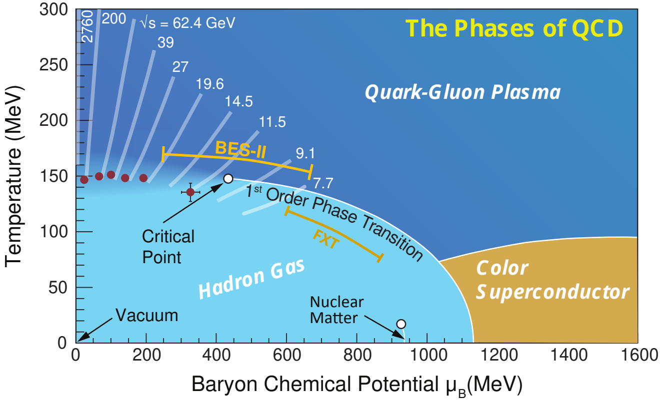
QCD critical point search:
fluctuations and hydrodynamics

M. Stephanov

Phase diagram of QCD

Where on the QCD phase boundary is the Critical Point?

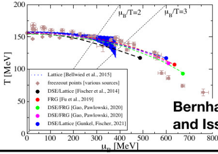


Theory estimates for the QCD CP location

- CP location: estimates converging around $\mu_B \sim 400 - 650$ MeV.

  Caution: systematic errors poorly known.
  Sign problem is still a challenge.

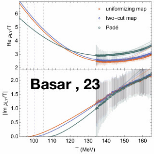
Latest theory developments on locating CP



FRG/
DSE

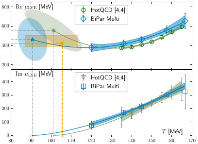
Bernhardt, Fischer
and Isserstedt, 23

$$(\mu_{BC}, T_c) = (495 - 654, 108 - 119)\,\mathrm{MeV}$$



Basar , 23

$$(\mu_{BC}, T_c) \approx (580, 100)\,\mathrm{MeV}$$



Clarke et al.,
24

$$(\mu_{BC}, T_c) = (422^{+80}_{-35}, 105^{+8}_{-18})\,\mathrm{MeV}$$

Extrapolations of Lee-Yang edge singularities to
real axis

Font Color

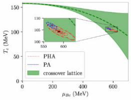
Bayesian Holography + Lattice input at $\mu = 0$

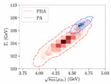
Hippert et al, e-Print: 2309.00579 [nucl-th]

Predict CEP (95% confidence level):

$$T_c = 101 - 108\ \mathrm{MeV} \qquad \mu_c = 560 - 625\ \mathrm{MeV}$$

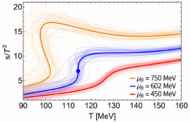
$$\sqrt{s} = 4.0 \ - \ 4.8\ \mathrm{GeV}$$





Thermodynamic analysis of a
lattice QCD extrapolated EoS

Shah et al., 24



$$(\mu_{BC}, T_c) = (602.1 \pm 62.1\,,\ 114.3 \pm 6.9)$$

From Maneesha Pradeep's talk, CPOD 2024

Different approaches, but broadly similar results: $\mu_{\rm CP} \sim 420 - 650$ MeV.

All critical points are equal, but some a more equal . . .

- While critical equation of state is universal, there are also important non-universal characteristics.
- Critical point is characterized not only by its location, but also by the strength and the shape of the singularity.
- BEST collaboration standardized the parameters quantifying these non-universal properties: w, ρ, α_1, α_2 — Ising-to-QCD mapping parameters.
- Equation of state with given critical point parameters *and* matching lattice QCD data: *Parotto et al 18', Kahangirwe et al 24'*

Quantifying theory predictions for fluctuations

- Translating fluctuations in hydrodynamics into fluctuations of particle multiplicites observed in expeiments had been a major challenge.
- A solution: **maximum entropy freezeout**

  (Pradeep-MS 22')

  - Satisfies all conservation laws (baryon number, energy, etc.)
  - Maximizes fluctuation entropy of correlated hadron gas.
  - Model-independent, minimally biased prediction.

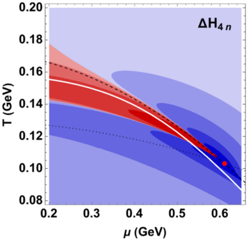
Recent applications of the maximum entropy freezeout

- Quantifying effects of non-universal parameters on fluctuations using maximum entropy approach to freezeout *(Karthein et al 25')*
  - w controls magnitude (width of critical region);
  - ρ — location of signatures in μ_B (shape of the CR)
- Estimates for critical fluctuations with non-universal parameters constrained to the region indicated by Padé approach
  - Padé constrains combination $\bar{\rho} \equiv \rho w^{1-1/\beta\delta}$ *(Basar et al 26')*
- Extracting baryon susceptibilities from proton fluctuations

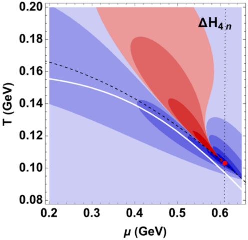
  G. Pihan's talk

Qualitative features depend on parameters too

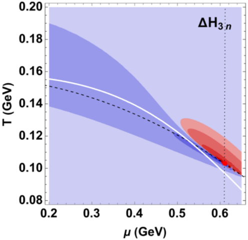
$\alpha_2 = 5°$:



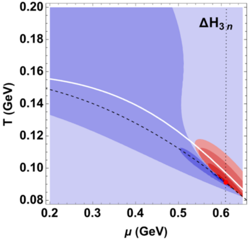
$\alpha_2 = -89°$:



"Hot CP":



"Cool CP":



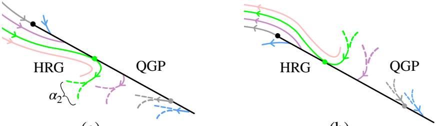
- Signatures could differ depending on non-universal parameters

(Basar et al, 26')

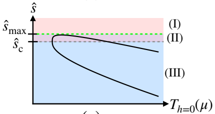
Critical point and non-trivial hydro trajectories

Pradeep, Sogabe, MS, Yee 2402.09519, PRC

- $\hat{s} \equiv s/n$ is non-monotonic along coexistence (1st order) line
- non-trivial deformation of trajectories



(a) (b)



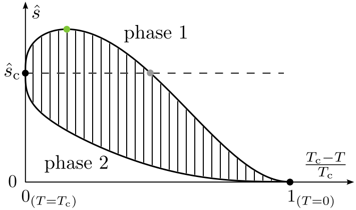
(c)

	(a)	(b)
(I)	Crossover	
(II)	HRG → HRG	QGP → QGP
(III)	QGP → HRG	

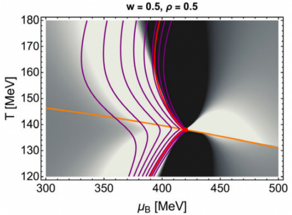
(d)

- depending on $\hat{s}_c - \cot\alpha_1$,

  or $\frac{1}{n}\left(\frac{\partial P}{\partial T}\right)_n$, at CP



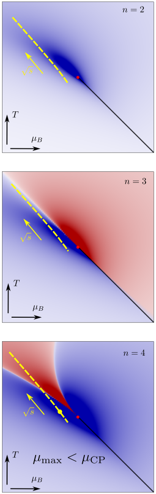
explains "lensing", "cusp"



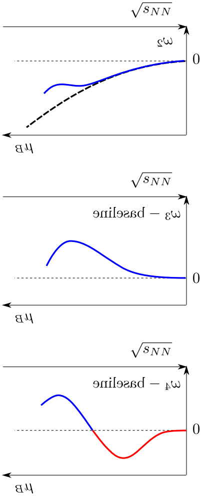
Critical lensing~Dore et al,22, Nonaka&Asakawa, 05

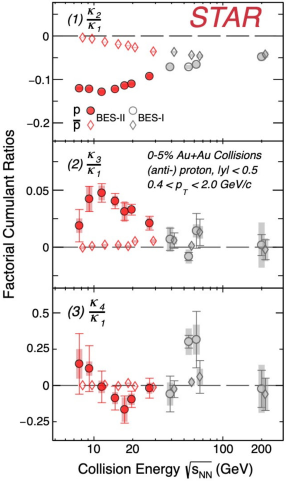
Theory vs BES-II data

(universal EOS) critical χ_n:



(irreducible correlations) $\mathrm{FC}_n[N_p] \sim \chi_n$ (Pradeep, MS 2211.09142), $\omega_n \equiv \mathrm{FC}_n/\mathrm{FC}_1$





Bzdak et al review 1906.00936

Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

2410.02861

more

Observations

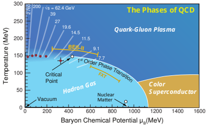
- Significant deviations from monotonic baseline(s) in all FC.
- Qualitatively, consistent with expectations from CP.
- To produce such signatures the CP has to be at $\mu_B > 420$ MeV.
- In line with recent theory estimates by different approaches.

But

- Non-critical baseline is important for interpretation of signals.
- Non-equilibrium effects — conservation laws, memory, i.e., fluctuation *dynamics* — are not yet taken into account.

more

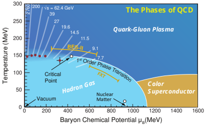
Connection between theory and experiment



- QCD phase diagram:
  - Lattice; Taylor; Padé; YL singularities.
  - FRG;
  - Models
- Fluctuations are singular *in equilibrium*.

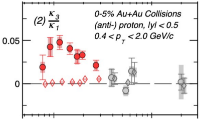
Connection between theory and experiment

- QCD phase diagram:
  - Lattice; Taylor; Padé; YL singularities.
  - FRG; - Models
- Fluctuations are singular *in equilibrium*.

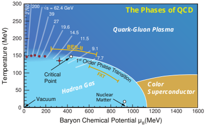


Gap

- Experiment:
  - BES at RHIC
  - Future experiments



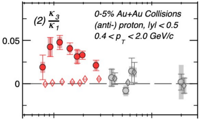
Connection between theory and experiment



- QCD phase diagram:
  - Lattice; Taylor; Padé; YL singularities.
  - FRG;
  - Models
- Fluctuations are singular *in equilibrium*.

  - **Hydrodynamics**
  - **Freezeout**

- Experiment:
  - BES at RHIC
  - Future experiments



Deterministic approach to non-Gaussian fluctuations

Andreev, Akamatsu et al, Yin et al, Martinez et al, . . .

An et al 1902.09517, 1912.13456, 2009.10742 (PRL), 2212.14029

- **Model independent** description based on hydrodynamics
- In *stochastic* hydrodynamics,

  equal-time correlators $H_{a_1 \dots a_N} \equiv \langle \delta\psi_{a_1} \dots \delta\psi_{a_N} \rangle^{\mathrm{c}}$

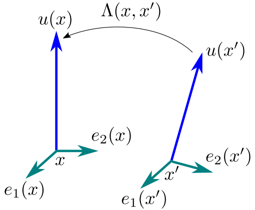
  of fluctuating hydrodynamic fields $\psi_a = \{\epsilon, n_B, \dots\}$

  obey *deterministic* evolution equations.
- The equations describe relaxation of the correlators towards (evolving) equilibrium values. more
- In *equilibrium*, the correlators are given by themrodynamic EOS.
- *Relativistic* flow presents interesting challenges. Equal time?

Main ingredients of the relativistic formalism

An et al 2604.14110

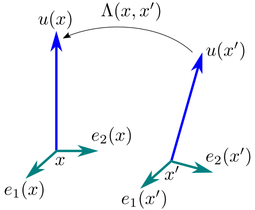
- With $u(x)$ – *average* flow velocity, i.e., Landau frame for $\langle T^{\mu\nu}\rangle$:
  - define connection $\widetilde{\omega} = \nabla\Lambda$ and *confluent derivative*: $\widetilde{\nabla}_\mu u^\nu = 0$.
  - supply *local basis* $e_a(x)$, also confluently constant: $\widetilde{\nabla}_\mu e_a^\nu = 0$. *Local* $SO(3)$ *(gauge) invariance.*



Main ingredients of the relativistic formalism

An et al 2604.14110

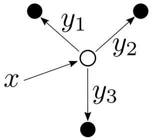
- With $u(x)$ – *average* flow velocity, i.e., Landau frame for $\langle T^{\mu\nu}\rangle$:
  - define connection $\widetilde{\omega} = \nabla\Lambda$ and *confluent derivative*: $\widetilde{\widetilde{\nabla}}_\mu u^\nu = 0$.
  - supply *local basis* $e_a(x)$, also confluently constant: $\widetilde{\widetilde{\nabla}}_\mu e_a^\nu = 0$. *Local* $SO(3)$ *(gauge) invariance.*



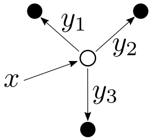
- Define *fluctuating* densities in this local frame/basis.

  Momentum 3-vector π^a, energy $\tilde{\varepsilon}$, (baryon) charge $\tilde{n}$.

- Define N-point correlator $H_{(N)}$ which is
  - *equal time* w.r.t. to midpoint, x:
  - $SO(3)$ *covariant*, i.e., transforms as a local tensor at the midpoint.



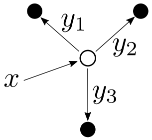
- Define N-point correlator $H_{(N)}$ which is
  - *equal time* w.r.t. to midpoint, x:
  - $SO(3)$ *covariant*, i.e., transforms as a local tensor at the midpoint.



- Define a *confluent derivative* of the correlator, $\widetilde{\nabla} H_{(N)}$.

  $\widetilde{\nabla} H_{(N)}$ is also $SO(3)$ covariant.

- Define N-point correlator $H_{(N)}$ which is
  - *equal time* w.r.t. to midpoint, x:
  - $SO(3)$ *covariant*, i.e., transforms as a local tensor at the midpoint.



- Define a *confluent derivative* of the correlator, $\widetilde{\nabla} H_{(N)}$.

  $\widetilde{\nabla} H_{(N)}$ is also $SO(3)$ covariant.
- Define generalized Wigner transform,

  $H_{(N)}(x_1, \ldots, x_N) \longrightarrow W_{(N)}(x; \boldsymbol{q}_1, \ldots, \boldsymbol{q}_N)$,

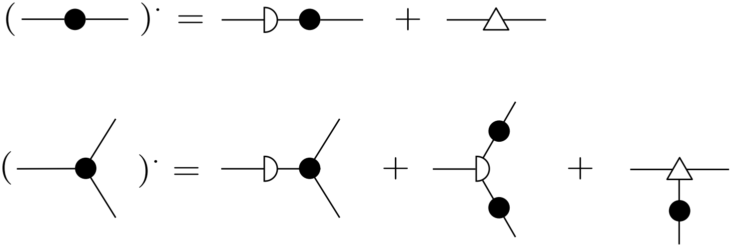
  — N-ple Fourier integral with midpoint fixed *An et al., 2009.10742*

  Takes advantage of scale separation:

  average gradients vs fluct. wavenumbers: $k \sim q^2 \ll q$.

Evolution equations

- Diagrammatic representation:



● – correlator; ᗡ – evolution; △ – noise.

- For all modes (incl. velocity) known for $N = 2$.
- For $N \geq 3$ — known for scalar, diffusive mode. *An et al., 2009.10742*
- All modes, $N = 3$, — recent result — *An et al, 2604.14110*

Physical meaning of W

- $N = 2$: matrix equations have intuitive nontrivial interpretation:
  - Wigner function (projected onto sound channel) obeys kinetic equation with local group velocity $c_s(x)|\boldsymbol{q}|$ — phonons.
  - In an inhomogeneous, expanding, accelerating, and rotating fluid – all inertial effects are accurately reproduced.
  - Distribution relaxes to equilibrium at the well-known rate of sound attenuation (times two).
  - The equilibrium is given by classical (Rayleigh-Jeans) limit of Bose-Einstein distribution.
- For $N = 3$, the same Liouville operator, and its field derivatives, appear in evolution equation.

Summary

- RHIC BES-II data shows interesting qualitative features (deviations from baseline) in line with theoretical expectations for the CP at $\mu_B > 400$ MeV.

But

- Understanding non-equilibrium effects and the baseline — important for interpretation.

  Fluctuation hydrodynamics + **maximum entropy freezeout** = model-independent formalism to address these issues.

- To obtain constraints on the QCD EOS from experimental data more work is needed and is underway.

More

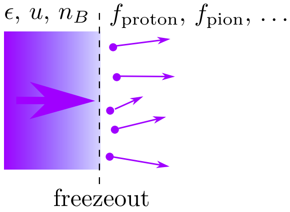
Freezeout of fluctuations and conservation laws

- Freezeout: translation of correlators of hydrodynamic fluctuations ($\psi = \pi, \epsilon, n_B$)

  $\langle \delta\psi \ldots \delta\psi \rangle = H_{(N)}(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N)$

  to particle correlators

  $\langle \delta f \ldots \delta f \rangle = G_{(N)}(\boldsymbol{x}_1, \boldsymbol{p}_1, \ldots, \boldsymbol{x}_N, \boldsymbol{p}_N)$.



- *Conservation laws* relate $\boldsymbol{p}$ integrals of $G_{(N)}$ to $H_{(N)}$,

  since $\psi(x) = \int_{\boldsymbol{p}} P_\psi f(x, \boldsymbol{p})$, where P_ψ is single particle contribution to ψ.

- But these relations are not enough to fix $\boldsymbol{p}$ dependence in $G_{(N)}$.

  There are ∞ many possibilities/solutions ($G_{(N)}$) satisfying conservation laws.

Maximum entropy freezeout

Pradeep, MS, 2211.09142, PRL

- There is a unique solution which maximizes the entropy!
  - Relative Entropy with respect to uncorrelated gas,

    as a functional of particle correlations $G_{(N)}$.
  - Maximized under constraints imposed by *conservation laws*.

Maximum entropy freezeout

Pradeep, MS, 2211.09142, PRL

- There is a unique solution which maximizes the entropy!
  - Relative Entropy with respect to uncorrelated gas,
    as a functional of particle correlations $G_{(N)}$.
  - Maximized under constraints imposed by *conservation laws*.
  - not only baryon number is conserved, also energy, etc.
  - for $N = 1$ equivalent to Cooper-Frye
  - in agreement with, but more general than earlier approaches
  - **model independent**, i.e., determined by QCD EOS + hydro

Particle correlators from hydrodynamic correlators

- The maximum entropy solution is most intuitive in terms of *irreducible* correlators $\hat{\Delta}G_{(N)}$.

  For example, for $N = 3$,

  $$\underbrace{\hat{\Delta}G_{ABC}}_{\text{irreducible particle correlations (}\sim\text{FC)}} = \underbrace{\hat{\Delta}H_{abc}}_{\text{hydrodynamic correlations}} \times \underbrace{P_A^a P_B^b P_C^c}_{\text{kinematic factors}}$$

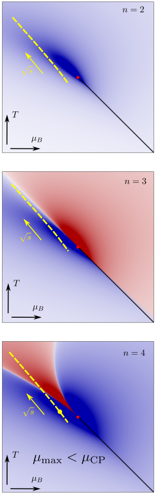
  P_A^a are determined by the amount of the conserved quantity a carried by particle state A and by the corresponding susceptibility.

  - $\hat{\Delta}G_{ABC}$ — vanishes for an uncorrelated hadron gas, or for a gas with only 2-particle correlations. more

  - $\int_A \int_B \int_C \hat{\Delta}G_{ABC}$ is a factorial cumulant more

Theory vs BES-II data

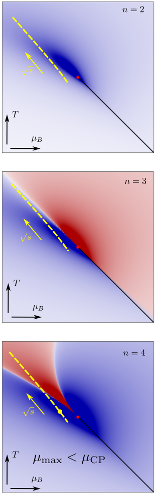
(universal EOS) critical χ_n:



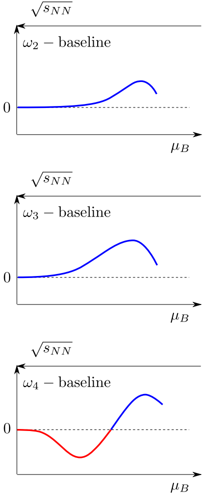
Bzdak et al review 1906.00936

Theory vs BES-II data

(universal EOS) critical χ_n:



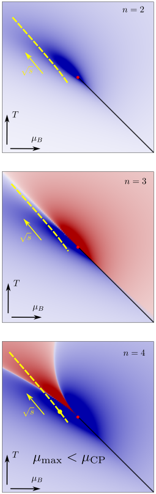
(irreducible correlations) $\mathrm{FC}_n[N_p] \sim \chi_n$ (Pradeep, MS 2211.09142), $\omega_n \equiv \mathrm{FC}_n/\mathrm{FC}_1$



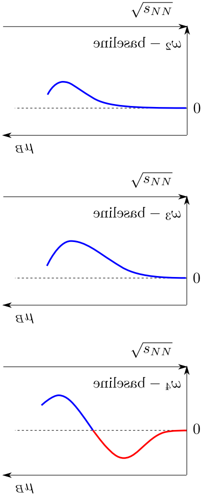
Bzdak et al review 1906.00936

Theory vs BES-II data

(universal EOS) critical χ_n:



(irreducible correlations) $\mathrm{FC}_n[N_p] \sim \chi_n$ (Pradeep, MS 2211.09142), $\omega_n \equiv \mathrm{FC}_n/\mathrm{FC}_1$

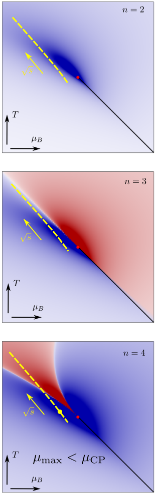


Bzdak et al review 1906.00936

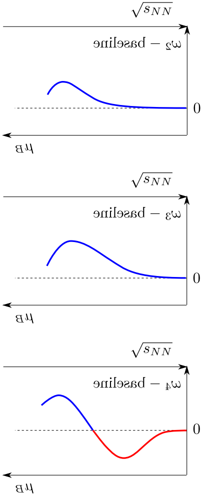
Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4

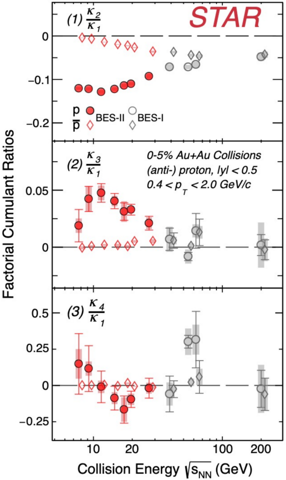
Theory vs BES-II data

(universal EOS) critical χ_n:



(irreducible correlations) $\mathrm{FC}_n[N_p] \sim \chi_n$ (Pradeep, MS 2211.09142), $\omega_n \equiv \mathrm{FC}_n/\mathrm{FC}_1$





Bzdak et al review 1906.00936

Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

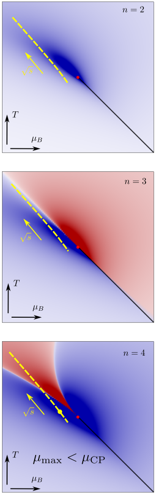
2410.02861

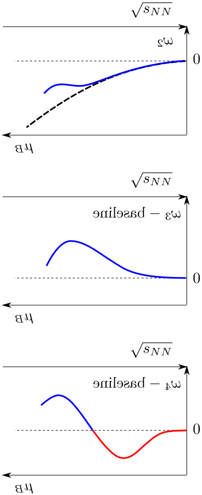
back

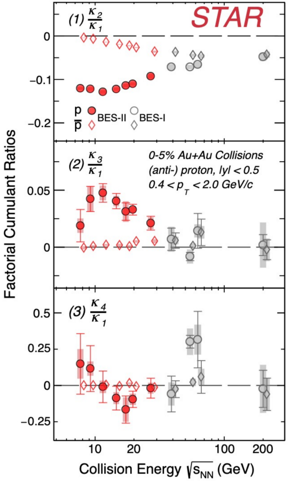
Theory vs BES-II data

(universal EOS) critical χ_n:

(irreducible correlations) $\mathrm{FC}_n[N_p] \sim \chi_n$ (Pradeep, MS 2211.09142), $\omega_n \equiv \mathrm{FC}_n/\mathrm{FC}_1$







Bzdak et al review 1906.00936

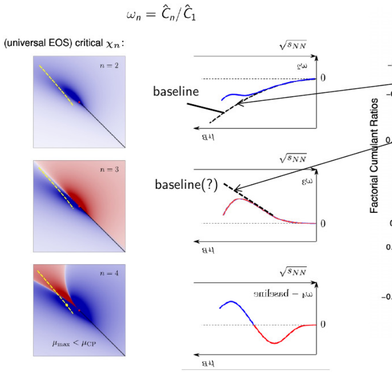
Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV 2410.02861

back

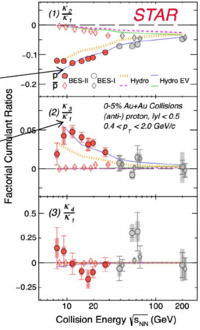
Baseline and non-equilibrium effects are important

From Vovchenko, BES seminar October 2024:

From M. Stephanov, SQM2024 & arXiv:2410.02861



STAR data:



A. Pandav, CPOD2024

baseline (hydro EV):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

- describes right side of the peak in $\hat{C}_3$
- **signal relative to baseline:**
  - *positive* $\hat{C}_2 > 0$
  - *negative* $\hat{C}_3 < 0$

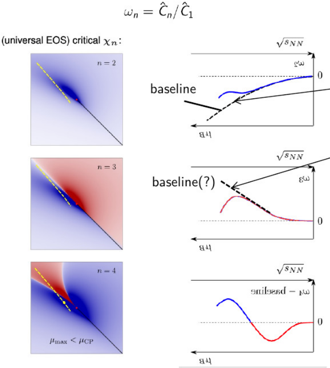
Conclusion 2:

Controlling the non-critical baseline is essential

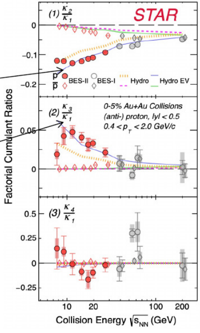
Baseline and non-equilibrium effects are important

From Vovchenko, BES seminar October 2024:

From M. Stephanov, SQM2024 & arXiv:2410.02861



STAR data:



A. Pandav, CPOD2024

CP *below* the freezeout curve?

Dynamics, memory effect?

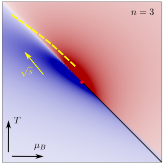
baseline (hydro EV):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

- describes right side of the peak in $\hat{C}_3$
- **signal relative to baseline:**
  - *positive* $\hat{C}_2 > 0$
  - *negative* $\hat{C}_3 < 0$

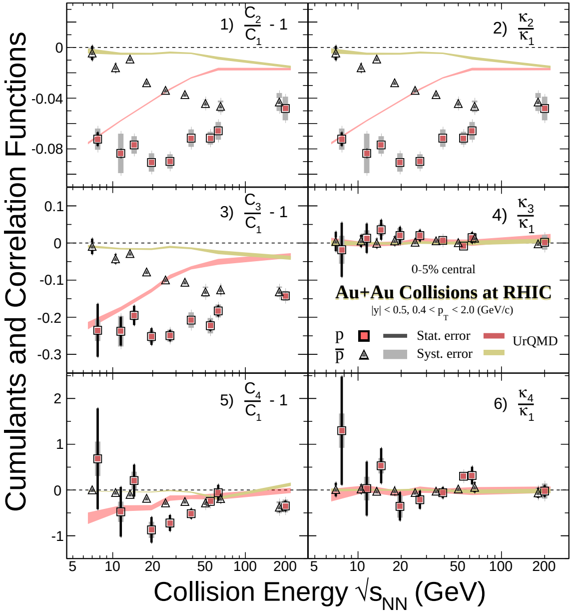
Conclusion 2:

Controlling the non-critical baseline is essential



back

BES-I data

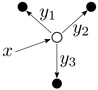


Cumulants and Correlation Functions
1) $\frac{C_2}{C_1} - 1$
2) $\frac{\kappa_2}{\kappa_1}$
3) $\frac{C_3}{C_1} - 1$
4) $\frac{\kappa_3}{\kappa_1}$
5) $\frac{C_4}{C_1} - 1$
6) $\frac{\kappa_4}{\kappa_1}$
0-5% central
Au+Au Collisions at RHIC
$|y| < 0.5$, $0.4 < p_T < 2.0$ (GeV/c)
p
$\bar{p}$
Stat. error
Syst. error
UrQMD
Collision Energy $\sqrt{s_{NN}}$ (GeV)

Generalizing Wigner transform

An et al 2009.10742 PRL

- Definition:

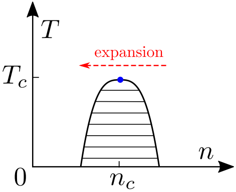


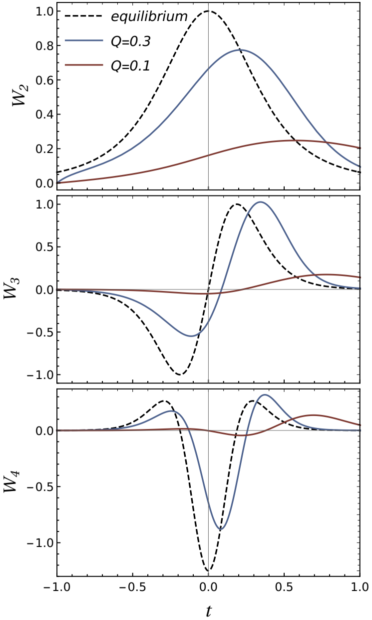
$$W_n(\boldsymbol{x};\boldsymbol{q}_1,\ldots,\boldsymbol{q}_n) \equiv \int d\boldsymbol{y}_1^3 \ldots \int d\boldsymbol{y}_n^3\, H_n\left(\boldsymbol{x}+\boldsymbol{y}_1,\ldots,\boldsymbol{x}+\boldsymbol{y}_n\right) \delta^{(3)}\left(\frac{\boldsymbol{y}_1+\ldots+\boldsymbol{y}_n}{n}\right) e^{-i(\boldsymbol{q}_1\cdot\boldsymbol{y}_1+\ldots+\boldsymbol{q}_n\cdot\boldsymbol{y}_n)};$$

- W_n's quantify magnitude and non-gaussianity of fluctuation harmonics with wave-vectors $\boldsymbol{q}_i$.

Example: expansion through a critical region

An et al 2009.10742*, PRL*



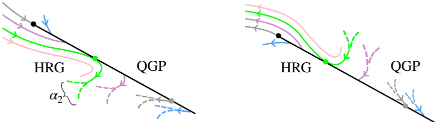


- Two main features:
  - Lag, "memory".
  - Smaller Q – slower evolution. Conservation laws.

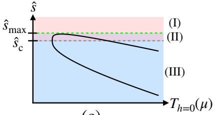
Critical point and non-trivial hydro trajectories

Pradeep, Sogabe, MS, Yee 2402.09519, PRC

- $\hat{s} \equiv s/n$ is non-monotonic along coexistence (1st order) line
- non-trivial deformation of trajectories



(a) (b)

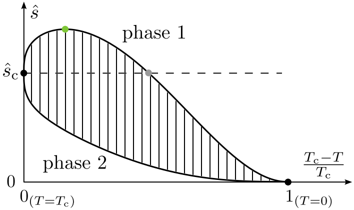


(c)

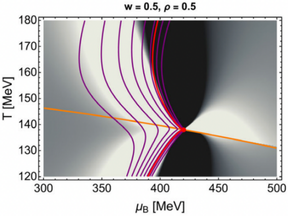
	(a)	(b)
(I)	Crossover	
(II)	HRG → HRG	QGP → QGP
(III)	QGP → HRG	

(d)

depending on $(\partial P/\partial T)_n$ at CP



explains “lensing”, “cusp”



Critical lensing~Dore et al,22, Nonaka&Asakawa, 05

Fluctuation evolution equations

An et al 2009.10742 PRL

- In hydrodynamics, fluctuations are *small* and *almost* Gaussian.
- There is a small expansion parameter ε, so that $\delta\breve{\psi} \sim \sqrt{\varepsilon}$.

  Then $H_n \equiv \varepsilon^{n-1}$ and to leading order in ε:

$$\partial_t \psi = -\nabla \cdot (\text{Flux}[\psi] + \mathcal{O}(\varepsilon));$$

$$\partial_t H = -2\Gamma(H - \bar{H}[\psi]) + \mathcal{O}(\varepsilon^2);$$

$$\vdots$$

$$\partial_t H_n = -n\Gamma(H_n - \bar{H}_n[\psi, H, \ldots, H_{n-1}]) + \mathcal{O}(\varepsilon^n);$$

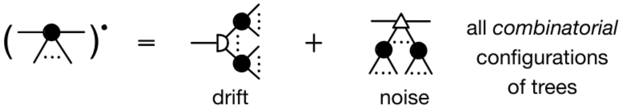
  To leading order, the equations are iterative and “linear”. back

- In hydrodynamics the small parameter is $(q/\Lambda)^3$, i.e., fluctuation wavelength $1/q \gg$ size of hydro cell $1/\Lambda$ (UV cutoff).

Diagrammatic representation

An et al 2009.10742, 2212.14029, An's talk at CPOD 2024

- Leading order in ε $\Leftrightarrow$ tree diagrams.



- Loops describe feedback of fluctuations (renormalization and long-time tails).

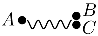
back

Non-gaussian correlators ($n \geq 3$ particles)

Linearied equations are simple and intuitive in terms of $\hat{\Delta} G_n$:

$$G_{AB} = \bar{G}_{AB} + \hat{\Delta} G_{AB}, \quad H_{ab} = \bar{H}_{ab} + \hat{\Delta} H_{ab},$$

$$G_{ABC} = \Big[\underbrace{\bar{G}_{ABC}} + \underbrace{3\hat{\Delta} G_{AD} \delta_{DBC}} + \underbrace{\widehat{\Delta} G_{ABC}}_{\text{irreducible correlation}} \Big]_{\overline{ABC}} \leftarrow \text{permutation average}$$



$$H_{abc} = \Big[\bar{H}_{abc} + 3\Delta H_{ad} \delta_{dbc} + \widehat{\Delta} H_{abc} \Big]_{\overline{abc}}$$

Maximum entropy method gives:

$$\Delta G_{AB} = \Delta H_{ab} (\bar{H}^{-1} P \bar{G})^a_A (\bar{H}^{-1} P \bar{G})^b_B$$

$$\hat{\Delta} G_{ABC} = \hat{\Delta} H_{abc} (\bar{H}^{-1} P \bar{G})^a_A (\bar{H}^{-1} P \bar{G})^b_B (\bar{H}^{-1} P \bar{G})^c_C$$

back

Why Factorial Cumulants

Factorial Cumulants (FC) are better measures of particle correlations

Three reasons:

- Maximum Entropy freezeout: *Pradeep, MS 2211.09142*

  FC$_N$ are integrals of $\hat{\Delta}G_{(N)}$ (the irreducible particle correlators), which are related by ME freezeout to hydrodynamic correlators.

Why Factorial Cumulants

Factorial Cumulants (FC) are better measures of particle correlations

Three reasons:

- Maximum Entropy freezeout: *Pradeep, MS 2211.09142*

  FC$_N$ are integrals of $\hat{\Delta}G_{(N)}$ (the irreducible particle correlators), which are related by ME freezeout to hydrodynamic correlators.

- Acceptance dependence: *Ling, MS 1512.09125, Bzdak, Koch, Strodthoff 1607.07375*

  FC are monomials of Δy for small Δy; NC are polynomials.

Why Factorial Cumulants

Factorial Cumulants (FC) are better measures of particle correlations

Three reasons:

- Maximum Entropy freezeout: *Pradeep, MS 2211.09142*

  FC$_N$ are integrals of $\hat{\Delta}G_{(N)}$ (the irreducible particle correlators), which are related by ME freezeout to hydrodynamic correlators.

- Acceptance dependence: *Ling, MS 1512.09125, Bzdak, Koch, Strodthoff 1607.07375*

  FC are monomials of Δy for small Δy; NC are polynomials.

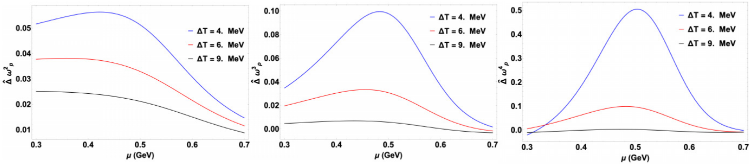
- Normal Cumulants (NC) – quantify non-gaussianity;
  FC – quantify non-poissonianity, i.e., *irreducible* correlations.

  NC are for densities (continuous);
  FC are for multiplicities (discrete).

back

Equilibrium estimates for critical contribution to factorial cumulants of proton multiplicities

Karthein, MP, Rajagopal, Stephanov, Yin (in preparation)



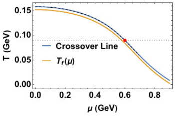
$$T_f(\mu) = T_{co}(\mu) - \Delta T$$

$$\mu_c = 600\,\mathrm{MeV}\,,\ \alpha_2 = 0^\circ\,,\ \rho = 1\,,\ w = 20$$

Example **choice of Mapping Parameters**

$$\hat{\Delta}\omega_p^k = \frac{\hat{\Delta}\,\langle \delta N_p^k \rangle}{\langle N_p \rangle}$$

23



Freeze-out curve

Signatures depend sensitively on ΔT, w, ρ, e.g.,

(approximately) the amplitude $\sim w^{-6/5}$, the width $\sim (w\rho)^{-1}$.

Thus, these parameters (T_c, μ_c, w, ρ) could be constrained by data.