

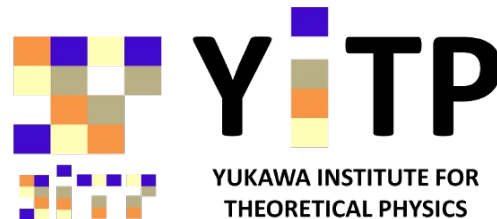
QCD Phase Diagram at Finite Volume via Lee–Yang Zeros and Lee–Yang Edge Singularities

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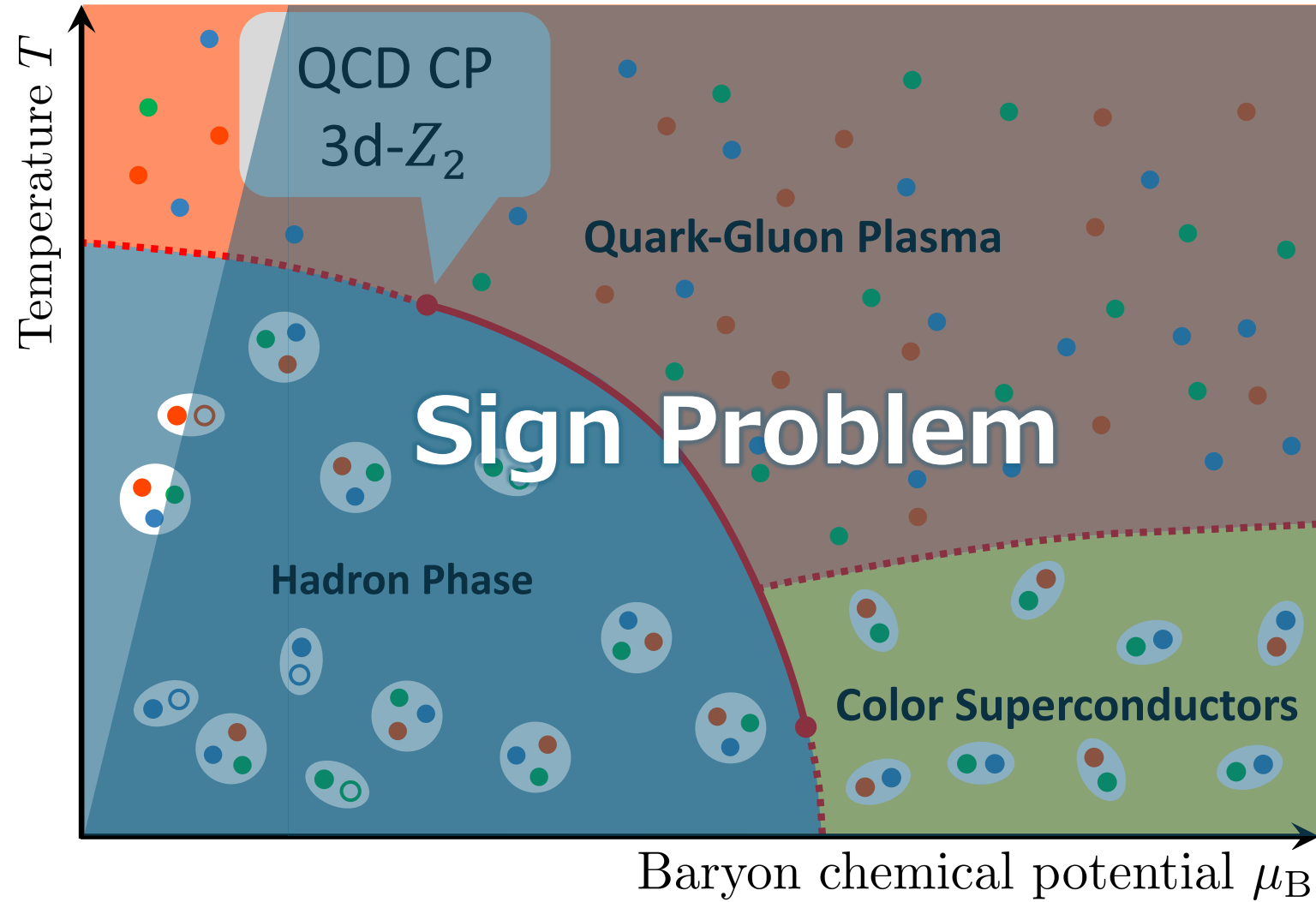
This talk is based on 2605.19964

QCD Critical Point and Hydrodynamic Evolution @ YITP June 01, 2026



QCD Critical Point

QCD Phase diagram



QCD Critical Point(CP)

◆ Constraint to behavior of physical quantities

◆ Classification by universality class

QCD is asymptotic-free theory
Non-perturbative in low energy.
Analytic approach is too difficult!

◆ Effective Model Approach

➤ NJL model etc.

◆ FRG Approach

◆ Numerical Approach

➤ Monte Carlo simulation

➤ Tensor Network

➤ Quantum Computation

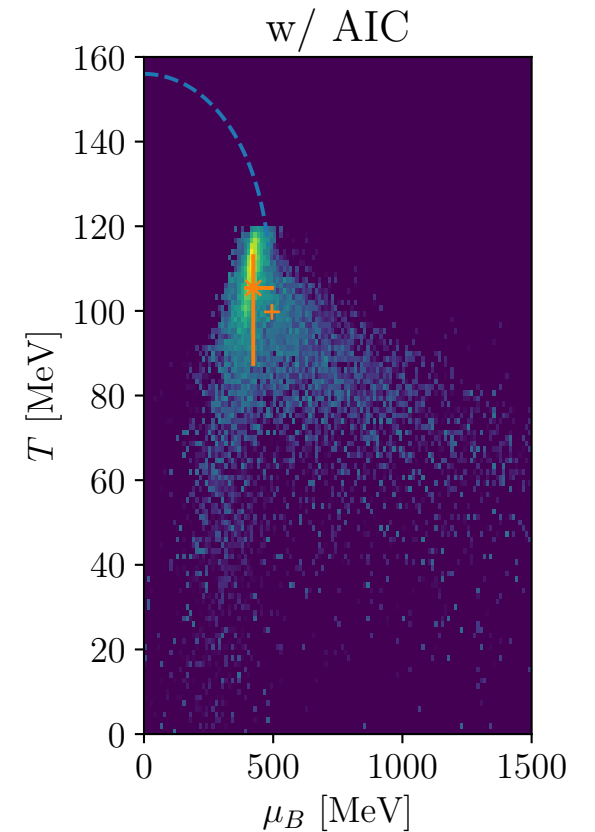
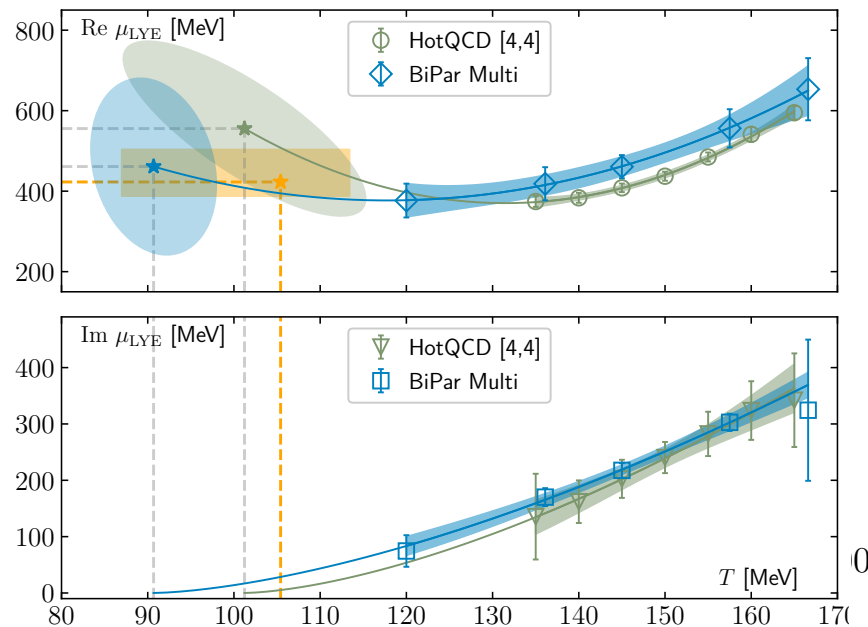
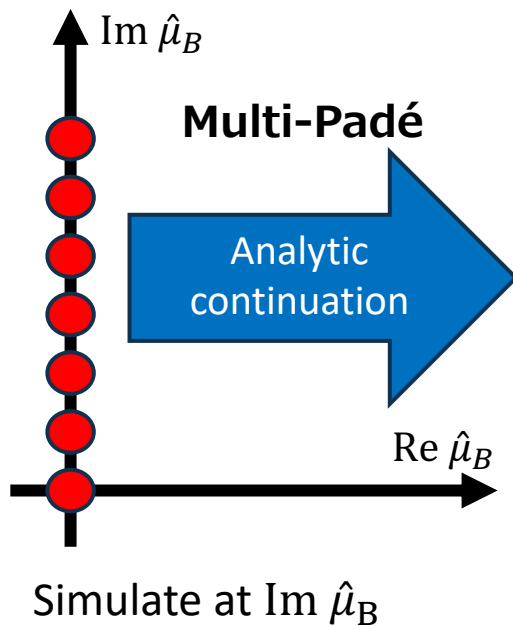
Recent progress of QCD-CP: Lee-Yang zeros

Lattice QCD

- Pure imaginary chemical potential + Padé approximation
- Extrapolate effective potential method

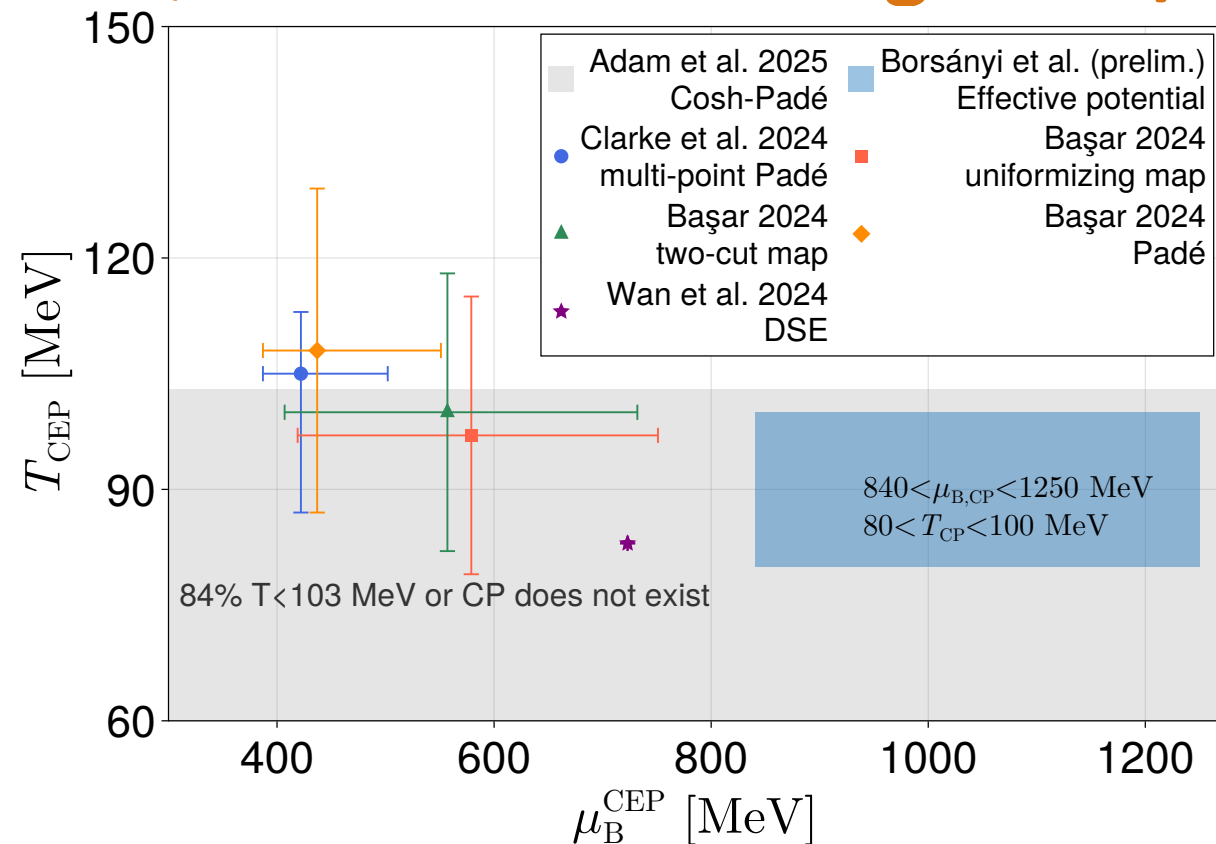
→ Pure imaginary chemical potential + Padé approx.

➤ Clarke, *et al.* (2024)



Recent progress of QCD-CP: Lee-Yang zeros

QCD-CP via Lee-Yang zeros /edge singularity approach



Lattice QCD

- Pure imaginary chemical potential + Padé approximation
- Extrapolate effective potential method

Lee-Yang zero \simeq Lee-Yang edge singularity

Finite V ? $V \rightarrow \infty$

Lee-Yang zero approach is powerful.
 But, in practice we always work at finite V .
 How do we treat finite volume effect?

**For the simple argument,
 we use the mean field approach**

Contents of This Talk

0. Introduction
1. Review : Lee-Yang Zeros/ Edge Singularity
2. Review Finite Volume Mean Field
3. Effective model w/ LYZ, LYES
 - Comparison to Lattice
 - LYZR Method
4. Summary

Review : Lee-Yang Zero

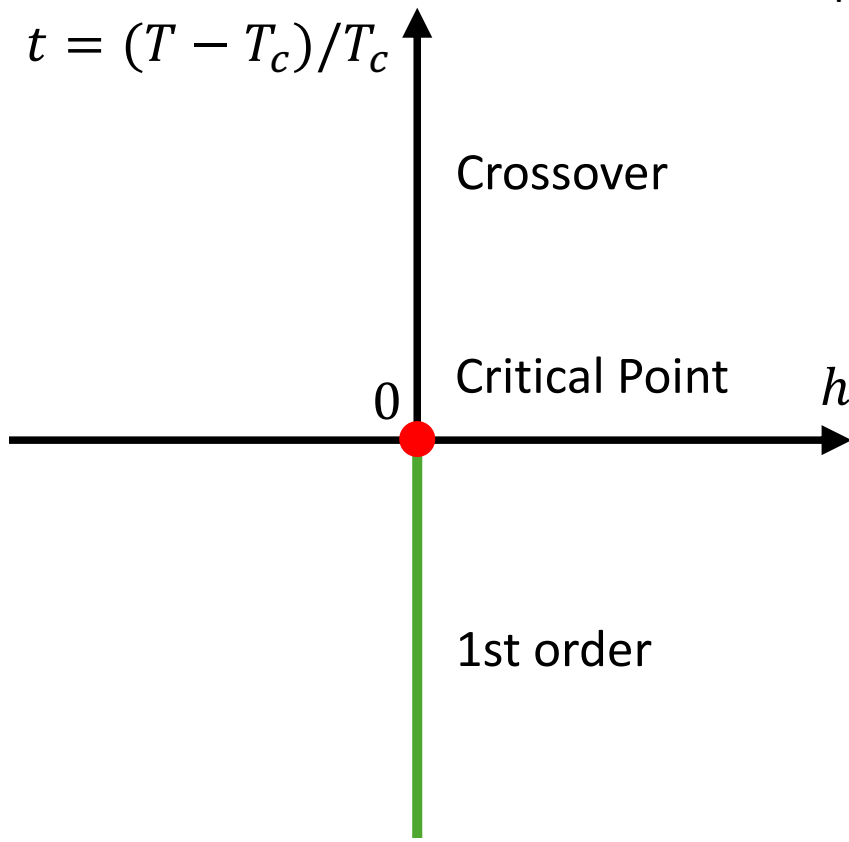
Lee, Yang 1952

Lee-Yang Zeros

= Zeros of Z in Complex Parameter space

Phase diagram ($d \geq 2$)

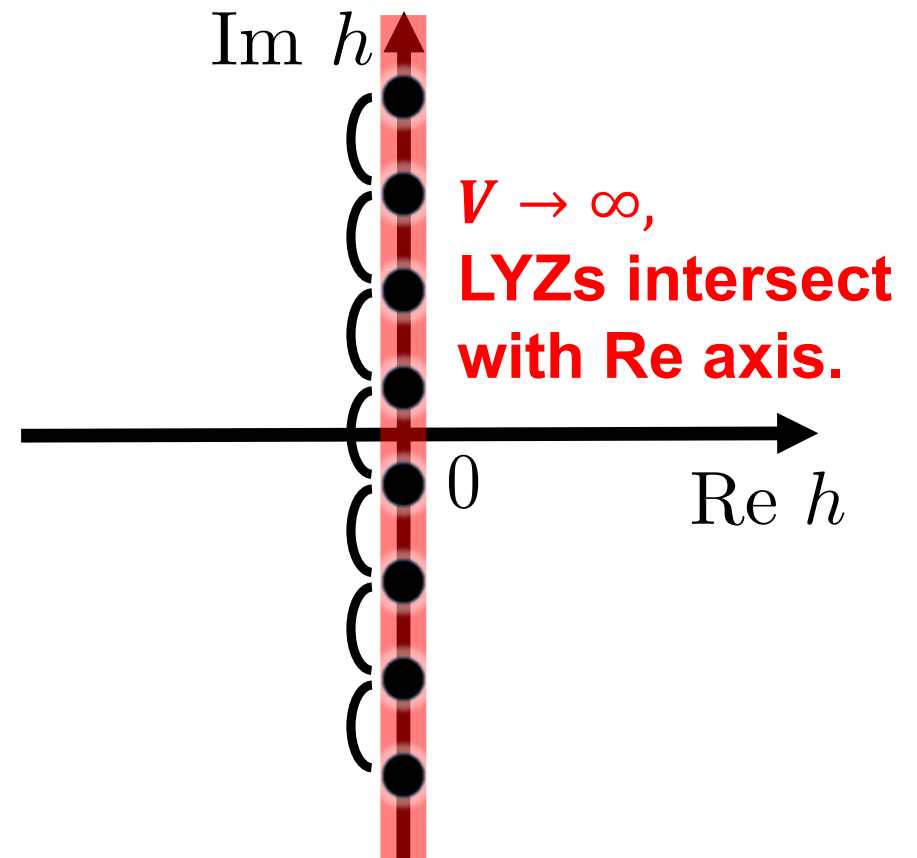
$t = (T - T_c)/T_c$
: reduced temperature



Ex.) d-dimensional Ising model $s_i = \pm 1$

$$Z(t, h, L) = \text{Tr} \exp \left(\frac{1}{T} \sum_{\langle i, j \rangle} s_i s_j + \frac{h}{T} \sum_i s_i \right)$$

$t < 0$: 1st order



Review : Lee-Yang Zero

Lee, Yang 1952

Lee-Yang Zeros

= Zeros of Z in Complex Parameter space

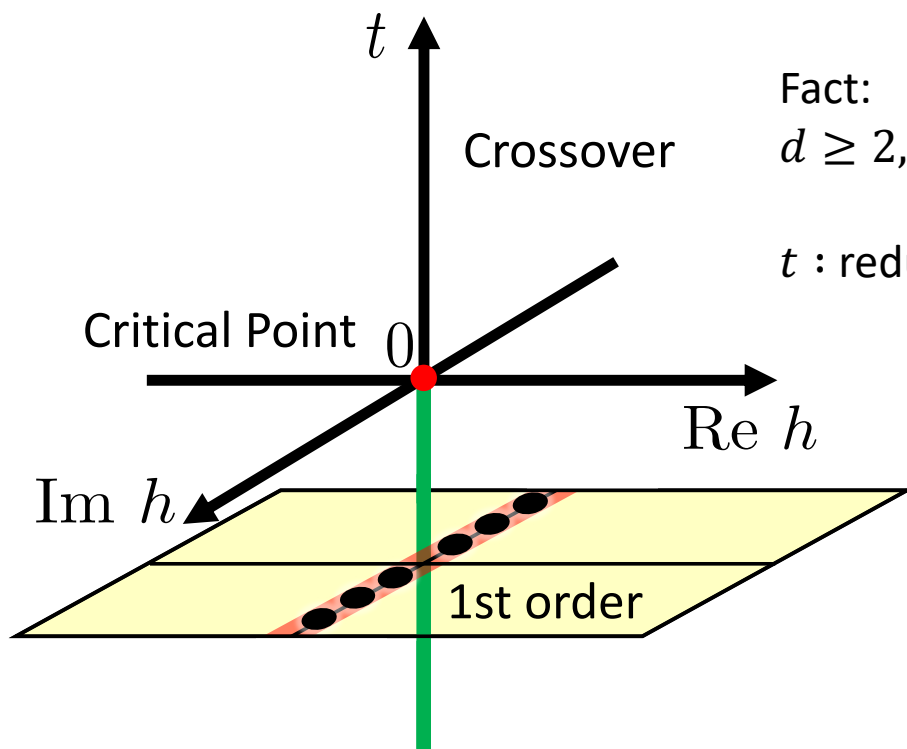
Lee-Yang's circle theorem

LYZs exist only on imaginary axis.

$t < 0$: 1st order

$V \rightarrow \infty$

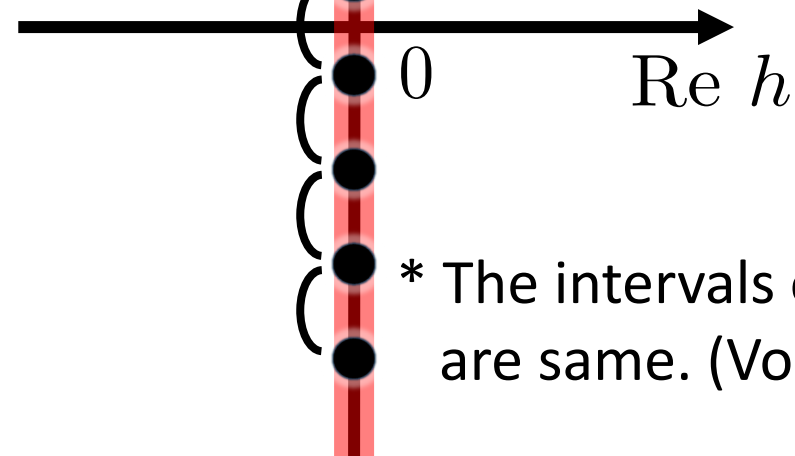
LYZs intersect with Re axis.



Im h

$V \rightarrow \infty$

LYZs intersect with Re axis.

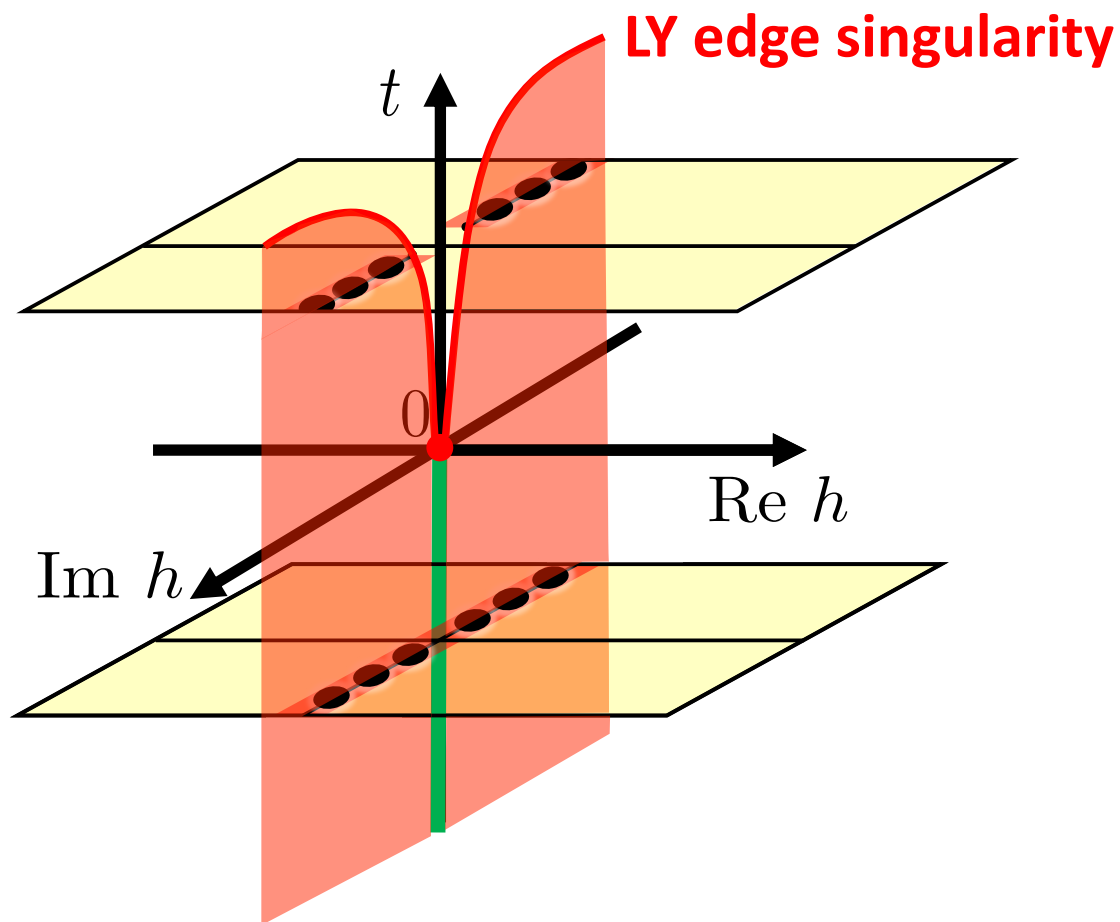


Review : Lee-Yang Zero

Lee, Yang 1952

Lee-Yang Zeros

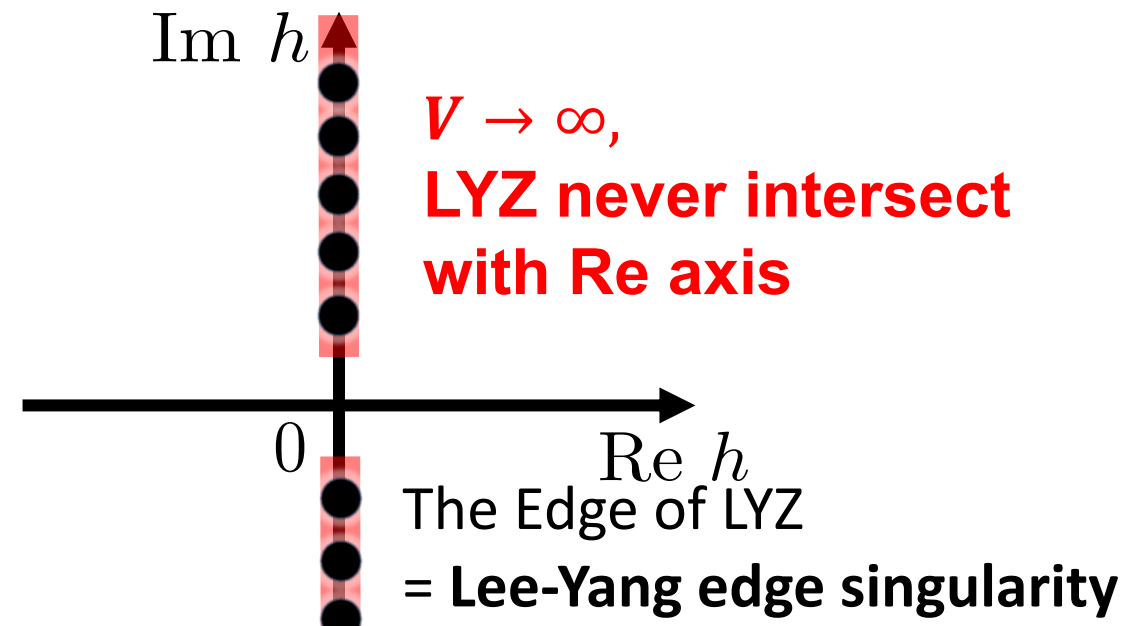
= Zeros of Z in Complex Parameter space



Lee-Yang's circle theorem

LYZs exist only on imaginary axis.

$t > 0$: Crossover



LY edge singularity [Fisher 1978]

$$\rightarrow h_{\text{LYES}}(t) = z_c^{-\beta\delta} t^{\beta\delta}$$

z_c, β, δ : universal constants

Lee-Yang Zero Ratio

TW, Kitazawa, Kanaya (2025)

LYZ Ratio

$$R_{nm}(t, L) \equiv \frac{h^{(n)}(t, L)}{h^{(m)}(t, L)}$$



$$R_{nm}(t, L) \xrightarrow{L \rightarrow \infty} \begin{cases} \frac{2n-1}{2m-1} & (t < 0) \\ 1 & (t > 0) \end{cases}$$

◆ $t < 0$: 1st order phase transition

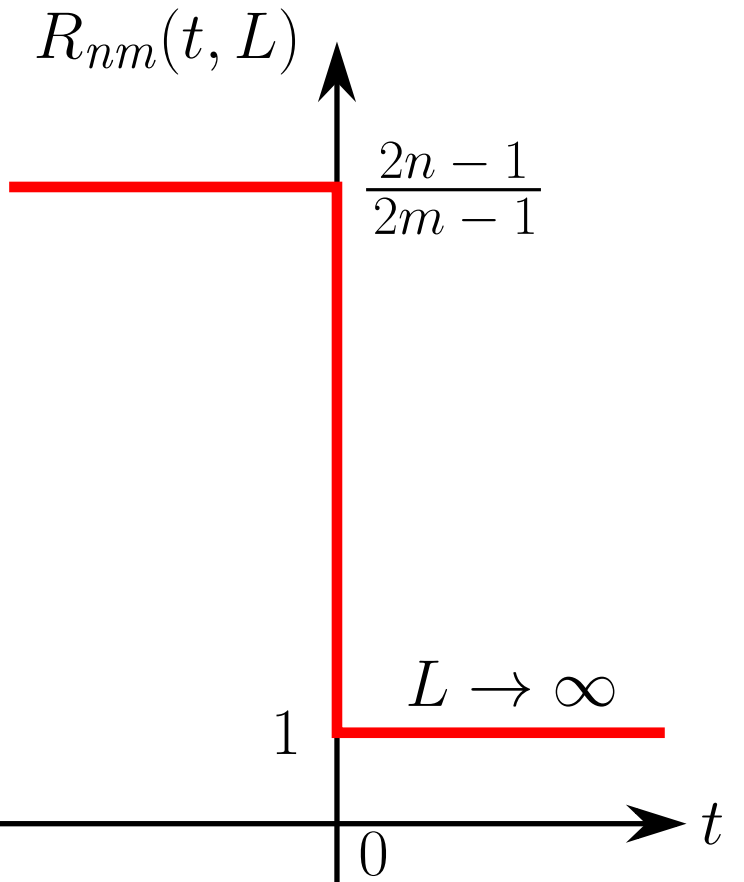
The distance between LYZ and Re-axis is 1:3:5... in $V \rightarrow \infty$

→ Ratio = $(2n-1)/(2m-1)$

◆ $t > 0$: Crossover

All LYZ accumulate at LYES in $V \rightarrow \infty$

→ Ratio = 1



How is the behavior in finite volume?

Finite Size Scaling (FSS)

FSS of Free Energy $\beta F_{\text{sing}}(t, h, L^{-1}) = -\log Z(t, h, L^{-1})$

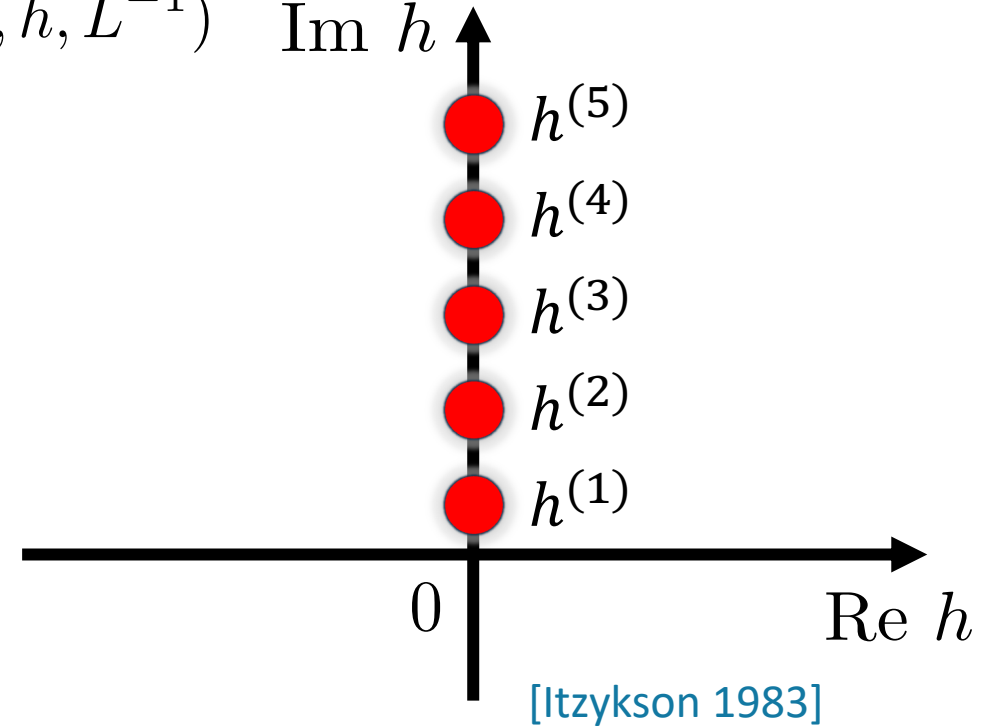
$$F_{\text{sing}}(t, h, L^{-1}) = \tilde{F}_{\text{sing}}(L^{y_t} t, L^{y_h} h)$$

FSS of Partition function

$$Z_{\text{sing}}(t, h, L^{-1}) = \tilde{Z}_{\text{sing}}(L^{y_t} t, L^{y_h} h)$$

Factorization of Partition function

$$Z_{\text{sing}}(t, h, L^{-1}) = \prod_{n=1}^N (h - h^{(n)}(t, L^{-1}))$$



LYZ function

$$h = h_{\text{LY}}^{(n)}(t, L^{-1})$$

FSS of LYZ

$$L^{y_h} h_{\text{LY}}^{(n)}(t, L^{-1}) = \tilde{h}_{\text{LY}}^{(n)}(L^{y_t} t)$$

We can treat finite volume using FSS in the vicinity of the CP!

LYZR near CP

TW, Kitazawa, Kanaya (2025)

FSS & Taylor Expansion near the CP

$$L^{y_h} h_{LY}^{(n)}(t, L^{-1}) = X_n + Y_n L^{y_t} t + \mathcal{O}(t^2)$$

$$\gg R_{nm}(t, L) \equiv \frac{h^{(n)}(t, L)}{h^{(m)}(t, L)} = \frac{X_n + Y_n L^{y_t} t + \mathcal{O}(t^2)}{X_m + Y_m L^{y_t} t + \mathcal{O}(t^2)}$$

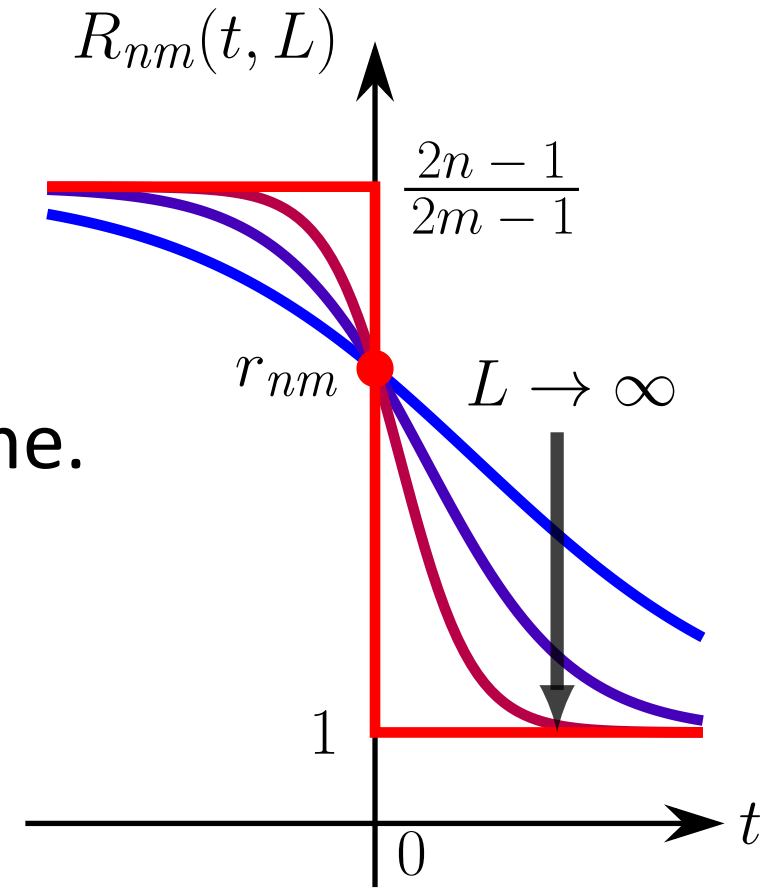
$$\gg R_{nm}(t, L) = (r_{nm} + c_{nm} L^{y_t} t + \mathcal{O}(t^2))$$

$R_{nm}(t = 0)$ does not depend on the volume.

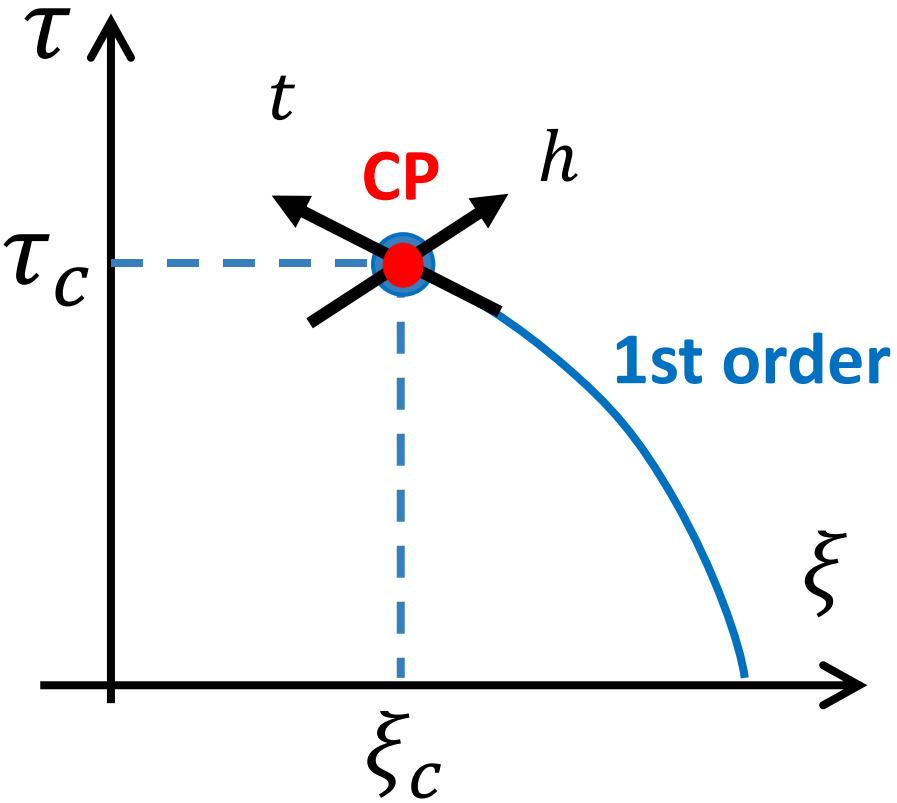
→ Useful to locate CP

Cf. Similar to Binder Cumulant $B_4(t = 0) = b_4$

Crossing Point = Critical Point



General System w/ Z_2 -CP TW, Kitazawa, Kanaya (2025)



Linear Approximation

$$\begin{pmatrix} t \\ h \end{pmatrix} = A \begin{pmatrix} \delta\tau \\ \delta\xi \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \tau - \tau_c \\ \xi - \xi_c \end{pmatrix}$$

$$h^{(n)} L^{y_h} = \tilde{h}_{LY}^{(n)} (t L^{y_t}) \simeq i(X_n + Y_n t L^{y_t})$$



Consistent with general LYES

$$\begin{cases} \xi_R^{(n)} = \xi_c - \frac{a_{21}}{a_{22}} \delta\tau \\ \xi_I^{(n)} = \text{const.} \times \delta\tau^{\beta\delta} \end{cases}$$

$$\begin{cases} \xi_R^{(n)} = \xi_c - \frac{a_{21}}{a_{22}} \delta\tau + \mathcal{O}(L^{2\bar{y}}) \\ \xi_I^{(n)} = \frac{X_n}{a_{22}} L^{-y_h} + \frac{\det A}{a_{22}^2} Y_n \delta\tau L^{\bar{y}} + \mathcal{O}(L^{2\bar{y}}) \end{cases} \quad \bar{y} = y_t - y_h < 0$$

$L \rightarrow \infty$

[Stephanov 2006]

General System w/ Z_2 -CP

TW, Kitazawa, Kanaya (2025)


$$\xi_I^{(n)} L^{y_h} = \frac{X_n}{a_{22}} + \frac{\det A Y_n}{a_{22}^2} L^{y_t} \delta\tau + \mathcal{O}(L^{2\bar{y}})$$

$$\bar{y} = y_t - y_h = -0.894 \quad \text{for 3d-Z(2)}$$

$$C = \frac{\det A}{a_{22}} \left(\frac{Y_2}{X_2} - \frac{Y_1}{X_1} \right), \quad D = \frac{a_{12}^2}{a_{22}^2} (Y_1^2 - Y_2^2)$$

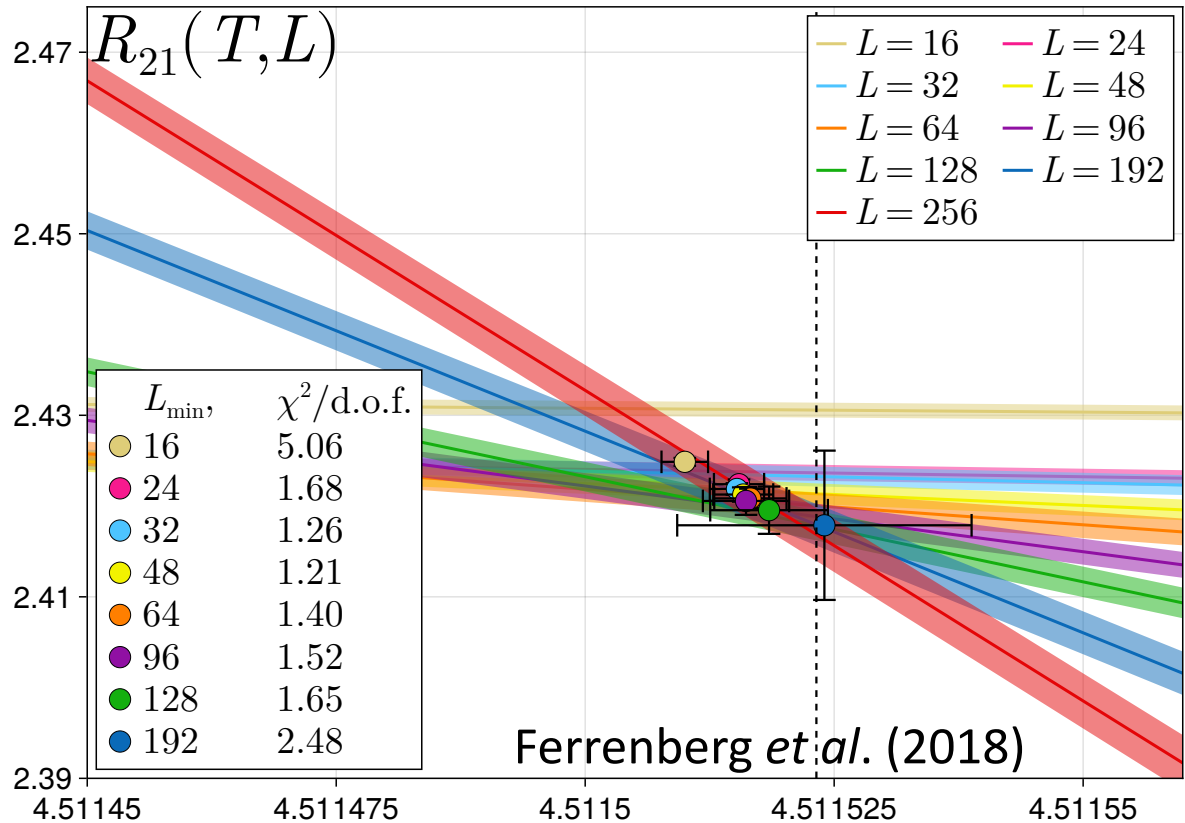
$$\mathcal{R}_{nm}(\tau, L) = \frac{\xi_I^{(n)}(\tau, L)}{\xi_I^{(m)}(\tau, L)} = \underbrace{\left(r_{nm} + C_{nm}(L^{y_t} \delta\tau) + \mathcal{O}(\tau^2) \right)}_{\text{Ising term}} \underbrace{\left(1 + D_{nm} L^{2\bar{y}} + \mathcal{O}(L^{4\bar{y}}) \right)}_{\text{Mixed term from } a_{12} \neq 0}$$

Ising Model $R_{nm}(t, L) = \left(r_{nm} + c_{nm} L^{y_t} t + \mathcal{O}(t^2) \right)$

- In the $L \rightarrow \infty$ limit, **Crossing Point = Critical Point !**
- r_{nm} is dependent on only universality class  Guideline for critical point search
Determination of universality class

LYZR method in Spin Model

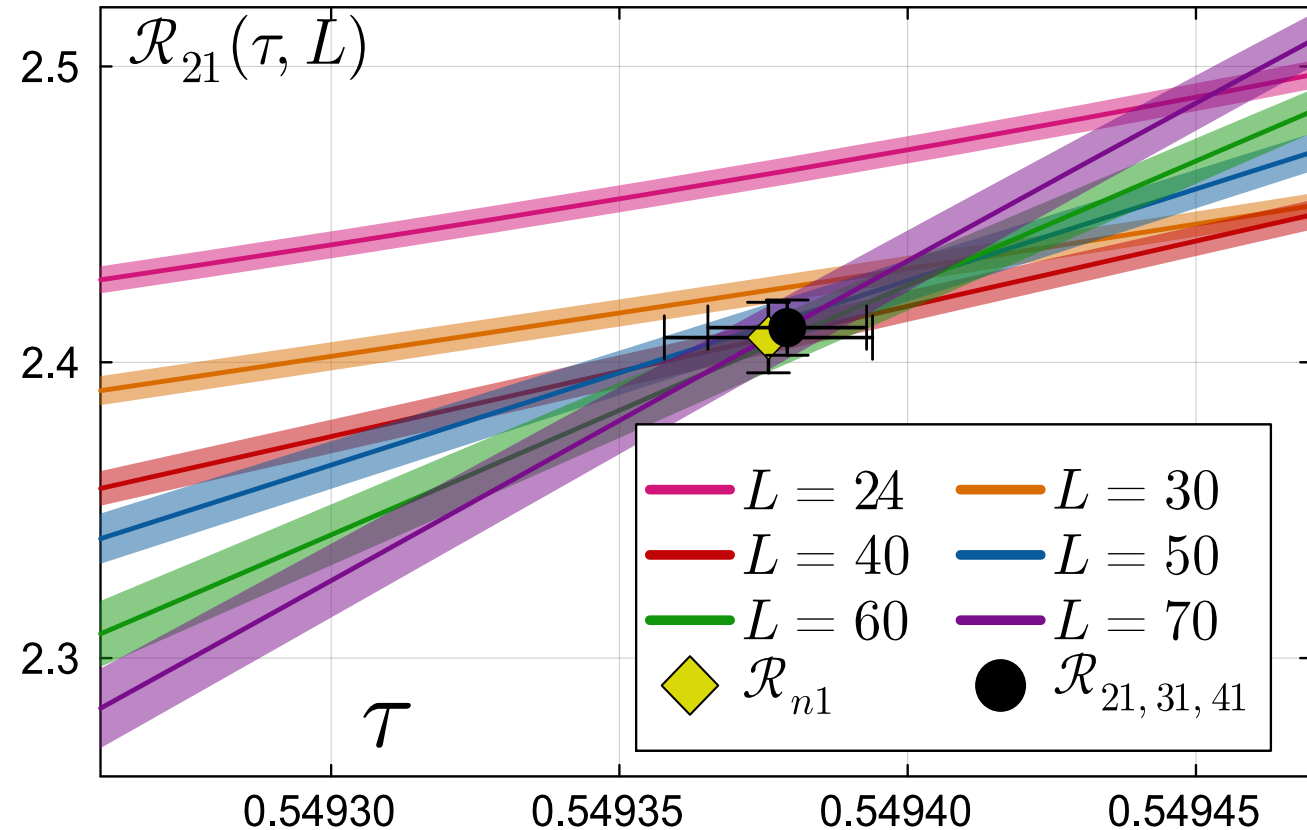
2nd/1st LYZR in 3d Ising



Ferrenberg *et al.* (2018)

TW, Kitazawa, Kanaya (2026)

2nd /1st LYZR in Potts model



TW, Kitazawa, Kanaya (2025)

LYZR method is valid! → Next, Discussion of Effective model

Finite-Volume Mean-Field Approach

General Argument in Mean Field

Gyozo, et al. (2025)

$$\mathcal{Z} = \int \mathcal{D}\phi e^{-S_{\text{E}}(\phi)} \quad S_{\text{E}}(\phi) : \text{Euclidian Action}$$

◆ Conventional mean field approach

Assumption : ϕ is temporally and spatially uniform $\bar{\phi} = \phi(x)$

The potential is given by

$$\mathcal{U}(\bar{\phi}) = \frac{T}{V} S_{\text{E}}(\bar{\phi})$$

$\bar{\phi}$ is determined by minimalizing potential $\mathcal{U}(\bar{\phi})$  Free energy : $f = \mathcal{U}(\bar{\phi})$

◆ Finite volume mean field approach

Assumption :

$$\mathcal{Z} = \int d\bar{\phi} e^{-V\mathcal{U}(\bar{\phi})/T} \quad \img alt="red arrow" data-bbox="628 800 662 876"/> f = -\frac{T}{V} \ln \mathcal{Z}$$

Model

Gyozo, et al. (2025)

Single component $N_c = 3$ quark-meson model in 3d

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - U_{\text{cl}}(\phi) + \bar{\psi} (i\gamma^\mu \partial_\mu - \gamma_0 \mu_q - g\phi) \psi$$

ψ : Quark

ϕ : Mesonic field

μ_q : quark chemical potential

$$U_{\text{cl}}(\phi) = \frac{m^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 - h\phi$$

Taking mesonic field constant, $\phi(x) = \bar{\phi}$ and thermodynamic limit

$$U_{\bar{q}q}(\bar{\phi}, T, \mu_B) = U_{\bar{q}q}^{\text{vac}}(\bar{\phi}) + U_{\bar{q}q}^{\text{mat}}(\bar{\phi}, T, \mu_B)$$

$$U_{\bar{q}q}^{\text{vac}}(\bar{\phi}) = -2N_c \int \frac{d^3p}{(2\pi)^3} E(p)$$

$$E(p) = \sqrt{p^2 + g^2 \bar{\phi}^2}$$

$$U_{\bar{q}q}^{\text{mat}}(\bar{\phi}; T, \mu_B) = -2N_c T \int \frac{d^3p}{(2\pi)^3} (\log g^+ + \log g^-)$$

$$g^\pm = 1 + e^{-(E(p) \mp \mu_q)/T}$$

Effective potential

$$\mathcal{U}(\bar{\phi}; T, \mu_B) = U_{\text{cl}}(\bar{\phi}) + U_{\bar{q}q}^{\text{mat}}(\bar{\phi}; T, \mu_B)$$

We assume that the vacuum terms is encoded in parameters of U_{cl}

Phase Diagram

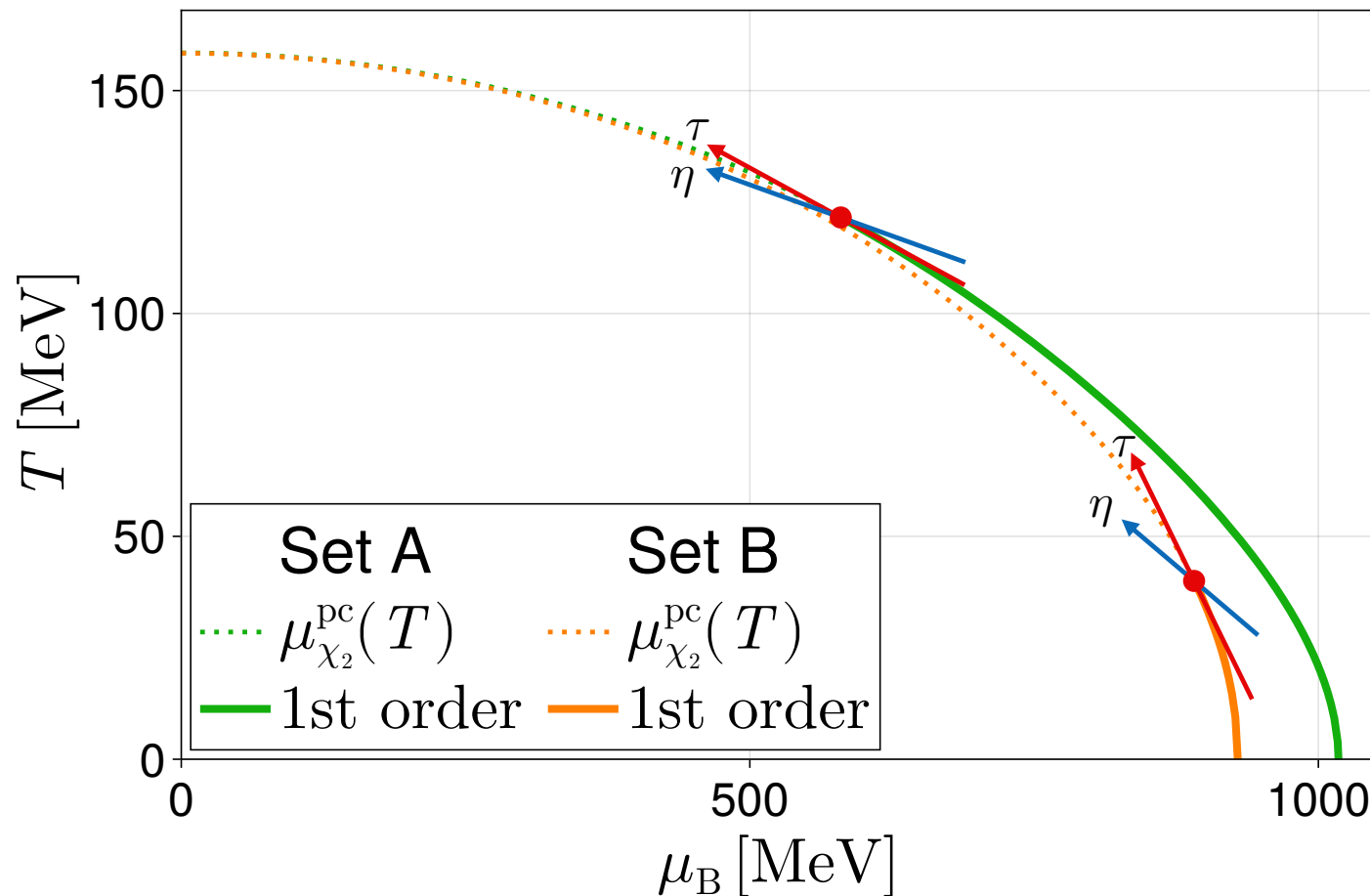
Gyozo, et al. (2025)

	λ	g	T_{CP}	μ_{CP}	T_0^{pc}
Set A	29.0	4.55	121.53	579.97	158.46
set B	46.0	4.35	40.04	890.84	158.38

* Dimensionful quantities are given in units of MeV.

$$m^2 = -0.155 \text{ GeV}^2$$

$$h = 0.0018 \text{ GeV}^3$$



$$\chi_n = \frac{V}{T} \frac{\partial^n f(T, \mu_B, L)}{\partial (\mu_B/T)^n}$$

$$\mu_{\chi_2}^{pc}(T) \equiv \max_{\mu_B} \chi_2(T, \mu_B)$$

- ◆ CPs belong to mean field universality class
→ Mapping

$$\begin{pmatrix} \tau \\ \eta \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} T - T_{CP} \\ \mu_B - \mu_{CP} \end{pmatrix}$$

a_{ij} are determined by effective potential.

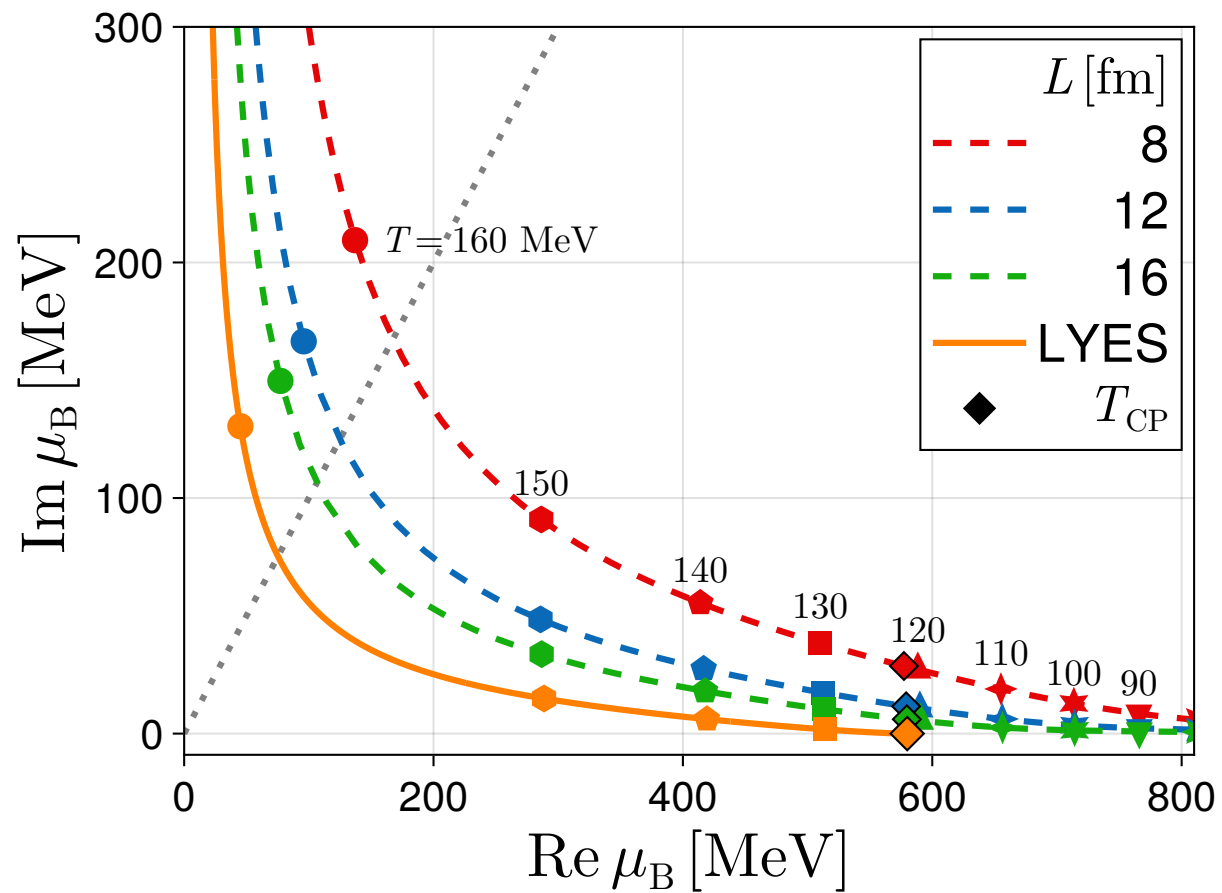
Z is calculable numerically

➔ We can calculate LYZ/LYES

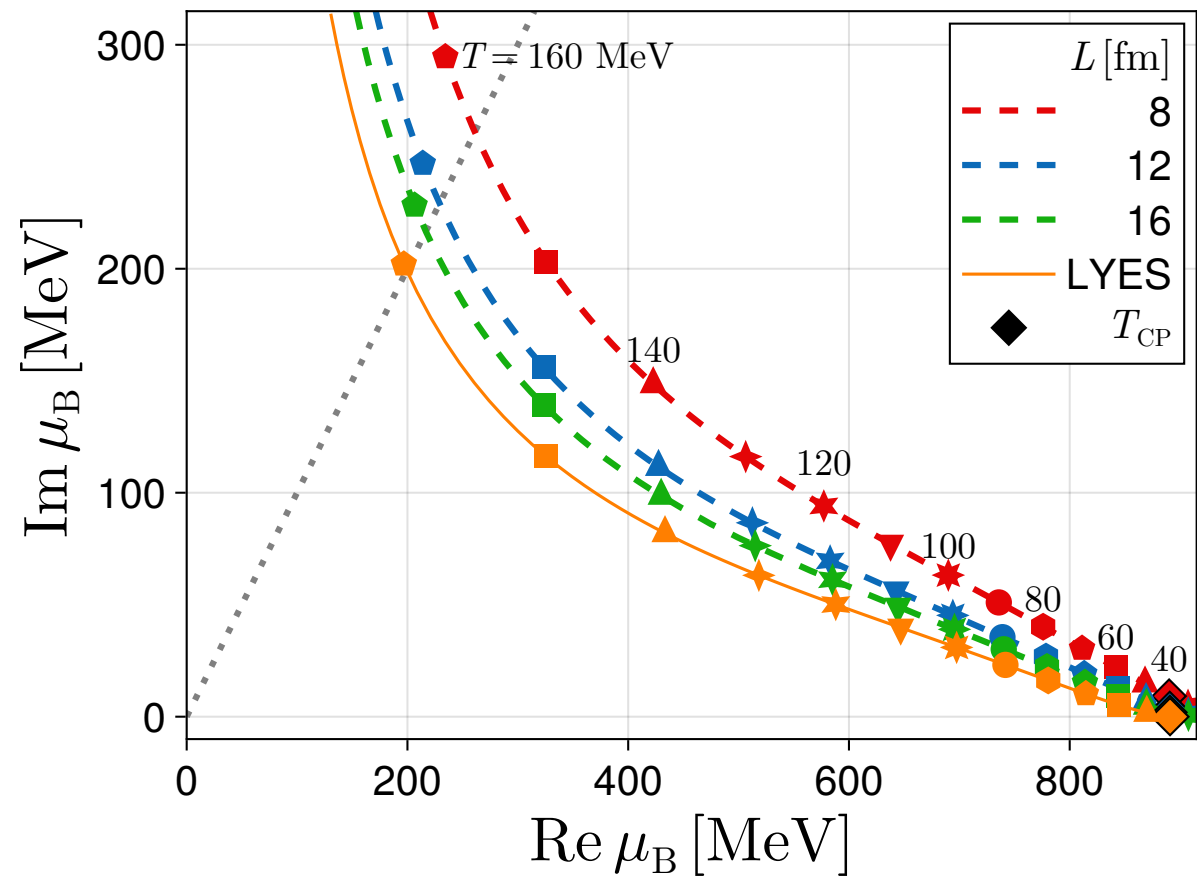
LYZ and LYES

 μ_B - plane

Set A



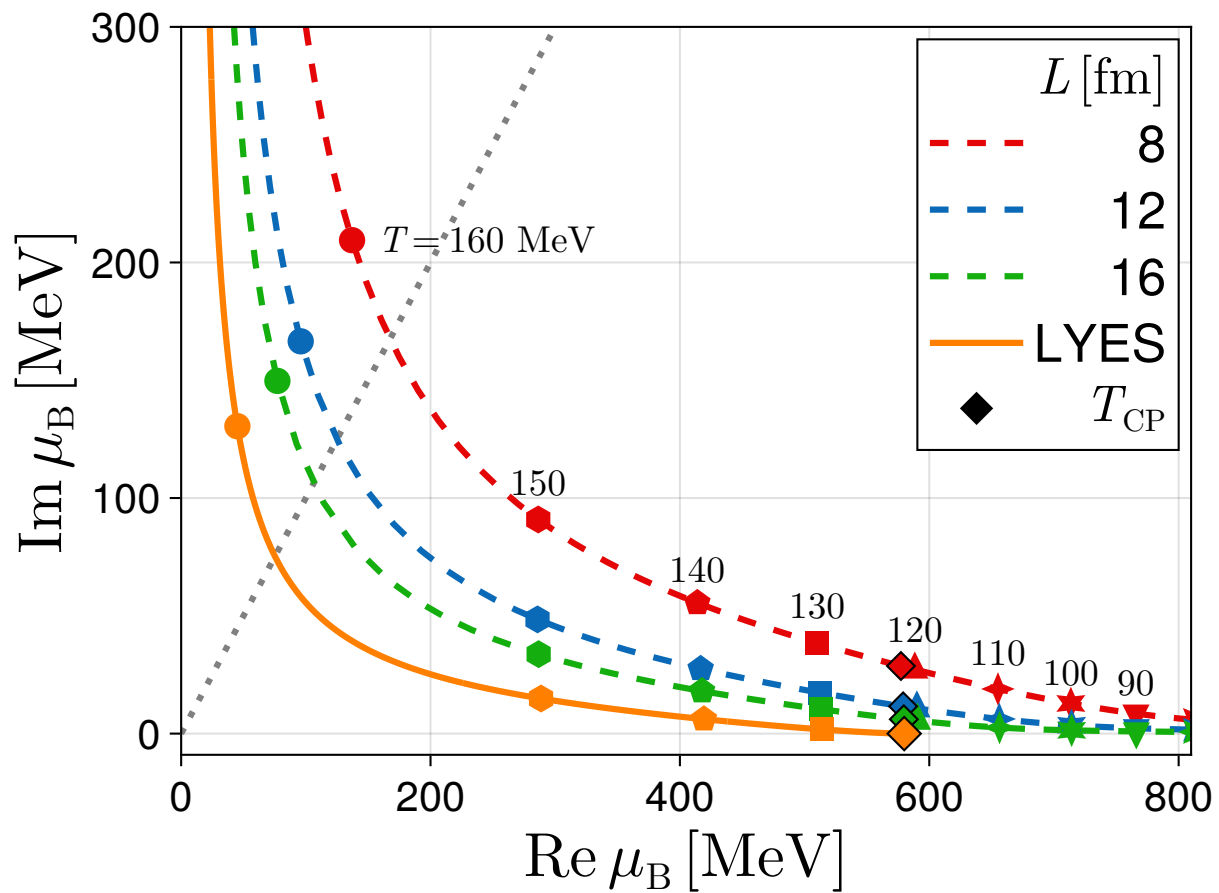
Set B



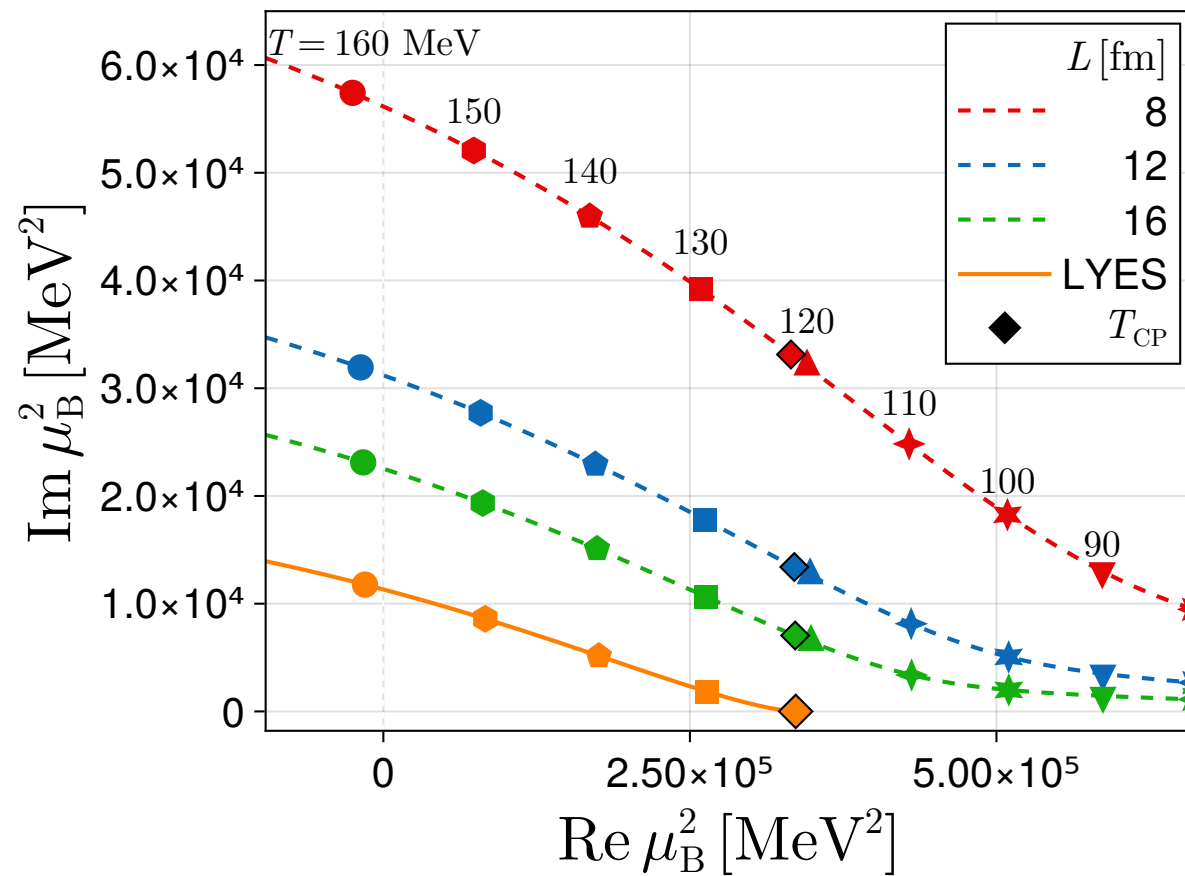
LYZ and LYES

 μ_B - plane

Set A

 μ_B^2 - plane

Set A



LYZ and LYES

Charge conjugation symmetry

$$Z(\mu_B) = Z(-\mu_B)$$

➤ Z is expressed as function of μ_B^2

Mapping relation is replaced

$$\begin{pmatrix} \tau \\ \eta \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} T - T_{\text{CP}} \\ \mu_B - \mu_{\text{CP}} \end{pmatrix}$$

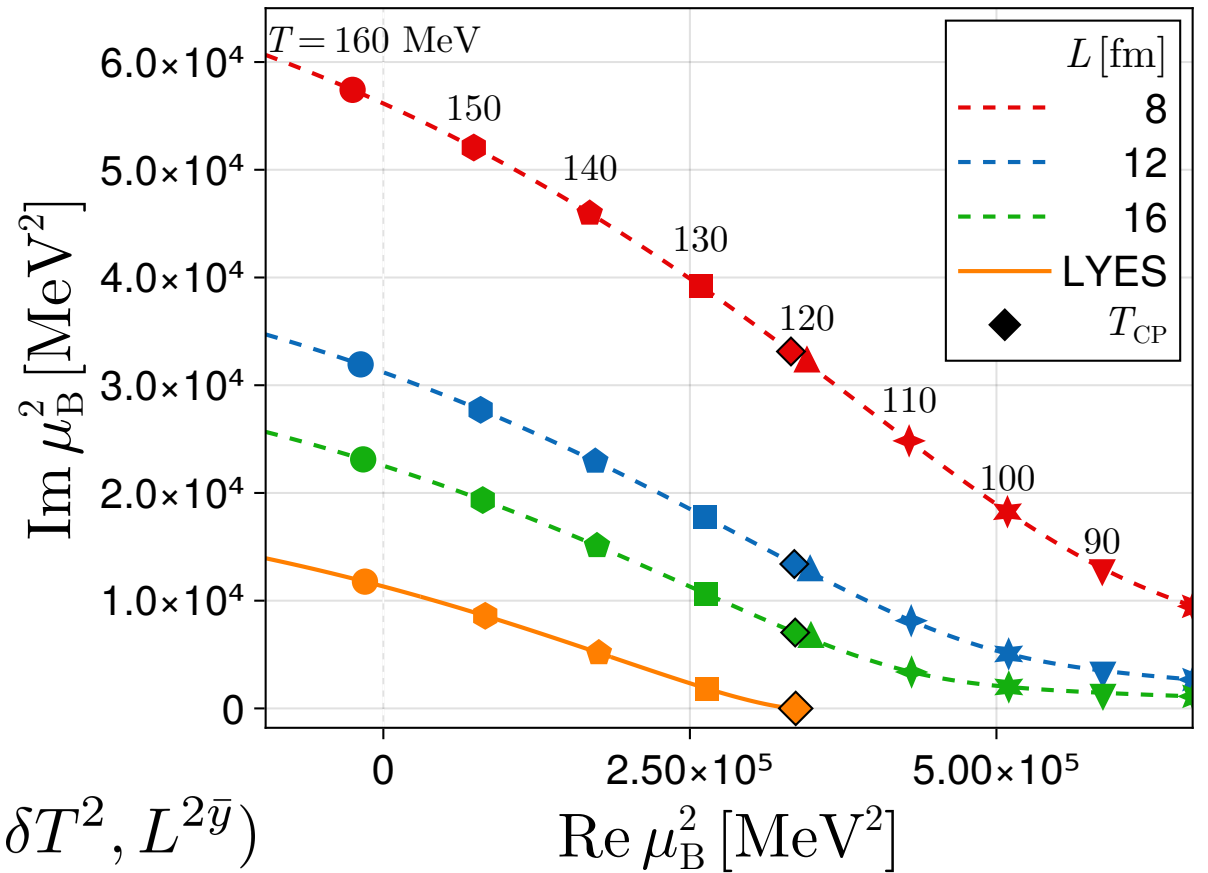
$$\begin{pmatrix} \tau \\ \eta \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12}/2\mu_{\text{CP}} \\ a_{21} & a_{22}/2\mu_{\text{CP}} \end{pmatrix} \begin{pmatrix} T - T_{\text{CP}} \\ \mu^2 - \mu_{\text{CP}}^2 \end{pmatrix}$$

Modified scaling function of LYZ

$$\text{Re}(\mu_{\text{LY}}^{(n)}(T, L))^2 = \mu_{\text{CP}}^2 - 2\mu_{\text{CP}} \frac{a_{21}}{a_{22}} \delta T + \mathcal{O}(\delta T^2, L^{2\bar{y}})$$

μ_B^2 - plane

Set A



LYZ and LYES

Charge conjugation symmetry

$$Z(\mu_B) = Z(-\mu_B)$$

➤ Z is expressed as function of μ_B^2

Mapping relation is replaced

$$\begin{pmatrix} \tau \\ \eta \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} T - T_{CP} \\ \mu_B - \mu_{CP} \end{pmatrix}$$

$$\begin{pmatrix} \tau \\ \eta \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12}/2\mu_{CP} \\ a_{21} & a_{22}/2\mu_{CP} \end{pmatrix} \begin{pmatrix} T - T_{CP} \\ \mu_B^2 - \mu_{CP}^2 \end{pmatrix}$$

Modified scaling

$$\text{Re}(\mu_{LY}^{(n)}(T, L))$$

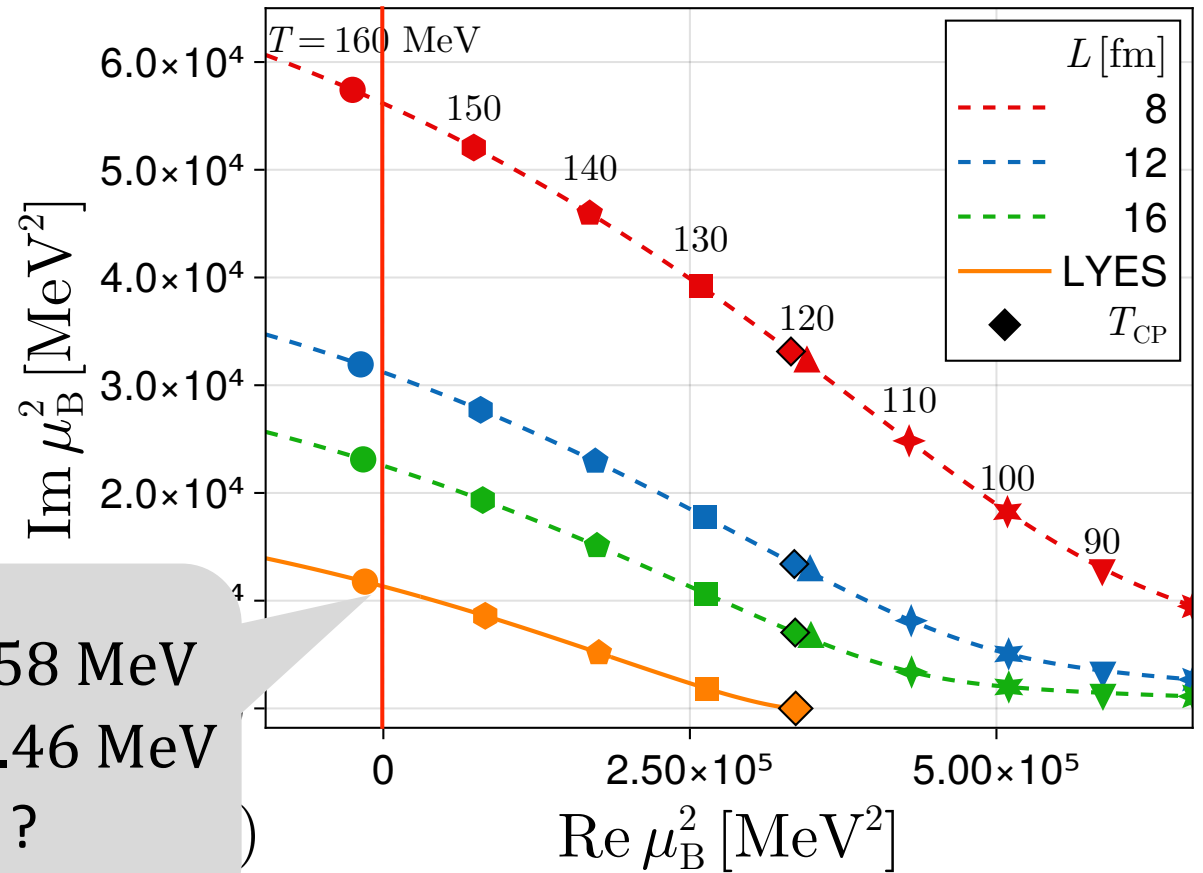
Crossing with $0 \text{ MeV}^2 \sim T=158 \text{ MeV}$

This value is close to $T_0^{\text{pc}} = 158.46 \text{ MeV}$

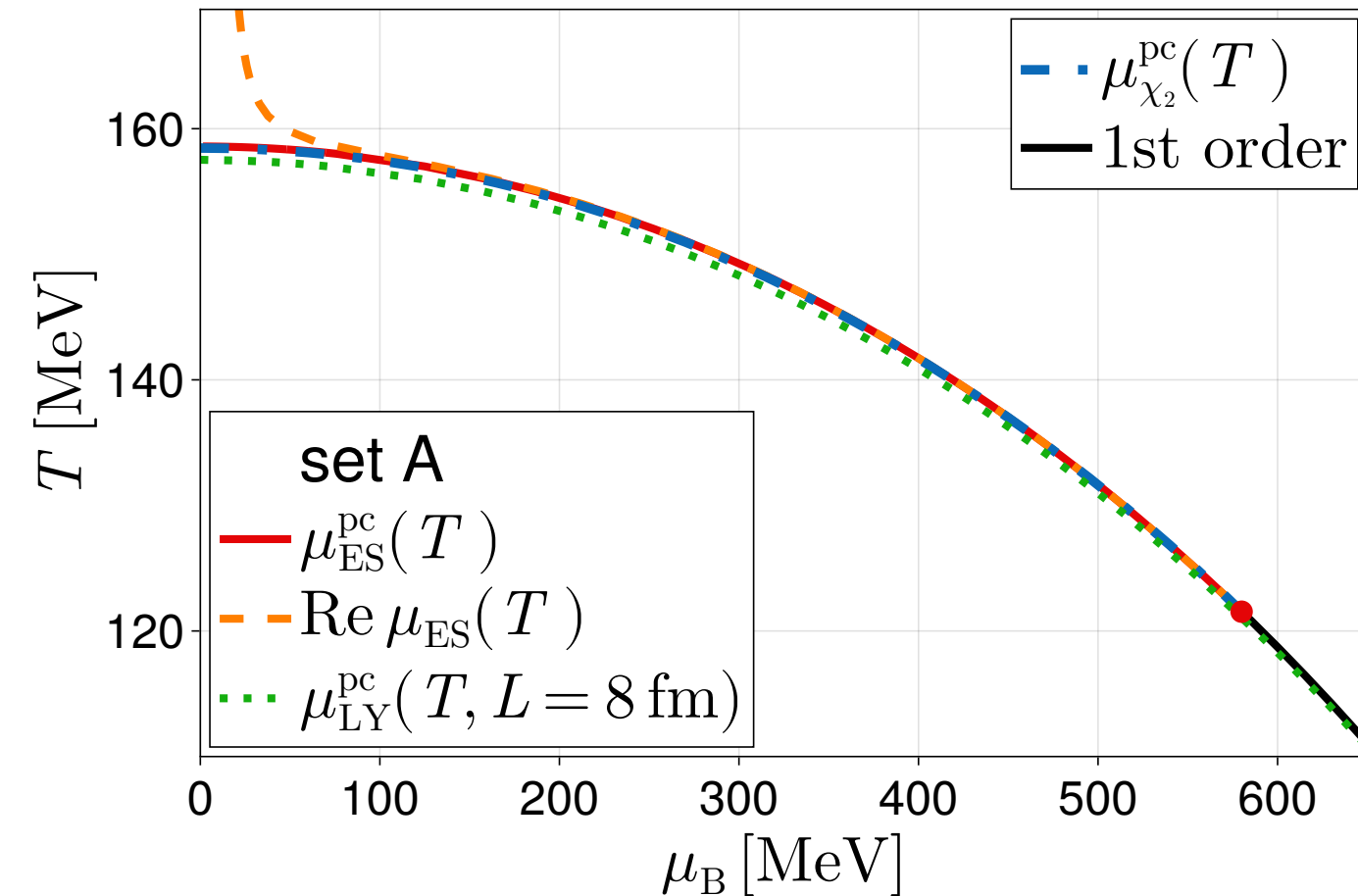
New crossover definition ?

μ_B^2 - plane

Set A



Crossover line defined LYES



Crossover μ_B by LYES

$$\mu_{\text{ES}}^{\text{pc}}(T) \equiv \sqrt{\text{Re } \mu_{\text{ES}}(T)^2}$$

Crossover μ_B by LYZ

$$\mu_{\text{LY}}^{\text{pc}}(T, L) \equiv \sqrt{\text{Re}(\mu_{\text{LY}}^{(1)}(T, L))^2}$$

Consistent with χ_2 def.

→ One of the crossover criterion

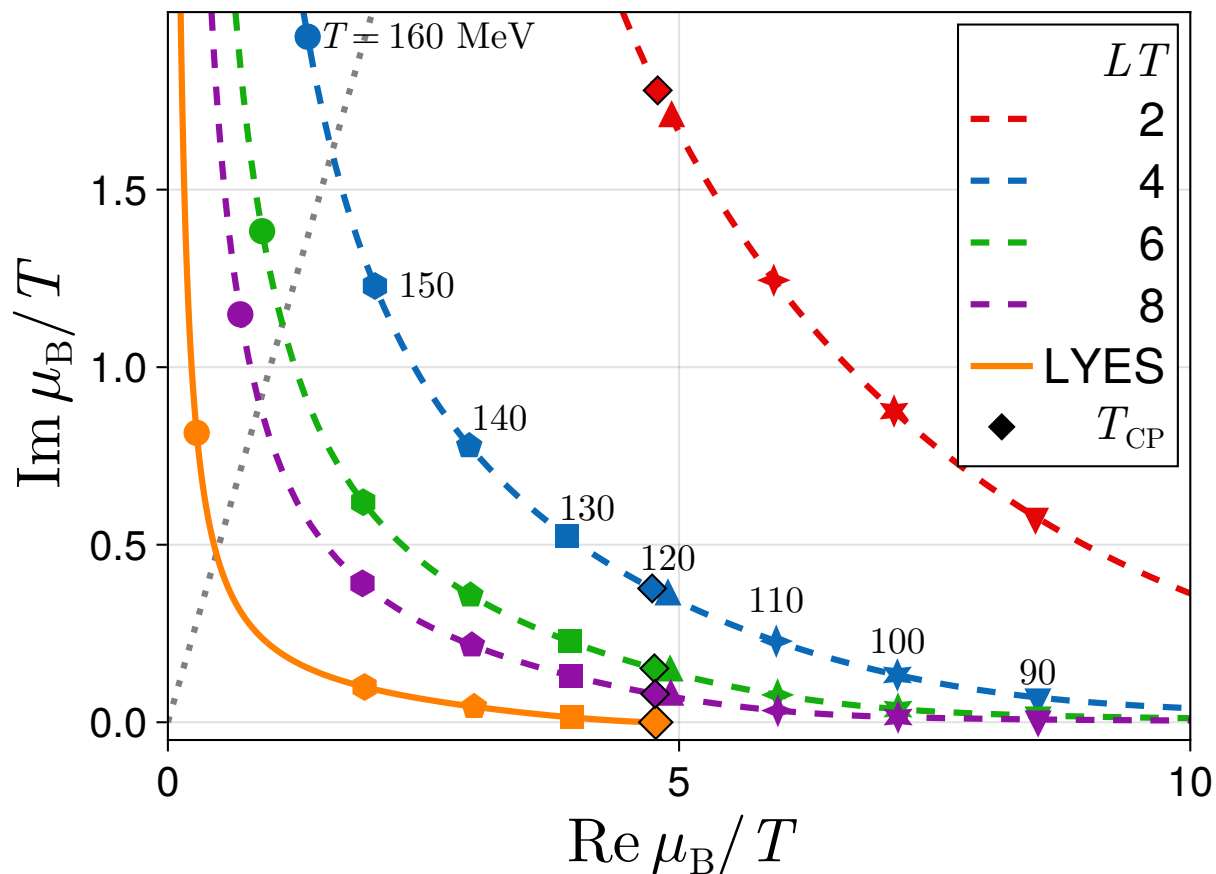
◆ Volume dependence of real part of LYZ is small.

→ This may be useful for tracing phase boundaries.

Comparison to Lattice Result

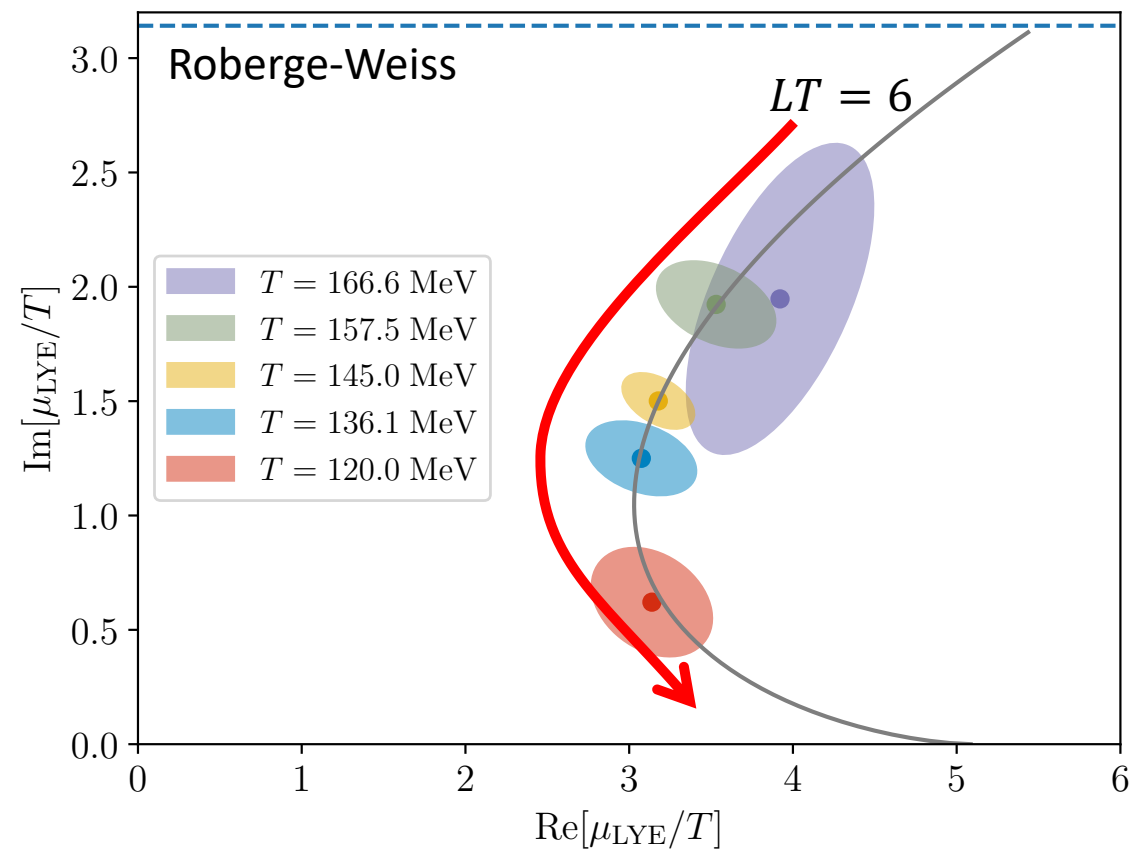
μ_B/T - plane

Set A



$N_f = 2 + 1$ QCD on Lattice

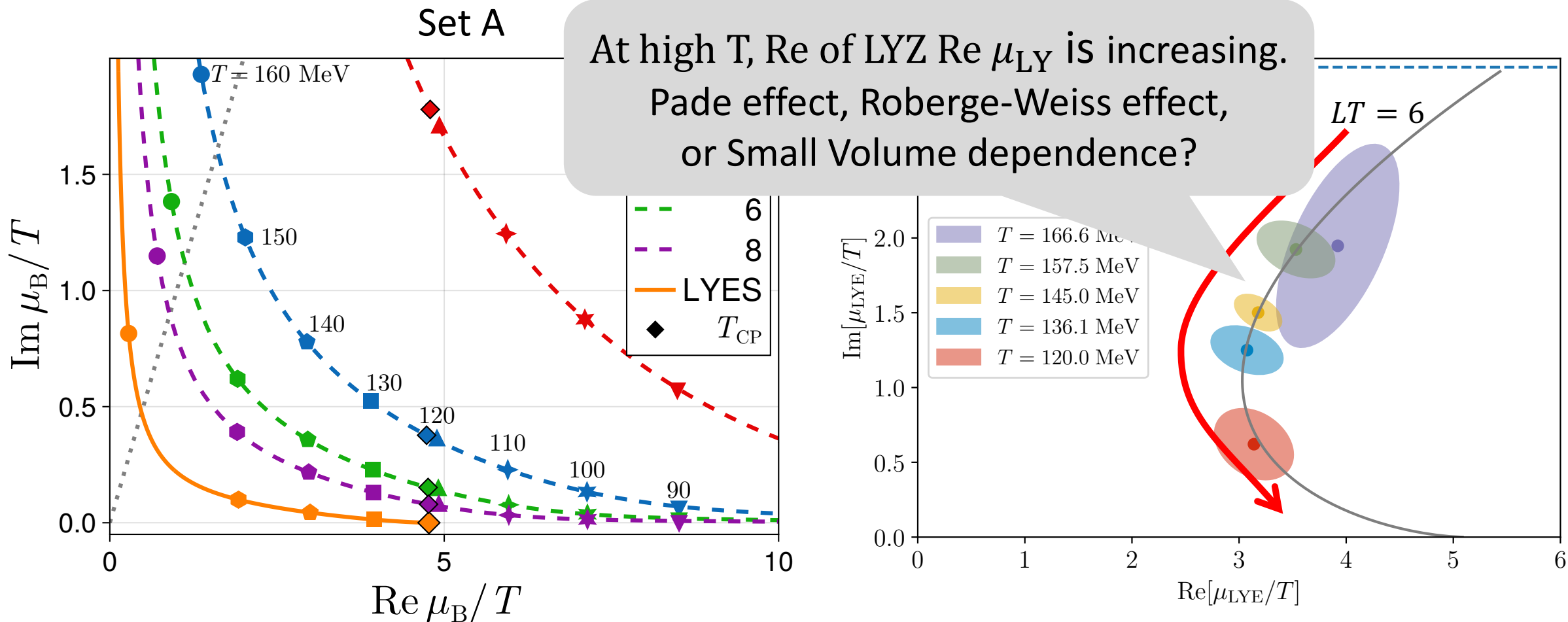
[Clarke, et. al, 2024]



Comparison to Lattice Result

μ_B/T - plane

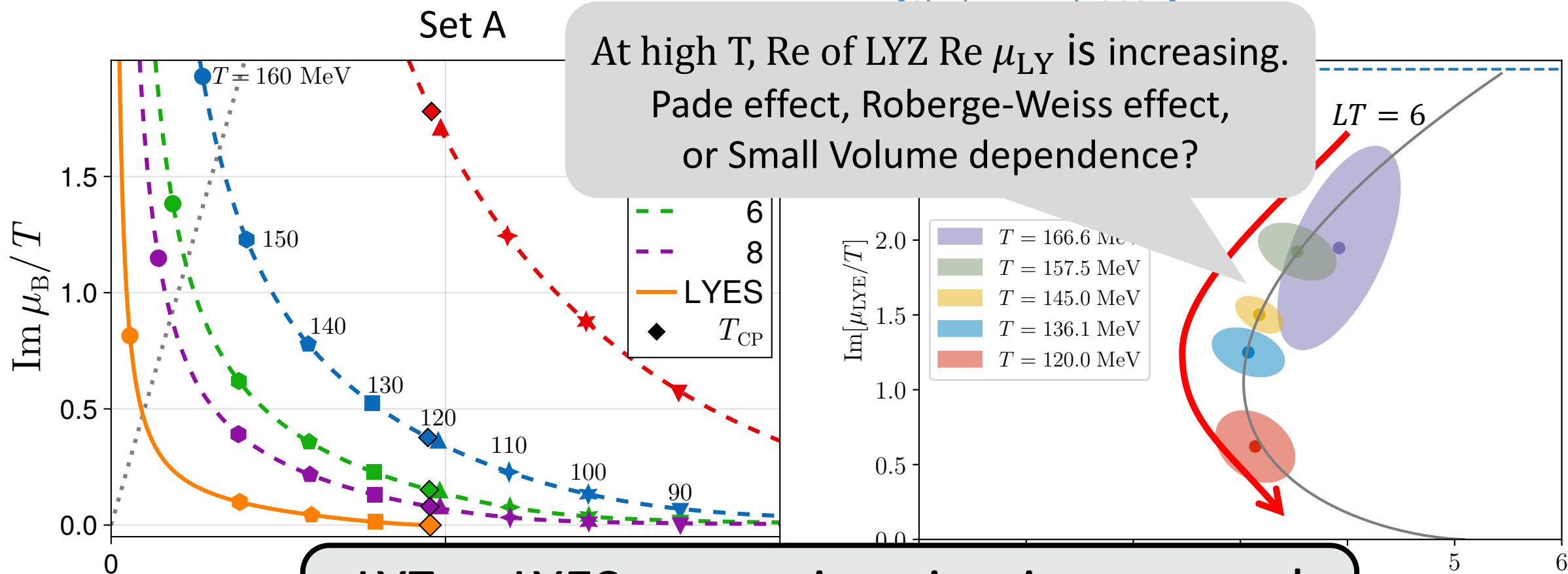
$N_f = 2 + 1$ QCD on Lattice



Comparison to Lattice Result

μ_B/T - plane

$N_f = 2 + 1$ QCD on Lattice

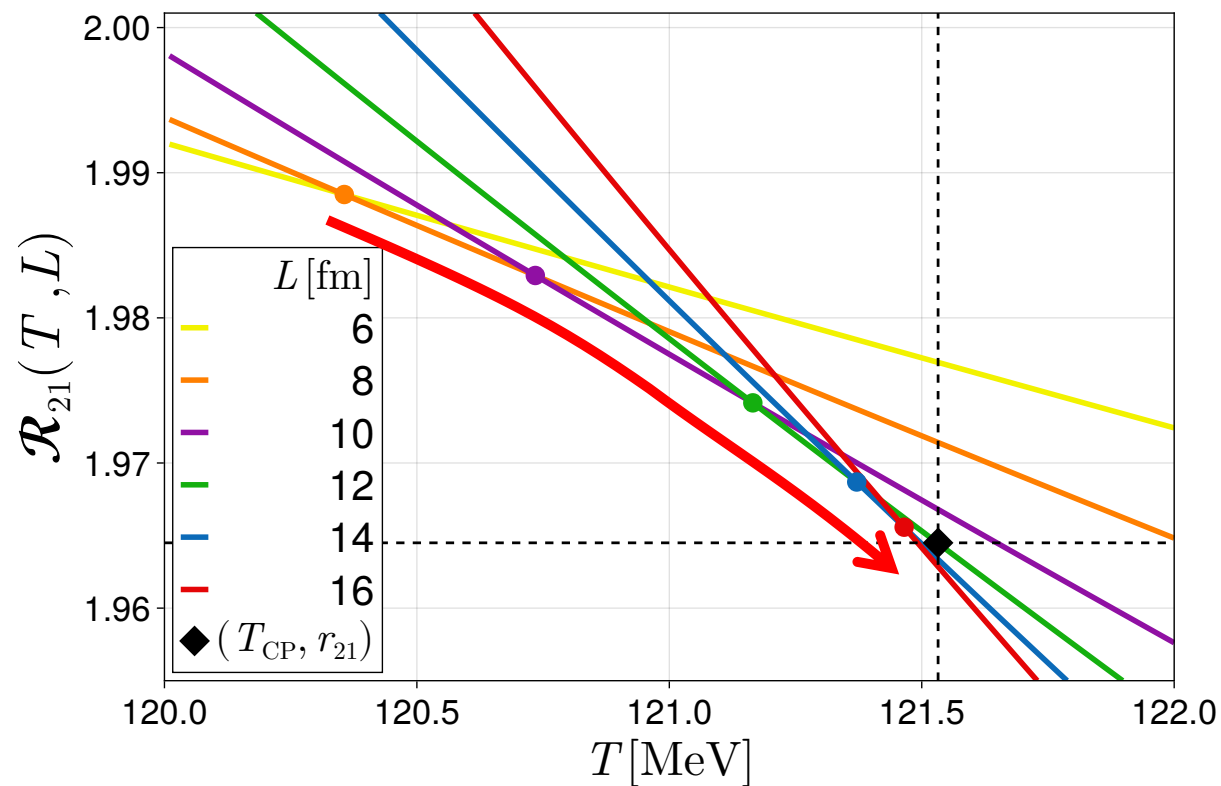


LYZ = LYES approximation is not good
in this lattice size.

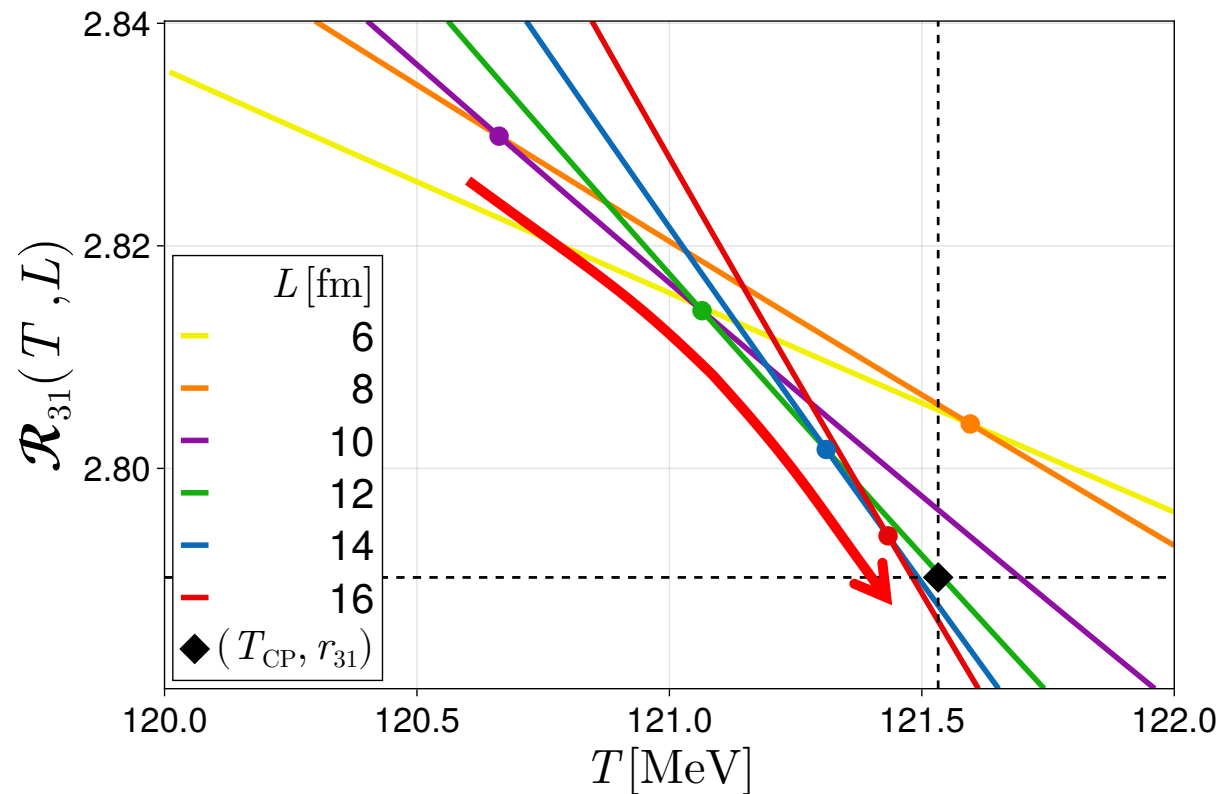
Lee-Yang Zero Ratio Method

$$\mathcal{R}_{nm}(T, L) = \frac{\text{Im} \mu_{\text{LY}}^{(n)}(T, L)}{\text{Im} \mu_{\text{LY}}^{(m)}(T, L)} = (r_{nm} + C_{nm}(L^{y_t} \delta T) + \mathcal{O}(\delta T^2))(1 + D_{nm}L^{2\bar{y}} + \mathcal{O}(L^{4\bar{y}}))$$

2nd / 1st LYZR



3rd / 1st LYZR



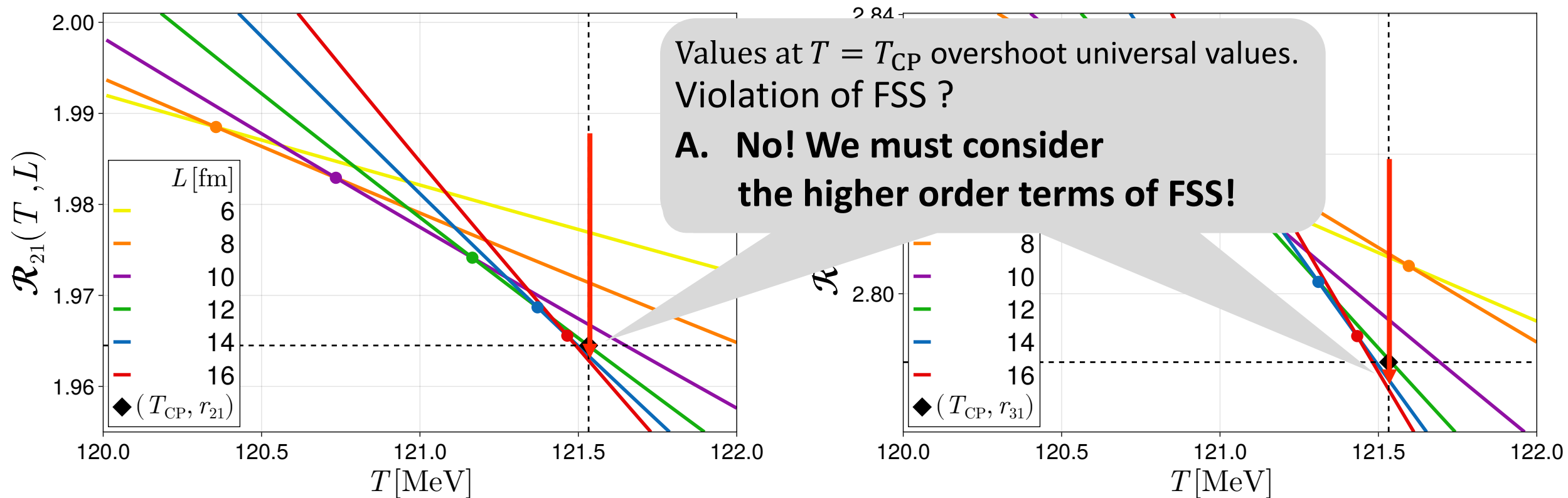
Intersection Points converge to CP!

Lee-Yang Zero Ratio Method

$$\mathcal{R}_{nm}(T, L) = \frac{\text{Im} \mu_{\text{LY}}^{(n)}(T, L)}{\text{Im} \mu_{\text{LY}}^{(m)}(T, L)} = (r_{nm} + C_{nm}(L^{y_t} \delta T) + \mathcal{O}(\delta T^2))(1 + D_{nm}L^{2\bar{y}} + \mathcal{O}(L^{4\bar{y}}))$$

2nd / 1st LYZR

3rd / 1st LYZR



Single LYZ method

$N_f = 2 + 1$ QCD

1st LYZ is investigated, but more than 2nd is not.

LYZ function

$$h = h_{\text{LY}}^{(n)}(t, L^{-1})$$



FSS of LYZ in Ising model

$$L^{y_h} h_{\text{LY}}^{(n)}(t, L^{-1}) = \tilde{h}_{\text{LY}}^{(n)}(L^{y_t} t)$$

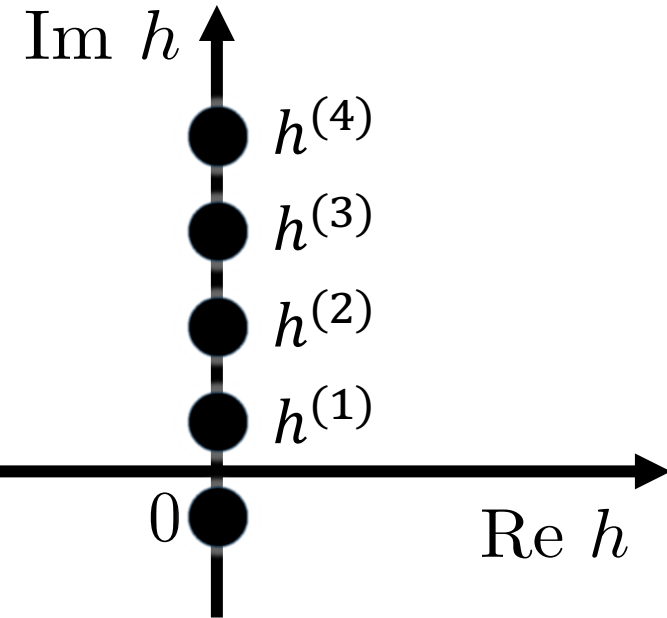
[Itzykson 1983]

Expansion



$$\tilde{h}_{\text{LY}}^{(n)}(L^{y_t} t) = X_n + Y_n L^{y_t} t + \mathcal{O}(t^2)$$

This value is independent to V at $t=0$

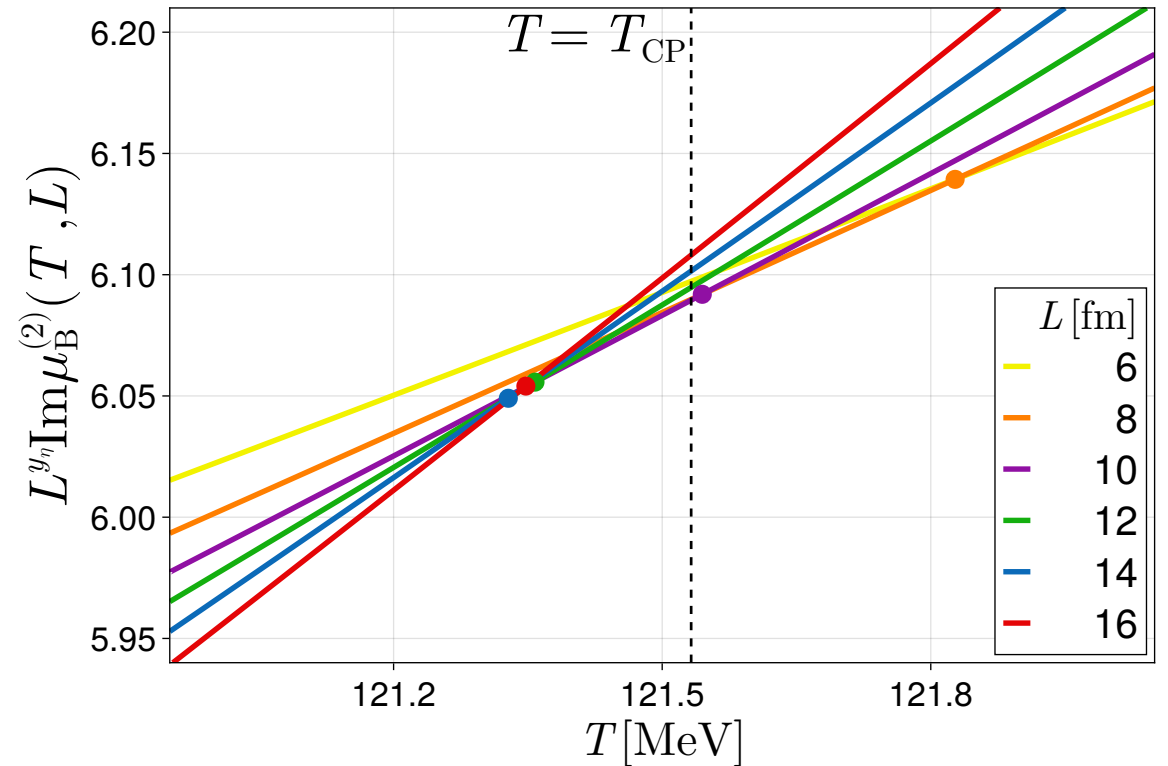
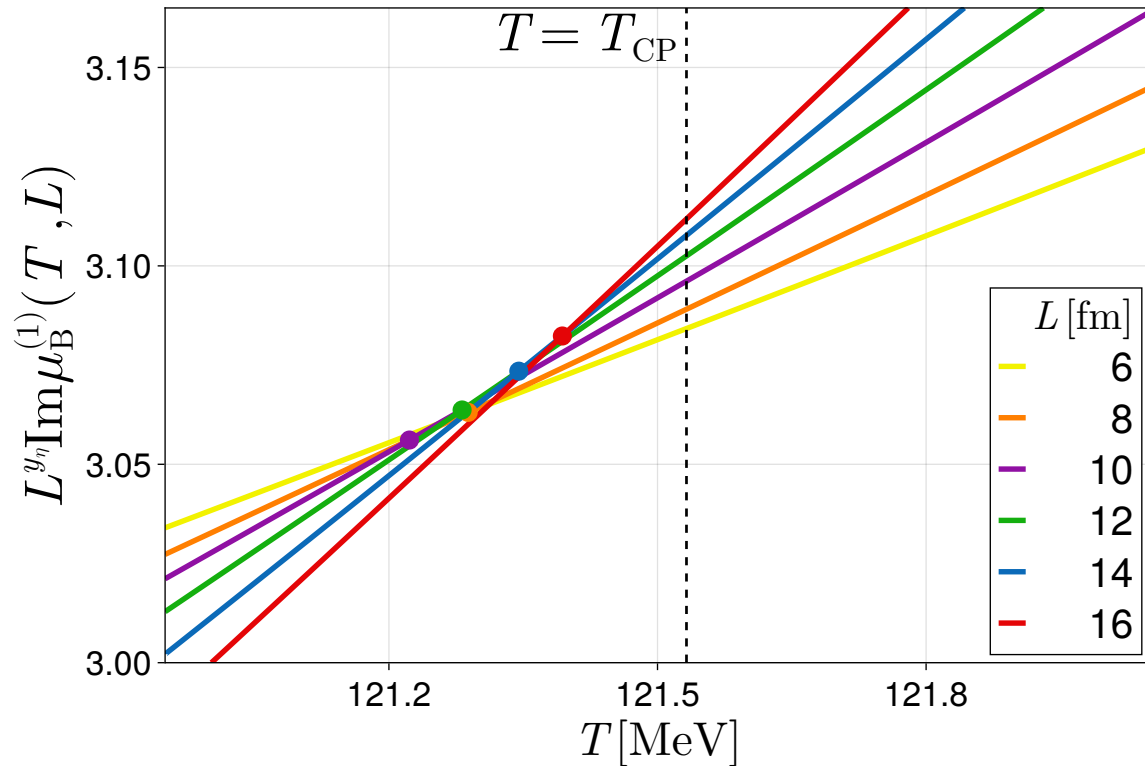


Crossing point = Critical Point

Single LYZ method in Effective model

$$L^{y_\eta} \mu_{\text{LY}}^{(n)}(L^{y_\tau} \delta T) = X_n + Y_n L^{y_\tau} \delta T + \mathcal{O}(\delta T^2)$$

* crossing value = X_n are not universal



Finite volume effect is large.

However, this value is a good indicator of CP.

(If we know the universality class.)



Next :

Application to $N_f = 2 + 1$ Lattice Result

Single LYZ method in $N_f=2+1$ QCD

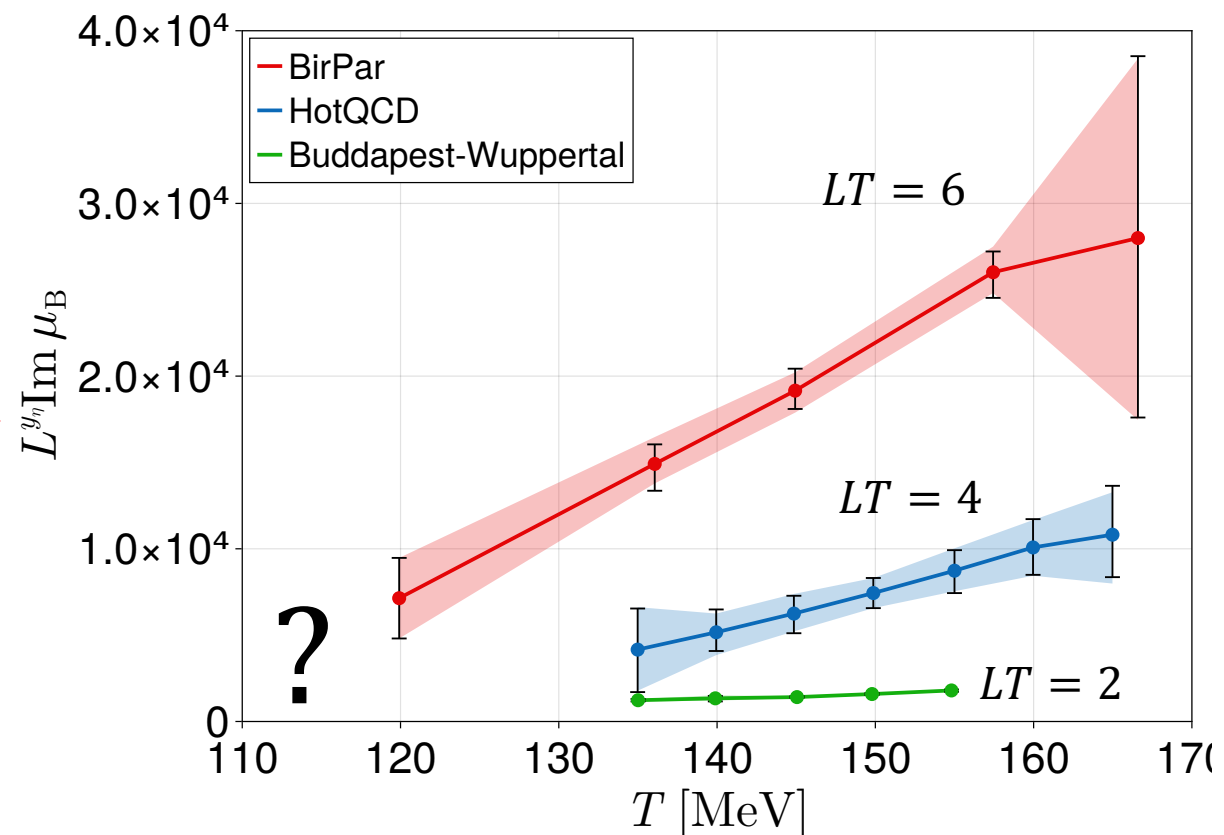
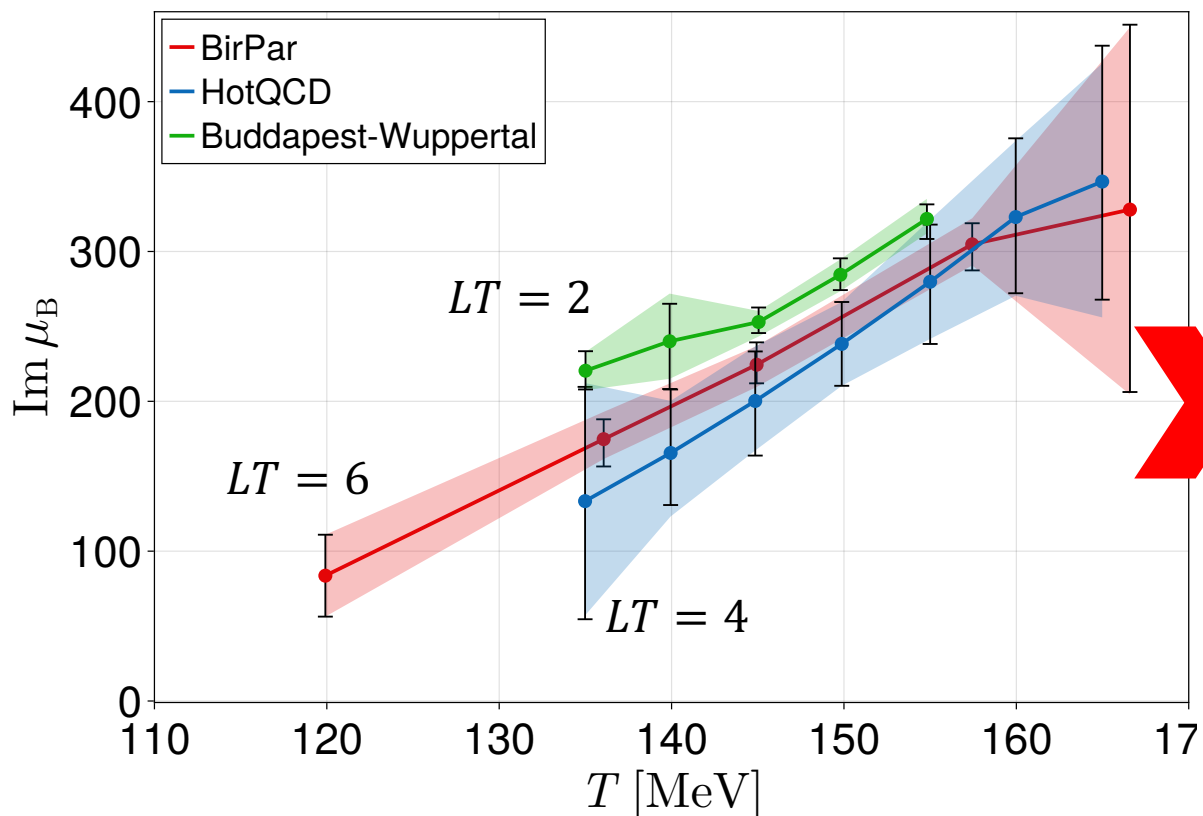
$$L^{y_\eta} \mu_{\text{LY}}^{(n)}(L^{y_\tau} \delta T) = X_n + Y_n L^{y_\tau} \delta T + \mathcal{O}(\delta T^2)$$

Speculation:

Lattice result is still high temperature.

$T_c = 90 - 100$ MeV?

1st LYZ



Conclusion

- ✓ We investigate **Lee-Yang zeros** and **Lee-Yang edge singularity** using **finite-volume mean field** approach.
- ✓ We obtain the qualitative properties of LYZ and LYES.
 - Behaviors LYES and LYZs.
 - New criteria of crossover line (squared definition)
- ✓ Application to LYZR and single LYZ method

Future Works

- ❑ Final Goal : Application to 2+1 flavor QCD
QCD critical point (We need 2nd LYZ!)
- ❑ Application to systems including Roberge Weiss transition (Imaginary μ_B)
- ❑ Application to systems with more richer phases (HQ-QCD+RW)

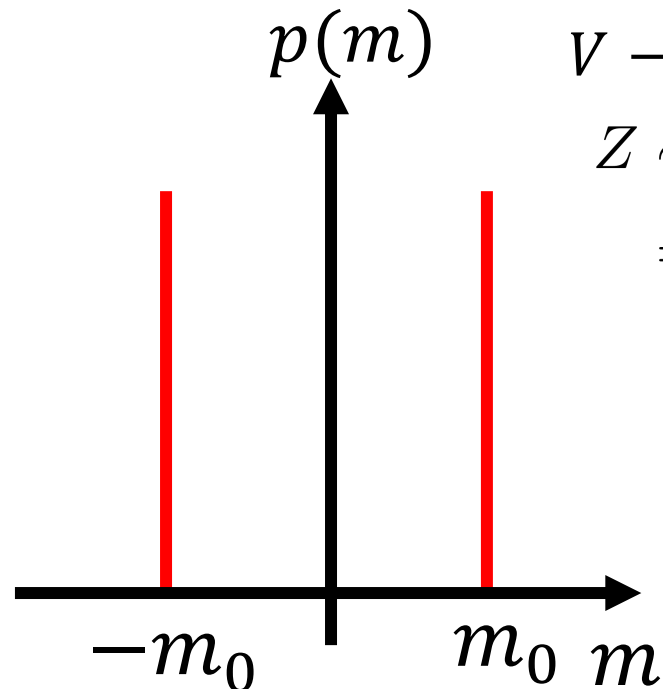
Review : Lee-Yang Zero

Lee, Yang 1952

Lee-Yang Zeros

= Zeros of Z in Complex Parameter space

The intervals of LYZs are same.



$$V \rightarrow \infty$$

$$Z \sim e^{-V\beta hm_0} + e^{V\beta hm_0}$$

$$= e^{-V\beta hm_0} (1 + e^{2V\beta hm_0})$$

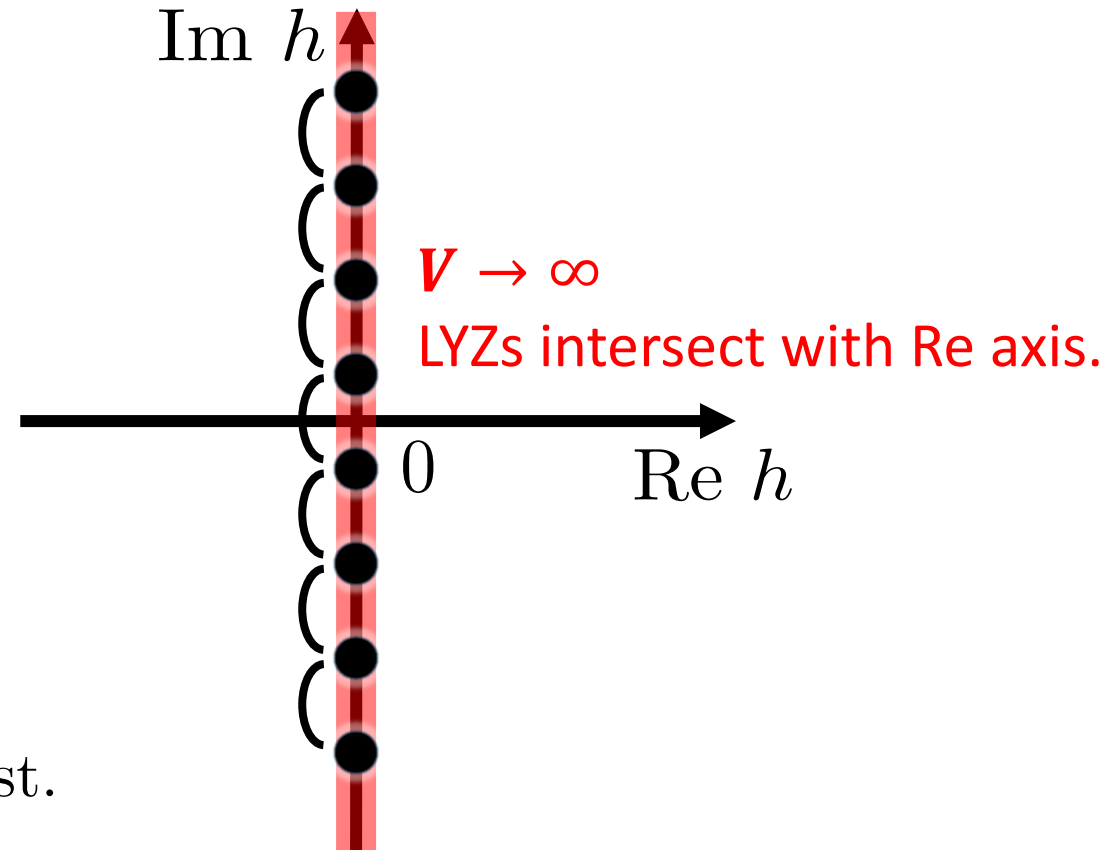
$$h_{\text{LY}}(t) = \frac{(2n+1)\pi i}{2\beta V m_0}$$

$$\sim i(2n+1) \times \text{const.}$$

Lee-Yang's circle theorem

LYZs exist only on imaginary axis.

$t < 0$: 1st order



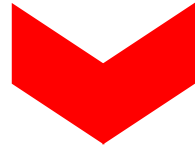
Short Summary

Lee-Yang edge singularity is defined in the **infinite volume limit**.

Lattice simulation is always performed in **finite volume**.

Problem : Exclude the finite volume effect

1st LYZ on finite volume $\stackrel{?}{\simeq}$ LY edge singularity ($V \rightarrow \infty$)



Lee-Yang zero ratio method

A new method to locate a CP using **Finite size scaling**

TW, Kitazawa, Kanaya, PRL, 134, 162302 2025



Finite Volume Mean Field approach + Effective Model

How to obtain LYZ and LYES

- ◆ Lee-Yang edge singularity = critical point in complex-plane

$$\blacktriangleright \frac{\partial \mathcal{U}(\bar{\phi}; T, \mu_B)}{\partial \bar{\phi}} = 0, \quad \frac{\partial^2 \mathcal{U}(\bar{\phi}; T, \mu_B)}{\partial \bar{\phi}^2} = 0 \quad \text{in complex } \mu_B \text{ plane}$$

For fixed T , the solution is $(\text{Re } \phi, \text{Im } \phi, \text{Re } \mu_{\text{ES}}(T), \text{Im } \mu_{\text{ES}}(T))$

- ◆ Lee-Yang zero

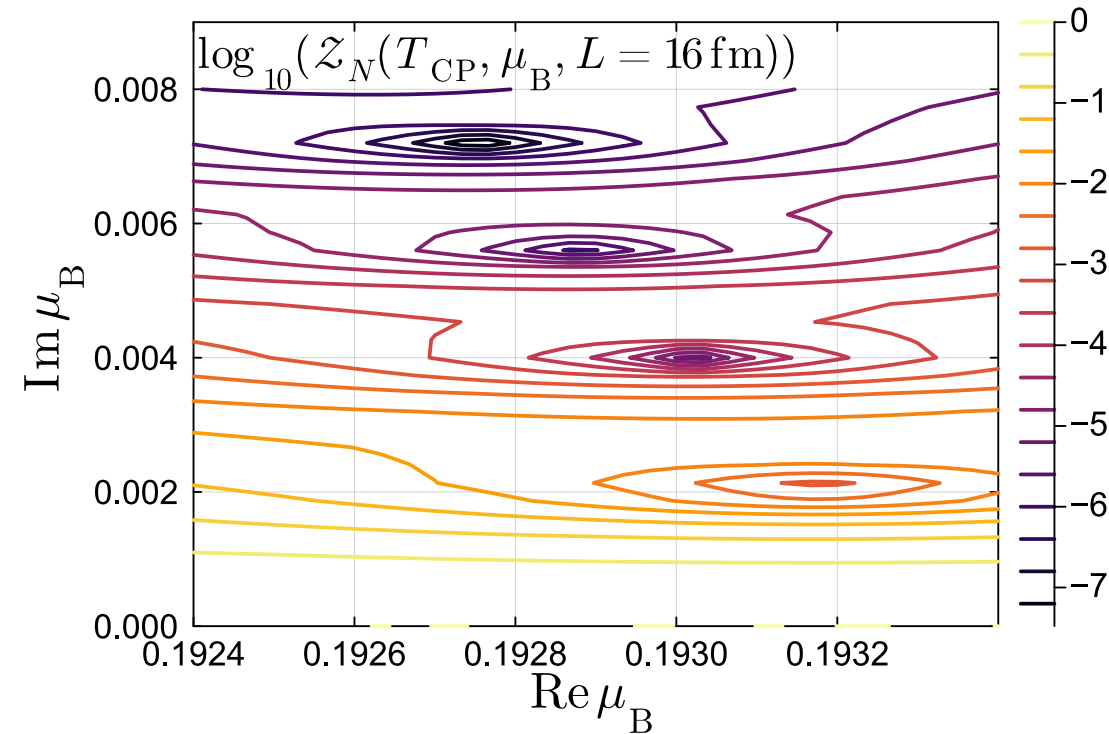
$$\mathcal{Z}(T, \mu_B, L) = \int d\bar{\phi} e^{-L^3 \mathcal{U}(\bar{\phi}, T, \mu_B) / T}$$

For numerical convenience,

we define normalized partition function \mathcal{Z}_N

$$\mathcal{Z}_N(T, \mu_B, L) = \frac{\mathcal{Z}(T, \mu_B, L)}{\mathcal{Z}(T, \text{Re } \mu_B, L)}$$

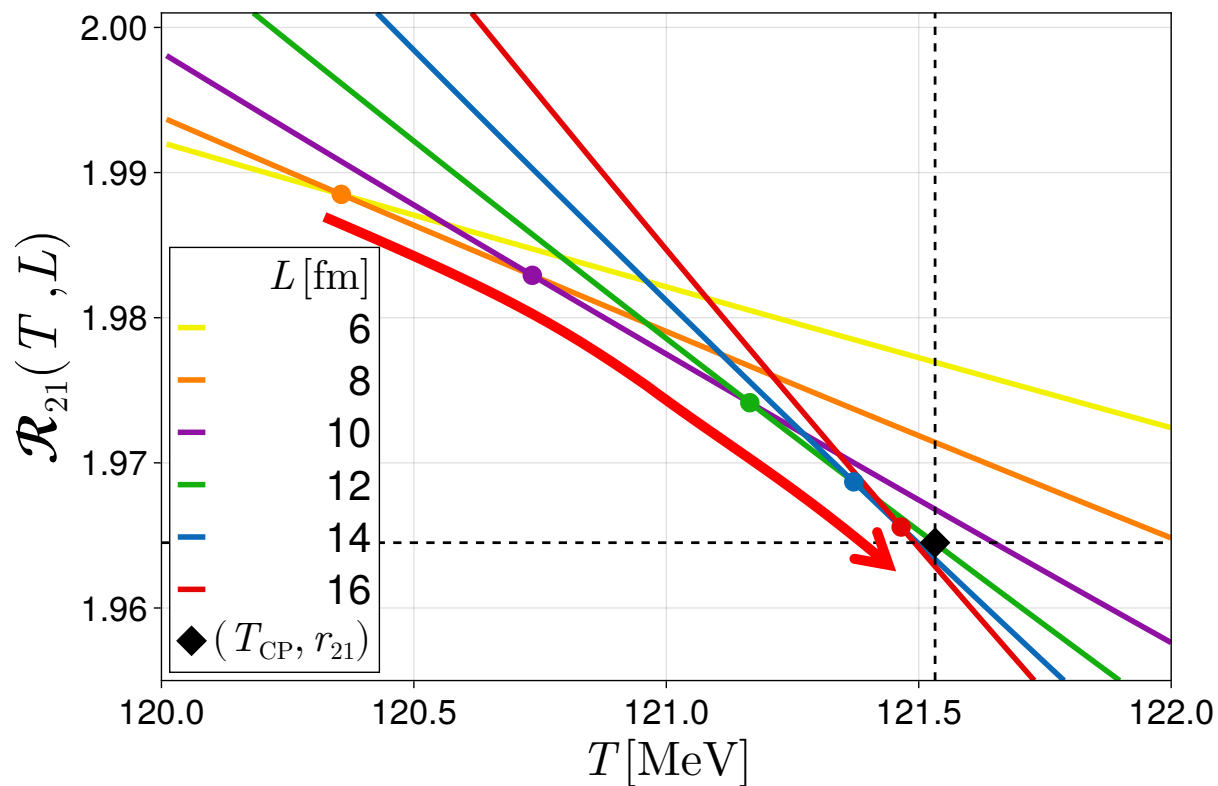
$$\blacktriangleright \begin{cases} \text{Re } \mathcal{Z}_N(T, \mu_B, L) = 0 \\ \text{Im } \mathcal{Z}_N(T, \mu_B, L) = 0 \end{cases}$$



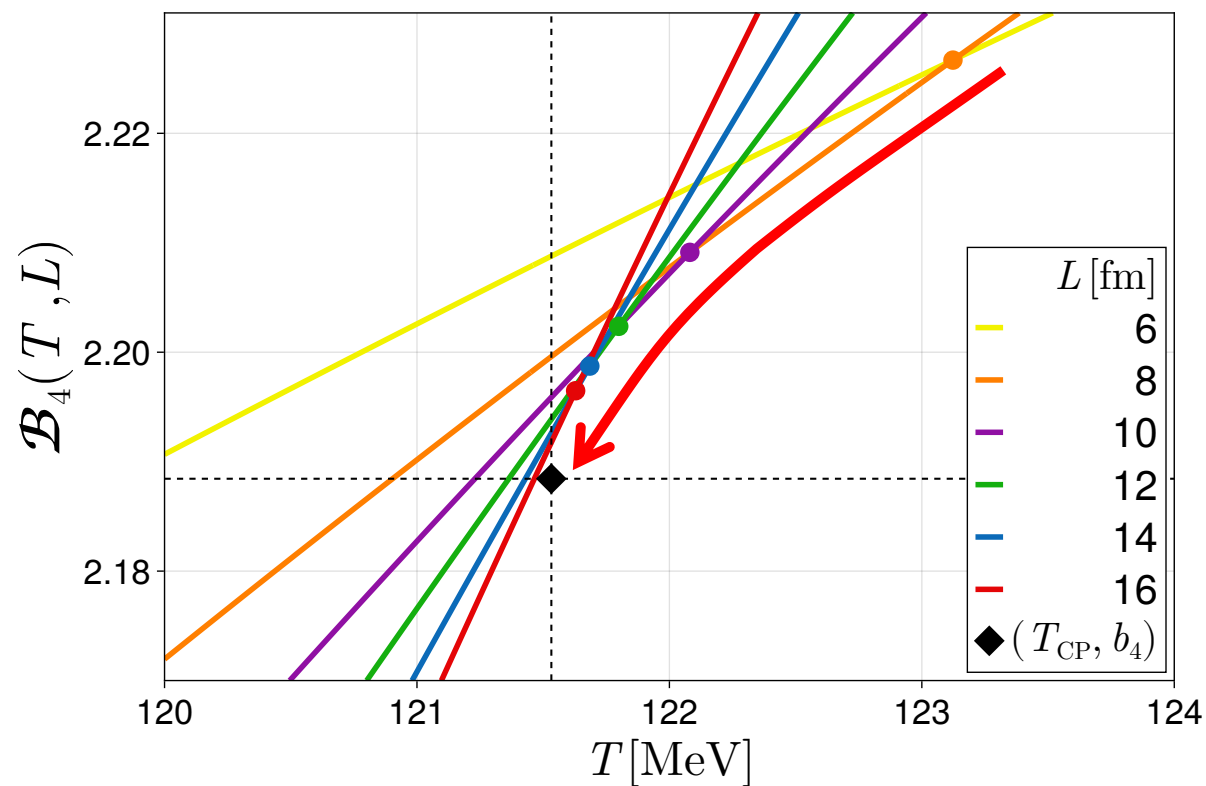
Lee-Yang Zero Ratio Method

$$\mathcal{R}_{nm}(T, L) = \frac{\text{Im} \mu_{\text{LY}}^{(n)}(T, L)}{\text{Im} \mu_{\text{LY}}^{(m)}(T, L)} = (r_{nm} + C_{nm}(L^{y_t} \delta T) + \mathcal{O}(\delta T^2))(1 + D_{nm}L^{2\bar{y}} + \mathcal{O}(L^{4\bar{y}}))$$

2nd / 1st LYZR



4th Binder cumulant



Intersection Points converge to CP

Higher Order Term of FSS

The origin of Higher order terms

◆ Parameter mixing

$$\begin{pmatrix} \tau \\ \eta \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} T - T_{\text{CP}} \\ \mu_{\text{B}} - \mu_{\text{CP}} \end{pmatrix}$$

Lowest order

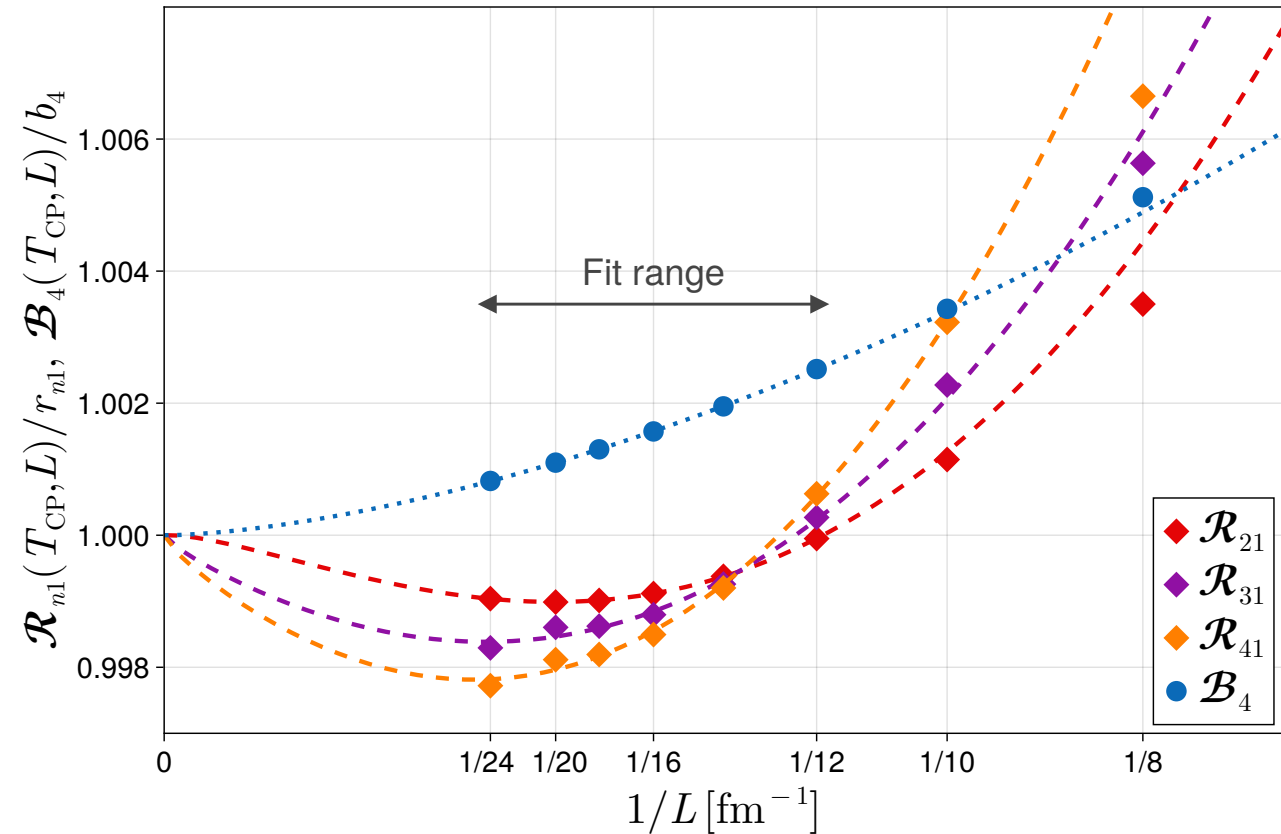
$$\text{LYZR} : 2\bar{y} = 2(y_{\tau} - y_{\eta}) = -3/2$$

$$\text{Binder} : \bar{y} = 2(y_{\tau} - y_{\eta}) = -3/4$$

◆ Irrelevant operator

$$\begin{aligned} U'_{\text{Landau}}(\bar{\phi}; \tau, \eta, \{\lambda\}) &= U'_{\text{Landau}}(\bar{\phi}; \tau, \eta, \lambda_5, \lambda_6, \dots) \\ &\equiv U_{\text{Landau}}(\bar{\phi}; \tau, \eta) + \sum_{i=5}^N \lambda_i \bar{\phi}^i \end{aligned}$$

$$\text{Exponent} : y_5 = -3/4, y_6 = -3/2 \dots$$



Fitting function $(A, a) = (\mathcal{R}_{n1}, r_{n1}), (\mathcal{B}_4, b_4)$

$$\frac{A(T_{\text{CP}}, L)}{a} = 1 + w_1 L^{-3/4} + w_2 L^{-3/2} + w_3 L^{-9/4}$$

Recent progress: Lee-Yang zeros

◆ 2+1 flavor QCD-CP

➤ Clarke, *et al.* (2024)

$$\begin{cases} \mu_c = 422^{+80}_{-35} \text{ MeV} \\ T_c = 105^{+8}_{-18} \text{ MeV} \end{cases}$$

➤ Basar (2024)

$$\begin{cases} \mu_c \sim 580 \text{ MeV} \\ T_c \sim 100 \text{ MeV} \end{cases}$$

➤ Alexander(2025)

$$T_c \sim 70 \text{ MeV}$$

or CP does not exist!

