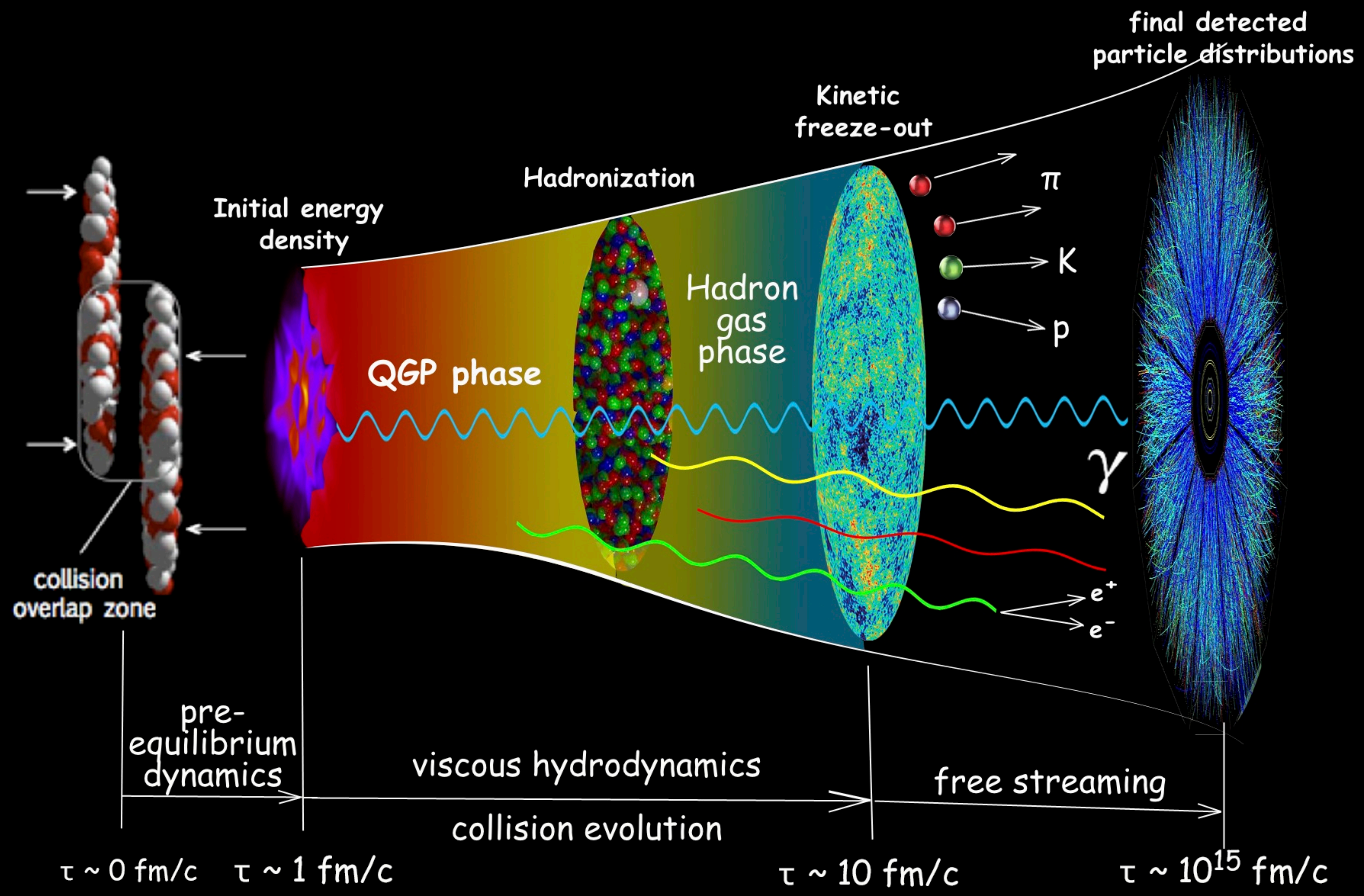
Bayesian Approach to RHIC Beam Energy Scan Program

Chun Shen (Wayne State University)

RELATIVISTIC HEAVY-ION COLLISIONS

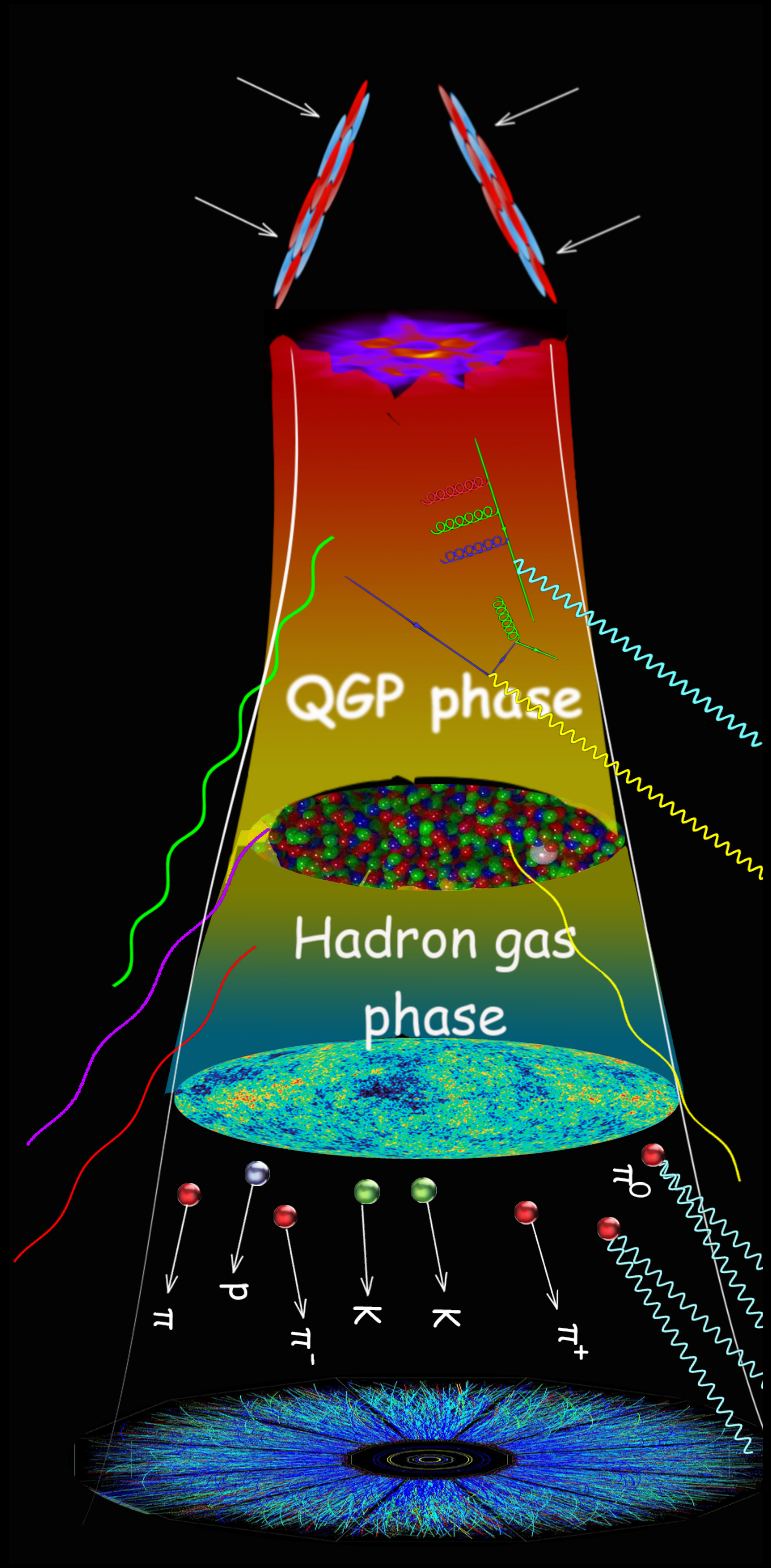


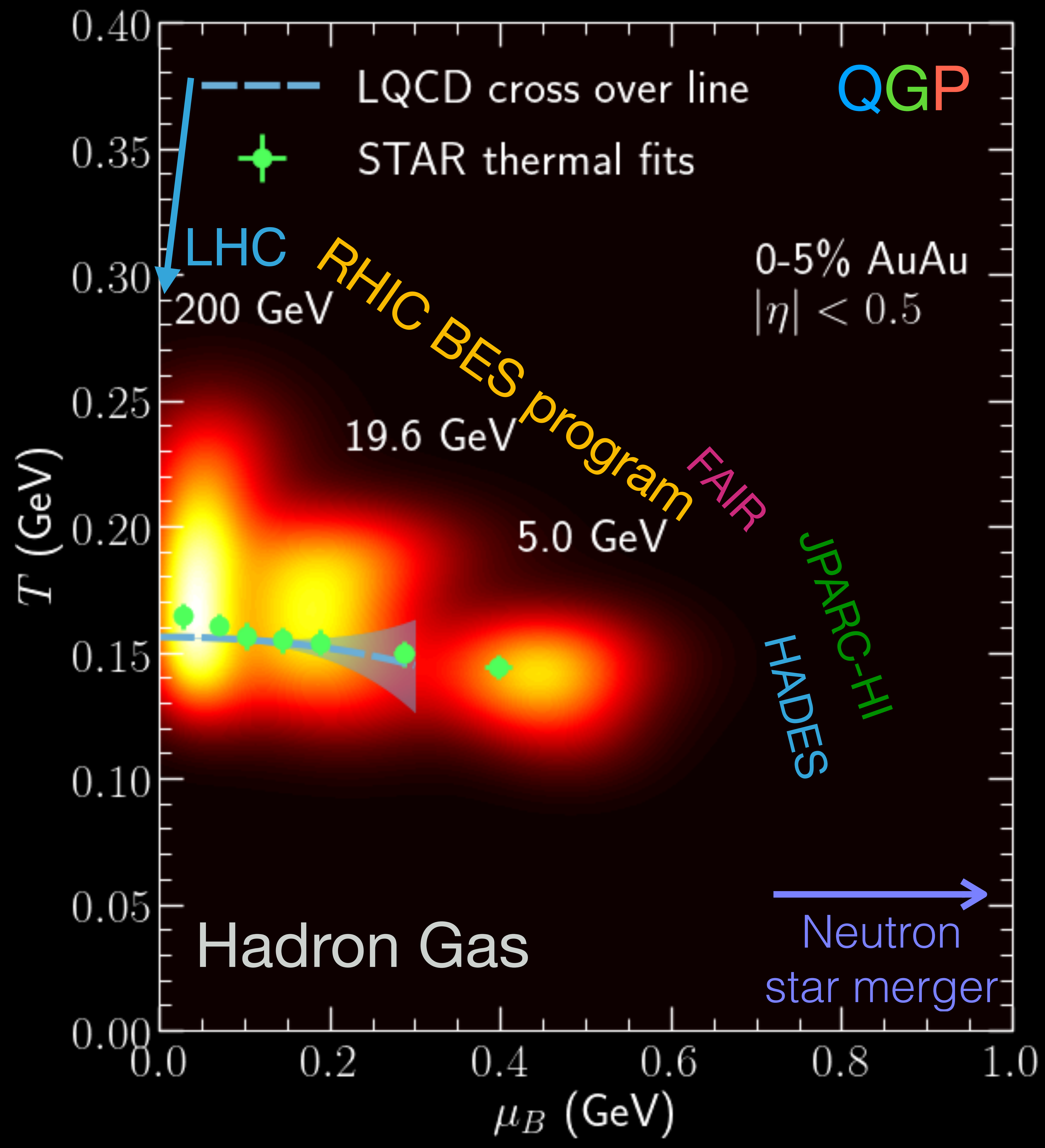
Relativistic heavy-ion collisions are tiny and have ultra-fast dynamics

It serves as a multi-purpose laboratory for many-body QCD

Physics insights reply on phenomenological theory-to-model comparisons

PROBING THE NUCLEAR MATTER PHASE DIAGRAM





- Search for a critical point & 1st order phase transition

  $c_s^2(T, \{\mu_q\})$

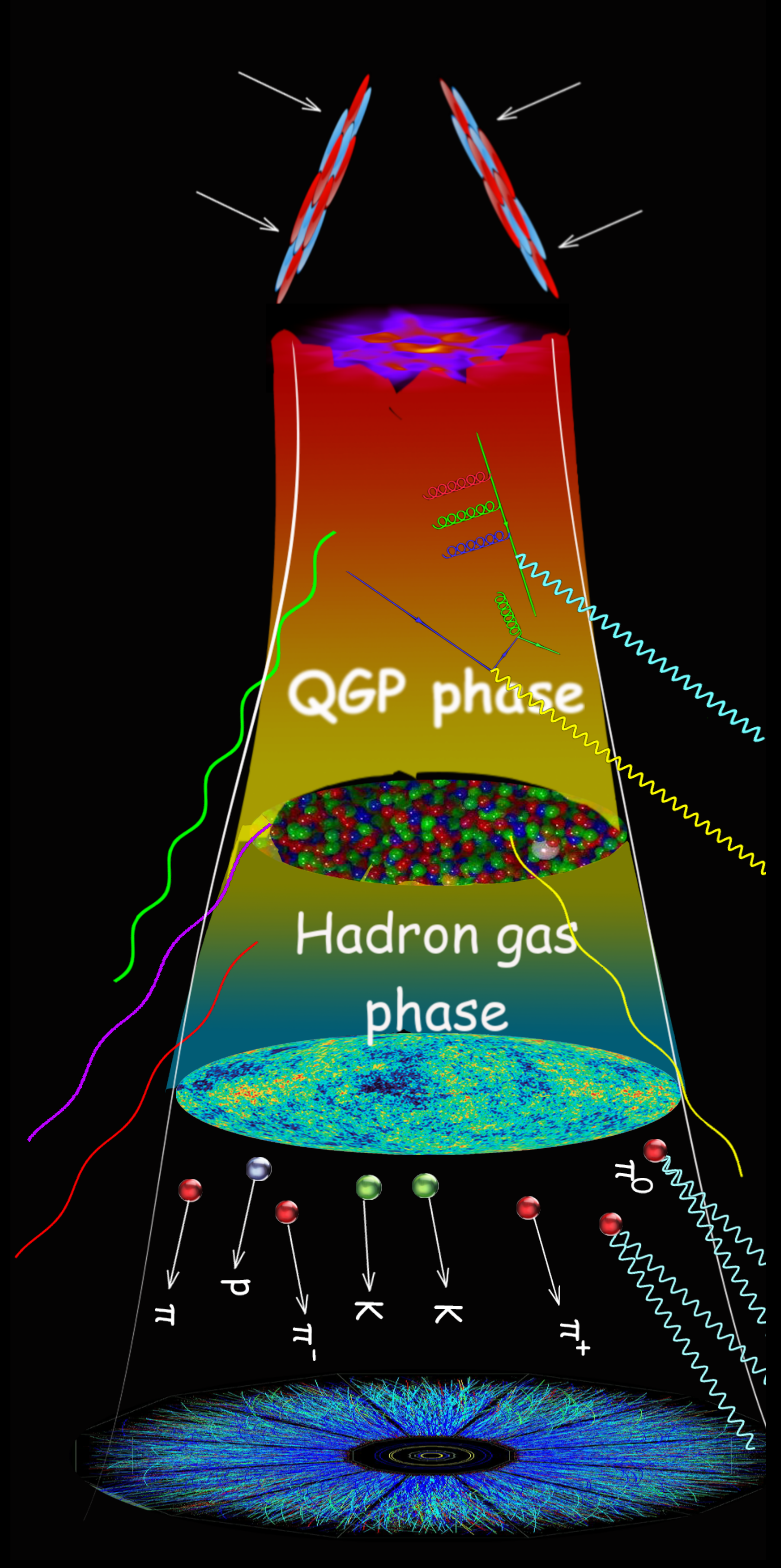
- How do the QGP transport properties change with baryon doping?

  $(\eta/s)(T, \{\mu_q\})$, $(\zeta/s)(T, \{\mu_q\})$

- Access to charge transport phenomena

  Charge diffusion

DEFINING THE QUARK-GLUON PLASMA



Which **properties of hot QCD matter** can we determine from relativistic heavy ion data (LHC, RHIC, HADES, and future FAIR/NICA/JPAC)?

Equation of State $T^{\mu\nu} \Longleftrightarrow e, P, s$

$$c_s^2 = \partial P/\partial e|_{s/n}$$

Shear and bulk viscosities $\eta/s(T, \mu_B), \zeta/s(T, \mu_B)$

Charge diffusion D_B, D_Q, D_S

Electromagnetic emissivity

Energy-momentum transport $\hat{q}, \hat{e}, \hat{e}_2, \ldots$

Spectra, collective flow, femtoscopy

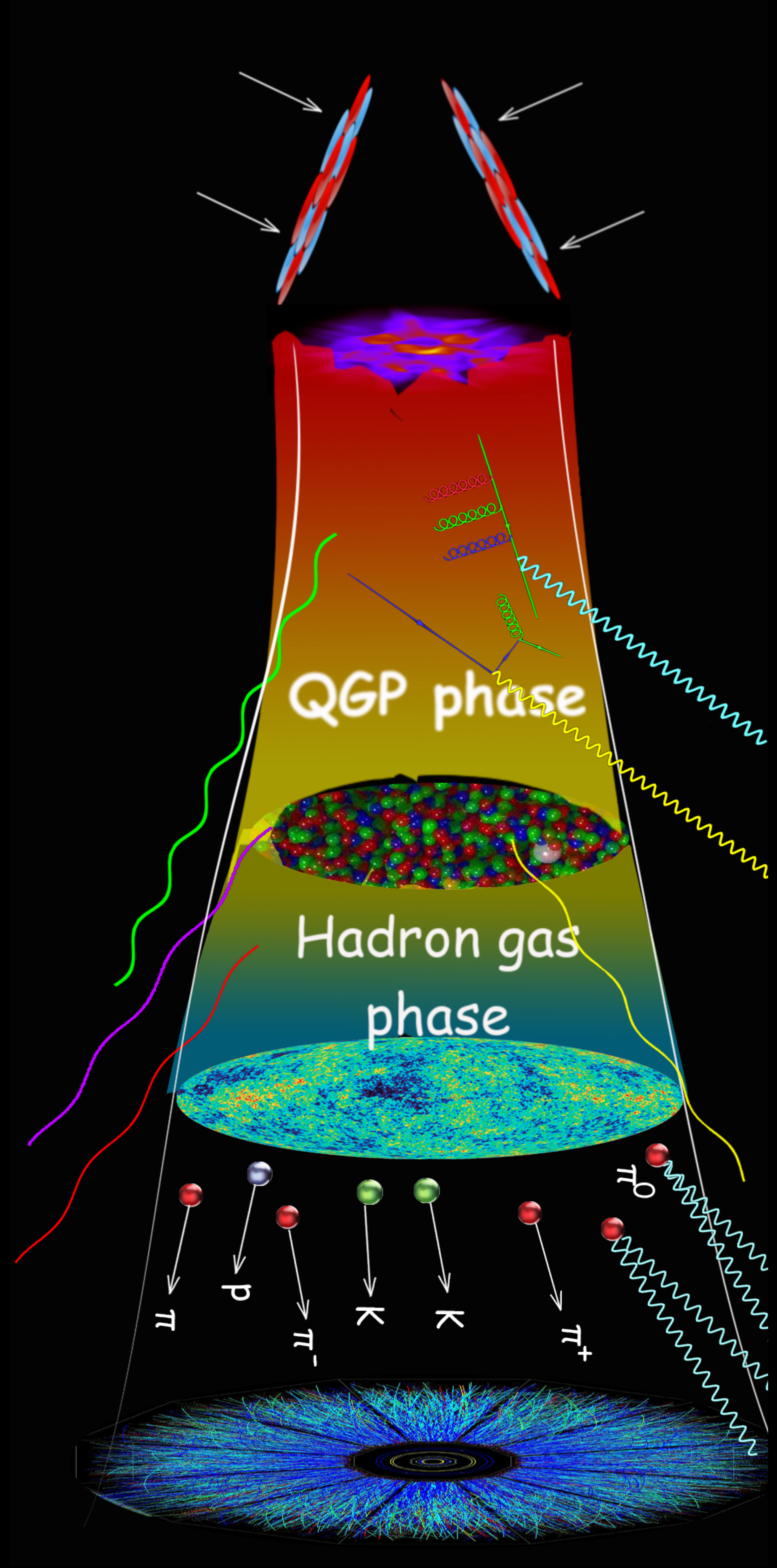
Anisotropic flow v_n
Flow correlations

Balance functions

Photons and dileptons

Jets and heavy-quarks

DEFINING THE QUARK-GLUON PLASMA



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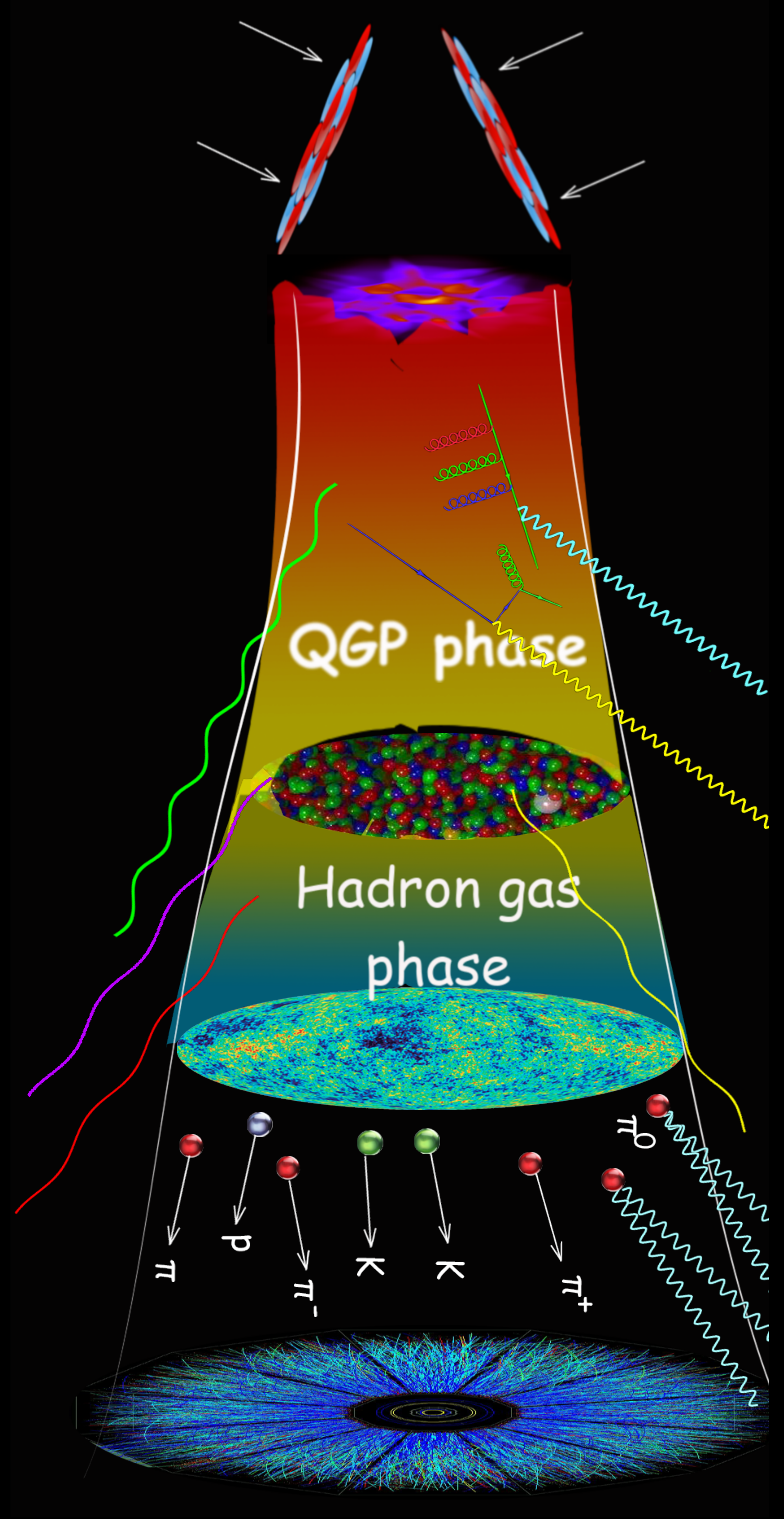

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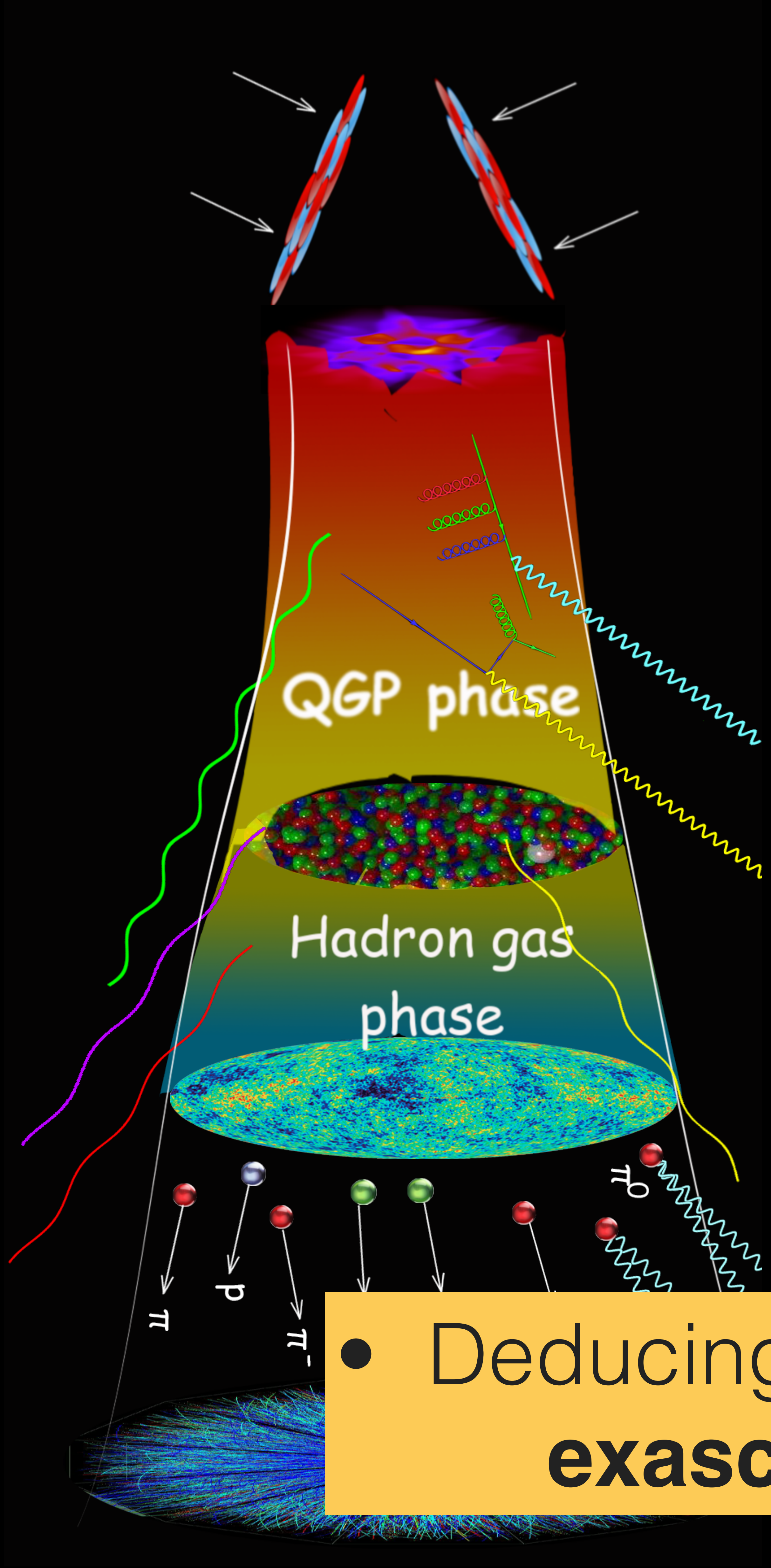

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Anisotropic flow v_n

Flow correlations

Balance functions

Photons and dileptons

quarks

- Deducing the QGP properties from experimental data requires **exascale computing** with advanced statistical methods

BAYESIAN ANALYSIS: CONNECTING MODELS AND DATA

Bayes' Theorem:

$$P(\theta \,|\, y_{\mathrm{exp}}) = \frac{P(y_{\mathrm{exp}} \,|\, \theta) P(\theta)}{P(y_{\mathrm{exp}})}$$

$P(\theta)$: Prior distribution for model parameters

BAYESIAN ANALYSIS: CONNECTING MODELS AND DATA

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$P(y_{\mathrm{exp}} | \theta)$: Likelihood
y_{exp} is described by $f(\theta)$; data + theory uncertainty

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BAYESIAN ANALYSIS: CONNECTING MODELS AND DATA

Bayes' Theorem:

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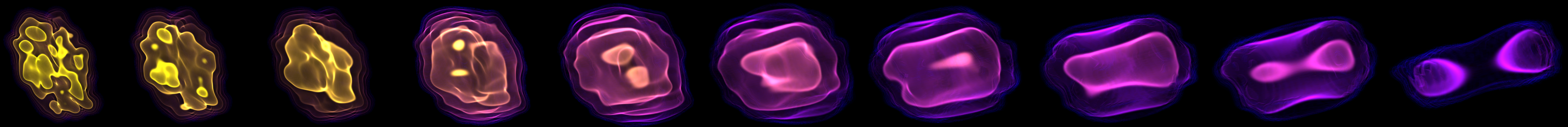
$P(\theta \,|\, y_{\rm exp})$: Posterior distribution of θ given $y_{\rm exp}$

$P(y_{\rm exp} \,|\, \theta)$: Likelihood $y_{\rm exp}$ is described by $f(\theta)$; data + theory uncertainty

$P(\theta)$: Prior distribution for model parameters

- Transformative techniques across different fields in nuclear physics
- Become a standard approach in nuclear physics for uncertainty quantification

THE MULTI-STAGE THEORETICAL FRAMEWORK



$T^{\mu\nu}$ 0^+ 0.5-1 τ (fm)

Initial State +
Pre-equilibrium dynamics

Hydrodynamics

Hadronic Transport

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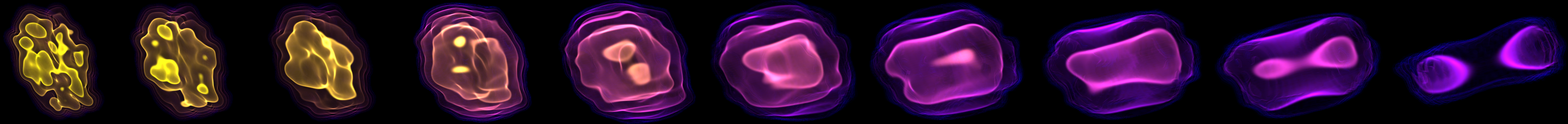

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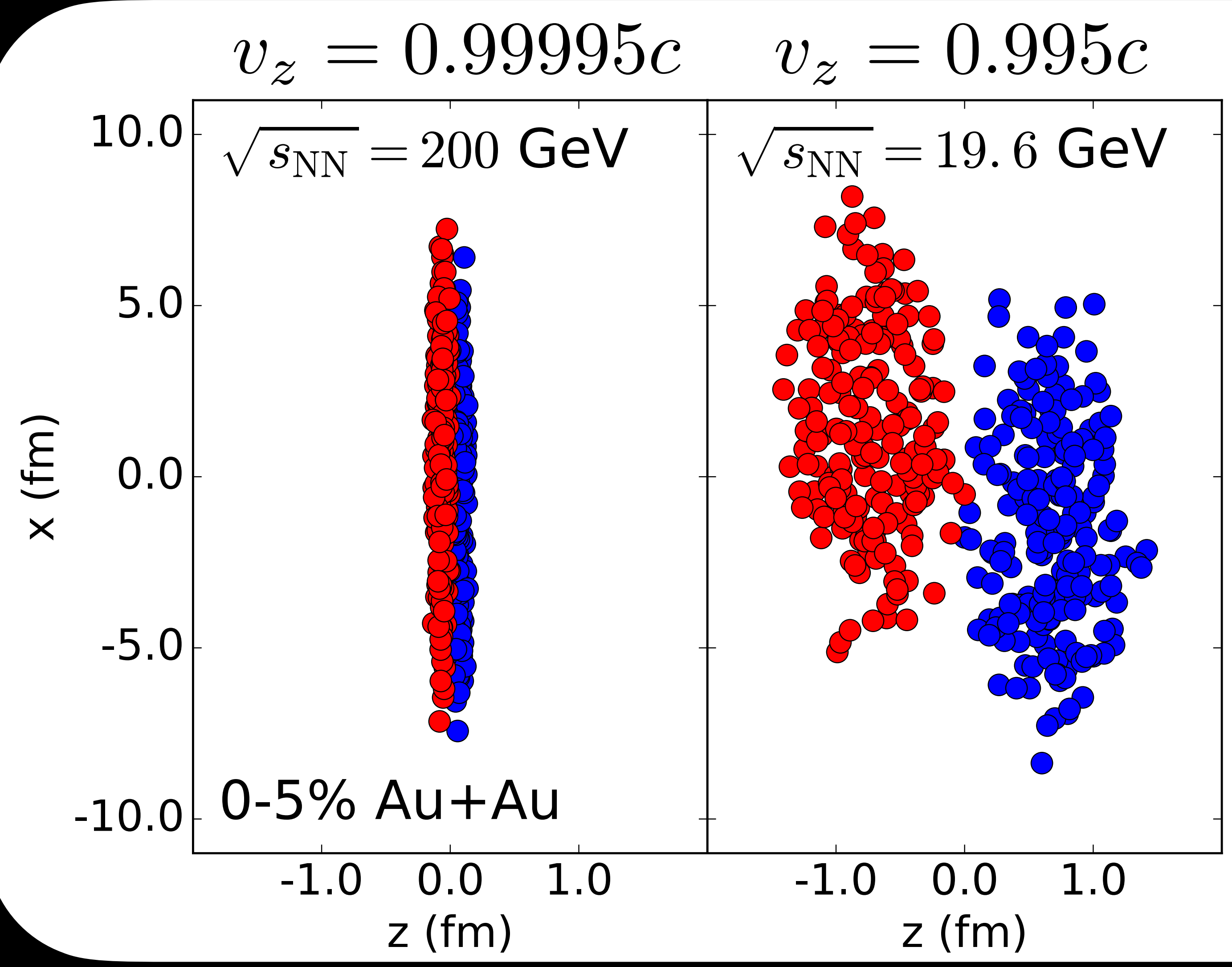
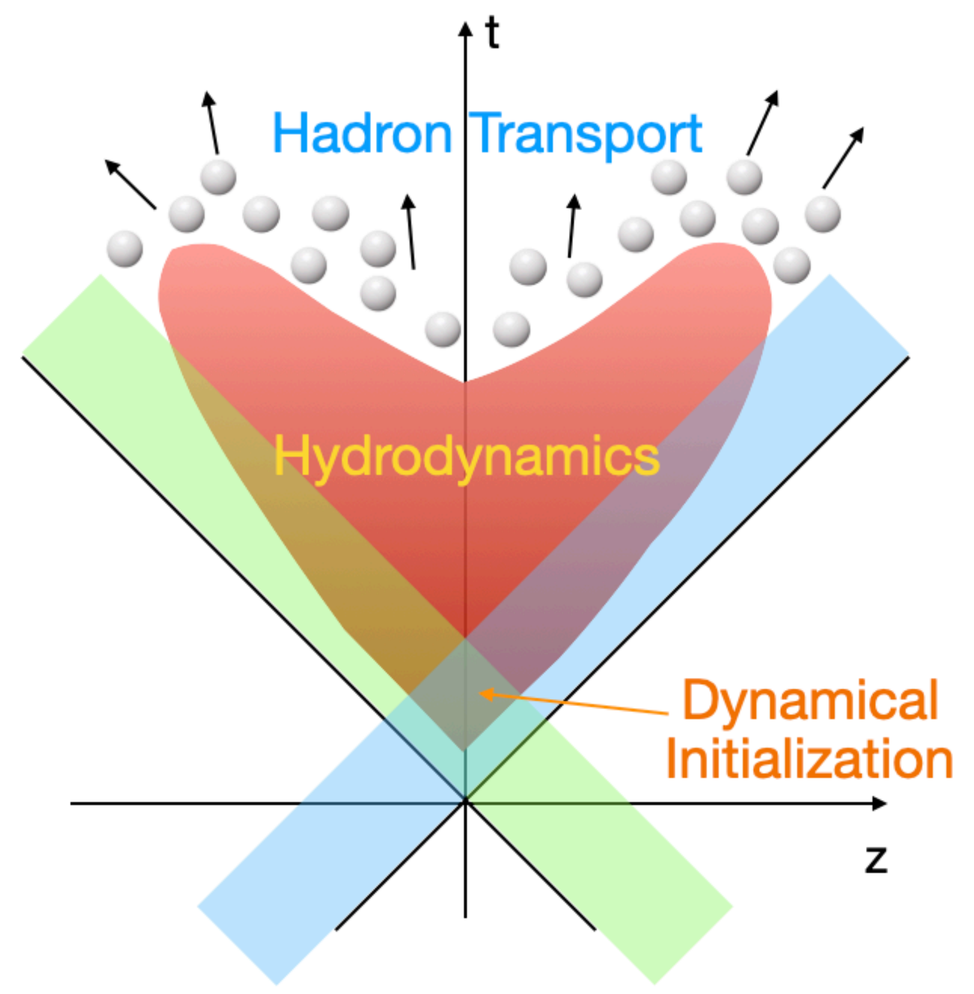
RHIC BES energies





$$\partial_\mu T^{\mu\nu} = J^\nu_{\rm source}$$

$$\partial_\mu J^\mu = \rho_{\rm source}$$

3D MC-GLAUBER MODEL WITH STRING DECELERATION

- Transverse collision geometry is determined by MC-Glauber model
- 3 valence quarks are sampled from PDF with

$$\sum_i x_i \leq 1$$

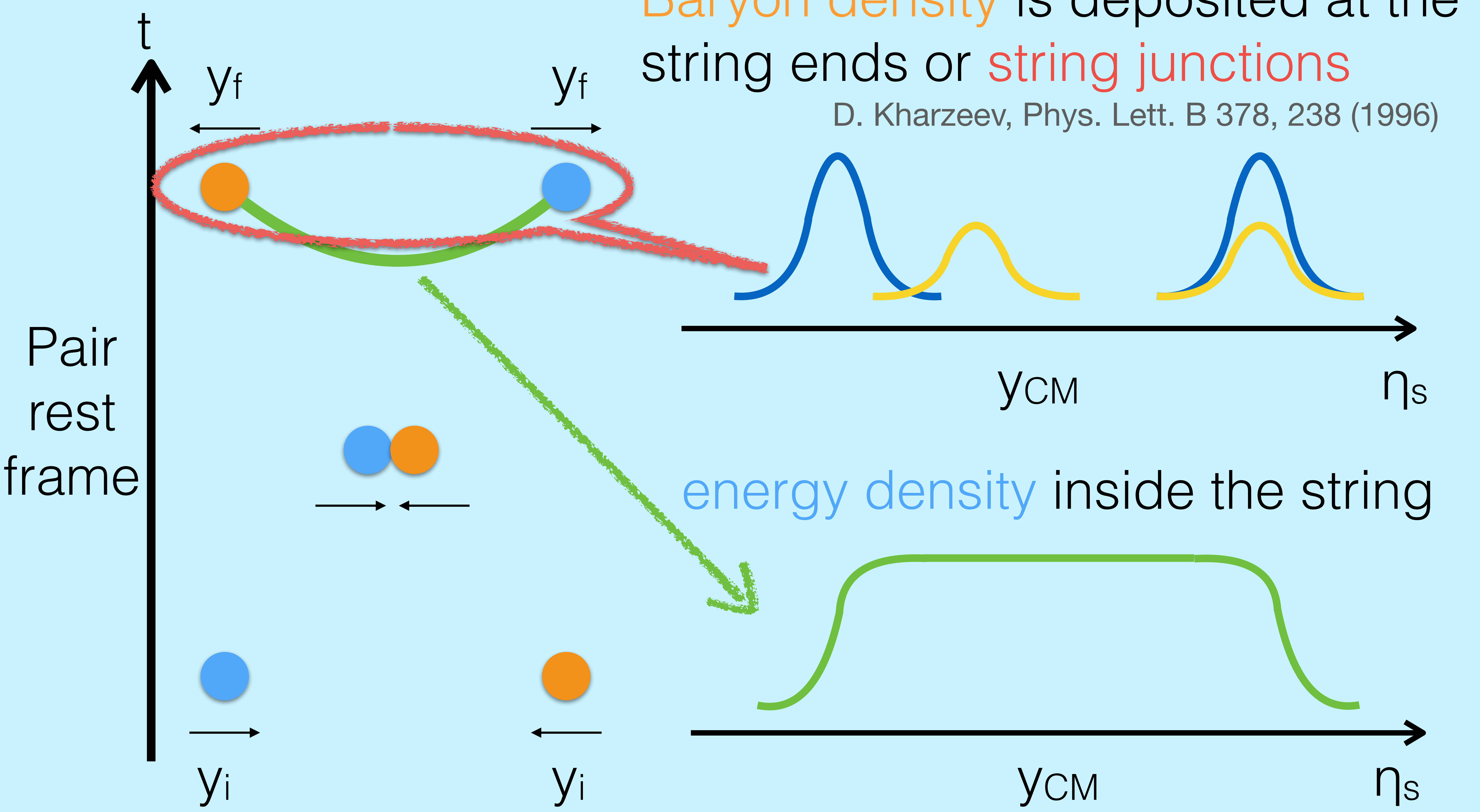
- Incoming quarks are decelerated with a string tension σ,

$$dp^z/dt = -\sigma$$

C. Shen and B. Schenke, Phys.Rev. C97 (2018) 024907 + Phys.Rev.C 105 (2022) 064905

Baryon density is deposited at the string ends or string junctions

D. Kharzeev, Phys. Lett. B 378, 238 (1996)



Imposed conservation for energy, momentum, and net baryon density

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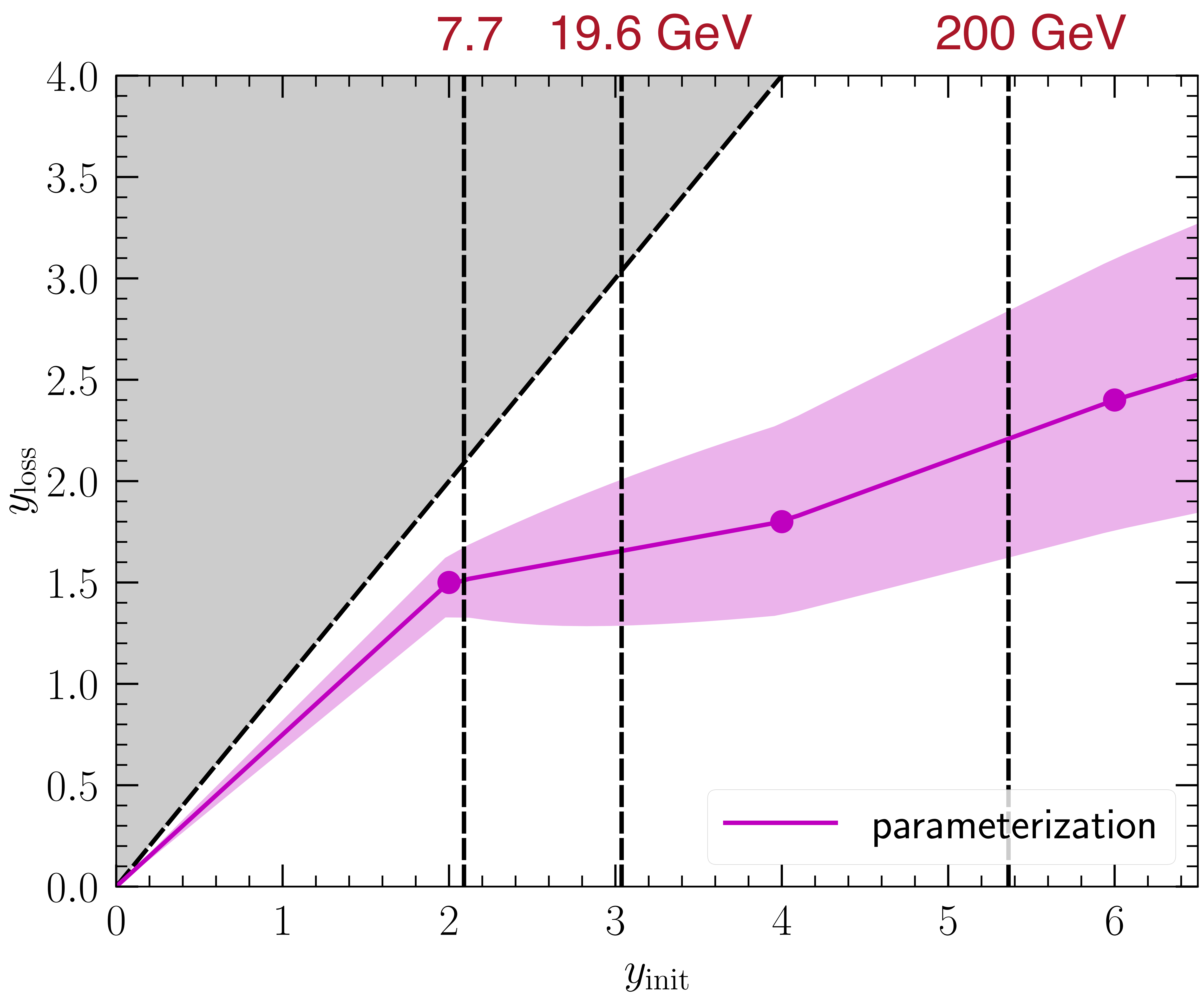


A piece-wise parameterization with

$y_{loss}(y_{init} = 2)$
$y_{loss}(y_{init} = 4)$
$y_{loss}(y_{init} = 6)$
$\sigma_{y_{loss}}$

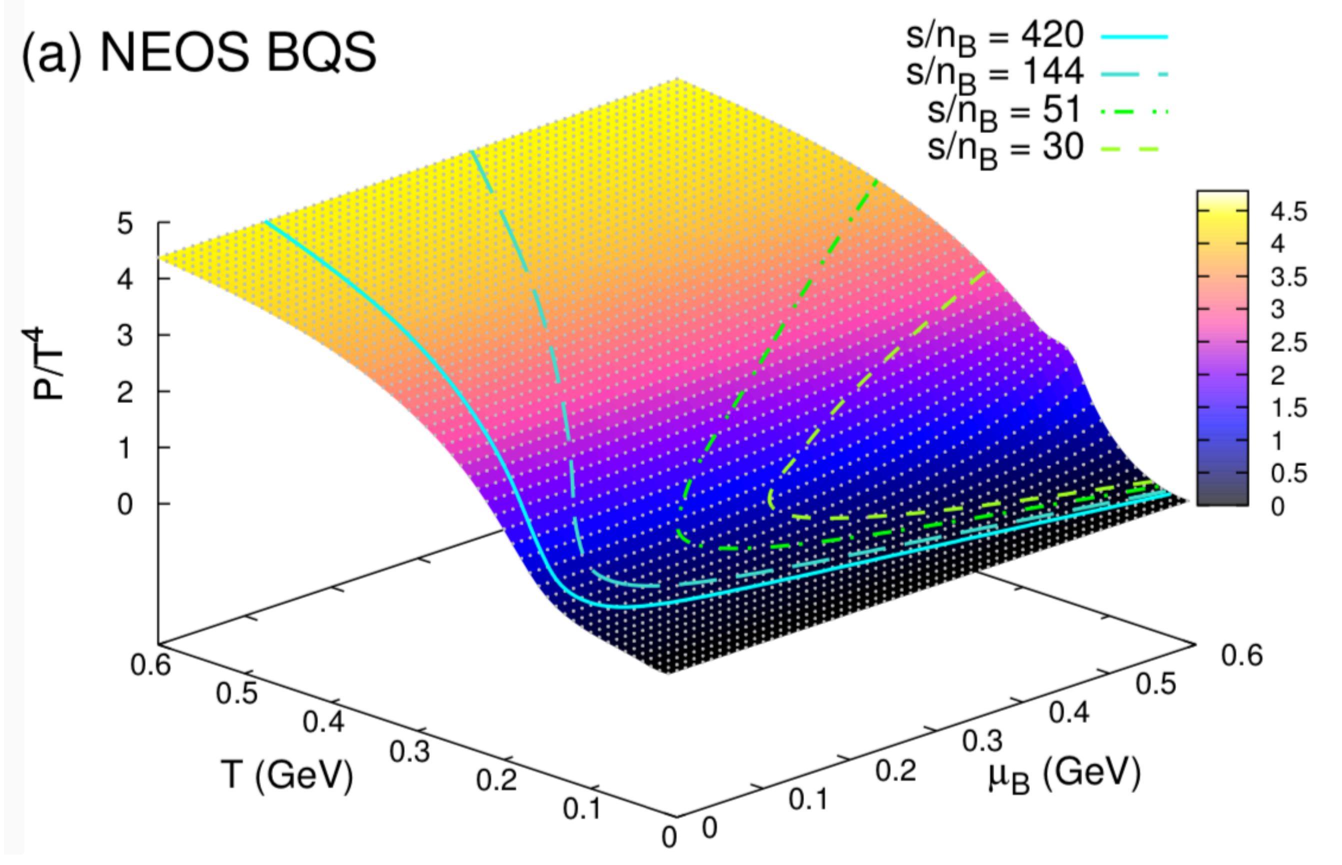
Imposed conservation for energy, momentum, and net baryon density

3D HYDRODYNAMICS WITH FINITE BARYON CURRENT

$$\partial_\mu T^{\mu\nu} = J^\nu_{\text{source}}$$
$$\partial_\mu J^\mu = \rho_{\text{source}}$$

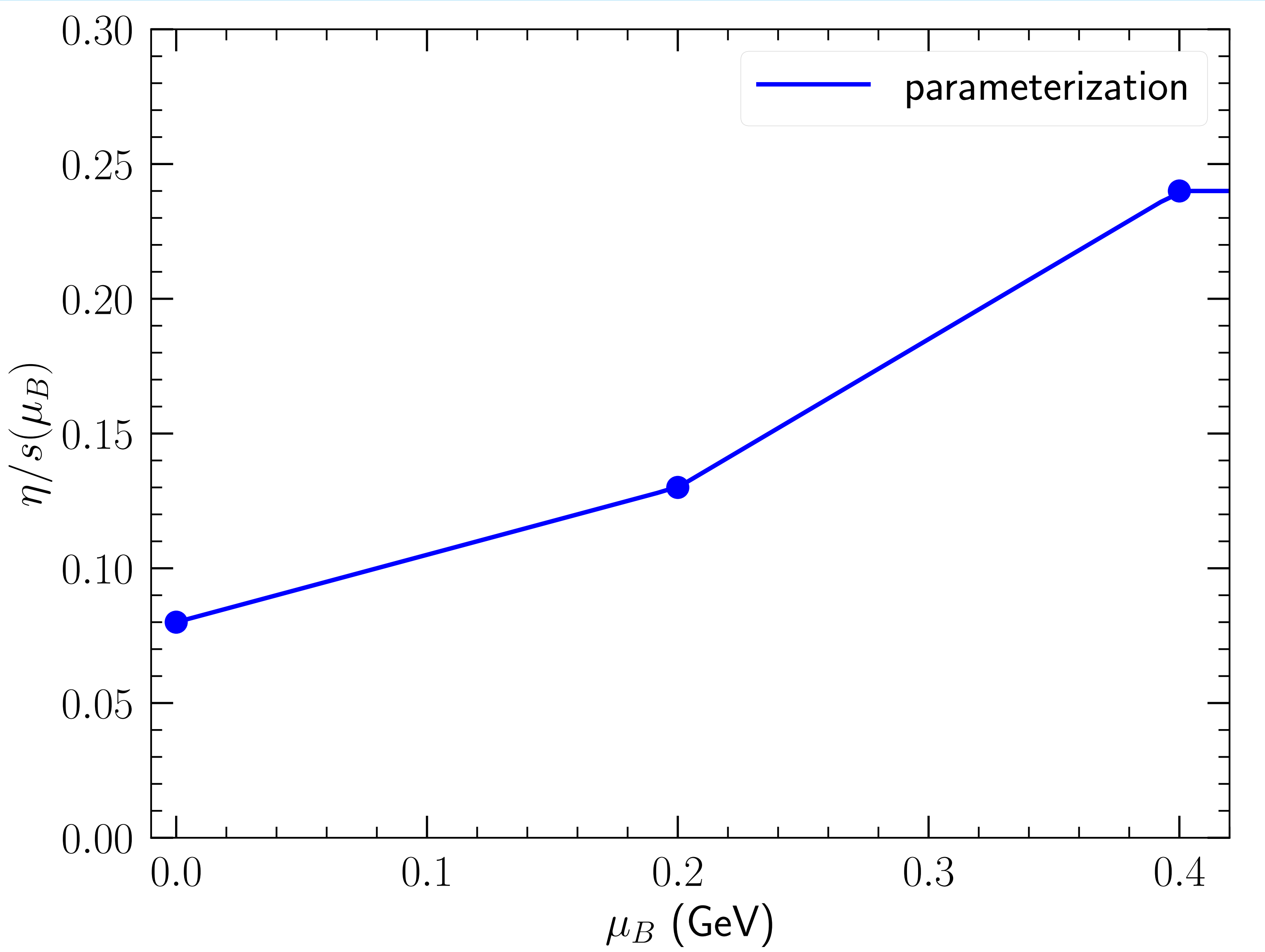
+

$n_s = 0 \quad n_Q = 0.4 n_B$

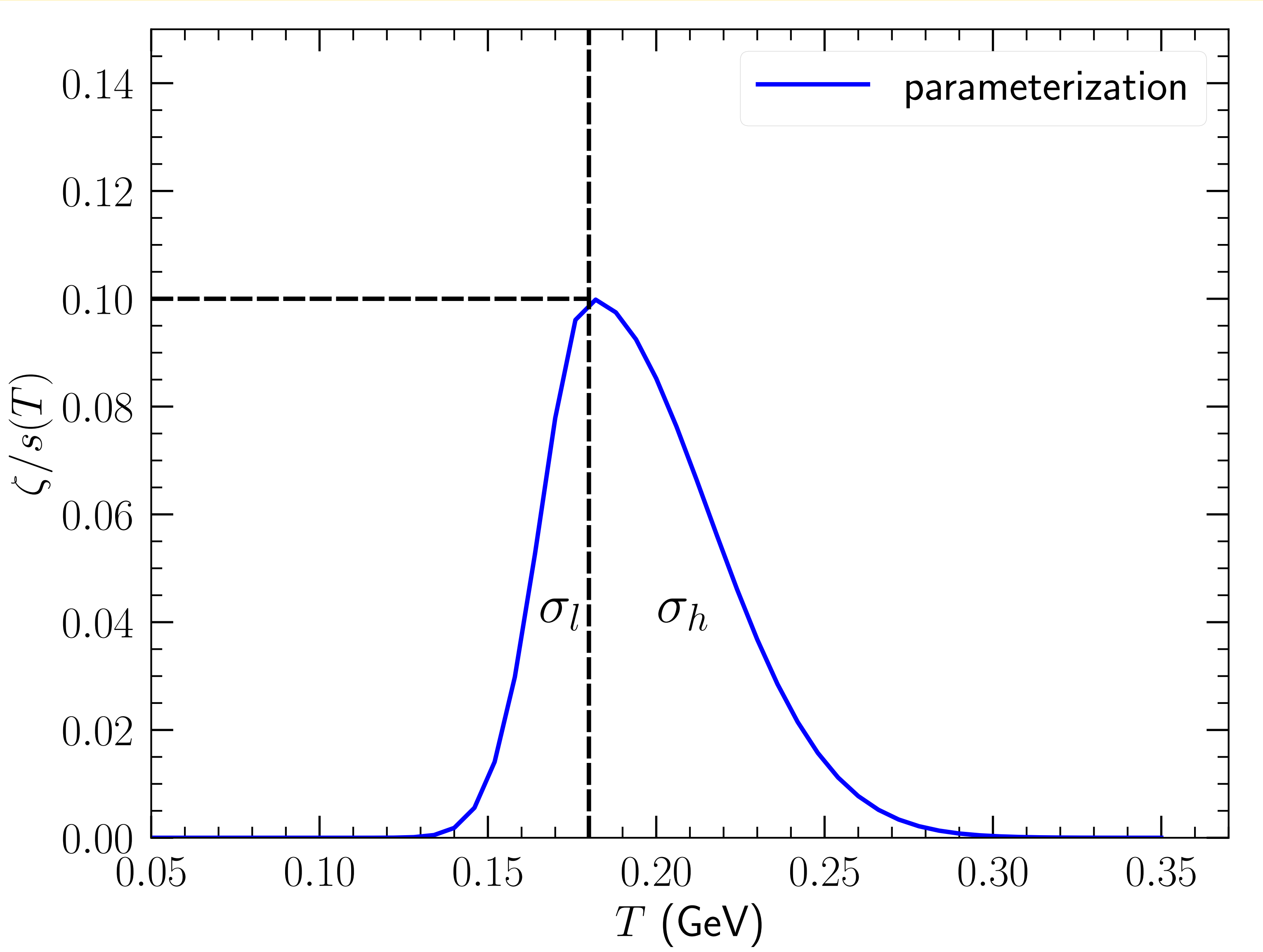


A. Monnai, B. Schenke and C. Shen, Phys. Rev. C100, 024907 (2019)

$\eta/s(\mu_B)$ has a piece-wise parameterization



$\zeta/s(T)$ is parameterized with an two-piece asymmetric Gaussian



THE INVERSE PROBLEM: BAYESIAN INFERENCE

A 20-dimensional model parameter space

Parameter	Prior	Parameter	Prior
B_G (GeV^{-2})	[1, 25]	$\alpha_{\rm string\ tilt}$	[0, 1]
$\alpha_{\rm shadowing}$	[0, 1]	$\alpha_{\rm preFlow}$	[0, 2]
$y_{\rm loss,2}$	[0, 2]	η_0	[0.001, 0.3]
$y_{\rm loss,4}$	[1, 3]	η_2	[0.001, 0.3]
$y_{\rm loss,6}$	[1, 4]	η_4	[0.001, 0.3]
$\sigma_{y_{\rm loss}}$	[0.1, 0.8]	$\zeta_{\rm max}$	[0, 0.2]
$\alpha_{\rm Rem}$	[0, 1]	$T_{\zeta,0}$ (GeV)	[0.15, 0.25]
λ_B	[0, 1]	$\sigma_{\zeta,+}$ (GeV)	[0.01, 0.15]
$\sigma_x^{\rm string}$ (fm)	[0.1, 0.8]	$\sigma_{\zeta,-}$ (GeV)	[0.005, 0.1]
$\sigma_\eta^{\rm string}$	[0.1, 1]	$e_{\rm sw}$ (GeV/fm^3)	[0.15, 0.5]

$P(y_{\rm exp}|\theta)$ →

Likelihood

Posterior

← $P(\theta|y_{\rm exp})$

~600 experimental data points

Au+Au	STAR midrapdity data vs. centrality	PHOBOS rapidity distribtion
200 GeV	$dN/dy(\pi^+, K^+, p, \bar{p})$ $\langle p_T\rangle(\pi^+, K^+, p, \bar{p})$ $v_2^{\rm ch}\{2\}, v_3^{\rm ch}\{2\}$	$dN^{\rm ch}/d\eta$ $v_2(\eta)$
19.6 GeV	$dN/dy(\pi^+, K^+, p)$ $\langle p_T\rangle(\pi^+, K^+, p, \bar{p})$ $v_2^{\rm ch}\{2\}, v_3^{\rm ch}\{2\}$	$dN^{\rm ch}/d\eta$
7.7 GeV	$dN/dy(\pi^+, K^+, p)$ $\langle p_T\rangle(\pi^+, K^+, p, \bar{p})$ $v_2^{\rm ch}\{2\}, v_3^{\rm ch}\{2\}$	

Bayes' Theorem:

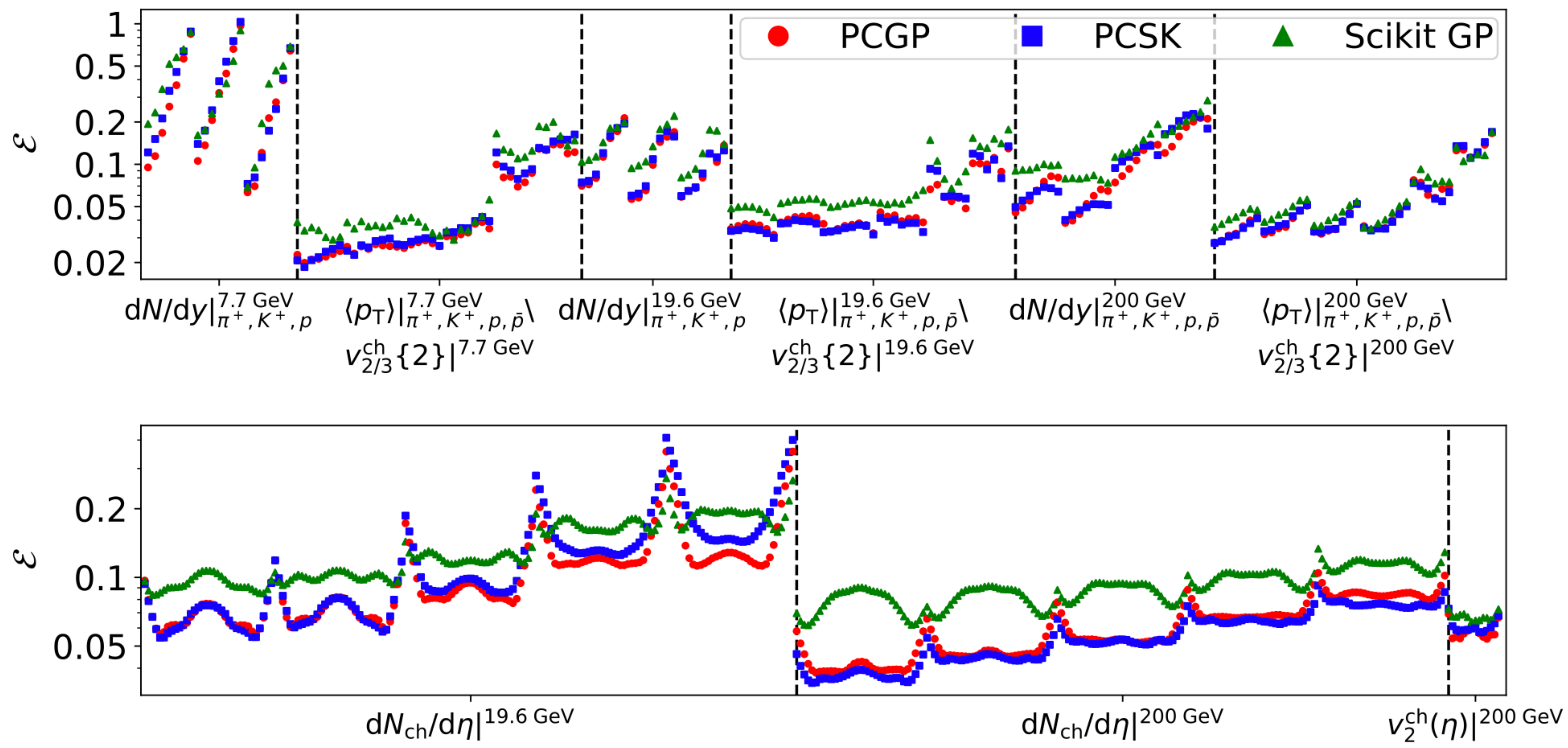
$$P(\theta|y_{\rm exp}) = \frac{P(y_{\rm exp}|\theta)P(\theta)}{P(y_{\rm exp})}$$

- Expensive models require training *emulators*

EVALUATE THE QUALITY OF MODEL EMULATION

H. Roch, S. A. Jahan and C. Shen, Phys. Rev. C 110, 044904 (2024)

$$\mathscr{E} = \sqrt{\left\langle \left(\frac{\text{prediction} - \text{truth}}{\text{truth}} \right)^2 \right\rangle}$$



Bayesian Analysis of Nuclear Dynamics

- With the same amount of training data, the PCGP and PCSK emulators are more accurate than the standard GP from the scikit-learn package

EVALUATE THE QUALITY OF MODEL EMULATION

H. Roch, S. A. Jahan and C. Shen, Phys. Rev. C 110, 044904 (2024)

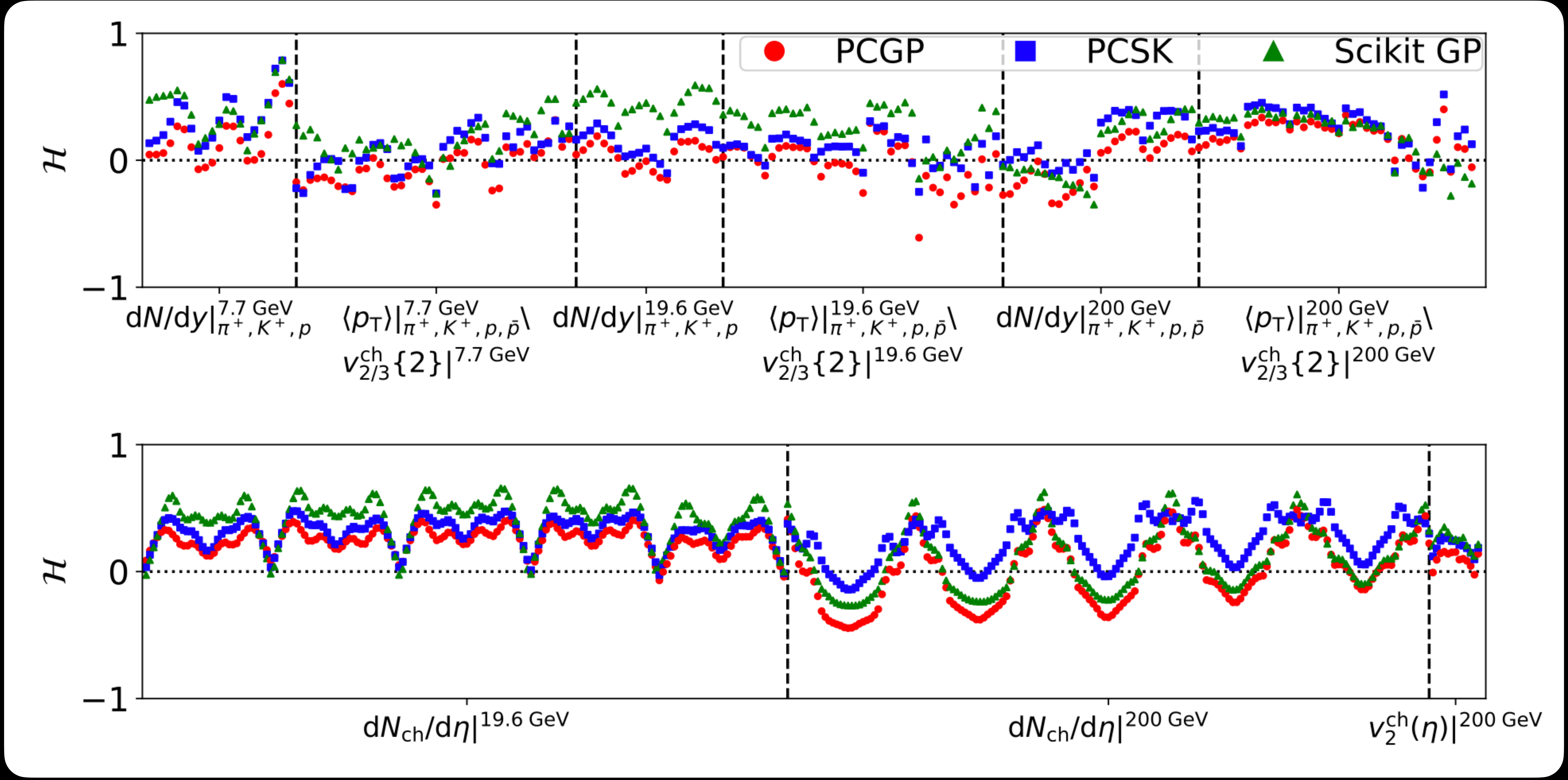
$$\mathscr{H} = \ln\left(\sqrt{\left\langle\left(\frac{\text{prediction} - \text{truth}}{\text{pred . uncertainty}}\right)^2\right\rangle}\right)$$

$\mathscr{H} = 0$: Best

$\mathscr{H} > 0$: Predicted uncertainty is too small

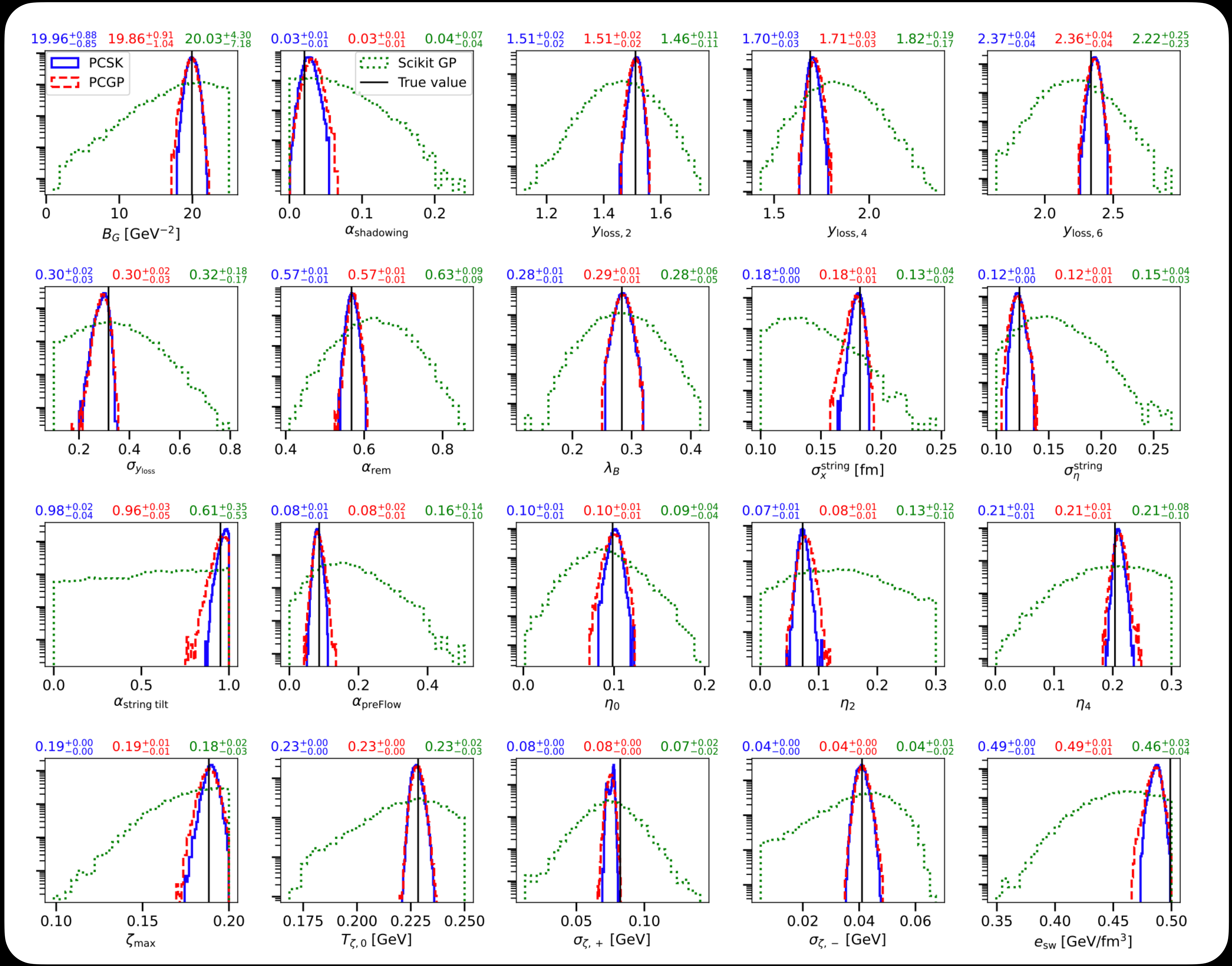
$\mathscr{H} < 0$: Predicted uncertainty is too large

Bayesian Analysis of Nuclear Dynamics



- The PCGP and PCSK emulators also give more reliable uncertainty estimation than that from the Scikit GP

IMPACTS OF EMULATOR PRECISION ON CLOSURE TESTS



- The more accurate PCGP and PCSK emulators give tighter posterior on model parameters than that from the Scikit GP

$$\Delta \equiv \frac{1}{N_{\mathrm{param.}}} \int \left| \frac{\boldsymbol{\theta} - \boldsymbol{\theta}_{\mathrm{truth}}}{\boldsymbol{\theta}_{\mathrm{max}} - \boldsymbol{\theta}_{\mathrm{min}}} \right|^2 p(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta},$$

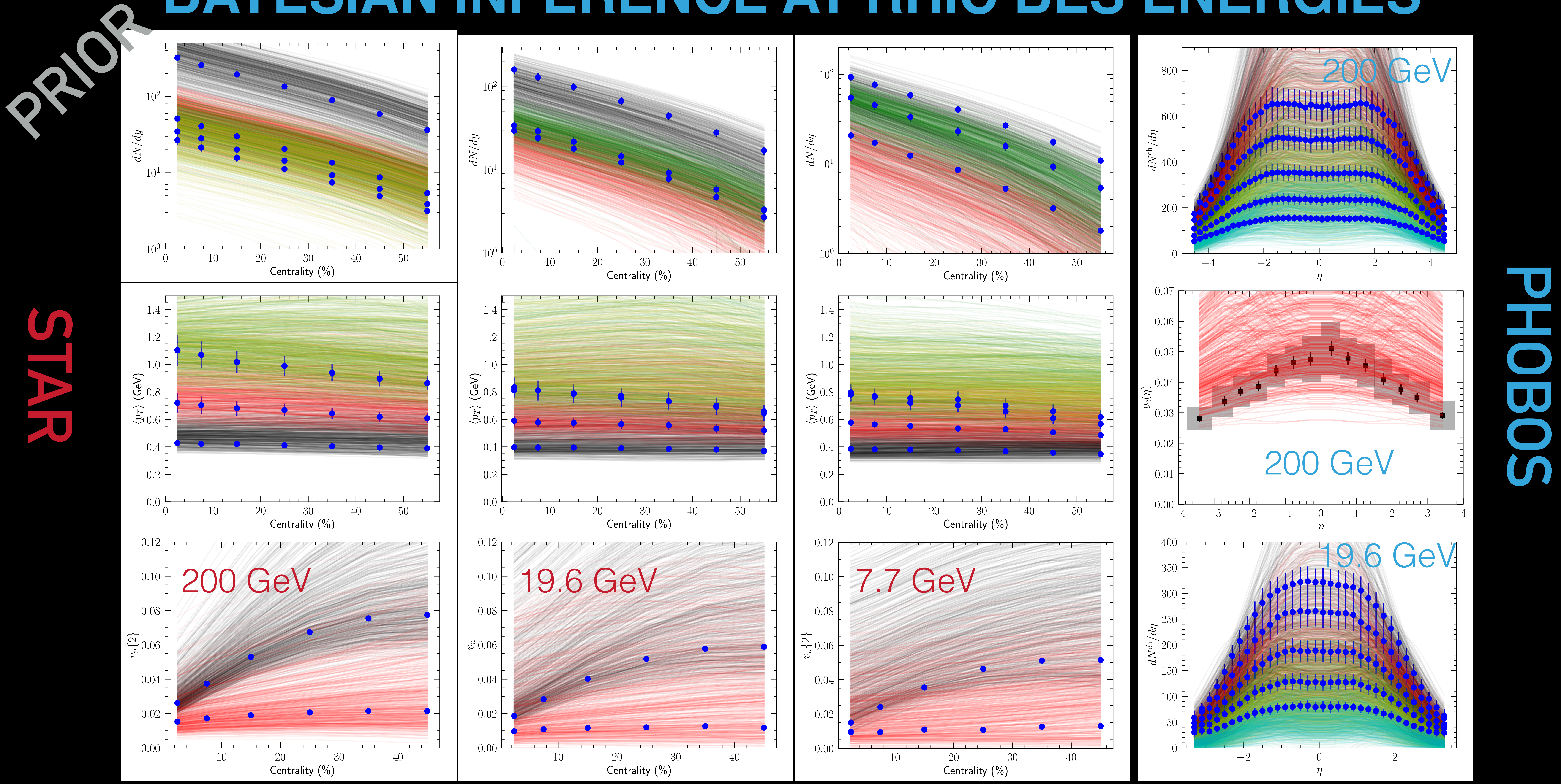
Emulator	Δ
PCGP	4.5×10^{-4}
PCSK	5.7×10^{-4}
Scikit GP	2.5×10^{-2}

The smaller the better

BAYESIAN VALIDATION: CLOSURE TEST

- Model emulation with Markov Chain Monte Carlo (MCMC) is verified with a closure test for initial-state stopping $y_{\rm loss}(y_{\rm init})$, $\eta/s(\mu_B)$, and $\zeta/s(T)$
- The selected observables can give strong constraints on the QGP properties at RHIC BES energies

BAYESIAN INFERENCE AT RHIC BES ENERGIES



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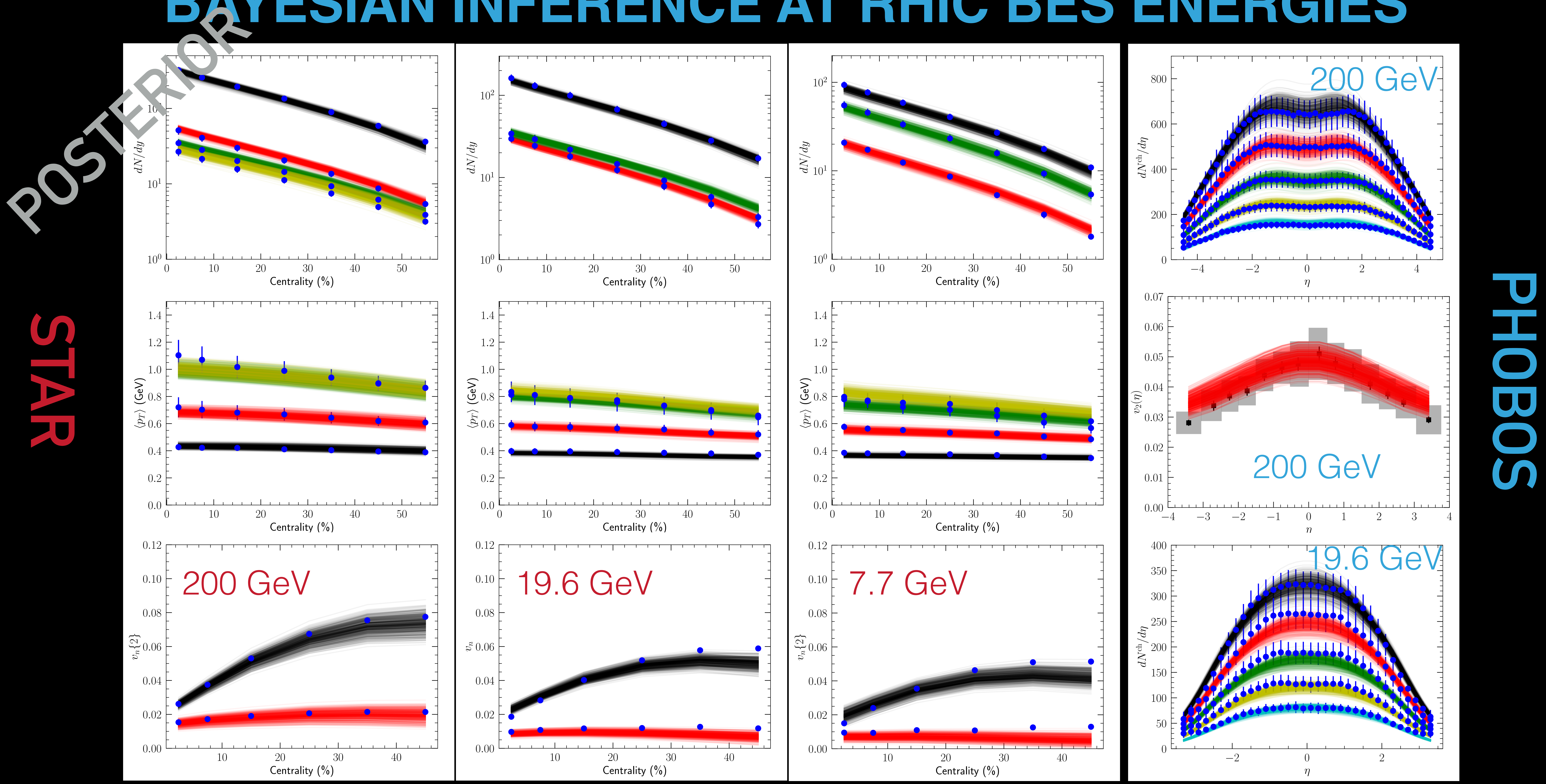
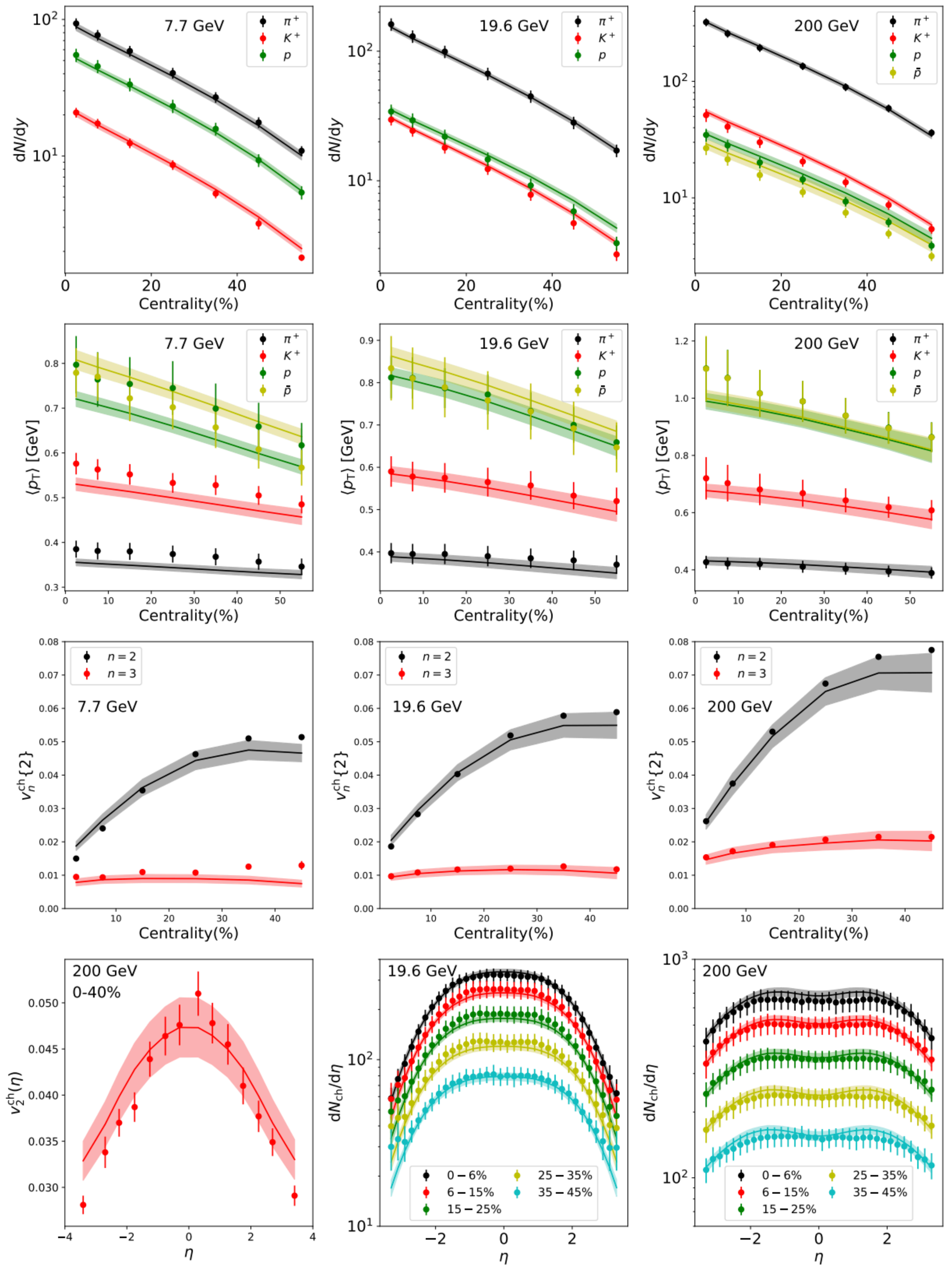
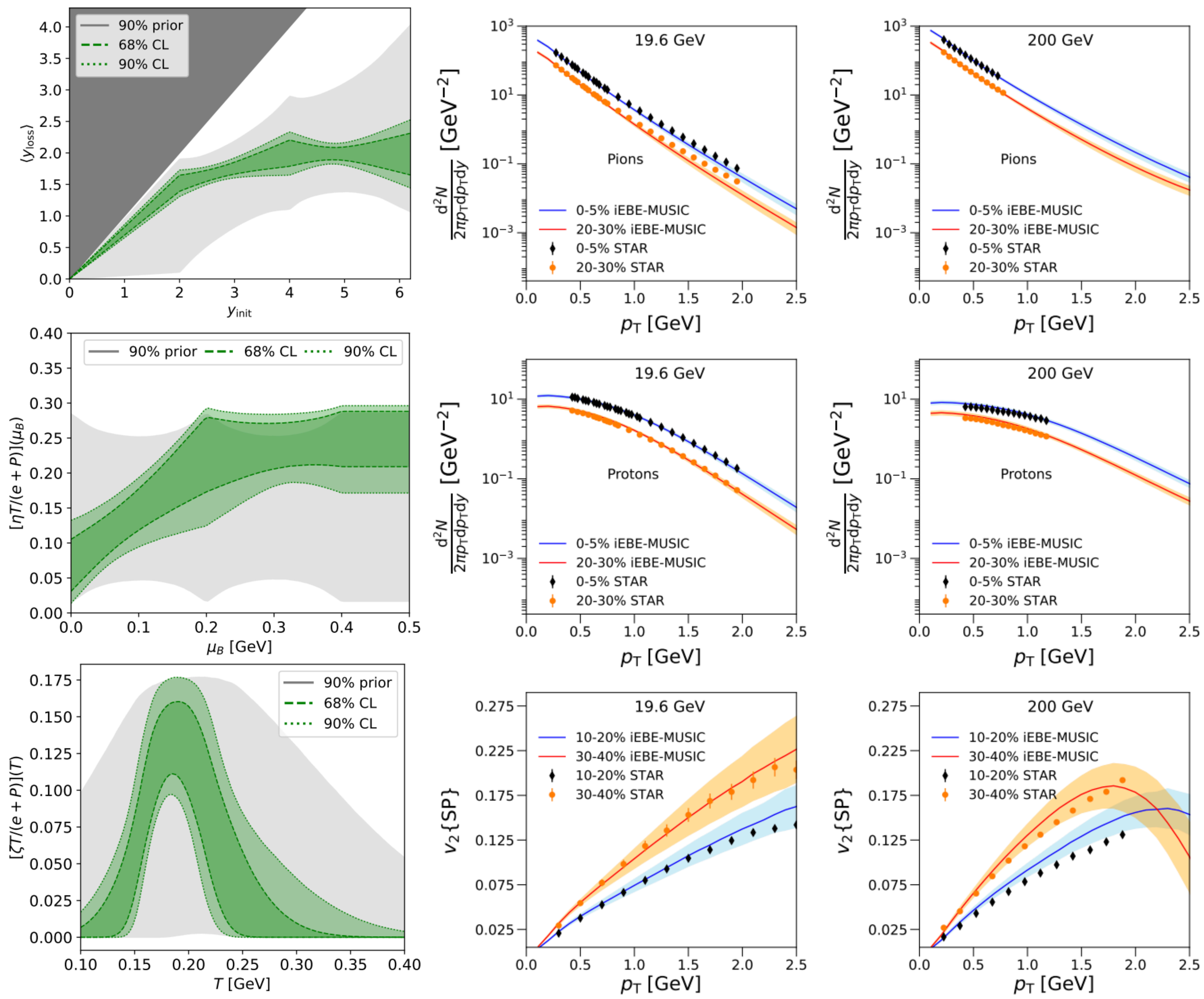


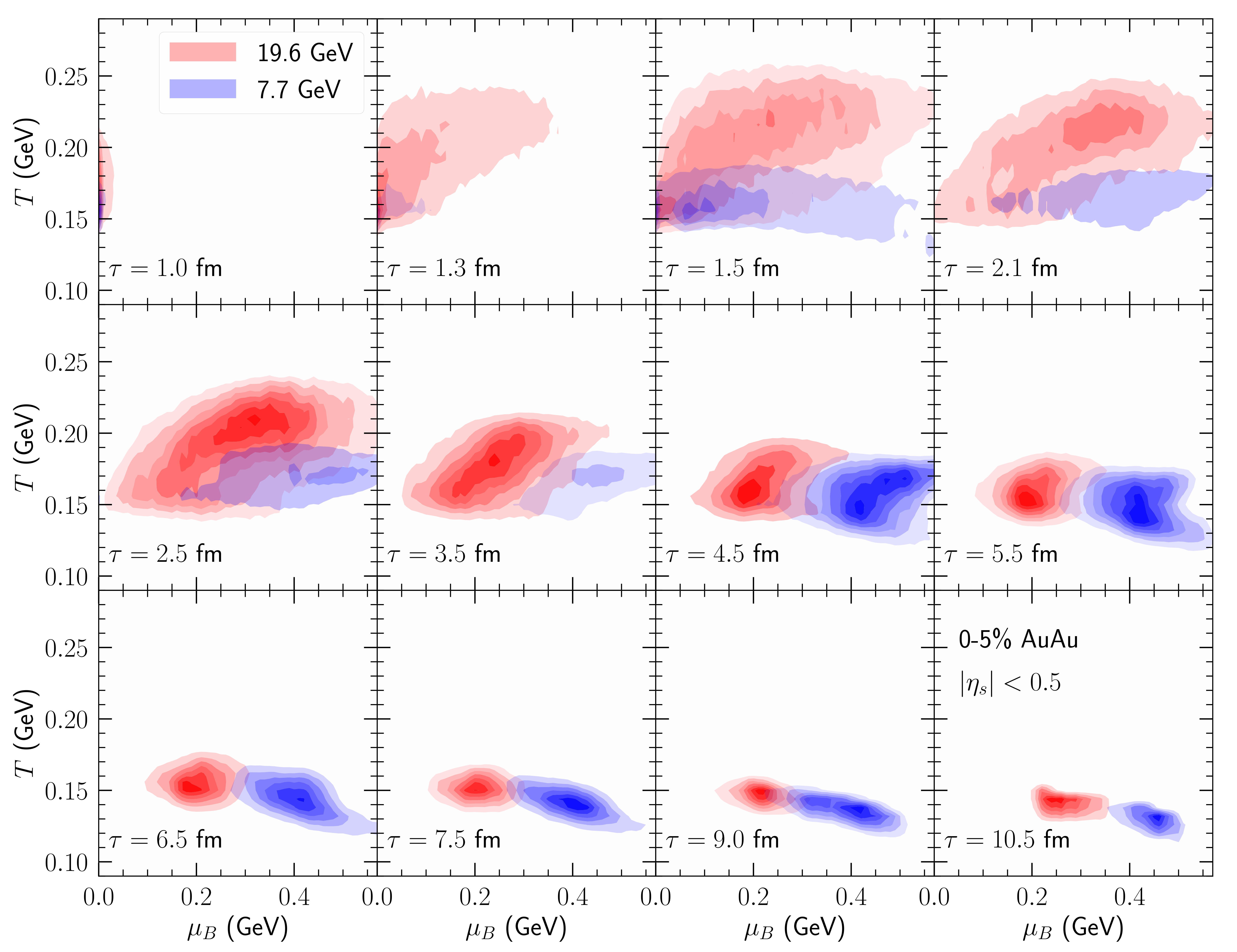
GLOBAL BAYESIAN CONSTRAINTS ON QGP PROPERTIES

S. A. Jahan, H. Roch and C. Shen, Phys. Rev. C 110, 054905



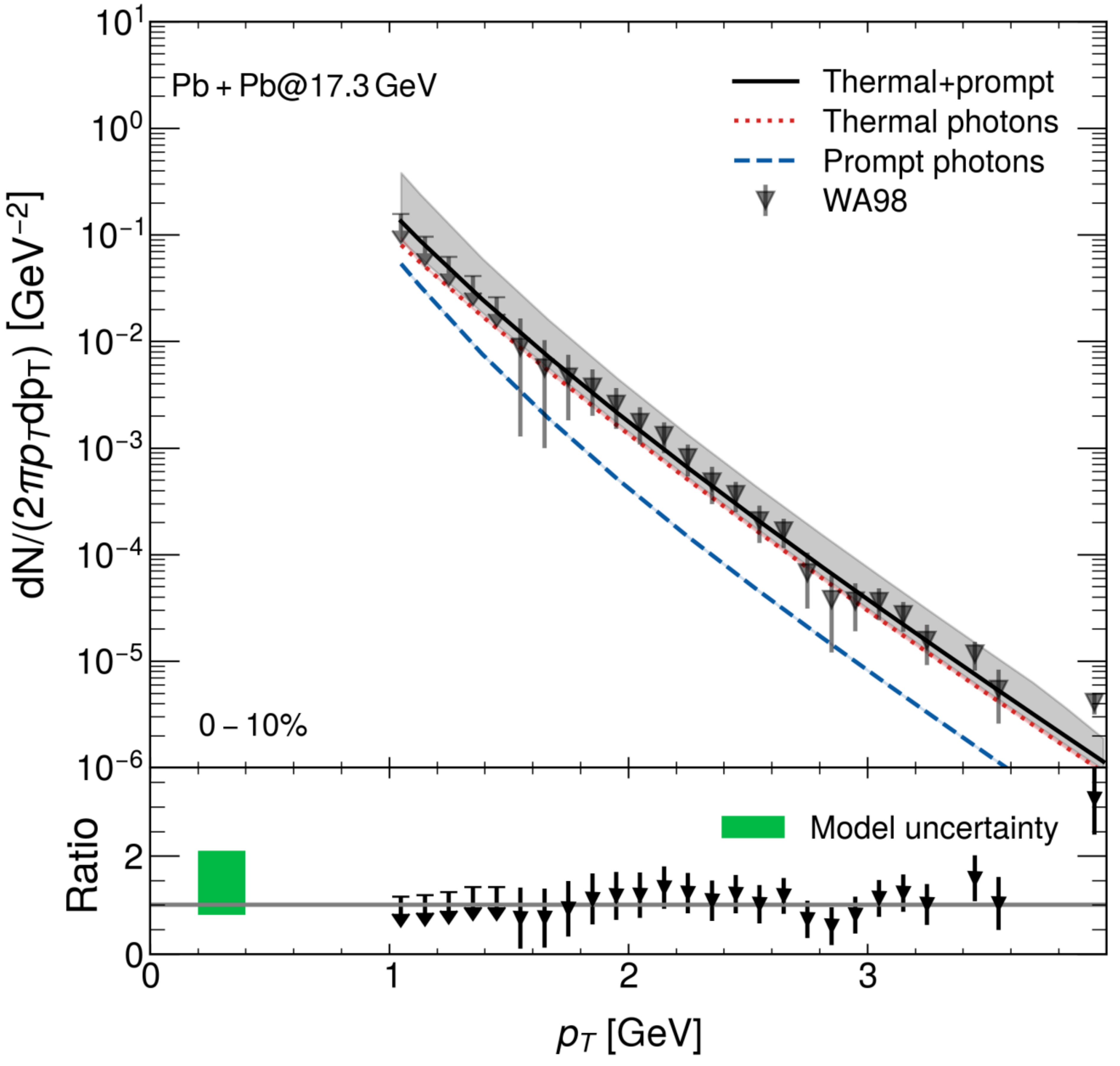


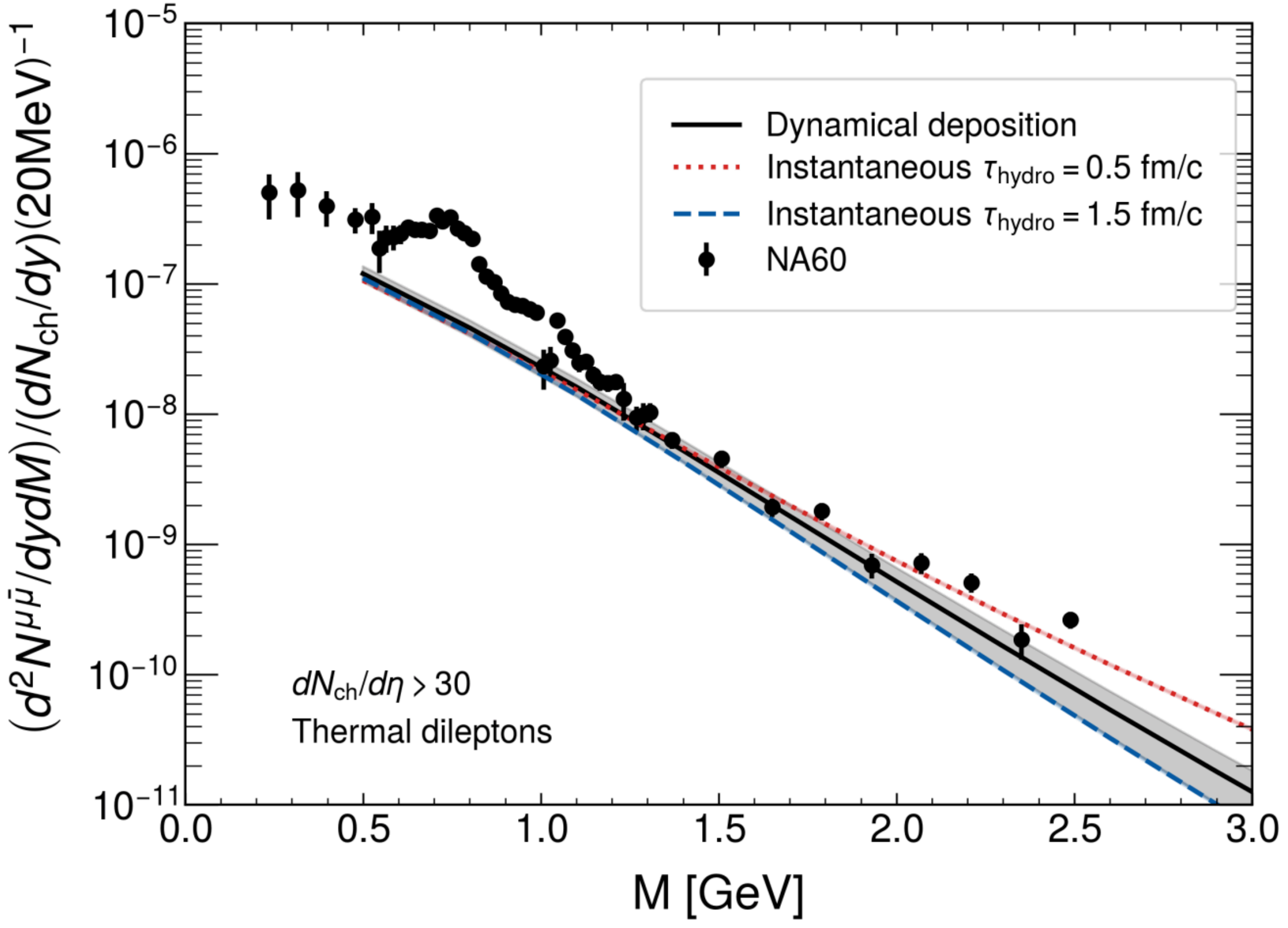
PHASE TRAJECTORIES FOR STOCHASTIC FLUCTUATIONS



EM RADIATION AT FINITE BARYON DENSITIES

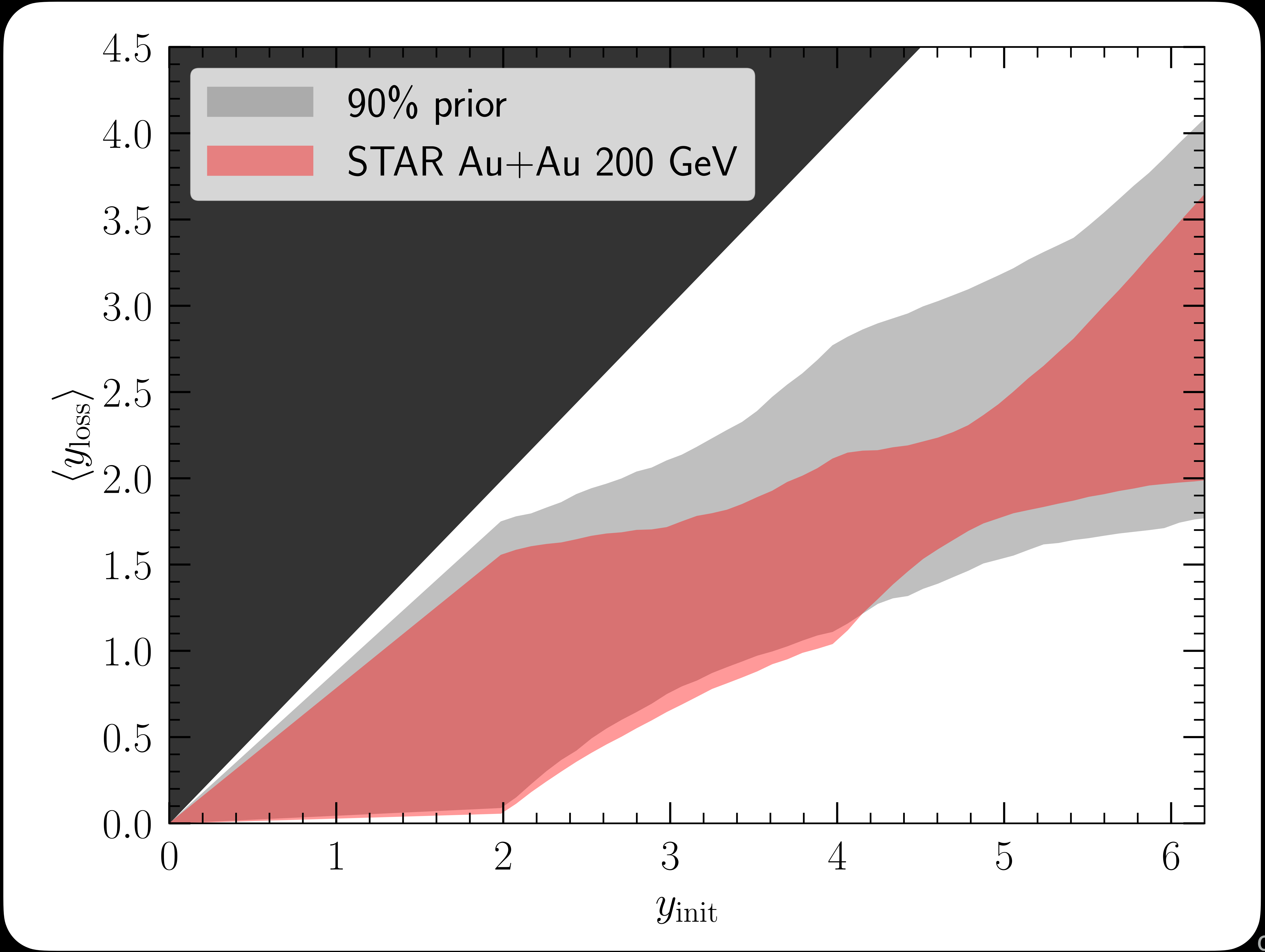
X. Y. Wu, C. Gale, S. Jeon, J. F. Paquet, B. Schenke, and C. Shen, arXiv:2511.08773 [nucl-th]





- Predictions of EM radiation verified the space-time evolution of heavy-ion collisions at finite net baryon densities

INITIAL-STATE STOPPING



- Mid-rapidity particle productions at 200 GeV yields $y_{\rm loss} \sim 2$ for $y_{\rm init} \sim 5$

color bands indicate 90% credible interval in the posterior

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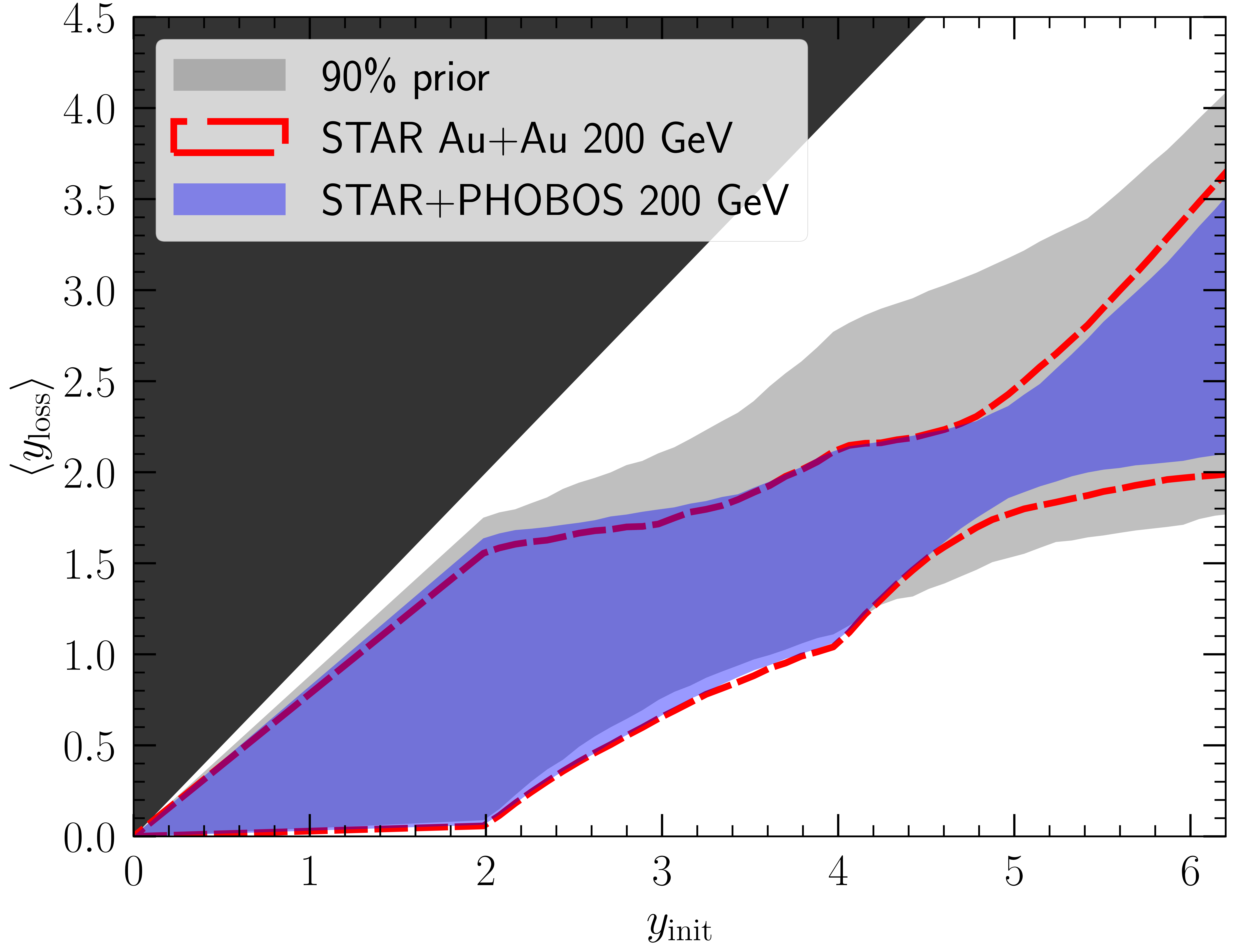

- Mid-rapidity particle productions at 200 GeV yields $y_{\rm loss} \sim 2$ for $y_{\rm init} \sim 5$
- The rapidity distributions from PHOBOS give small improvements to the constraint

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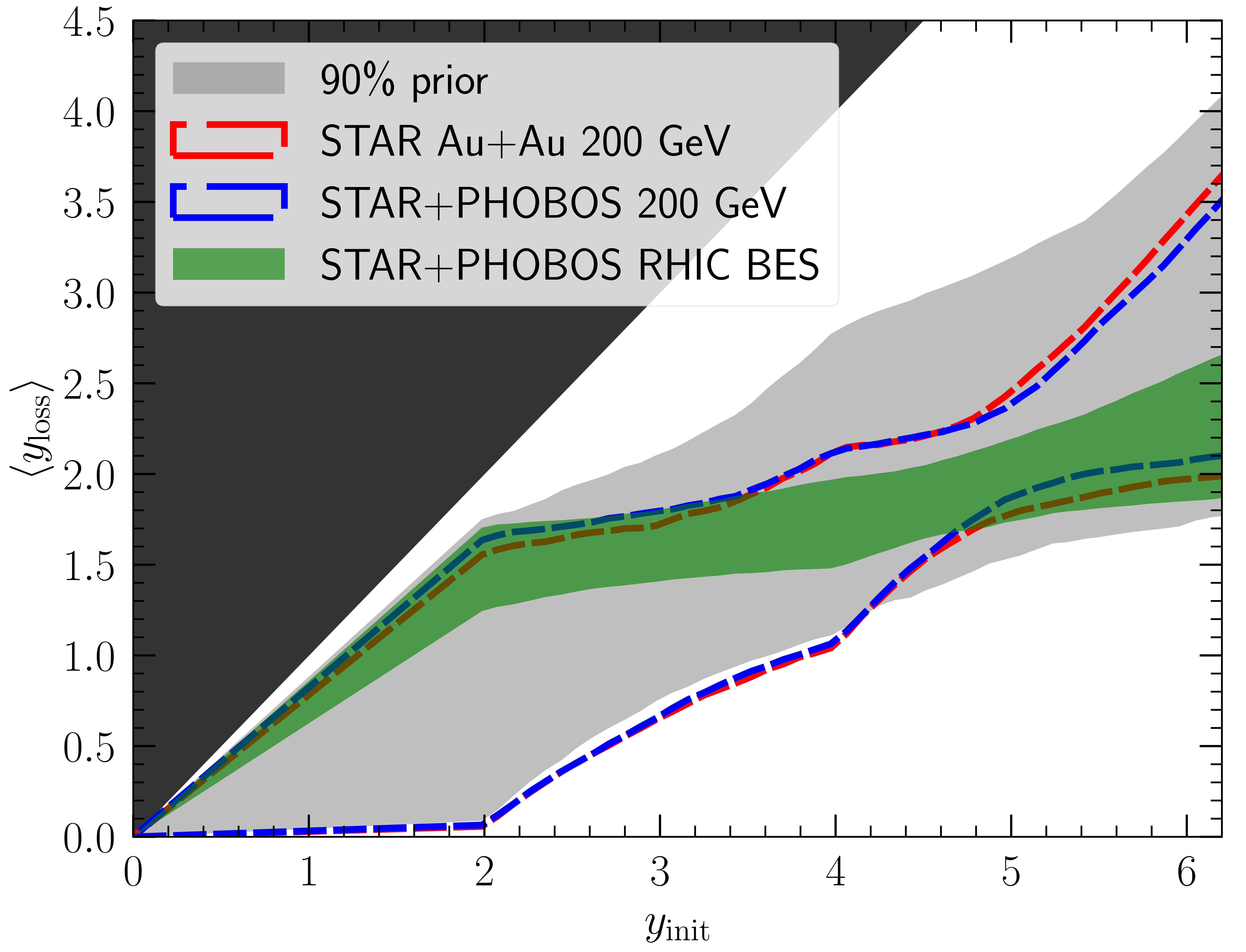

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- Particle production from 7.7, 19.6, and 200 GeV sets strong constrain on $y_{loss}(y_{init})$ for $y_{init} \in [0,6]$

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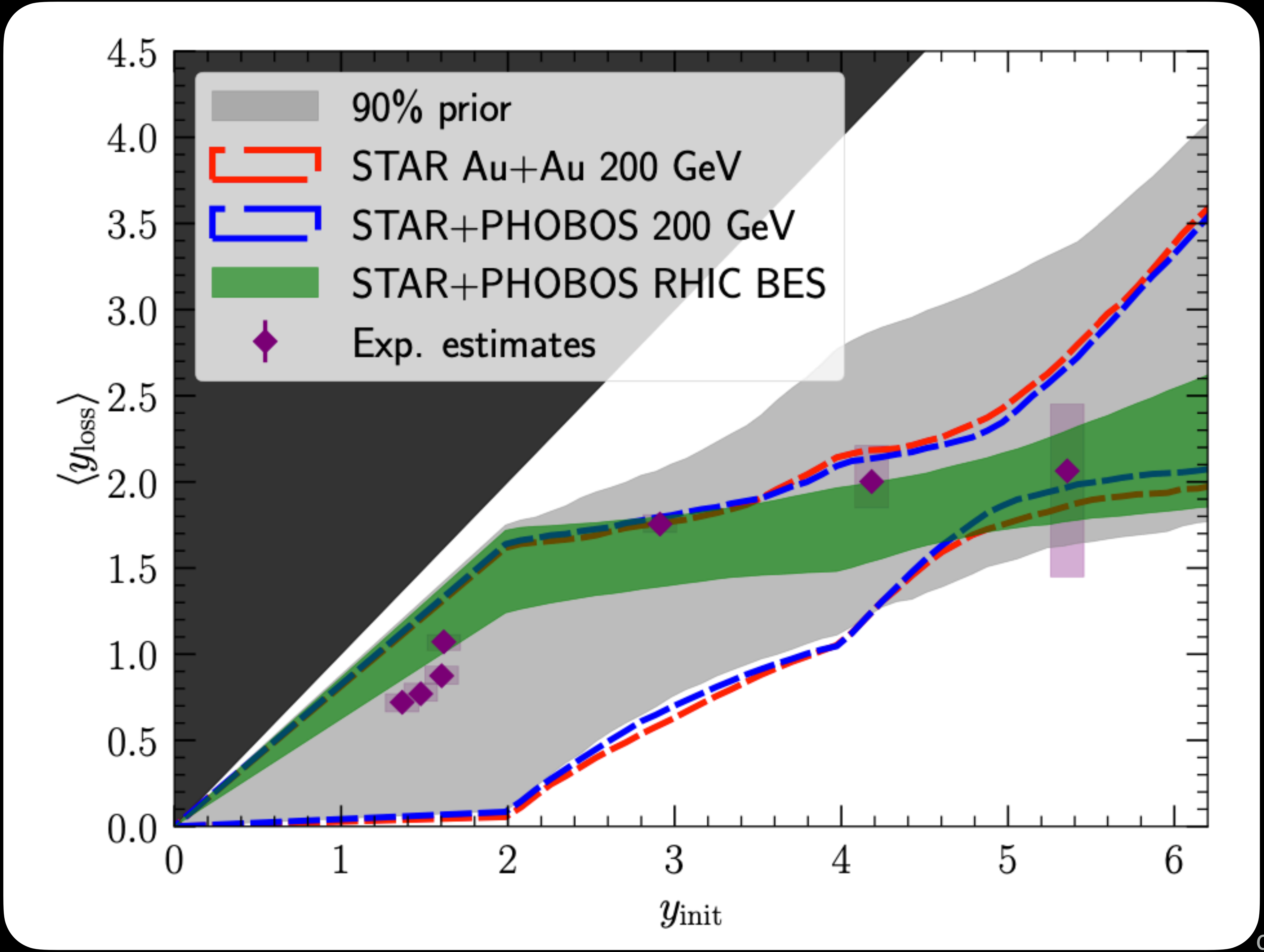

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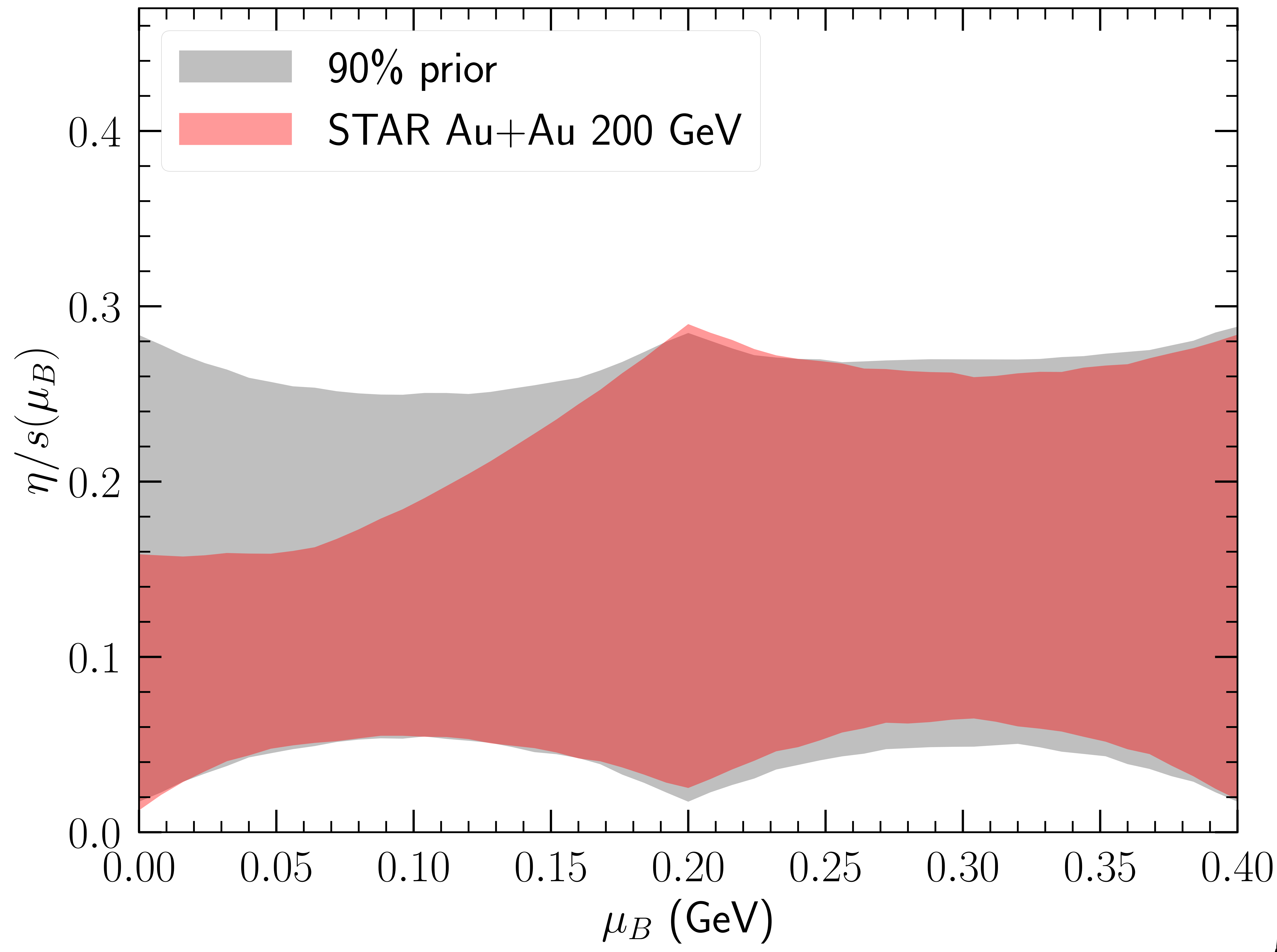
SHEAR VISCOSITY $\eta/s(\mu_B)$



- Mid-rapidity data at 200 GeV can constrain η/s around $\mu_B = 0$

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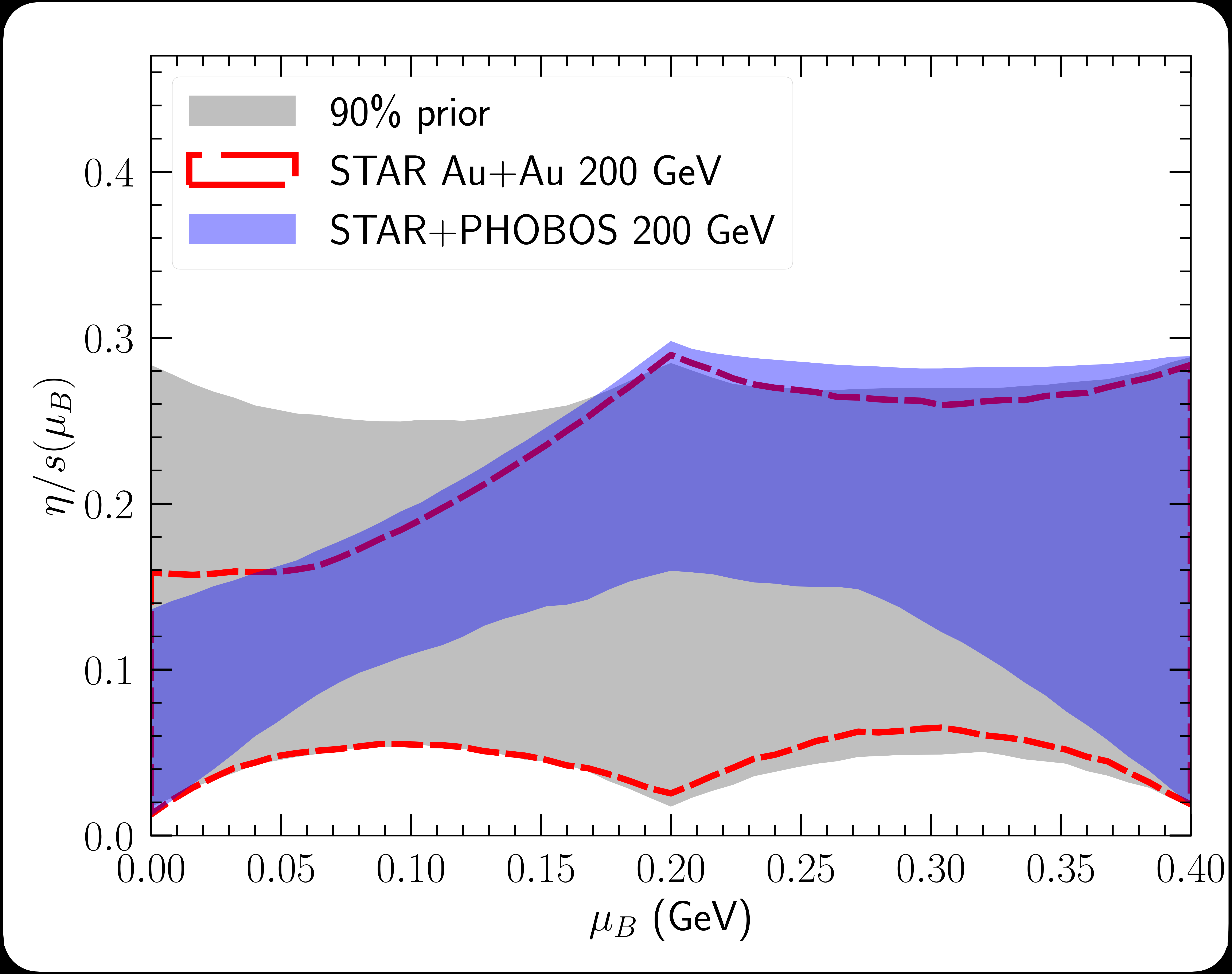

- Mid-rapidity data at 200 GeV can constrain η/s around $\mu_B = 0$
- The $dN^{\rm ch}/d\eta$ and $v_2(\eta)$ at 200 GeV *significantly* improve the η/s constraint at $\mu_B \sim 0.2$ GeV

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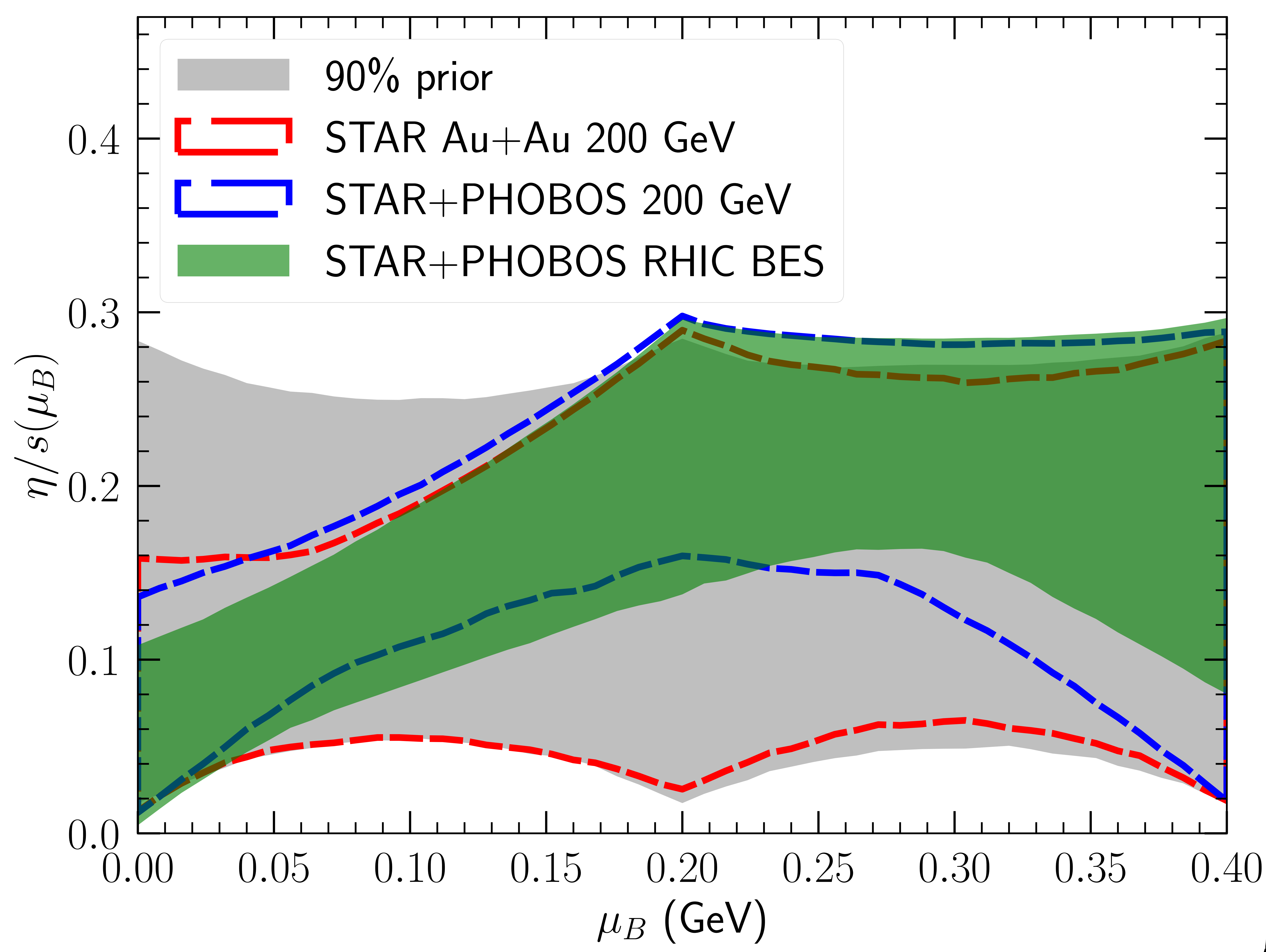
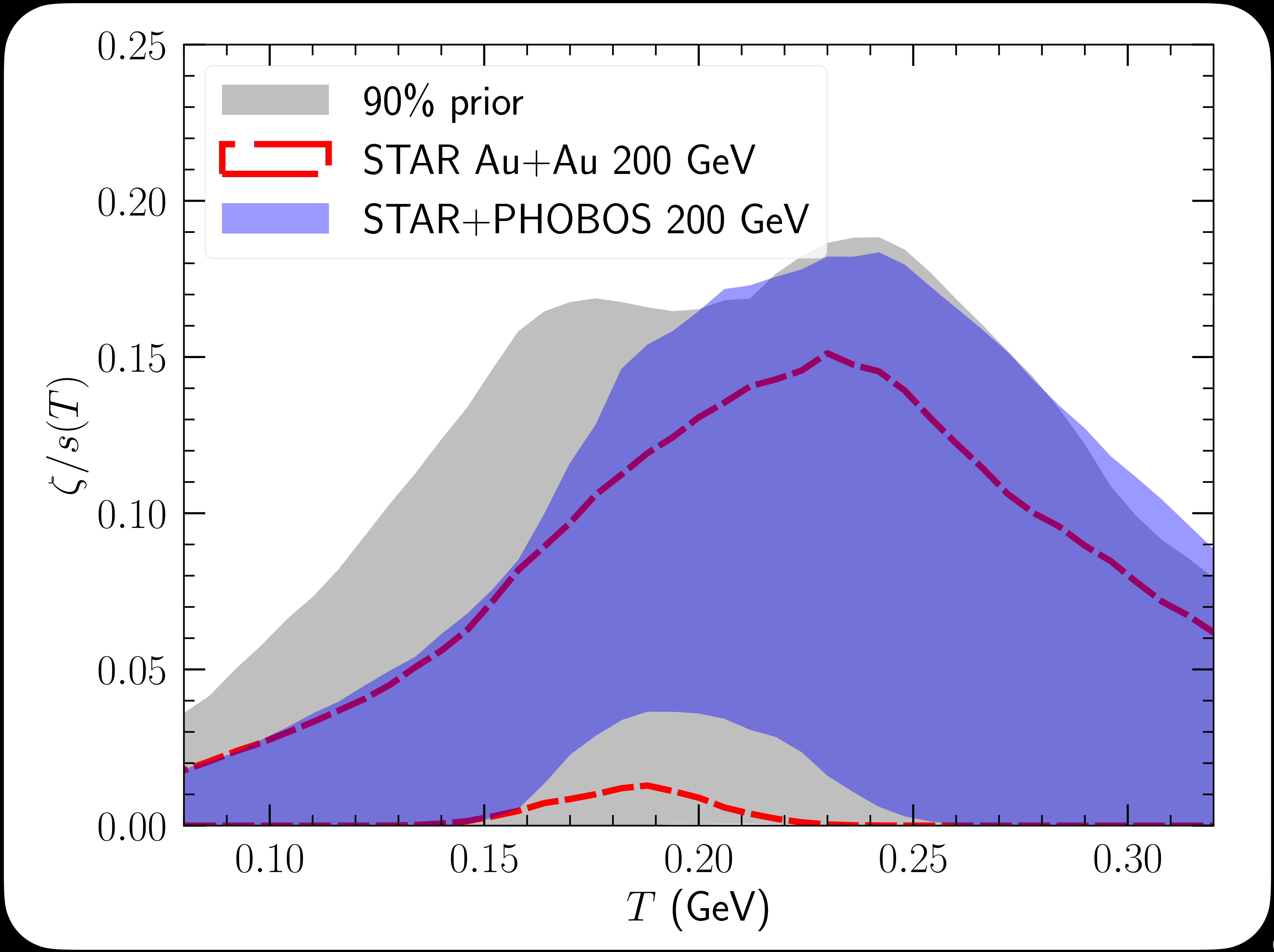


- Mid-rapidity data at 200 GeV can constrain η/s around $\mu_B = 0$
- The $dN^{\rm ch}/d\eta$ and $v_2(\eta)$ at 200 GeV *significantly* improve the η/s constraint at $\mu_B \sim 0.2$ GeV
- The full RHIC BES data (STAR+PHOBOS) shows that the QGP η/s is **larger** at finite μ_B than that at $\mu_B = 0$

color bands indicate 90% credible interval in the posterior

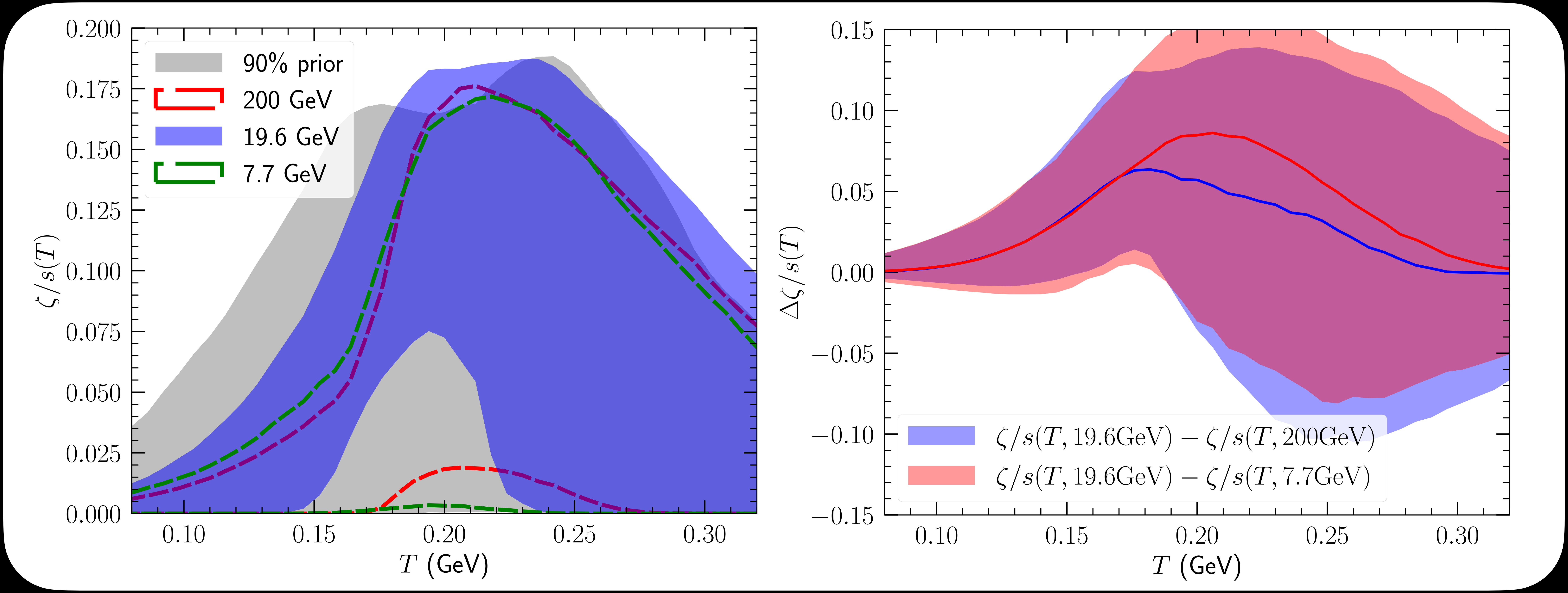
BULK VISCOSITY $\zeta/s(T)$



- Mid-rapidity identified particle yields and their $\langle p_T \rangle$ at 200 GeV set constraints on the temperature dependence of the QGP bulk viscosity
- The additional PHOBOS data shifts the posterior $\zeta/s(T)$ to larger values

color bands indicate 90% credible interval in the posterior

BULK VISCOSITY $\zeta/s(T,\sqrt{s})$



color bands indicate 90% credible interval in the posterior

- Allowing $\zeta/s(T)$ to be an independent function for the three collision energies, our calibration suggests a larger $\zeta/s(T)$ at 19.6 GeV than those at 200 and 7.7 GeV for $T \in [0.15, 0.2]$ GeV

Hint for softening(hardening) EoS at $\mu_B = 0.2(0.4)$ *GeV?*

BAYESIAN MODEL SELECTION

Can we do better by introducing energy-dependence on model parameters?

$$\theta \rightarrow \theta(\sqrt{s})$$

Select the optimal model using the Bayes factor $\mathscr{B}_{A/B} = \dfrac{P(y_{\rm exp}|A)}{P(y_{\rm exp}|B)}$

Model A	Model B	$\ln(\mathcal{B}_{A/B})$
no $\sqrt{s_{\rm NN}}$	$\alpha_{\rm shadowing}(\sqrt{s_{\rm NN}})$	0.88 ± 0.09
no $\sqrt{s_{\rm NN}}$	$\alpha_{\rm rem}(\sqrt{s_{\rm NN}})$	0.2 ± 0.3
no $\sqrt{s_{\rm NN}}$	$\lambda_B(\sqrt{s_{\rm NN}})$	0.2 ± 0.08
no $\sqrt{s_{\rm NN}}$	$\sigma_x(\sqrt{s_{\rm NN}})$	-2.00 ± 0.10
no $\sqrt{s_{\rm NN}}$	$\sigma_\eta(\sqrt{s_{\rm NN}})$	-1.6 ± 0.2
no $\sqrt{s_{\rm NN}}$	$\alpha_{\rm preFlow}(\sqrt{s_{\rm NN}})$	-0.2 ± 0.2
no $\sqrt{s_{\rm NN}}$	$e_{\rm sw}(\sqrt{s_{\rm NN}})$	0.57 ± 0.08
no $\sqrt{s_{\rm NN}}$	$\sigma_x(\sqrt{s_{\rm NN}}), \sigma_\eta(\sqrt{s_{\rm NN}})$	-3.01 ± 0.08

Introducing $\sigma_x(\sqrt{s})$ and $\sigma_\eta(\sqrt{s})$ are favored, while the Bayes factor penalizes other irrelevant parameters

- $\ln(\mathscr{B}) > 0$: Model B disfavored
- $-5 < \ln(\mathscr{B}) < 0$: Model B moderately favored
- $\ln(\mathscr{B}) < -5$: Model B strongly favored

BAYESIAN MODEL SELECTION

S. A. Jahan, H. Roch and C. Shen, Phys. Rev. C113, 024919 (2026)

Can we do better by introducing energy-dependence on model parameters?

$$\theta \to \theta(\sqrt{s})$$

Select the optimal model using the Bayes factor $\mathscr{B}_{A/B} = \frac{P(y_{\exp} | A)}{P(y_{\exp} | B)}$

Model A	Model B	$\ln(\mathscr{B}_{B/A})$
$\eta/s(T)$	$\eta/s = 0$	11.7
$\eta/s(T)$	$(\eta/s)_{\rm eff}$	-0.2

D. Everett *et al.* [JETSCAPE], Phys. Rev. C103, 054904 (2021)

Model A	Model B	$\ln(\mathscr{B}_{B/A})$
$\eta/s(\mu_B)$	$(\eta/s)_{\rm eff}$	1.9

- $\ln(\mathscr{B}) > 0$: Model B disfavored
- $-5 < \ln(\mathscr{B}) < 0$: Model B moderately favored
- $\ln(\mathscr{B}) < -5$: Model B strongly favored

OBSERVABLE RESPONSES TO MODEL PARAMETERS

Red: Positive correlation; Blue: Negative correlation

OBSERVABLE RESPONSES TO MODEL PARAMETERS

- Particle yields provide strong constraints on initial-state energy (rapidity) loss parameters
- The opposite correlations between mid-rapidity and forward in PHOBOS pseudo-rapidity distributions can provide complement constraints to those from STAR mid-rapidity measurements at the multiple collision energies

Red: Positive correlation; Blue: Negative correlation

OBSERVABLE RESPONSES TO MODEL PARAMETERS

- Shear viscosity shows positive correlation with particle yields — entropy production; negative correlation with anisotropic flow — viscous damping
- PHOBOS pseudo-rapidity distributions at 200 GeV offers additional constrains on $\eta/s(\mu_B)$

Red: Positive correlation; Blue: Negative correlation

OBSERVABLE RESPONSES TO MODEL PARAMETERS

- Bulk viscosity shows positive correlation with particle yields — entropy production; negative correlation with mean p_T — resistance to expansion
- Particle yields at forward and backward pseudo-rapidity shows stronger correlation with $(\zeta/s)_{\rm max}$ than those with the mid-rapidity particle yields

Red: Positive correlation; Blue: Negative correlation

MOVING TOWARDS NEXT-GENERATION BAYESIAN INFERENCE

THEORETICAL UNCERTAINTY

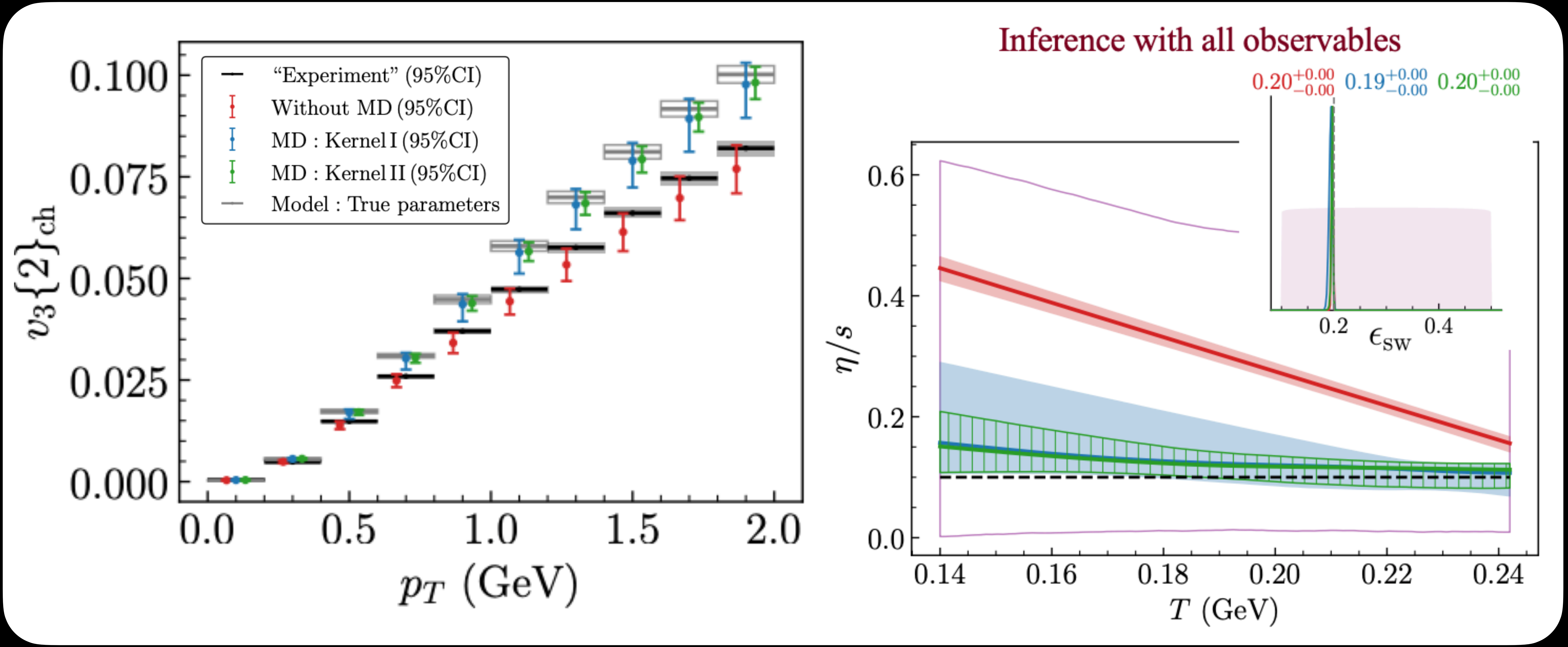
S. Jaiswal, C. Shen, R. J. Furnstahl, U. Heinz and M. T. Pratola, arXiv:2504.13144 [hep-ph]

Theoretical models are not perfect. Modeling its error $\delta(p_T)$ with Gaussian Process in a data-driven approach,

$$y(p_T) = \eta(p_T, \theta) + \epsilon_{\rm exp} \quad \Longrightarrow \quad y(p_T) = \eta(p_T, \theta) + \delta(p_T) + \epsilon_{\rm exp}$$

Validate this framework with mock data (w. Grad δf)

Enabled robust inference with observables with different theoretical fidelities



SEQUENTIAL BAYESIAN INFERENCE

Today's posterior is tomorrow's prior

$$P(\theta|D_1,D_2) = \frac{P(D_1,D_2|\theta)P(\theta)}{P(D_1,D_2)}$$

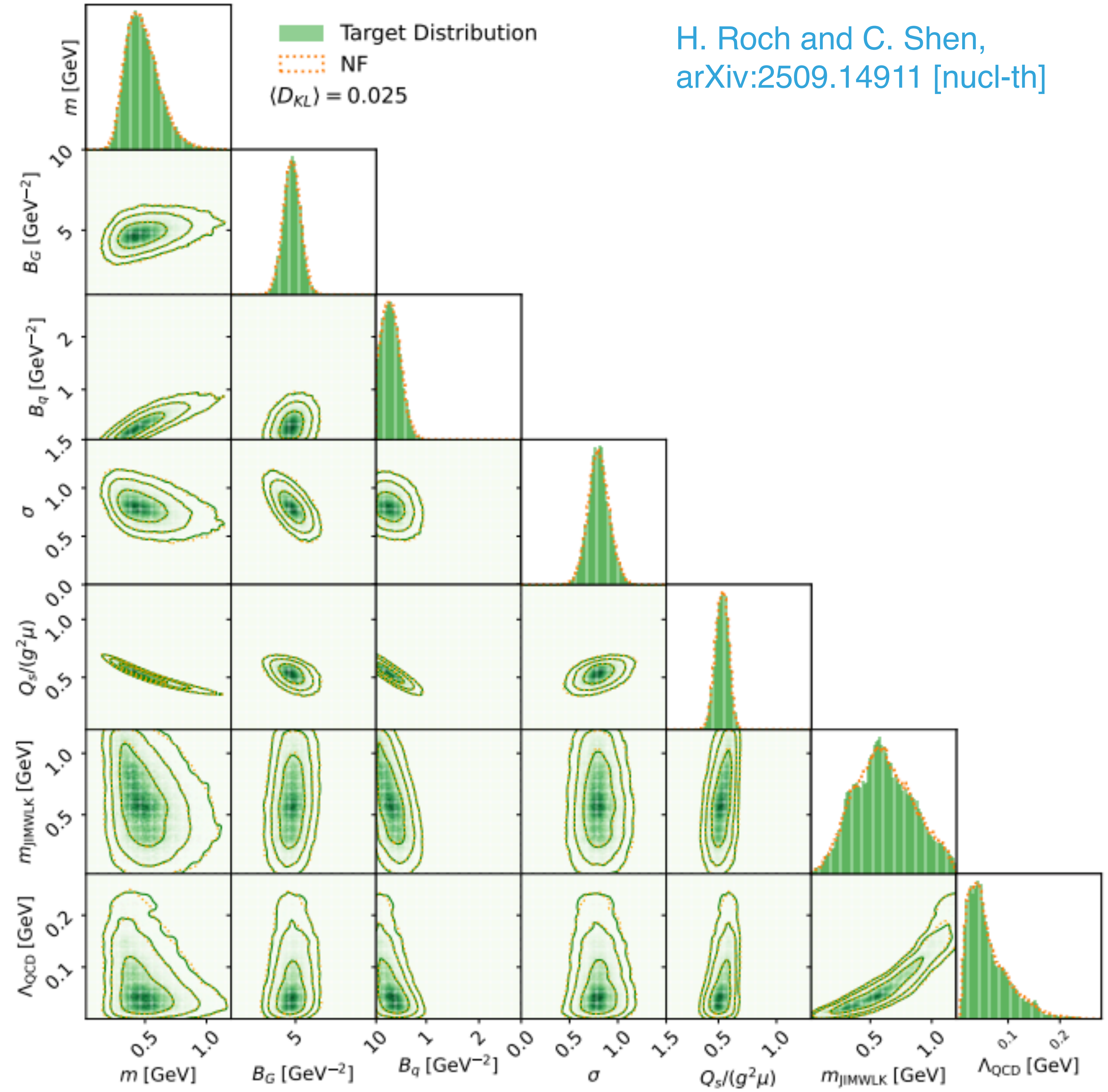
$$= \frac{P(D_2|\theta)P(D_1|\theta)P(\theta)}{P(D_2)P(D_1)}$$

$$= \frac{P(D_2|\theta)P(\theta|D_1)}{P(D_2)}$$

The distribution $P(\theta|D_1)$ is highly correlated and non-trivial to sample

A normalizing-flow generative model learns $P(\theta|D_1)$

Y. Yamauchi, L. Buskirk, P. Giuliani and K. Godbey, arXiv:2310.04635 [nucl-th]



SEQUENTIAL BAYESIAN INFERENCE

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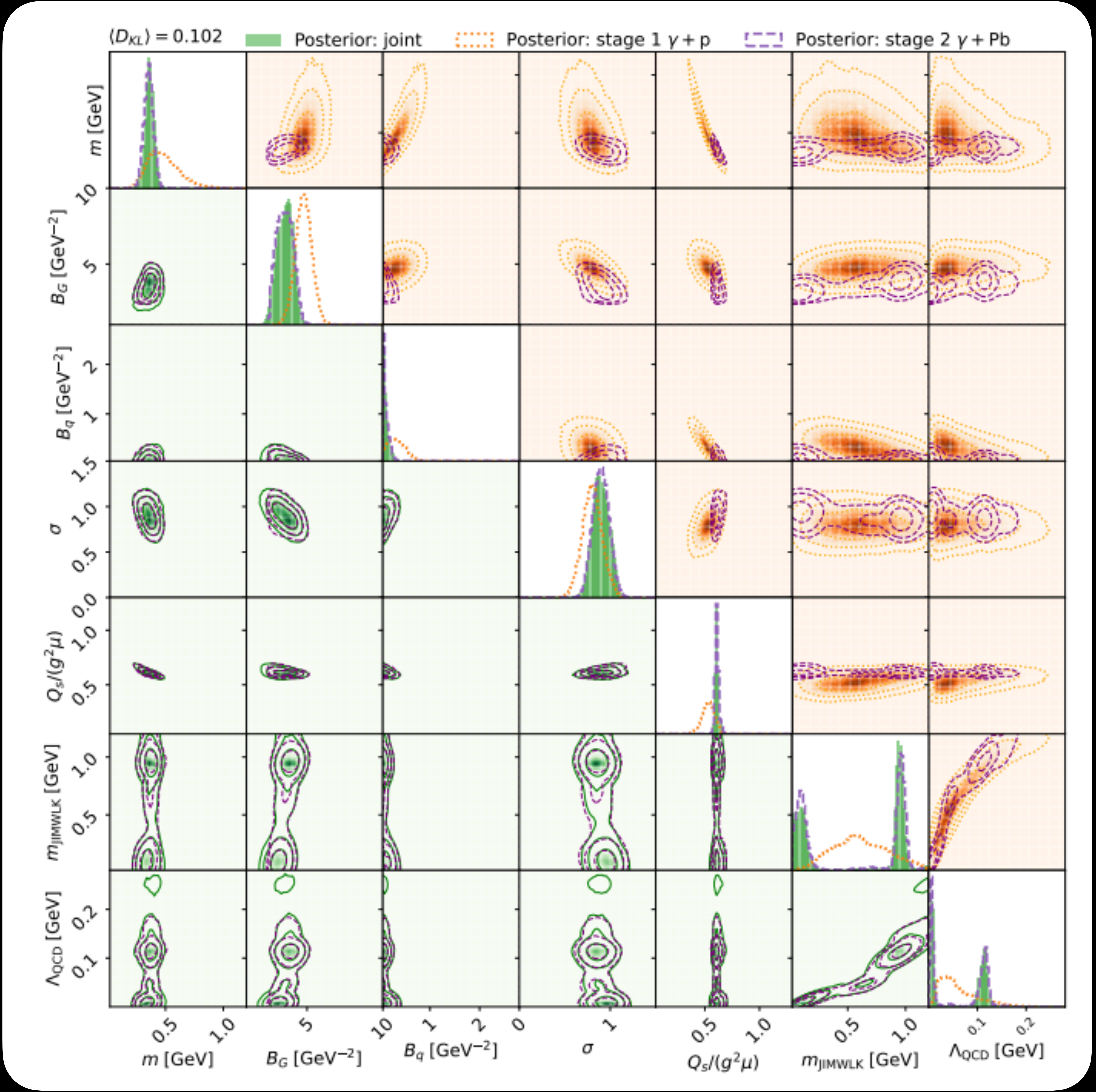
$$= \frac{P(D_2|\theta)P(\theta|D_1)}{P(D_2)}$$

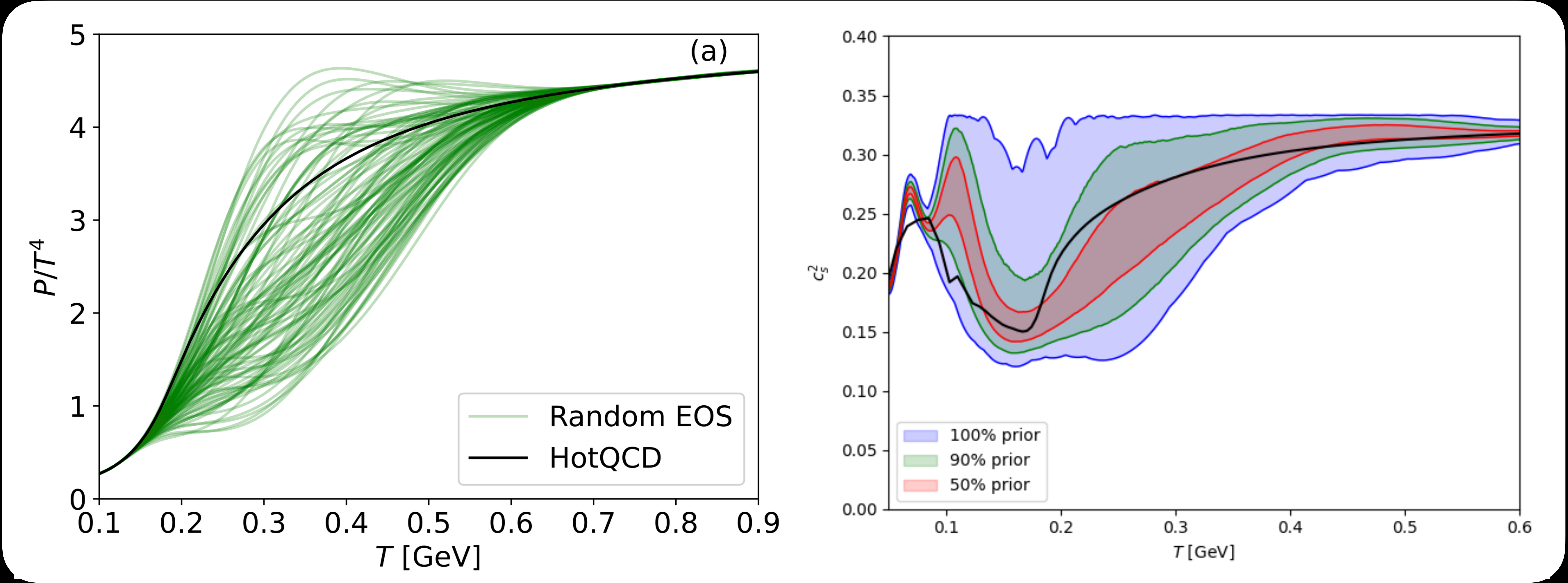
Robust Markov chain Monte Carlo (MCMC) sampler (pocoMC) is required for iterative Bayesian inference when the posterior is bimodal



CONSTRAIN QCD EQUATION OF STATE

- Confront with the RHIC BES phase II data
  - ➡ Constrain the speed of sound $c_s^2(T, \mu_B)$
- First step: introduce nonparametric GP prior to reproduce lattice QCD EoS at $\mu_B = 0$

J. Gong, H. Roch and C. Shen, arXiv:2410.22160 [nucl-th]



SUMMARY

- The vibrant experimental programs at the RHIC and LHC have been driving heavy-ion physics to a precision era

    Unified theoretical models + Bayesian statistical frameworks are *essential* to elucidate the many-body QCD physics

- We performed a comprehensive Bayesian Inference study at multiple RHIC BES energies with a state-of-the-art event-by-event (3+1)D hybrid framework

    Initial stopping, $\eta/s(\mu_B)$, $\zeta/s(T, \mu_B)$

- Looking forward to the next-generation of Bayesian studies in heavy-ion physics

    Quantify theoretical uncertainties

    non-parametric representation, Sequential Bayesian constraints and more machine learning enhancements