Illinois Center for Advanced Studies of the Universe

Relativistic Hydrodynamics with a Critical Point

Isabella Danhoni

In collaboration with Kevin Pala, Jacquelyn Noronha-Hostler, Surkhab Kaur, Dekrayat Almaalol, Christopher Plumberg and Fernando Gardim.

QCD Critical Point and Hydrodynamic Evolution 2026

muses

Heavy-Ion Collisions: Equilibrium

Energy-Momentum Tensor

$$T^{\mu\nu} = T_0^{\mu\nu} + \Pi + \pi^{\mu\nu}$$

Equation of State
(EOS)

$$T_0^{\mu\nu} = \begin{bmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{bmatrix}$$

Baryon Current $N_B^\mu = n_B u^\mu + q_B^\mu$

Strangeness Current $N_S^\mu = n_S u^\mu + q_S^\mu$

Electric Charge Current $N_Q^\mu = n_Q u^\mu + q_Q^\mu$

Heavy-Ion Collisions: Equilibrium vs Out-of-Equilibrium

Energy-Momentum Tensor

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Electric Charge Current $N_Q^\mu = n_Q u^\mu + q_Q^\mu$

Contains diffusion matrix and gradients of μ/T

$$\kappa = \begin{bmatrix} \kappa_{BB} & \kappa_{BS} & \kappa_{BQ} \\ \kappa_{SB} & \kappa_{SS} & \kappa_{SQ} \\ \kappa_{QB} & \kappa_{QS} & \kappa_{QQ} \end{bmatrix}$$

Bulk Pressure

$$\Pi = Tr\left[T^{\mu\nu} - T_0^{\mu\nu}\right]$$

Shear stress tensor $\pi^{\mu\nu} = T^{\mu\nu} - T_0^{\mu\nu} - \Pi$

NuClearConfectionery FRAMEWORK + MUSES EOS

CCAKE

MUSES EOS

Grefa et al PRD 104 3, 034002

Yang et al arxiv 2601.07987

Plumberg et al PRC 111 4, 044905

Lin and Ko PRC 65 034904

Bin Zhang et al PRC 61 067901

Baochi Fu et al PRC 103 2, 024903

Weil et al PRC 94 054905

SMEARED AMPT

INITIAL CONDITIONS | PRE EQUILIBRIUM

→ CCAKE 2.0

HYDRODYNAMICS

→ BQSSAMPLER

PARTICLIZATION

→ SMASH

DECAYS

Plumberg et al PRC 111 4, 044905

P. Pala, **I. Danhoni** et al arxiv 2511.22852

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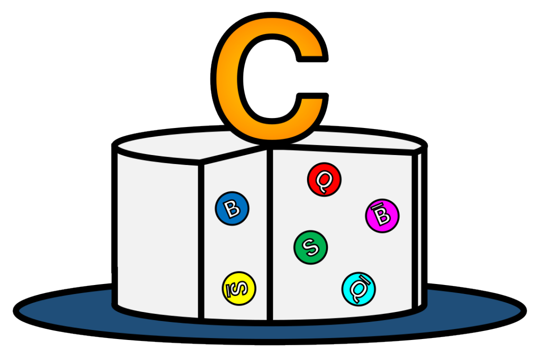

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SMEARED AMPT		CCAKE 2.0	BQSSAMPLER	SMASH
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P. Pala et al arxiv 2511.22852

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P. Pala, **I. Danhoni** et al arxiv 2511.22852

Muses Equation of State

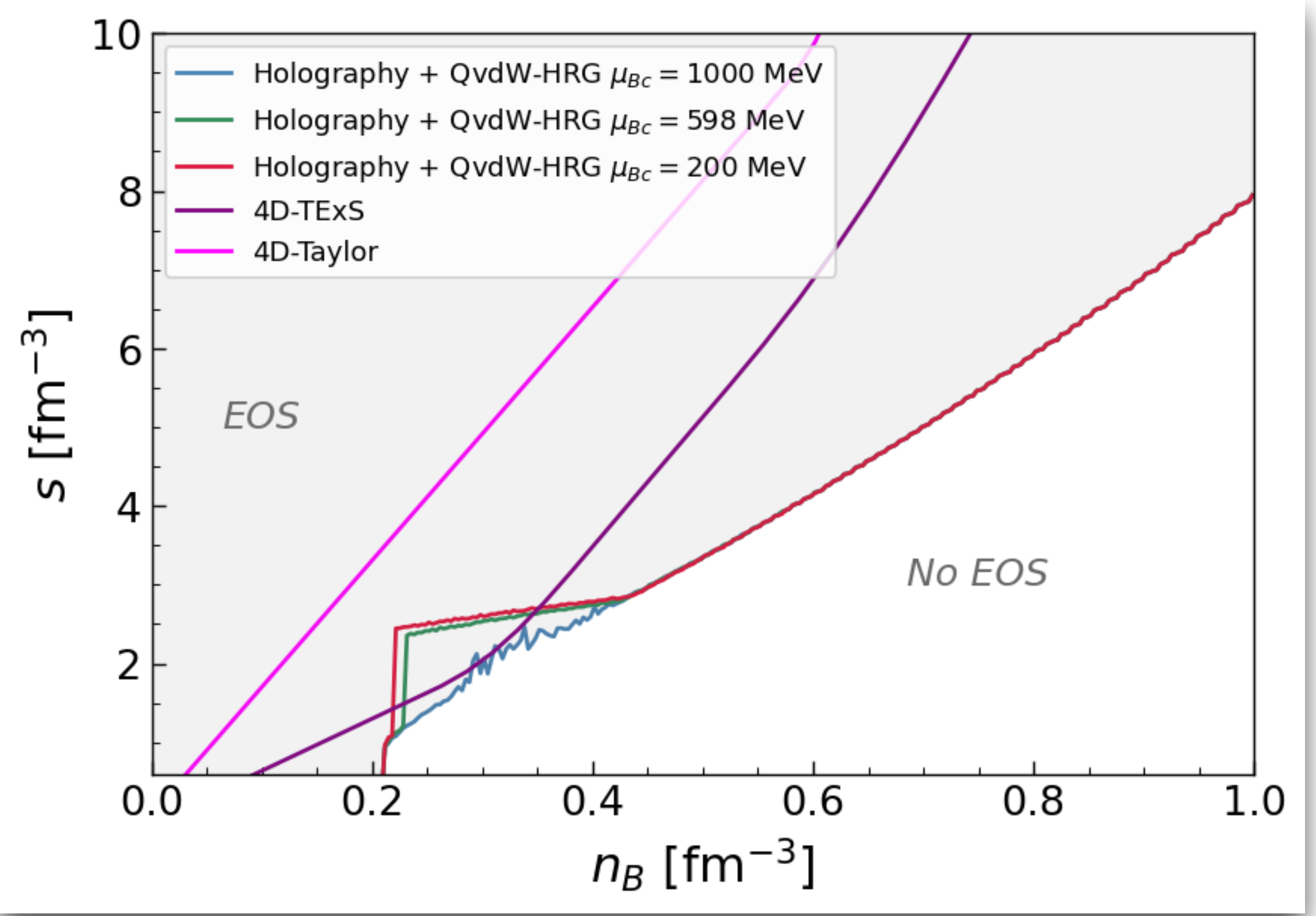
- Lower beam energies:

System explores regions higher baryon chemical potential (μ_B)

↓

EoS tables have limited coverage

- We plot the range of validity in terms of s, n_B of all MUSES EoSs made available ($\mu_S = \mu_Q = 0$) and we ensure causality and stability.



[MUSES] "Studying the QCD Matter produced in Heavy-Ion Collisions using the MUSES Calculation Engine" Soon to appear!

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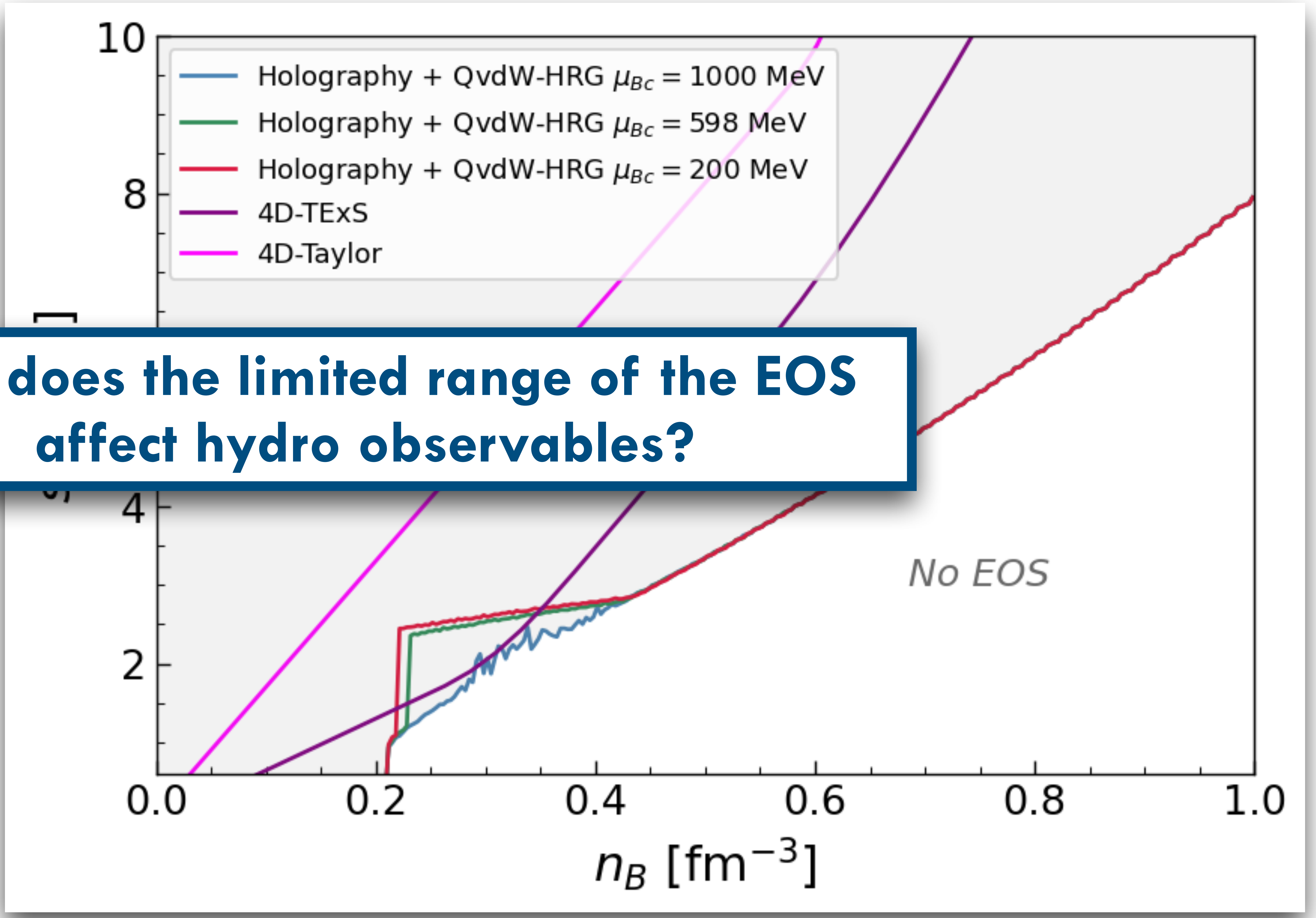

How does the limited range of the EOS affect hydro observables?

- We plot the range of validity in terms of s, n_B of all MUSES EoSs made available $(\mu_S = \mu_Q = 0)$ and we ensure causality and stability.

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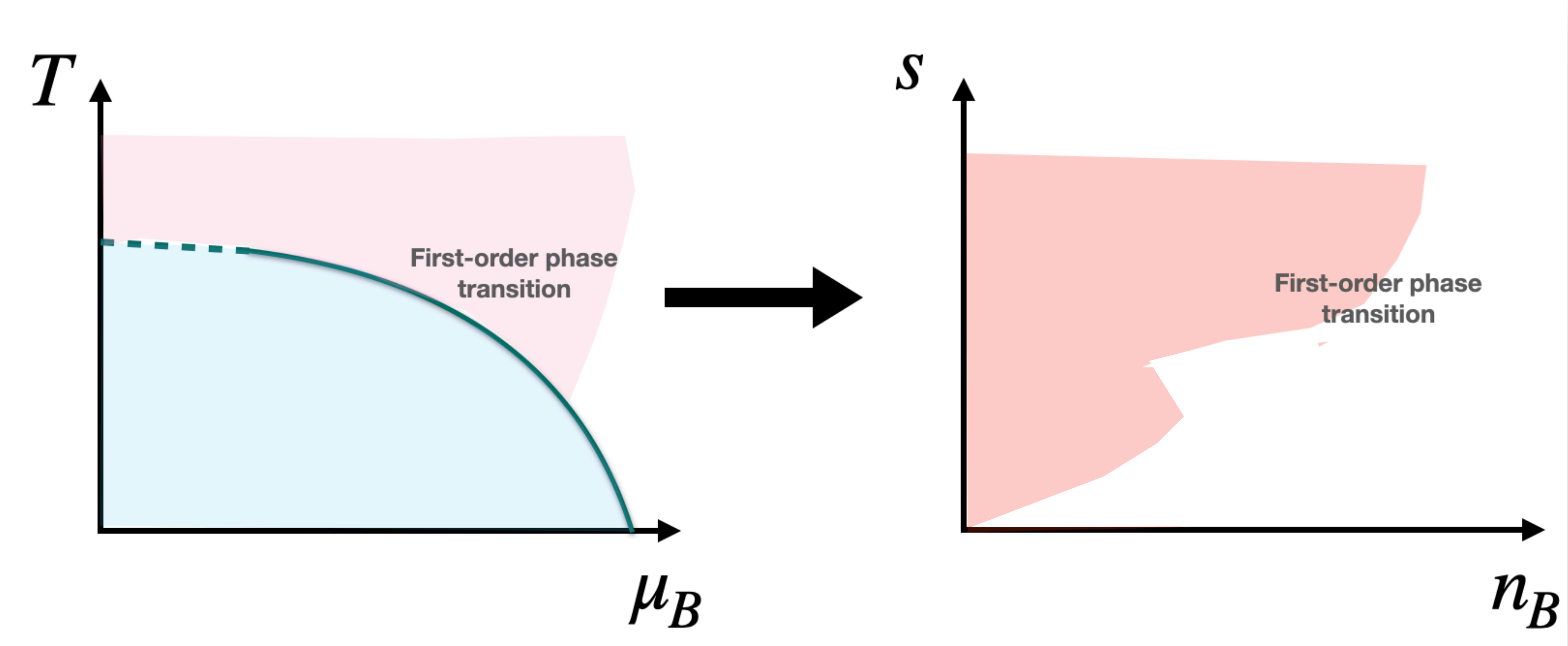
Muses Equation of State

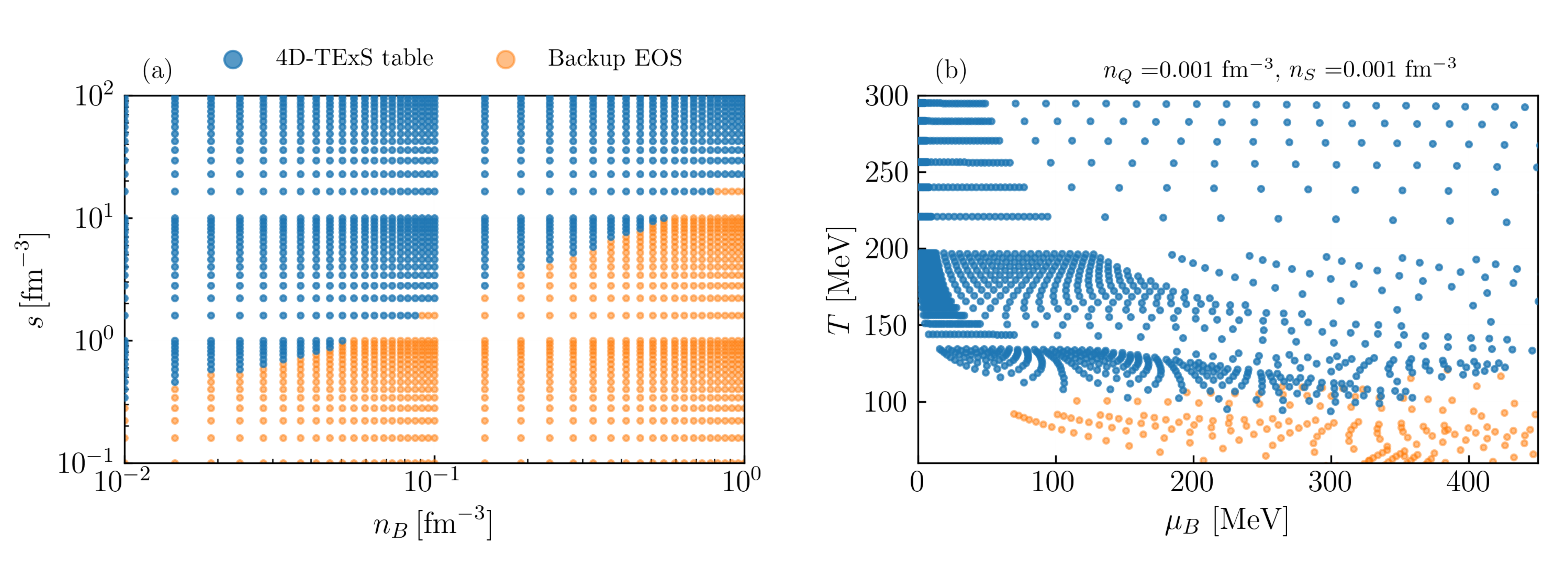
For the inversion algorithm: [arXiv:2405.09648 [nucl-th]]

Natural Hydrodynamic Variables

Hydrodynamical simulations require a mapping of the EoS in the T, μ_B - plane into s, n_B - plane:



This mapping is in general highly non-linear!



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Problem!

- First-order phase transitions cause a gap in the s, n_B- plane

- In this region, hydro simulations either rely on backup (conformal) EoS tables or uncontrolled extrapolations, to ensure that every fluid cells has an EoS.

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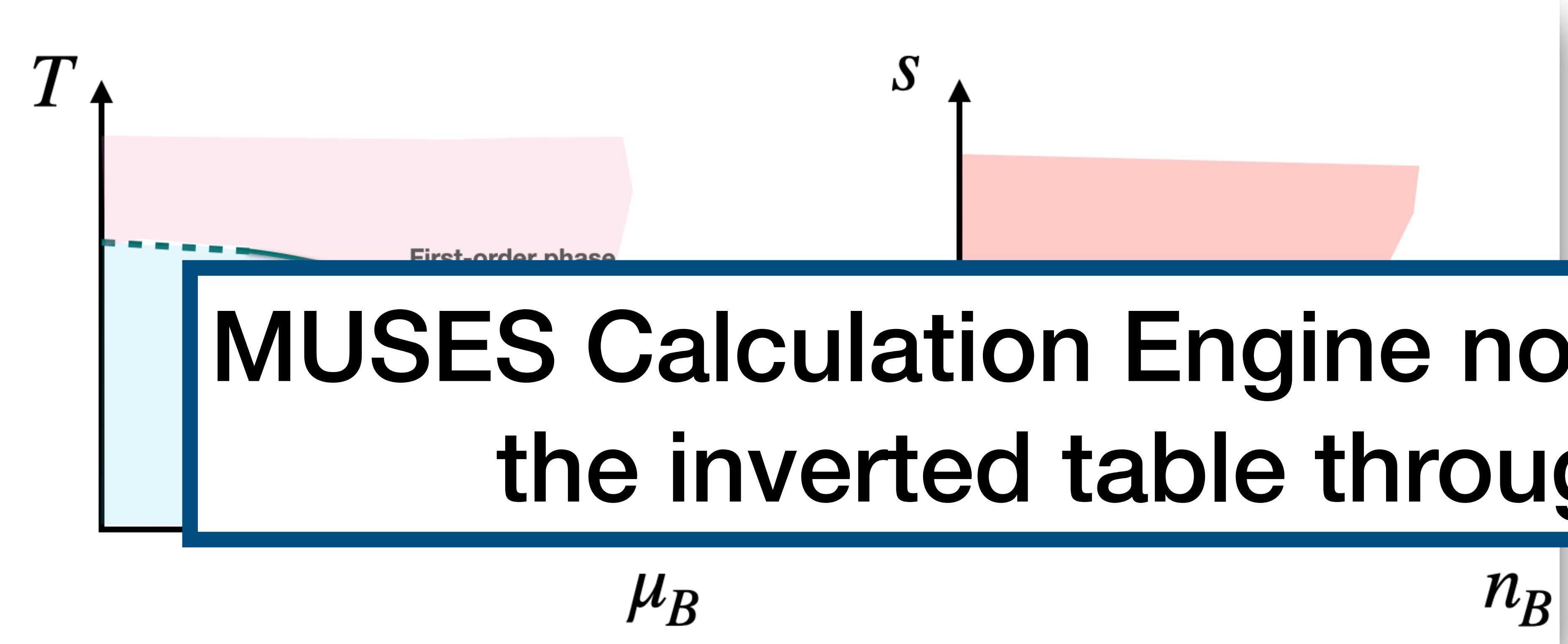

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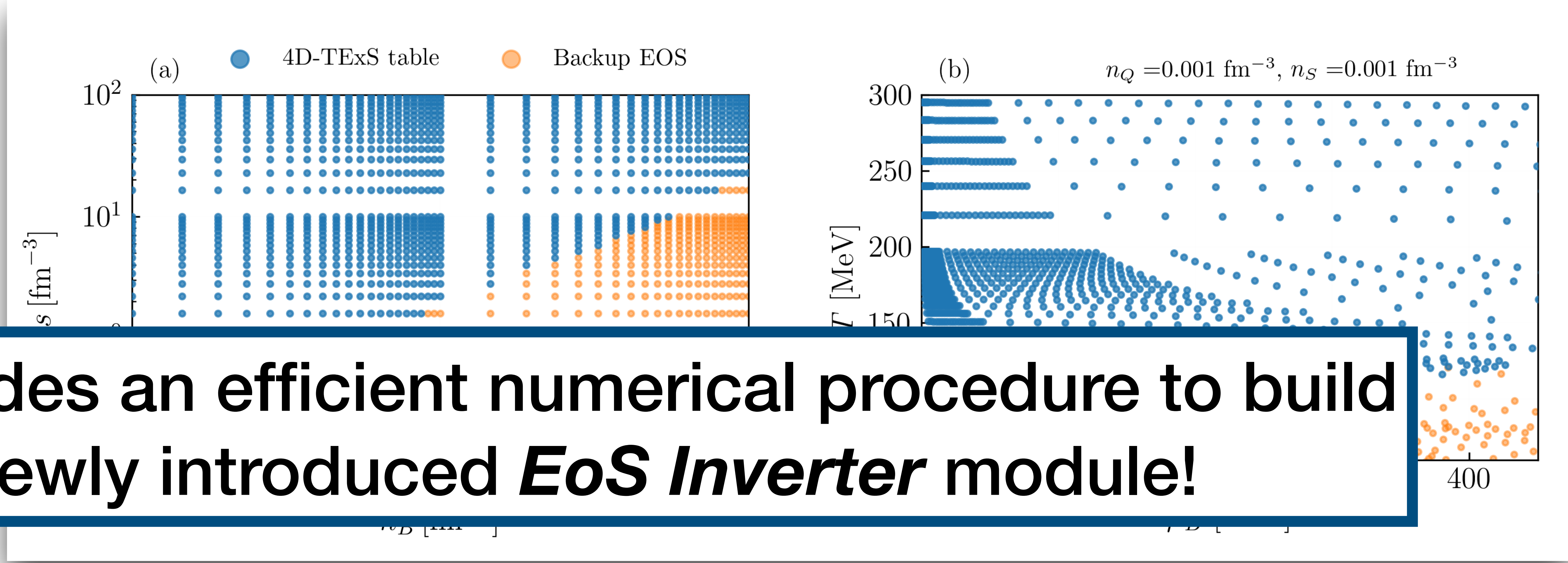

MUSES Calculation Engine now provides an efficient numerical procedure to build the inverted table through the newly introduced *EoS Inverter* module!

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MUSES EOS and Backup EOS

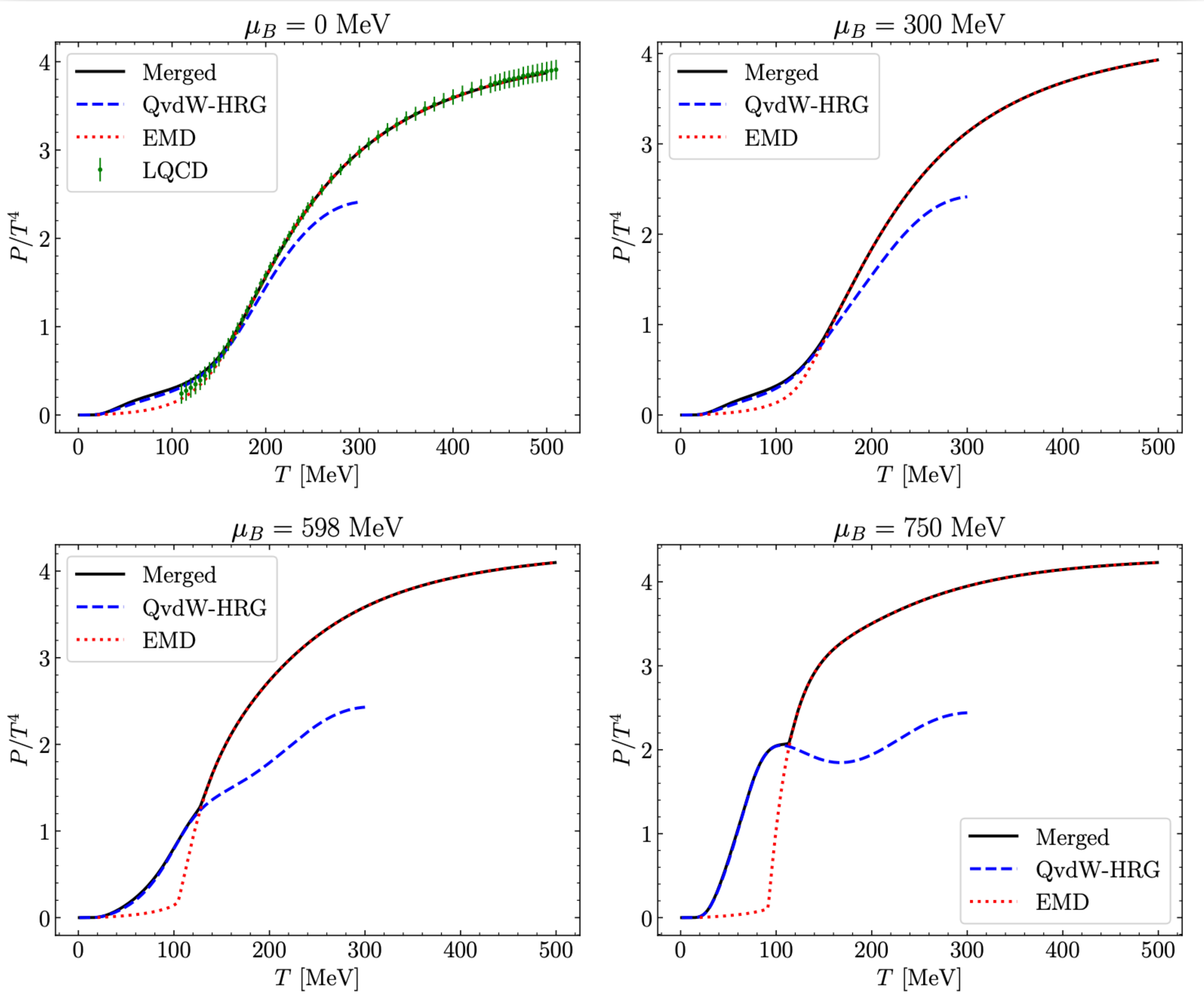
Holography + QvdW-HRG

Grefa et al PRD 104 3, 034002

Yang et al arxiv 2601.07987

Reproduces lattice QCD, was constrained by a Bayesian analysis.

- Holographic correspondence for QCD
- Lattice at $\mu_B = 0$ (single charge)
- Merged with QvdW-HRG $\rightarrow$ $T \in [0, 800]$, $\mu_B \in [-1000, 1000]$
- Moving critical point



Backup EoS

TABLE → TANH-CONFORMAL → CONFORMAL → CONFORMAL DIAGONAL

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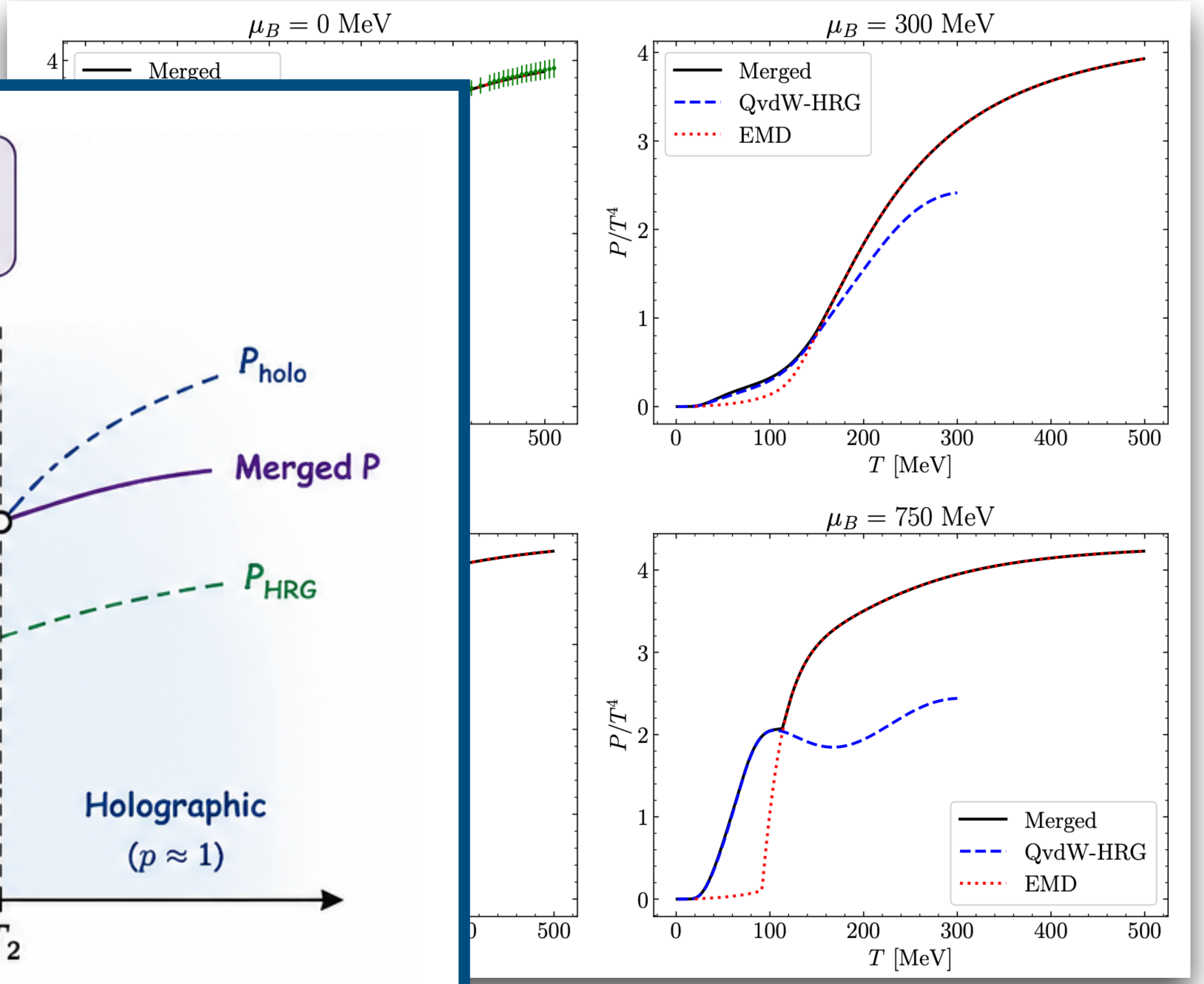

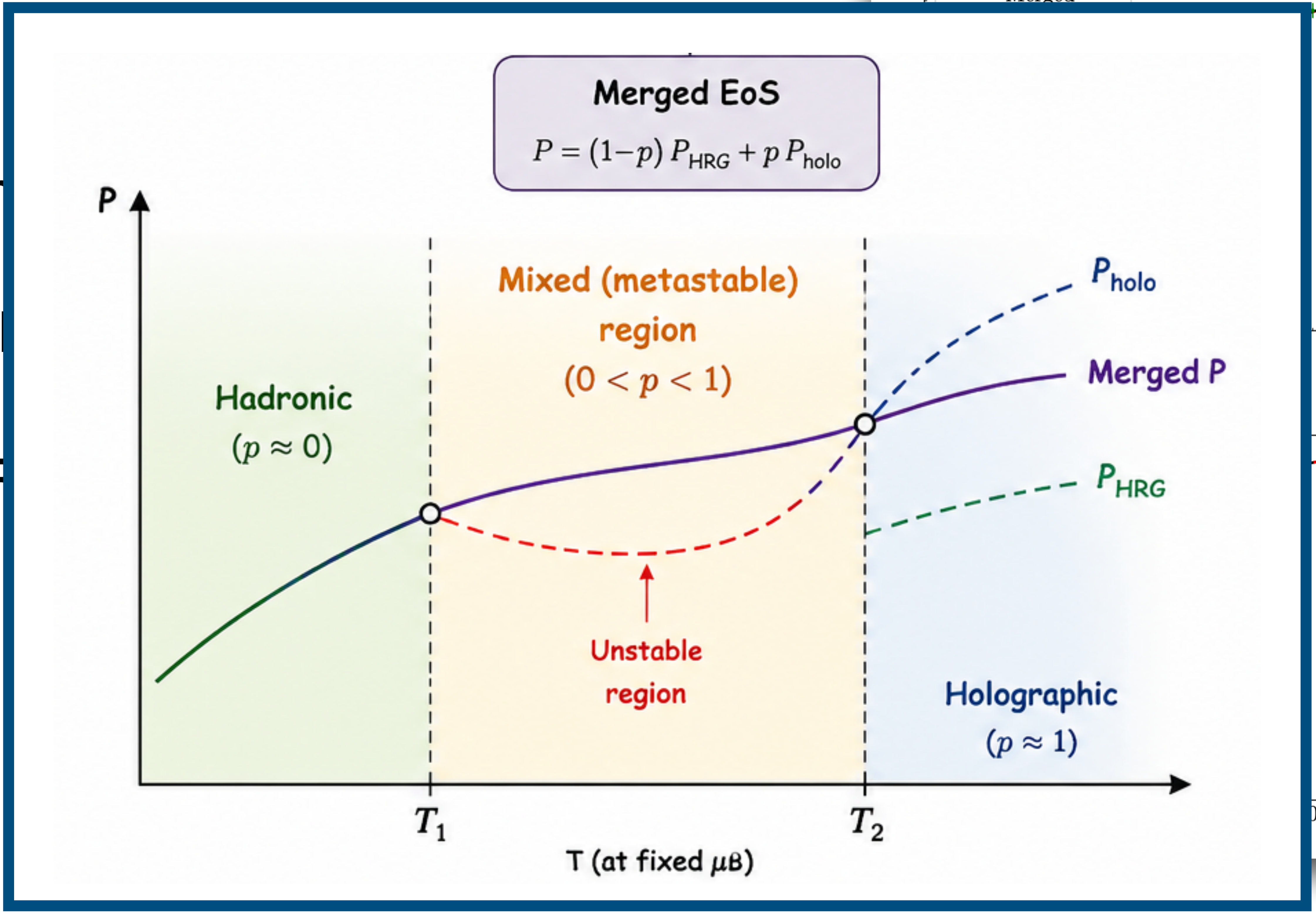

Backup EoS

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MUSES EOS and Backup EOS

4D-Taylor

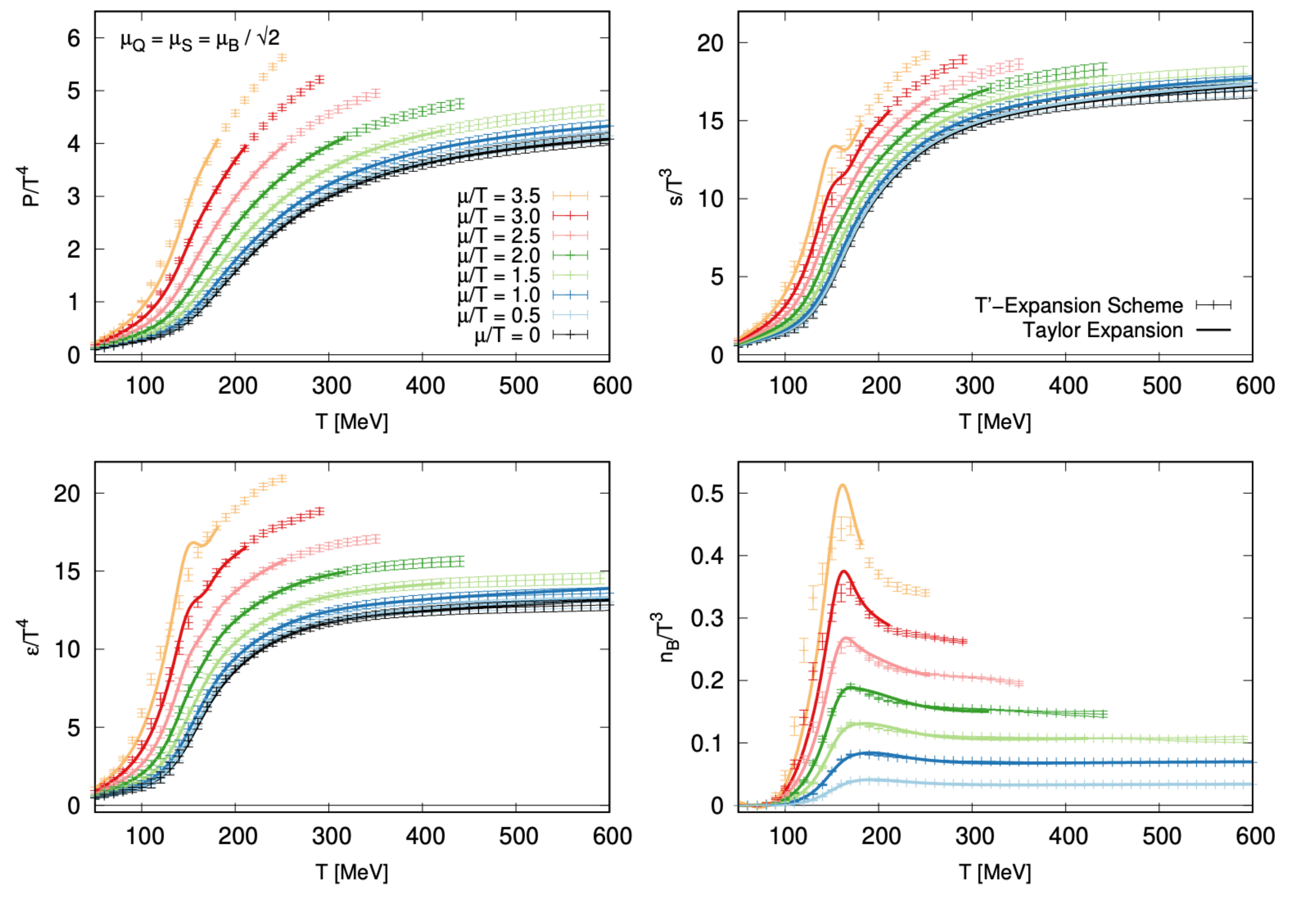
Plumberg et al PRC 111 4, 044905

- Based on lattice
- Taylor series expansion up to O(μ4)
- μ_B, μ_S and μ_Q
- Not Merged

4D-TExS

Abuali et al Phys. Rev. D 112, no.5, 054502

- Based on lattice
- Alternative expansion scheme
- μ_B, μ_S and μ_Q
- Not Merged



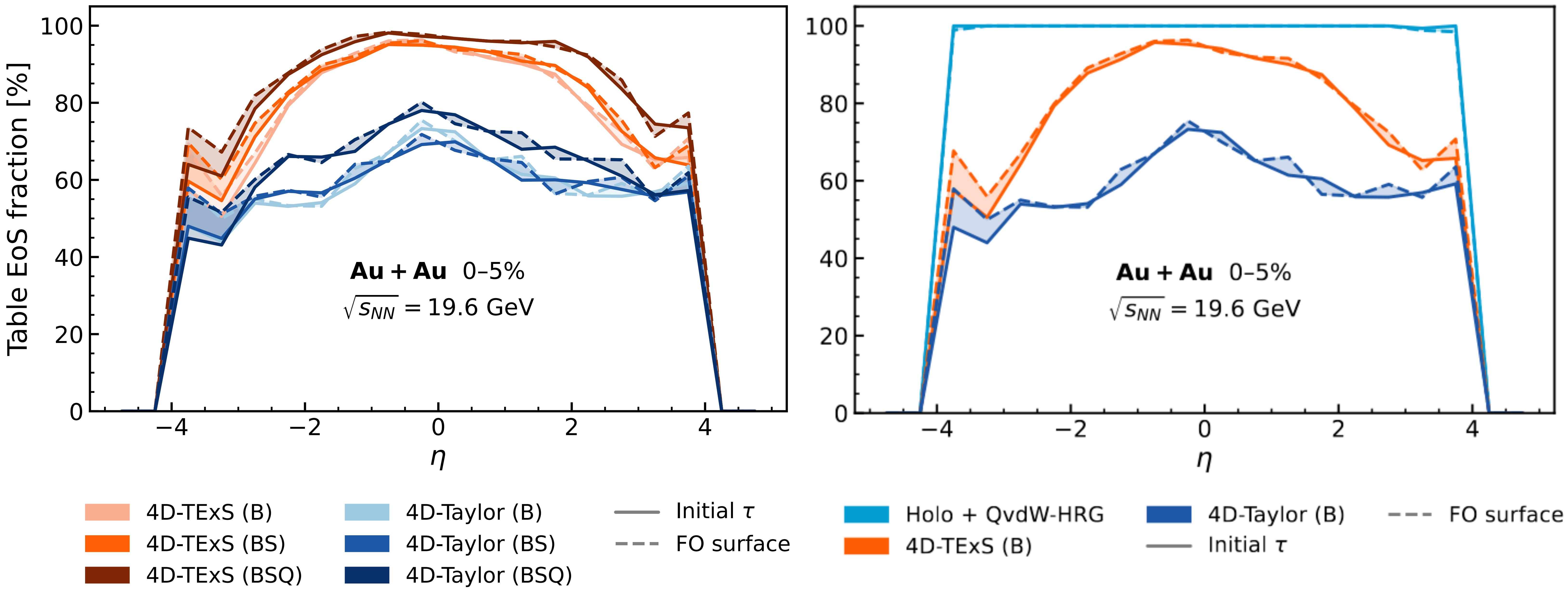
Backup EoS

TABLE → TANH-CONFORMAL → CONFORMAL → CONFORMAL DIAGONAL

How does the limited range of the EOS affect different beam energies?

Muses Equation of State

- Spatial rapidity distribution of fluid cells within the EoS validity range, where trajectories are tracked until freeze-out.
- The μ_B range explored by the trajectories is strongly correlated with the fluid-cell rapidity.



Schematically:

Lower μ_B → mid-rapidity
Larger μ_B → forward/backward rapidity

Backup EoS

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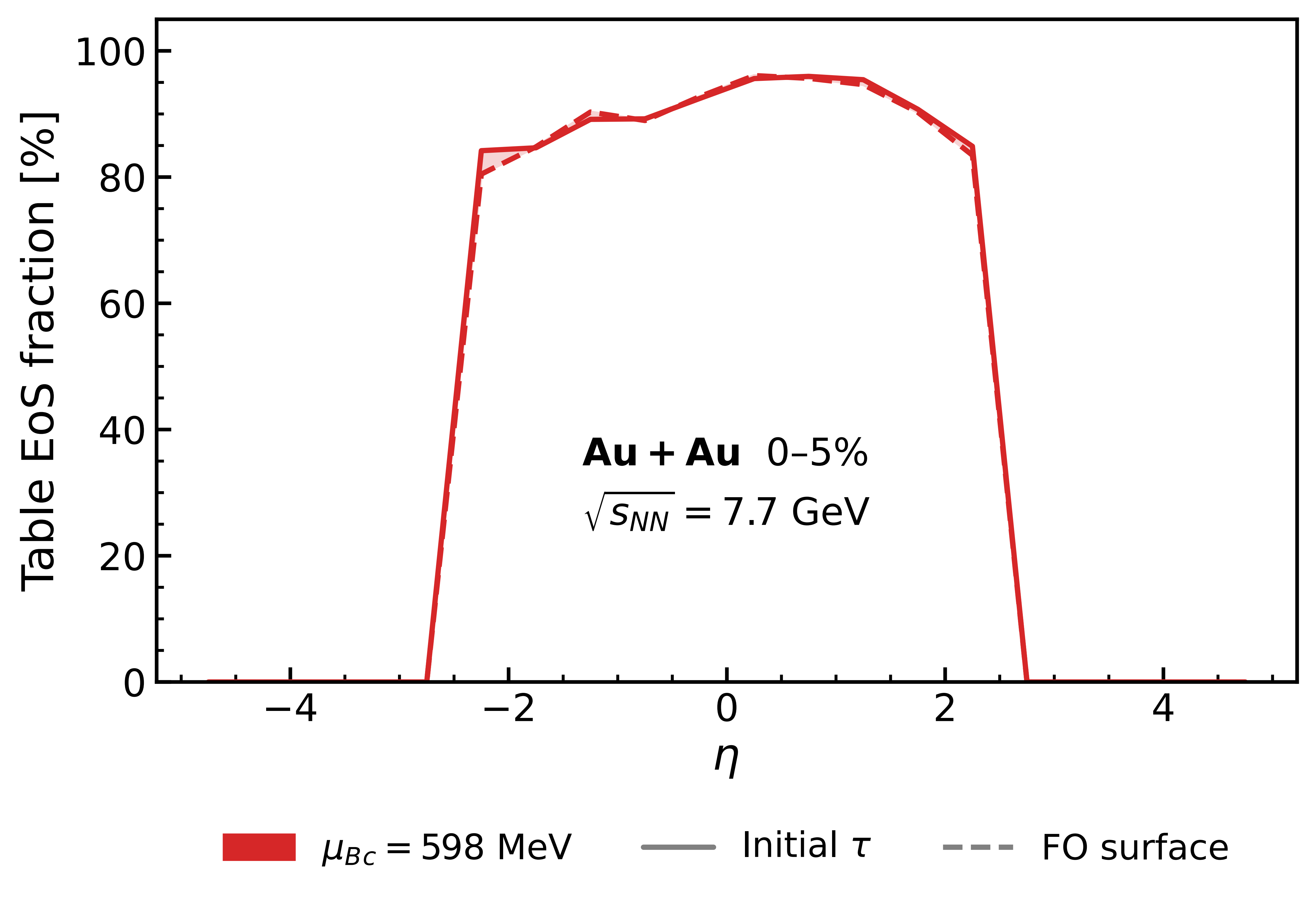
TABLE → TANH-CONFORMAL → CONFORMAL → CONFORMAL DIAGONAL

Muses Equation of State

What about $\sqrt{s_{NN}} = 7.7$ GeV?

- Same plot for the Holography+QvdW-HRG EoS in a $\sqrt{s_{NN}} = 7.7$ GeV event.
- The percentage of fluid cells covered by the table is lower $\rightarrow$ larger μ_B/T ratios.
- **This indicates that one must be careful when using the Holography+QvdW-HRG EoS at lower energies.**



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Holography + QvdW-HRG

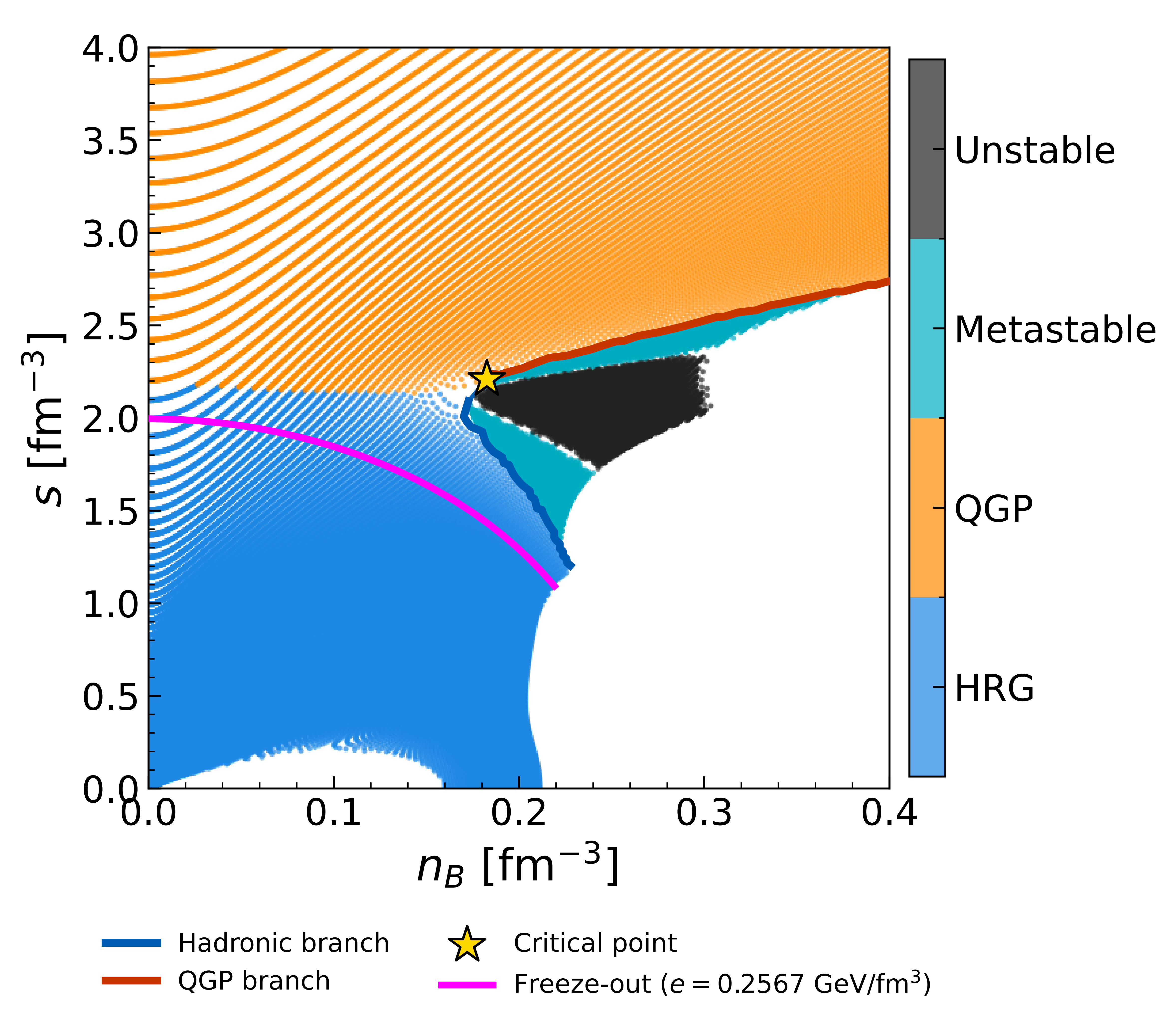
Muses Equation of State

Holography + QvdW-HRG

Grefa et al PRD 104 3, 034002

Yang et al arxiv 2601.07987

- $\mu_B^{CP} = 598$ MeV in natural hydrodynamic variables.
- Table of fixed T, μ_B for the stable QGP and HRG phases.
- Within the first-order phase transition we show also the metastable and unstable regions.
- An example freeze-out line is shown assuming a constant energy-density at freeze-out.



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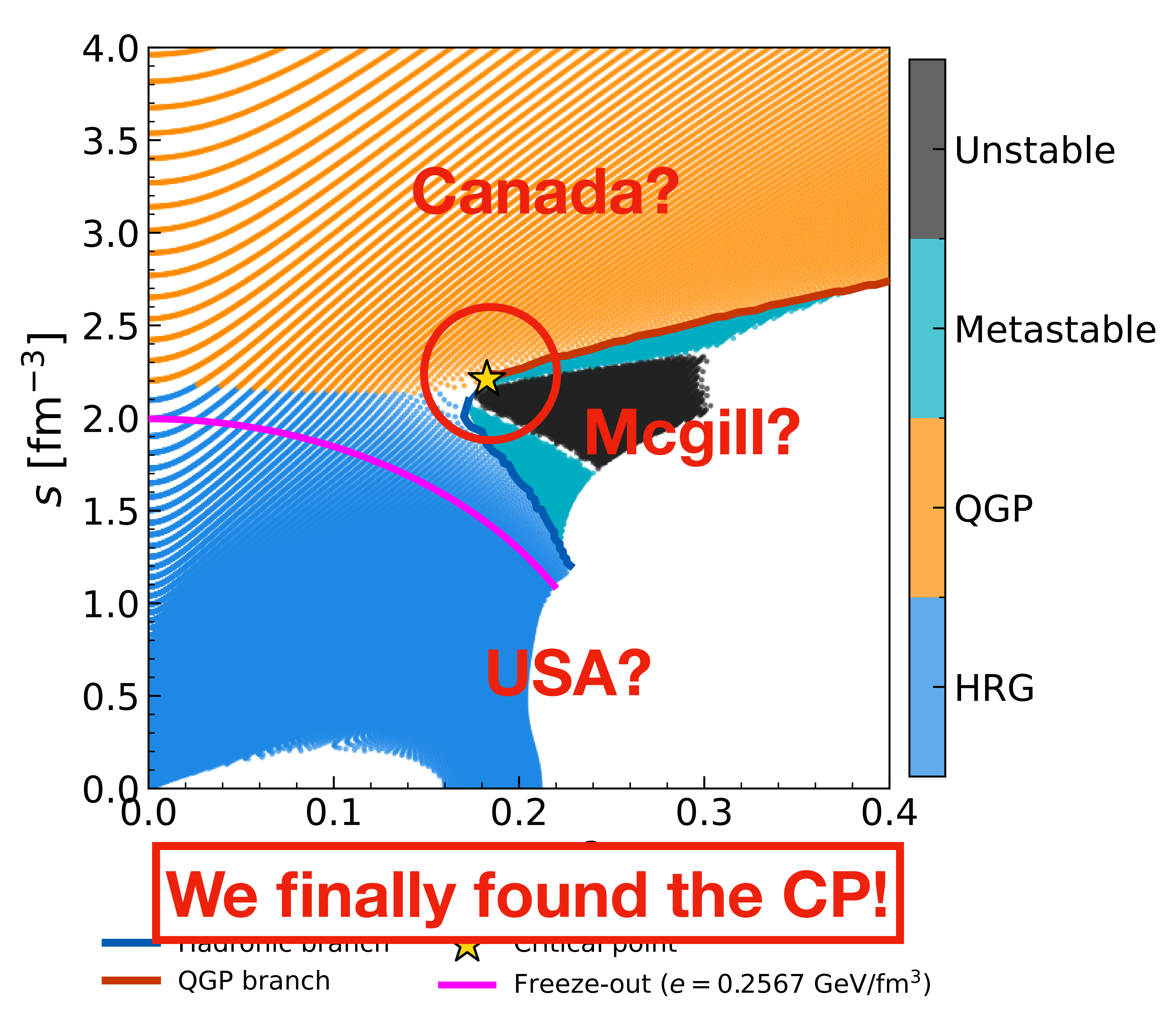

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P. Pala et al arxiv 2511.22852

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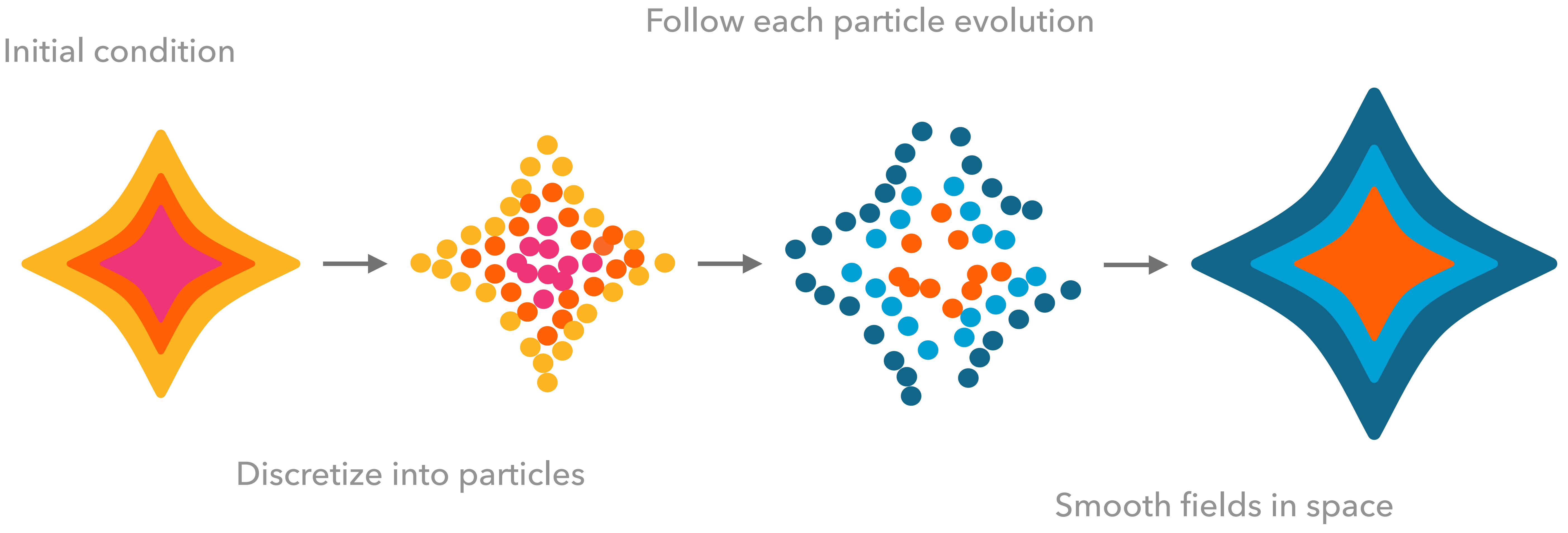
CCAKE 2.0

- Smoothed Particle Hydrodynamics
- Performance portability (CPUs and GPUS)
- Source terms
- BSQ charges and (3+1)D



$$\nabla_\mu T^{\mu\nu} = I^\nu$$

$$\nabla_\mu N^\mu_q = I_q$$

Denicol et al PRD 85 114047
Denicol et al PRD 91 039902

$$\tau_\pi \Delta^{\mu\nu}_{\alpha\beta} D\pi^{\alpha\beta} + \pi^{\mu\nu} = 2\eta\,\sigma^{\mu\nu} - \delta_{\pi\pi}\,\pi^{\mu\nu}\,\Theta \quad + \text{DNMR}$$

$$\tau_\Pi\, D\Pi + \Pi = -\left(\zeta + \delta_{\Pi\Pi}\Pi\right)\Theta \quad + \text{DNMR}$$

$$\tau^{ab}_q\, \Delta^\mu{}_\nu\, Dq^\nu_b + q^\mu_a = \kappa^{ab}\,\nabla^\mu\left(\frac{\mu_b}{T}\right) - \tau^{ab}_q\, q^\mu_a\,\Theta$$

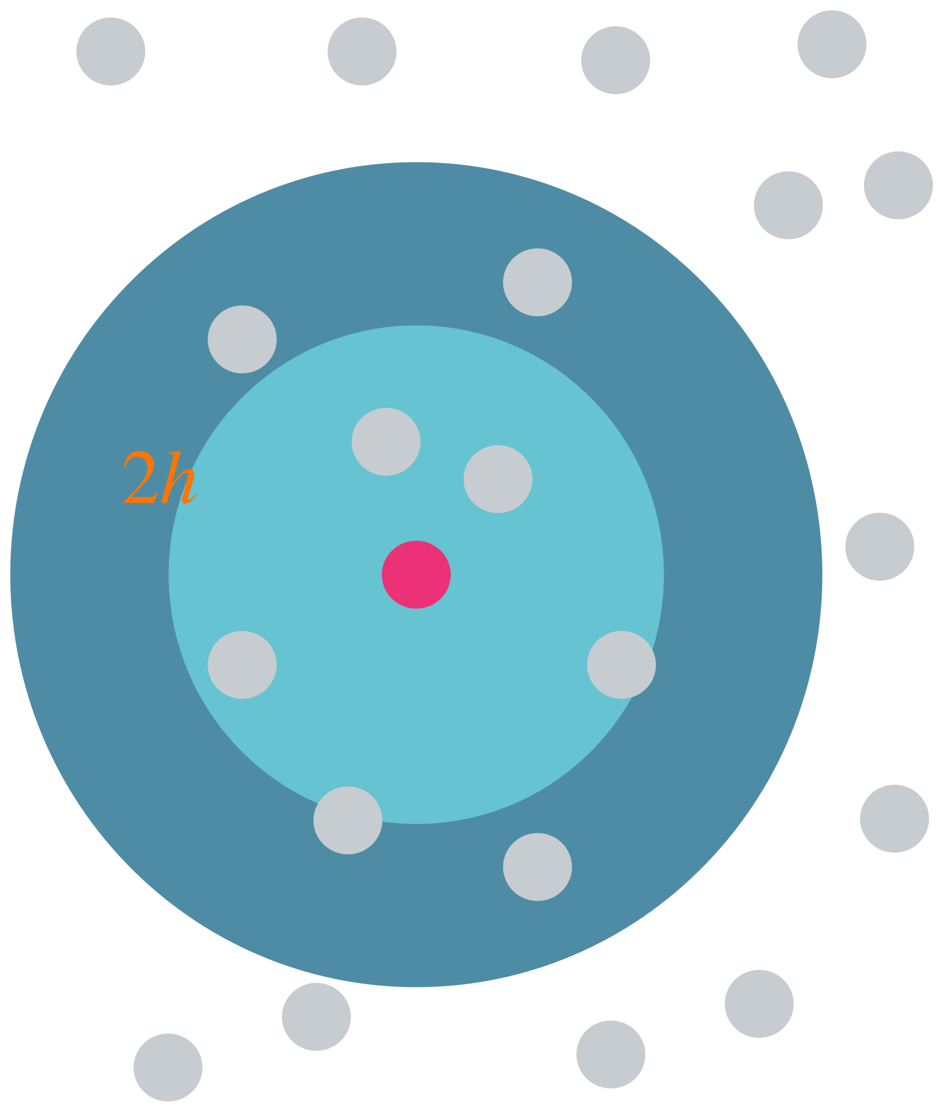
$$A(\mathbf{r}) = \sum_{i=1} A(\mathbf{r}_i) W(\mathbf{r}_i - \mathbf{r}, h)\Delta V_i$$

SPH Kernel

$$\nabla_j A(\mathbf{r}) = \sum_{i=1} A(\mathbf{r}_i)\,\nabla_j W(\mathbf{r}_i - \mathbf{r}, h)\Delta V_i$$

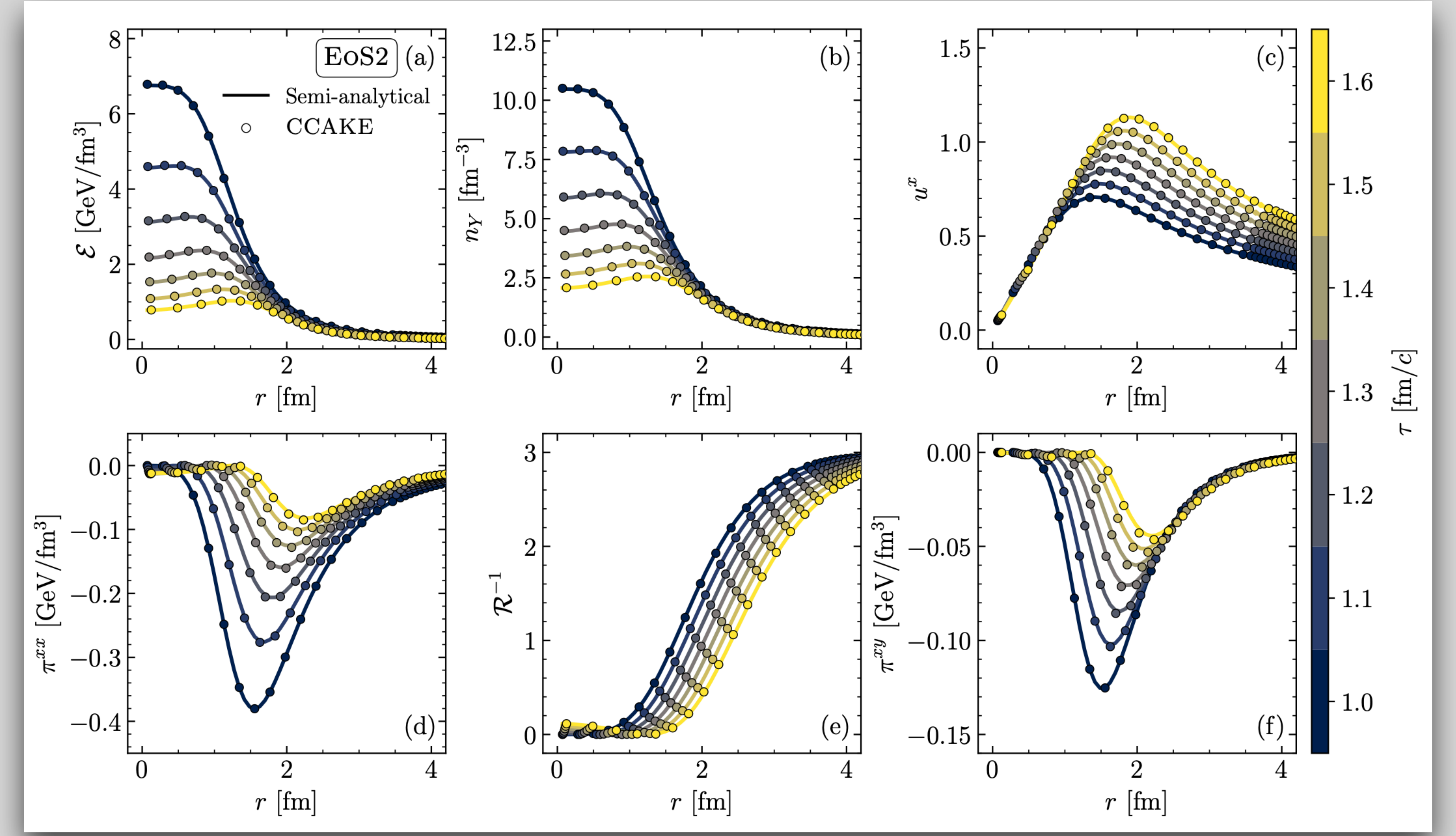
$$\lim_{h\to 0} W(\mathbf{r}, h) = \delta(\mathbf{r})$$

Smoothing scale



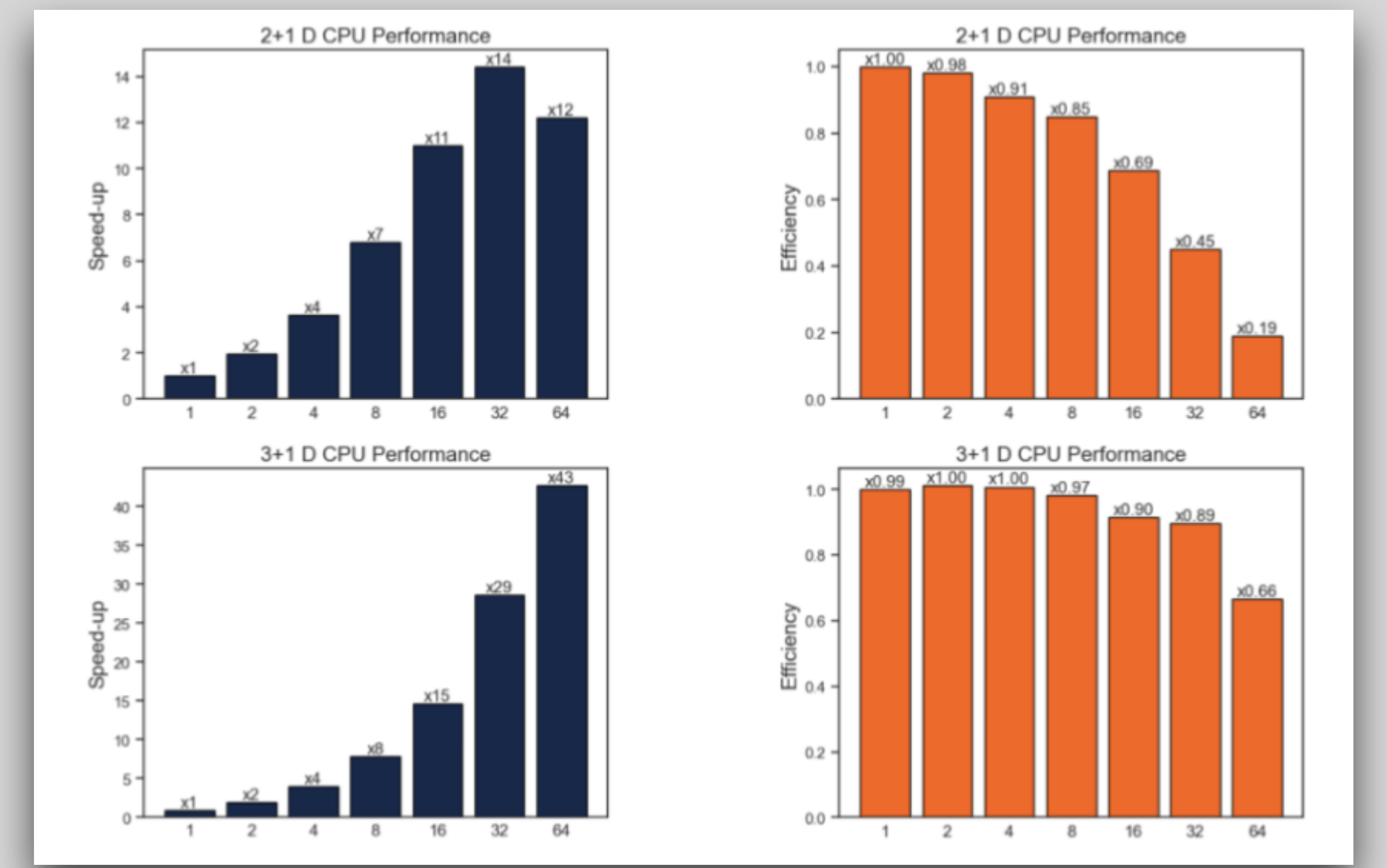
CCAKE 2.0

Gubser Test with conserved charges:



arXiv:2503.20021v3 [nucl-th]

It can run on both CPUs and GPUs!



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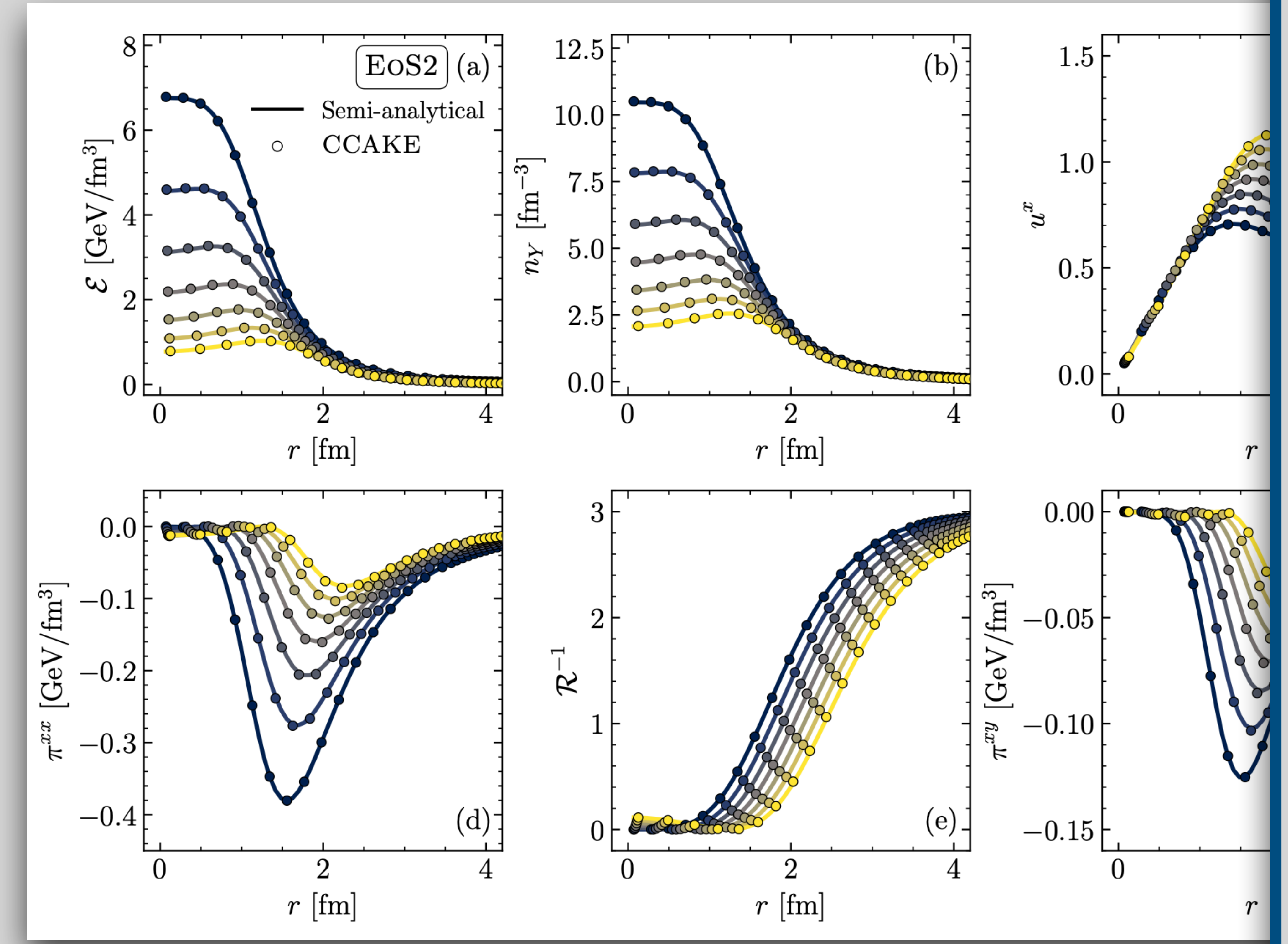

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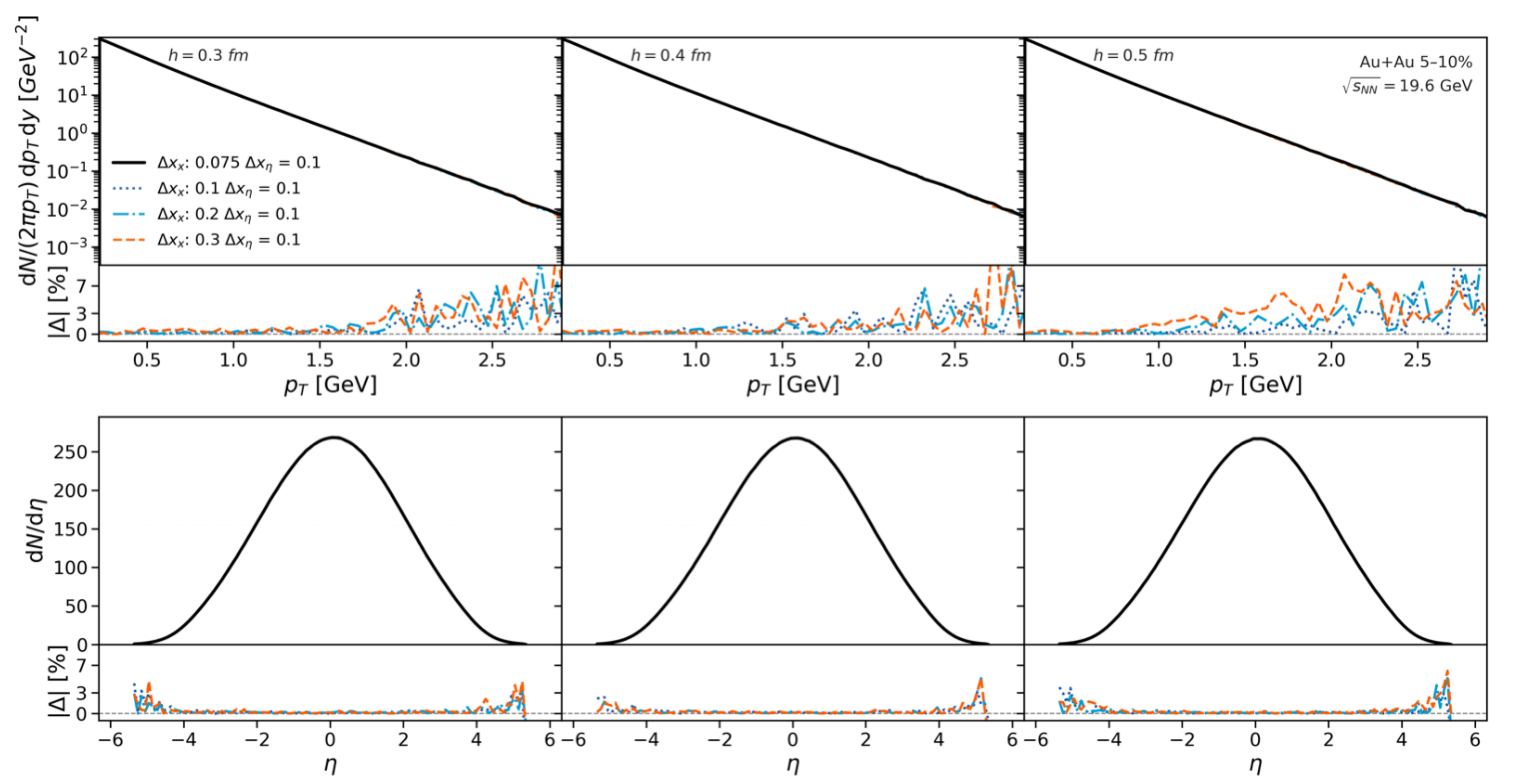
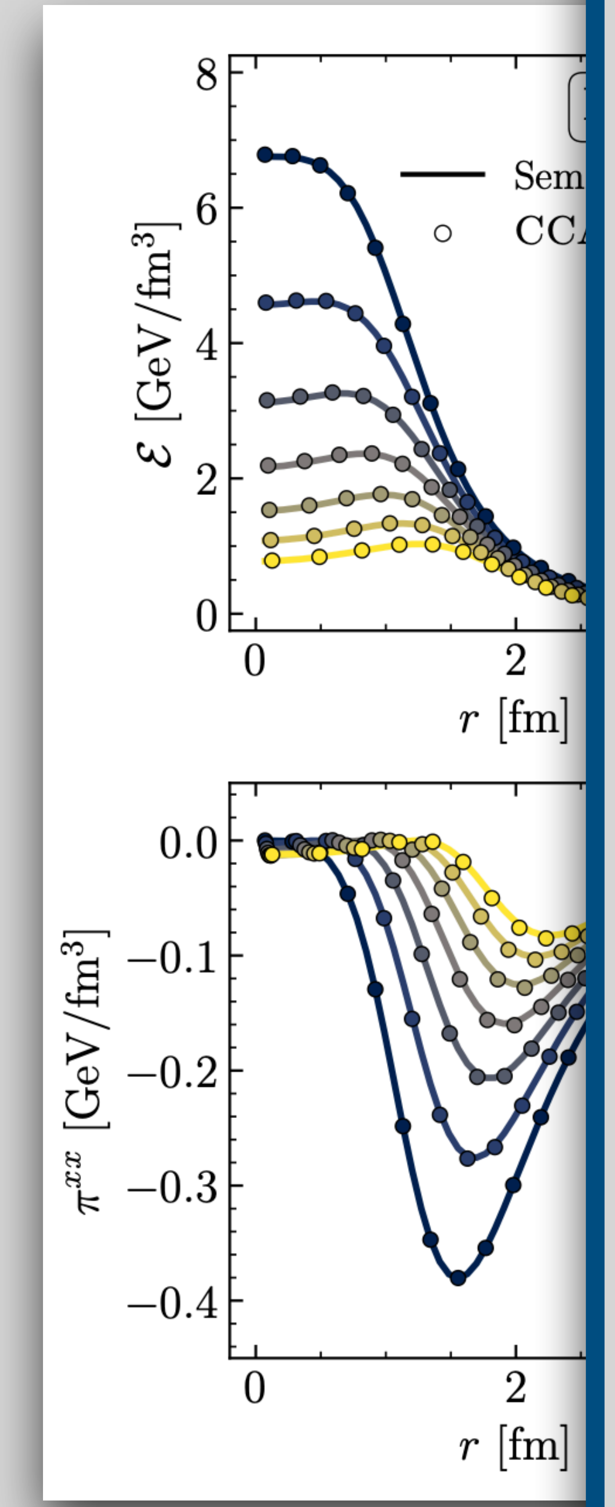
We have performed several convergence tests!



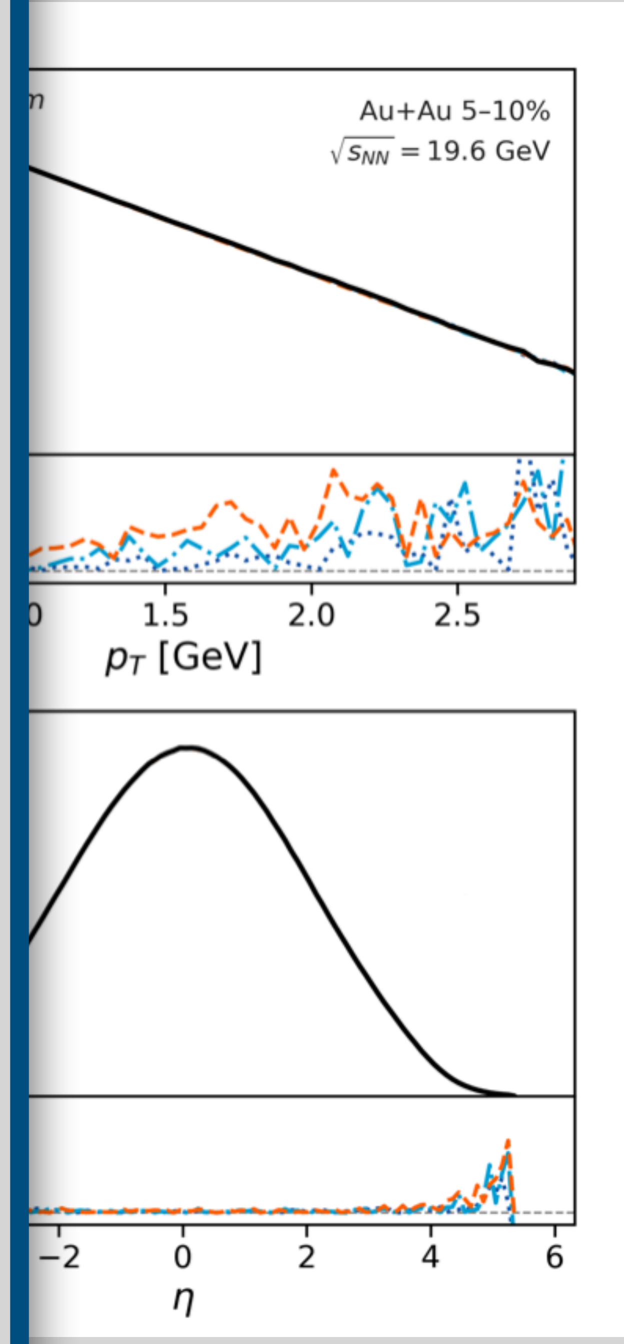
CCAKE 2.0

Gubser

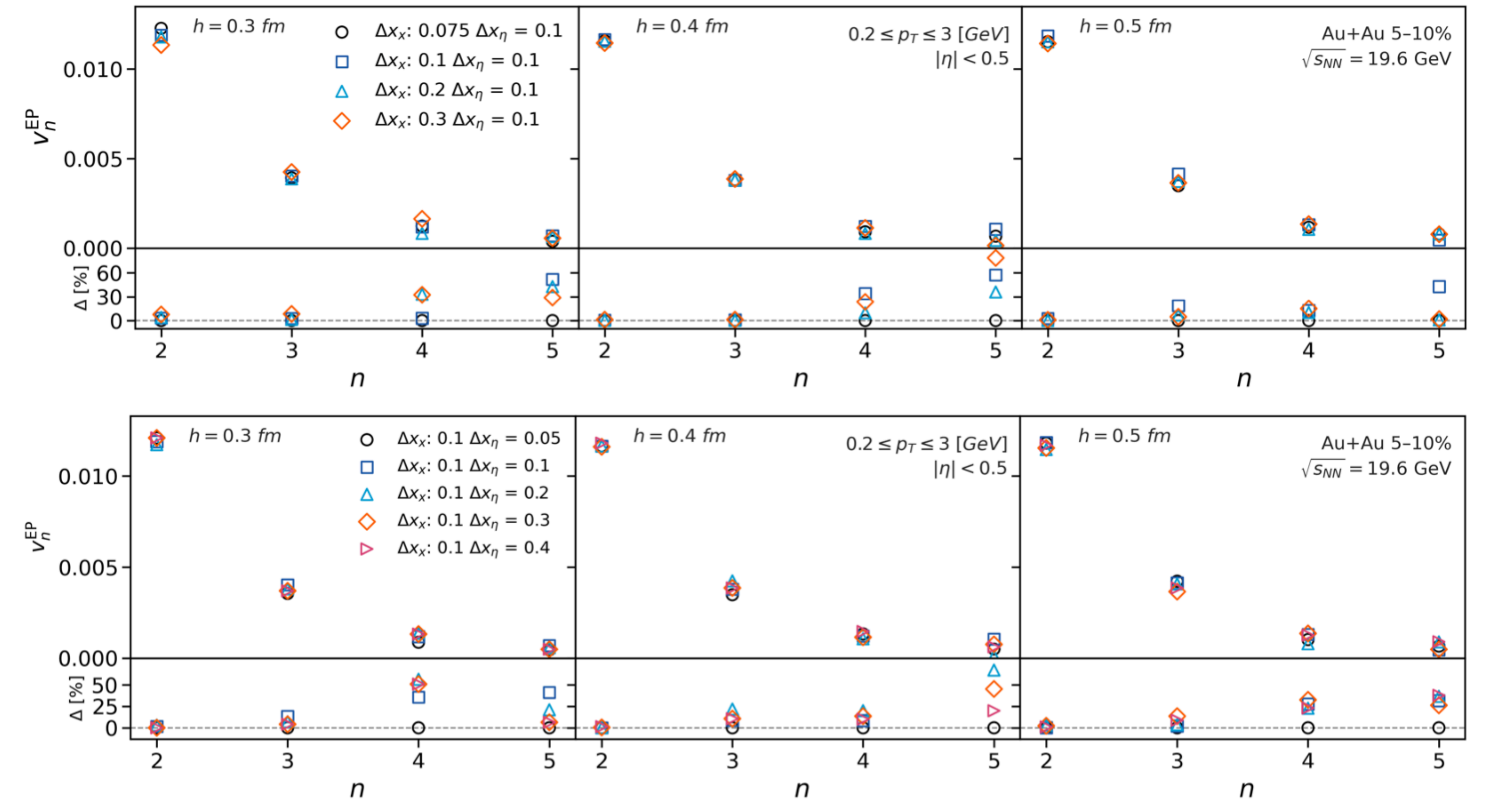


Us and GPUs!

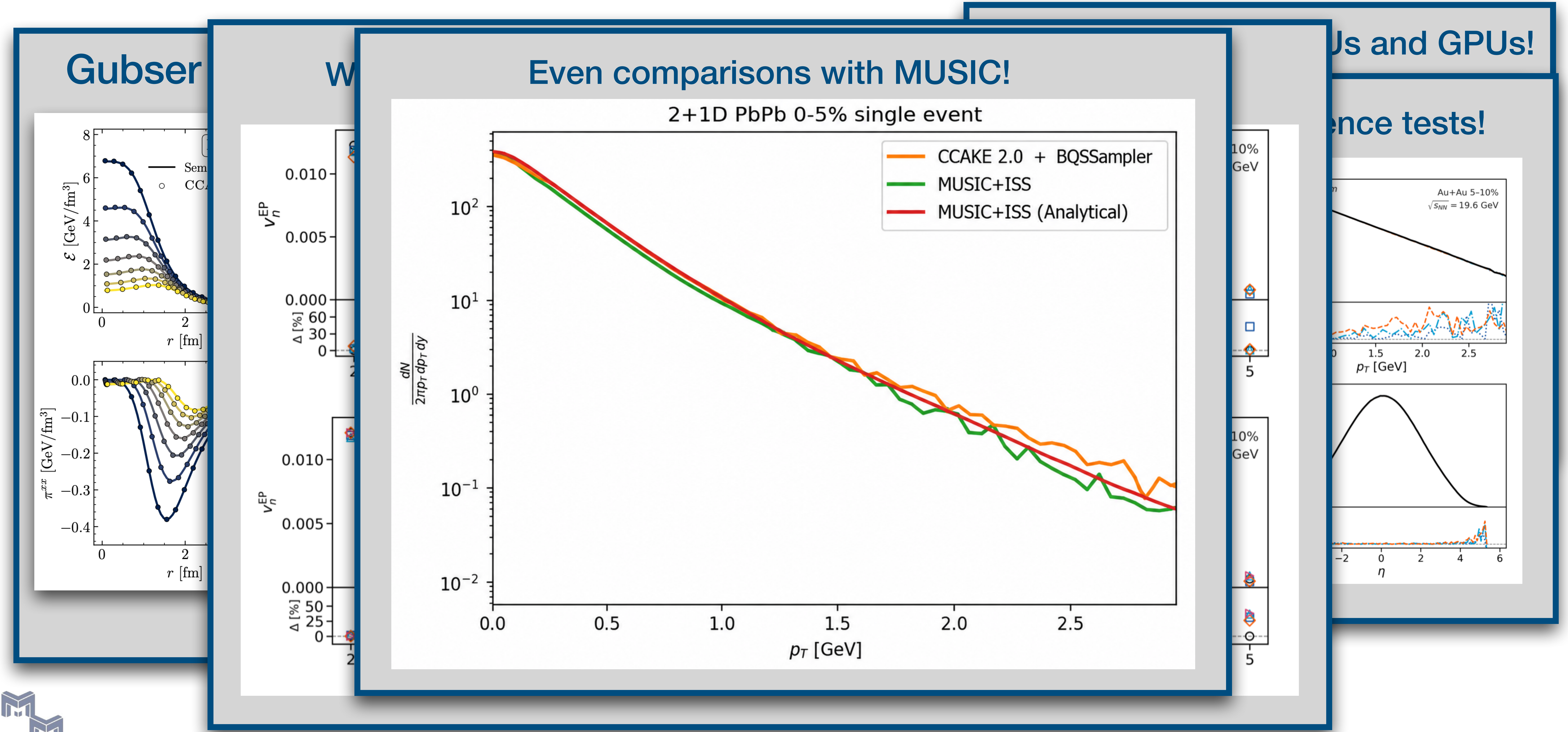
ence tests!



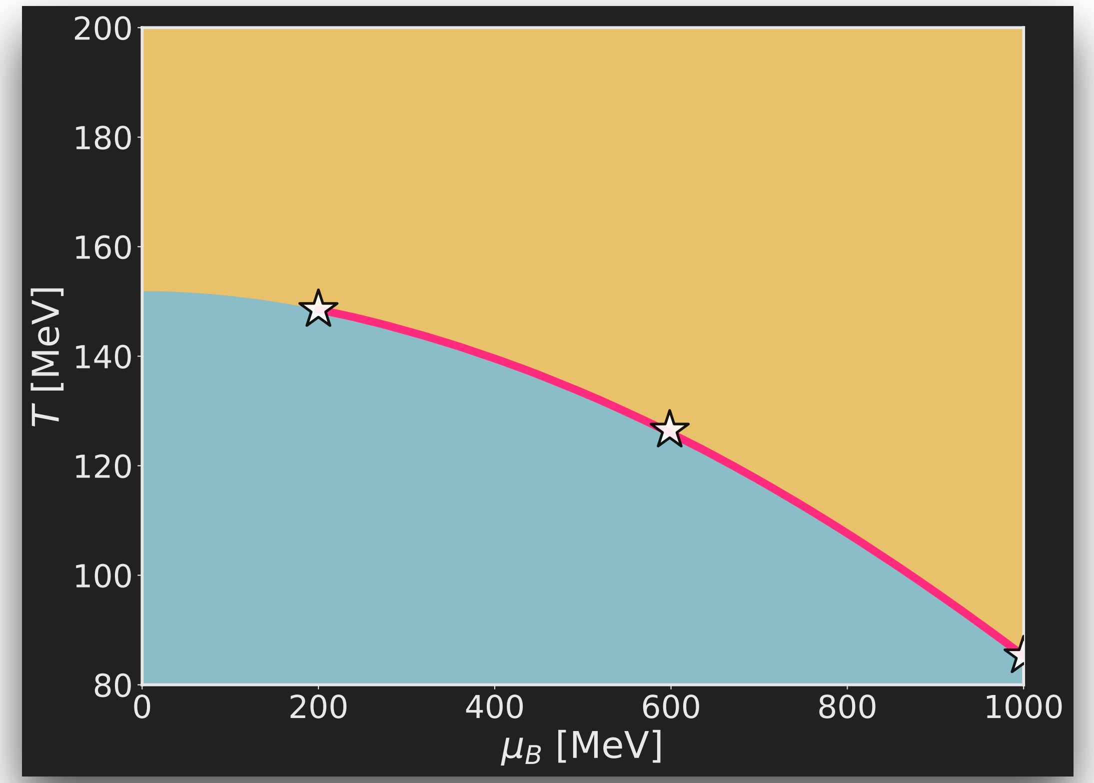
We have performed convergence tests on integrated flow!



CCAKE 2.0


Gubser
Us and GPUs!
ence tests!
Even comparisons with MUSIC!
2+1D PbPb 0-5% single event
CCAKE 2.0 + BQSSampler
MUSIC+ISS
MUSIC+ISS (Analytical)
$\frac{dN}{2\pi p_T dp_T dy}$
p_T [GeV]
Au+Au 5-10%
$\sqrt{s_{NN}}$ = 19.6 GeV
$\mathcal{E}$ [GeV/fm^3]
r [fm]
π^{xx} [GeV/fm^3]
v_n^{EP}
Δ [%]
η

Simulation Setup - Holography



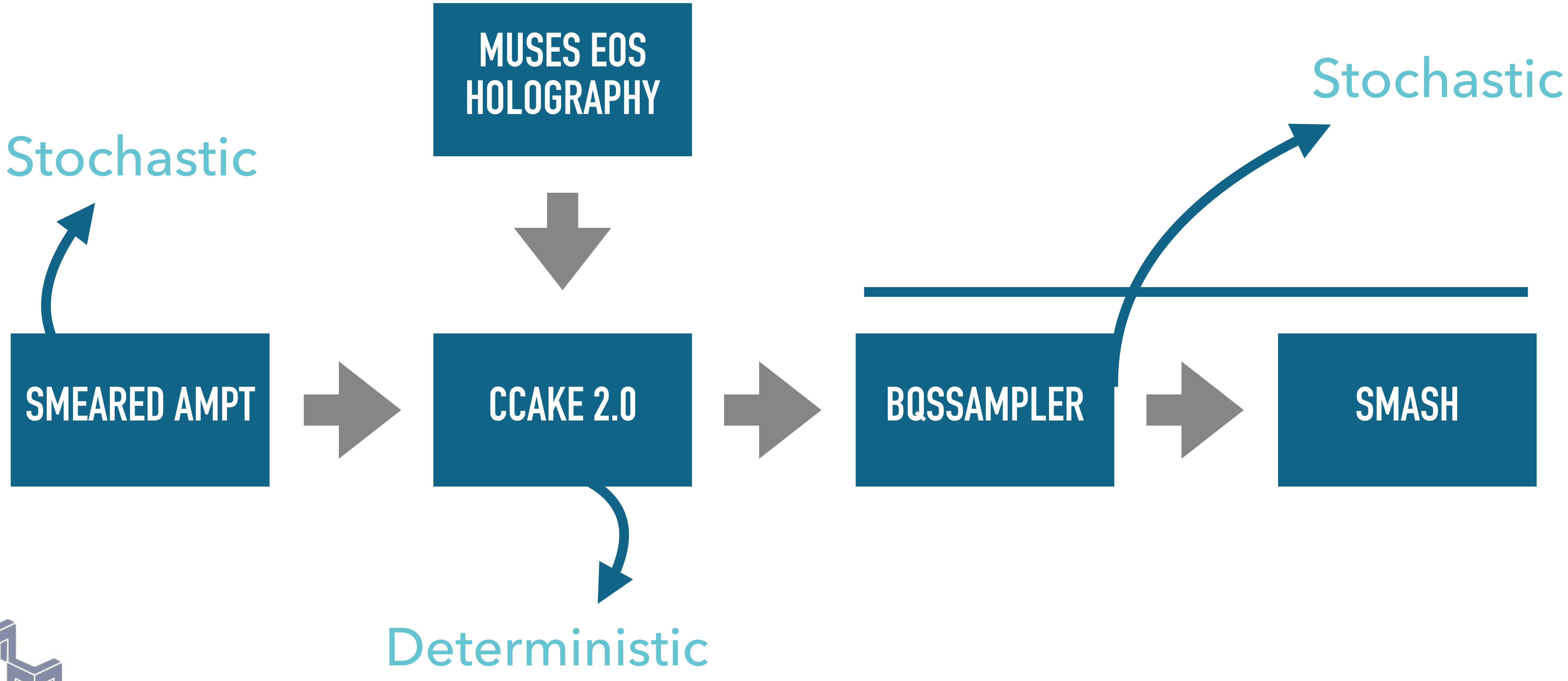
- Take the partons at desired x^0 (τ or t) and smear them

$$T^{\mu\nu}\left(x^1,x^2,x^3\right) = \sum_{i=partons} \frac{p_i^\mu p_i^\nu}{p_i^0}\phi_i\left(x^1,x^2,x^3\right)$$

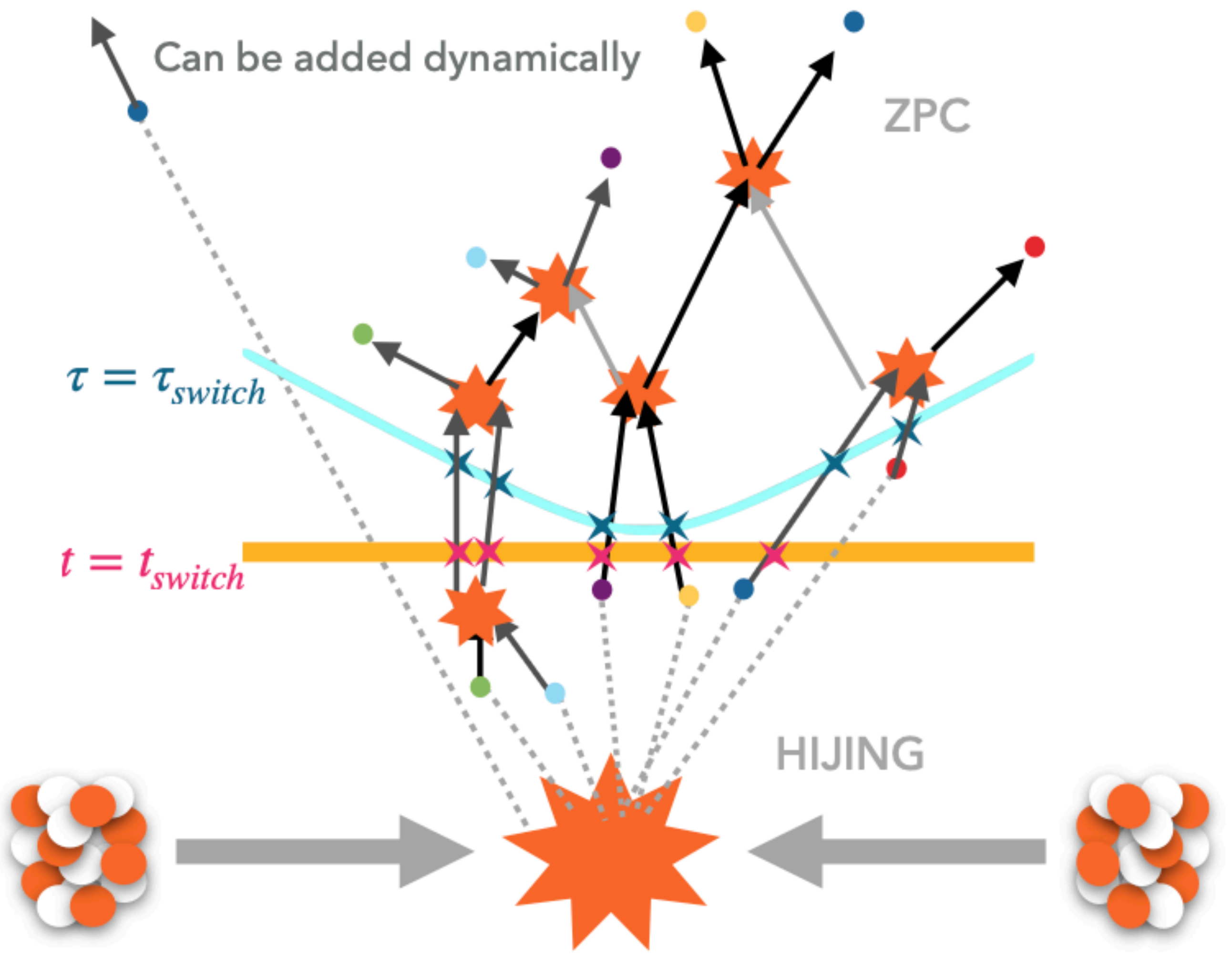
$$N_X^{\mu\nu}\left(x^1,x^2,x^3\right) = \sum_{i=partons} Q_X \frac{p_i^\mu}{p_i^0}\phi_i\left(x^1,x^2,x^3\right)$$

$$\phi_i\left(x^1,x^2,x^3\right) = \frac{K}{(2\pi)^{3/2}\sqrt{-g}\,\sigma_\perp^2\sigma_{x3}} e^{-\frac{\left(x^1-x_i^1\right)^2+\left(x^2-y_i^2\right)^2}{2\sigma_\perp^2}-\frac{\left(x^3-x_i^3\right)^2}{2\sigma_{x3}^2}}$$

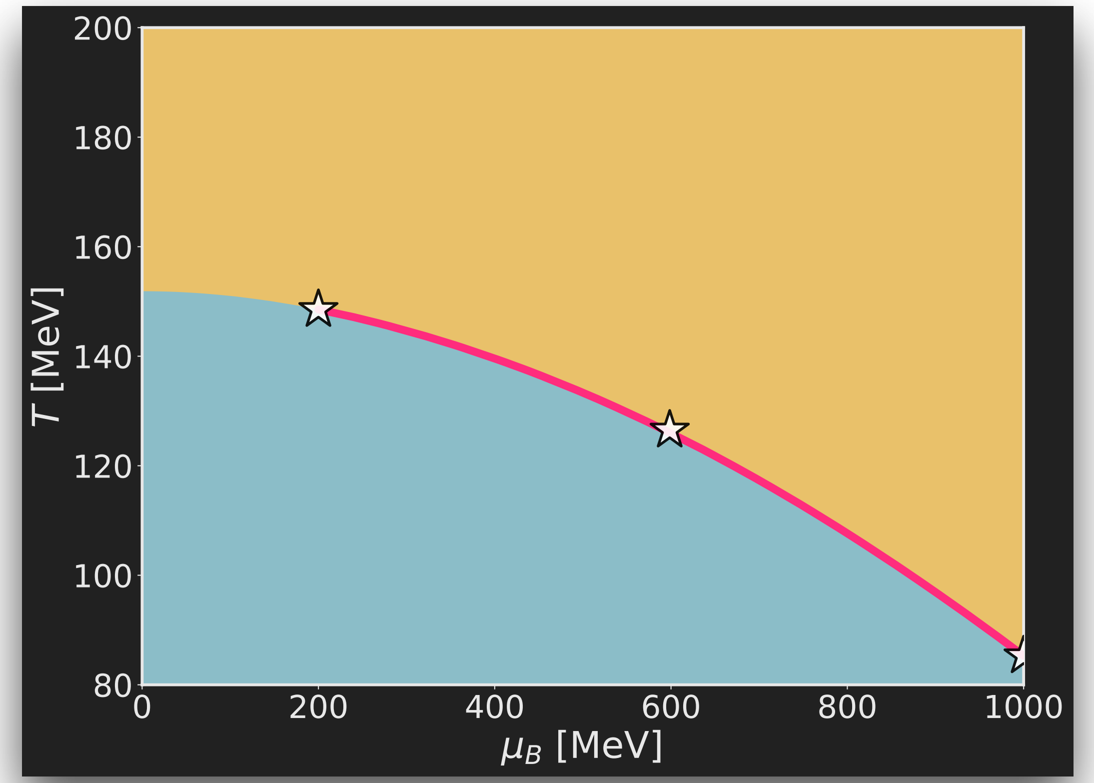
- 3+1D , cartesian or hyperbolic
- BSQ charges and initial currents
- Full energy momentum tensor



AMPT+SMEARING



Simulation Setup - Holography



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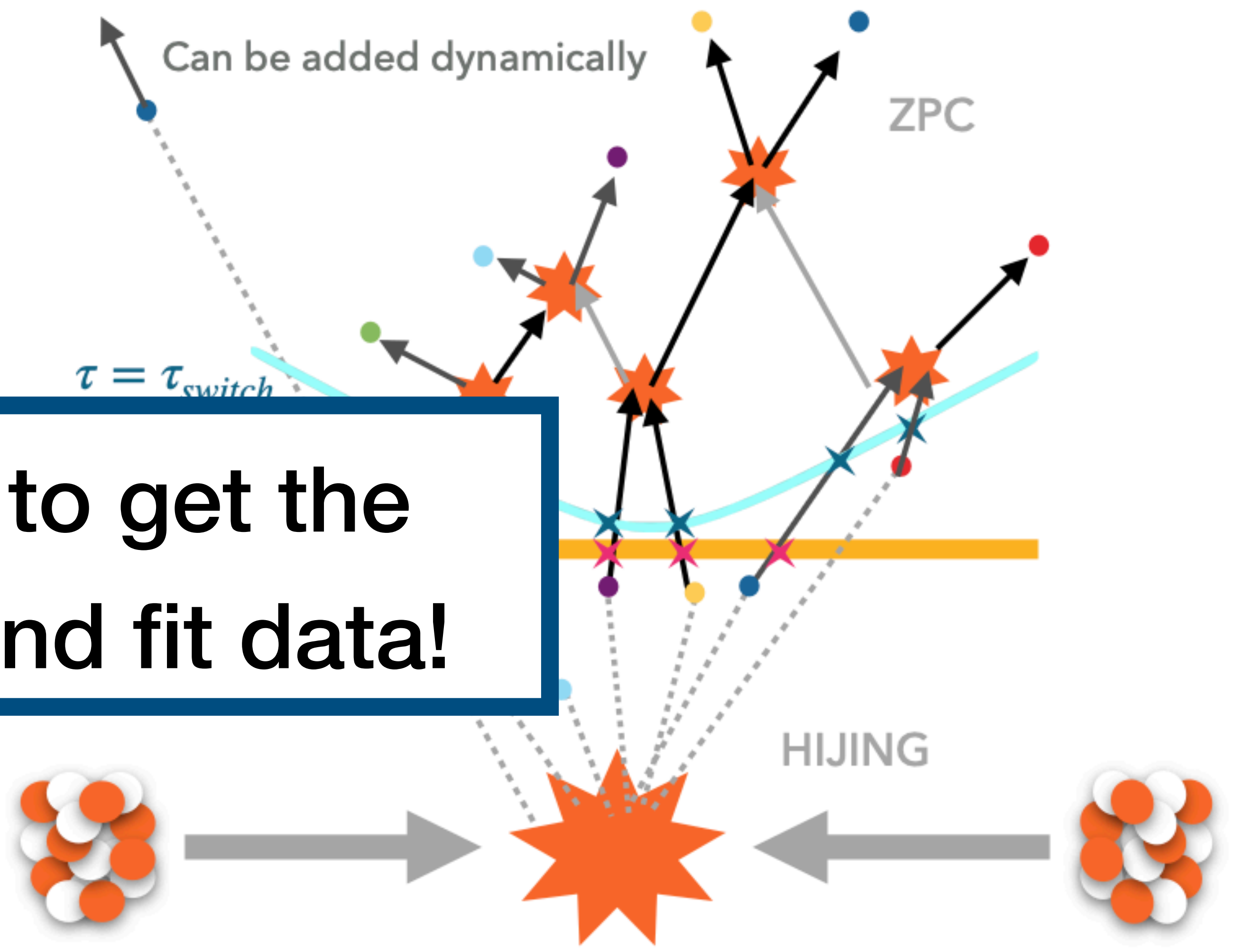


MUSES EOS HOLOGRAPHY

SMEARED AMPT → CCAKE 2.0 → BQSSAMPLER

Stochastic

Stochastic

Deterministic

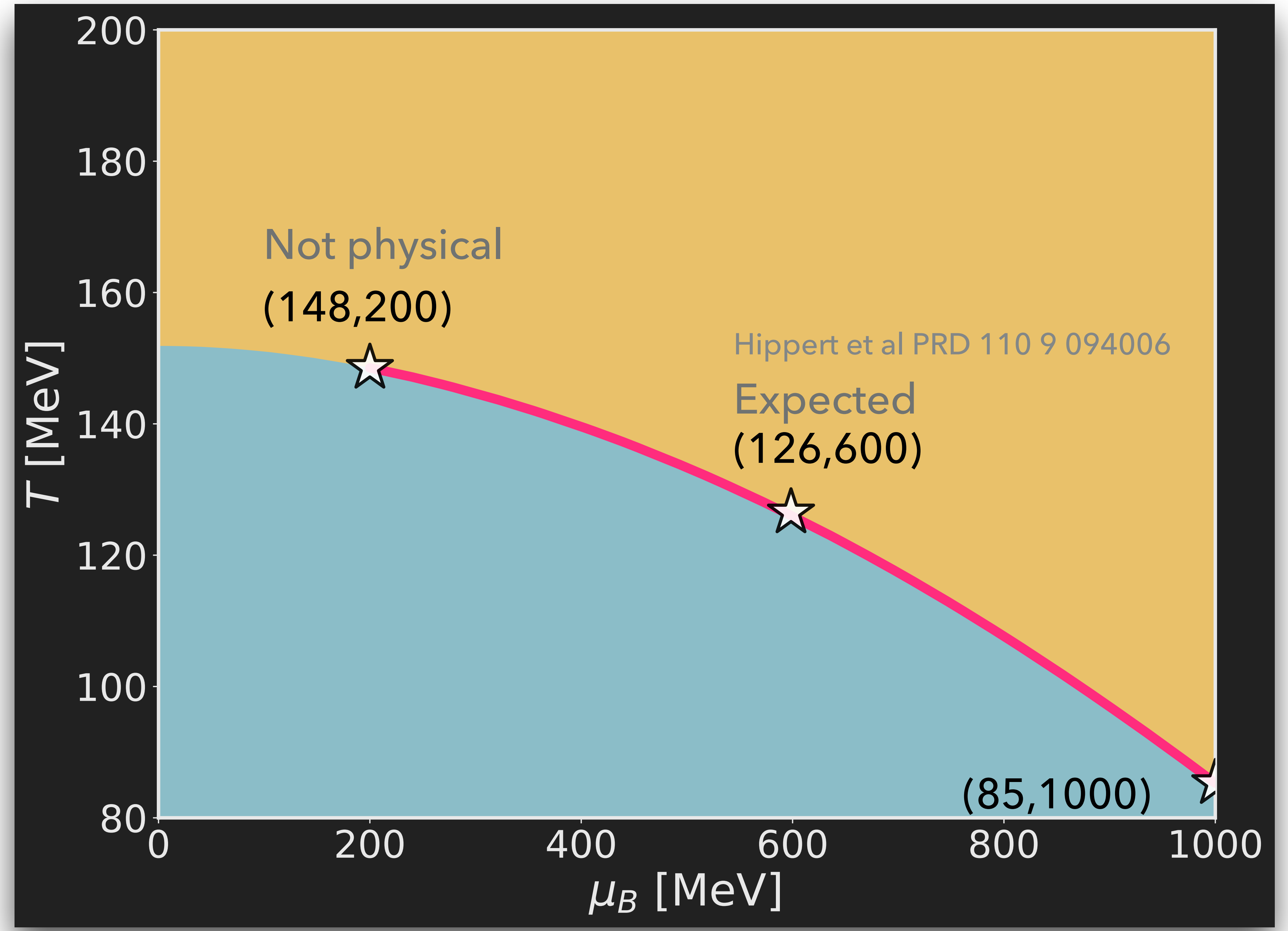
One can play with σ_η to get the correct distributions and fit data!

Simulation Setup - Holography

AuAu 0-5% $\sqrt{s_{NN}} =$ $7.7\ GeV$, $19.6\ GeV$, $39\ GeV$

3 CP positions

MUSES EOS HOLOGRAPHY



SMEARED AMPT → CCAKE 2.0 → BQSSAMPLER → SMASH

$$\frac{\zeta}{s} = 1.68\left(\frac{1}{3} - c_s^2\right)^2$$

$$\frac{\eta}{s} = 0.08$$

$$\frac{\kappa^{bb}}{T^2} = 0.025 \qquad \frac{\kappa^{bb}}{T^2} = 0.0$$

2000 samples

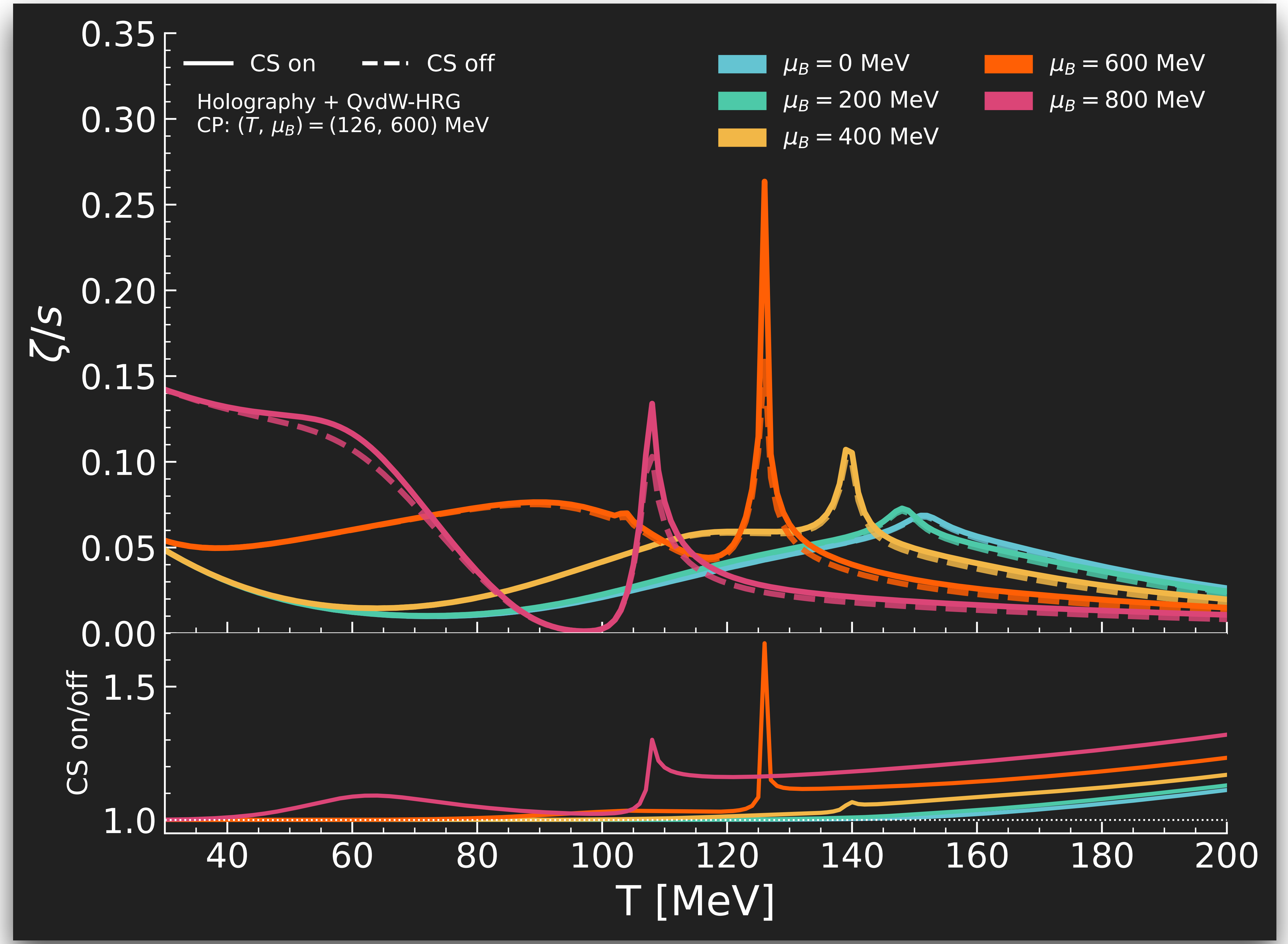
Decays only

CCAKE 2.0- Critical scaling

Bulk Viscosity

$$\tau_\pi \Delta^{\mu\nu}_{\alpha\beta} D\pi^{\alpha\beta} + \pi^{\mu\nu} = 2\eta\, \sigma^{\mu\nu} - \delta_{\pi\pi}\, \pi^{\mu\nu}\, \Theta$$ + DNMR

$$\tau_\Pi D\Pi + \Pi = -\left(\zeta + \delta_{\Pi\Pi}\Pi\right) \Theta$$ + DNMR



$$\zeta = \zeta_0 \left(1 + \frac{\xi}{\xi_0}\right)^3$$

$$\xi_0 \sim \sqrt{\langle \chi_{BB} \rangle_{table}}$$

$$\xi \sim \sqrt{\chi_{BB}}$$

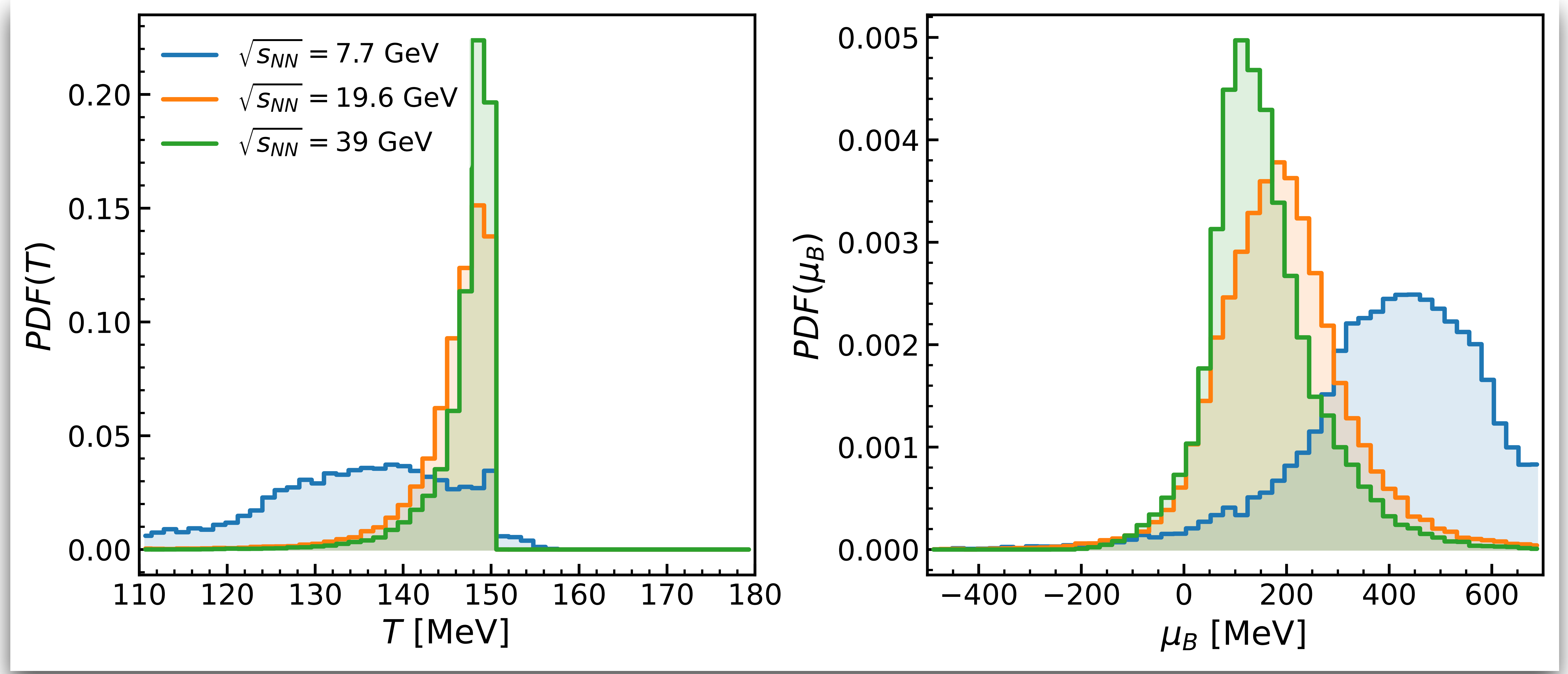
For the case of no charge diffusion!

Holography + QvdW-HRG

- Critical point at (126, 598) MeV
- Probability distribution function (PDF) of T (left panel) and μ_B (right panel) at freeze-out, for a single event!

[MUSES] "Studying the QCD Matter produced in Heavy-Ion Collisions using the MUSES Calculation Engine"

Soon to appear!

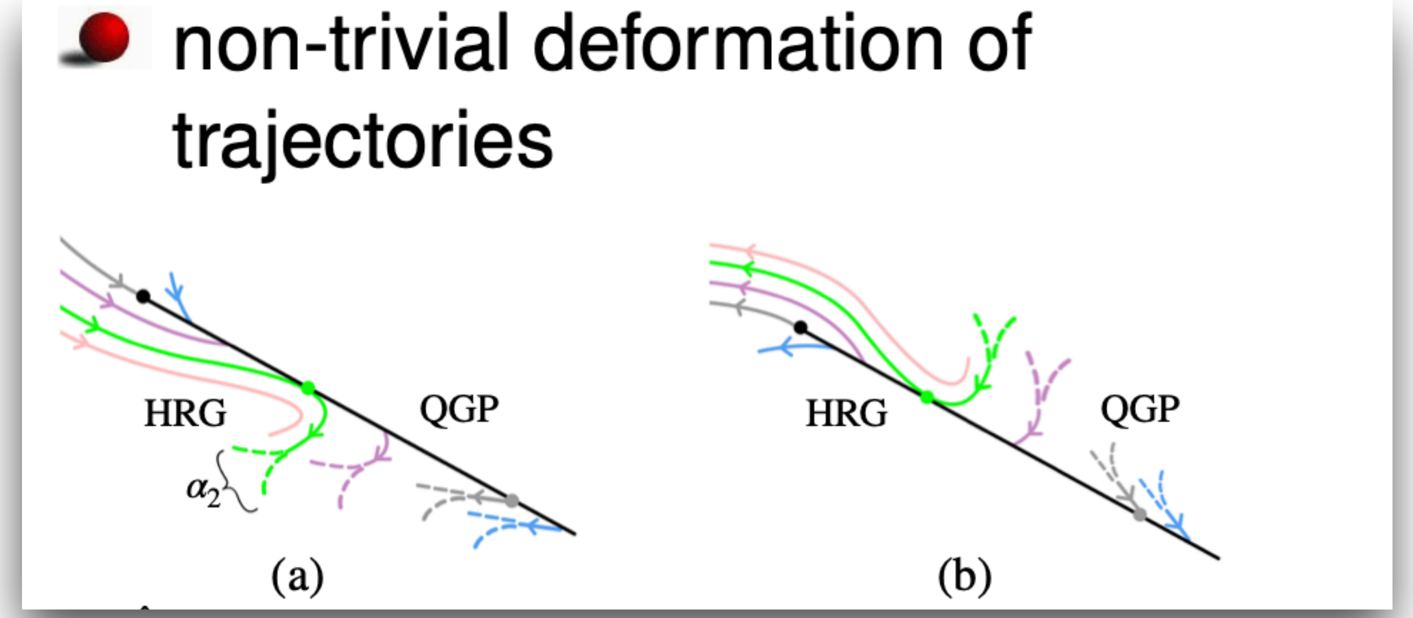


- Comparing to thermal fit data from the STAR, the average thermal fit values for $0-5\,\%$ central are $\langle\langle\mu_B^{FO}\rangle\rangle\sim 100$ MeV at $\sqrt{s_{NN}}\gtrsim 39$ GeV, $\langle\langle\mu_B^{FO}\rangle\rangle\sim 190$ MeV at $\sqrt{s_{NN}}\gtrsim 19.6$ GeV, and $\langle\langle\mu_B^{FO}\rangle\rangle\sim 400$ MeV at $\sqrt{s_{NN}}\gtrsim 7.7$ GeV

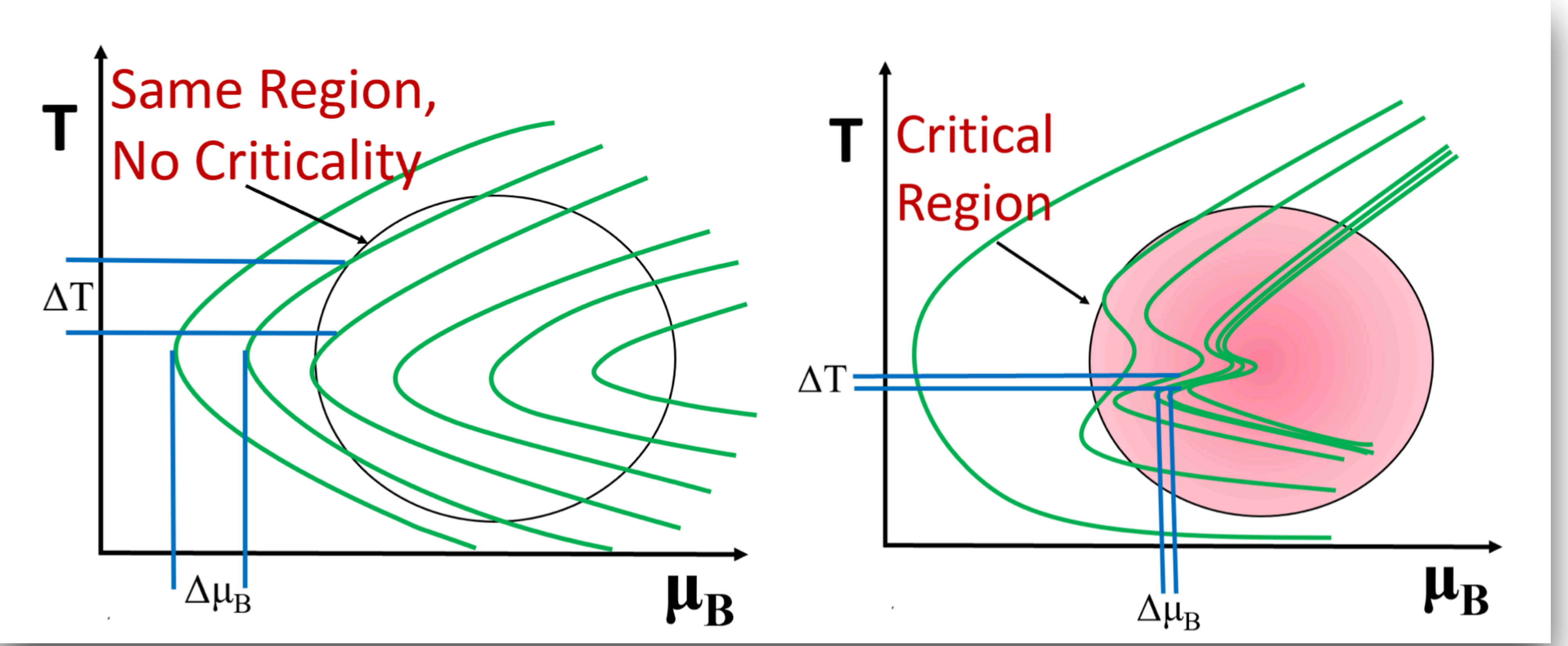
L. Adamczyk et al. (STAR), Phys. Rev. C 96, 044904 (2017)

Trajectories and Critical Lensing

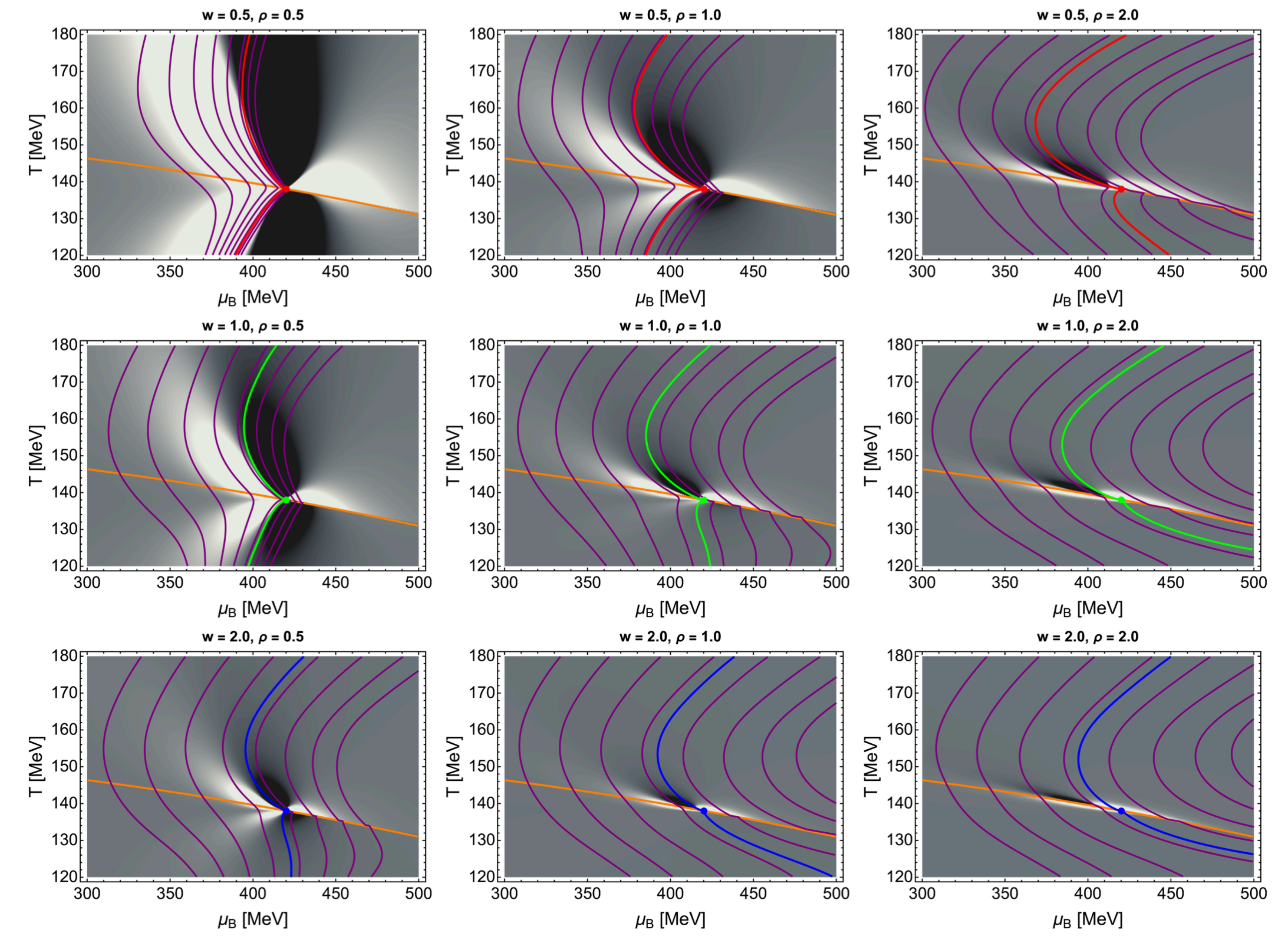
From M. Stephanov's talk on Monday!



Isentropic trajectories with and without criticality:



Dore et all, Phys. Rev. D 106 (2022) no.9, 094024

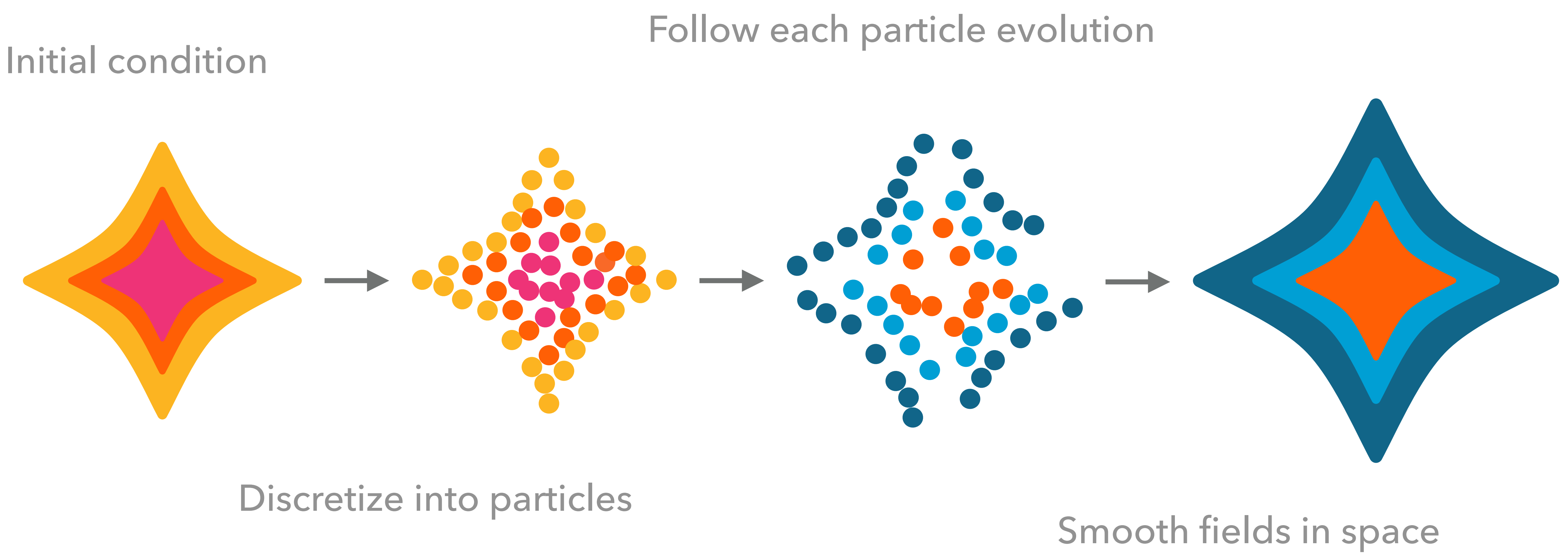


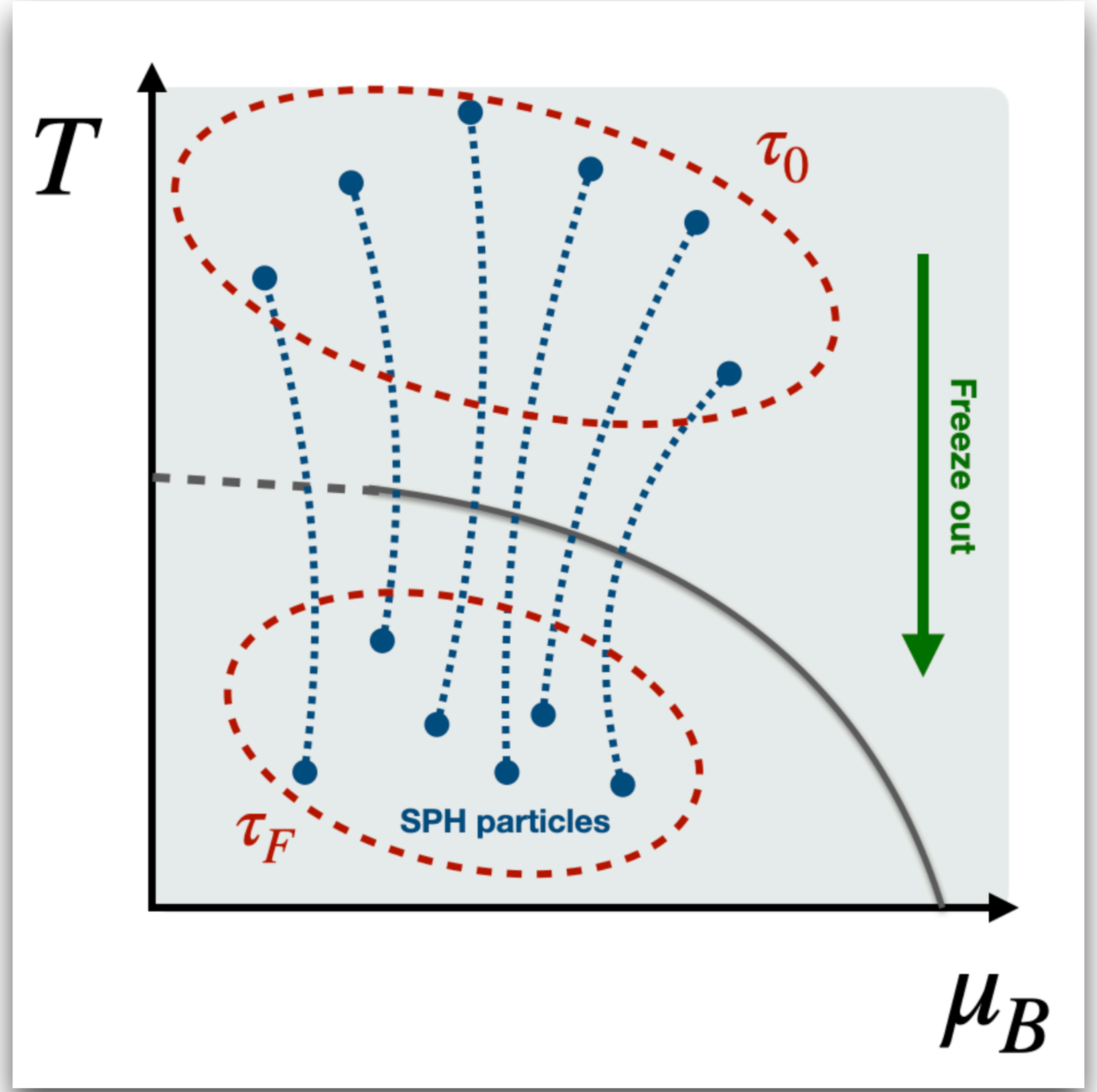
Dore et all, Phys. Rev. D 106 (2022) no.9, 094024

Critical regions that extend farther along the T direction have a more of a kink near the CP than along the μ_B direction

Trajectories in SPH Formalism

- Individual particles are not on a fixed grid.
- We can track from the initial time τ_0 how the SPH particles cool and expand in time until their final time step τ_F.





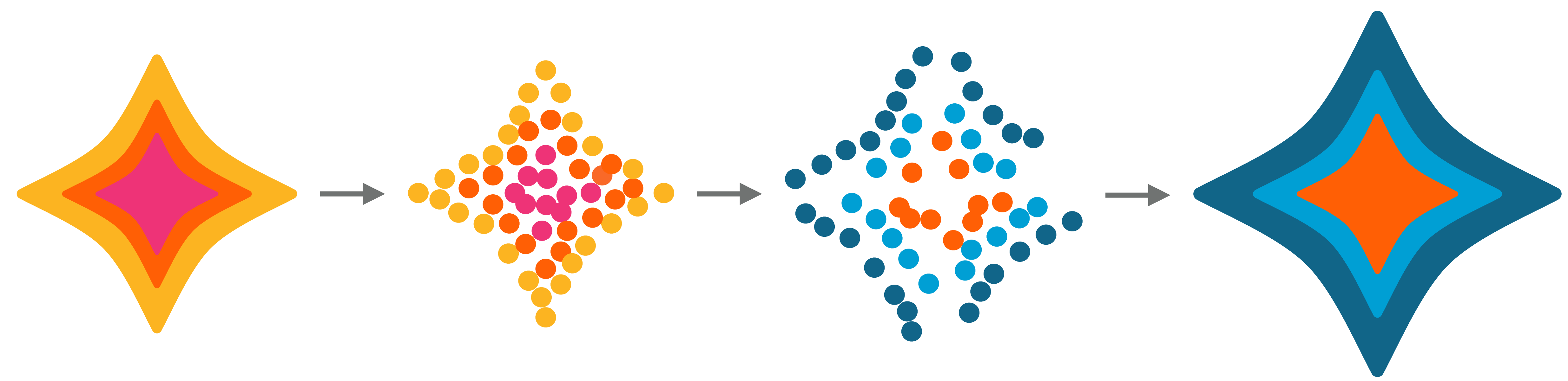
Trajectories

Holography + QvdW-HRG

- Individual particles are not on a fixed grid.
- We can track from the initial time τ_0 how the SPH particles cool and expand in time until their final time step τ_F.

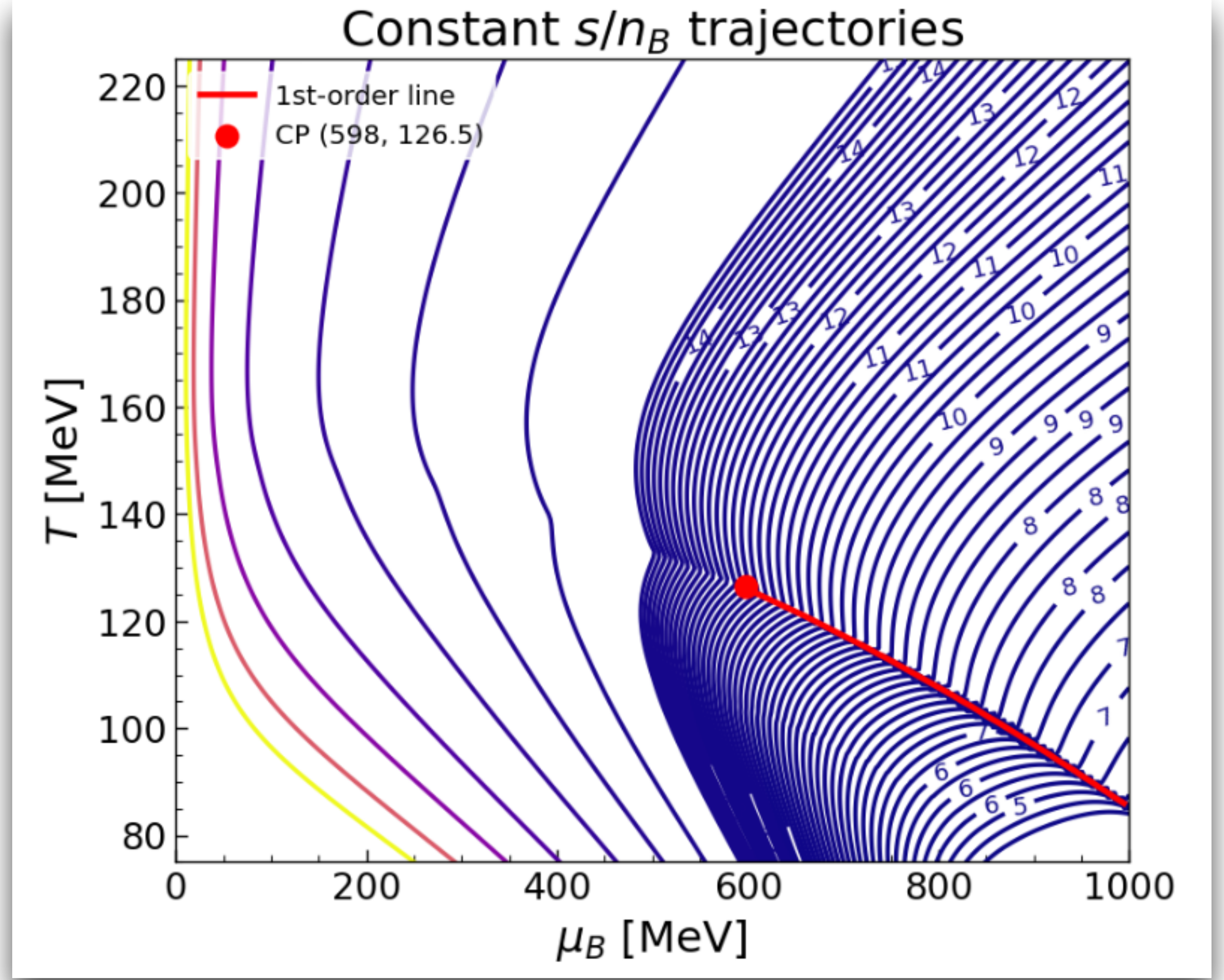
Initial condition

Follow each particle evolution



Discretize into particles

Smooth fields in space



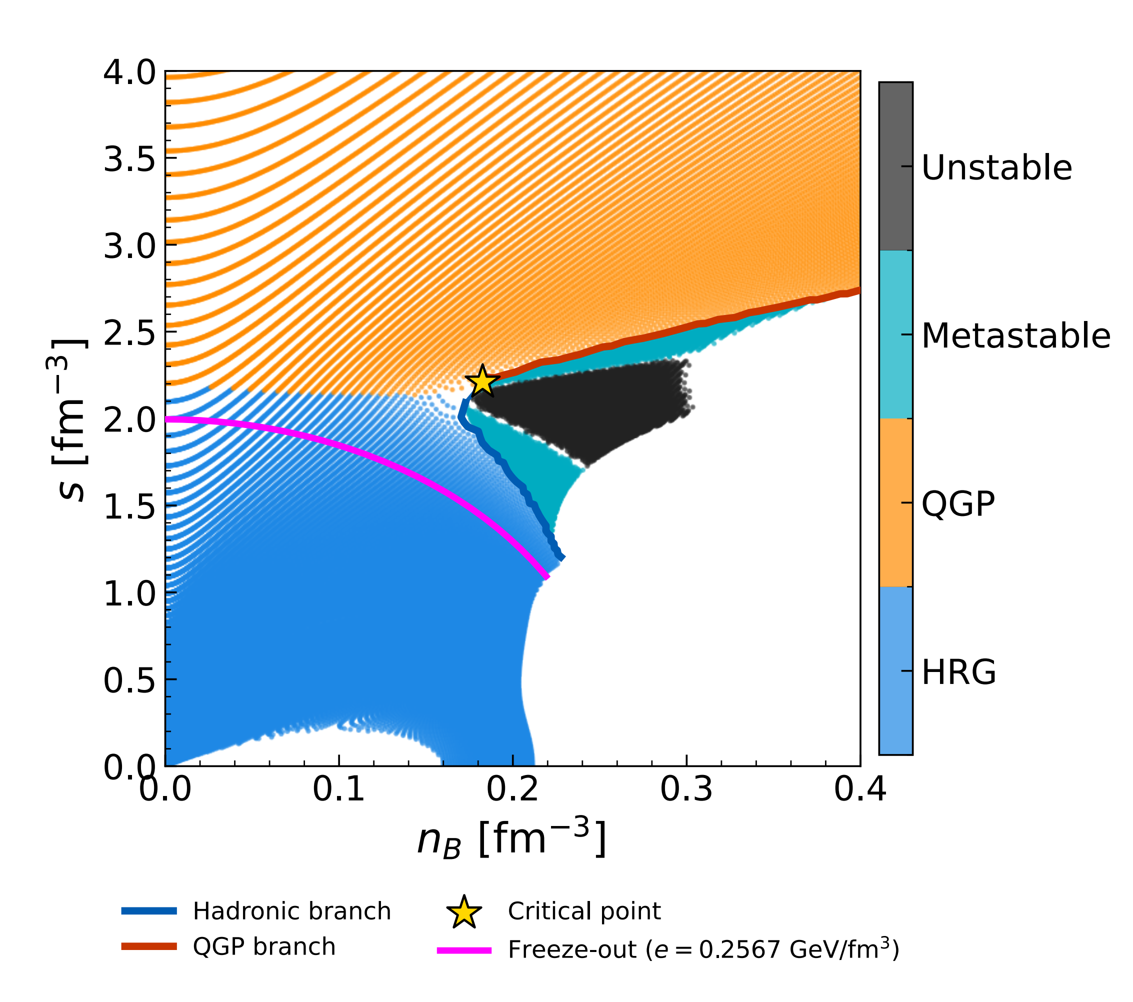
[MUSES] "Studying the QCD Matter produced in Heavy-Ion Collisions using the MUSES Calculation Engine" Soon to appear!

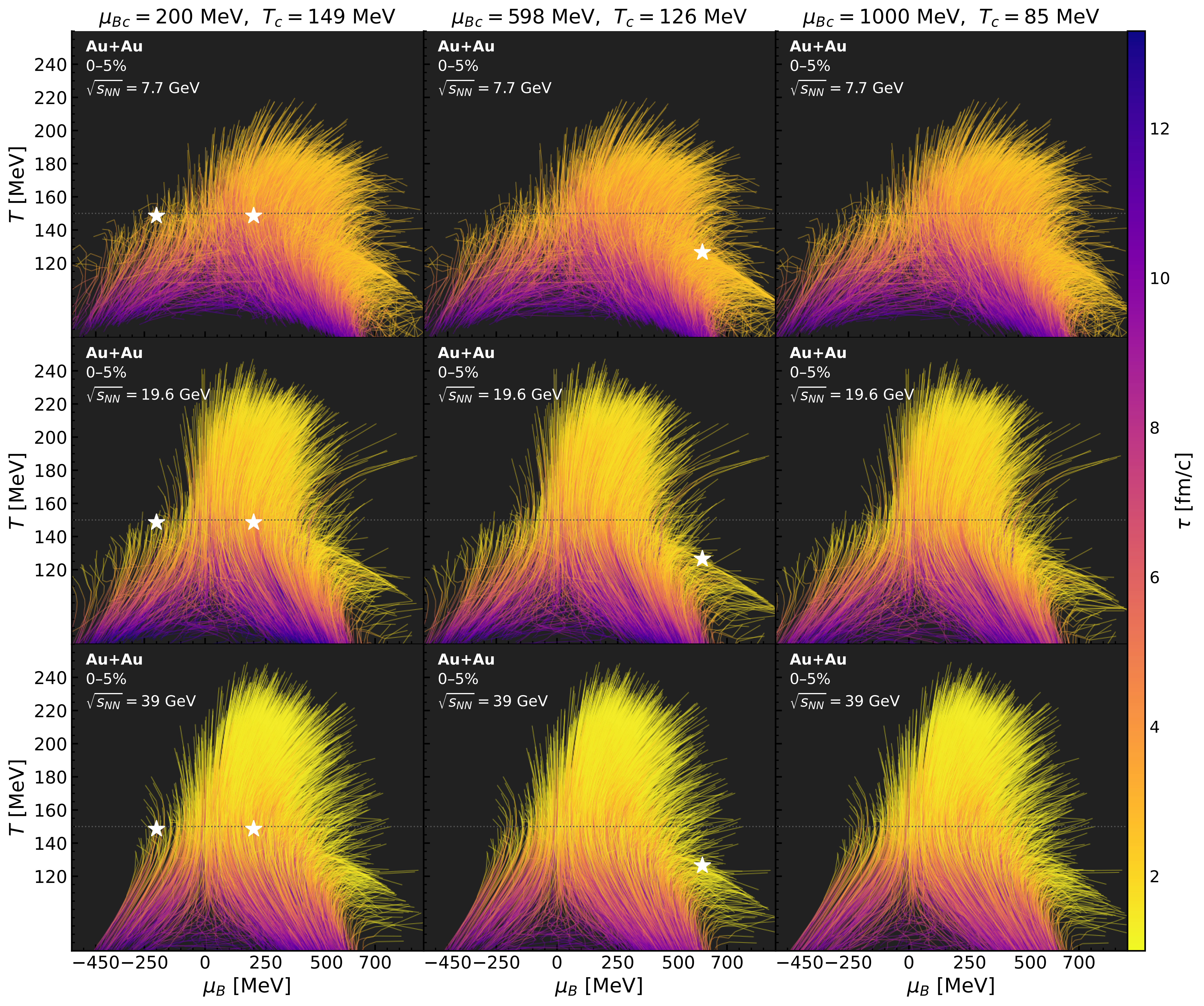
- Isentropes emulate the path that an ideal fluid would travel along across the QCD phase diagram.

Trajectories on CCAKE 2.0

Holography + QvdW-HRG

- For fixed, single initial condition at each $\sqrt{s_{NN}} = \{39, 19.6, 7.7\}$ GeV, we plot the SPH trajectories across the T, μ_B plane for $5{,}000$ SPH particles
- Three different critical point positions at $\mu_{B_C} = \{200, 598, 1000\}$ MeV (here the metastable and unstable are included in the simulations).





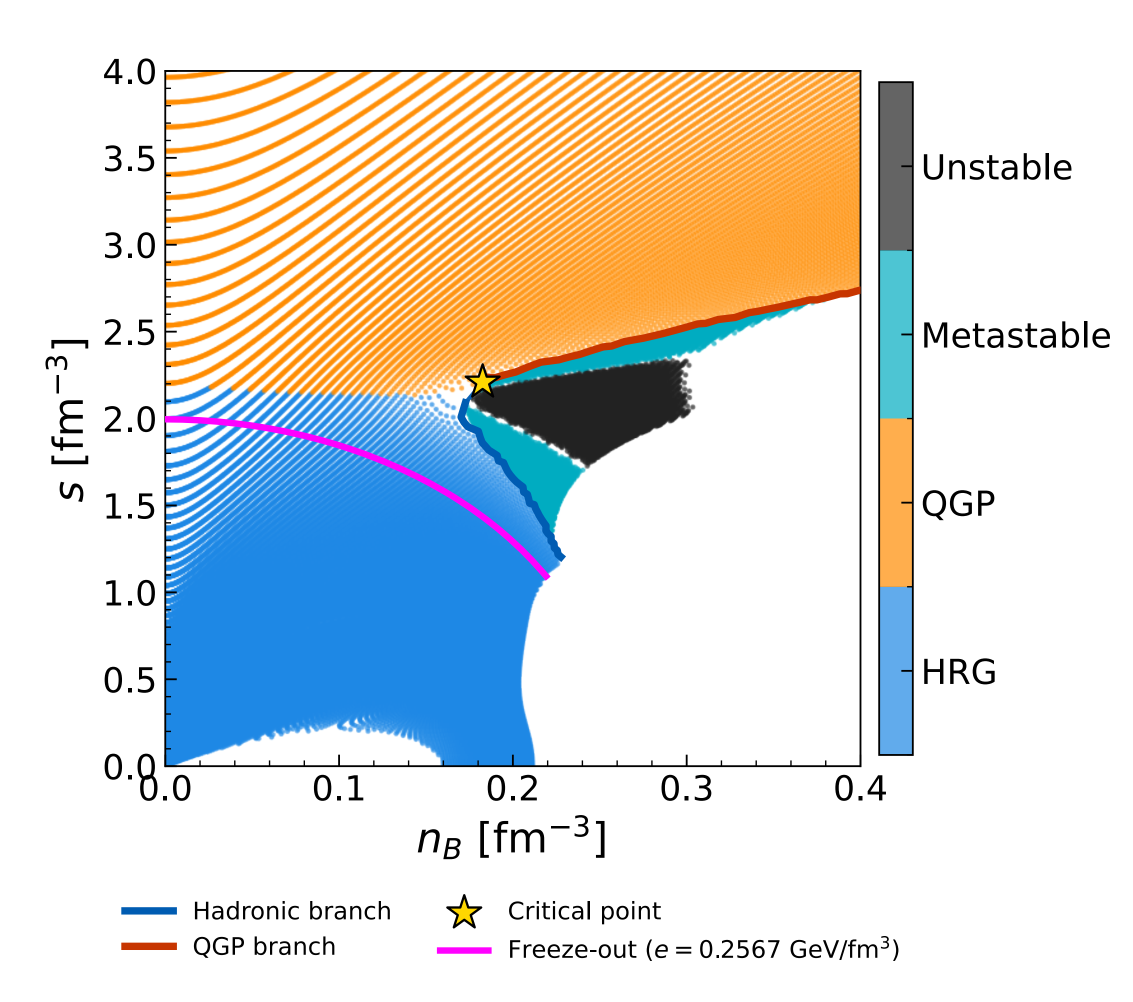
[MUSES] "Studying the QCD Matter produced in Heavy-Ion Collisions using the MUSES Calculation Engine"

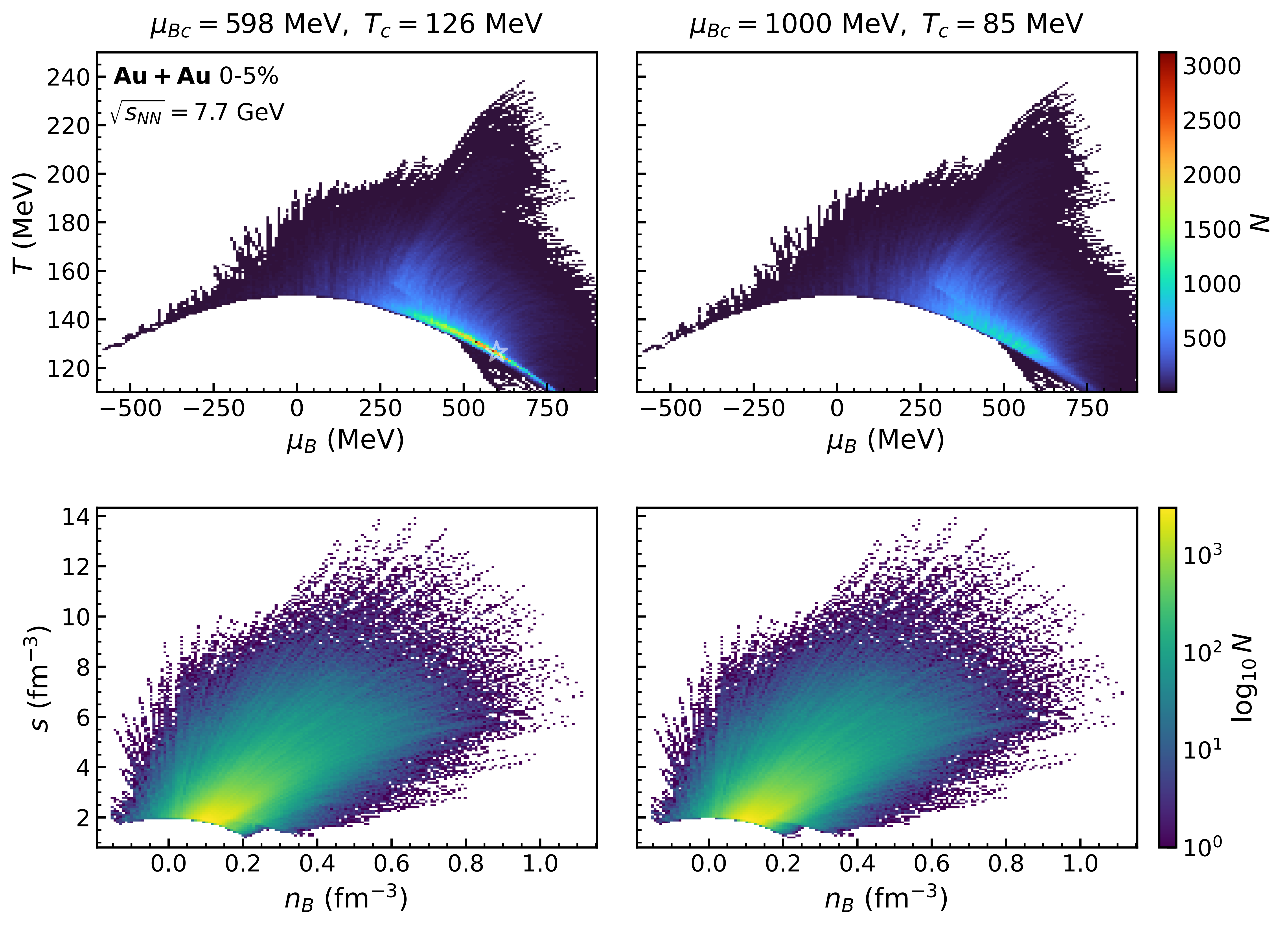
Soon to appear!

Trajectories on CCAKE 2.0

Holography + QvdW-HRG

- Fixed, single initial condition at each $\sqrt{s_{NN}} = 7.7$GeV.
- Heat map over all SPH particles across all time steps (including the metastable and unstable regime).
- Two critical point positions at $\mu_{B_c} = \{598,1000\}$MeV.
- The critical point is indicated by a star.





[MUSES] "Studying the QCD Matter produced in Heavy-Ion Collisions using the MUSES Calculation Engine"

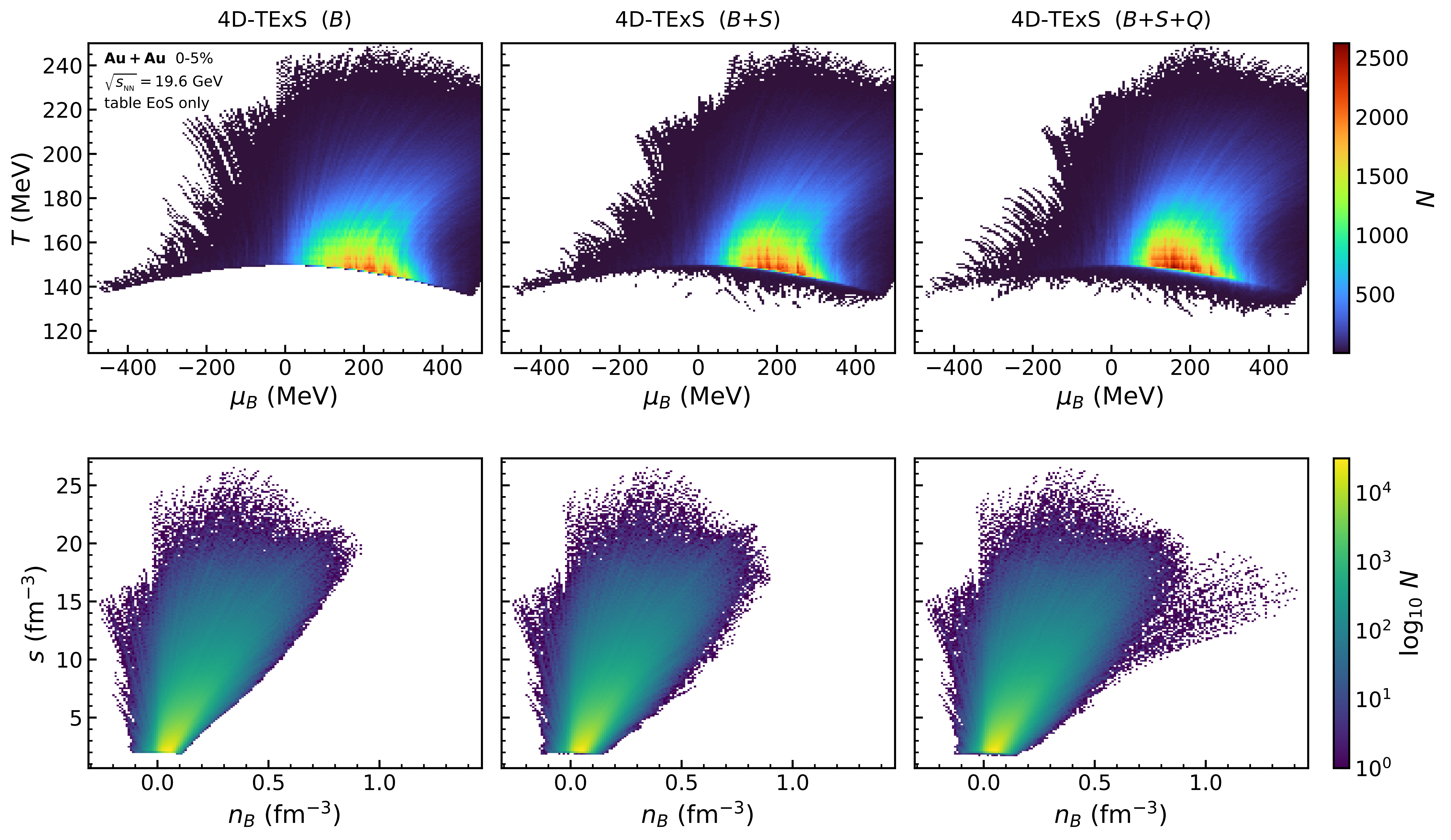
Soon to appear!

Trajectories on CCAKE 2.0

4D-TExS EoS

[arXiv:2504.01881 [hep-lat]]

- For fixed, single initial condition at each $\sqrt{s_{NN}} = 19.6$ GeV,
- We only include regimes of the simulation above freeze-out.
- The top row shows the distribution in the T, μ_B plane and the bottom row shows the distribution in the s, n_B plane.



[MUSES] "Studying the QCD Matter produced in Heavy-Ion Collisions using the MUSES Calculation Engine"

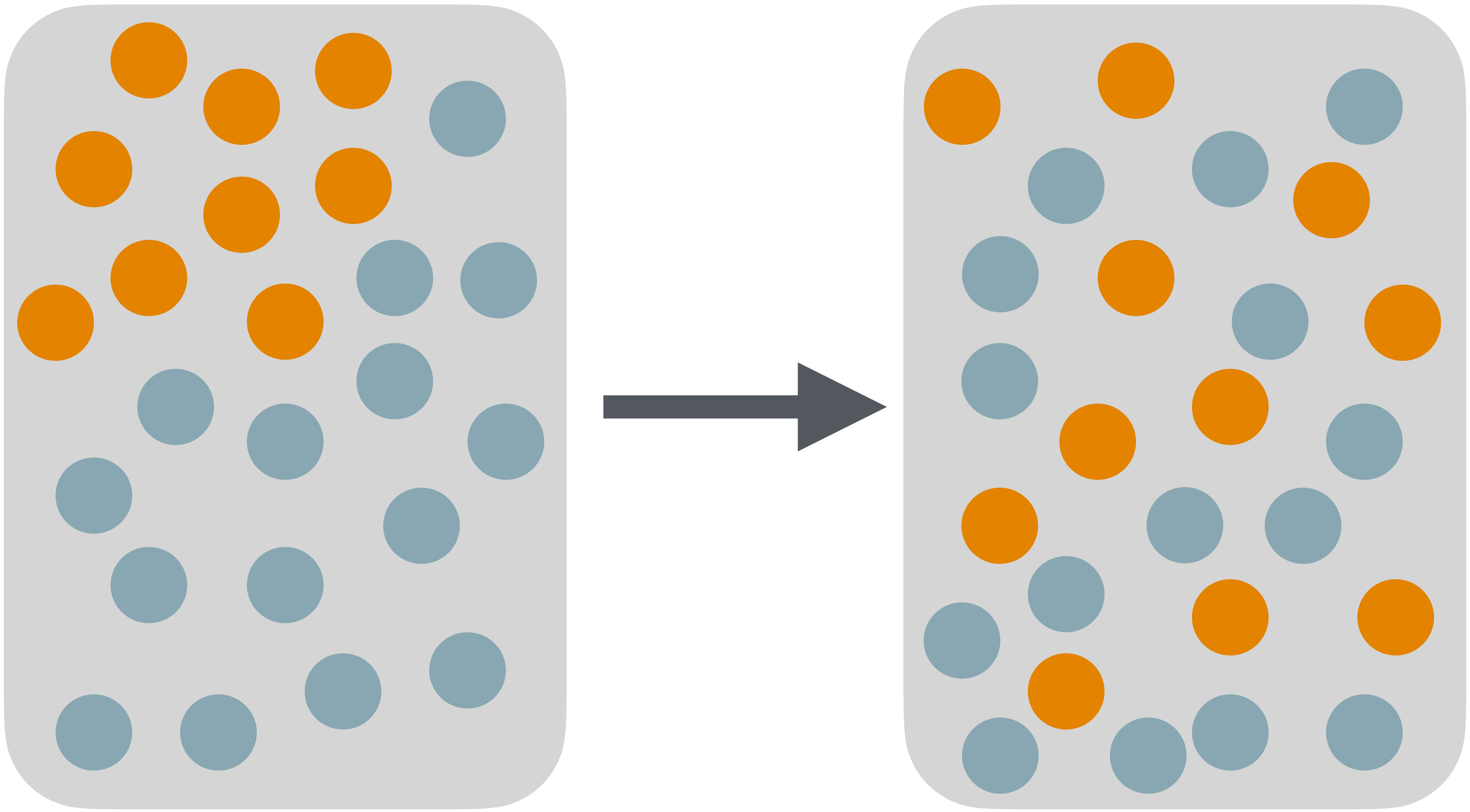
Soon to appear!

How does BSQ charge diffusion affect hydrodynamic simulations?

BSQ Charge Diffusion

In Israel–Stewart hydrodynamics, diffusion currents respond to gradients of chemical potentials:

$$j_q^\mu = \kappa_q \nabla^\mu \alpha_q, \qquad \alpha_q = \frac{\mu_q}{T}$$



with multiple conserved charges, this leads to coupled diffusion:

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

$\underbrace{\kappa_{ab}}_{\text{couples charges}}$ $\qquad$ $\underbrace{\tau_{ab}}_{\text{diagonal (no coupling)}}$

diffusion currents relax to their Navier–Stokes values over a finite timescale,

$$\tau_{ab}\, \dot{j}_b^\mu + j_a^\mu = \kappa_{ab} \nabla^\mu \alpha_b \quad \longrightarrow \quad \tau_{ab} = \frac{1}{T} \mathrm{diag}(0.2, 0.2, 0.2)$$

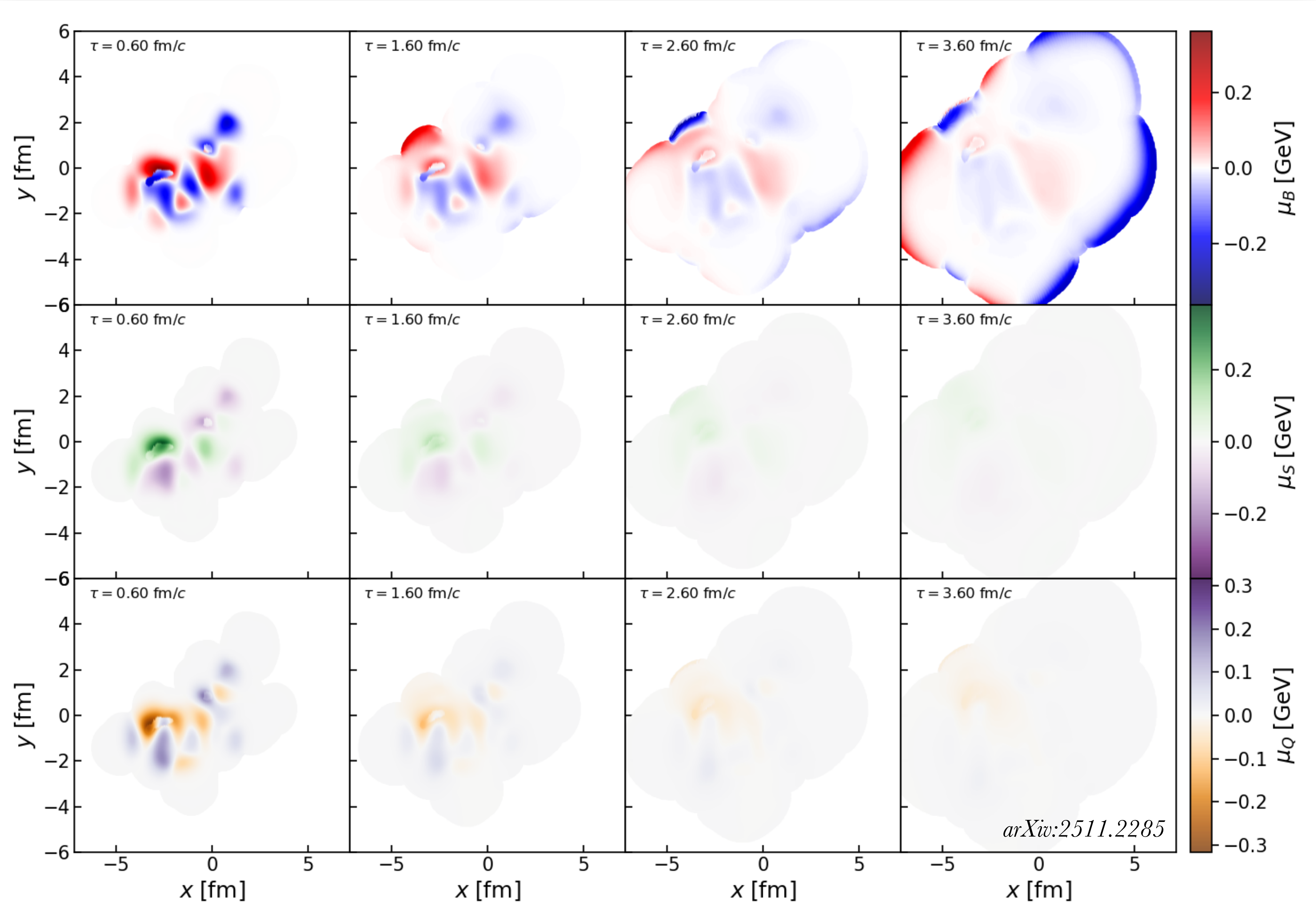
Fotakis, Greif, Greiner, Denicol and Niemi, Phys. Rev. D 101, no.7, 076007 (2020)

BSQ Charge Diffusion

BSQ fluctuations ran with ideal BSQ currents compared different scenarios for the BSQ diffusion matrix:

$$\kappa = \begin{bmatrix} \kappa_{BB} & \kappa_{BS} & \kappa_{BQ} \\ \kappa_{SB} & \kappa_{SS} & \kappa_{SQ} \\ \kappa_{QB} & \kappa_{QS} & \kappa_{QQ} \end{bmatrix}$$

- Diffusion matrix that has constant values of $\kappa_{qq'}/T^2$
- Diagonal: self-diffusion
- Off-diagonal: charge mixing



P. Pala, **Danhoni** et al arxiv 2511.22852

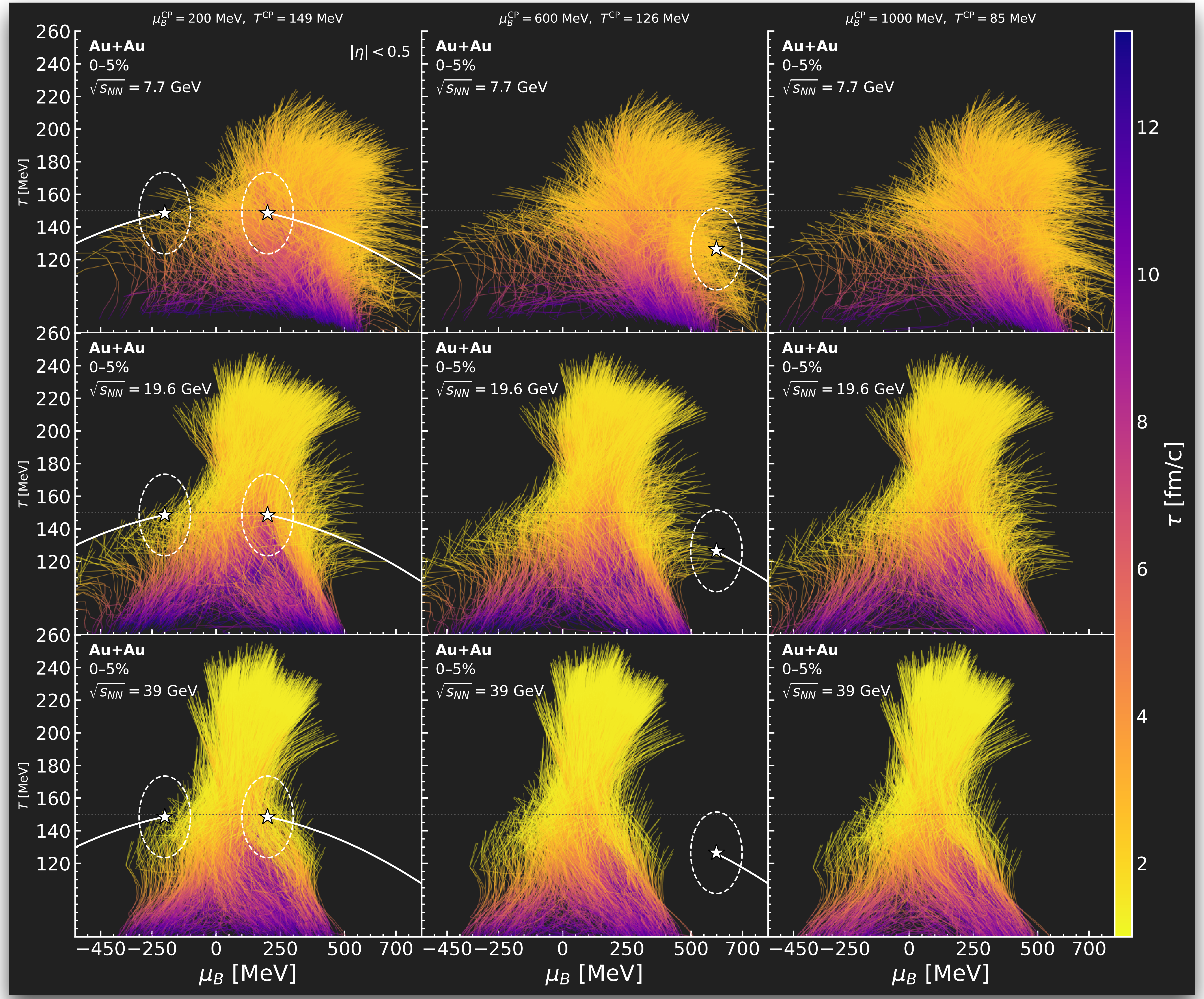
trento+iccing+ccake simulations with the same initial condition.

BSQ Charge Diffusion

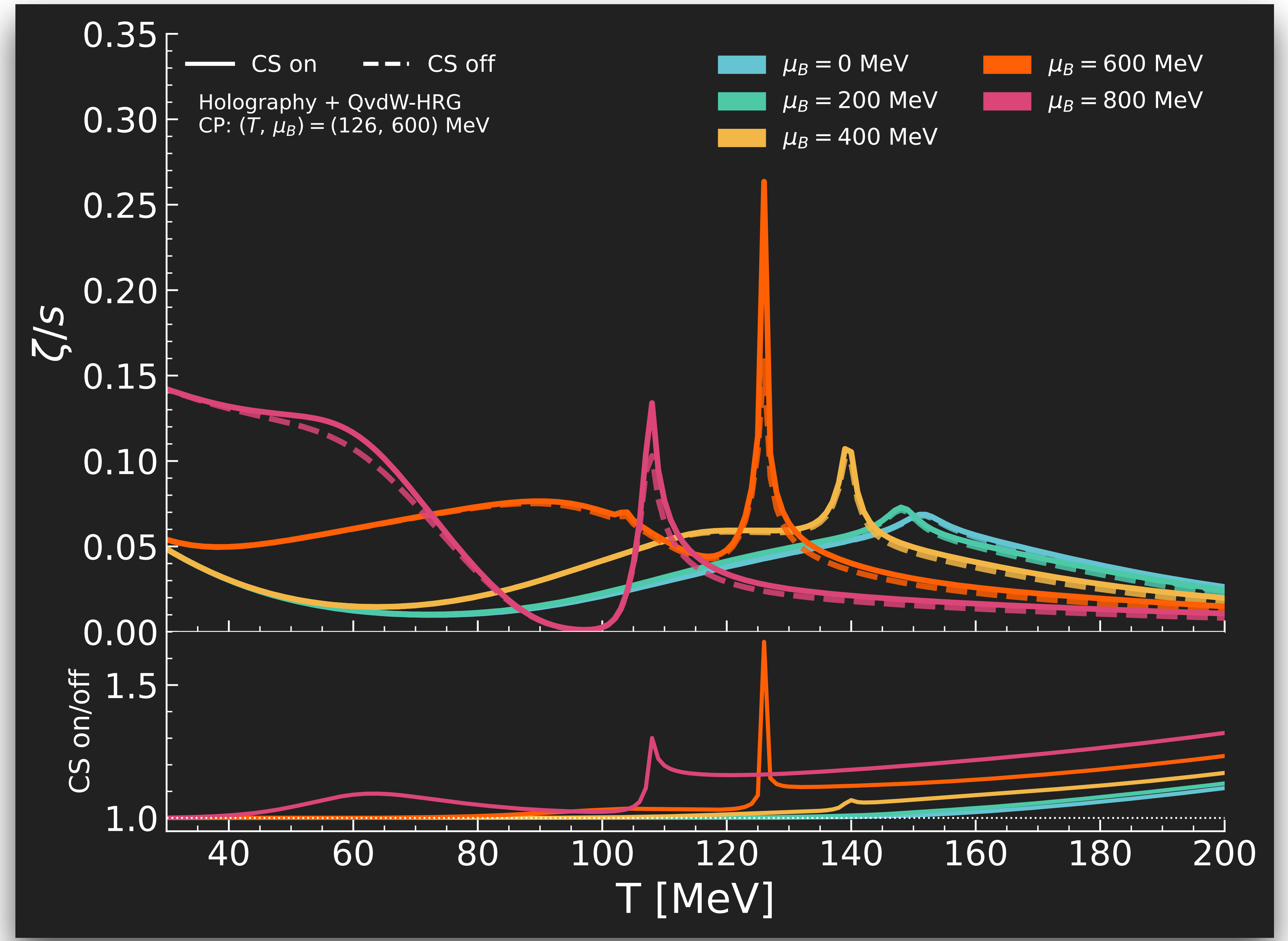
BSQ fluctuations ran with ideal BSQ currents compared different scenarios for the BSQ diffusion matrix:

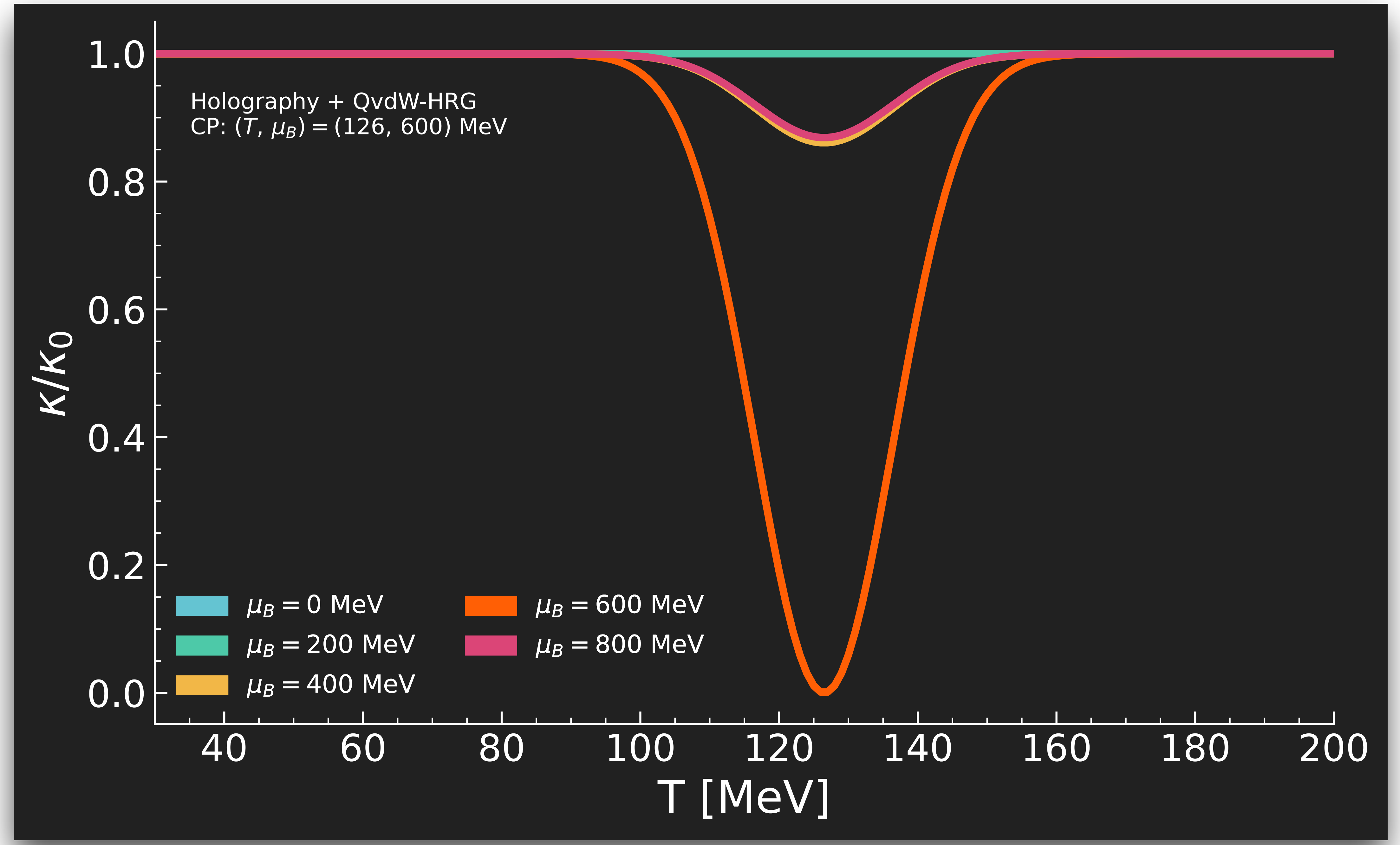
$$\kappa = \begin{bmatrix} \kappa_{BB} & \kappa_{BS} & \kappa_{BQ} \\ \kappa_{SB} & \kappa_{SS} & \kappa_{SQ} \\ \kappa_{QB} & \kappa_{QS} & \kappa_{QQ} \end{bmatrix}$$

- Diffusion matrix that has constant values of $\kappa_{qq'}/T^2$
- Diagonal: self-diffusion
- Off-diagonal: charge mixing



CCAKE 2.0- Critical scaling





$$\tau_q^{ab}\,\Delta^{\mu}_{\ \nu}\,Dq_b^{\nu} + q_a^{\mu} = \kappa^{ab}\nabla^{\mu}\left(\frac{\mu_b}{T}\right) - \tau_q^{ab}\,q_a^{\mu}\,\Theta$$

$$\kappa = \kappa_0\left(1 - \exp\left[-\frac{1}{2}\left(\frac{(T-T_c)^2}{\sigma_T^2} + \frac{(\mu_B-\mu_B^c)^2}{\sigma_\mu^2}\right)\right]\right)$$

Same critical scaling for bulk!

NuClearConfectionery FRAMEWORK + MUSES EOS

CCAKE

MUSES EOS

Grefa et al PRD 104 3, 034002

Yang et al arxiv 2601.07987

Plumberg et al PRC 111 4, 044905

Lin and Ko PRC 65 034904

Bin Zhang et al PRC 61 067901

Baochi Fu et al PRC 103 2, 024903

Weil et al PRC 94 054905

SMEARED AMPT		CCAKE 2.0	BQSSAMPLER	SMASH
INITIAL CONDITIONS	PRE EQUILIBRIUM	HYDRODYNAMICS	PARTICLIZATION	DECAYS

Plumberg et al PRC 111 4, 044905

P. Pala, **I. Danhoni** et al arxiv 2511.22852

P. Pala, **Danhoni** et al arxiv 2511.22852

Conserved Charge Sampling

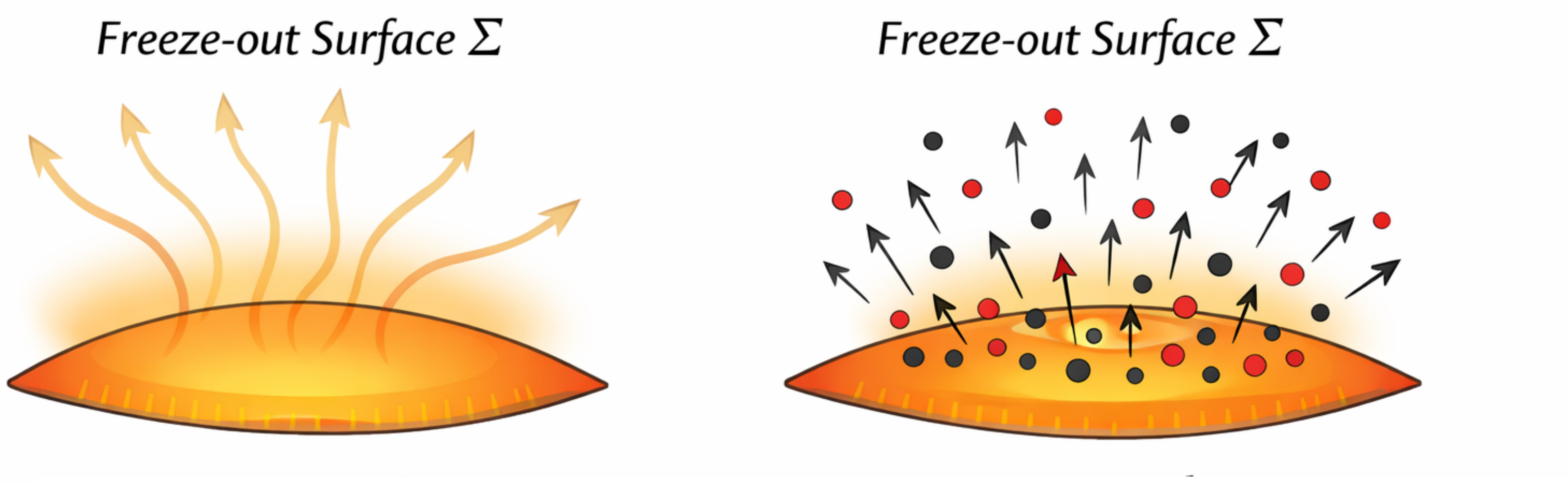
New code to sample at finite densities!

Regular sampling

$$E\frac{d^3N_i}{dk^3}(x^\mu,k^\mu) = \int_\sigma k^\nu d^3\sigma_\nu \left(f_{0i}(x^\mu,k) + \delta f_i(x^\mu,k^\mu)\right)$$

$$f = \frac{g_i}{(2\pi)^3}\frac{1}{\exp\left(\frac{u(x)\cdot k + b_i\mu_b(x)}{T}\right)\pm 1}$$

Sampling algorithm from iS3D extended to finite μ_B, μ_S, μ_Q!



Cooper–Frye (continuous)

Monte Carlo sampling

AI generated

Conserved Charge Sampling

New code to sample at finite densities!

P. Pala, **Danhoni** et al arxiv 2511.22852

Algorithm to conserve charge event by event (and sample by sample):

Sample baryons freely → Sample anti-baryons exactly

$$N^{\{s,ev\}}_{+1,a,a} \geq N^{\text{net},\{ev\}}_B$$

$$N^{\{s,ev\}}_{-1,a,a} = N^{\{s,ev\}}_{+1,a,a} - N^{\text{net},\{ev\}}_B$$

Sample strange mesons freely → Sample anti-strange mesons exactly

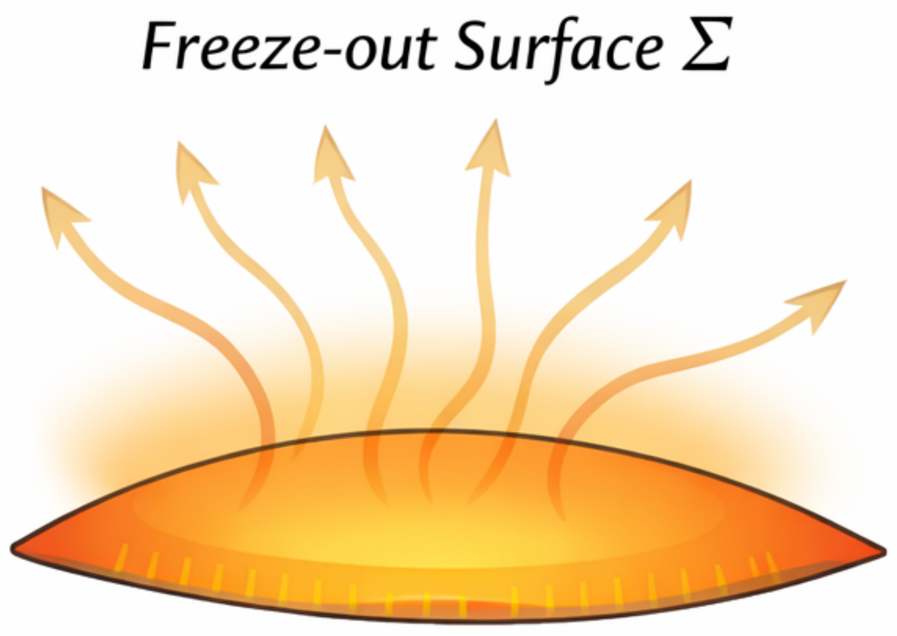
$$N^{\{s,ev\}}_{0,-1,a}$$

$$N^{\{s,ev\}}_{0,+1,a} = N^{\text{net},\{s,ev\}}_{SB} - N^{\{s,ev\}}_{0,-1,a} - N^{\text{net},\{ev\}}_S$$

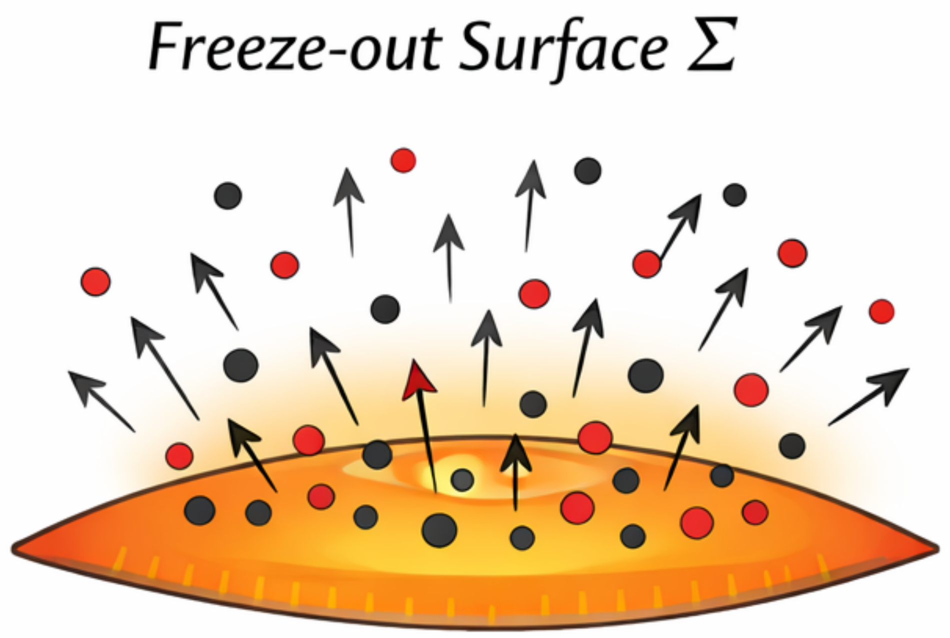
Sample + charged mesons freely → Sample - charged mesons exactly

$$N^{\{s,ev\}}_{0,0,+1} \geq N^{\text{net},\{ev\}}_Q$$

$$N^{\{s,ev\}}_{0,0,-1} = N^{\text{net},\{s,ev\}}_{BQ} + N^{\text{net},\{s,ev\}}_{SMQ} + N^{\{s,ev\}}_{0,0,+1} - N^{\text{net},\{ev\}}_Q$$



Cooper–Frye (continuous)



Monte Carlo sampling

AI generated

NuClearConfectionery FRAMEWORK + MUSES EOS

CCAKE

MUSES EOS

Grefa et al PRD 104 3, 034002

Yang et al arxiv 2601.07987

Plumberg et al PRC 111 4, 044905

↓

Lin and Ko PRC 65 034904

Bin Zhang et al PRC 61 067901

Baochi Fu et al PRC 103 2, 024903

Weil et al PRC 94 054905

SMEARED AMPT		→	CCAKE 2.0	→	BQSSAMPLER	→	SMASH
INITIAL CONDITIONS	PRE EQUILIBRIUM		HYDRODYNAMICS		PARTICLIZATION		DECAYS

Plumberg et al PRC 111 4, 044905

P. Pala, **I. Danhoni** et al arxiv 2511.22852

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NuClearConfectionery ~~FRAMEWORK~~ + MUSES EOS

Grefa et al PRD 104 3, 034002
Yang et al arxiv 2601.07987
Plumberg et al PRC 111 4, 044905

MUSES EOS

CCAKE

Lin and Ko PRC 65 034904
Bin Zhang et al PRC 61 067901
Baochi Fu et al PRC 103 2, 024903

Weil et al PRC 94 054905

Preliminary Results!

SMEARED AMPT — INITIAL CONDITIONS, PRE EQUILIBRIUM

→ HYDRODYNAMICS

→ …AMPLER — PARTICLIZATION

→ SMASH — DECAYS

Plumberg et al PRC 111 4, 044905
P. Pala, **I. Danhoni** et al arxiv 2511.22852

P. Pala et al arxiv 2511.22852

Critical scaling for bulk with no diffusion

[MUSES] "Studying the QCD Matter produced in Heavy-Ion Collisions using the MUSES Calculation Engine"

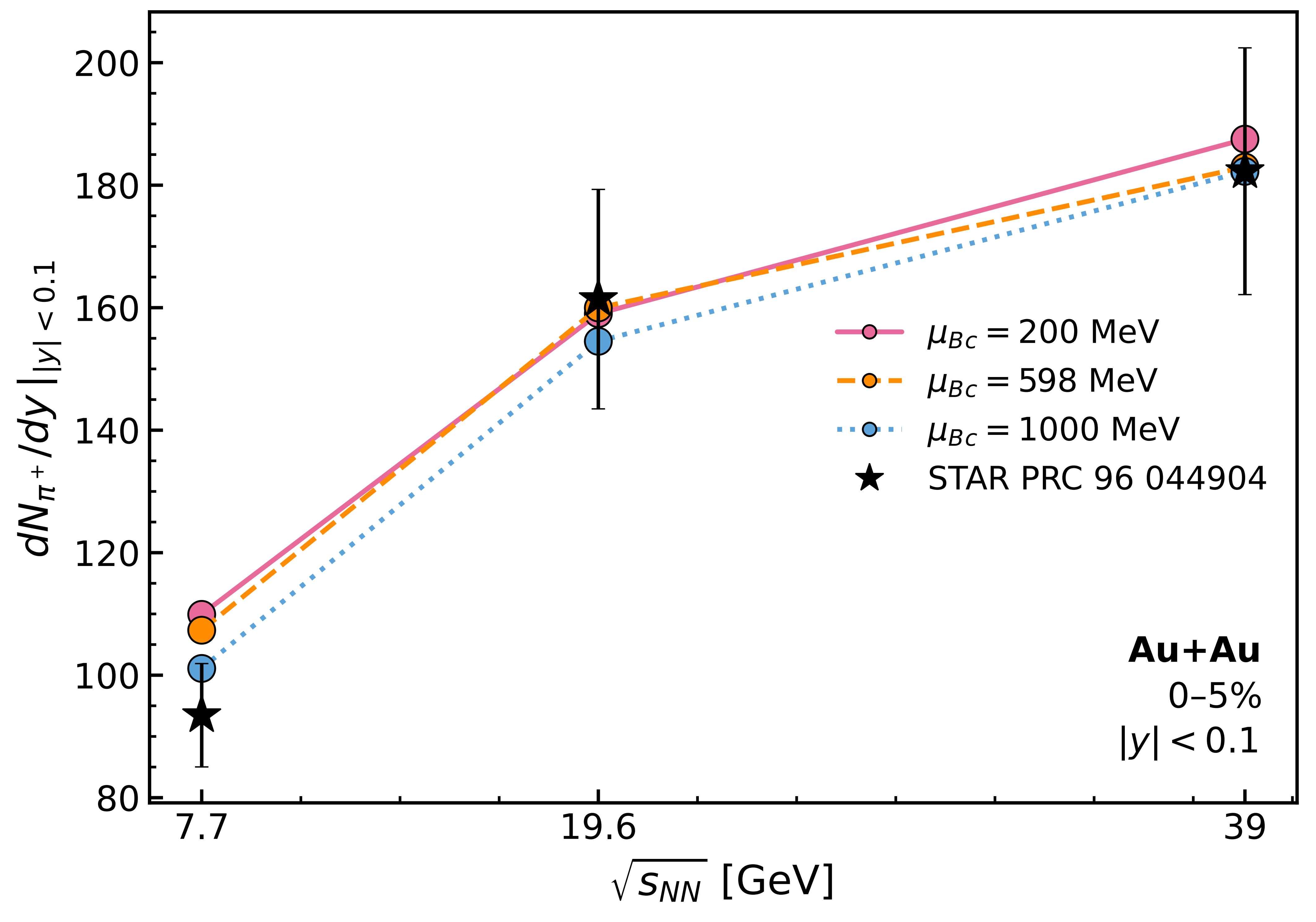
Soon to appear!

Multiplicity

Holography + QvdW-HRG

- dN/dy at mid-rapidity for positive pions!
- The choice of critical point location has only a small effect on the total pion yield
- Normalization is mostly insensitive to details of the phase structure in the EoS.

Description	Symbol	$\sqrt{s_{NN}}$ [GeV] 39	19.6	7.7
Initial time	τ_0	1.0	1.3	2.0
Smearing radius	$\sigma_\perp$ and σ_η	0.8	1.0	1.2
Normalization	$\mathcal{N}$	1.43	1.6	1.11
Freeze-out energy	ε_{FO}	$\varepsilon(T=150, \mu_X=0)$		
Shear viscosity	$\eta T/w$	0.08		
Bulk viscosity magnitude	A	1.67552		
Bulk viscosity scaling	p	2		
Diffusion	κ_{XX}	0		
Dynamical initialization		OFF		
Hadron List		PDG21		

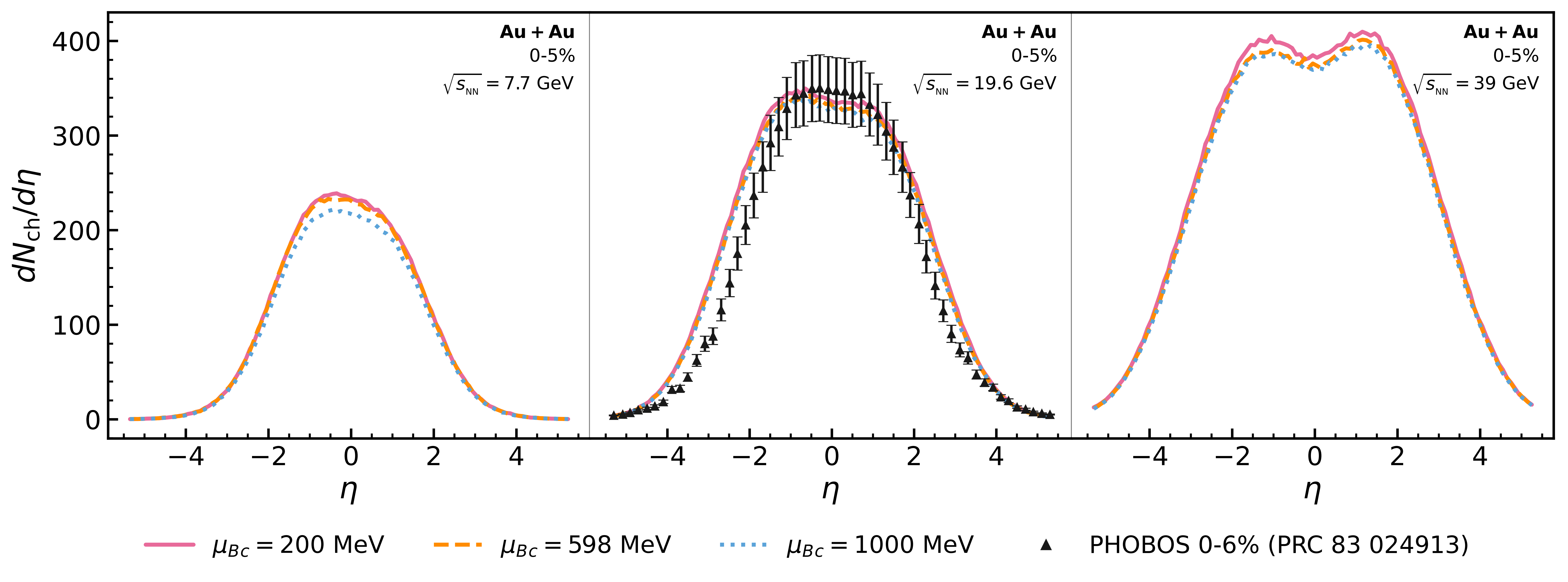


- The critical point located at $\mu_{B_c} = 1000$ MeV yields a somewhat larger multiplicity of pions in this setup.

Multiplicity

- We show the $dN/d\eta$ for all charged particles at AuAu $\sqrt{s_{NN}} = 19.6$ GeV compared to PHOBOS data for $0 - 6\,\%$ centralities. B. B. Back et al., Phys. Rev. Lett. 91, 052303 (2003).
- $\sqrt{s_{NN}} = 39{,}7.7$ GeV for $0 - 5\,\%$ centrality class.

Holography + QvdW-HRG



[MUSES] "Studying the QCD Matter produced in Heavy-Ion Collisions using the MUSES Calculation Engine"

Soon to appear!

- The width of this distribution is strongly dependent on the choice of the smearing length in AMPT.
- Smaller σ decreases the width of the distribution and large σ broaden it.

$\sigma = 1.2$ fm for $\sqrt{s_{NN}} = 19.6$GeV

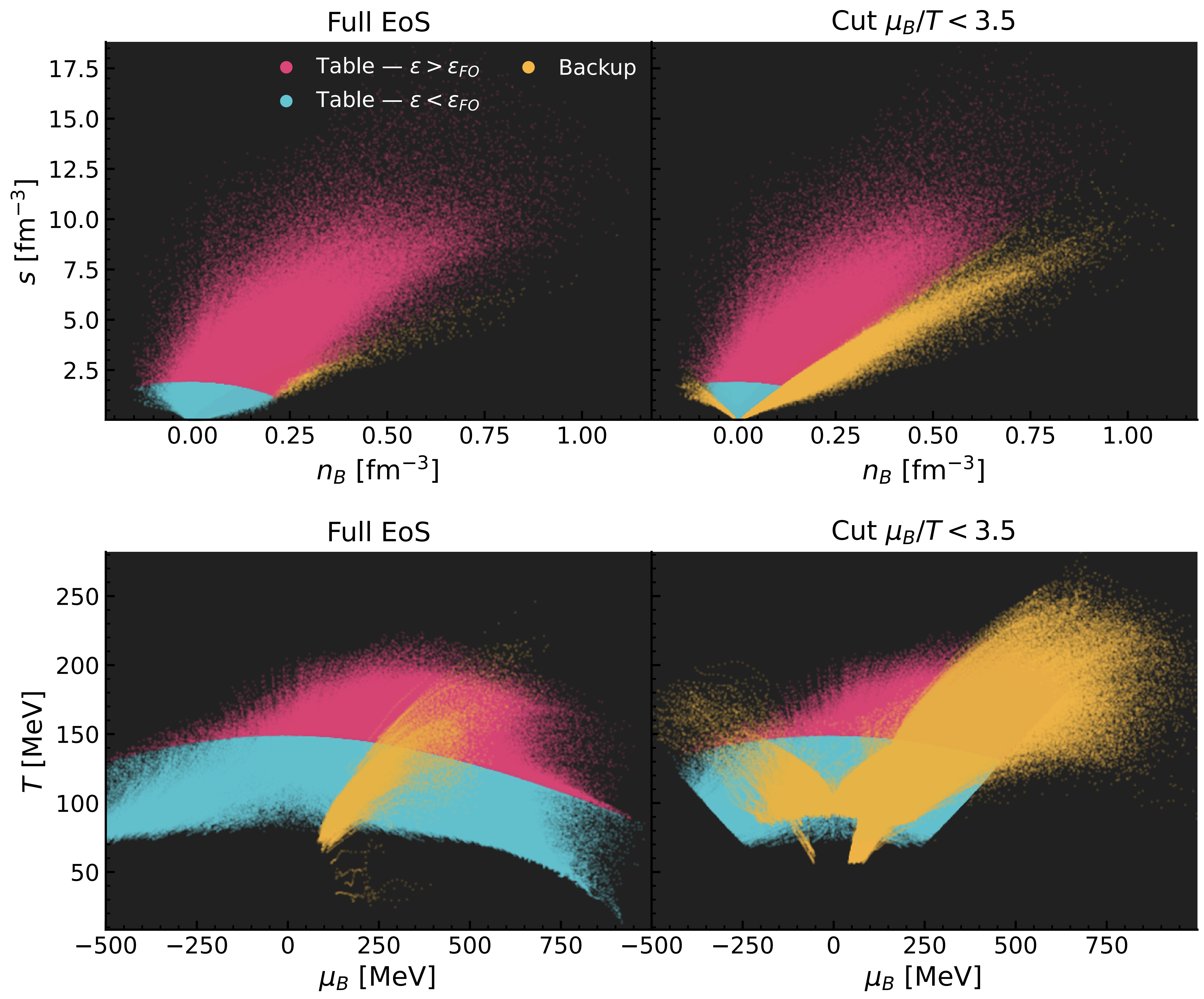
Quantifying out-of-bound fluid cell contamination

- **CCAKE** simulation for a single event:

Holography+QvdW-HRG with $\mu_B^{CP} = 1000$ MeV
vs
cut off at $\mu_B/T < 3.5$

- Even for the full EoS there are still of out-of-bound fluid cells that are concentrated in the low s, large n_B regime.
- Out-of-bound → limitations of the QvdW-HRG EoS in terms of its range in n_B.
- Fluid cells that switch to the backup EoS continue and complete their hydrodynamic evolution using the backup table.

This can affect observables



[MUSES] "Studying the QCD Matter produced in Heavy-Ion Collisions using the MUSES Calculation Engine"
Soon to appear!

Quantifying out-of-bound fluid cell contamination

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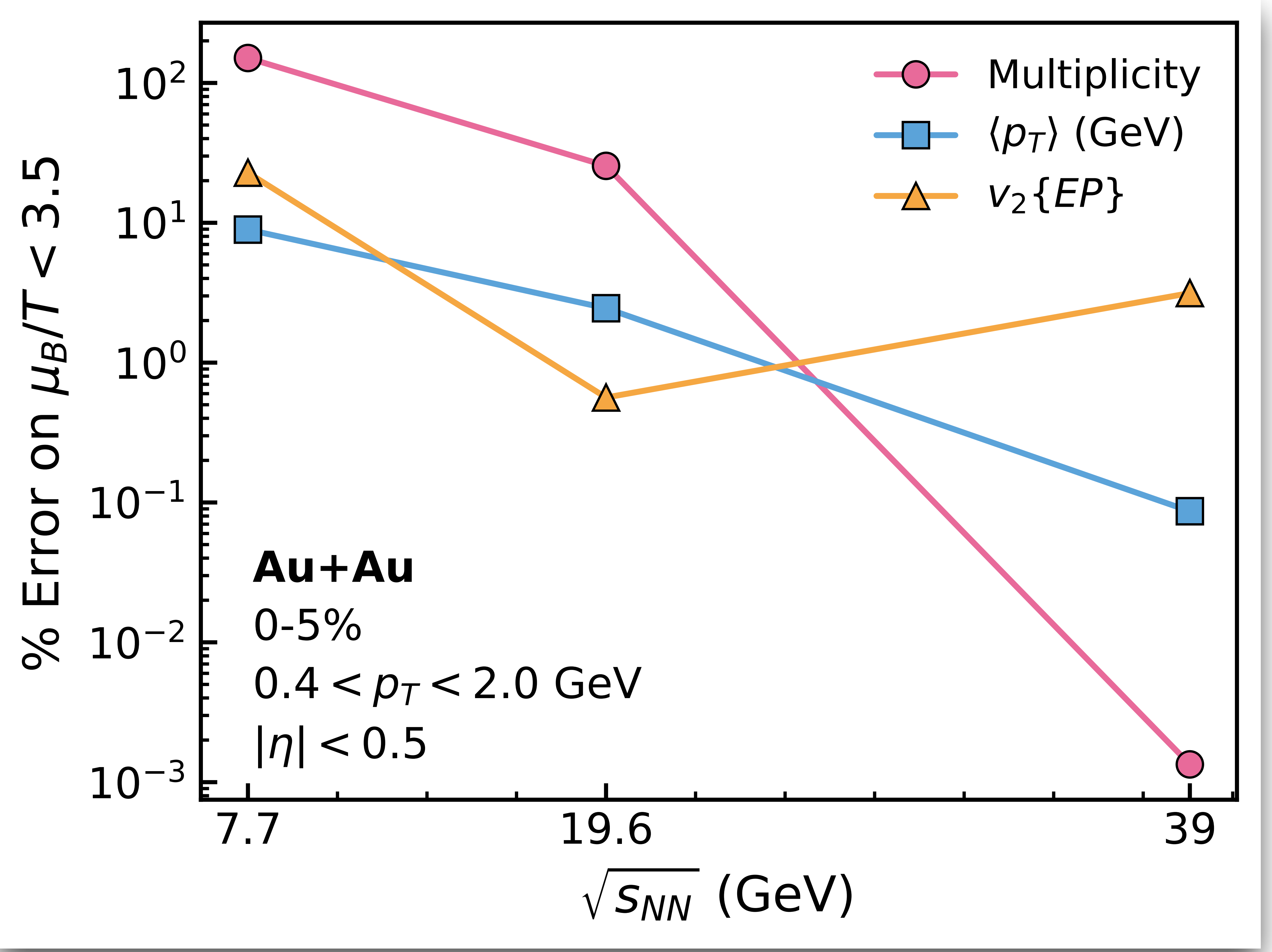


[MUSES] "Studying the QCD Matter produced in Heavy-Ion Collisions using the MUSES Calculation Engine"
Soon to appear!

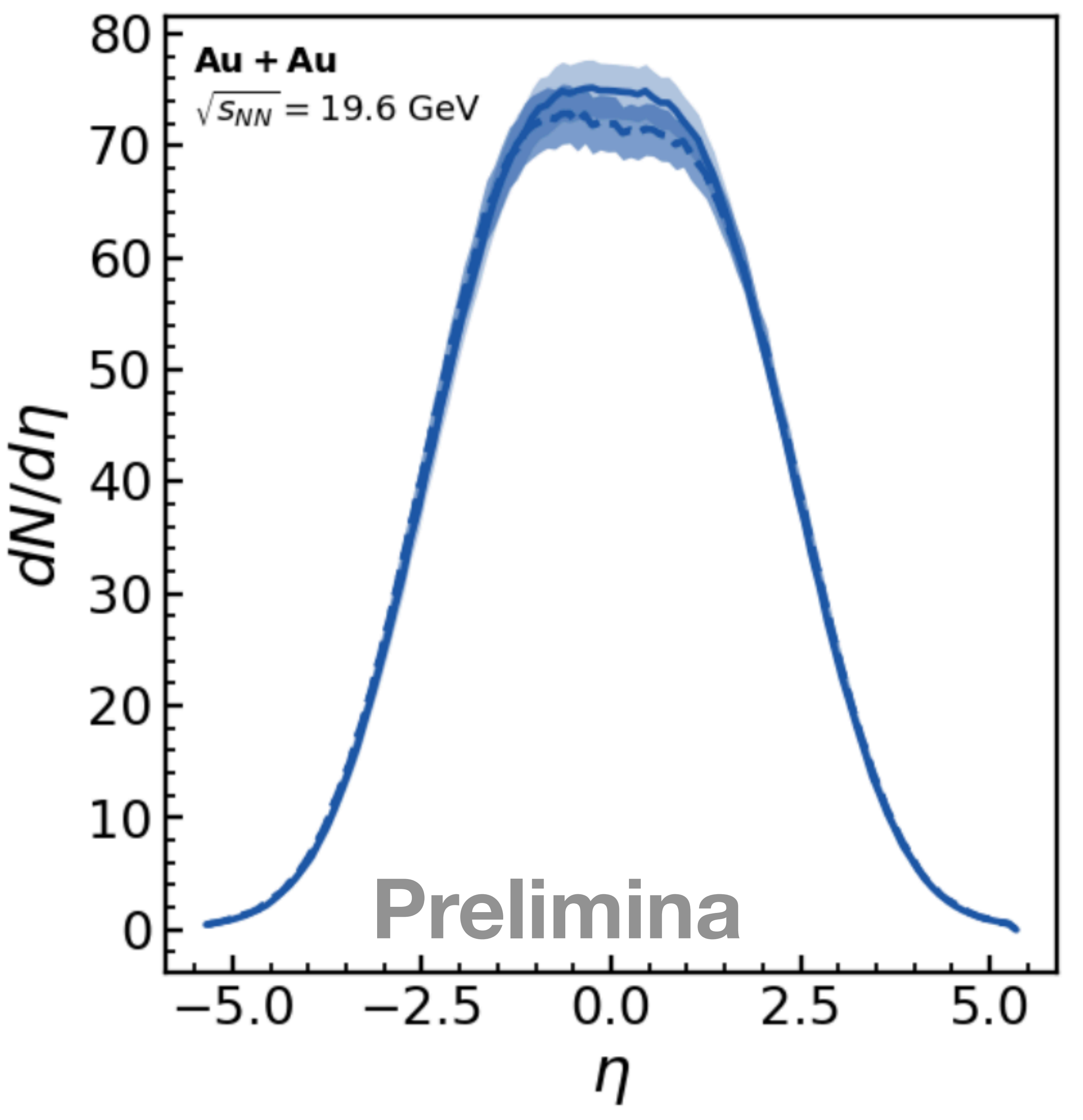
Total multiplicity, average transverse momentum and elliptical flow using STAR's rapidity cuts of $|\eta| < 0.5$

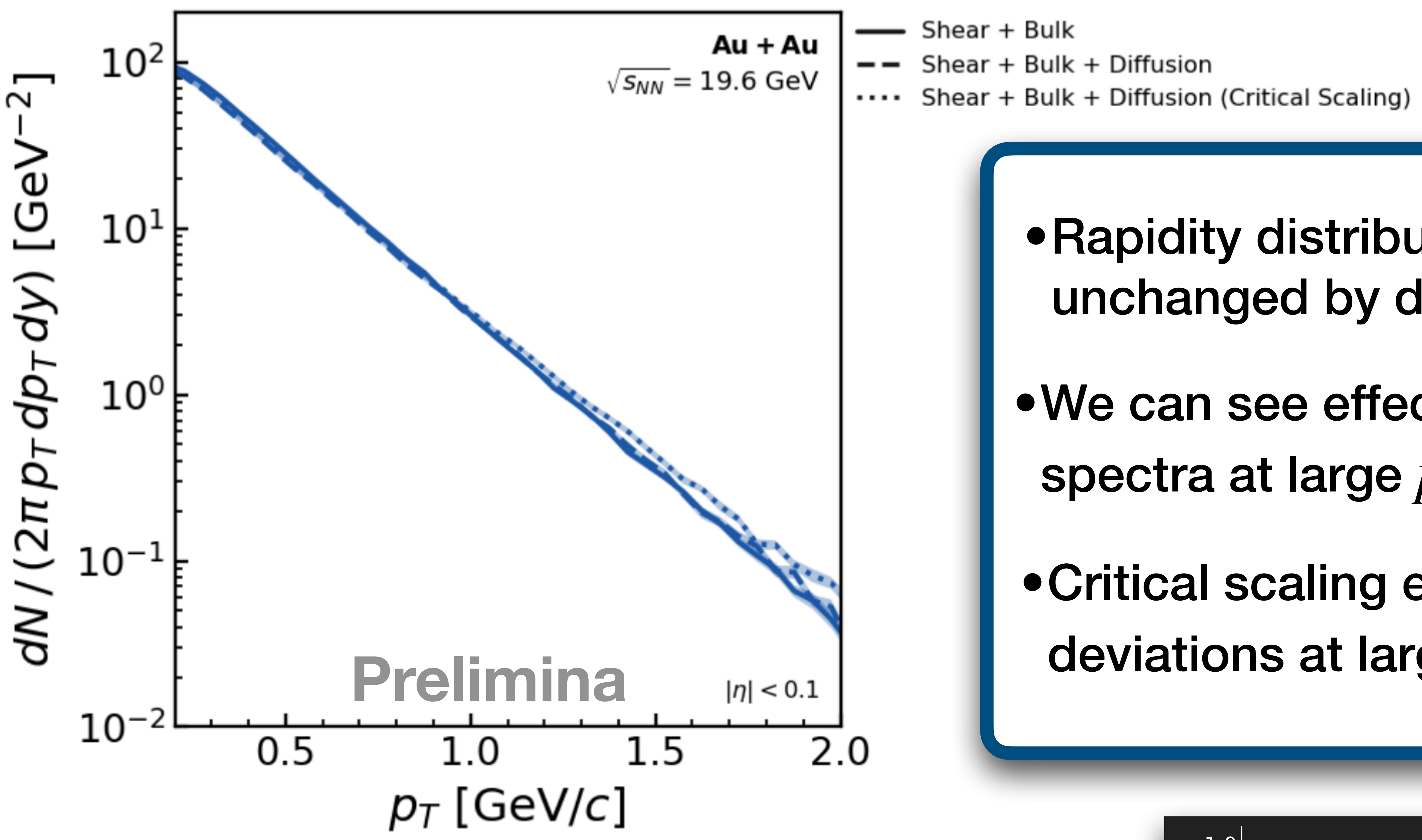
- Even for the full EoS there are still of out-of-bound fluid cells that are concentrated in the low s, large n_B regime.
- Out-of-bound → limitations of the QvdW-HRG EoS in terms of its range in n_B.
- Fluid cells that switch to the backup EoS:

complete their hydrodynamic evolution using the backup table!

Critical scaling for bulk and diffusion

Au + Au collisions with BSQ conservation

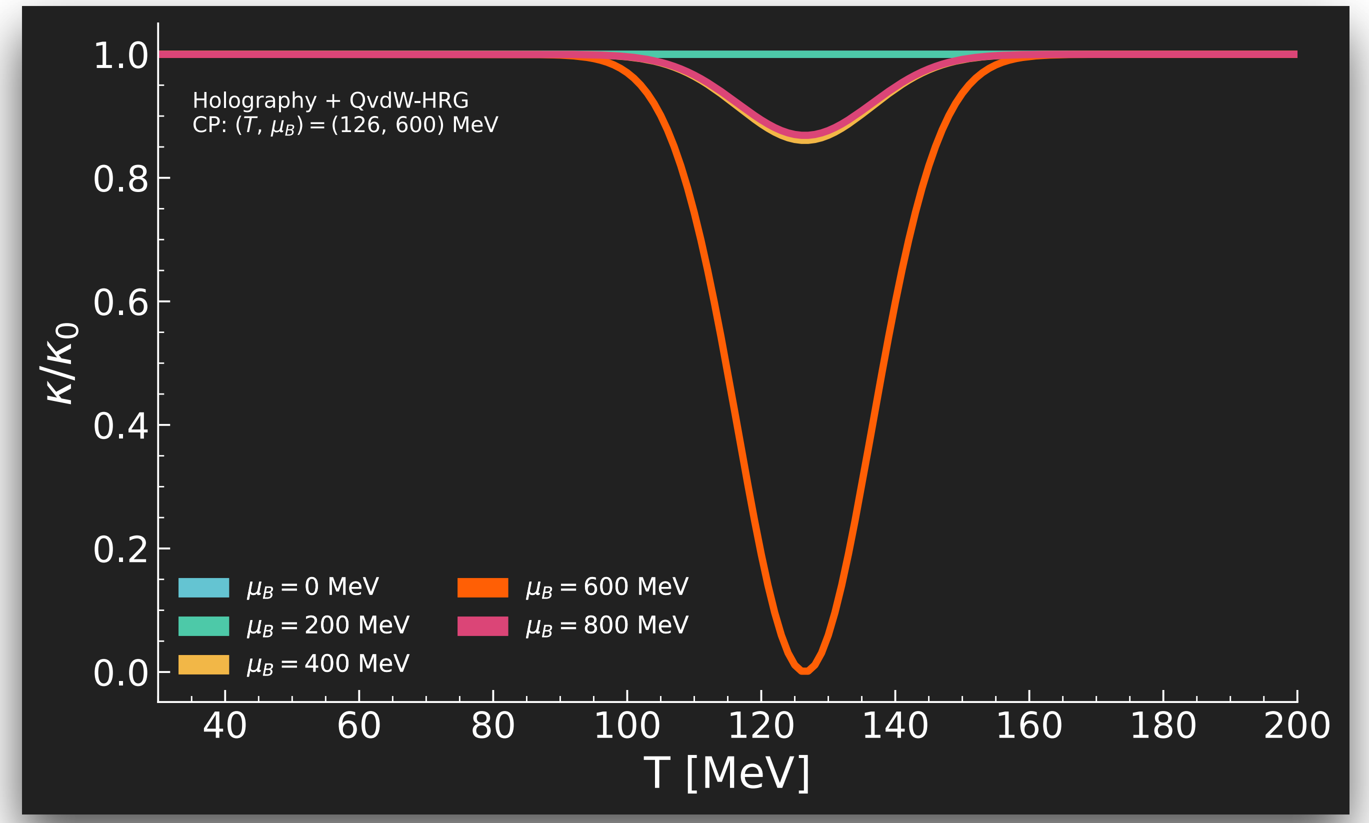




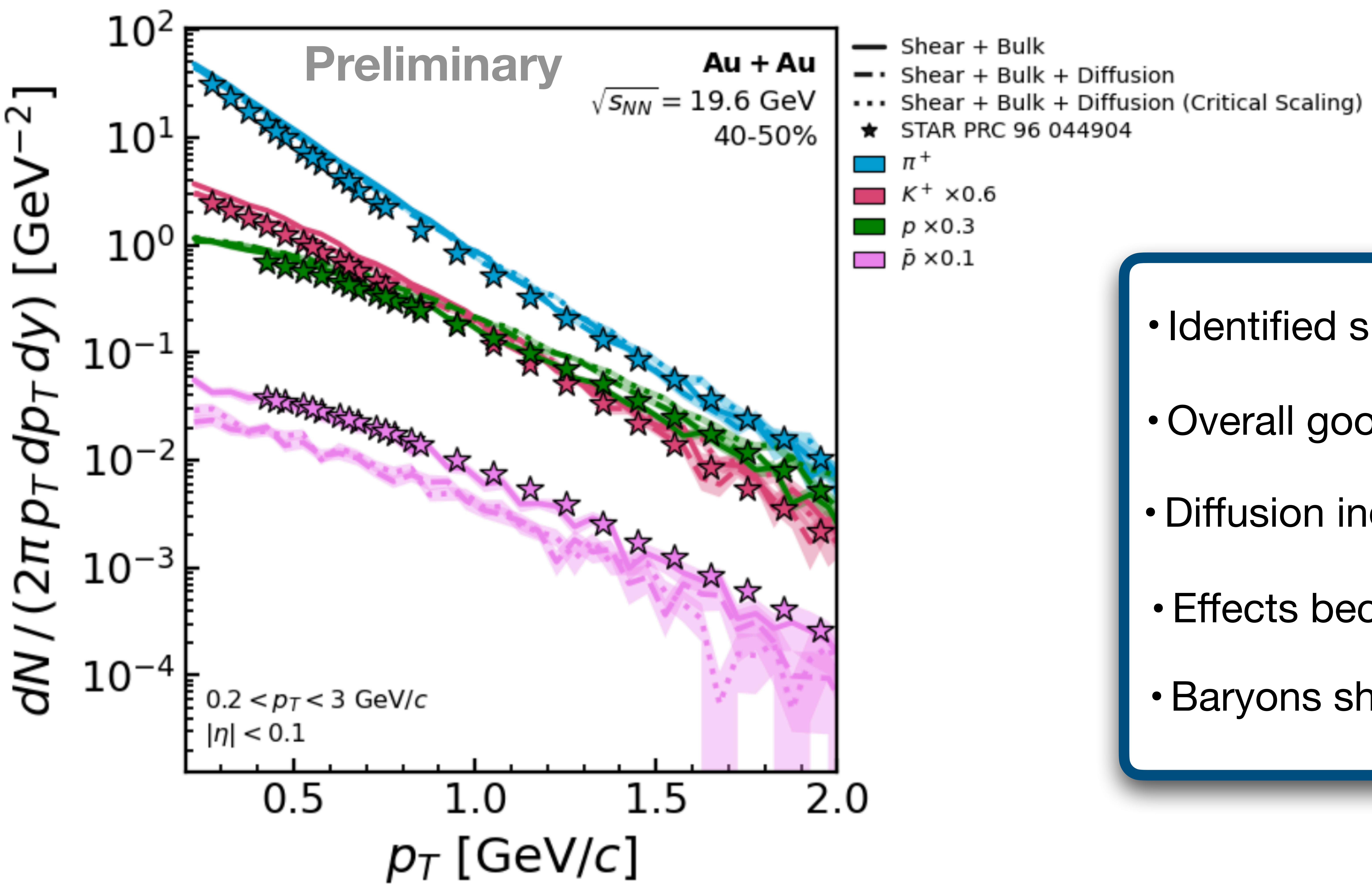
- Rapidity distribution largely unchanged by diffusion
- We can see effects of diffusion in spectra at large p_T
- Critical scaling enhances deviations at large p_T

Diffusion has a small impact on bulk observables but becomes visible in differential spectra!

Remember the critical scaling for diffusion!



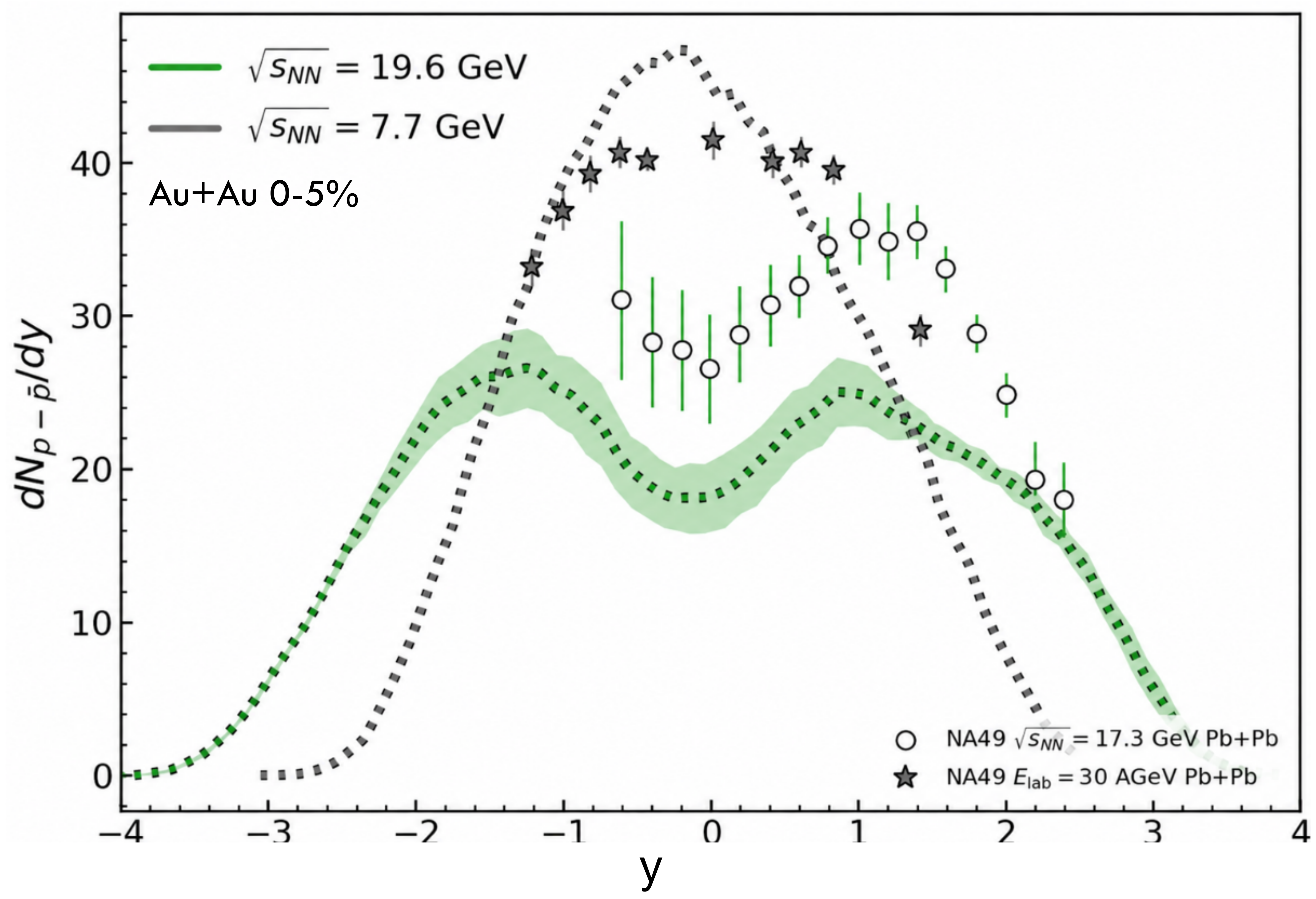
Au + Au collisions with BSQ conservation



- Identified spectra: π^+, K^+, p, $\bar{p}$
- Overall good agreement with STAR measurements!
- Diffusion induces species-dependent modifications
- Effects become visible at intermediate and high p_T
- Baryons show slightly stronger sensitivity than mesons

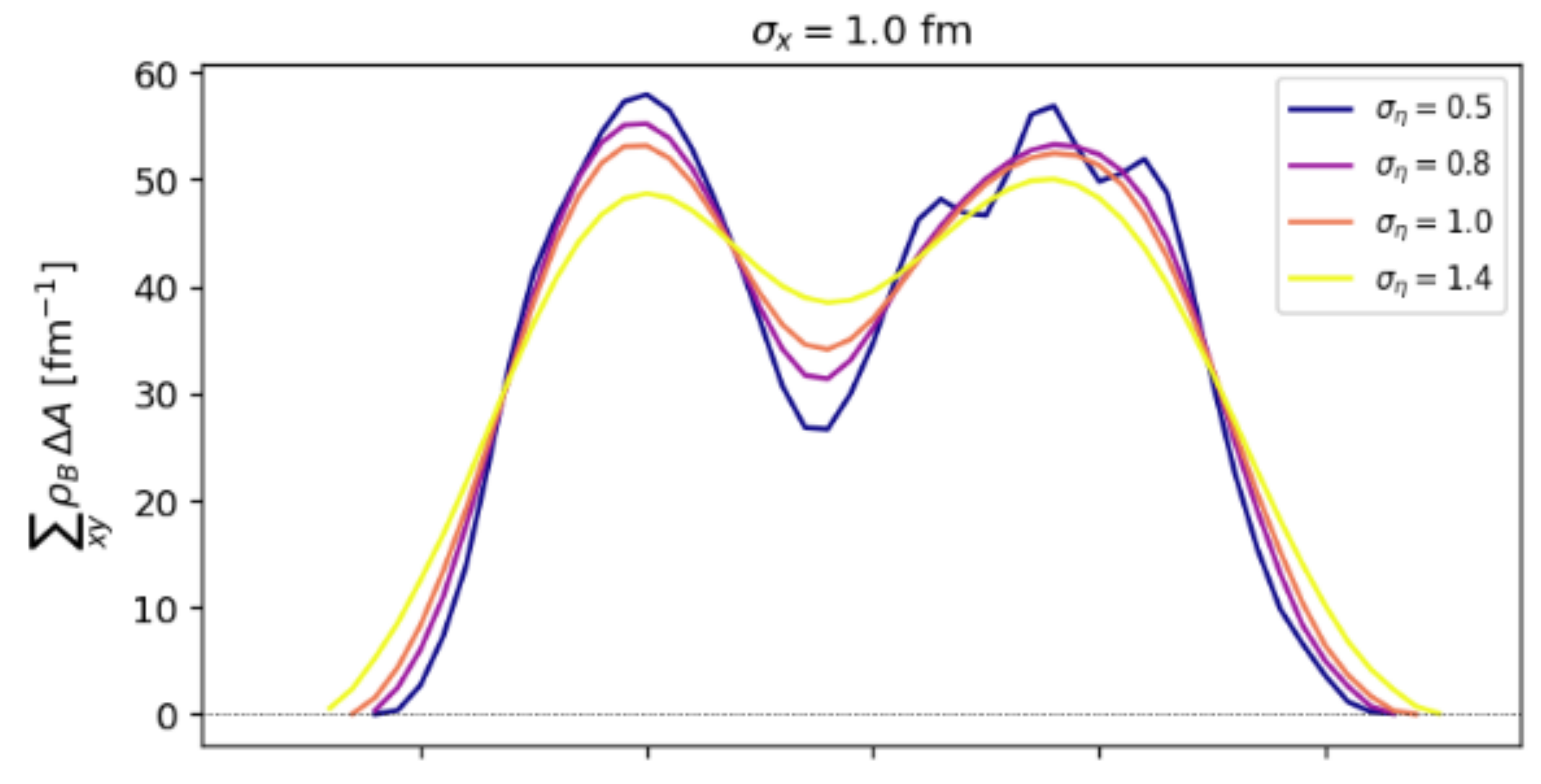
These results are obtained with a minimal, out-of-the-box setup!

Au + Au collisions with BSQ conservation



Plot with BSQ diffusion on!

This affects the baryon stopping



Same IC as a function of the longitudinal smearing lenght

We are working on an analysis of how these parameter of the IC affect the distribution!

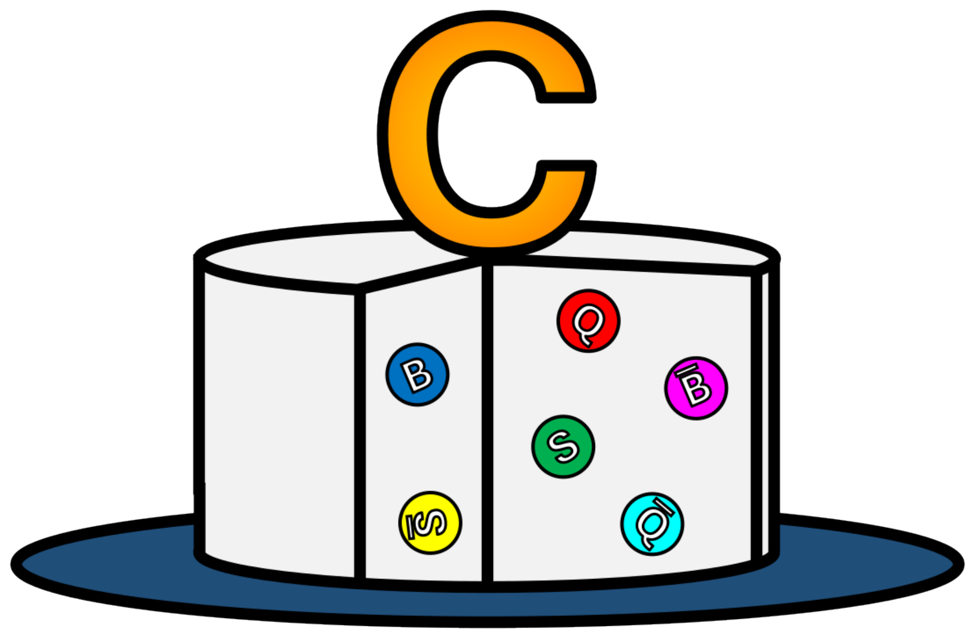
Summary

- Nuclear Confectionery Framework: 3+1D Relativistic viscous hydrodynamics with (B, Q, S) diffusion in hydrodynamic + sampling framework!
- Even at intermediate energies full rapidity coverage requires an EoS with a very large range of μ_B!
- We observe potential limitations of lattice-QCD-based EoS models in dynamical frameworks at $\sqrt{s_{NN}} \lesssim 19.6$ GeV and below, with possible implications for QCD critical point searches.
- No significant effects of BSQ diffusion, but more statistics is necessary
- Interplay between rapidity, beam energy and critical point position

More statistics

BSQ + critical point

Importance of forward/backward rapidity



CCAKE

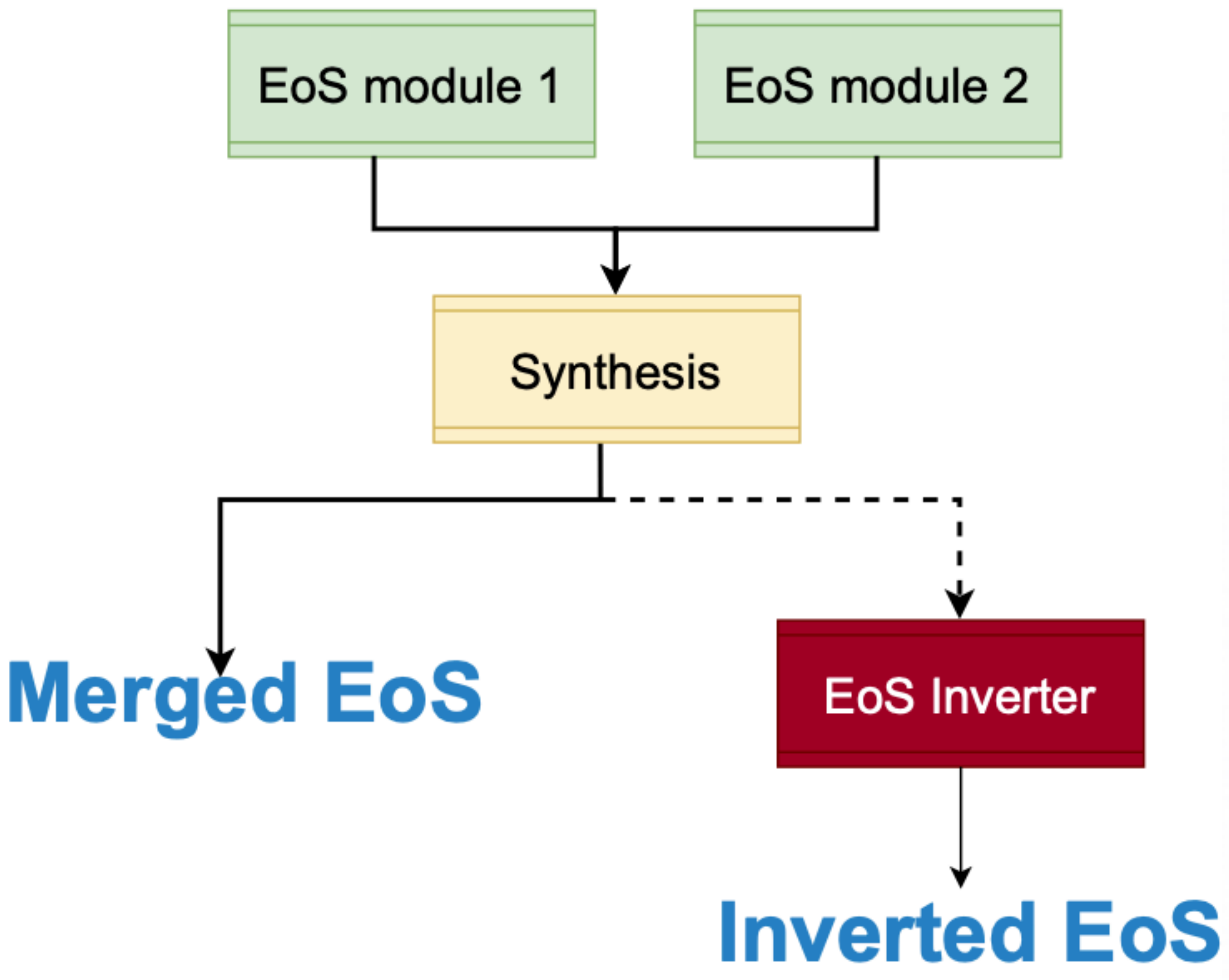
Pala, Kaur Virk, Almaalol, Danhoni, Yao, Long, Serenone, Salinas San Martin, Yared, Plumberg, Gardim, JNH, "NuclearConfectionery: Multi-stage simulation framework for modeling relativistic heavy-ion collisions," *arXiv:2511.2285*

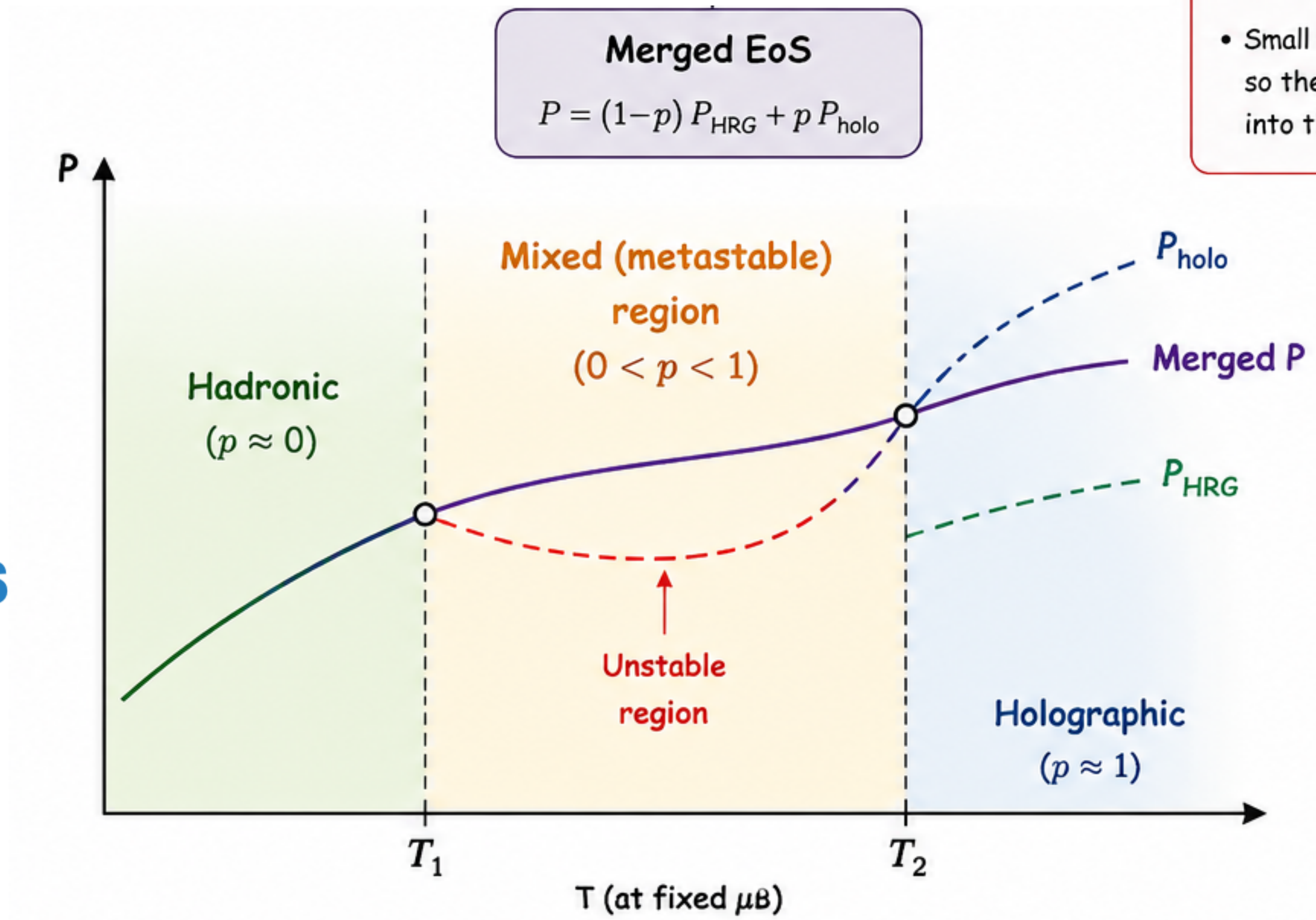
github.com/the-nuclear-confectionery/confectionery_chain

Backup

Merging Procedure

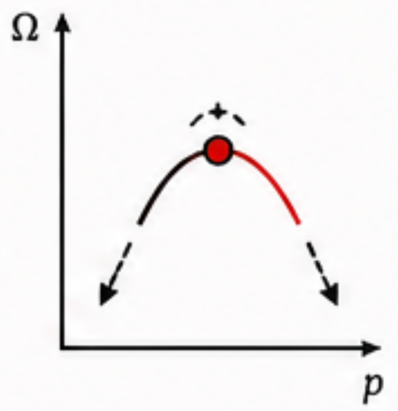
EoS generators





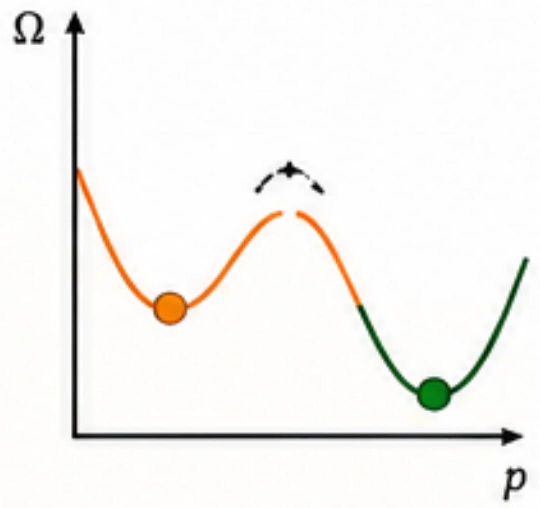
Unstable phase

- Occurs in the mixed region where $\partial P/\partial p|_{T,\mu_B} < 0$.
- Small fluctuations lower the free energy, so the system spontaneously separates into the two stable phases.



Metastable phase

- Occurs in the mixed region where $\partial P/\partial p|_{T,\mu_B} > 0$.
- The phase is locally stable (small fluctuations cost free energy) but not the global minimum.
- The system can stay in this state for a long time (nucleation barrier) before transitioning.

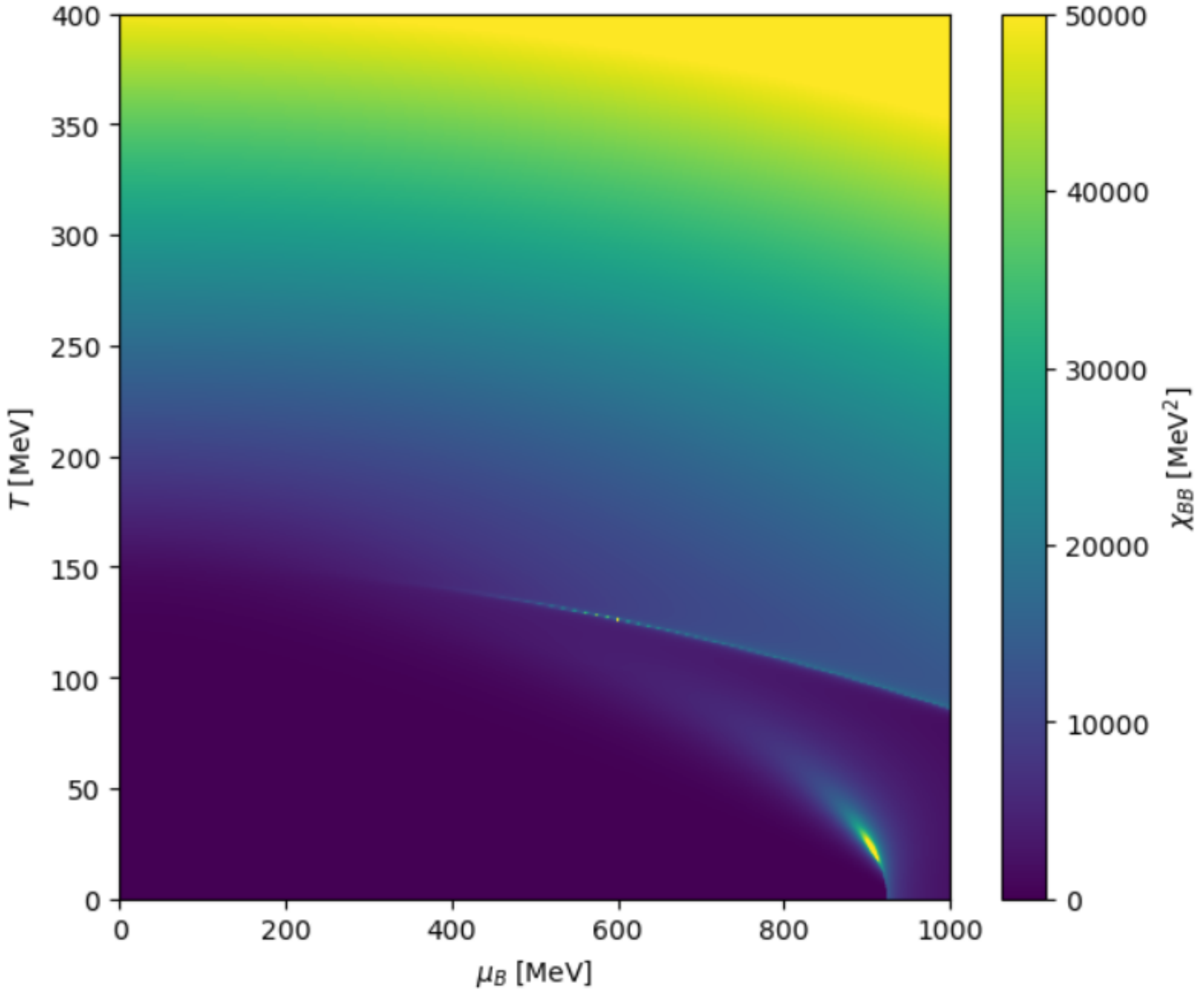


Backup EoS

TABLE → TANH-CONFORMAL → CONFORMAL → CONFORMAL DIAGONAL

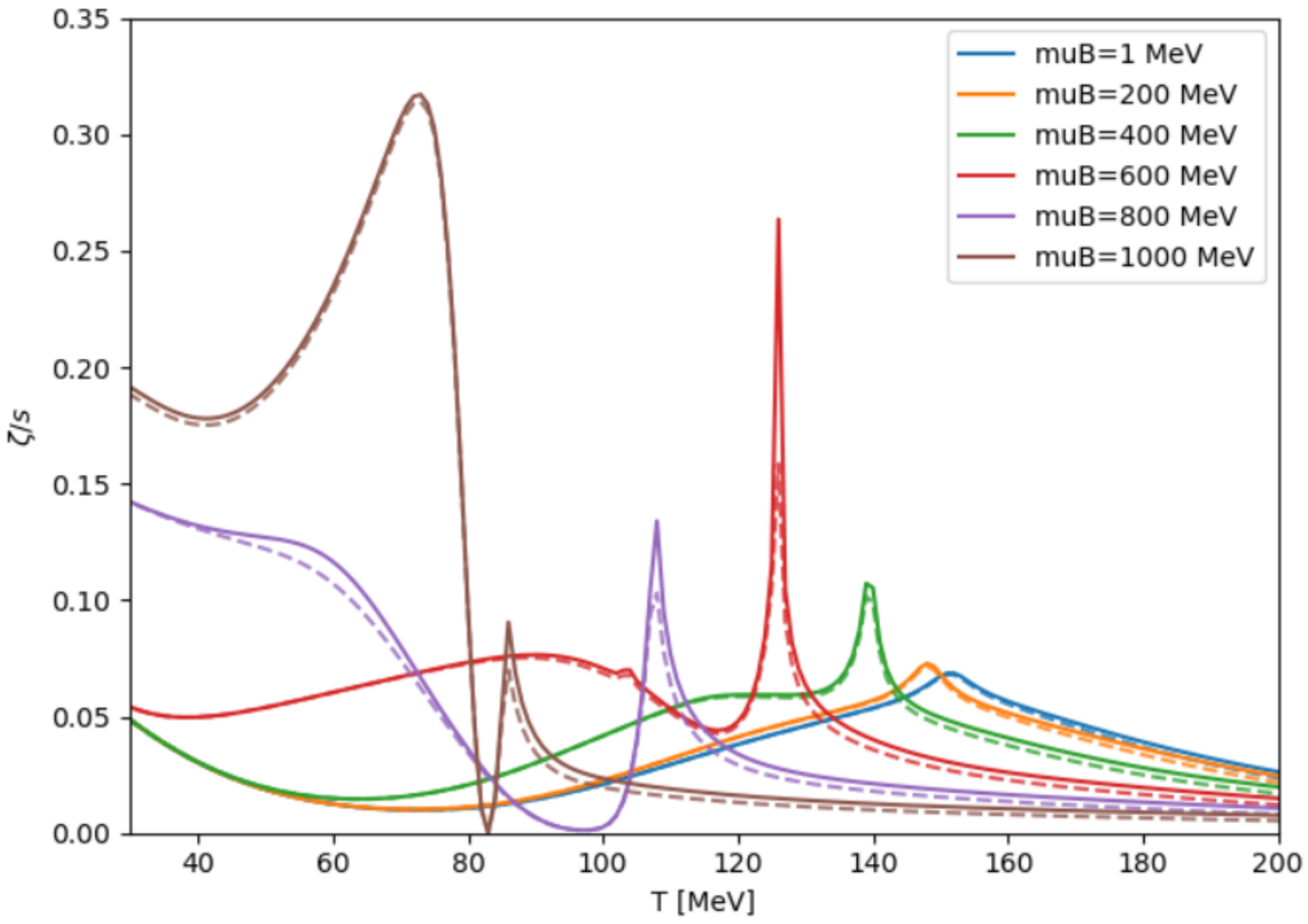
Critical Scaling for Bulk

Critical Point at $T = 128$ MeV, $\mu_B = 600$ MeV



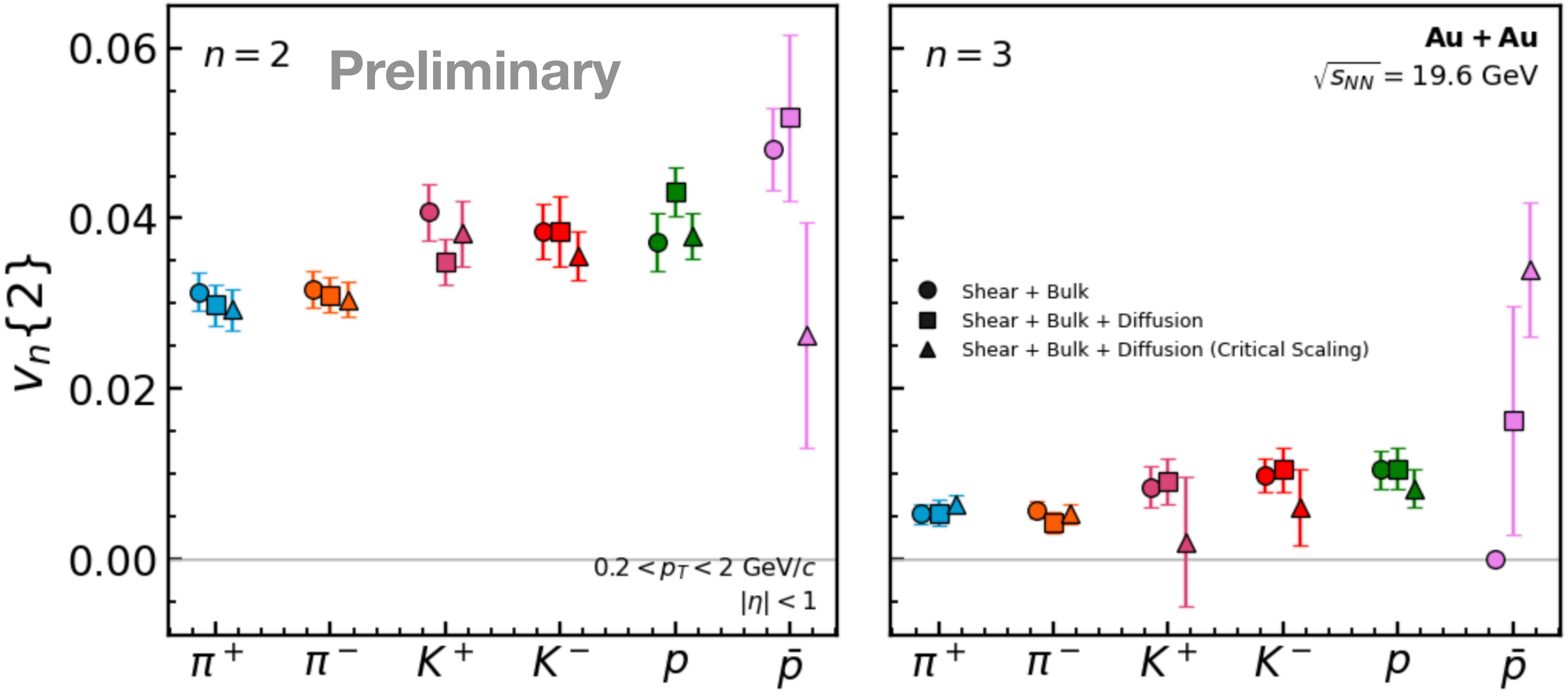
Dashed lines are regular ζ/s with $c_s^2 \to 0$ at CP of the EoS

Solid lines have critical scaling



Collective flow

Identified particle flow $v_n\{2\}$ for $n = 2,3$ in Au+Au collisions at $\sqrt{s_{NN}} = 19.6$ GeV



To appear soon in the MUSES collaboration HIC paper!

- Clear species dependence: baryons vs mesons
- Diffusion induces moderate modifications across particle species
- Effects seem to be larger for v_3 than for v_2
- Critical scaling increases the deviations in flow, especially for $\bar{p}$

EOS: Holography + HRG

BQS Sampler

Higher-order flow (v_3) seems to be more sensitive to conserved-charge dynamics!

HOLOGRAPHY+QVDW-HRG