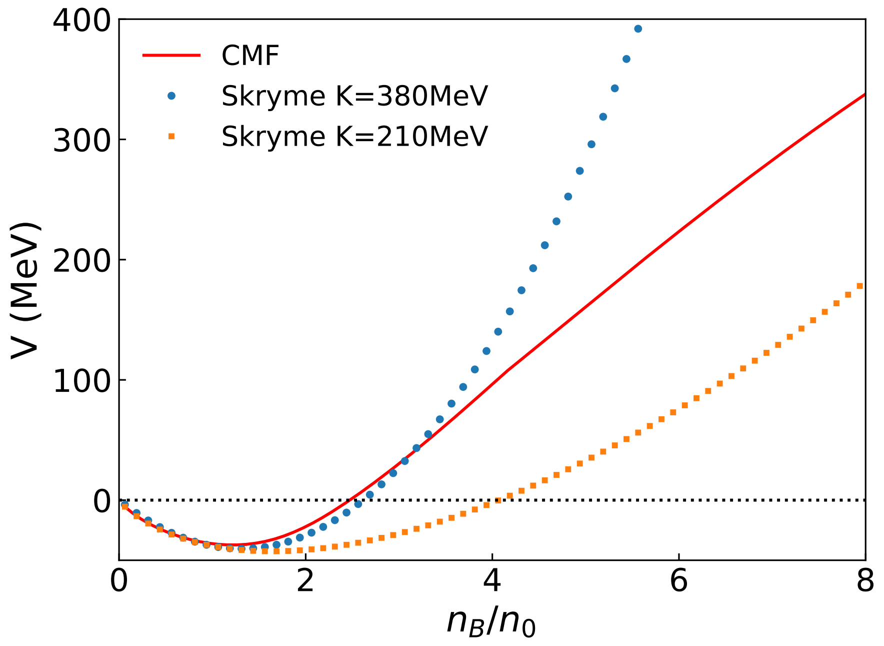


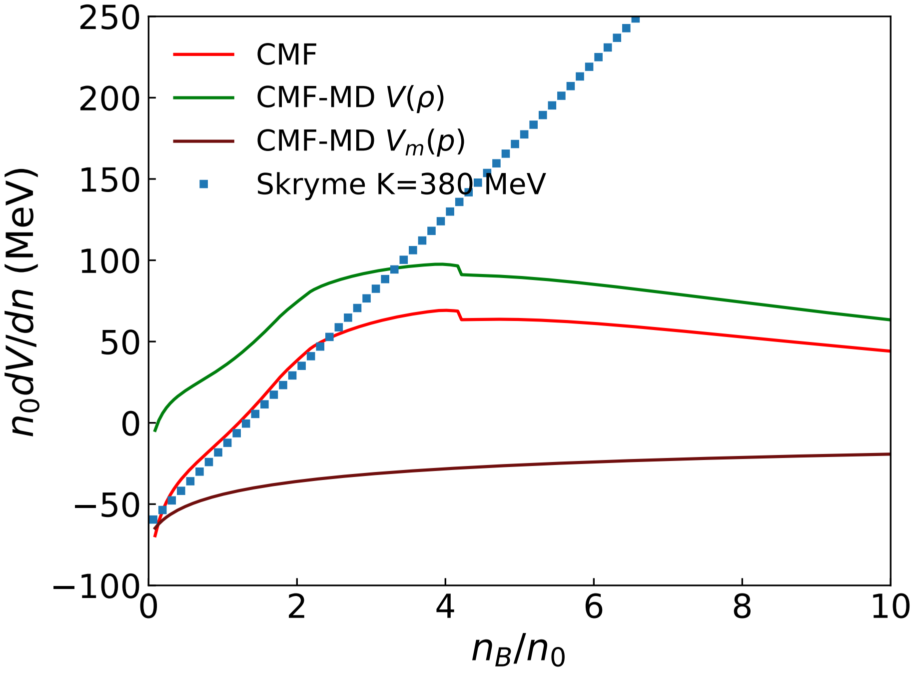
See also V.Koch,M.Marczenko, K.Redlich,and C.Sasaki, Hep-ph2308.15794 for the studies with PDM.

Delta and its isomer state has large effects on the Net-baryon number fluctuations.
What about the dynamical effect? We can do it by RQMD.

QMD potential from the Chiral Mean Field model (CMF)

A. Motornenko,et.al PRC103,054908(2021), J.Steinheimer,et.al, EPJC82,911(2022)





$$V = \frac{1}{n}\left[\epsilon - \epsilon_{\rm free}\right] \qquad n\frac{dV}{dn} = U - V \qquad U = \text{single particle energy} - \sqrt{m_N^2 + k_F^2}$$

v2 from RQMD.RMF

Beam dependence of proton v1 at mid-rapidity