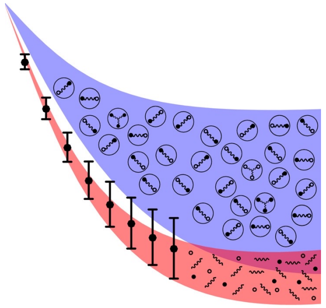
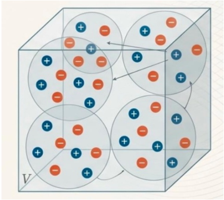


Probing fractional quark charges and remnants of chiral criticality with fluctuations at the LHC



Volodymyr Vovchenko (University of Houston)

YITP workshop "QCD Critical Point and Hydrodynamic Evolution", Kyoto, Japan

June 1, 2026

- **J. Parra**, **R. Poberezhniuk**, V. Koch, C. Ratti, VV, Phys. Rev. Lett. 135, 242302 (2025)
- **M. Ciacco, V. Kuznietsov**, S. Kundu, M. Puccio, VV, arXiv:2605.30710

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

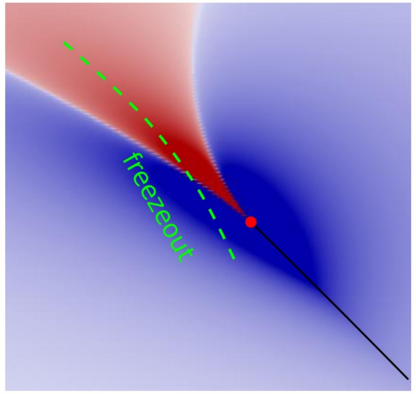
$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

Grand partition function

$$\ln Z^{\mathrm{gce}}(T, V, \mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\mathrm{ce}}(T, V, N)\right]$$

$$\boxed{\kappa_n \propto \frac{\partial^n(\ln Z^{\mathrm{gce}})}{\partial \mu^n}}$$

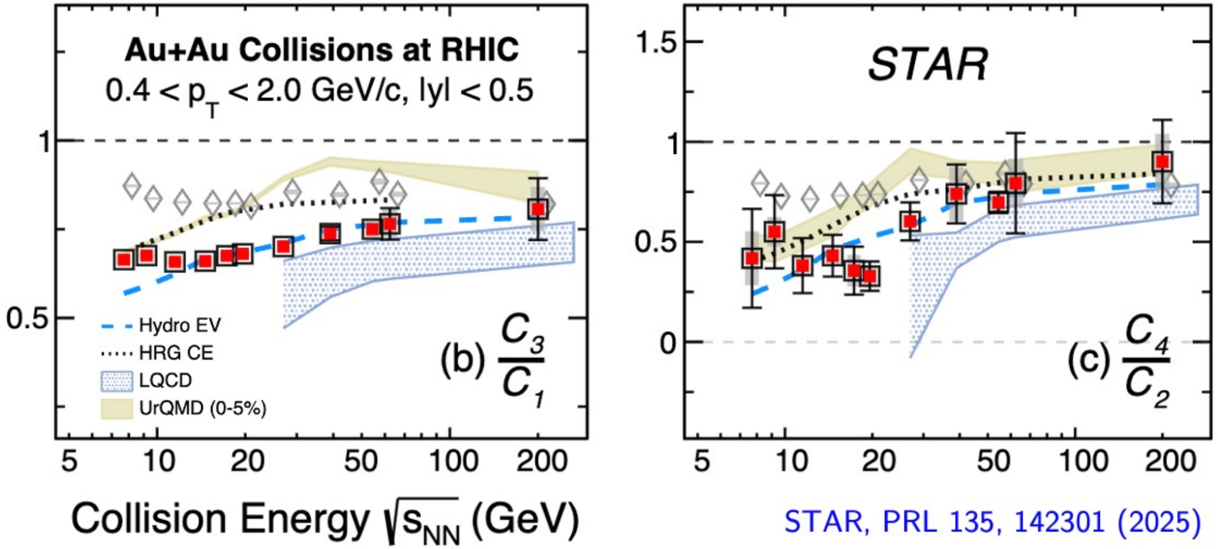
Cumulants measure chemical potential derivatives of the (QCD) equation of state



Finite density and
QCD critical point
with beam energy scan

M. Stephanov, PRL '09, '11
Motivation for RHIC-BES

Talks by G. Pihan & J. Karthein on Fri

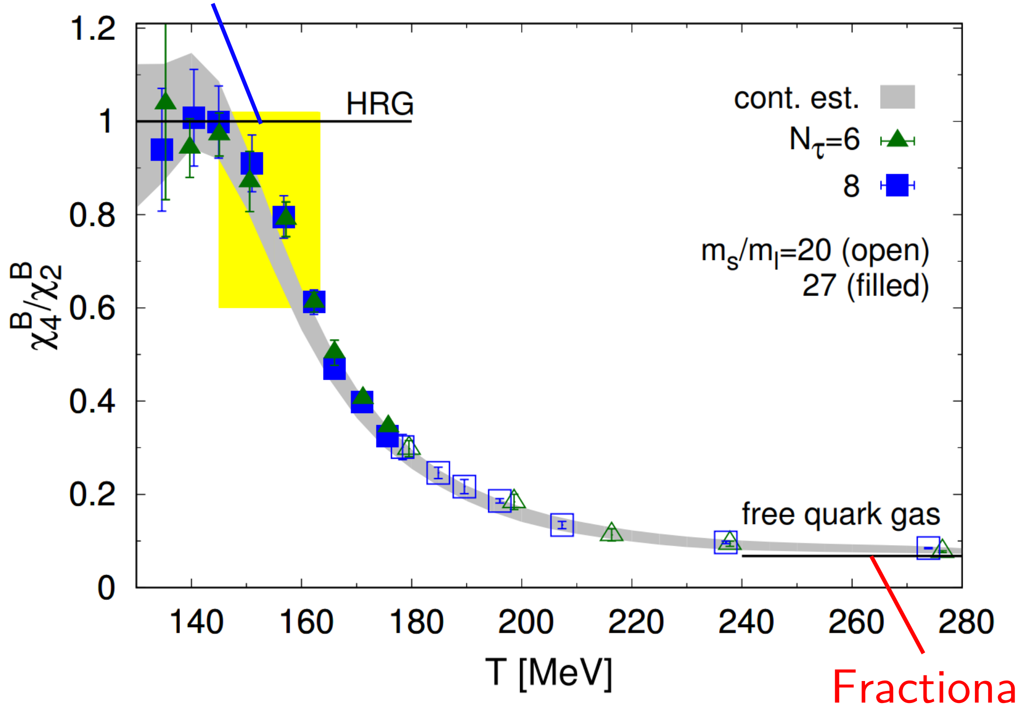


STAR, PRL 135, 142301 (2025)

QCD phase structure with fluctuations

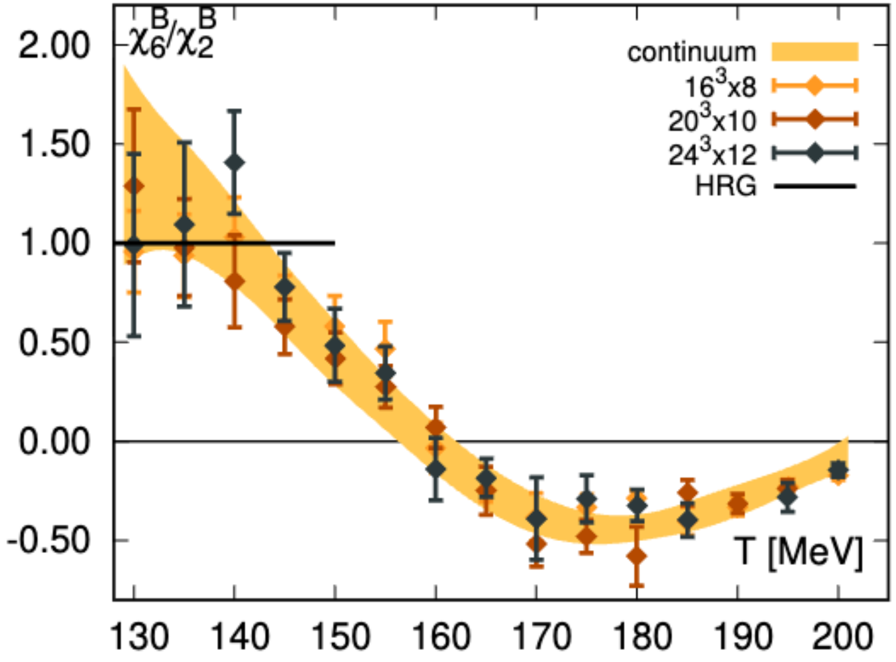
This talk: Fluctuation signatures at LHC

Integer charge carriers



Fractional charge carriers

HotQCD Collaboration, PRD 95, 054504 (2017)



Wuppertal-Budapest, PRD 110, L011501 (2024)

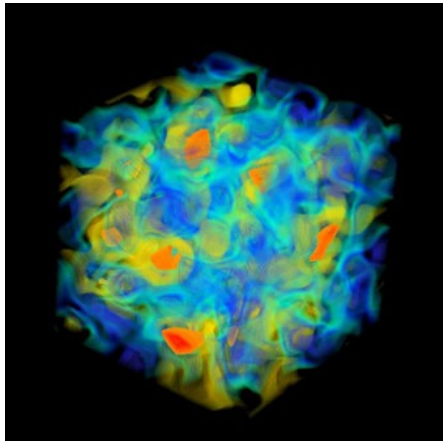
- Suppression of fluctuations: fractional charge carriers [Jeon, Koch, PRL 85, 2076 (2000), Asakawa, et al., PRL 85, 2072 (2000)]
- Chiral crossover and remnants of chiral criticality with high-order baryon cumulants at $\mu_B = 0$

Friman, Karsch, Redlich, Skokov, EPJC 71, 1694 (2011)

Can it be seen in experiment?

Theory vs experiment: Challenges for fluctuations

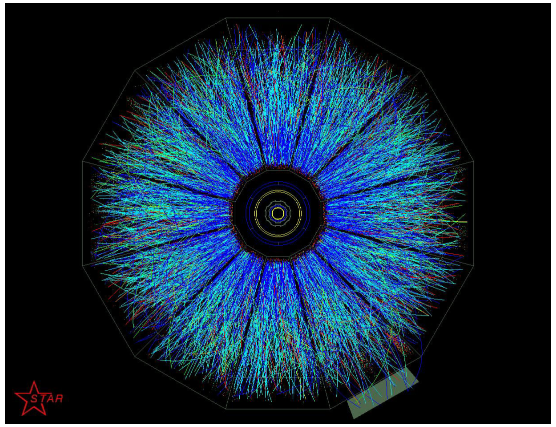
Theory



© Lattice QCD@BNL

- Coordinate space
- In contact with the heat bath
- Equilibrium and uniform
- Fixed volume

Experiment



STAR event display

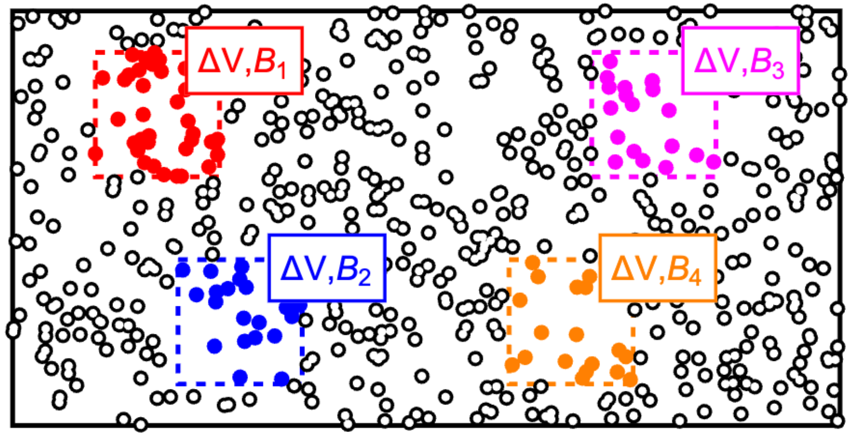
- Momentum space
- Expanding in vacuum
- Non-uniform, out-of-equilibrium
- Centrality

From RHIC we learned that baryon conservation is the primary driver of the baseline

Density correlations framework

VV, PRC 110, L061902 (2024)

Split a thermal system into multiple subvolumes ΔV and consider joint distribution of charge B inside the subvolumes



$$G_{\boldsymbol{B}}(\boldsymbol{t}) = \ln \left\langle e^{\sum_{i=1}^{n} t_i B_i} \right\rangle = \ln \left[\sum_{\boldsymbol{B}} \exp \left(\sum_{i=1}^{n} t_i B_i \right) P(\boldsymbol{B}) \right]$$

$$\boldsymbol{B} = (B_1, \ldots, B_N), \qquad B_1 + \cdots + B_N = B_{tot}$$

In large system (thermodynamic limit), the joint probability factorizes into a product of partition functions

$$P(\boldsymbol{B}) \propto \left[\prod_{j=1}^{n} Z(\Delta V, B_j) \right] Z(V - n\Delta V, B - \sum_{j=1}^{n} B_j) \propto \left[\prod_{j=1}^{n} e^{-\Delta V f(\rho_j)} \right] e^{-(V - n\Delta V) f(\rho_{n+1})}$$

$f(\rho)$ – free energy density

$$(\partial \mu_B / \partial \rho_B)_T = [T^3 \chi_2^B]^{-1}$$

$$\left\langle \delta B_1^{k_1} \ldots \delta B_n^{k_n} \right\rangle_c = \left. \frac{\partial^{k_1 + \ldots + k_n} G_{\boldsymbol{B}}(\boldsymbol{t})}{\partial t_1^{k_1} \ldots \partial t_n^{k_n}} \right|_{t_1 = \ldots = t_n = 0} \xrightarrow{\Delta V \to 0} \mathcal{C}_n(\eta_1, \ldots, \eta_n) \equiv \left\langle \prod_{i=1}^{n} \delta \rho_i \right\rangle_c, \qquad n \geq 2,$$

Density correlations framework

VV, PRC 110, L061902 (2024)

$$\mathcal{C}_2(\eta_1,\eta_2) = \underset{\textbf{GCE}}{\boldsymbol{\chi_2^B \delta_{1,2}}} - \underset{\text{2-point}}{\frac{\chi_2^B}{V}}$$

$$\mathcal{C}_3(\eta_1,\eta_2,\eta_3) = \underset{\textbf{GCE}}{\boldsymbol{\chi_3^B \delta_{1,2,3}}} - \underset{\text{2-point}}{\frac{\chi_3^B}{V}} [\delta_{1,2} + \delta_{1,3} + \delta_{2,3}] + \underset{\text{3-point}}{\frac{2\chi_3^B}{V^2}}$$

$$\mathcal{C}_4(\eta_1,\eta_2,\eta_3,\eta_4) = \underset{\textbf{GCE}}{\boldsymbol{\chi_4^B \delta_{1,2,3,4}}} - \underset{\text{2-point}}{\frac{\chi_4^B}{V}} [\delta_{1,2,3} + \delta_{1,2,4} + \delta_{1,3,4} + \delta_{2,3,4}] - \underset{\text{2-point}}{\frac{(\chi_3^B)^2}{\chi_2^B V}} [\delta_{1,2}\delta_{3,4} + \delta_{1,3}\delta_{2,4} + \delta_{1,4}\delta_{2,3}]$$

$$+ \frac{1}{V^2}\left[\chi_4^B + \frac{(\chi_3^B)^2}{\chi_2^B}\right] \underset{\text{3-point}}{[\delta_{1,2} + \delta_{1,3} + \delta_{1,4} + \delta_{2,3} + \delta_{2,4} + \delta_{3,4}]} - \underset{\text{4-point}}{\frac{3}{V^3}\left[\chi_4^B + \frac{(\chi_3^B)^2}{\chi_2^B}\right]}$$

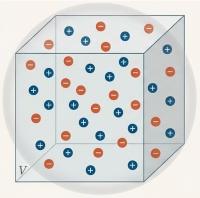
All terms apart from the local one are *balancing contributions*

Integrals yield canonical ensemble cumulants in subvolume $\prod_{i=1}^{n} \int d\eta_i \, \mathcal{C}_n(\eta_1, \ldots, \eta_n) = \kappa_n[B].$

Introducing local charge conservation

VV, PRC 110, L061902 (2024)

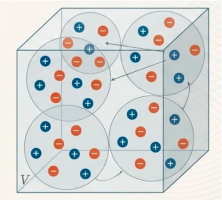
Introduce Gaussian (spatial) rapidity correlation into charge-conservation balancing term



global conservation

$$C_2^B(\eta_1,\eta_2) = \langle n_B + n_{\bar B}\rangle \left[\delta(\eta_1-\eta_2) - \frac{1}{2\eta_{\max}}\right]$$

local correlation **balancing contribution (e.g. baryon conservation)**

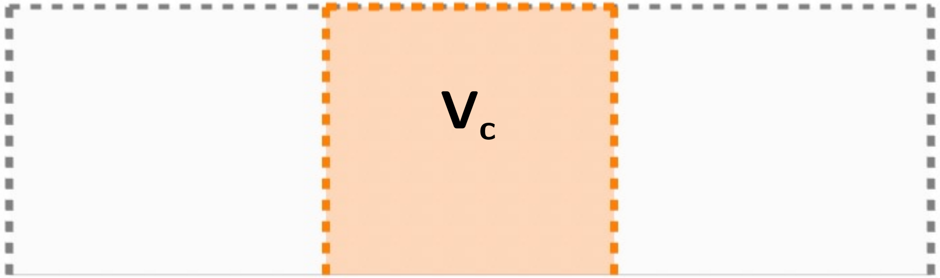


+ local conservation

$$C_2^B(\eta_1,\eta_2) = \langle n_B + n_{\bar B}\rangle \left[\delta(\eta_1-\eta_2) - \frac{\tilde{A}\, e^{-\frac{(\eta_1-\eta_2)^2}{2\sigma_\eta^2}}}{2\eta_{\max}}\right]$$

local correlation **local balancing contribution**

truncated fireball



VV, Donigus, Stoecker, PRC 100, 054906 (2019)

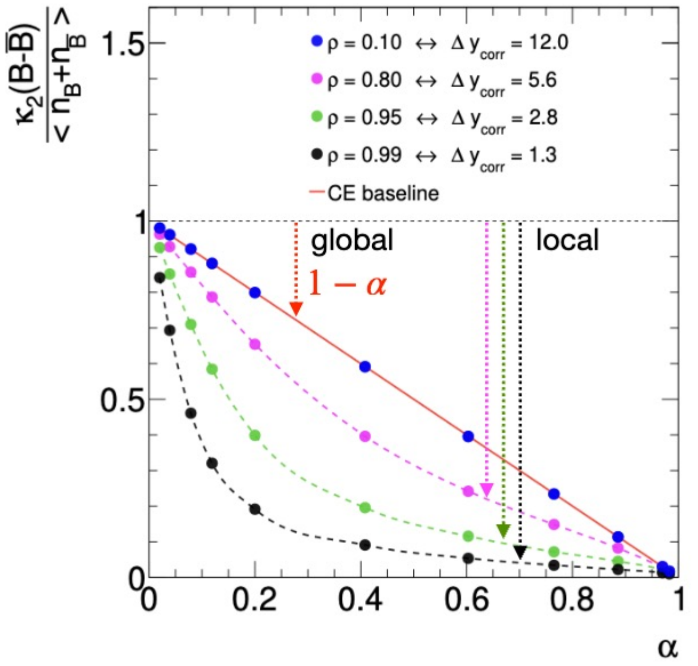
vs

Gaussian correlation

Gaussian correlation captures the diminishing contributions of hadrons at forward/backward rapidities

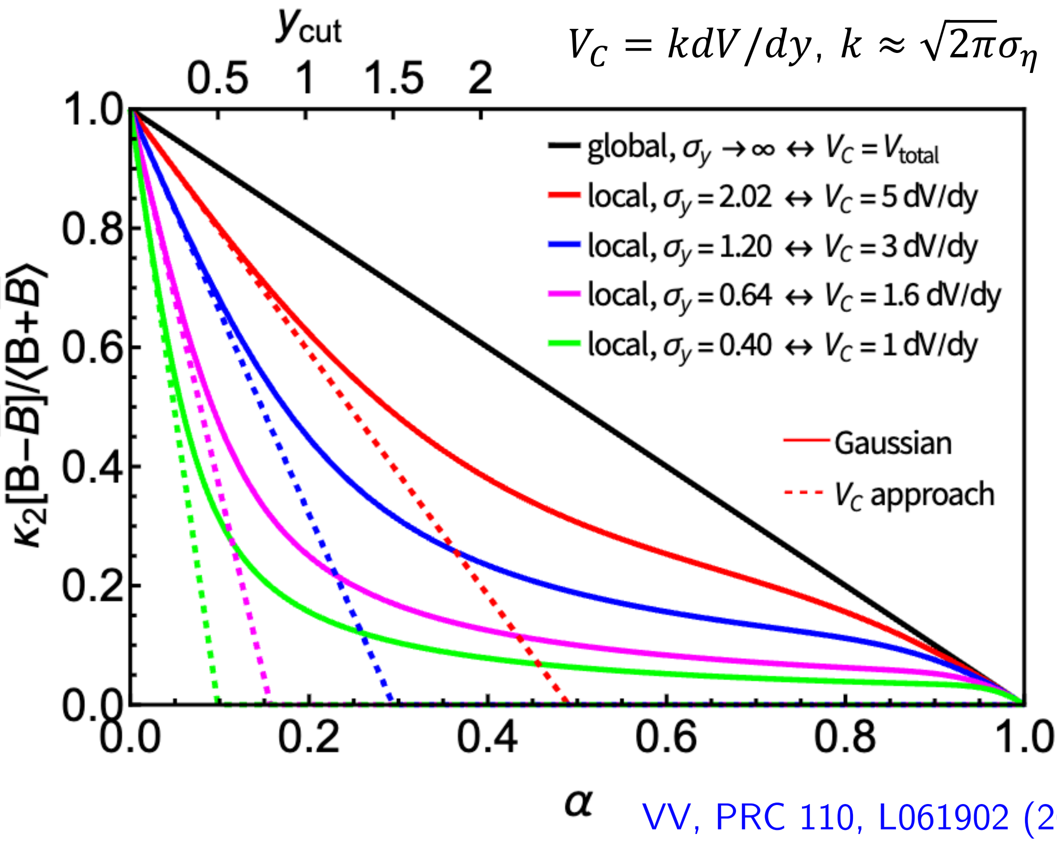
Local charge conservation in coordinate space

correlated sampling



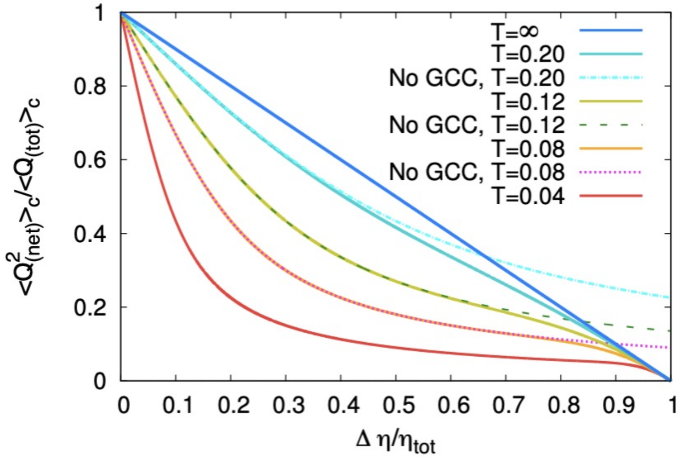
Braun-Munzinger et al., JHEP 08, 113 (2024)

$V_C = k dV/dy,\ k \approx \sqrt{2\pi}\sigma_\eta$



VV, PRC 110, L061902 (2024)

diffusion equation

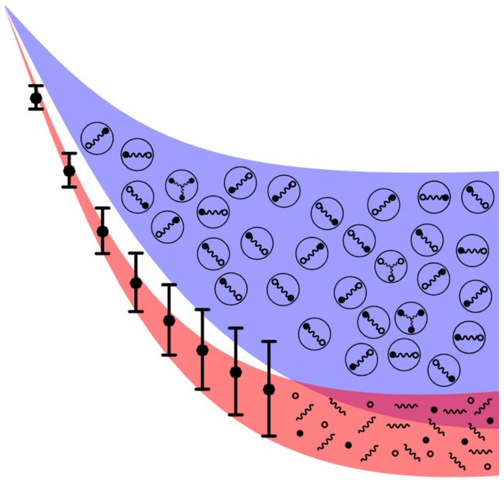


Sakaida, Asakawa, Kitazawa, PRC 90, 064911 (2014)

- Good agreement between different implementations of local charge conservation at 2nd order
- V_c approach works at small α, experiment (LHC) corresponds to effective $\alpha \approx$ 0.025-0.1

Braun-Munzinger et al., NPA 1008, 122141 (2021)

- Complementary observables: balance function (not studied in this work)



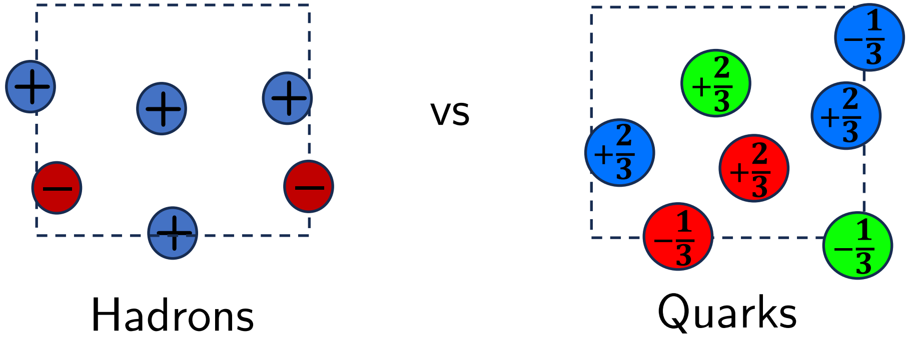
Charge fluctuations as a signature of fractional quark charges

J. Parra, R. Poberezhniuk, V. Koch, C. Ratti, VV, Phys. Rev. Lett. 135, 242302 (2025)

Charge fluctuations

An old idea: Hadrons carry integer electric charges, quarks carry *fractional* electric charges.

Jeon, Koch, PRL (2000);
Asakawa, Muller, Heinz, PRL (2000)



$D_{QGP} < D_{HG}$ → Distinct signal for QGP in heavy-ion collisions

Quantified by:

$$D = 4\frac{\kappa_2[Q]}{\langle N_{\rm ch}\rangle}$$

Here $\kappa_2[Q] = \langle Q^2\rangle - \langle Q\rangle^2$

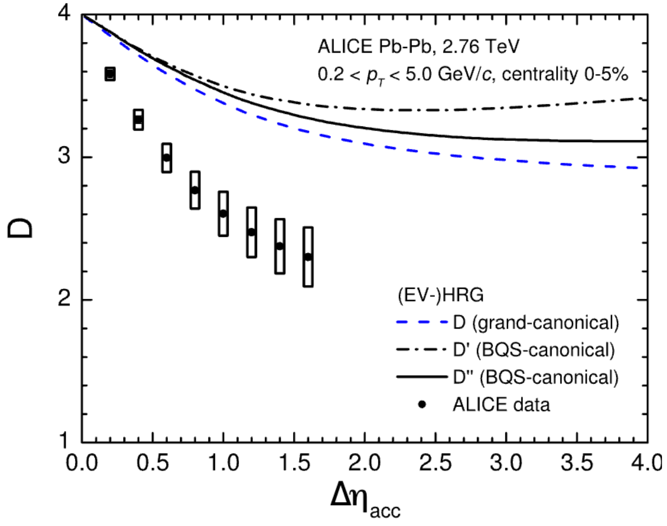
$$N_{\rm ch} = N_+ + N_-$$

GCE estimates:

- $D_{HG} \approx 2.8 - 4$
- $D_{QGP} \approx 1 - 1.5$

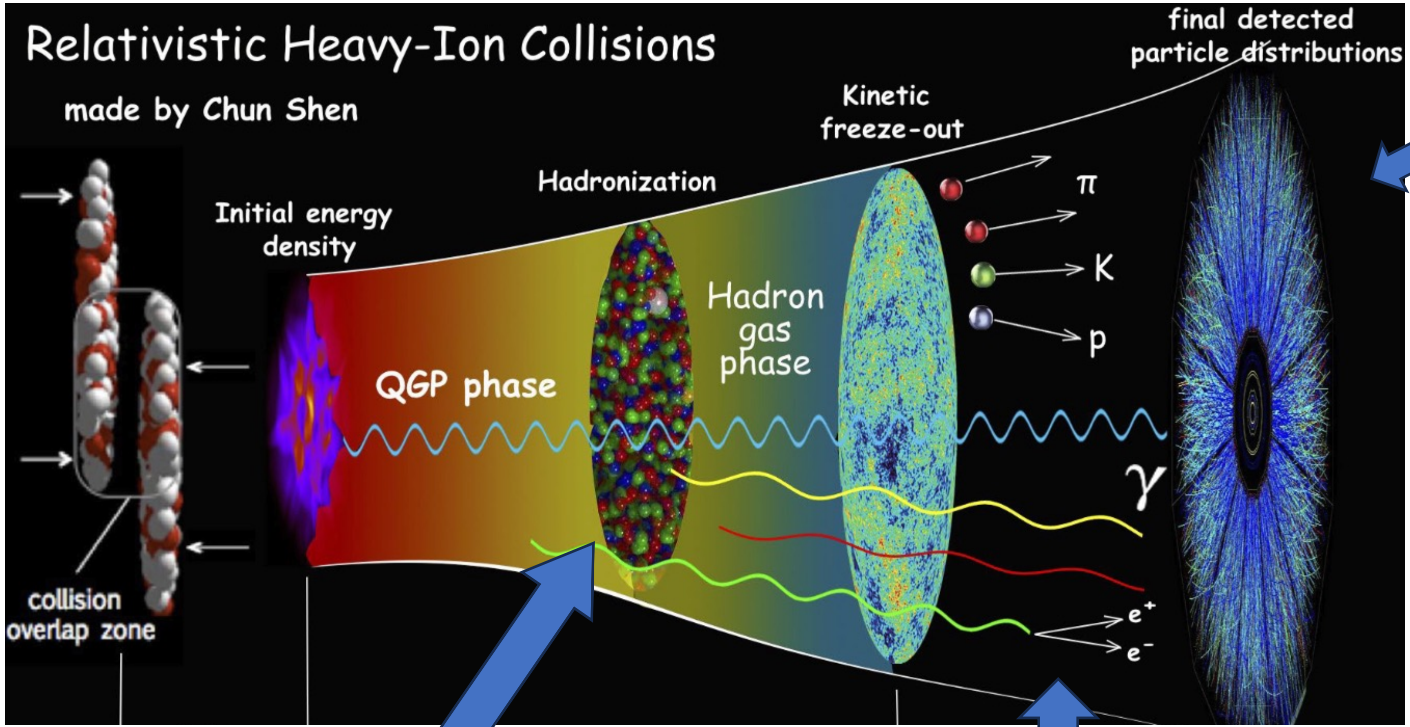
No quantitative calculations have been done for QGP outside the GCE limit

Prev. analyses are for HG



ALICE, PRL 110, 152301 (2013)
VV, Koch, PRC 103, 044903 (2021)

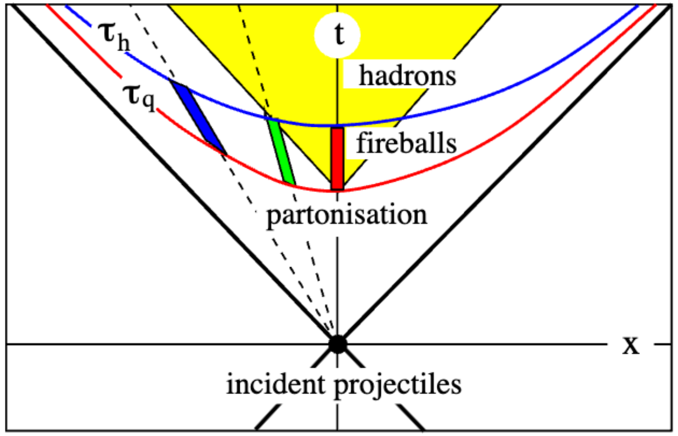
Charge fluctuations: stages



4. Kinematical cuts

never done for QGP

3. (Local) charge conservation



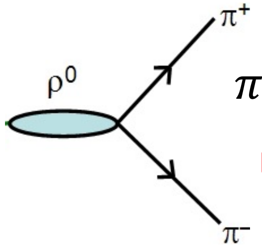
Castorina, Satz, IJMPE '14

never done for QGP

1. Fluctuations at hadronization (primordial charges)

$\boldsymbol{\omega}$ – Distinguishes hadron gas ($\boldsymbol{\omega \approx 1}$) from QGP ($\omega \approx 0.25 - 0.40$)

2. Resonance decays



$\pi^+\pi^-$ are correlated!

never done for QGP

1. Hadronization

$$\omega = \frac{\kappa_2[Q]}{\langle N_{\rm ch}^{\rm prim}\rangle}$$

variance at hadronization

charged multiplicity

Hadron gas: $\omega_{HG} \approx 1$ (Poisson statistics + Bose)

Free QGP*: $\omega_{QGP} \approx 0.36$ (Stefan-Boltzmann limit)

More generally:

$$\omega = \frac{\kappa_2[Q]}{\langle N_{\rm ch}^{\rm prim}\rangle} = \frac{VT^3\chi_2^Q}{S}\frac{S}{\langle N_{\rm ch}^{\rm prim}\rangle}$$

$$= \frac{\chi_2^Q}{s/T^3}\frac{S}{\langle N_{\rm ch}\rangle}\frac{\langle N_{\rm ch}\rangle}{\langle N_{\rm ch}^{\rm prim}\rangle}$$

The EoS

e.g. lattice QCD

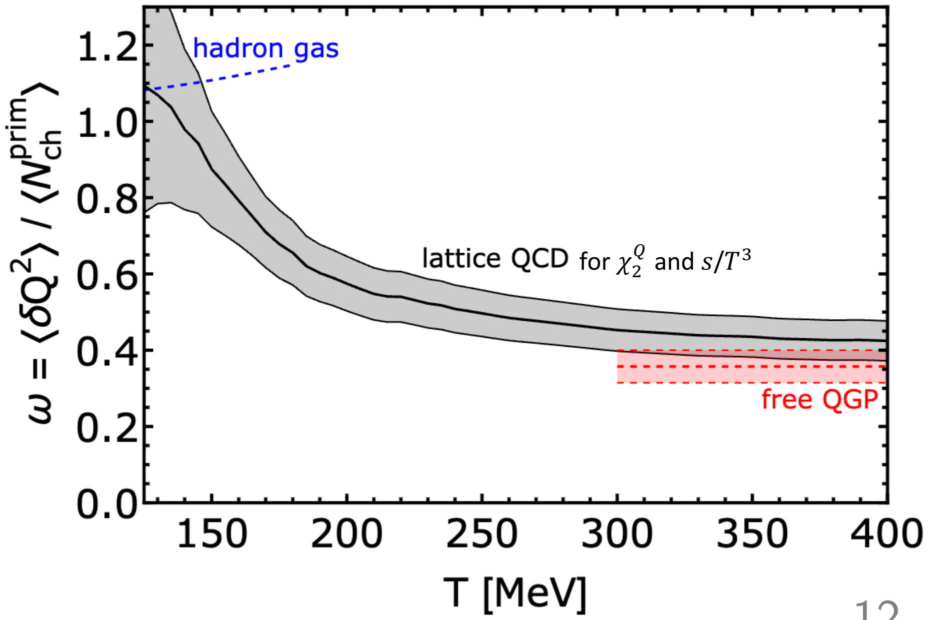
$S/N_{\rm ch} = 6.7 \pm 0.8$ (LHC)

$\gamma_Q \approx 1.67$ (decays)

from thermal model

Data-driven [P. Hanus, A. Mazeliauskas, K. Reygers, PRC (2019)]

ω from lattice QCD



*Same/similar for SQGB scenario of Fujimoto et al., PRD 112, 074006 (2025)

2. Decays and decomposition of charge susceptibility

GCE: $\kappa_2[Q] = V\chi_2^Q$

$$\chi_2^Q = \langle n_{\rm ch}^{\rm prim} \rangle + \varphi_2^{Q,{\rm prim}}$$

Skellam baseline (self-correlation)

Correction (2-particle correlations)

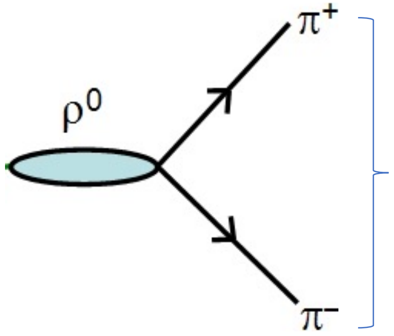
At hadronization (before decays) the strength of interactions is parametrized by ω:

$$\chi_2^Q = \omega \langle n_{\rm ch}^{\rm prim} \rangle \qquad \omega = \frac{\kappa_2[Q]}{\langle N_{\rm ch}^{\rm prim} \rangle} \qquad \varphi_2^{Q,{\rm prim}} = (\omega - 1)\langle n_{\rm ch}^{\rm prim} \rangle$$

After decays, net charge remains conserved but multiplicities of $+$ and – charges increase:

$$\langle n_{\rm ch} \rangle = \gamma_Q \langle n_{\rm ch}^{\rm prim} \rangle$$

$\gamma_Q \approx 1.67$ (from HRG)



Correlated!

$$\chi_2^Q = \langle n_{\rm ch} \rangle + \left(\frac{\omega}{\gamma_Q} - 1 \right) \langle n_{\rm ch} \rangle$$

Decays reshuffle self-correlation and 2-particle correlation terms

Other stages

3. Local charge conservation

[VV, PRC 110, L061902 (2024)]

- 2-point charge density correlator with a balancing term
- Local charge conservation introduced through modulation of the balancing term

$$\mathcal{C}_2^Q(\mathbf{r}_1,\mathbf{r}_2) \equiv \langle \delta\rho_Q(\mathbf{r}_1)\delta\rho_Q(\mathbf{r}_2)\rangle$$

$$\mathcal{C}_2^Q(\mathbf{r}_1,\mathbf{r}_2) = \chi_2^Q \left[\underbrace{\delta(\mathbf{r}_1-\mathbf{r}_2)}_{\text{local correlation}} - \underbrace{\frac{\varkappa(\mathbf{r}_1,\mathbf{r}_2)}{V_{\text{tot}}}}_{\text{balancing contribution}} \right] \qquad \underbrace{\mathbf{r} = \eta}_{\text{spatial rapidity}}$$

$$\underbrace{\varkappa(\eta_1,\eta_2) \propto \exp\left[-\frac{(\eta_1-\eta_2)^2}{2\sigma_\eta^2}\right]}_{\text{local charge conservation}}$$

4. Kinematical cuts

$$\kappa_2[Q_{\text{acc}}] = \int d\,\eta_1 \int d\,\eta_2 \mathcal{C}_2^Q(\eta_1,\eta_2)\, p(\eta_1)p(\eta_2)$$

$$\mathcal{C}_2^Q(\mathbf{r}_1,\mathbf{r}_2) = \chi_2^Q \left[\delta(\mathbf{r}_1-\mathbf{r}_2) - \frac{\varkappa(\mathbf{r}_1,\mathbf{r}_2)}{V_{\text{tot}}} \right]$$

$$\chi_2^Q = \langle n_{\text{ch}} \rangle + \left(\frac{\omega}{\gamma_Q} - 1 \right) \langle n_{\text{ch}} \rangle$$

Acceptance probabilities $p(\eta)$: weighted (π, K, p) average from the **blast-wave model**

NB: The local self-correlation term is multiplied by a single $p(\eta_1)$

Putting everything together

$$D = 4\left\{1-\left(1-\frac{\overset{\text{hadronization}}{\omega}}{\underset{\text{decays}}{\gamma_Q}}\right)\frac{\overset{\text{pair acceptance}}{\langle p^2(\eta)\rangle}}{\underset{\text{acceptance}}{\langle p(\eta)\rangle}} - \frac{\overset{\text{hadronization}}{\omega}}{\underset{\text{decays}}{\gamma_Q}}\frac{\overset{\text{local charge conservation}}{\langle p(\eta_1)p(\eta_2)\rangle_{\varkappa}}}{\underset{\text{acceptance}}{\langle p(\eta)\rangle}}\right\}$$

ω - Charge fluctuations at hadronization

$\omega_{HG} = 1$ $\qquad$ $\omega_{QGP} = 0.36$

γ_Q - Resonance decays

$\langle p(\eta_1)p(\eta_2)\rangle_{\varkappa}$ - Pair acceptance weighted with Local Charge Conservation

$\frac{\langle p^2(\eta)\rangle}{\langle p(\eta)\rangle}$ - Momentum Acceptance Cuts

p(η) from the blast-wave model

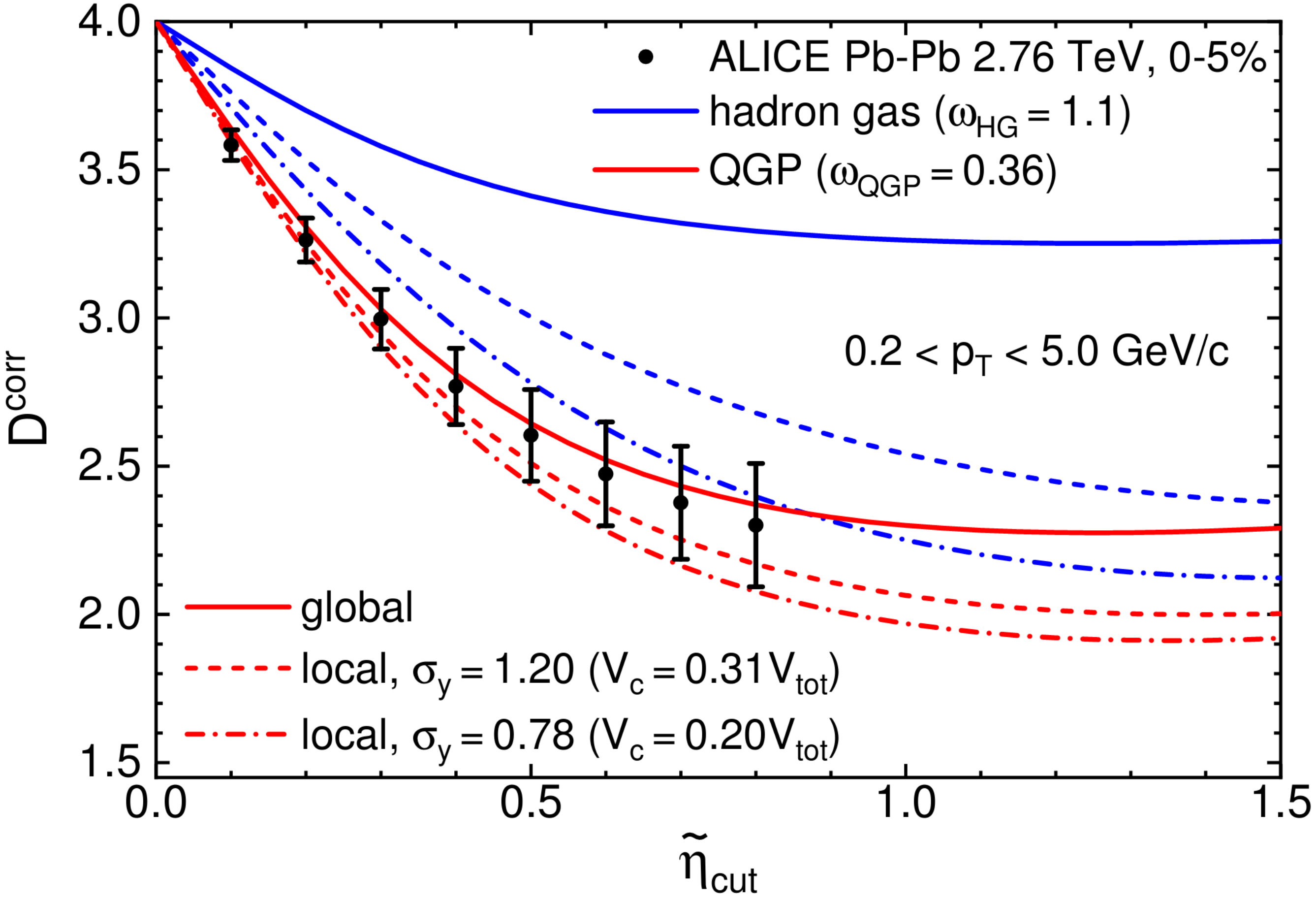
Experiment applies additional correction for global charge conservation

$$D^{\text{corr}} = \frac{D' + D''}{2}$$

$$D' = D + 4\langle p(\eta)\rangle, \qquad D'' = \frac{D}{1-\langle p(\eta)\rangle}$$

D-measure at LHC: comparison with experiment

$$D = 4\left\{1 - \left(1 - \frac{\omega}{\gamma_Q}\right)\frac{\langle p^2(\eta)\rangle}{\langle p(\eta)\rangle} - \frac{\omega}{\gamma_Q}\frac{\langle p(\eta_1)p(\eta_2)\rangle_\varkappa}{\langle p(\eta)\rangle}\right\}$$



Parameters used:

$\omega_{HG} = 1.1$ $\omega_{QGP} = 0.36$

$\gamma_Q = 1.67$

Vary σ_η to accommodate global vs local charge conservation, based on VV, PRC 110, L061902 (2024)

Experiment applies additional correction for global charge conservation which we repeat

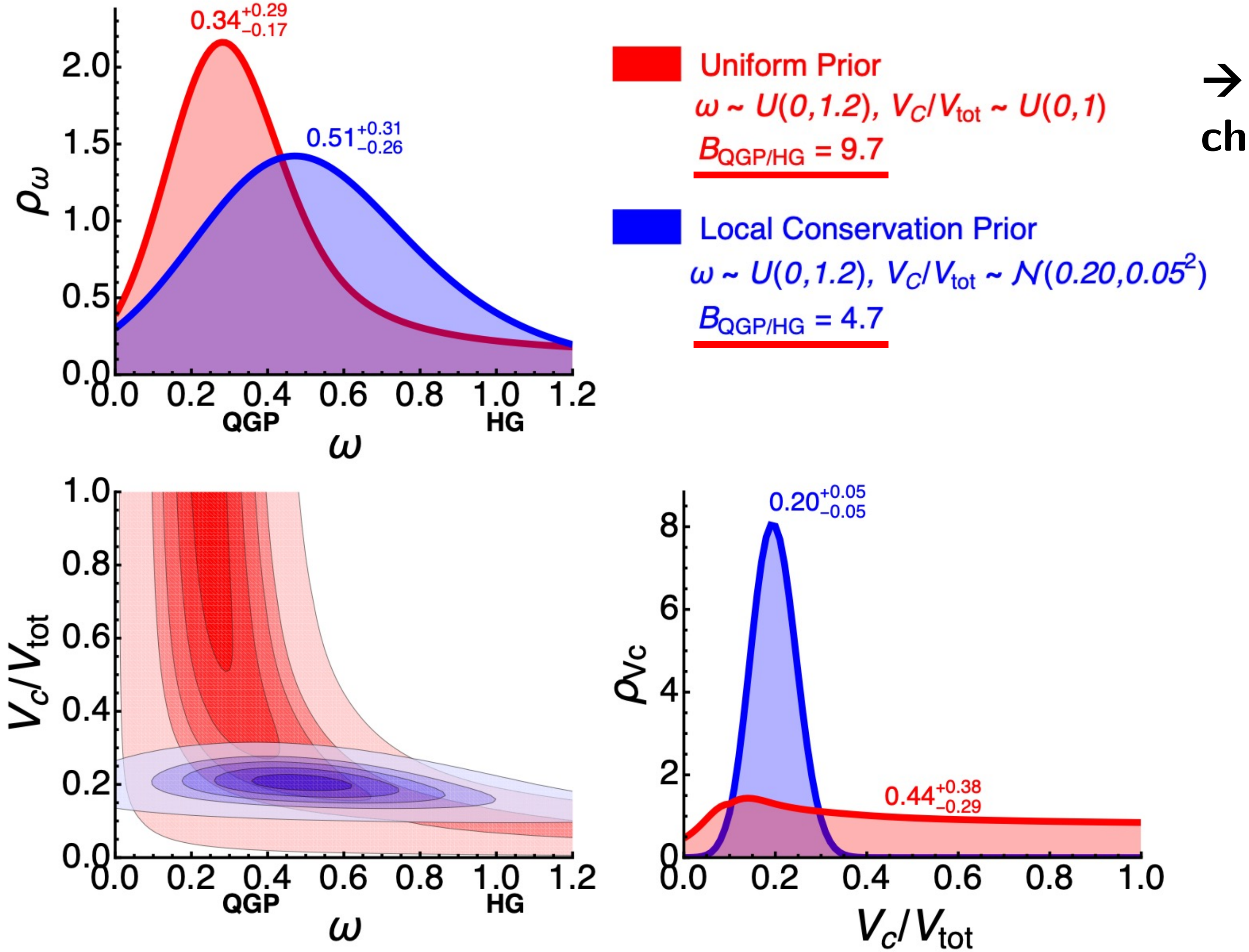
$$D^{\rm corr} = \frac{D' + D''}{2}$$

$$D' = D + 4\langle p(\eta)\rangle, \qquad D'' = \frac{D}{1 - \langle p(\eta)\rangle}$$

Hadron gas scenario requires a very local charge conservation range

D-measure at LHC: Bayesian analysis

Vary primordial fluctuation ω (HG vs QGP) and correlation volume V_C (local conservation) freely



→ **Moderate evidence for freeze-out of charge fluctuations in the QGP phase (ω).**

Bayes factor BF_{12} for H_1 over H_2	**Evidence category**
> 100	Extreme evidence for H_1 over H_2
30 - 100	Very strong evidence for H_1 over H_2
10 - 30	Strong evidence for H_1 over H_2
3 - 10	Moderate evidence for H_1 over H_2
1 - 3	Anecdotal evidence for H_1 over H_2
1	No evidence over H_2

D-measure at LHC: Run 2

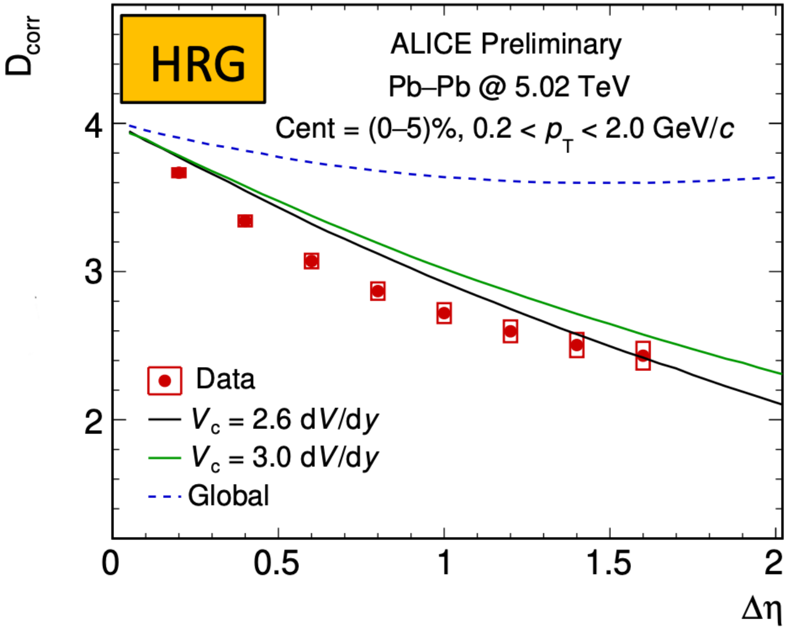
LHC Run 2 analysis is nearing completion

From M. Arslandok (ALICE preliminary), QM2025

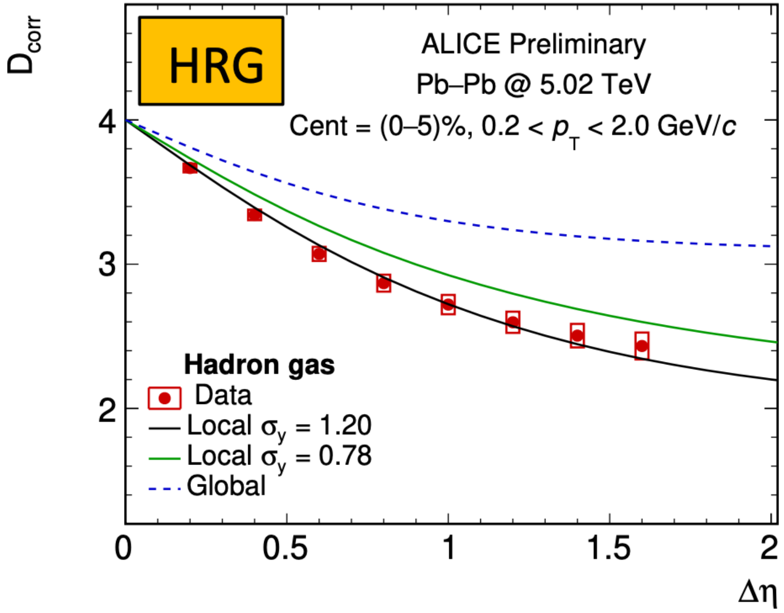
$$D^{\text{corr}} = \frac{D' + D''}{2}$$

$$D' = D + 4\langle p(\eta)\rangle, \qquad D'' = \frac{D}{1 - \langle p(\eta)\rangle}$$

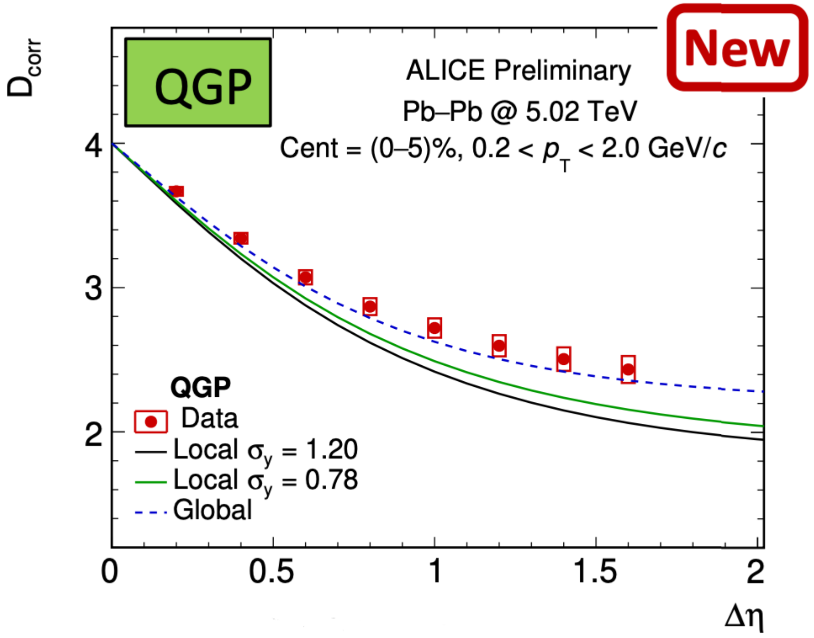
V_c approach



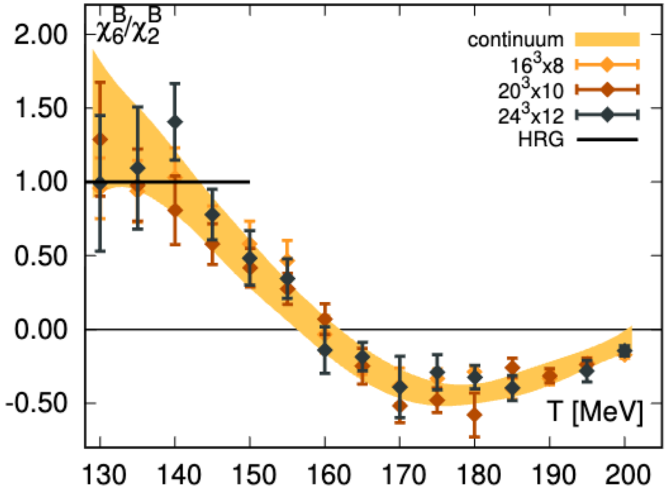
Gaussian correlation

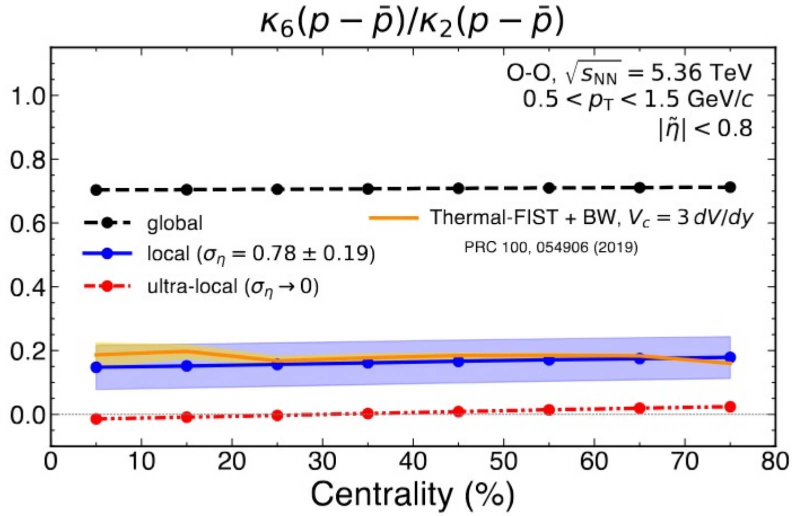


Gaussian correlation



Experiment still uses $\mathrm{D}^{\mathrm{corr}}$ which diminishes discriminating power
We advocate for direct comparisons with uncorrected D-measure (in progress)





Canonical statistical hadronization with local baryon conservation for higher-order cumulants

M. Ciacco, V.A. Kuznietsov, S. Kundu, M. Puccio, VV, arXiv:2605.30710

Local charge conservation: high-order cumulants

Ciacco, Kuznietsov, et al., to appear

Introduce **n-point local conservation kernel**

$$\mathcal{C}_2(\eta_1,\eta_2) = \chi_2^B \left[\delta_{1,2} - \frac{\varkappa_2(\eta_1,\eta_2)}{V}\right]$$

second-order

$$\mathcal{C}_3(\eta_1,\eta_2,\eta_3) = \chi_3^B\,\delta_{1,2,3} - \frac{\chi_3^B}{V}\left[\delta_{1,2}\,\varkappa_2(\eta_1,\eta_3) + \delta_{1,3}\,\varkappa_2(\eta_1,\eta_2) + \delta_{2,3}\,\varkappa_2(\eta_2,\eta_1)\right] + \frac{2\chi_3^B}{V^2}\,\varkappa_3(\eta_1,\eta_2,\eta_3)\Big]$$

third-order

$$\mathcal{C}_4(\eta_1,\ldots,\eta_4) = \chi_4^B\delta_{1,2,3,4} - \frac{\chi_4^B}{3!V}\sum_{\sigma\in S_4}\delta_{\sigma_1,\sigma_2,\sigma_3}\varkappa_2(\eta_{\sigma_1},\eta_{\sigma_4}) - \frac{(\chi_3^B)^2}{(2!)^2\chi_2^B V}\sum_{\sigma\in S_4}\delta_{\sigma_1,\sigma_2}\delta_{\sigma_3,\sigma_4}\varkappa_2(\eta_{\sigma_1},\eta_{\sigma_3})$$

fourth-order

$$+\frac{1}{2!V^2}\left[\chi_4^B + \frac{(\chi_3^B)^2}{\chi_2^B}\right]\sum_{\sigma\in S_4}\delta_{\sigma_1,\sigma_2}\varkappa_3(\eta_{\sigma_1},\eta_{\sigma_3},\eta_{\sigma_4}) - \frac{3}{V^3}\left[\chi_4^B + \frac{(\chi_3^B)^2}{\chi_2^B}\right]\varkappa_4(\eta_1,\eta_2,\eta_3,\eta_4).$$

Sum rule:

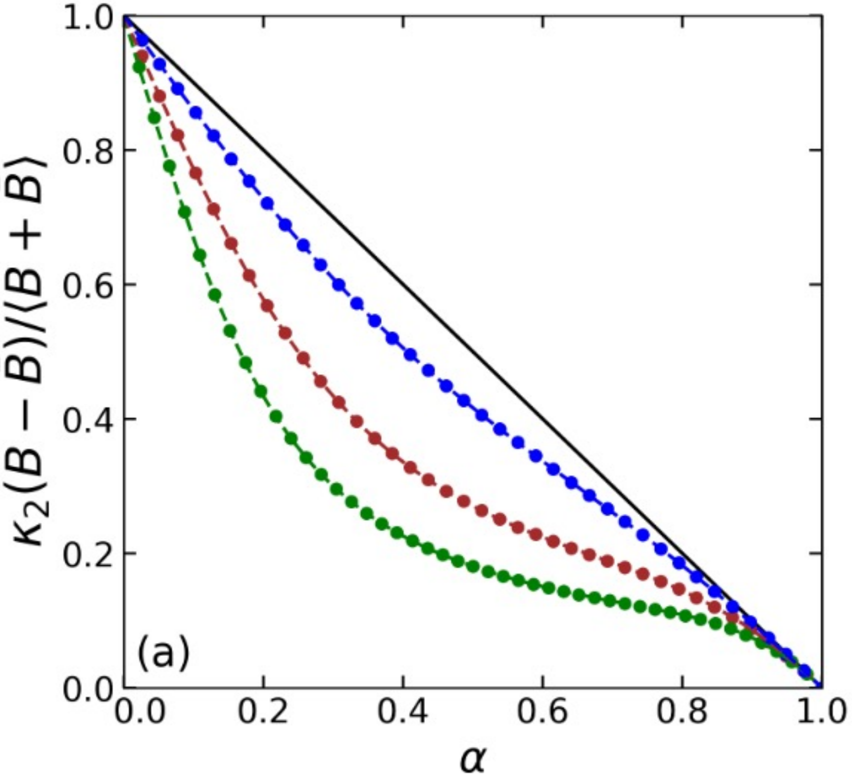
$$\int d\eta_n\,\mathcal{C}_n = 0$$ for any *n* $\Longleftrightarrow$ $$\int d\eta_n\,\varkappa_n(\eta_1,\ldots,\eta_n) = V\,\varkappa_{n-1}(\eta_1,\ldots,\eta_{n-1})$$

Symmetric n-point Gaussian kernel:

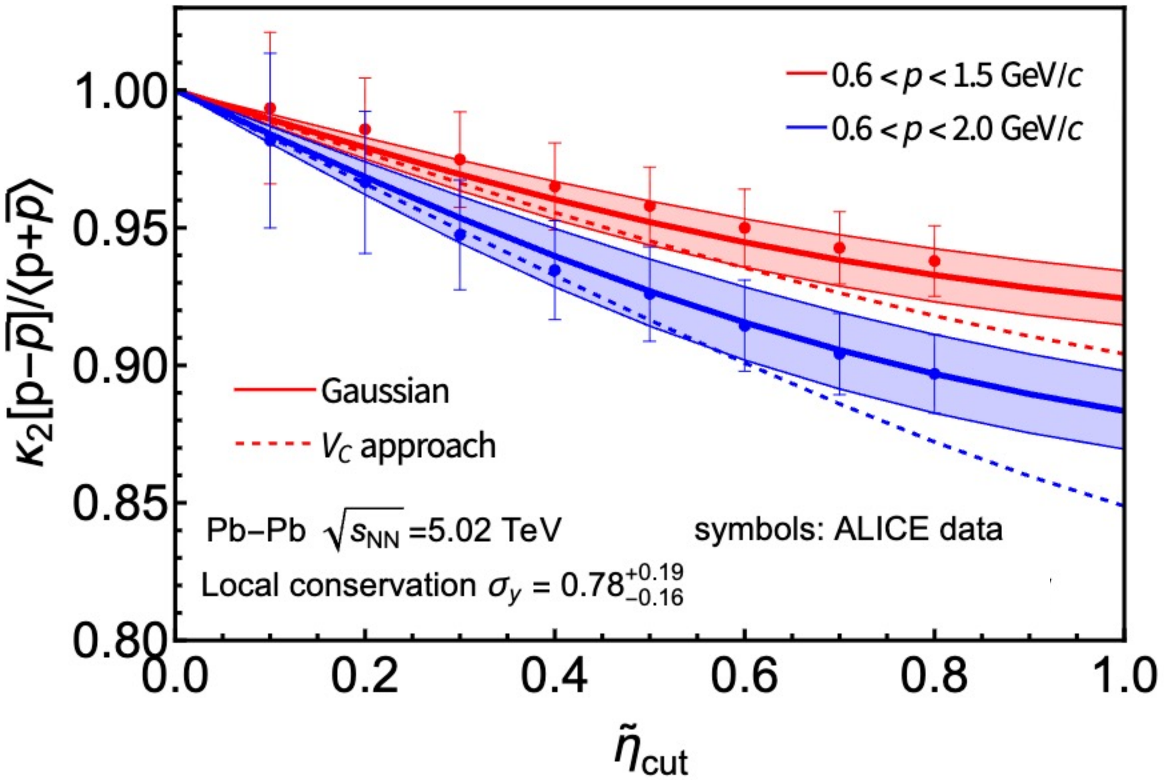
$$\varkappa_2(\eta_1,\eta_2) \propto \exp\left[-\frac{(\eta_1-\eta_2)^2}{2\sigma_\eta^2}\right] \Longrightarrow \varkappa_n(\eta_1,\ldots,\eta_n) \propto A_n\exp\left[-\frac{1}{n\sigma_\eta^2}\sum_{1\le i<j\le n}(\eta_i-\eta_j)^2\right]$$

Baseline: 2nd order net-proton cumulants

coordinate space



momentum space



VV, PRC 110, L061902 (2024)

Agrees with hadronic diffusion model of Sakaida et al., PRC 90, 064911 (2014)

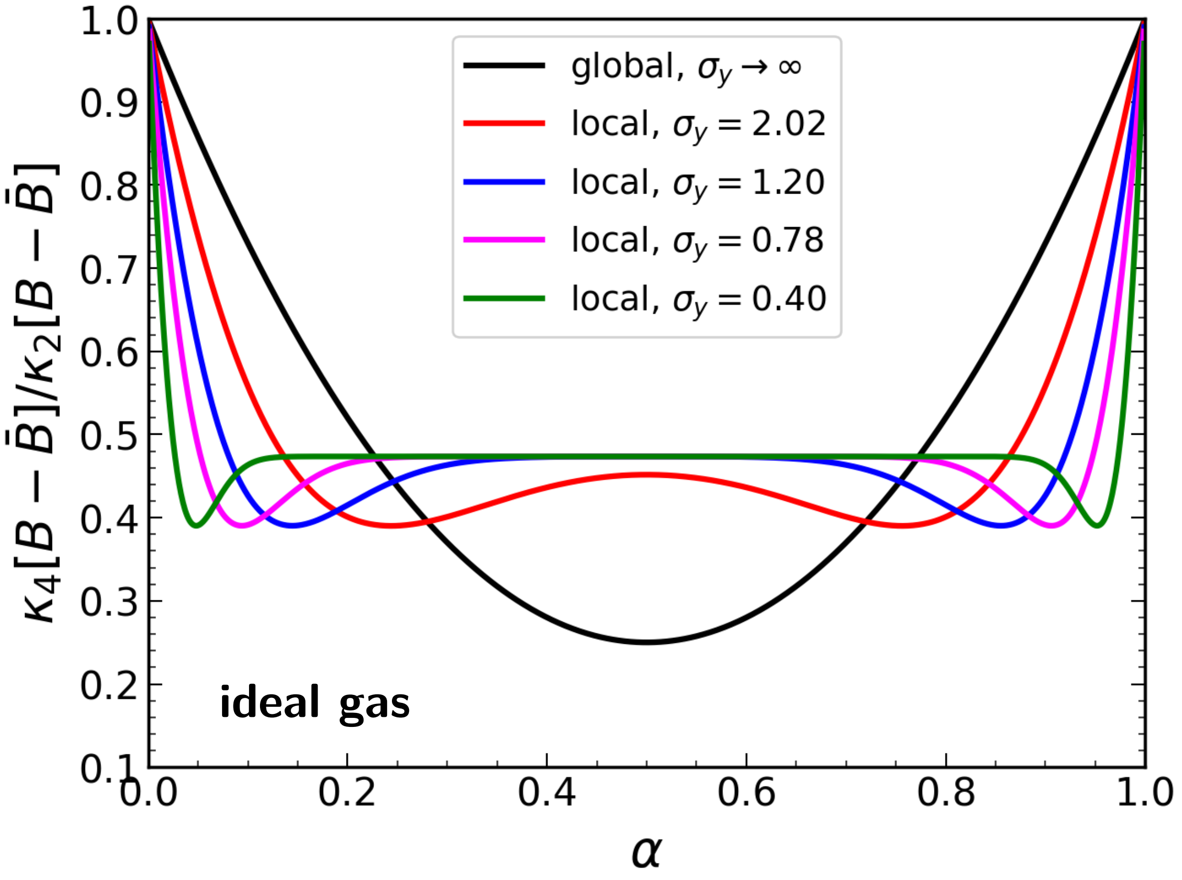
→ Describes hadronic diffusion in hadron gas limit

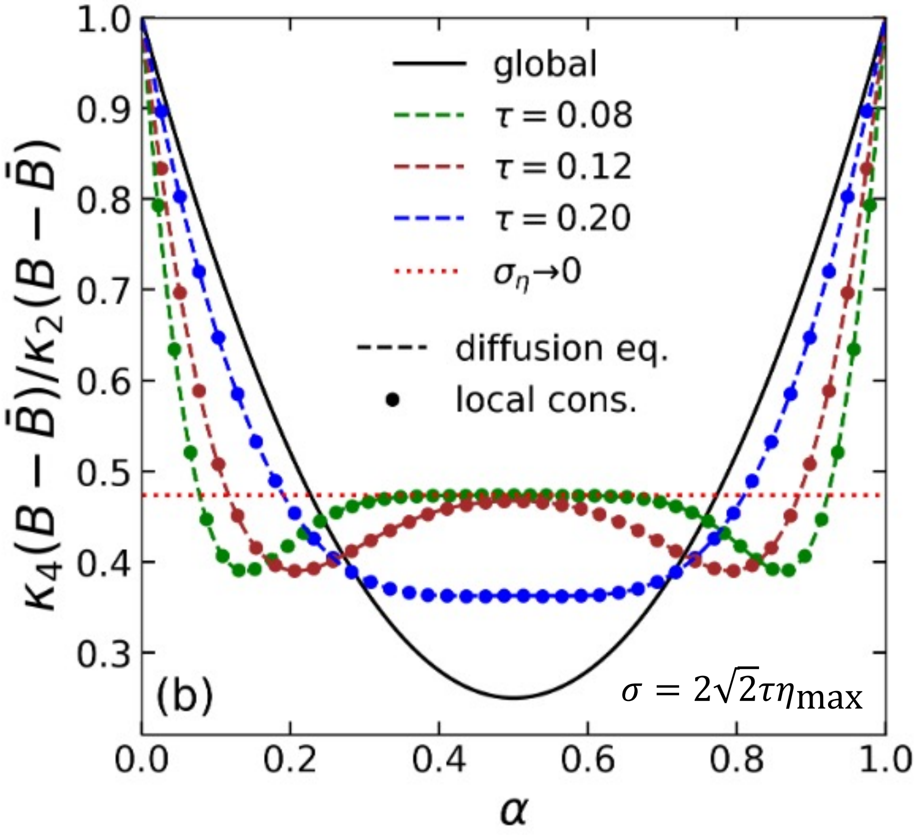
Hadronic scenario describes the data with $\sigma_\eta \sim 0.78$

Fourth-order net-baryon fluctuations

Ciacco, Kuznietsov, Kundu, Puccio, VV, arXiv:2605.30710

Coordinate space: cut in spatial rapidity $|\eta| < \eta_{cut}$





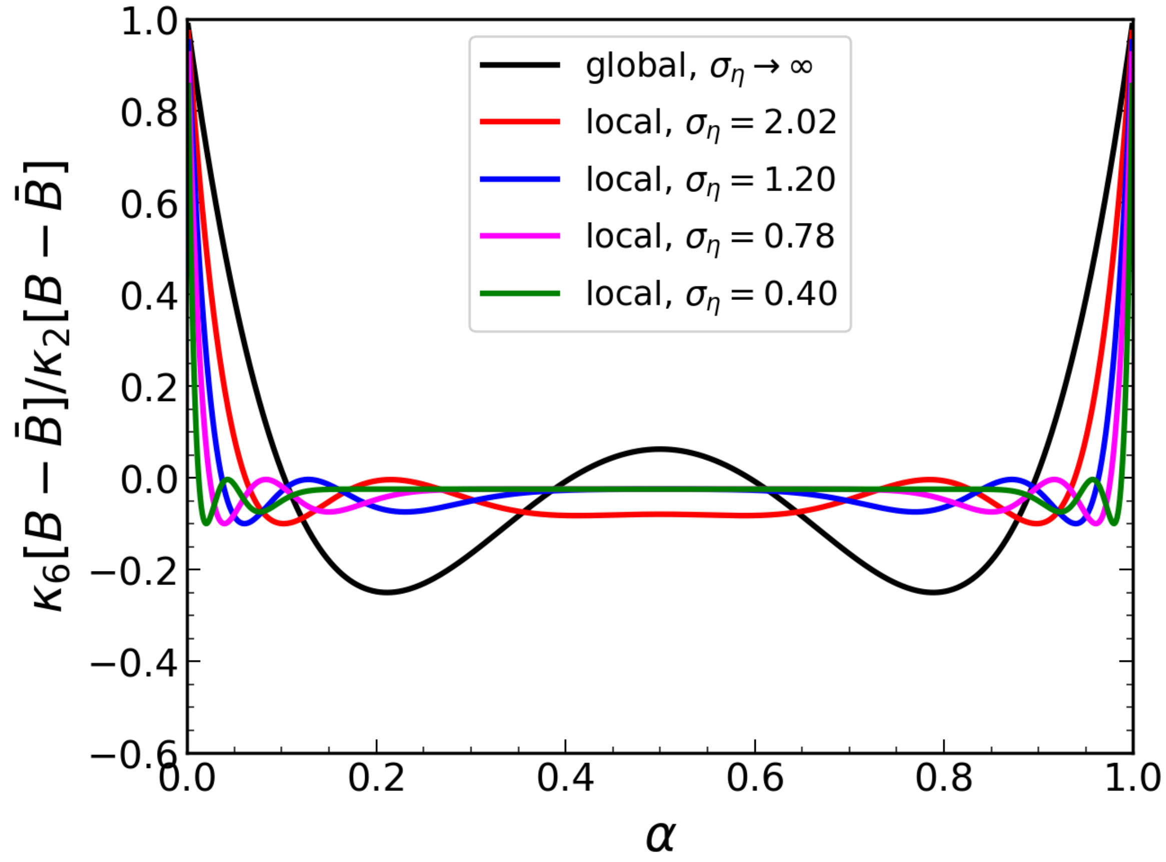
Small σ limit: plateau at $-\frac{1}{2}+\frac{9}{\pi}\arcsin(1/3) \approx 0.474$

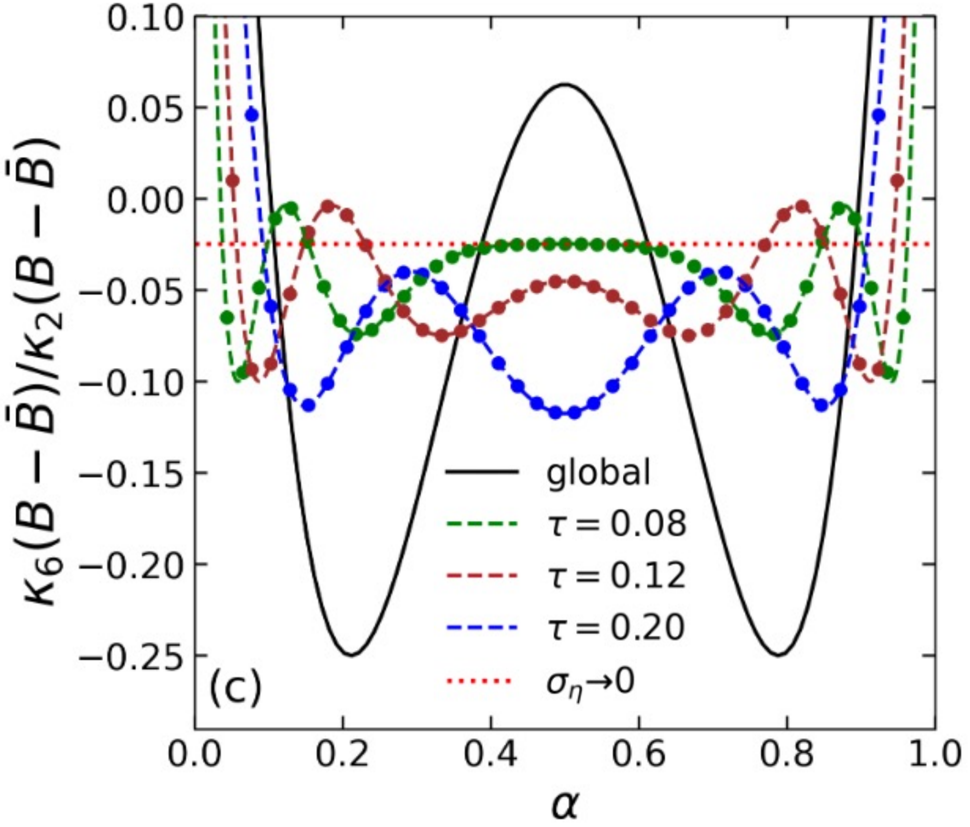
Excellent (exact?) agreement with hadronic diffusion model of Sakaida et al., PRC 90, 064911 (2014)

Sixth-order net-baryon fluctuations

Ciacco, Kuznietsov, Kundu, Puccio, VV, arXiv:2605.30710

Coordinate space: cut in spatial rapidity $|\eta| < \eta_{cut}$





Small σ limit: plateau at $\frac{31}{16} - \frac{225}{4\pi}\arcsin\left(\frac{1}{3}\right) + \frac{675}{16\sqrt{\pi}} I_4 \simeq -0.02465$

Excellent (exact?) agreement with hadronic diffusion model of Sakaida et al., PRC 90, 064911 (2014)

Predictions for O-O collisions

Ciacco, Kuznietsov, Kundu, Puccio, VV, arXiv:2605.30710

Momentum-space measurements → Acceptance factors $p(\eta)$ at each spatial rapidity

Input from the blast-wave model

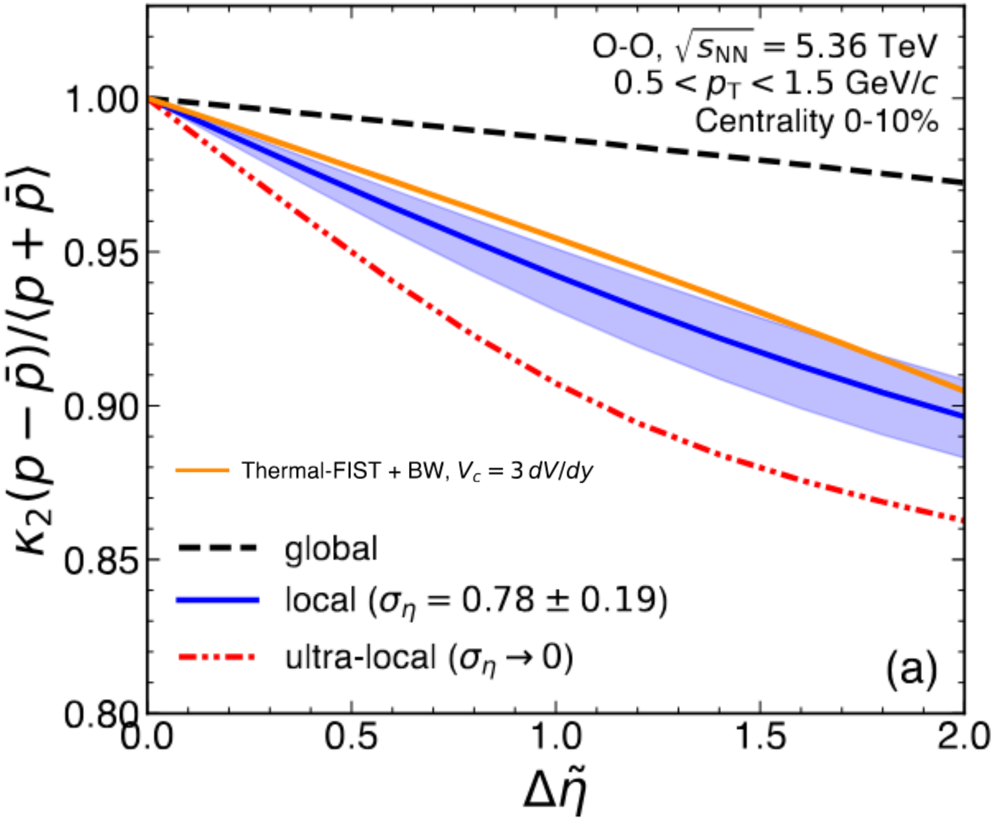
ideal gas

$$\kappa_2 = \frac{\langle B+\bar{B}\rangle}{V}\left[\mathcal{J}_1 - \mathcal{J}_2\right],$$

$$\kappa_4 = \frac{\langle B+\bar{B}\rangle}{V}\left[\mathcal{J}_1 - 4\,\mathcal{J}_2 + 6\,\mathcal{J}_3 - 3\,\mathcal{J}_4\right],$$

$$\kappa_6 = \frac{\langle B+\bar{B}\rangle}{V}\left[\mathcal{J}_1 - 16\,\mathcal{J}_2 + 75\,\mathcal{J}_3 - 150\,\mathcal{J}_4 + 135\,\mathcal{J}_5 - 45\,\mathcal{J}_6\right].$$

$$\mathcal{J}_n \equiv \frac{1}{V^{n-1}}\int d\eta_1 \dots d\eta_n\, p(\eta_1)\dots p(\eta_n)\, \varkappa_n(\eta_1,\dots,\eta_n)$$



Preferred scenario: $\sigma_\eta = 0.78$

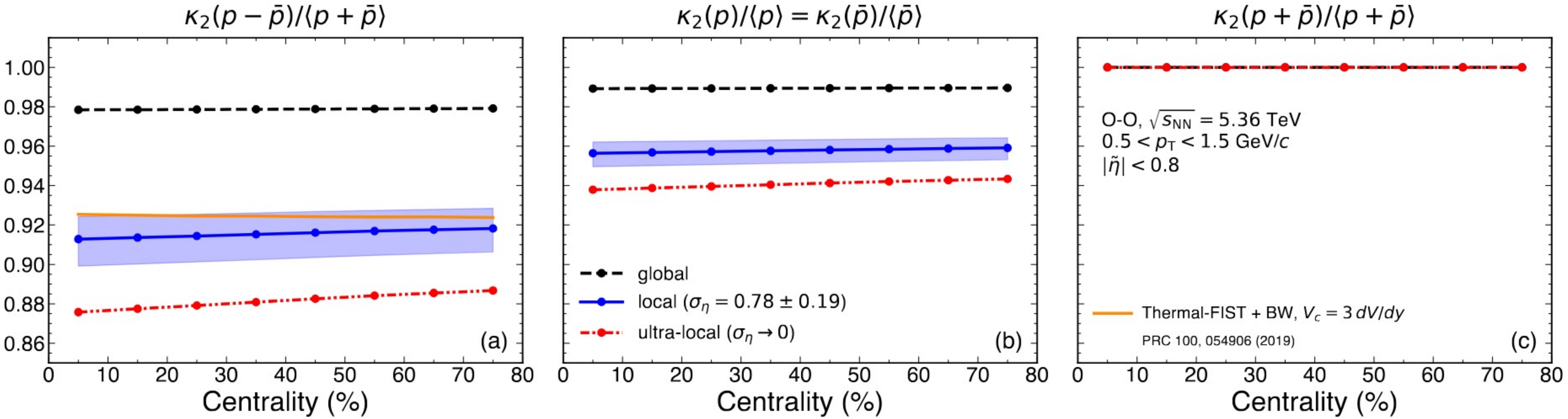
- Constraint from 5.02 TeV Pb-Pb data

Ultra-local limit ($\sigma_\eta \to 0$):

- maximum effect of baryon conservation
- non-zero cumulants in this limit due to momentum cut and absence of neutrons

Predictions for O-O collisions

Ciacco, Kuznietsov, Kundu, Puccio, VV, arXiv:2605.30710



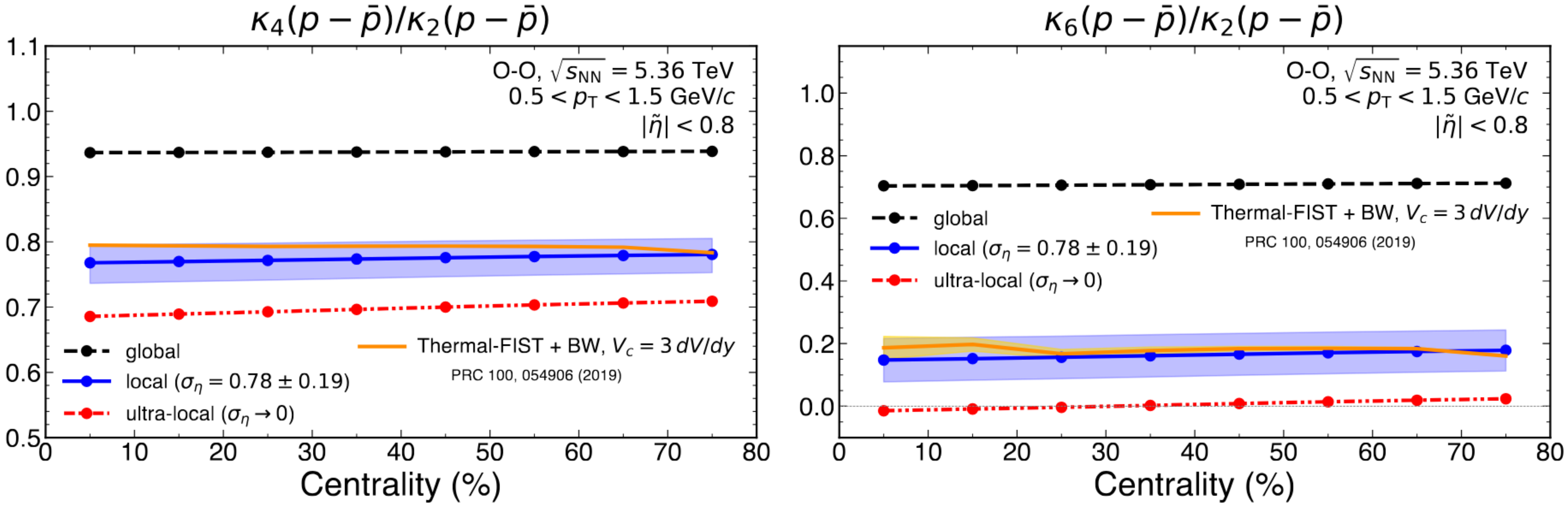
- Measurements of individual variances carry additional information

- $\kappa_2[p]/\langle p\rangle$ closer to Poisson than $\kappa_2[p-\bar{p}]/\langle p+\bar{p}\rangle$

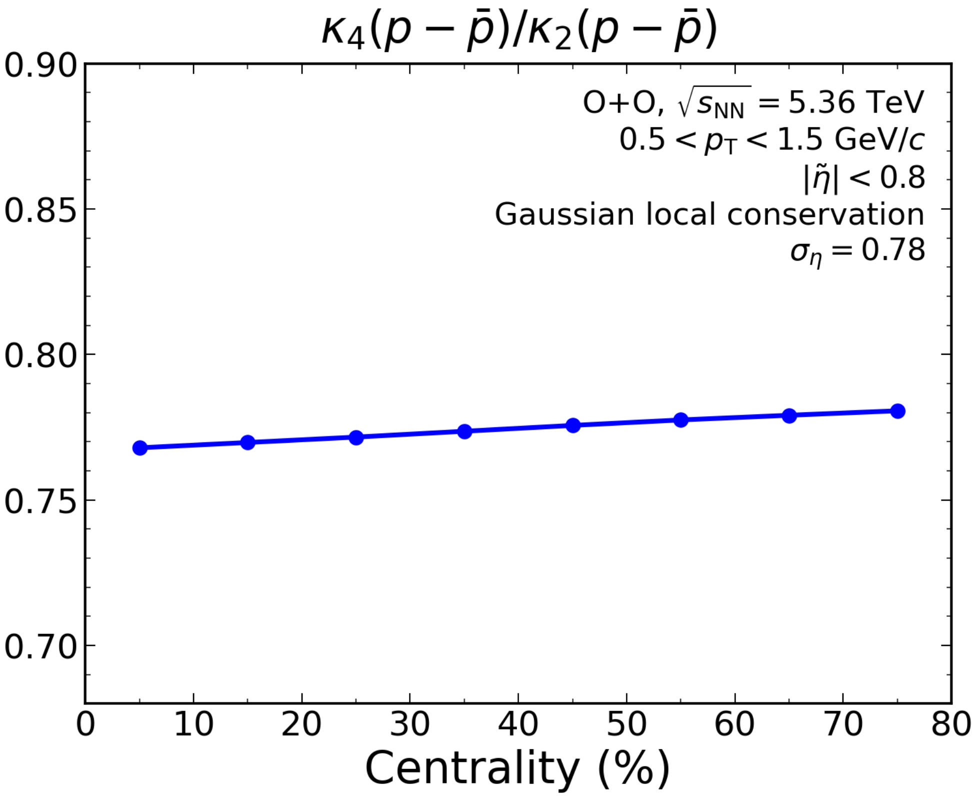
- $\boldsymbol{\kappa_2[p+\bar{p}]/\langle p+\bar{p}\rangle=1}$
  - consequence of symmetry at LHC: $\mathrm{cov}(p+\bar{p}, B_{\mathrm{tot}}-\bar{B}_{\mathrm{tot}}) = 0$
  - clear test of physics beyond baryon conservation

Predictions for O-O collisions: high-order cumulants

Ciacco, Kuznietsov, Kundu, Puccio, VV, arXiv:2605.30710

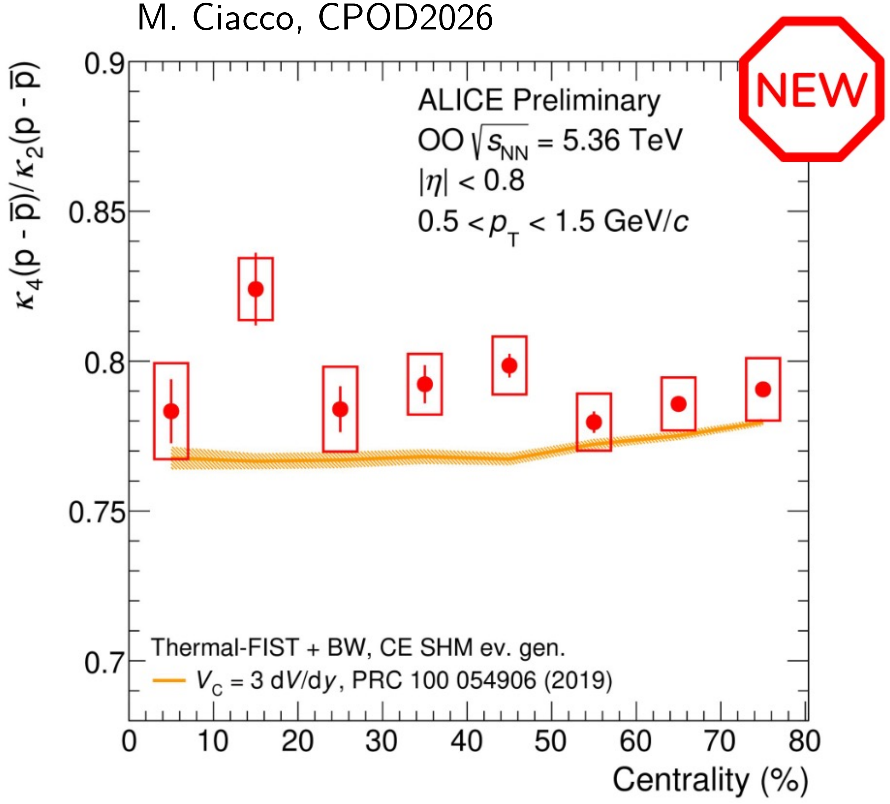


Comparison to data and other implementations: κ_4/κ_2



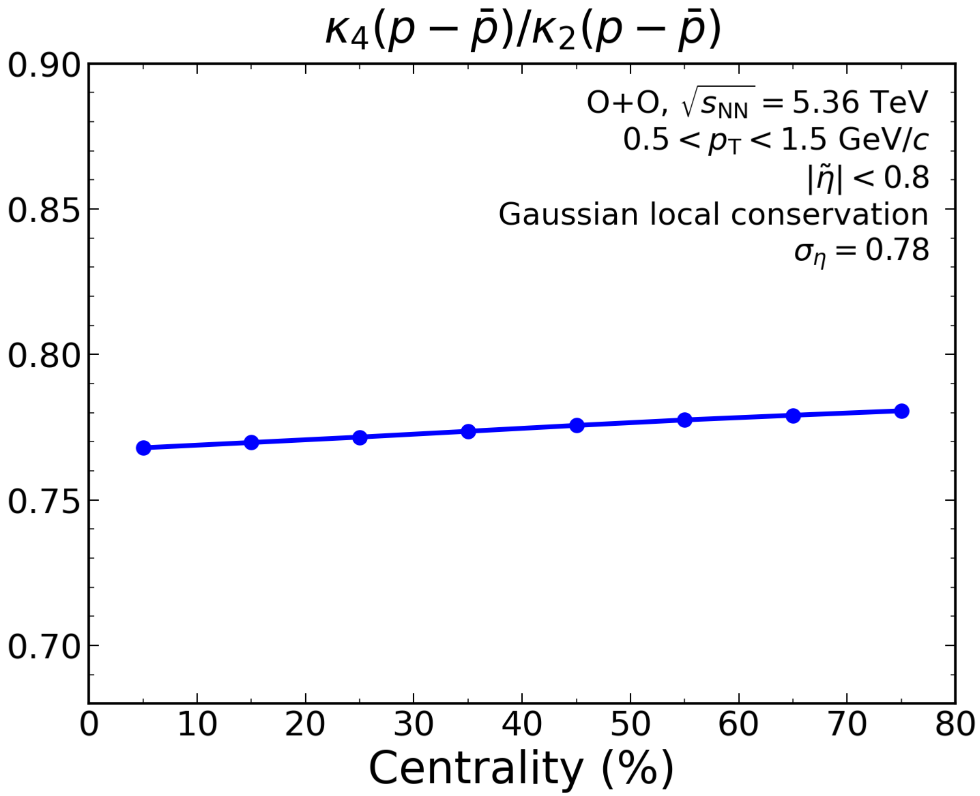
$\kappa_4(p-\bar{p})/\kappa_2(p-\bar{p})$

M. Ciacco, CPOD2026

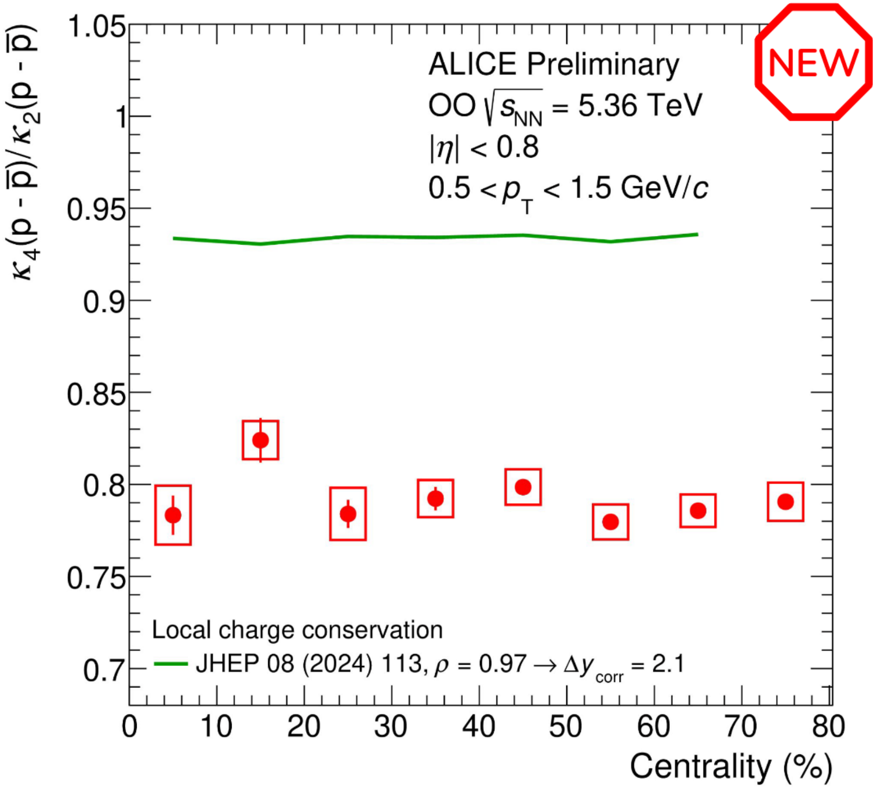


- ◐ Fair agreement with preliminary O-O data
- ✓ Excellent (exact?) agreement with the diffusion model of PRC 90, 064911 (2014)
- ✓ Good agreement with V_c approach (Thermal-FIST SHM $V_c = 3dV/dy$) PRC 100, 054906 (2019)

Comparison to data and other implementations: κ_4/κ_2



M. Ciacco, CPOD2026; A. Rustamov, CPOD2026



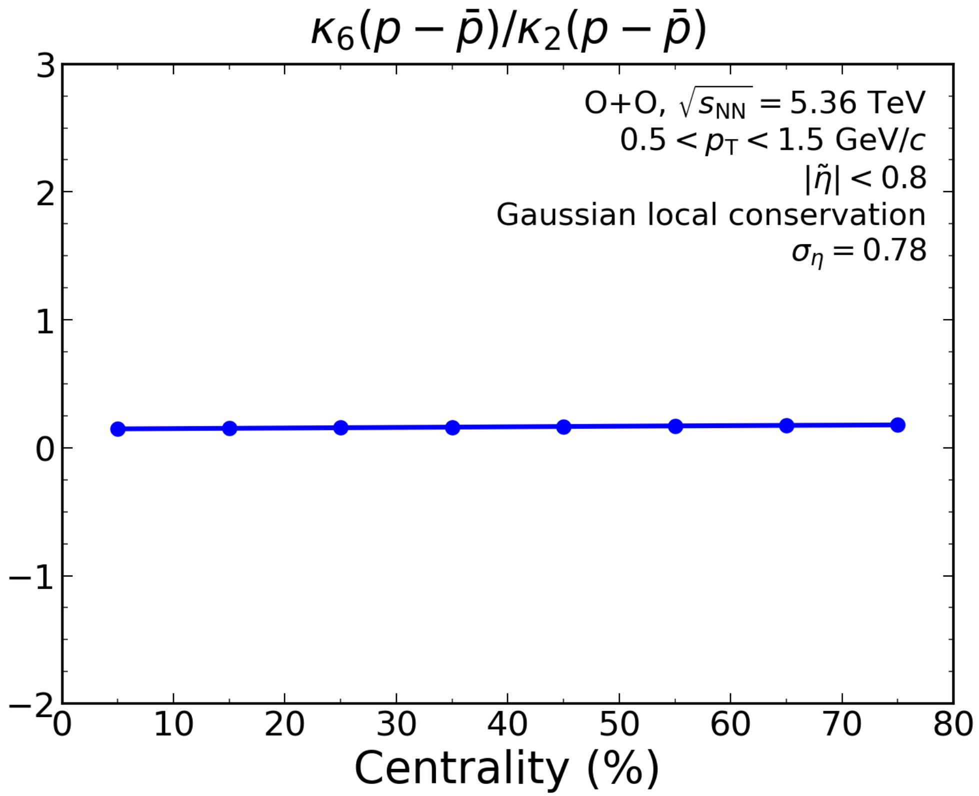
◐ Fair agreement with preliminary O-O data

✓ Excellent (exact?) agreement with the diffusion model of PRC 90, 064911 (2014)

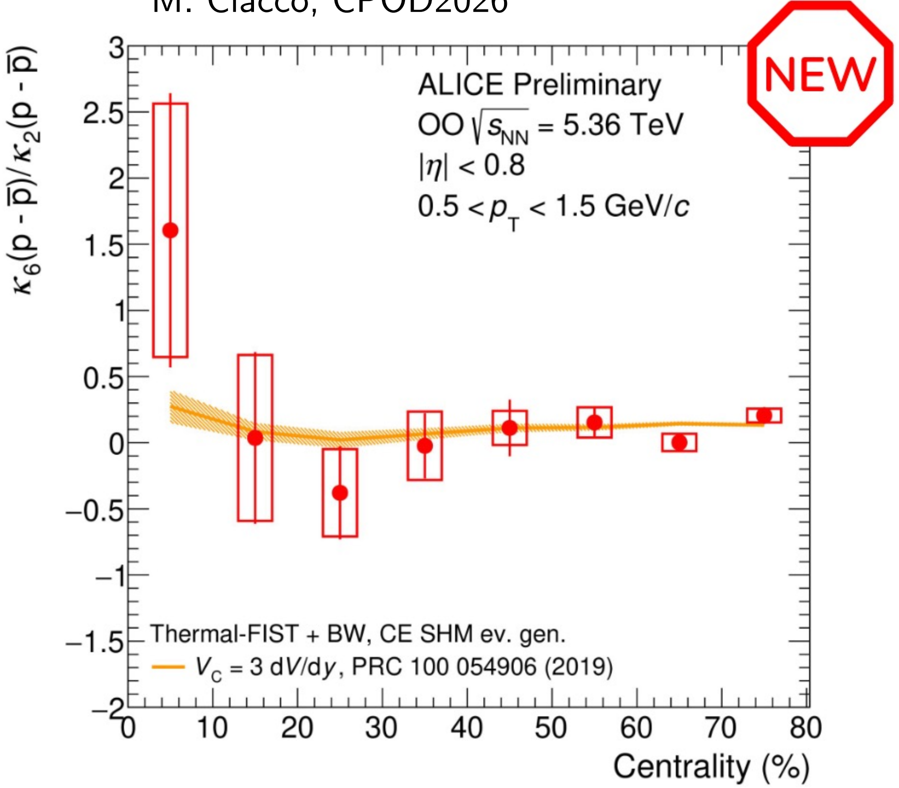
✓ Good agreement with V_c approach (Thermal-FIST SHM $V_c = 3dV/dy$) PRC 100, 054906 (2019)

✗ No agreement with the correlated sampling model of JHEP 08, 113 (2024)

Comparison to data and other implementations: κ_6/κ_2



M. Ciacco, CPOD2026

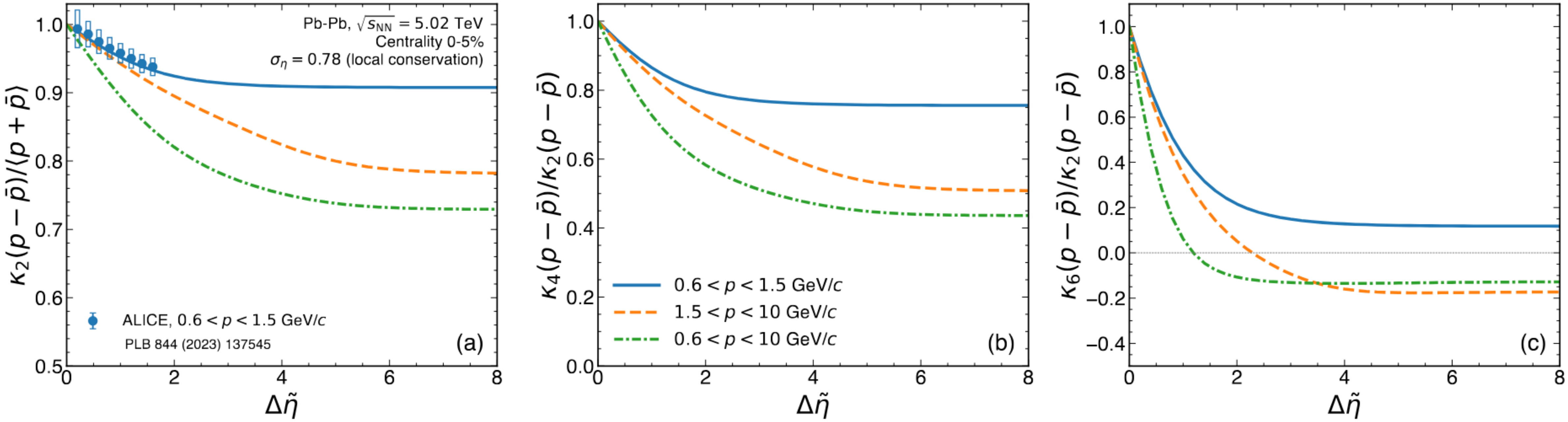


- ◐ Fair agreement with preliminary O-O data
- ✓ Excellent (exact?) agreement with the diffusion model of PRC 90, 064911 (2014)
- ✓ Good agreement with V_c approach (Thermal-FIST SHM $V_c = 3dV/dy$) PRC 100, 054906 (2019)

Looking further: ALICE3

Extended acceptance and statistics coverage with next-generation heavy-ion experiment at LHC

ALICE3 letter of intent, arXiv:2211.02491



- Calculations provide a robust baseline
  - Local baryon conservation alone can drive proton κ_6 negative
- **Outlook:** Direct calculation of proton cumulants based on lattice QCD susceptibilities, e.g. through maximum entropy method

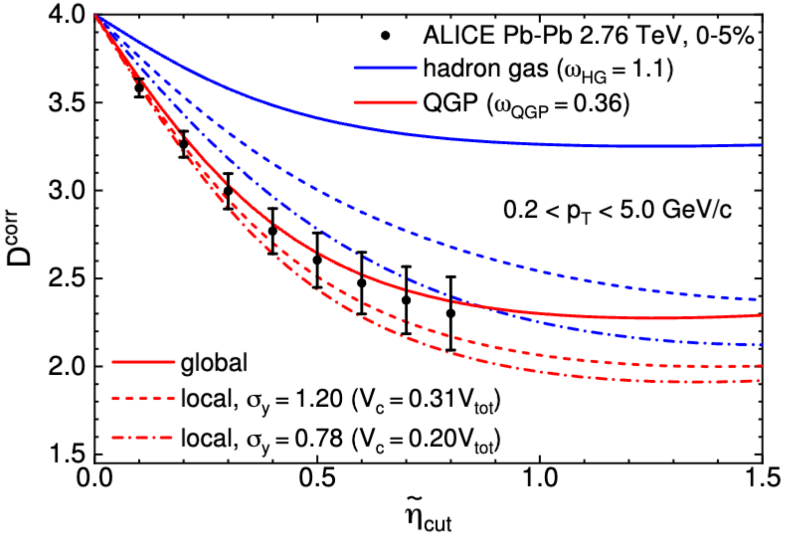
Summary

Density correlations framework:

local charge conservation for higher-order cumulants

$$\mathcal{C}_2(\eta_1,\eta_2) = \chi_2^B \left[\delta_{1,2} - \frac{\varkappa_2(\eta_1,\eta_2)}{V}\right]$$

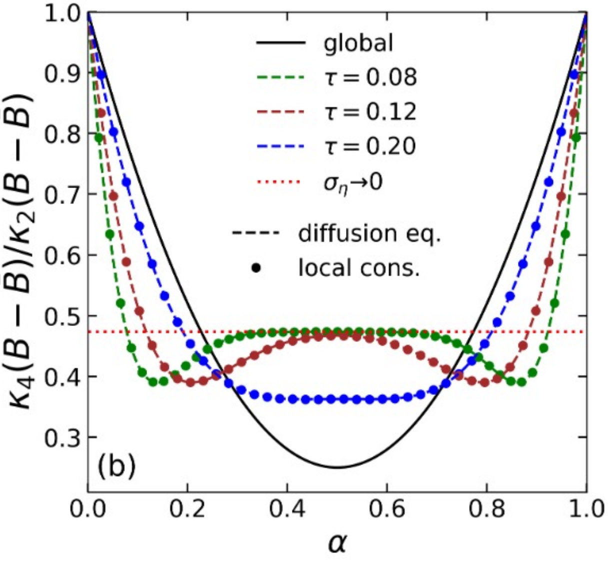
Moderate evidence for charge fluctuations at LHC in the QGP



Outlook:

- Higher-order charge cumulants
- Beyond ideal gas: MaxEnt for lattice QCD χ_n^B
- Balance functions and other observables
- Extensions to RHIC

Baselines for higher-order proton cumulants in search of chiral criticality



Baseline describes prelim. O-O data

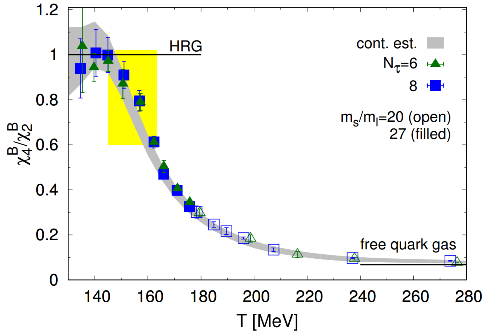
Thanks for your attention

Additional slides

Theory vs experiment

guidance from theory (e.g. lattice)

experiment (the real thing)

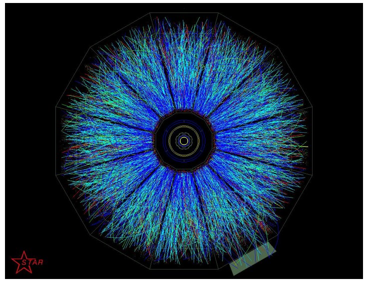


hydro simulation

hadronic phase

canonical effects

kinematic cuts



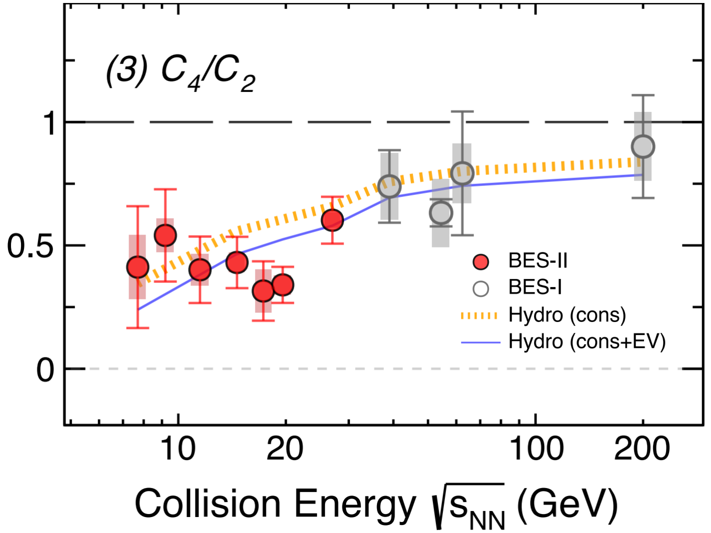
event selection

track reconstruction

efficiency correction

CBWC

purity



Hadron resonance gas in the canonical ensemble

Begun, Gazdzicki, Gorenstein, Zozulya, PRC 70, 034901 (2004)

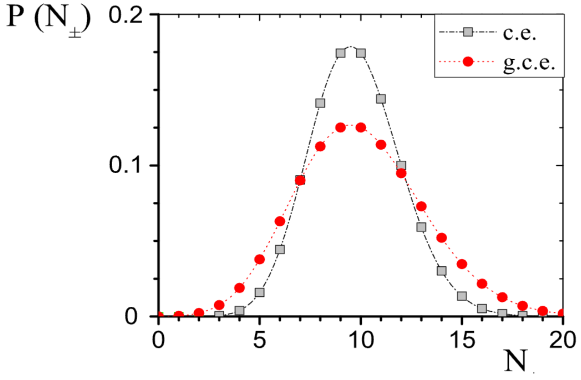
Canonical partition function of an ideal gas of **particles and antiparticles:**

$$Z_{c.e.}(V,T) = \sum_{N_+=0}^{\infty}\sum_{N_-=0}^{\infty} \frac{(\lambda_+ z)^{N_+}}{N_+!}\frac{(\lambda_- z)^{N_-}}{N_-!}\,\delta(N_+ - N_-) =$$

$$= \frac{1}{2\pi}\int_0^{2\pi} d\phi \;\; \exp\left[z\,(\lambda_+\, e^{i\phi} + \lambda_-\, e^{-i\phi})\right] = I_0(2z)$$

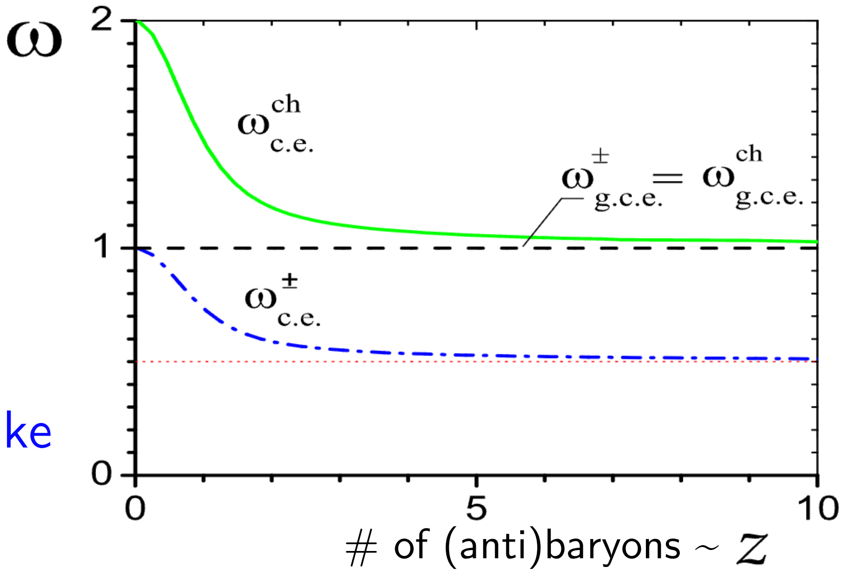
Skellam distribution

$$P_{c.e.}(N_+) = \frac{1}{I_0(2z)}\cdot\left(\frac{z^{N_+}}{N_+!}\right)^2$$



Fluctuations:

$$\omega^{\pm}_{c.e.} = \frac{\langle N_\pm^2\rangle_{c.e.} - \langle N_\pm\rangle^2_{c.e.}}{\langle N_\pm\rangle_{c.e.}} = 1 - z\left[\frac{I_1(2z)}{I_0(2z)} - \frac{I_2(2z)}{I_1(2z)}\right] \cong \frac{1}{2}$$



Exact (baryon) charge conservation introduces correlation among unlike charges ($B\bar{B}$) and anticorrelation among like charges (BB and $\bar{B}\bar{B}$)

Further developments evaluate high-order cumulants in acceptance

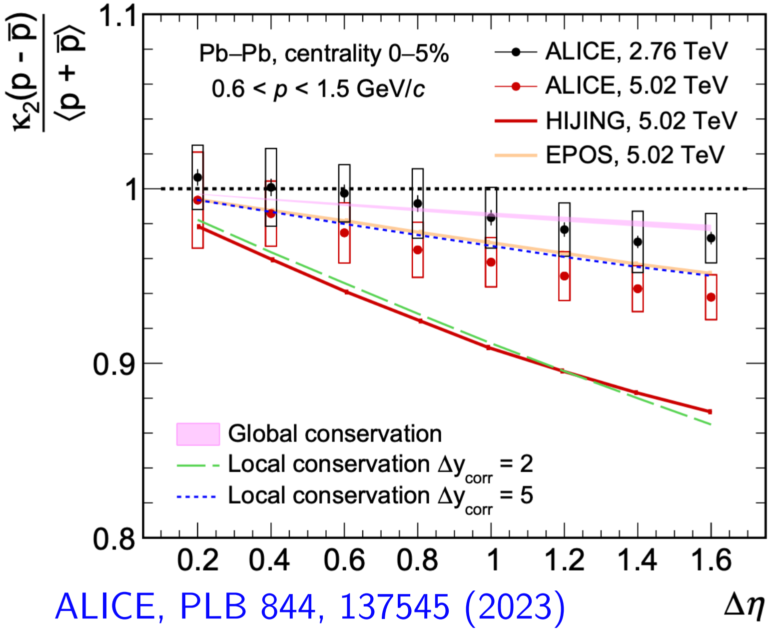
Bzdak, Koch, Skokov, PRC 87, 014901 (2013); Braun-Munzinger et al., NPA 1008, 122141 (2021)

LHC and local conservation

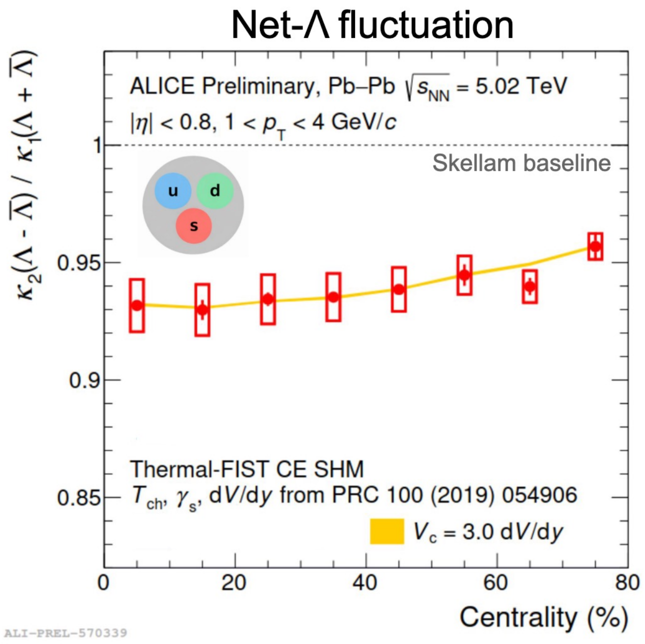
Global conservation correlates all baryons everywhere in the fireball

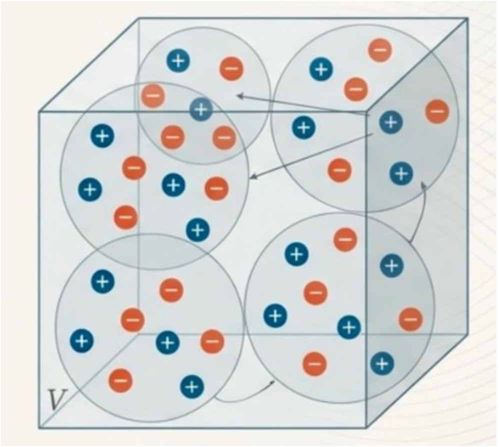
This requires very early production of the baryon charge...

or introducing **local baryon conservation**

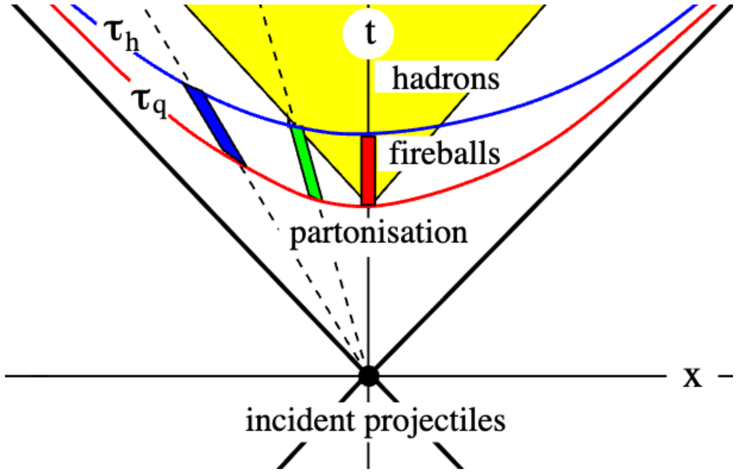


ALICE, PLB 844, 137545 (2023)





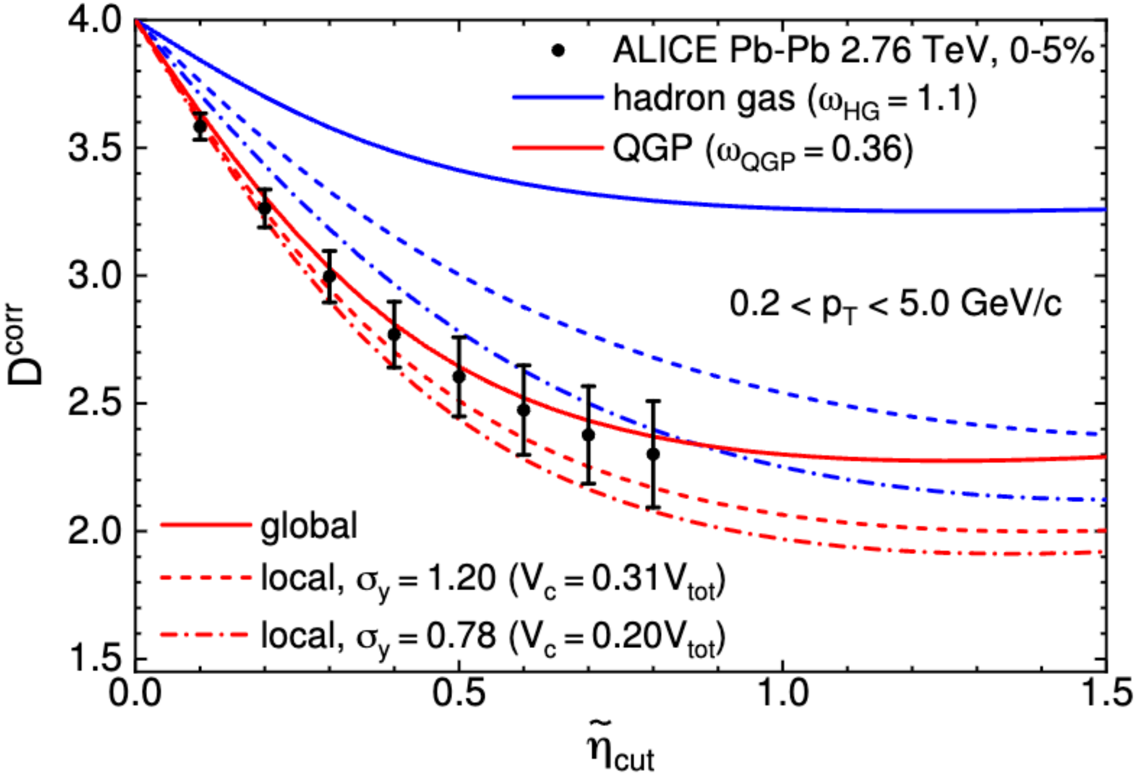
Castorina, Satz, IJMPE 23, 1450019 (2014)

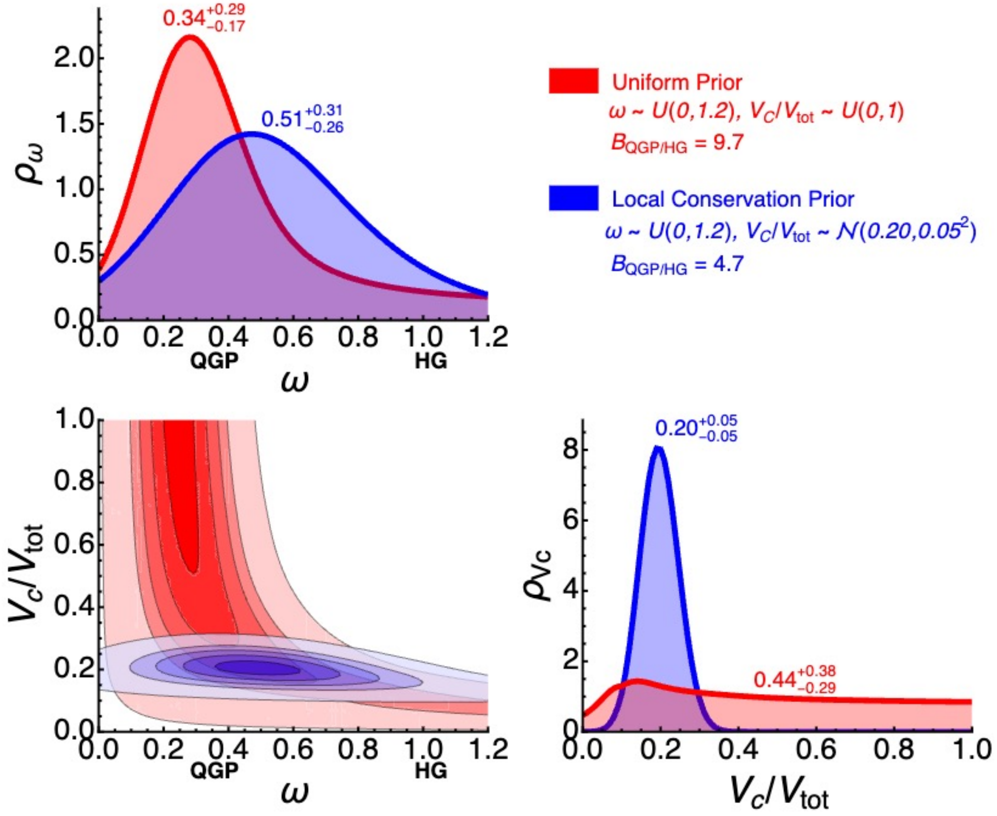


- balance functions [Schlichting, Pratt, PRC 83, 014913 (2011)]
- diffusion master equation [Sakaida et al., PRC 90, 064911 (2014)]
- V_c approach [VV, Donigus, Stoecker, PRC 100, 054906 (2019)]
- Correlated sampling [Braun-Munzinger, Redlich, Rustamov, Stachel, JHEP 08, 113 (2024)]

D-measure of charge fluctuations

$$D = 4\frac{\kappa_2[N_+ - N_-]}{\langle N_{\rm ch}\rangle} = 4\frac{\kappa_2[Q]}{\langle Q^+ + Q^-\rangle} = 4\left\{1 - \left(1 - \frac{\omega}{\gamma_Q}\right)\frac{\langle p^2(\eta)\rangle}{\langle p(\eta)\rangle} - \frac{\omega}{\gamma_Q}\frac{\langle p(\eta_1)p(\eta_2)\rangle_{\varkappa}}{\langle p(\eta)\rangle}\right\}$$





J. Parra, R. Poberezhniuk, V. Koch, C. Ratti, arXiv:2504.02085

Cumulants in the canonical ensemble

Net-baryon cumulants in the acceptance, non-zero total baryon number B

$$g(t) = \ln \left(\sum\nolimits_n P_B(n) e^{nt} \right) = \ln \left[\left(\frac{q_+}{q_-} \right)^{B/2} \frac{I_B(2z\sqrt{q_+ q_-})}{I_B(2z)} \right]$$

$$q_+ = 1 - p_B + p_B e^{t}$$

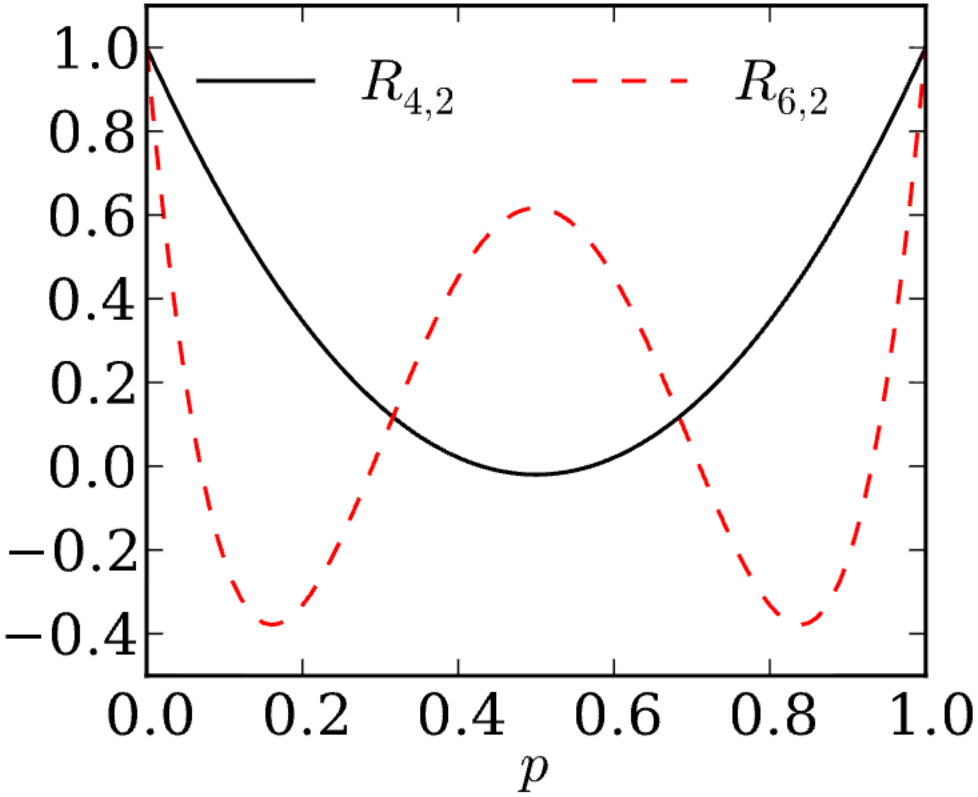
$$q_- = 1 - p_{\bar{B}} + p_{\bar{B}} e^{-t}$$

Bzdak, Koch, Skokov, PRC 87, 014901 (2013)

$$\frac{\kappa_2[B - \bar{B}]}{\langle \mathrm{B} + \bar{\mathrm{B}} \rangle} \approx 1 - p$$

$$\frac{\kappa_4[B - \bar{B}]}{\kappa_2[B - \bar{B}]} \approx 1 - 3p[1 - \mathrm{p}(1 + r_B^2)]$$

$$\frac{\kappa_6[B - \bar{B}]}{\kappa_2[B - \bar{B}]} \approx 1 - 15p(1-p)[1 + r_B^2 - p(1-p)](3 + 6r_B^2 - r_B^4)$$



see also Braun-Munzinger et al., NPA 1008, 122141 (2021)

Local charge conservation: 2nd order generalizations

- Non-conserved quantities correlated to a conserved charge

$$\mathrm{C}_{11}^{ij}(x_1,x_2)=\chi_{ij}\left[\delta(x_1-x_2)-\varkappa(x_1,x_2)\frac{\chi_{11}^{iB}\chi_{11}^{jB}}{\chi_2^B\,\mathrm{V}}\right]$$

$$\mathrm{C}_2^{\mathrm{B}-\bar{\mathrm{B}}}(x_1,x_2)=\langle\mathrm{B}+\bar{\mathrm{B}}\rangle\left[\delta(x_1-x_2)-\varkappa(x_1,x_2)\frac{1}{\mathrm{V}}\right]$$

self-correlation **balancing term**

$$\mathrm{C}_2^{\mathrm{B}+\bar{\mathrm{B}}}(x_1,x_2)=\langle\mathrm{B}+\bar{\mathrm{B}}\rangle\,\delta(x_1-x_2)$$

Local baryon conservation does not affect $\boldsymbol{B}+\bar{\boldsymbol{B}}$

$$\mathrm{C}_2^{\bar{\mathrm{B}}}(x_1,x_2)=\langle\bar{\mathrm{B}}\rangle\left[\delta(x_1-x_2)-\varkappa(x_1,x_2)\frac{1}{2\mathrm{V}}\right]$$

Anticorrelation among like baryons

$$\mathrm{C}_{11}^{\mathrm{B}\bar{\mathrm{B}}}(x_1,x_2)=\langle\bar{\mathrm{B}}\rangle\,\varkappa(x_1,x_2)\frac{1}{2\mathrm{V}}$$

Correlation among unlike baryons

- Multiple conserved charges, $\boldsymbol{Q}=(B,Q,S,\dots)$

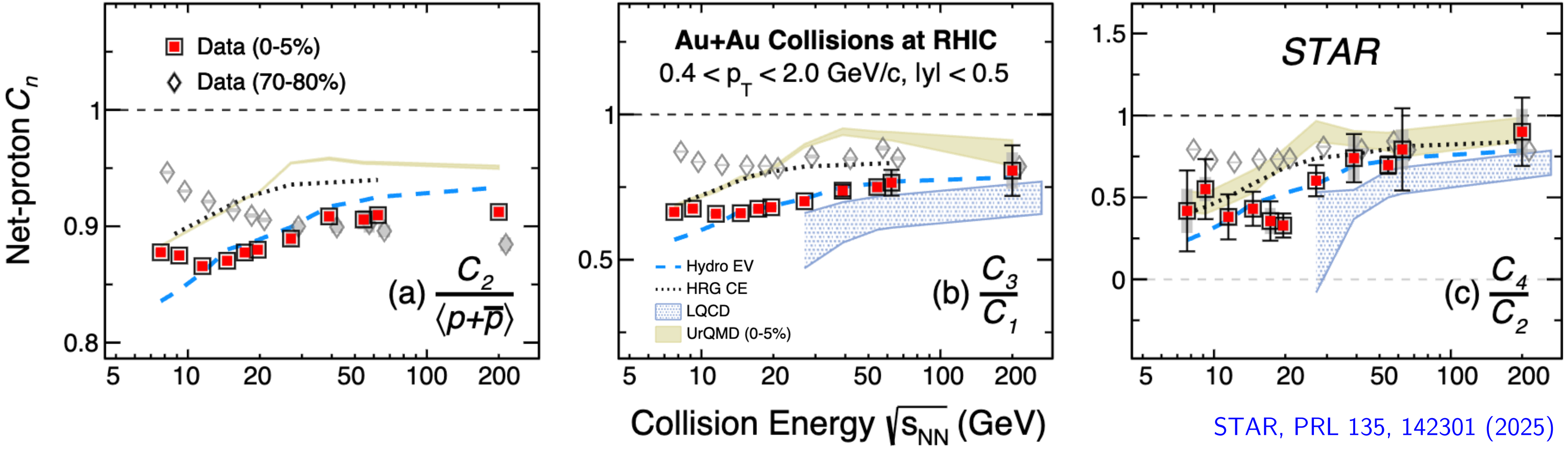
$$\mathrm{C}_{11}^{ij}(x_1,x_2)=\chi_{ij}\left[\delta(x_1-x_2)-\varkappa(x_1,x_2)\frac{\chi_{11}^{i\boldsymbol{Q}_\mathrm{k}}\left(\chi_{kl}^{\boldsymbol{Q}}\right)^{-1}\chi_{11}^{j\boldsymbol{Q}_\mathrm{l}}}{\mathrm{V}}\right]$$

- Balance functions (ideal gas, LHC)

$$B(x_1,x_2)=\frac{n_B+n_{\bar{B}}}{\langle B+\bar{B}\rangle}\varkappa(x_1,x_2)$$

Conservation, baselines, and data

Net-proton cumulant ratios



STAR, PRL 135, 142301 (2025)

HRG CE: Braun-Munzinger et al., NPA 1008, 122141 (2021)

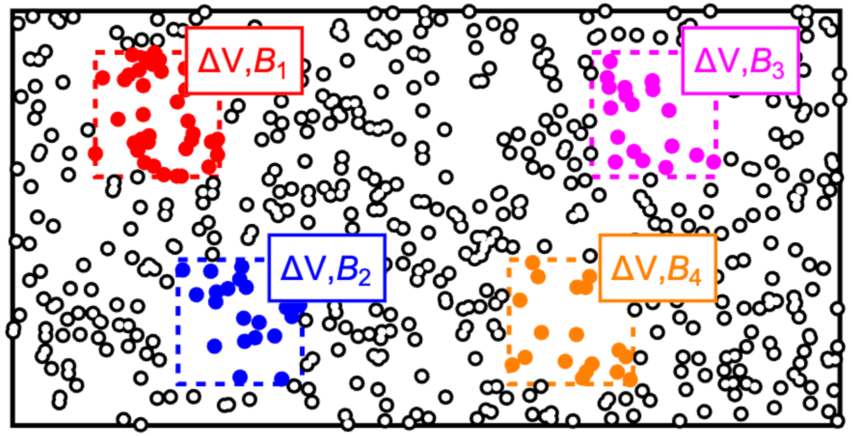
Hydro EV: VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Exact baryon conservation is a primary ingredient of non-critical baselines

Density correlations framework

VV, PRC 110, L061902 (2024)

Evaluate the cumulants in thermodynamic limit using maximum term (saddle-point) method



$$\langle \delta B_i \delta B_j \rangle = \Delta V \chi_2 \left[\delta_{ij} - \frac{\Delta V}{V}\right]$$

$$\langle \delta B_i \delta B_j \delta B_k \rangle = \delta_{ijk} \chi_3 \Delta V - (\delta_{ij} + \delta_{ik} + \delta_{kj}) \chi_3 (\Delta V)^2 + 2\chi_3 (\Delta V)^3$$

$$\langle \delta B_i \delta B_j \delta B_k \delta B_l \rangle_c = \Delta V \chi_4 \delta_{ijkl} - \chi_4 \frac{(\Delta V)^2}{V} [\delta_{ijk} + \delta_{ijl} + \delta_{ikl} + \delta_{jkl}] - \frac{(\chi_3)^2}{\chi_2} \frac{(\Delta V)^2}{V} [\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}]$$
$$+ \frac{(\Delta V)^3}{V^2} \left[\chi_4 + \frac{(\chi_3)^2}{\chi_2}\right] [\delta_{ij} + \delta_{ik} + \delta_{il} + \delta_{jk} + \delta_{jl} + \delta_{kl}] - \frac{3(\Delta V)^4}{V^3} \left[\chi_4 + \frac{(\chi_3)^2}{\chi_2}\right] .$$

Taking "continuum" limit ($\Delta V \to 0$) yields *n*-point density correlation functions

$$\mathcal{C}_n(\eta_1, \ldots, \eta_n) \equiv \left\langle \prod_{i=1}^{n} \delta\rho_i \right\rangle_c , \qquad n \geq 2, \qquad\qquad \prod_{i=1}^{n} \int d\eta_i \, \mathcal{C}_n(\eta_1, \ldots, \eta_n) = \kappa_n[B].$$

N-point local conservation kernel

$$\varkappa_2(\eta_1,\eta_2) \propto \exp\left[-\frac{(\eta_1-\eta_2)^2}{2\sigma_\eta^2}\right]$$

2-point Gaussian kernel

$$\varkappa_n(\eta_1,\ldots,\eta_n) \propto A_n \exp\left[-\frac{1}{n\sigma_\eta^2}\sum_{1\le i<j\le n}(\eta_i-\eta_j)^2\right]$$

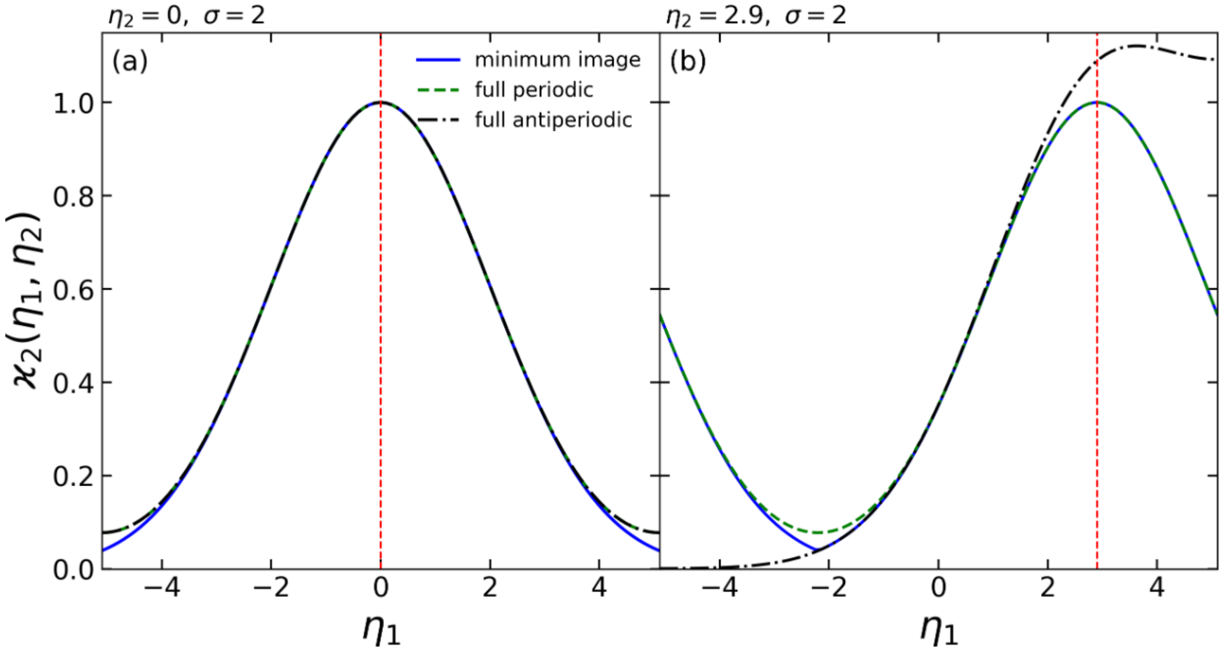
Symmetric n-point Gaussian kernel

Reflecting boundary conditions in a finite system → Sum over antiperiodic Gaussian images

$$\varkappa_n^{\mathrm{BC}}(\eta_1,\ldots,\eta_n) = A_n \sum_{k_2,\ldots,k_n=-\infty}^{\infty} \exp\left[-\frac{1}{n\sigma_\eta^2}\sum_{1\le i<j\le n}(\tilde{\eta}_i-\tilde{\eta}_j)^2\right],$$

$\tilde{\eta}_j = \eta_j + 4k_j\eta_{\max}$ and $\tilde{\eta}_j = 2\eta_{\max} - \eta_j + 4k_j\eta_{\max}$

In practice, these boundary conditions are largely irrelevant for midrapidity



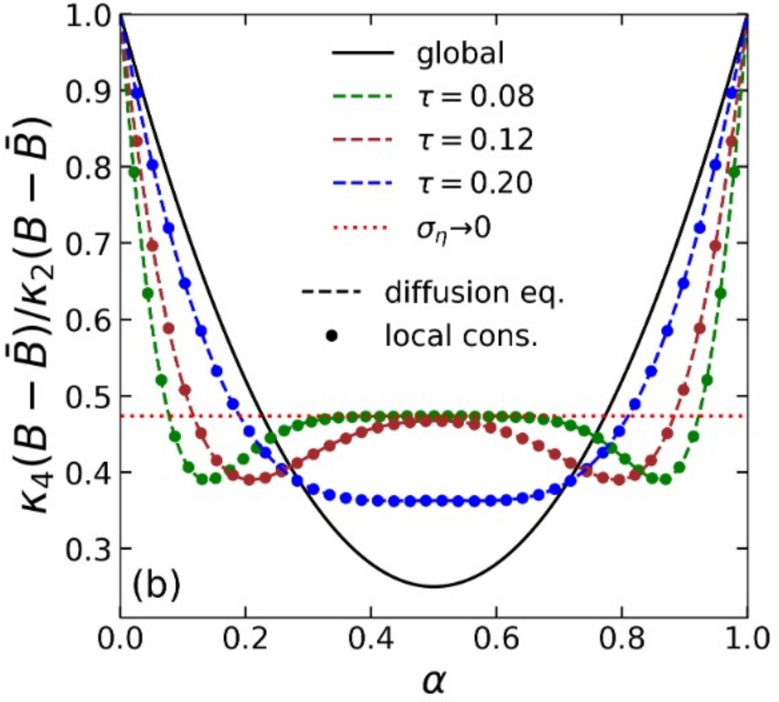
Fifth and sixth order

$$\begin{aligned}
\mathcal{C}_5(\eta_1,\ldots,\eta_5) &= \chi_5^B\delta_{1,2,3,4,5} - \frac{\chi_5^B}{4!V}\sum_{\sigma\in S_5}\delta_{\sigma_1,\sigma_2,\sigma_3,\sigma_4}\varkappa_2(\eta_{\sigma_1},\eta_{\sigma_5}) - \frac{\chi_3^B\chi_4^B}{3!2!\chi_2^B V}\sum_{\sigma\in S_5}\delta_{\sigma_1,\sigma_2,\sigma_3}\delta_{\sigma_4,\sigma_5}\varkappa_2(\eta_{\sigma_1},\eta_{\sigma_5}) \\
&+ \frac{1}{2!3!V^2}\left[\chi_5^B + \frac{\chi_3^B\chi_4^B}{\chi_2^B}\right]\sum_{\sigma\in S_5}\delta_{\sigma_1,\sigma_2,\sigma_3}\varkappa_3(\eta_{\sigma_1},\eta_{\sigma_4},\eta_{\sigma_5}) + \frac{2\chi_3^B\chi_4^B}{(2!)^3V^2}\sum_{\sigma\in S_5}\delta_{\sigma_1,\sigma_2}\delta_{\sigma_3,\sigma_4}\varkappa_3(\eta_{\sigma_1},\eta_{\sigma_4},\eta_{\sigma_5}) \\
&- \frac{1}{3!2!V^3}\left[\chi_5^B + 5\frac{\chi_3^B\chi_4^B}{\chi_2^B}\right]\sum_{\sigma\in S_5}\delta_{\sigma_1,\sigma_2}\varkappa_4(\eta_{\sigma_1},\eta_{\sigma_3},\eta_{\sigma_4},\eta_{\sigma_5}) + \frac{4}{V^4}\left[\chi_5^B + 5\frac{\chi_3^B\chi_4^B}{\chi_2^B}\right]\varkappa_5(\eta_{\sigma_1},\ldots,\eta_{\sigma_5})
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_6(\eta_1,\ldots,\eta_6) &= \chi_6^B\delta_{1,2,3,4,5,6} - \frac{\chi_6^B}{5!V}\sum_{\sigma\in S_6}\delta_{\sigma_1,\sigma_2,\sigma_3,\sigma_4\sigma_5}\varkappa_2(\eta_{\sigma_1},\eta_{\sigma_6}) - \frac{\chi_3^B\chi_5^B}{4!2!\chi_2^B V}\sum_{\sigma\in S_6}\delta_{\sigma_1,\sigma_2,\sigma_3,\sigma_4}\delta_{\sigma_1,\sigma_5}\varkappa_2(\eta_{\sigma_1},\eta_{\sigma_6}) \\
&- \frac{(\chi_4^B)^2}{2!(3!)^2V\chi_2^B}\sum_{\sigma\in S_6}\delta_{\sigma_1,\sigma_2,\sigma_3}\delta_{\sigma_4,\sigma_5,\sigma_6}\varkappa_2(\eta_{\sigma_1},\eta_{\sigma_6}) + \frac{1}{4!2!V^2}\left[\chi_6^B + \frac{\chi_3^B\chi_5^B}{\chi_2^B}\right]\sum_{\sigma\in S_6}\delta_{\sigma_1,\sigma_2,\sigma_3,\sigma_4}\varkappa_3(\eta_{\sigma_1},\eta_{\sigma_5},\eta_{\sigma_6}) \\
&+ \frac{1}{(2!)^2 3!V^2}\left[\frac{(\chi_4^B)^2 + \chi_3^B\chi_5^B}{\chi_2^B}\right]\sum_{\sigma\in S_6}\delta_{\sigma_1,\sigma_2,\sigma_3}\delta_{\sigma_4,\sigma_5}\varkappa_3(\eta_{\sigma_1},\eta_{\sigma_5},\eta_{\sigma_6}) + \frac{1}{3!(2!)^3V^2}\left[\frac{3\chi_2^B(\chi_3^B)^2\chi_4^B - (\chi_3^B)^4}{\chi_2^B}\right] \\
&\times\sum_{\sigma\in S_6}\delta_{\sigma_1,\sigma_2}\delta_{\sigma_3,\sigma_4}\delta_{\sigma_5,\sigma_6}\varkappa_3(\eta_{\sigma_1},\eta_{\sigma_5},\eta_{\sigma_6}) - \frac{1}{(3!)^2V^3}\left[\frac{2(\chi_4^B)^2 + 3\chi_3^B\chi_5^B + \chi_2^B\chi_6^B}{\chi_2^B}\right]\sum_{\sigma\in S_6}\delta_{\sigma_1,\sigma_2,\sigma_3}\varkappa_4(\{\eta_{\sigma_i}\}_{i=1,4,5,6}) \\
&+ \frac{1}{(2!)^5V^3}\left[\frac{(\chi_3^B)^4 - 3\chi_2^B(\chi_3^B)^2\chi_4^B - 2(\chi_2^B)^2(\chi_4^B)^2 - 2(\chi_2^B)^2\chi_3^B\chi_5^B}{(\chi_2^B)^3}\right]\sum_{\sigma\in S_6}\delta_{\sigma_1,\sigma_2}\delta_{\sigma_3,\sigma_4}\varkappa_4(\{\eta_{\sigma_i}\}_{i=1,4,5,6}) \\
&+ \frac{1}{4!2!V^4}\left[\frac{-3(\chi_3^B)^4 + 9\chi_2^B\chi_4^B(\chi_3^B)^2 + 9(\chi_2^B)^2\chi_5^B\chi_3^B + (\chi_2^B)^2(8(\chi_4^B)^2 + \chi_2^B\chi_6^B)}{(\chi_2^B)^3}\right]\sum_{\sigma\in S_6}\delta_{\sigma_1,\sigma_2}\varkappa_5(\{\eta_{\sigma_i}\}_{i=1,3,4,5,6}) \\
&- \frac{5}{V^5}\left[\frac{-3(\chi_3^B)^4 + 9\chi_2^B\chi_4^B(\chi_3^B)^2 + 9(\chi_2^B)^2\chi_5^B\chi_3^B + (\chi_2^B)^2(8(\chi_4^B)^2 + \chi_2^B\chi_6^B)}{(\chi_2^B)^3}\right]\varkappa_6(\{\eta_i\}_{i=1,2,3,4,5,6})
\end{aligned}$$

Comparison to data and other implementations: κ_2

Agrees with the diffusion model of PRC 90, 064911 (2014)



Opposite behavior in the correlated sampling model of JHEP 08, 113 (2024)

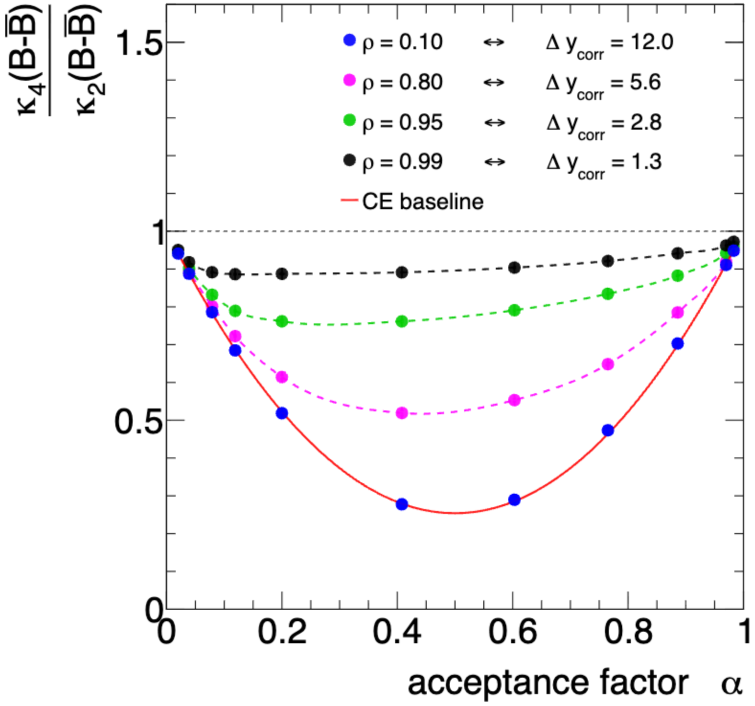
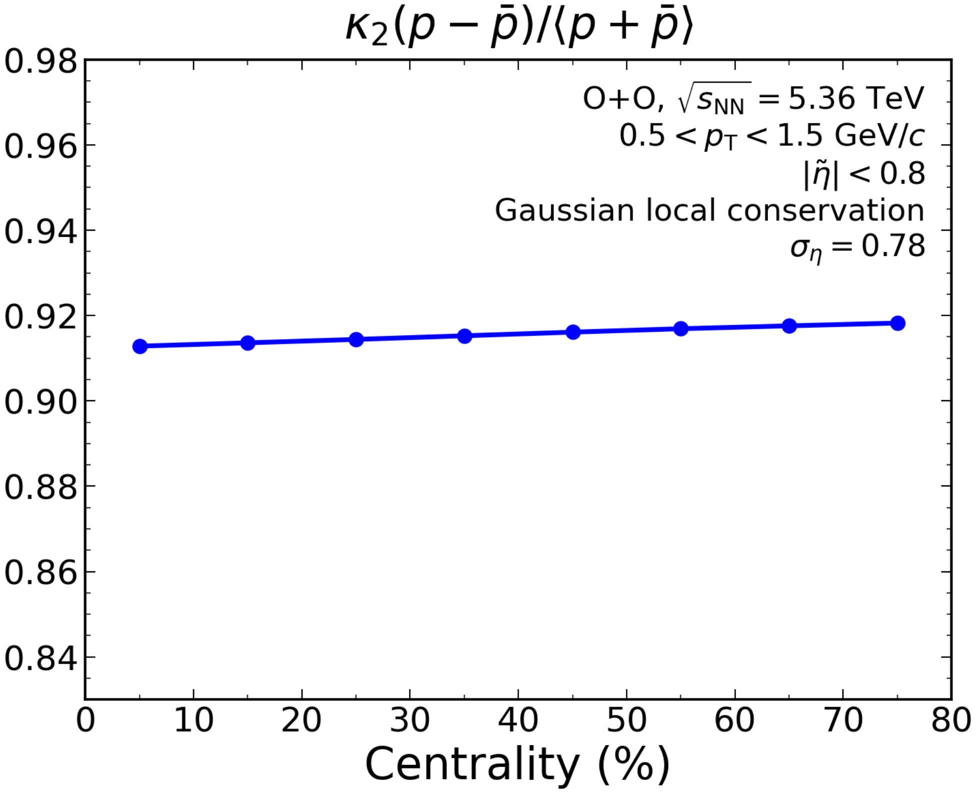
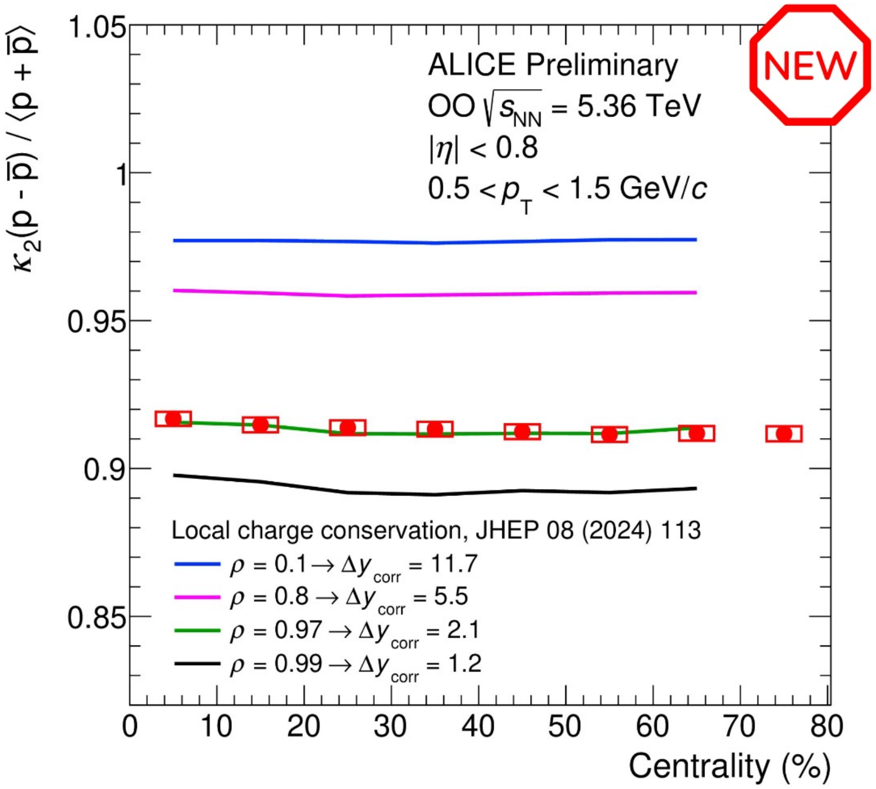


Figure from M. Arslandok, QM2026

Comparison to other implementations: κ_2

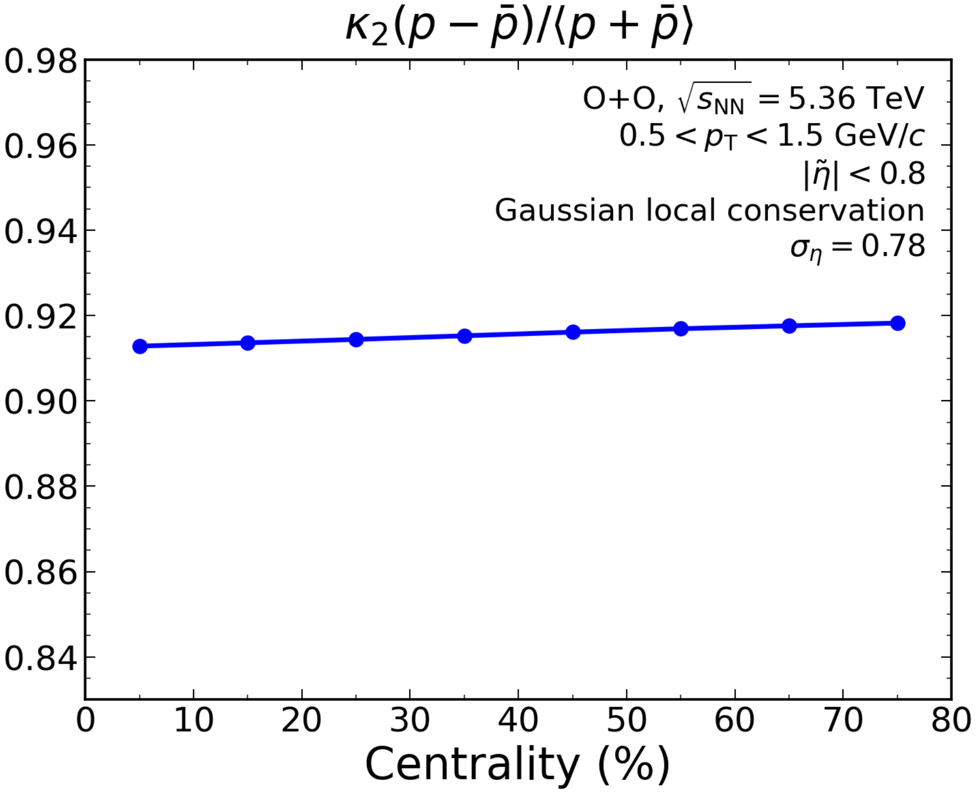


M. Ciacco, CPOD2026

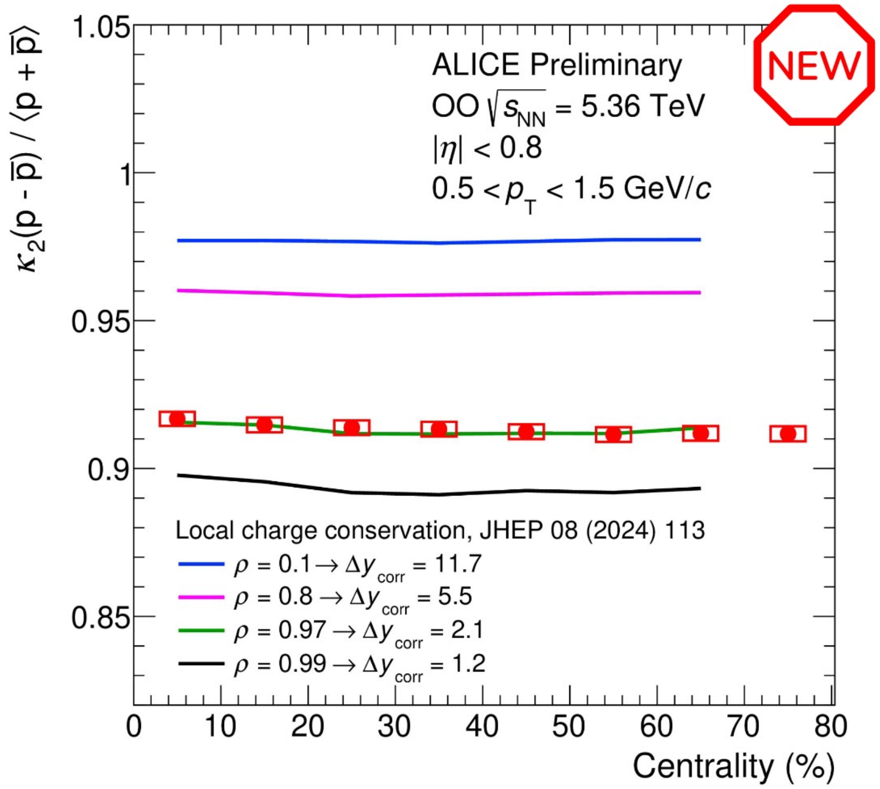


- ✓ Good agreement with preliminary O-O data (no need to retune σ_η from Pb-Pb)
- ✓ Excellent (exact?) agreement with the diffusion model of PRC 90, 064911 (2014)
- ✓ Good agreement with V_c approach (Thermal-FIST SHM $V_c = 3dV/dy$) PRC 100, 054906 (2019)
- ✓ Good agreement with the correlated sampling model of JHEP 08, 113 (2024)

Comparison to data and other implementations: κ_2

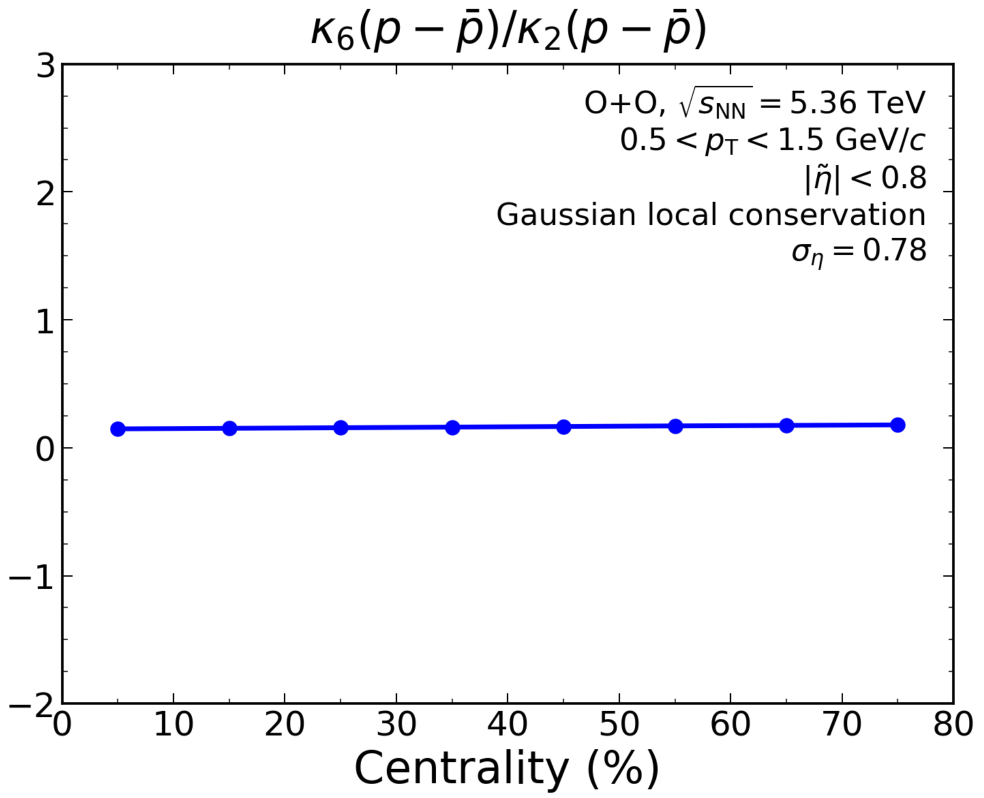


M. Ciacco, CPOD2026

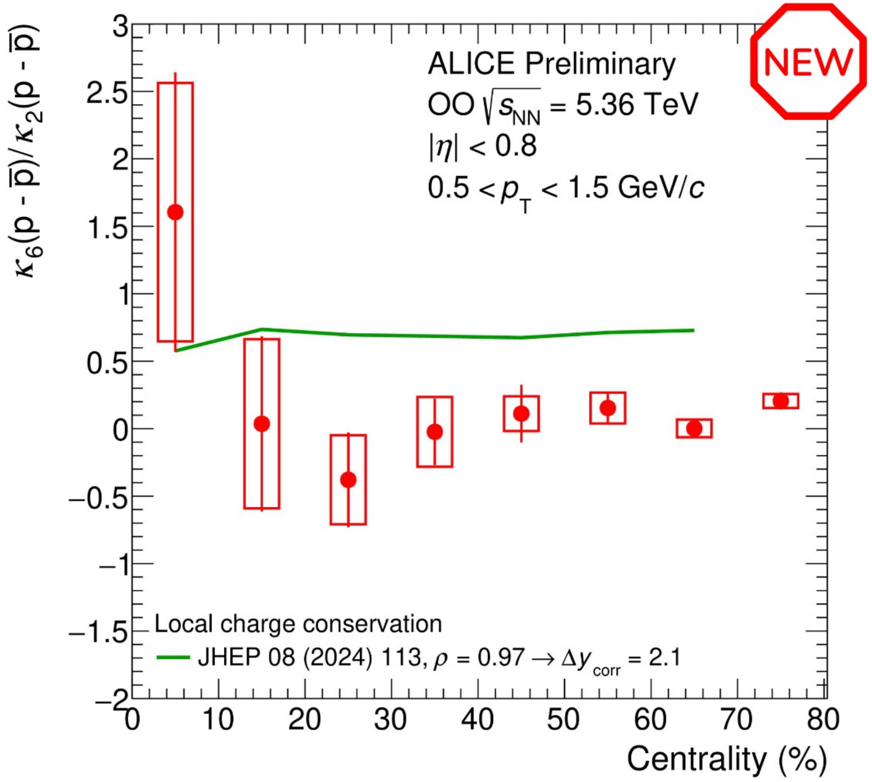


- ✓ Good agreement with preliminary O-O data (no need to retune σ_η from Pb-Pb)
- ✓ Excellent (exact?) agreement with the diffusion model of PRC 90, 064911 (2014)
- ✓ Good agreement with V_c approach (Thermal-FIST SHM $V_c = 3dV/dy$) PRC 100, 054906 (2019)
- ✓ Good agreement with the correlated sampling model of JHEP 08, 113 (2024)

Comparison to data and other implementations: κ_6/κ_2



M. Ciacco, CPOD2026; A. Rustamov, CPOD2026



- ◐ Fair agreement with preliminary O-O data
- ✓ Excellent (exact?) agreement with the diffusion model of PRC 90, 064911 (2014)
- ✓ Good agreement with V_c approach (Thermal-FIST SHM $V_c = 3dV/dy$) PRC 100, 054906 (2019)
- ✗ No agreement with the correlated sampling model of JHEP 08, 113 (2024)