Extracting QCD baryon susceptibilities from proton fluctuations at BES energies

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U.S. DEPARTMENT OF ENERGY | Office of Science

The QCD phase diagram

Different phase of nuclear matter

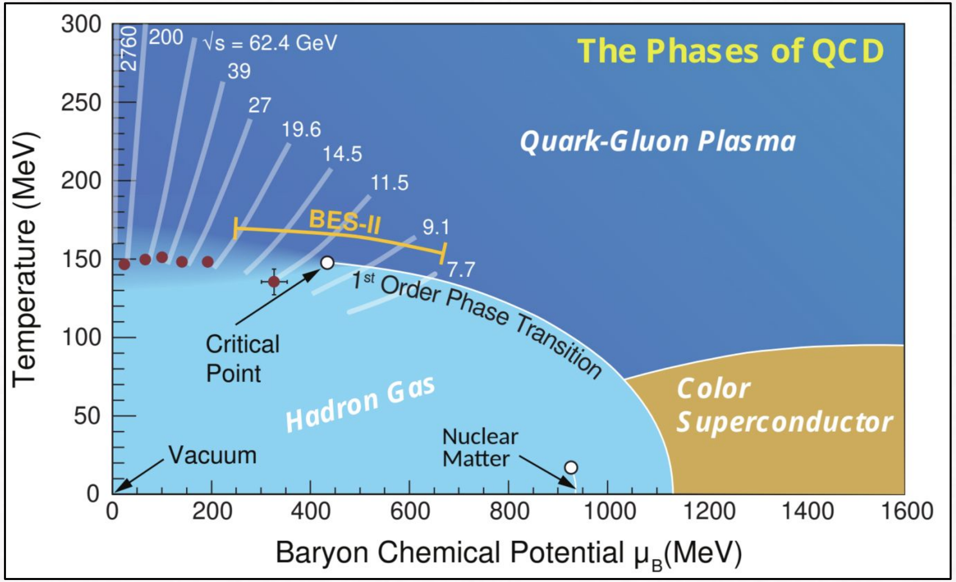
- Dilute hadron gas low T, μ_B
- Quark-gluon plasma high T, μ_B

Nature of the transition?

- Lattice: Crossover at muB = 0 GeV
  Y. Aoki et al., Nature 443, 675
- Effective QCD: 1st order?

Finite density?

- Taylor and T' expansions of P
  Hot QCD, Phys. Rev. D 95, 054504
  A. Abuali et al., Phys. Rev. D 112, 054502
- Analytic continuation from imaginary chemical potential
  Vovchenko et al, Phys. Rev. D 97, 114030
- Alternative expansion: Re(PL)



Critical points $\mu_B < 450$ MeV ruled out

The existence and location of the critical point remains largely unconstrained

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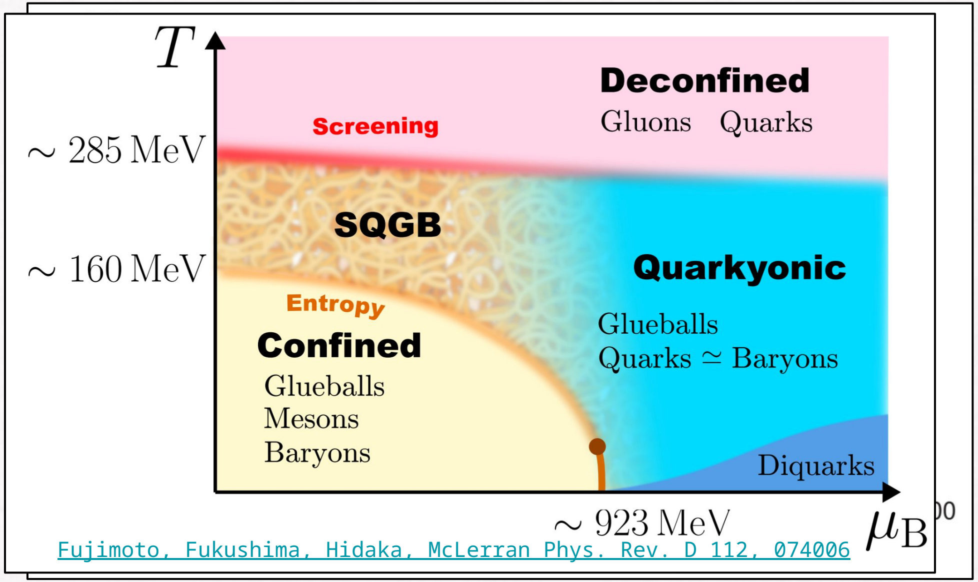

Fujimoto, Fukushima, Hidaka, McLerran Phys. Rev. D 112, 074006

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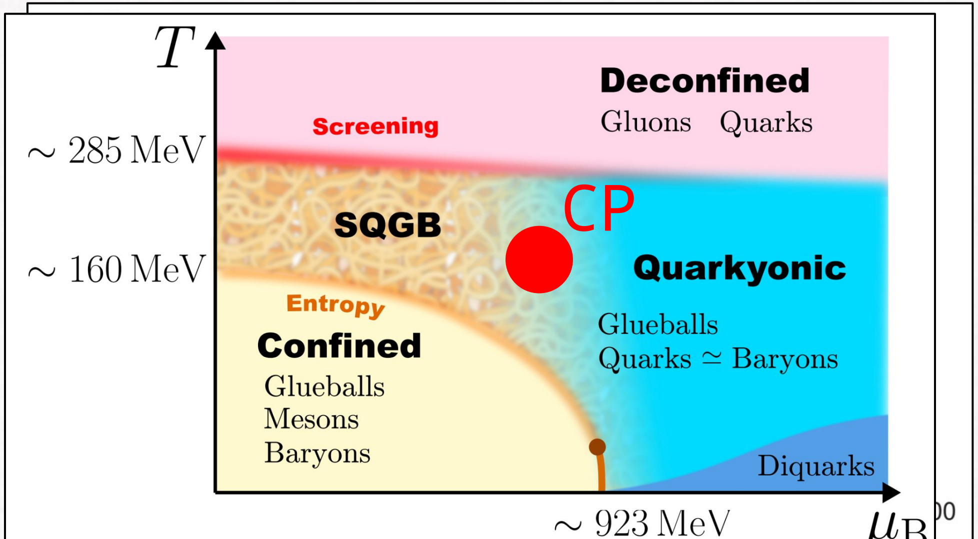

Fujimoto, Fukushima, Hidaka, McLerran Phys. Rev. D 112, 074006

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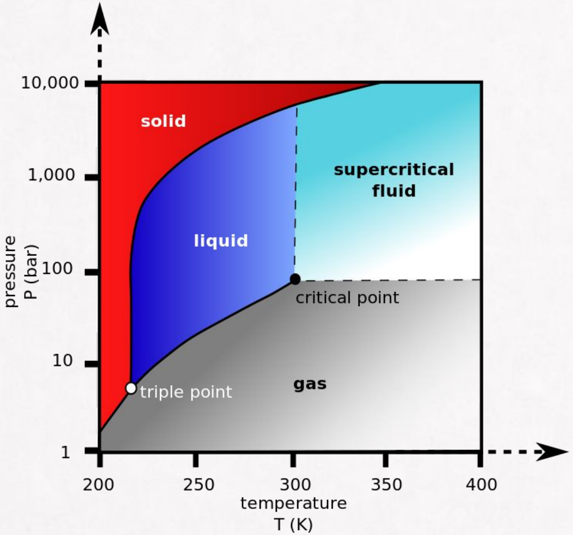
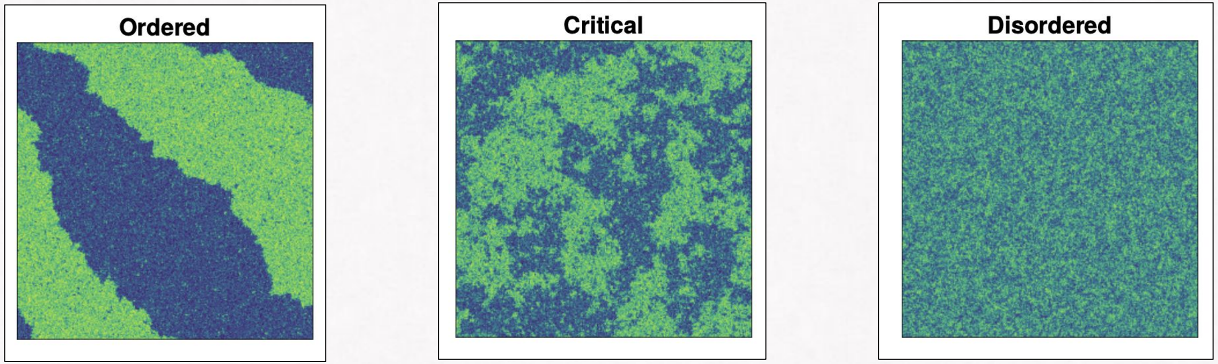
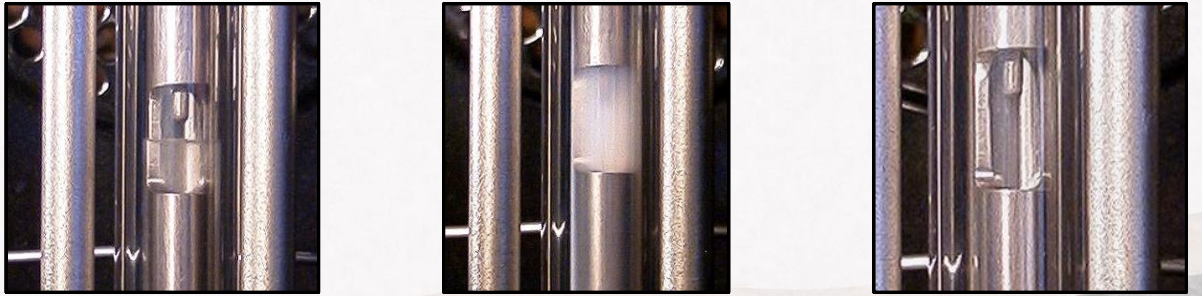
Critical opalescence

Enhancement of the correlation length of the density fluctuations







(pictures from Josh's talk on Monday!)

Conserved charges fluctuations

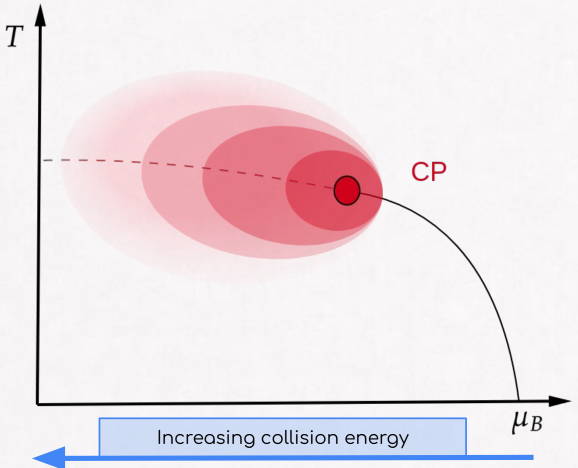
Grand partition function

$$\ln Z^{\text{gce}}(T,V,\mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\text{ce}}(T,V,N)\right]$$

Cumulant generating function

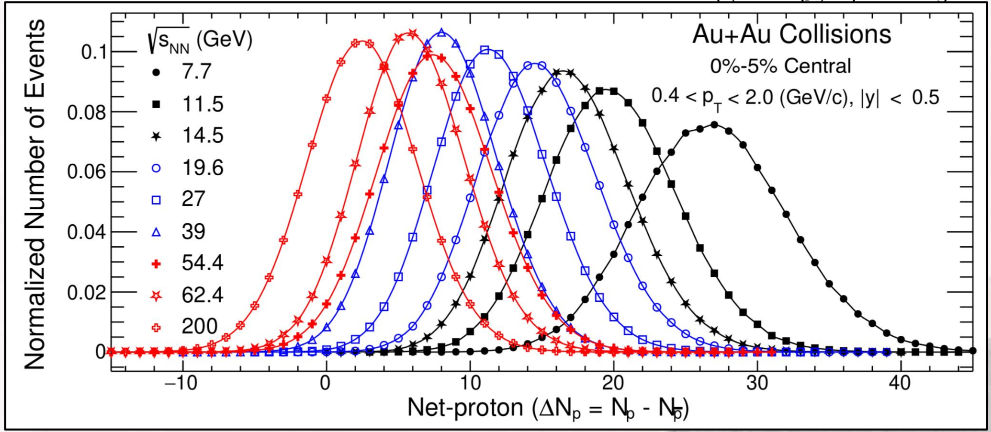
$$K_N(t) = \ln\left\langle e^{tN}\right\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

$$\kappa_n = \frac{\partial_n \ln Z^{GCE}}{\partial \hat{\mu}^n} \equiv VT^3\chi_n$$



M.A. Stephanov, Phys. Rev. Lett. 102, 032301

$$\langle(\delta N_p)^2\rangle \propto \xi^2 \quad \langle(\delta N_p)^3\rangle \propto \xi^{4.5} \quad \langle(\delta N_p)^4\rangle \propto \xi^7$$



STAR, Phys. Rev. Lett. **126**, 092301

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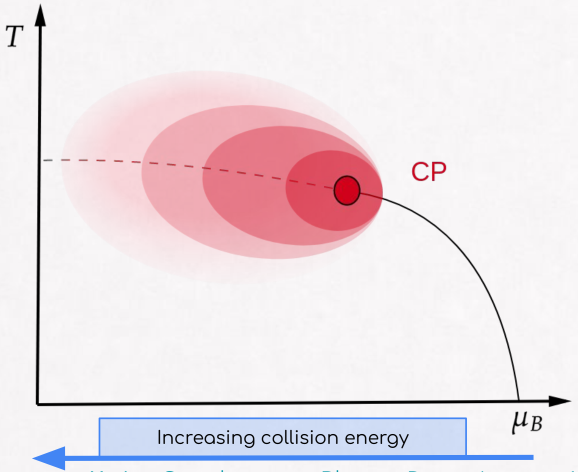

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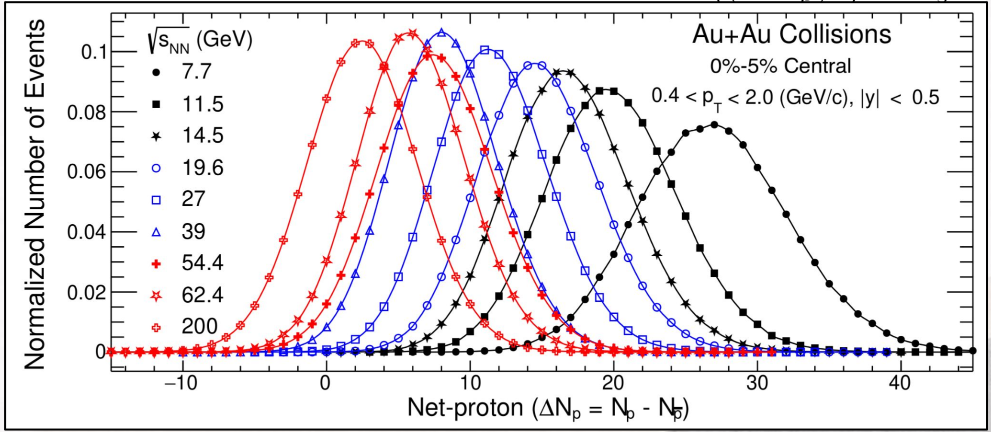

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κ_n

$VT^3\chi_n$

Collision dynamics

Evolving clouds

Du et al, Phys. Rev. C 104, 064904

Schenke et al Phys. Rev. C 82, 014903

Karpenko et al Comput. Phys.Commun. 185, 3016-3027

Global charge conservation

Finite size effects

Vovchenko,et al Phys.Lett.B 811, 135868

Thermal smearing

Coordinate / momentum space

Ohnishi et al, Phys. Rev. C 94, 044905

Detector effects

Acceptance/efficiency

Savchuk, et al Phys.Rev.C 101, 024917
Vovchenko et al, Phys.Rev.C 112 024901
Kitazawa et al Phys. Rev. C 93, 044911
Ling, Stephanov, PRC 93, 034915

Proxy observables

Baryon to proton

Kitazawa, Asakawa, Phys. Rev. C 86.024904

hadron gas dynamics

Late stage interactions

Steinheimer et al PLB 776, 32

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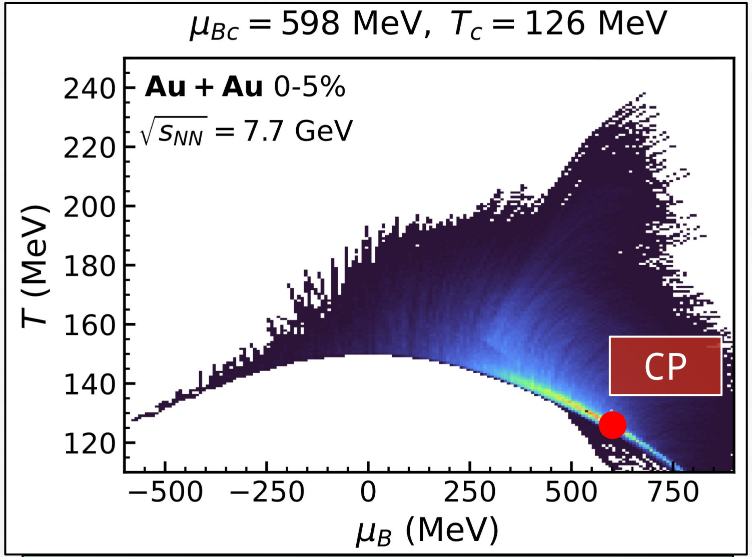
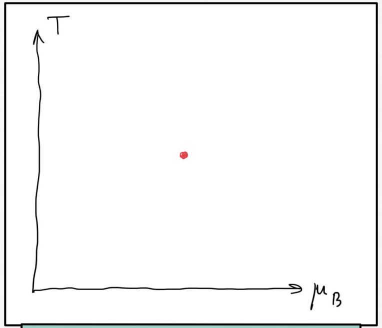
Pipeline tracker

Inhomogeneous system: $\sqrt{s_{\mathrm{NN}}} \neq (T_{\mathrm{fo}}, \mu_{B,\mathrm{fo}})$



See Isabella's talk on Wednesday!

- Different part of the system feel the EoS differently
- Not all part of the system goes close to the critical region
- Strong dependence on the collision energy



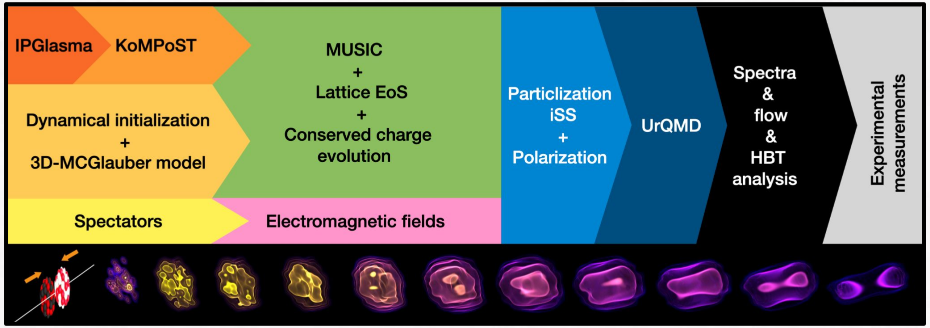
See Iurii's talk on Tuesday!

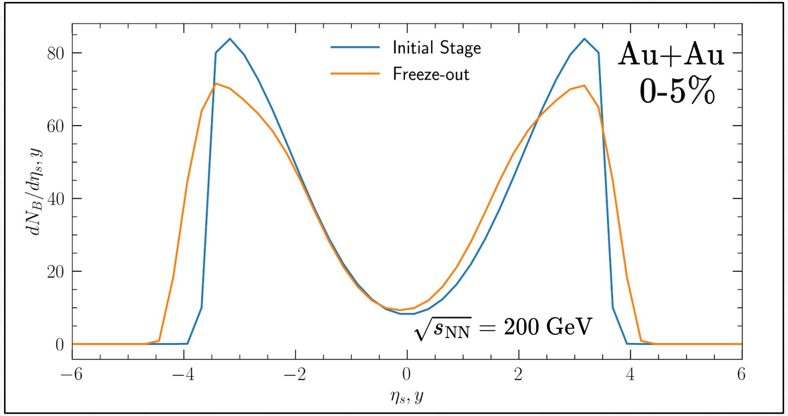
The distribution of temperature and chemical potential on the freeze out surface is essential to interpret experimental fluctuations observables.

Smooth collision event: iEBE-MUSIC

Collision event

Pipeline tracker





Transverse: Glauber model

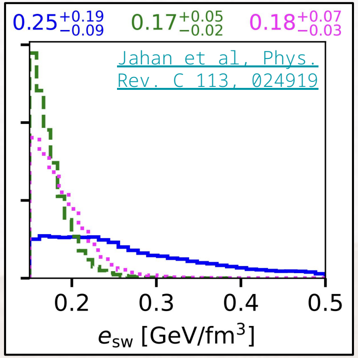
$$\epsilon(x, y, \eta_s) \propto e(\eta_s) T_{\text{Proj}}(x, y) T_{\text{Targ}}(x, y)$$

$$n_B(x, y, \eta_s) \propto f^{n_B}_{\text{Proj}}(\eta_s) T_{\text{Proj}}(x, y) + f^{n_B}_{\text{Targ}}(\eta_s) T_{\text{Targ}}(x, y)$$

Longitudinal: Density enveloppes

$$e(\eta_s, y_{CM}) \propto \exp\left[-\frac{(|\eta_s - y_{\text{CM}}| - \eta_0)^2}{2\sigma_\eta^2}\theta(|\eta_s - y_{\text{CM}}| - \eta_0)\right]$$

$$f^{n_B}_{\text{Proj}} \propto \theta(\eta_s - \eta_{B,0}) \exp[-\frac{(\eta_s - \eta_{B,0})^2}{2\sigma^2_{B,\text{out}}}] + \theta(\eta_s + \eta_{B,0}) \exp[-\frac{(\eta_s + \eta_{B,0})^2}{2\sigma^2_{B,\text{in}}}]$$



200 GeV 19.6 GeV 7.7 GeV

$$e_{\text{sw}} = 0.18\ \text{GeV/fm}^3$$

$$e_{\text{sw}} = 0.25\ \text{GeV/fm}^3$$

See Chun's talk on Tuesday!

Each hydro cell is in local equilibrium (GCE)

$$\kappa_n^{B,\mathrm{GCE}}(x) = \delta V(x) T(x)^3 \chi_n^B(x)$$

$\delta V = d\Sigma_\mu u^\mu \; (> 0) \qquad x_i \equiv (\tau_i, x_i, y_i, \eta_i)$

Cells are statistically independent

$$\kappa_n^B = \sum_{i \in \mathrm{cells}} \delta\kappa_n^B$$

Equation of state

$P(T, \mu, V)$

derivatives

$n_B \quad \chi_2 \quad \chi_3 \quad \chi_4$

cumulant on the surface

$\kappa_1^B \quad \kappa_2^B \quad \kappa_3^B \quad \kappa_4^B$

Densities to particles

Cooper-Frye formula

$$E_i \frac{dN_i}{d^3p} = \frac{g_i}{(2\pi)^3} \int_\Sigma d\Sigma_\mu p_i^\mu f_i(p_i^\nu u_\nu, T, \mu_i)$$

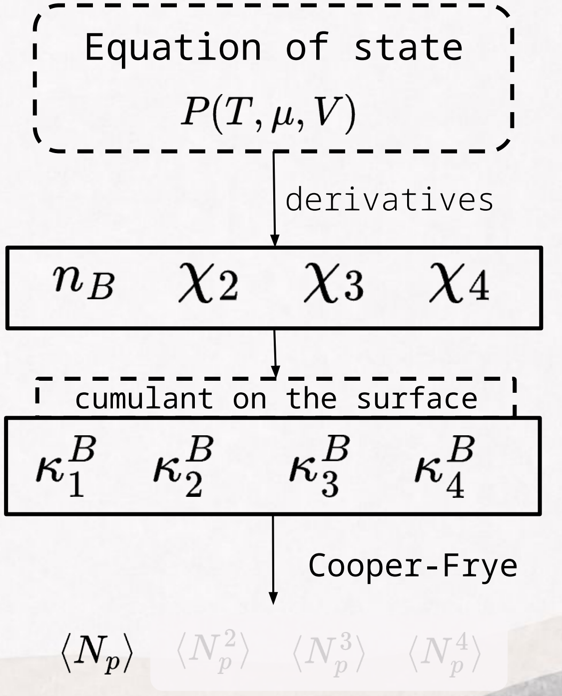
one particle distributions = ensemble average phase space densities

- Carries no information about the fluctuations

$$E_i \frac{dN_i}{d^3p} \equiv \langle E_i \frac{dN_i}{d^3p} \rangle$$

- Sampling on the surface, independent CF, reproduction of the mean but Poisson fluctuations

Koch, Vovchenko, Acta Phys.Polon.B 52



Collision event → Baryon number fluctuations → Proton number fluctuations

Pipeline tracker

See Jamie's talk on Thursday

The maximum entropy freeze-out

The Max Ent method provides the least-biased reconstruction of particle multiplicity fluctuations consistent with the hydrodynamic correlations.

$$\hat{\Delta} G_{A_1 \dots A_k} = \hat{\Delta} \mathcal{H}_{a_1 \dots a_k} \prod_{i=1}^{k} P_{A_i}^{a_i}$$

$\hat{\Delta}$ Irreducible relative cumulants (IRC) operator

M.S. Pradeep, M.A. Stephanov Phys. Rev. Lett. 130, 162301

$\hat{\Delta}\mathcal{H}_{a_1 \dots a_k}$ IRC of the K-points correlator of hydrodynamic fields $a_i = \epsilon, n_B$

$\hat{\Delta} G_{A_1 \dots A_k}$ IRC of the K-points correlator of particles $A_i \in \text{Particles}$

$P_{A_i}^{a_i}$ Hydro matching conditions projectors

The maximum entropy freezeout

Collision event | Baryon number fluctuations | Proton number fluctuations

Pipeline tracker

General definition of the particle cumulants

$$\kappa_{\hat{A}_1 \ldots \hat{A}_k} = \left\langle \prod_{i=1}^{k} \delta N_{\hat{A}_i} \right\rangle$$

$$\hat{A}_i \equiv (x_A, p_A, q_A, s_A, g_A, m_A, \ldots)_i$$

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Cooper-Frye Integrals

$$\delta N_{\hat{A}} = \int_{\hat{A}} \delta f_A = \sum_{q_A, s_A, \ldots} \int_{x_A} \mathrm{d}\sigma_\mu(x_A) \int_{p_A} p_A^\mu \mathrm{d}^4 p_A \delta(p_A^2 - m^2) 2\theta(p_A . n) \delta f_A$$

M.S. Pradeep, M.A. Stephanov Phys. Rev. Lett. 130, 162301

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Cumulants from single particle distributions correlators

$$\kappa_{\hat{A}_1 \ldots \hat{A}_k} = \int_{\hat{A}_1 \ldots \hat{A}_k} G_{A_1 \ldots A_k}$$

$$G_{A_1 \ldots A_k} = \langle \delta f_{A_1} \ldots f_{A_k} \rangle$$

M.S. Pradeep, M.A. Stephanov Phys. Rev. Lett. 130, 162301

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IRCs from single particle distributions IRCs

$$\hat{\Delta}\kappa_{\hat{A}_1 \dots \hat{A}_k} = \int_{\hat{A}_1 \dots \hat{A}_k} \hat{\Delta} G_{A_1 \dots A_k}$$

M.S. Pradeep, M.A. Stephanov Phys. Rev. Lett. 130, 162301

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M.S. Pradeep, M.A. Stephanov Phys. Rev. Lett. 130, 162301

Hydro matching condition

$$\hat{\Delta} G_{A_1\ldots A_k} = \hat{\Delta}\mathcal{H}_{a_1\ldots a_k} \prod_{i=1}^k P_{A_i}^{a_i}$$

↓

Particle IRCs from hydrodynamics

$$\hat{\Delta}\kappa_{\hat{A}_1\ldots\hat{A}_k} = \hat{\Delta}\mathcal{H}_{a_1\ldots a_k} \prod_{i=1}^k P_{\hat{A}_i}^{a_i}$$

Assumptions

- B^+ B^- joint cumulants $B^\pm = \left\{\hat{A}_i \;/\; q_{A_i} = \pm 1\right\}_{i\in\text{Particles}}$
- Boltzmann approximation
  Particle IRCs = factorial cumulants
- Net-baryon density field only $a_i = n_B \,\forall\, i$

Maximum Entropy B^+ B^- factorial cumulants

$$\tilde{\kappa}^{+-}_{nm}(x) = \bar{\tilde{\kappa}}^{+-}_{nm}(x) + \hat{\Delta}\mathcal{H}_{n+m}(x)\left(P_+\right)^n\left(P_-\right)^m$$

The ideal-HRG factorial cumulants

$$\bar{\tilde{\kappa}}^{+-}_{nm}(x) = \delta_{n,1}\delta_{m,0}\langle N_{B^+}\rangle(x) + \delta_{n,0}\delta_{m,1}\langle N_{B^-}\rangle(x)$$

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The hydrodynamics correlators IRCs

$$\hat{\Delta}\mathcal{H}_{n+m}(x) = \delta V(x) T(x)^3 \bar{\chi}^B_{n+m} F_{nm}(\chi^B_2, \chi^B_3, \chi^B_4)$$

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<u>The hydrodynamic matching condition projectors</u>

$$P_\pm(x) = \pm \frac{\langle \delta N^\pm \rangle^{\text{HRG}}}{\langle \delta N^+ \rangle^{\text{HRG}} + \langle \delta N^- \rangle^{\text{HRG}}}$$

Collision event → Baryon number fluctuations → Proton number fluctuations in acceptance

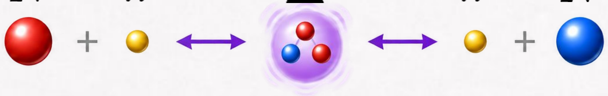
Pipeline tracker

Experiments measure charged particles

$$B \longrightarrow B^{+} \longrightarrow N_p$$

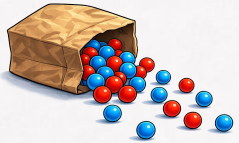
Isospin randomization

$N \quad \pi \quad \Delta \quad \pi \quad N$



At equilibrium in the hadronic phase

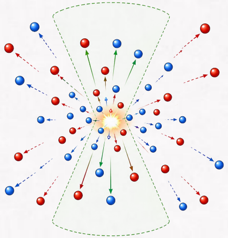
$$P(N_p|N_B^{\pm}) = \mathrm{Binomial}(N_N, q_{\pm})$$



$$q_{\pm}(x) = \frac{\bar{n}_p^{\mathrm{FD}}}{\bar{n}_{B^{\pm}}}$$

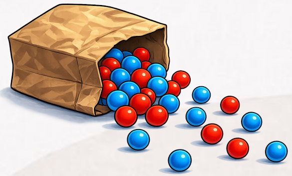
M. Kitazawa, M. Asakawa Phys. Rev. C 86, 024904

Experiments measure particles in a finite acceptance



$$N_p \longrightarrow N_p^{\mathrm{Acc}}$$

$$p(x_i) = \frac{\int_{\mathrm{acc}} \frac{d^3p}{E_p} d\Sigma_\mu(x_i) p^\mu(x_i) f_p[u^\nu p_\nu, T(x_i), \mu_p(x_i)]}{\int \frac{d^3p}{E_p} d\Sigma_\mu(x_i) p^\mu(x_i) f_p[u^\nu p_\nu, T(x_i), \mu_p(x_i)]}$$



$$P(N_{p,\mathrm{Acc}}|N_p) = \mathrm{Binomial}(N_p, p)$$

Bzdak, Koch, Phys. Rev. C 86, 044904

Collision event → Baryon number fluctuations → Proton number fluctuations in acceptance

Pipeline tracker

Experiments measure charged particles

$$B \longrightarrow B^{+} \longrightarrow N_p$$

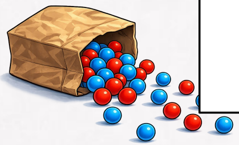
Iso

N π

 +

At equilib

$P(N_p|$



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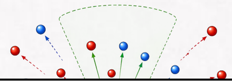


$\longrightarrow N_p^{\mathrm{Acc}}$

$T(x_i),\mu_p(x_i)]$

$T(x_i),\mu_p(x_i)]$

$\text{ial}(N_p, p)$

Total binomial distribution

$$\alpha_{\pm}(x) = q_{\pm}(x)p(x)$$

$$P(N_{p,\mathrm{Acc}}|N_B^{\pm}) = \mathrm{Binomial}(N_B^{\pm}, \alpha_{\pm})$$

M. Kitazawa, M. Asakawa Phys. Rev. C 86, 024904

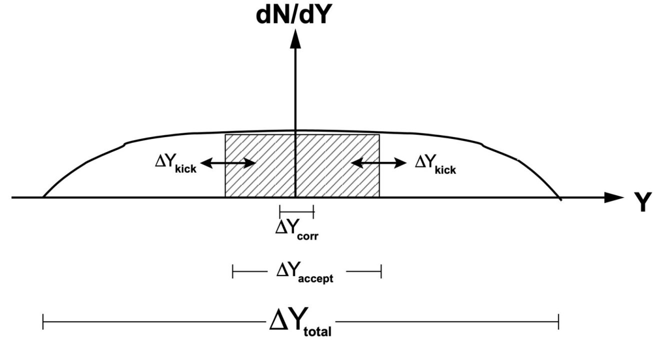
Bzdak, Koch, Phys. Rev. C 86, 044904

Pipeline tracker

In collisions events the net baryon number is exactly conserved

- ➢ In the full space: fluctuations vanishes
- ➢ Grand canonical ensemble if

$$\Delta Y_{\text{total}} \gg \Delta Y_{\text{accept}} \gg \Delta Y_{\text{phys}}$$



Koch, 0810.2520

In practice this might not be satisfied

The measured ensemble of particle cannot be seen as a closed system in contact with a particle bath

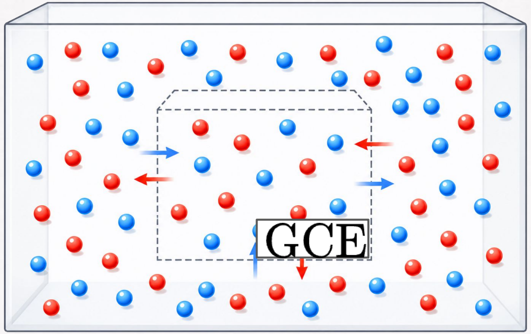
Global charge conservation modifies the measured fluctuations non-trivially.

Vovchenko et al., Phys. Lett. B 811, 135868
Phys. Rev. C 105, 014903

Collision event → Baryon number fluctuations → Proton number fluctuations in acceptance → Baryon conservation correction

Pipeline tracker

The subensemble acceptance method 3.0



The canonical ensemble is the grand-canonical ensemble conditioned on a fixed value of the total conserved charge.

$$P_{\mathrm{ce}}(X) \equiv P_{\mathrm{gce}}\left(X \,\middle|\, B_{\mathrm{tot}} = B_0\right) = \frac{P_{\mathrm{gce}}(X, B_{\mathrm{tot}}=B_0)}{P_{\mathrm{gce}}(B_{\mathrm{tot}}=B_0)}$$

X : A random variable, e.g. particle numbers

R. Poberezhniuk, V. Kuznietsov, GP, V. Vovchenko, in Prep

Relation between the cumulant generating functions

$$G_{\mathrm{ce}}(t) = \ln \frac{\int_0^{2\pi} \frac{d\phi}{2\pi} \exp\left[i\phi B_0 + G_{\mathrm{gce}}(t, -i\phi)\right]}{\int_0^{2\pi} \frac{d\phi}{2\pi} \exp\left[i\phi B_0 + G_{\mathrm{gce}}(0, -i\phi)\right]}$$

In the limit of high multiplicity event

Integrals can be solved by a saddle point approximation

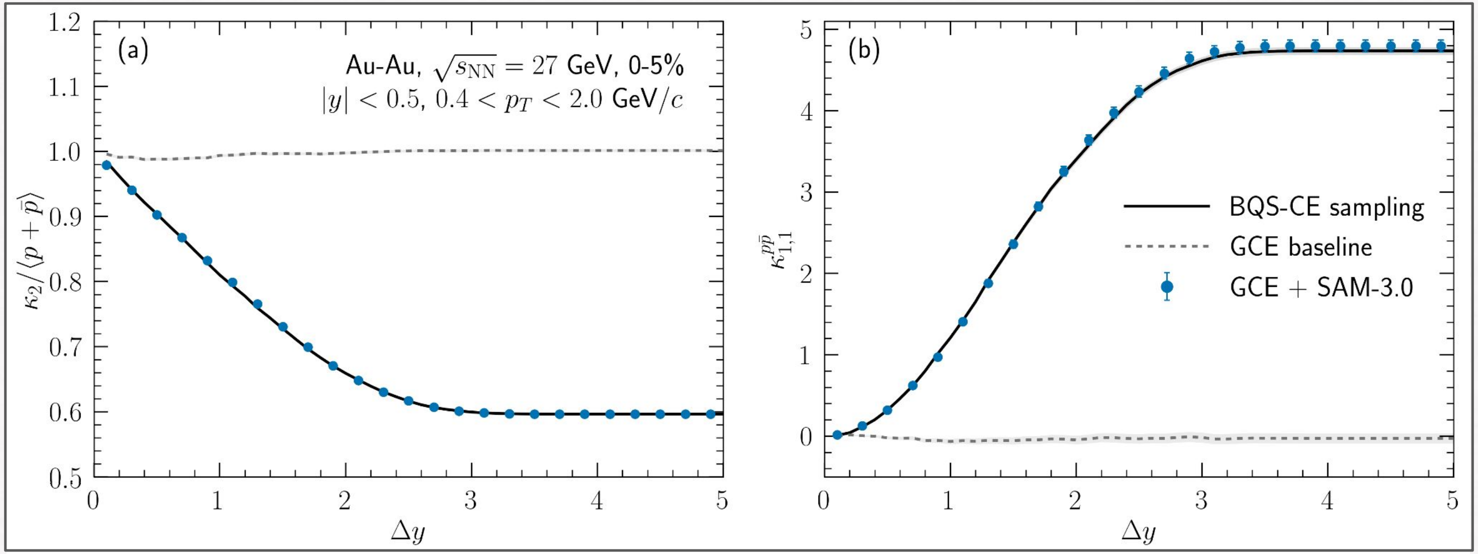
$$G_{\mathrm{ce}}(t) = G_{\mathrm{gce}}(t, \lambda_\star(t)) - \lambda_\star(t) B_0,$$

$$\frac{\partial G_{\mathrm{gce}}}{\partial \lambda}(t, \lambda_\star(t)) = B_0, \qquad \lambda_\star(0) = 0$$

$$\kappa_{\vec{n}}^{\vec{X}|\vec{B}} = F[\kappa_{\vec{n}}^{\vec{X}}]$$

Global charge conservation: SAM 3.0

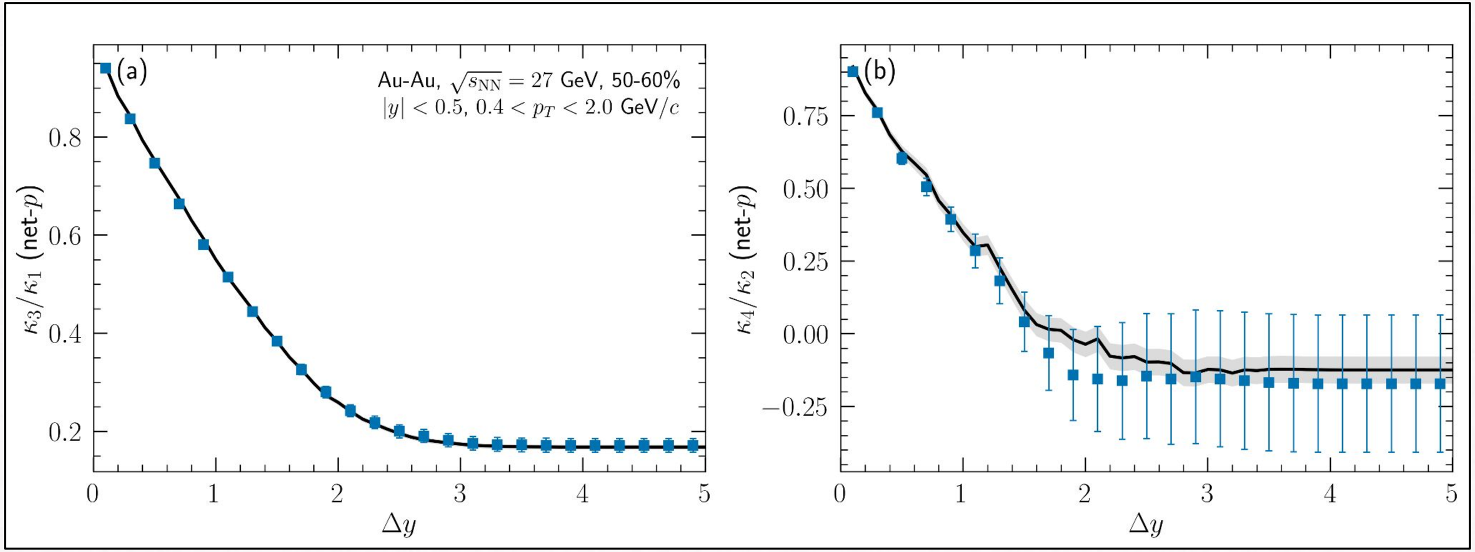
Example on GCE fluctuations on the hydrodynamics hypersurface



Excellent reproduction of the CE cumulants from GCE cumulants

Example on GCE fluctuations on the hydrodynamics hypersurface

Pipeline tracker



Same for higher order cumulants

Collision event → Baryon number fluctuations → Proton number fluctuations in acceptance → Baryon conservation correction

Pipeline tracker

In the specific case: $\vec{B} = (B)$ $\vec{n} = (n)$ $\vec{X} = (N_{p,\mathrm{Acc}})$

$$\kappa_n^{\mathrm{CE}}(N_{p,\mathrm{Acc}}) = \mathrm{SAM}\left[\kappa_{l,m}^{\mathrm{GCE}}(N_{p,\mathrm{Acc}}, B)\right]$$

One last ingredient to close the loop:

B^+, B^- $\quad G(t_+, t_-) = \ln\left\langle e^{\,t_+ B_+ + t_- B_-}\right\rangle$

$$B = B^+ - B^-$$

$N_{p,\mathrm{Acc}}, B$ $\quad \tilde{G}(t, t_B) = \ln\left\langle e^{\,t\, N_{p,\mathrm{Acc}} + t_B\, B}\right\rangle$

$$\tilde{G}(t, t_B) = G\big(t_B + \theta(t, \alpha_+),\, -t_B\big)$$

$$\theta(t, \alpha_+) \equiv \ln\big(1 - \alpha_+ + \alpha_+ e^t\big)$$

Collision event → Baryon number fluctuations → Proton number fluctuations in acceptance → Baryon conservation correction

Pipeline tracker

In the specific case: $\vec{B} = (B) \quad \vec{n} = (n) \quad \vec{X} = (N_{p,\mathrm{Acc}})$

$$\kappa_n^{\mathrm{CE}}(N_{p,\mathrm{Acc}}) = \mathrm{SAM}\left[\kappa_{l,m}^{\mathrm{GCE}}(N_{p,\mathrm{Acc}}, B)\right]$$

One last ingredient to close the loop:

$$\kappa_{l,m}^{\mathrm{GCE}}(N_{p,\mathrm{Acc}}, B) = f[\kappa_{nm}^{+-}]$$

Max Ent cumulants

Grand partition function

$$\ln Z^{\mathrm{gce}}(T,V,\mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\mathrm{ce}}(T,V,N)\right]$$

Cumulant generating function

$$K_N(t) = \ln\left\langle e^{tN}\right\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

$$\kappa_n$$

$$VT^3\chi_n$$

Collision dynamics

Evolving clouds

Du et al, Phys. Rev. C 104, 064904

Schenke et al Phys. Rev. C 82, 014903

Karpenko et al Comput. Phys.Commun. 185, 3016-3027

Global charge conservation

Finite size effects

Vovchenko,et al Phys.Lett.B 811, 135868

Thermal smearing

Coordinate / momentum space

Ohnishi et al, Phys. Rev. C 94, 044905

Detector effects

Acceptance/efficiency

Savchuk, et al Phys.Rev.C 101, 024917

Vovchenko et al, Phys.Rev.C 112 024901

Kitazawa et al Phys. Rev. C 93, 044911

Ling, Stephanov, PRC 93, 034915

Proxy observables

Baryon to proton

Kitazawa, Asakawa, Phys. Rev. C 86.024904

hadron gas dynamics

Late stage interactions

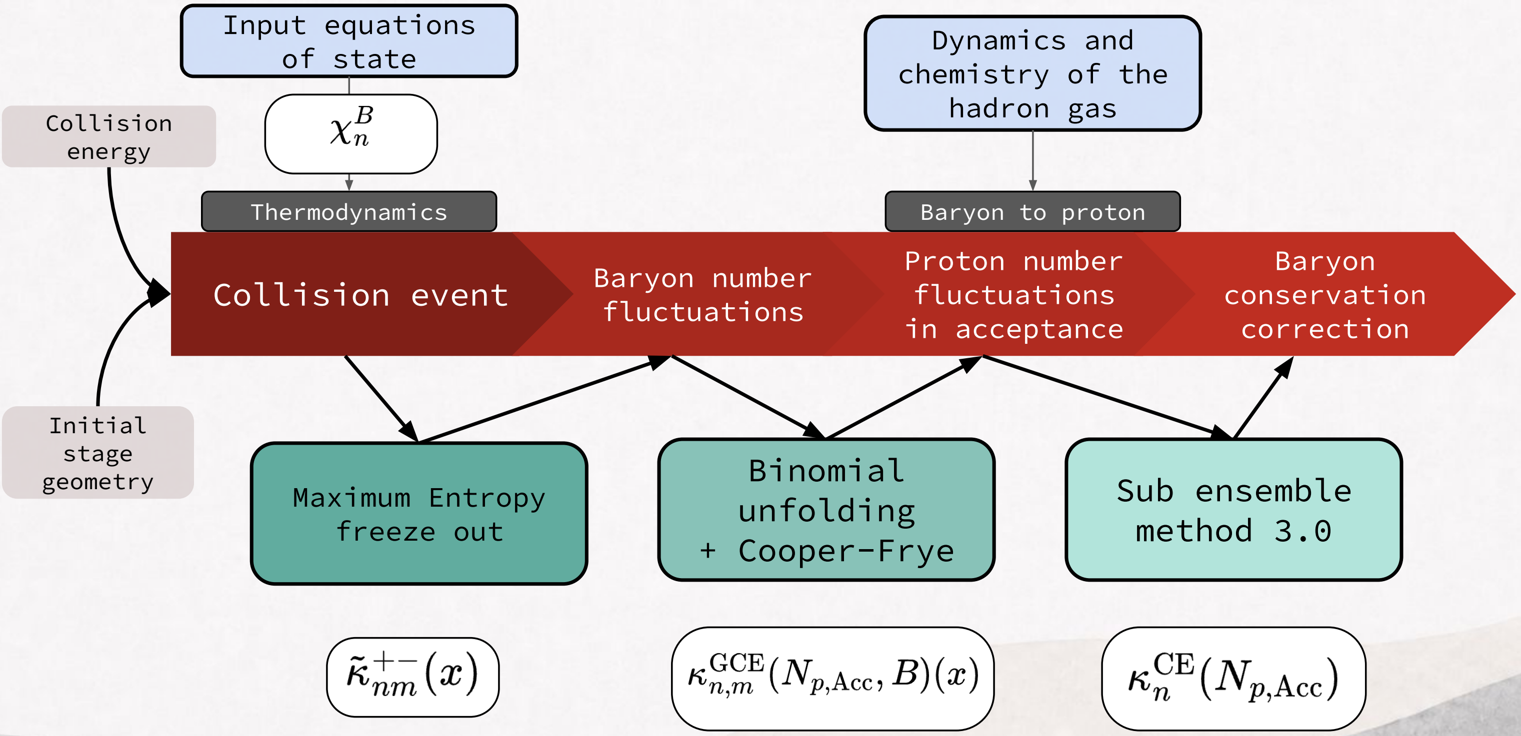
Steinheimer et al PLB 776, 32

$$\boldsymbol{\kappa}_n \longleftrightarrow \kappa_{l,m}^{\mathrm{GCE}}(N_{p,\mathrm{Acc}}, B) \longleftrightarrow \boldsymbol{\kappa}_{nm}^{+-} \longleftrightarrow \boldsymbol{\kappa}_n^{B,\mathrm{GCE}} \longleftrightarrow VT^3\chi_n$$

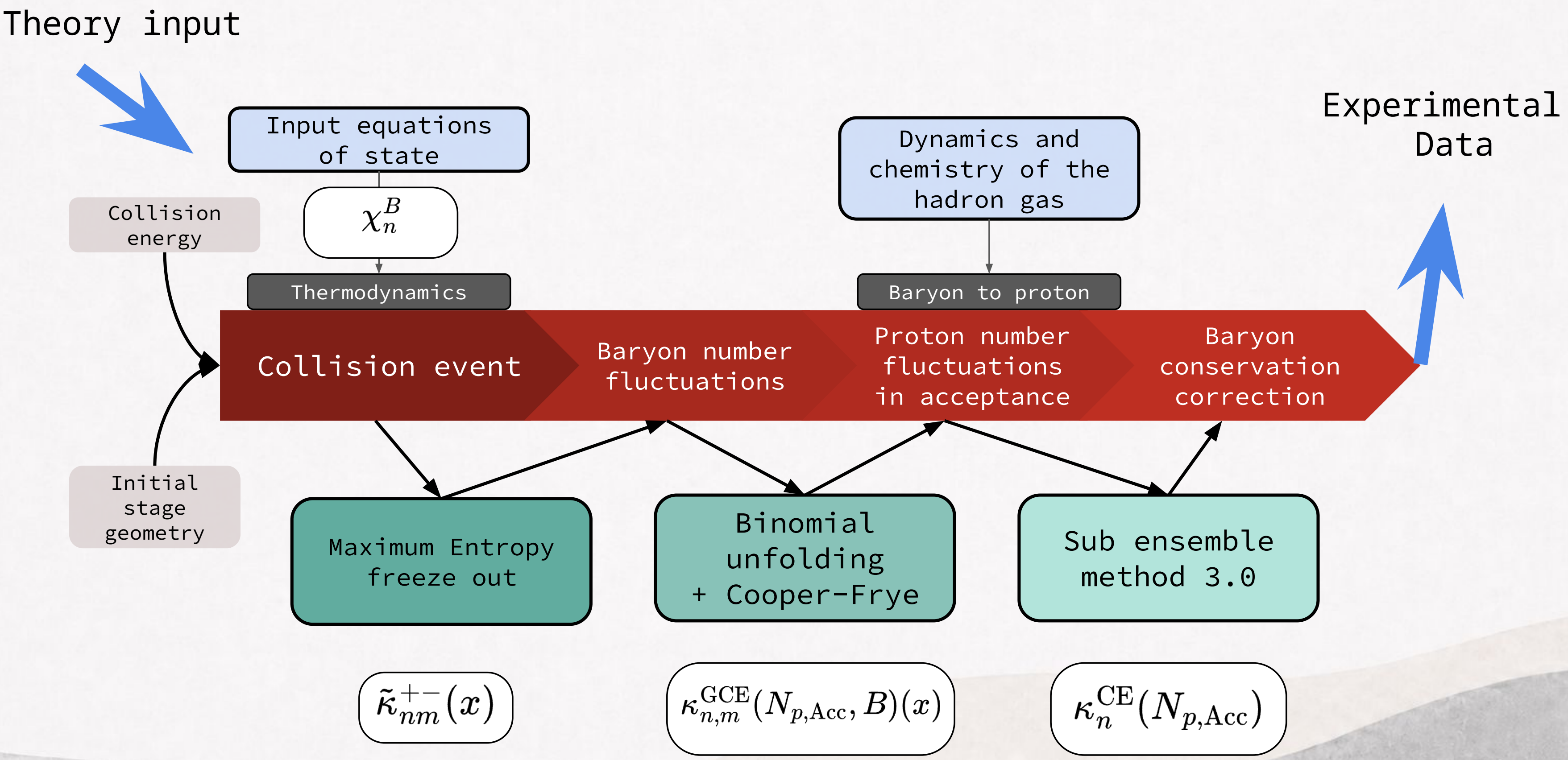
The roadmap from theory to experiment

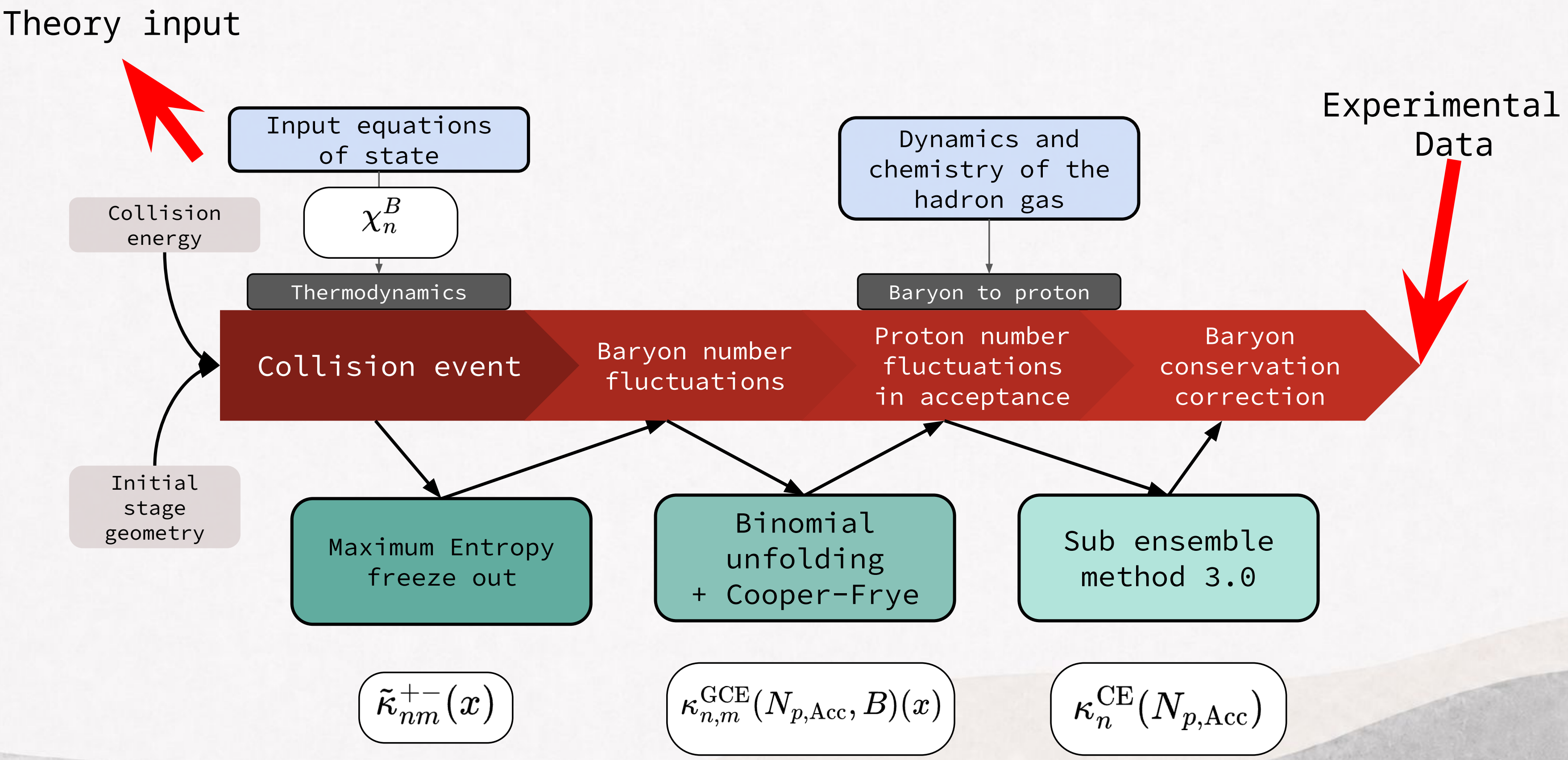
The MEME method

The Maximum Entropy Method Extended method



The roadmap from theory to experiment




Theory input
Experimental Data
Input equations of state
χ_n^B
Collision energy
Thermodynamics
Dynamics and chemistry of the hadron gas
Baryon to proton
Collision event
Baryon number fluctuations
Proton number fluctuations in acceptance
Baryon conservation correction
Initial stage geometry
Maximum Entropy freeze out
Binomial unfolding + Cooper-Frye
Sub ensemble method 3.0
$\tilde{\kappa}^{+-}_{nm}(x)$
$\kappa^{\mathrm{GCE}}_{n,m}(N_{p,\mathrm{Acc}}, B)(x)$
$\kappa^{\mathrm{CE}}_{n}(N_{p,\mathrm{Acc}})$

Bayesian inference: setup

The Max Ent cumulants $\tilde{\kappa}^{+-}_{nm}(x) = \tilde{\tilde{\kappa}}^{+-}_{nm}(x) + \hat{\Delta}\mathcal{H}_{n+m}(x)\left(P_+\right)^n\left(P_-\right)^m$

At second order

$$\hat{\Delta}\mathcal{H}_{n+m=2} = \Delta\mathcal{H}_2$$

Definition of susceptibility ratios

$$\Delta\mathcal{H}_2 = VT^3(\chi_2^B - \bar{\chi}_2^B)$$

$$\Delta\mathcal{H}_2 = VT^3\bar{\chi}_2^B(p_2 - 1)$$

Ideal HRG susceptibilities

$$p_2 = \frac{\chi_2^B}{\bar{\chi}_2^B}$$

Higher orders

$$p_3 = \frac{\chi_3^B}{\chi_1^B} \qquad p_4 = \frac{\chi_4^B}{\chi_2^B}$$

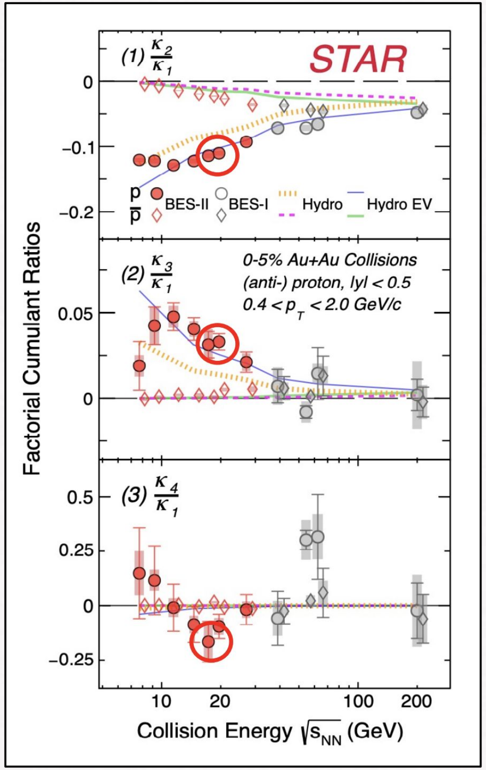
$$\hat{\Delta}\mathcal{H}_3 = V\,T^3\,\bar{\chi}_3^B\,(p_3 - 3p_2 + 2)$$

$$\hat{\Delta}\mathcal{H}_4 = \Delta V\,T^3\,\bar{\chi}_4^B\left[p_2(p_4 - 4) + 3 - 3\left(\frac{\bar{\chi}_3^B}{\bar{\chi}_2^B}\right)^2(2p_3 - 5p_2 + 3)\right]$$

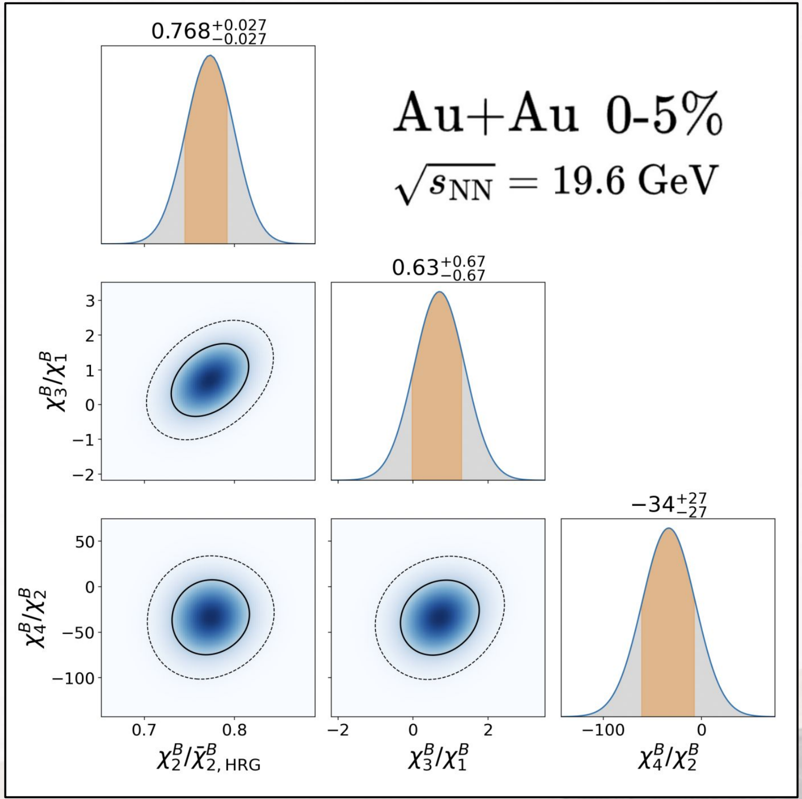
Bayesian setup

$$\hat{\kappa}_n^{\mathrm{CE}}(N_{p,\mathrm{Acc}}) = f(p_2, p_3, p_4, \ldots)$$

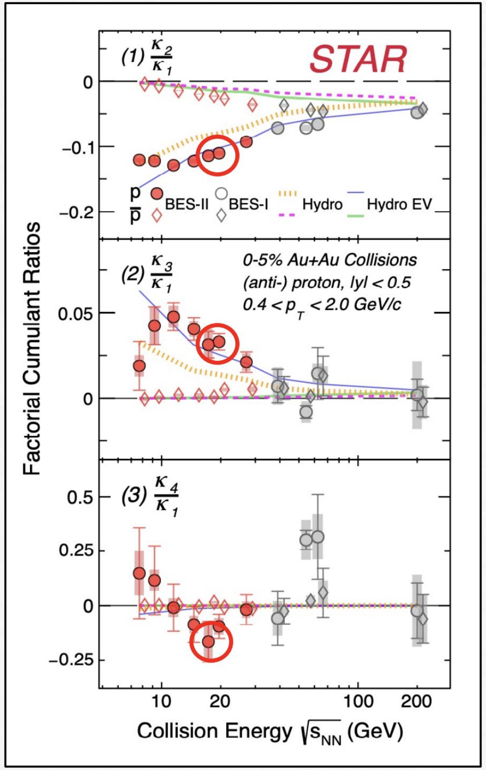
Bayesian inference: 19.6 GeV



Unfolding



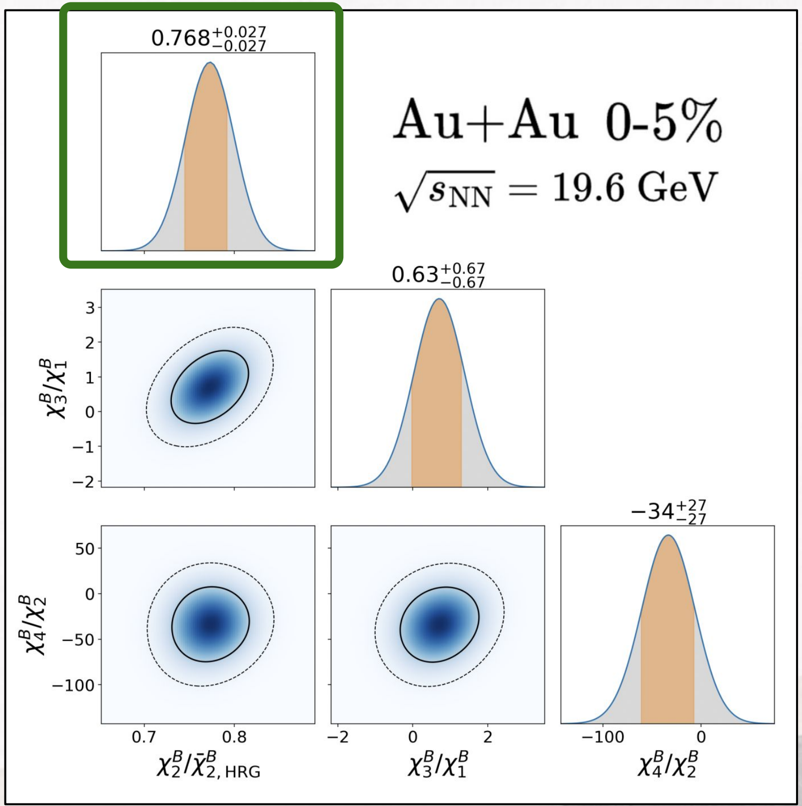
Bayesian inference: 19.6 GeV



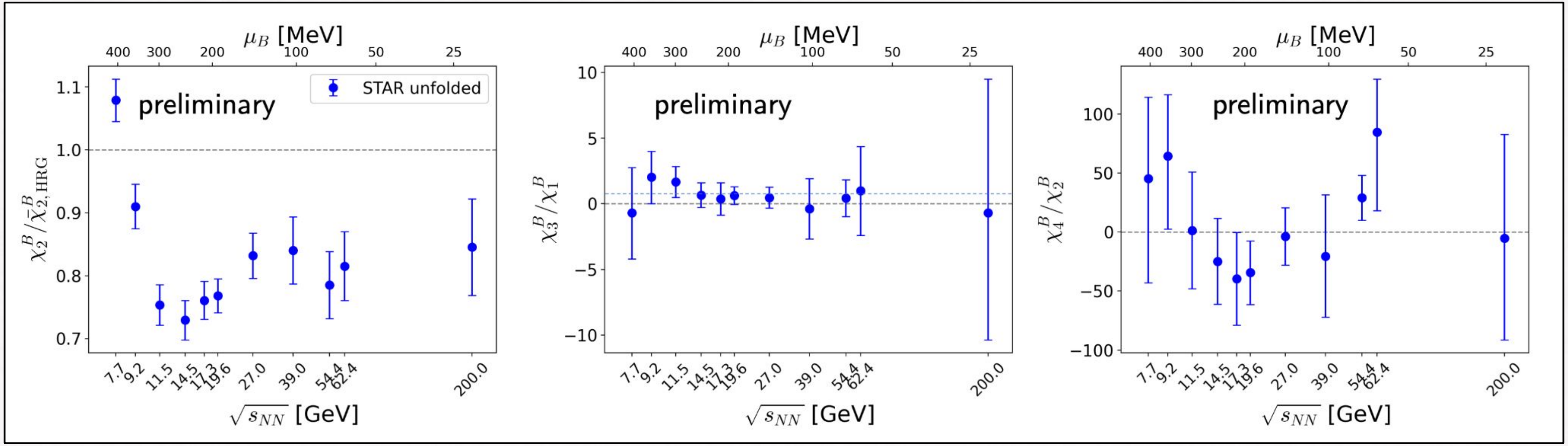
Unfolding →

Extraction of the QCD susceptibility relative to HRG

Possible at the 2nd order!



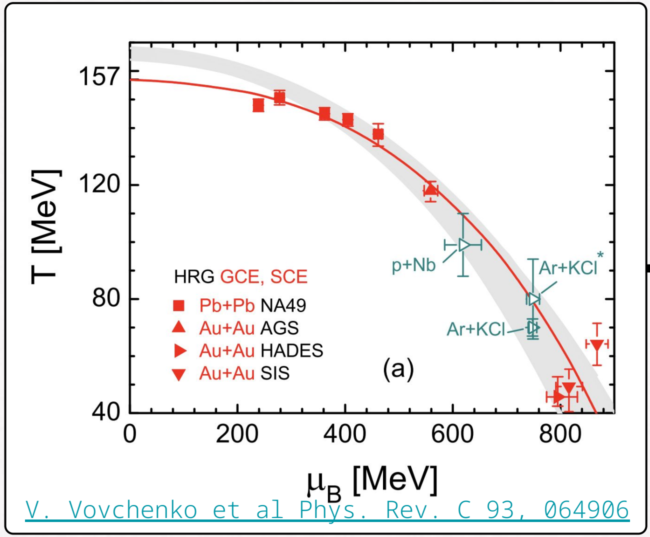
Extracted susceptibilities



- The deviation from ideal HRG at second order is tightly constrained overall
- At third and fourth order, the errors are large (propagated error from experimental data)
- The deviations from ideal HRG are mostly insensitive to the freezeout energy
-

Extracted susceptibilities

Freezeout line: strangeness neutral $n_Q \sim 0.4 n_B \quad n_S = 0$



V. Vovchenko et al Phys. Rev. C 93, 064906

Baseline EoS

Ideal HRG
With Quark Model particle list
QMHRG2020

Hot QCD, Phys. Rev. D 104, 074512

LQCD based EoS

The 4D T'ExS
(strangeness neutral)

A. Abuali et al., Phys. Rev. D 112, 054502

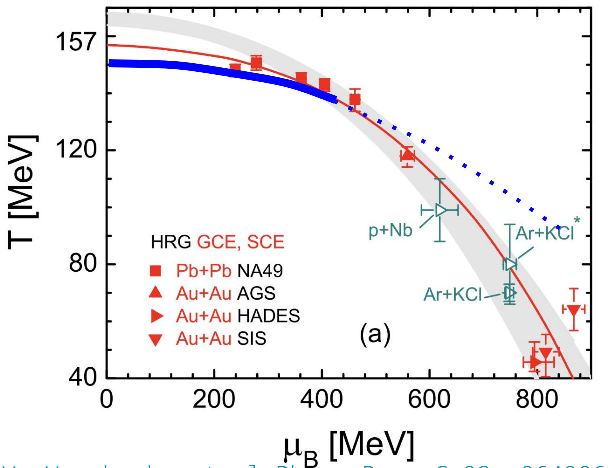
See Isabella's talk on Wednesday!

Energy density at freezeout

$$e_{\rm sw} = 0.18\ {\rm GeV/fm^3}$$
$$e_{\rm sw} = 0.25\ {\rm GeV/fm^3}$$

$$e_{\rm sw} \sim 0.35 - 0.4\ {\rm GeV/fm^3}$$

Freezeout line: strangeness neutral $n_Q \sim 0.4 n_B \quad n_S = 0$



V. Vovchenko et al Phys. Rev. C 93, 064906

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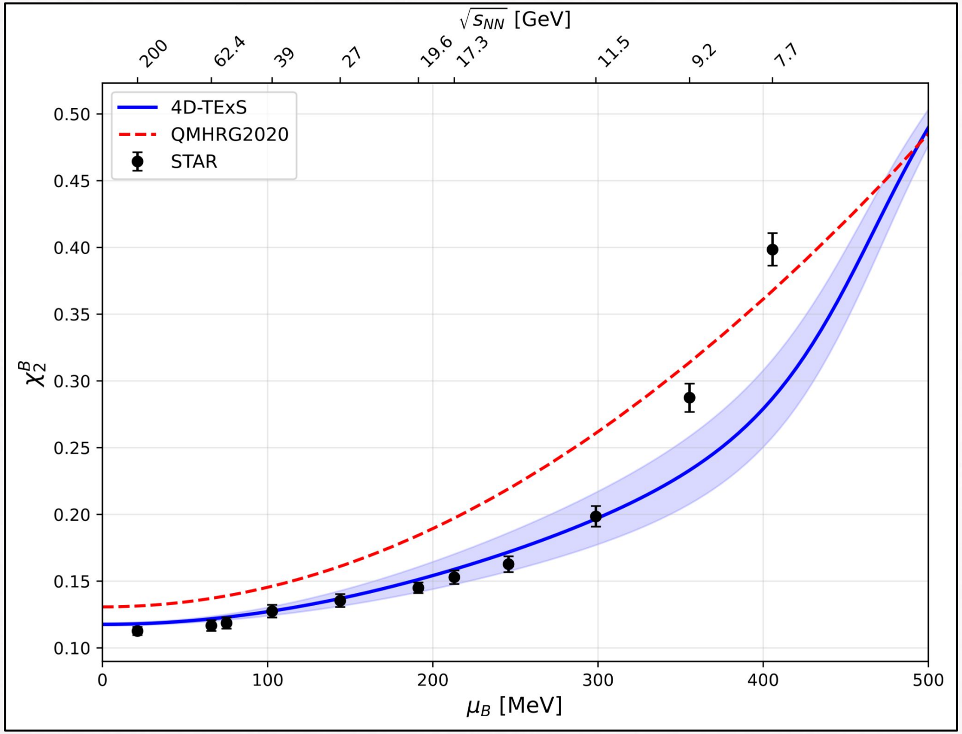
Extracted baryon number susceptibility

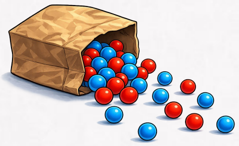


- ➢ Excellent agreement with lattice based expansion $\mu_B < 300\,\mathrm{MeV}$
- ➢ Statistically significant deviation at larger chemical potential

Caveats

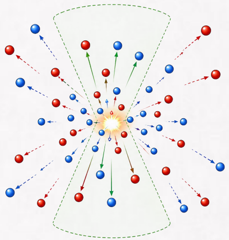
- Ideal HRG baseline sensitive to the choice of particle list
- Baseline and comparison EoS depends on the freezeout line
- 4D T'ExS EoS is extrapolated from vanishing chemical potential

Summary



Fluctuations: excellent tool to probe the QCD thermodynamics and critical point physics

Theory predictions cannot be compared directly to experimental measurements



Systematic corrections from Hydro, Max Ent and Subensemble acceptance method can be implemented!

Cleaner estimation of the net proton factorial cumulant allow for the extraction of the second order susceptibility from the STAR measurements.

The extracted agrees with LQCD based calculations at low chemical potential and deviates at large chemical potential

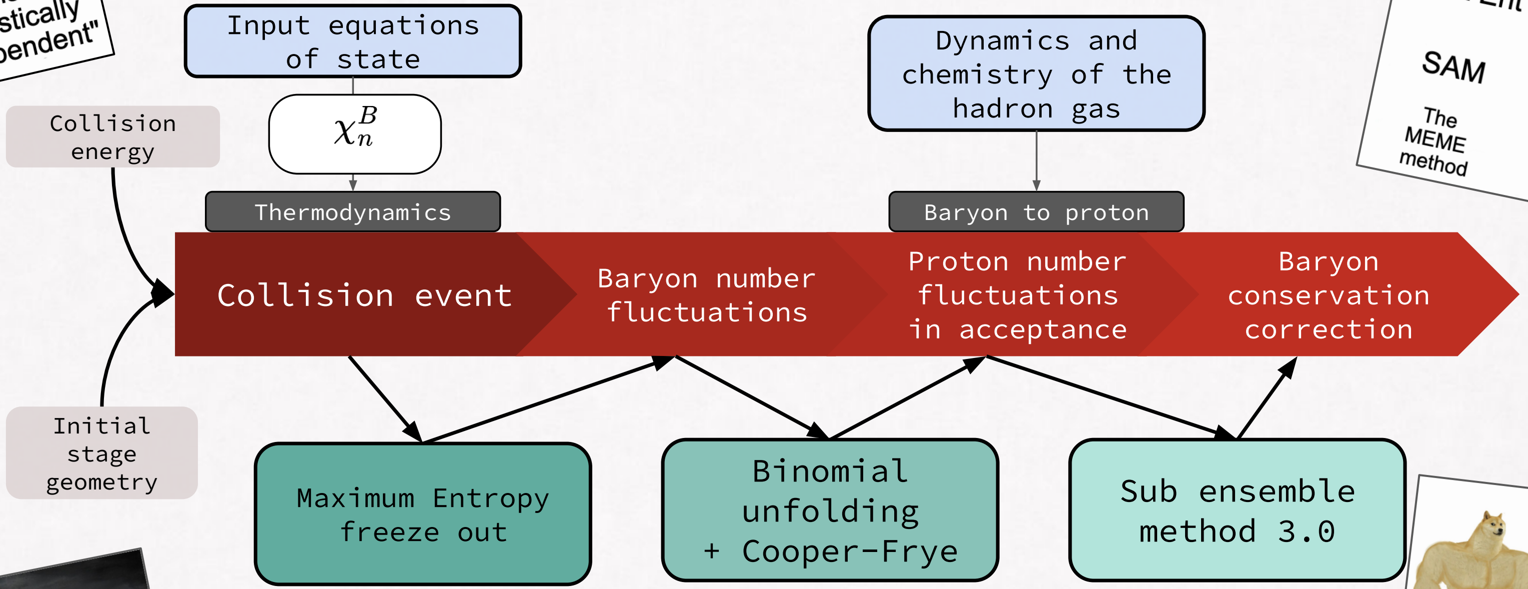
Higher order fluctuations requires more precision on the measurement

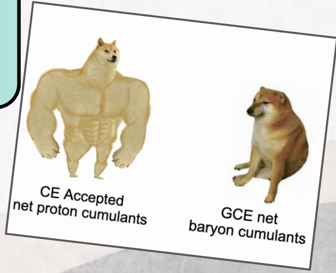
Outlook

- Include initial stage event by event fluctuations
- Consistent dynamical fluctuations framework to evolve fluctuations during the hydrodynamics -> consistency of the EoS
- Consider more realistic contributions from the hadronic cascades
- Extend the bayesian analysis to other available fluctuations observables: cumulants, peripheral collision, rapidity dependence, etc.

The MEME method

The Maximum Entropy Method Extended method





Thank you for your attention