Hydrodynamic Consequences of the QCD Critical Point

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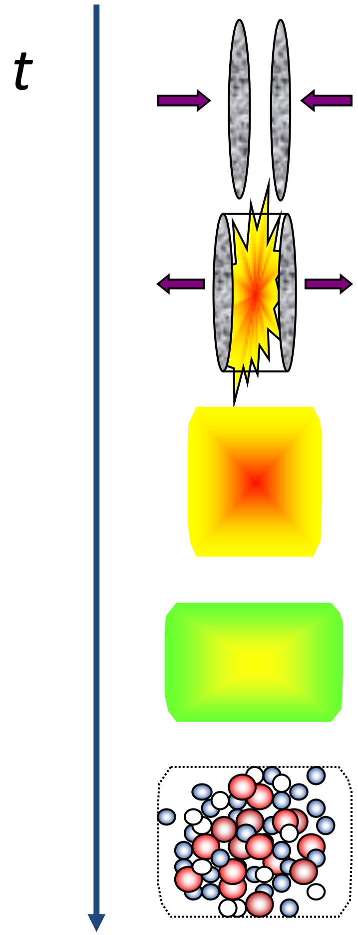
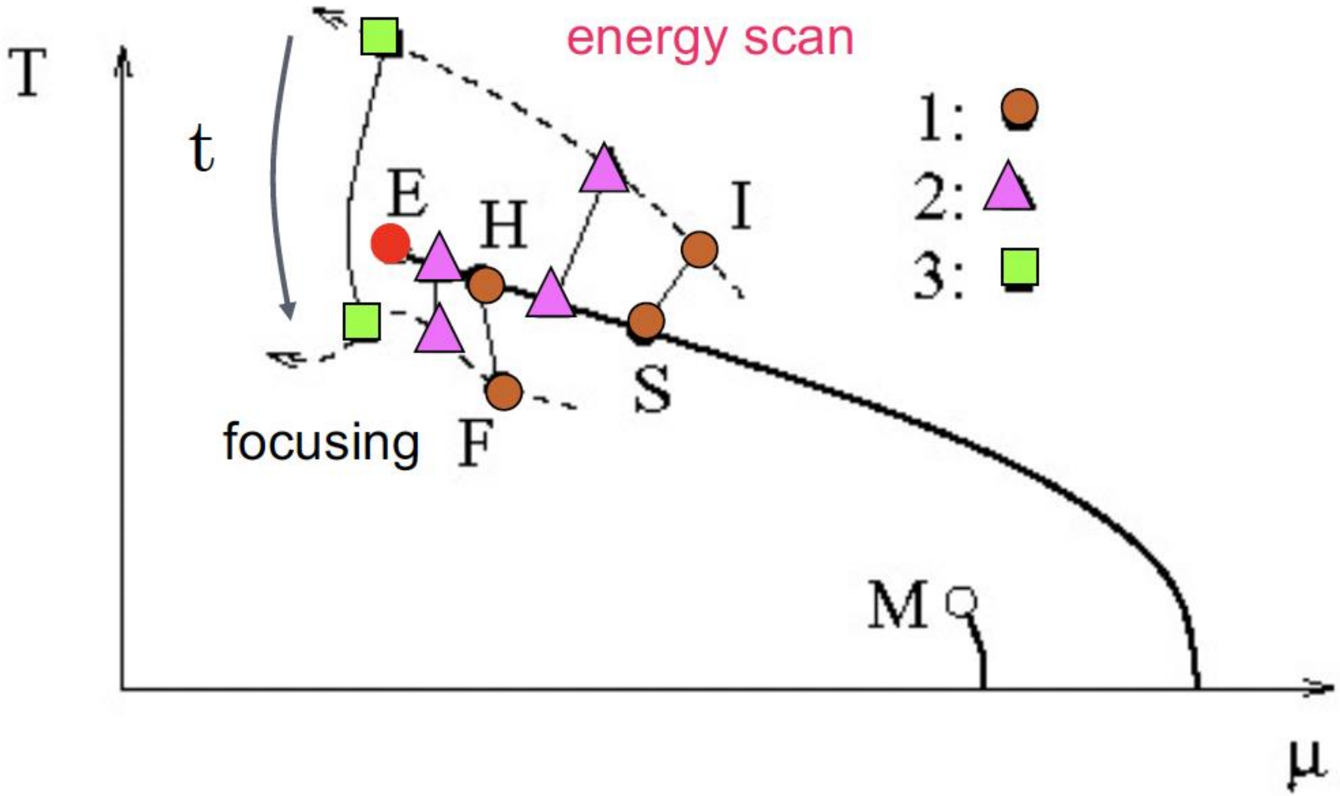
Kobayashi-Maskawa Institute for the Origin of Particles and the Universe

June 3, 2026 @QCD Critical Point and Hydrodynamic Evolution, YITP

QCD Phase Diagram and QCD Critical Point

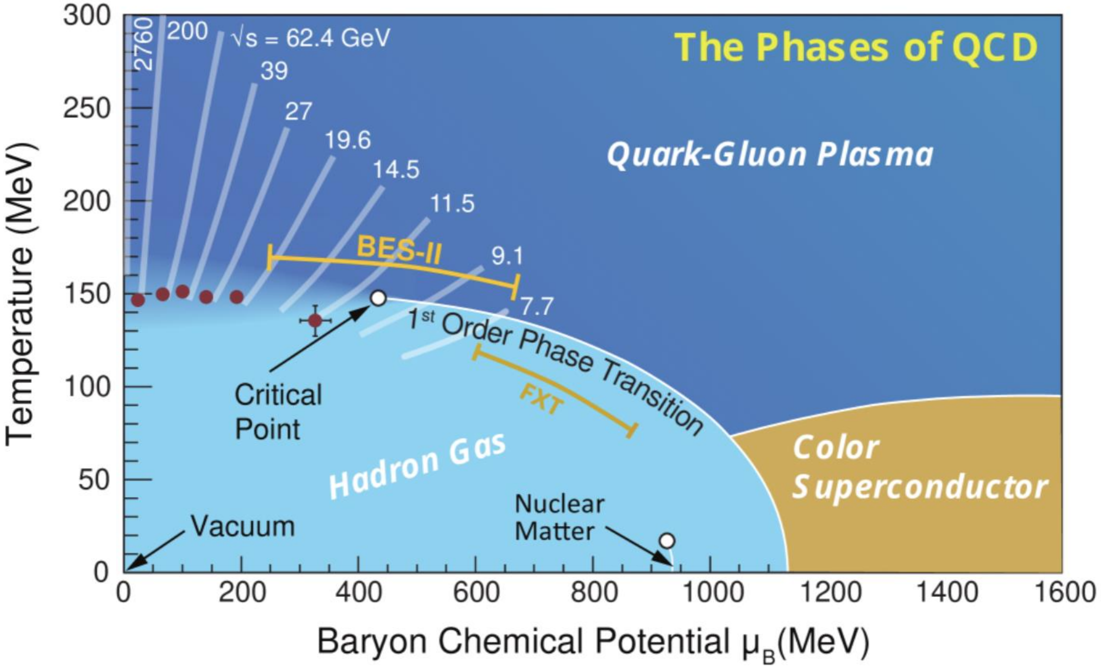




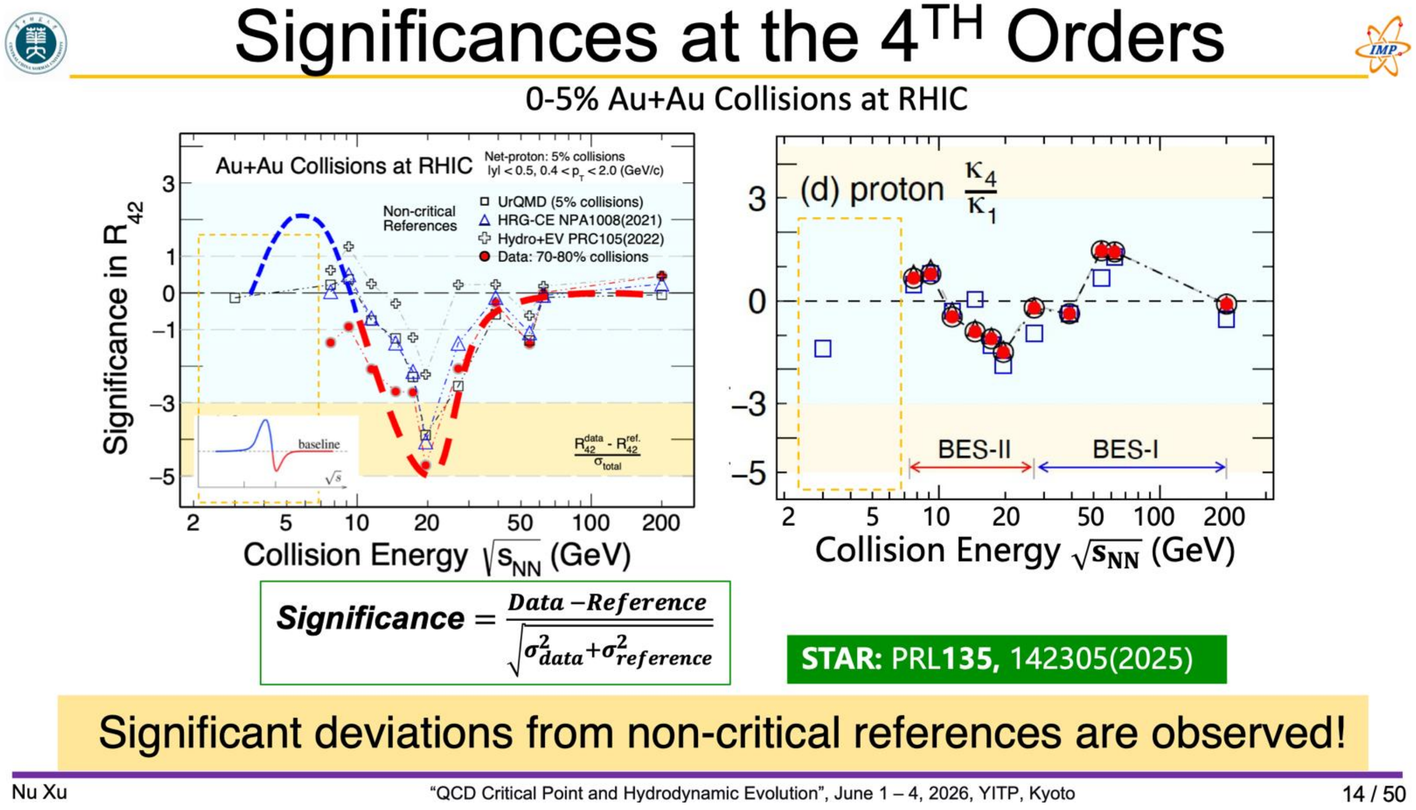
Stephanov, Rajagopal, Shuryak, PRL81 (1998) 4816

Experimental Search and the Critical Point

RHIC Beam Energy Scan Program



arXiv:2410.02861



From Nu Xu's talk

Two Faces of Criticality

Fluctuations

- Correlation length
- Susceptibilities
- Higher cumulants

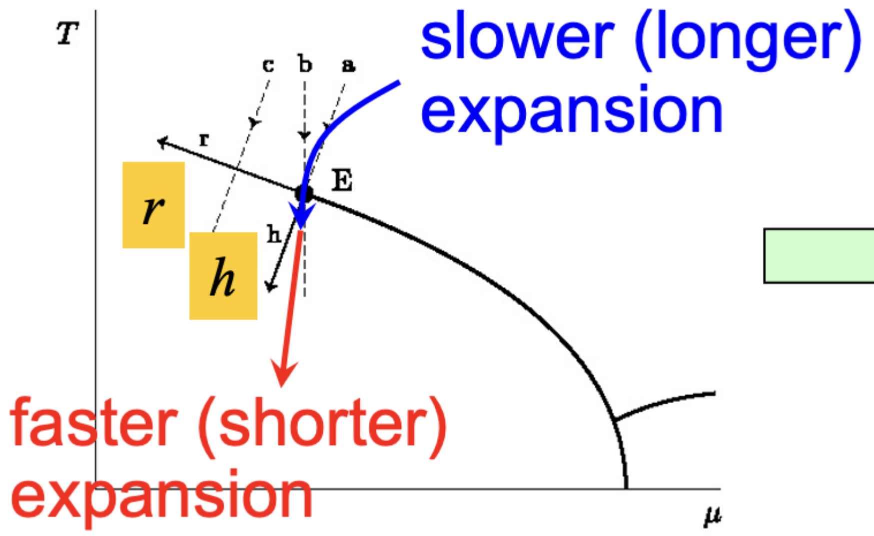
Hydrodynamic response

- Equation of state
- Entropy per baryon
- Isentropic trajectories
- Focusing

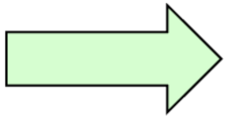
Criticality affects both fluctuations and the bulk evolution.

Time Evolution

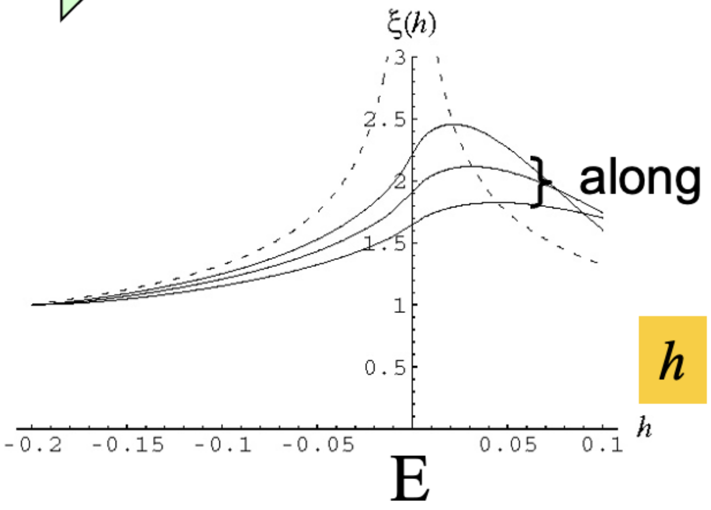
- Berdnikov and Rajagopal's Schematic Argument



B. Berdnikov and K. Rajagopal, Phys. Rev. D61 (2000) 105017



Correlation Length longer than L_{eq}



- What's missing:
  - Realistic Hydro Calculation with Realistic EOS

Constructing on EoS with a Critical Point

CN, Asakawa, Phys.Rev.C71:044904(2005)

- **Singular Part near CEP + Non-singular Part**
  - Non-singular part

    QGP phase, Hadron phase
  - Singular part of EOS near CEP

3d Ising Model ⟷ **QCD**

Same Universality Class

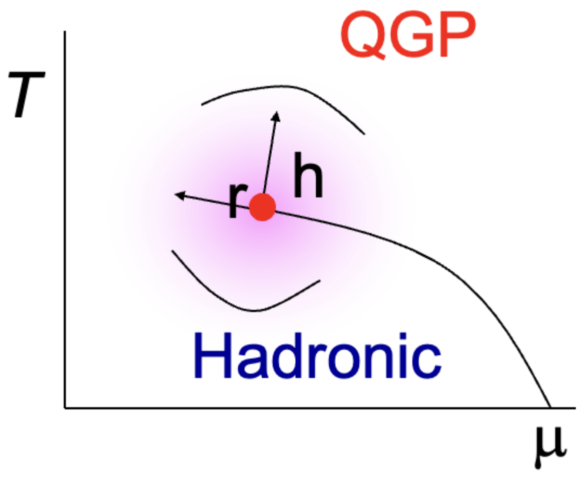
$r = \frac{T - T_C}{T_C}$

h: extermal magnetic field

$(r, h) \longleftrightarrow (T, \mu)$

T, μ

- Mapping $(r, h) \longleftrightarrow (T, \mu)$
- Matching with known *QGP* and *Hadronic* entropy density
- Thermodynamical quantities



Mapping the 3-D Ising Model to QCD

CN, Asakawa, Phys.Rev.C71:044904(2005)

- Parametric Representation of EOS

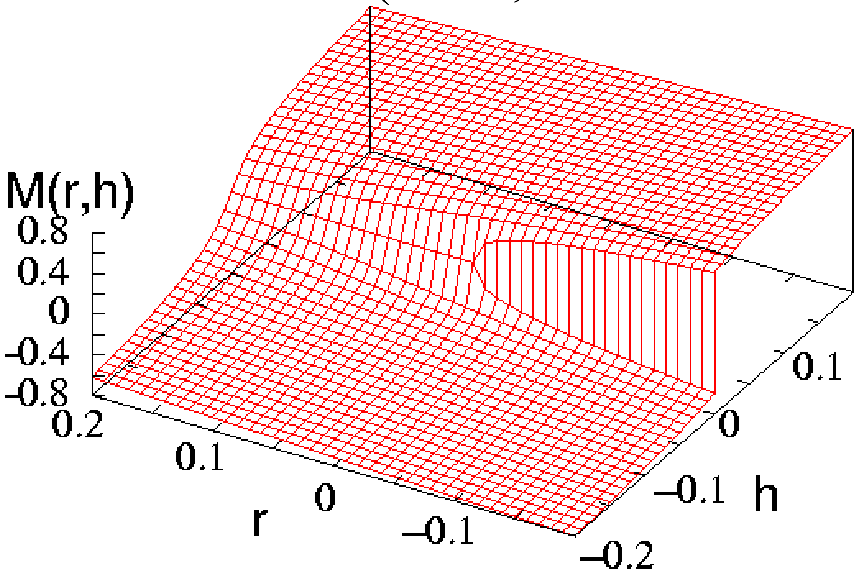
$$\begin{cases} M = M_0 R^{\beta}\theta \\ h = h_0 R^{\beta}\tilde{h}(\theta) = hR_0^{\beta\delta}(\theta - 0.76201\theta^3 + 0.00804\theta^5) \\ r = R(1-\theta^2) \qquad (R \geq 0, \ \ -1.154 \leq \theta \leq 1.154) \end{cases}$$

$$r = \frac{T - T_C}{T_C}$$

h : external magnetic field

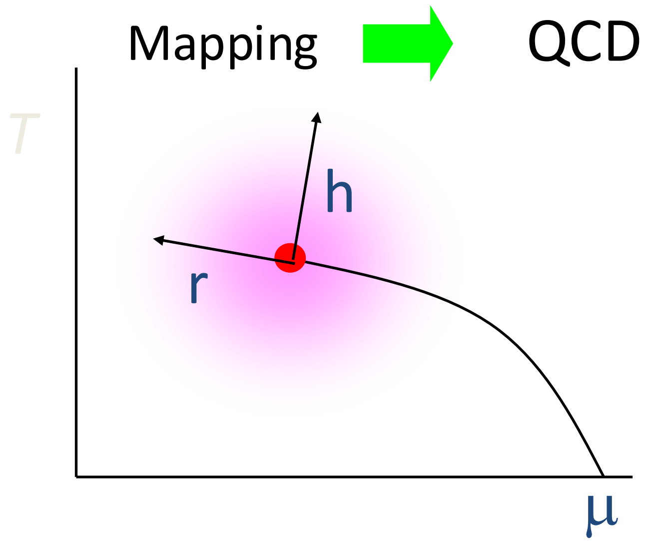
$$\beta = 0.326$$
$$\delta = 4.8$$

Guida and Zinn-Justin NPB486(97)626



Order parameter

$$M \leftrightarrow \langle \bar{q}q \rangle$$



Singular Part of EoS

CN, Asakawa, Phys.Rev.C71:044904(2005)

- **Gibbis Free Energy**

$$G(h,r) = F(M,r) - Mh$$

Guida and Zinn-Justin NPB486(97)626

Free energy: $F(M,r) = h_0 M_0 r^{2-\alpha} g(\theta)$ ← $h = \left(\frac{\partial F}{\partial M}\right)\bigg|_r$ $\alpha = 0.11$

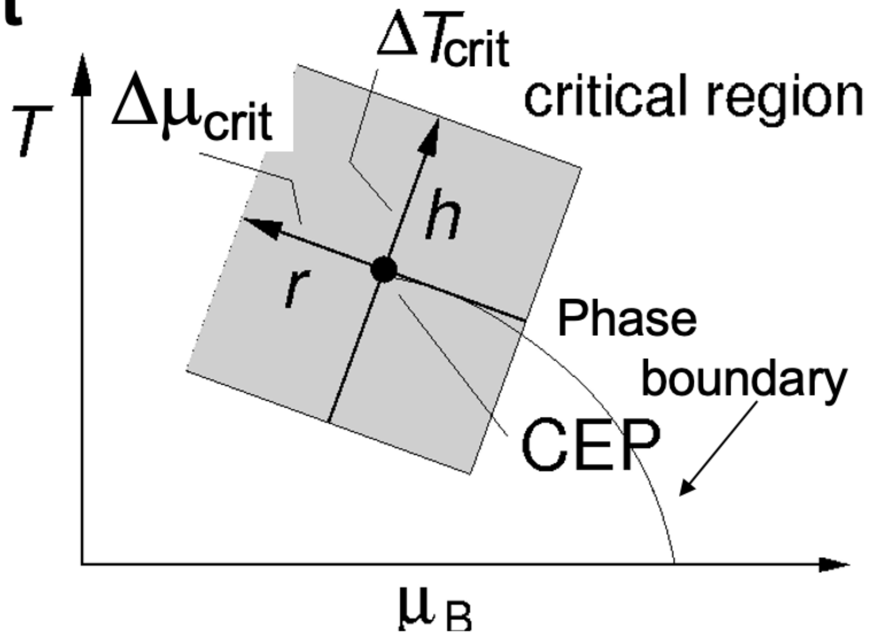
- **Entropy Density for Singular Part**

$$s_c = -\frac{\partial G}{\partial T}\bigg|_\mu = -\frac{\partial G}{\partial h}\bigg|_r \frac{\partial h}{\partial T} - \frac{\partial G}{\partial r}\bigg|_h \frac{\partial r}{\partial T}$$

$$\begin{cases} \frac{\partial G}{\partial h}\big|_r = -M \\ \frac{\partial G}{\partial r}\big|_h = \frac{\partial F}{\partial r}\big|_h - \frac{\partial M}{\partial r}\big|_h \end{cases}$$

mapping

(r, h) ↔ (T, μ)



Singular Part + Non-singular Part

CN, Asakawa, Phys.Rev.C71:044904(2005)

Entropy Density

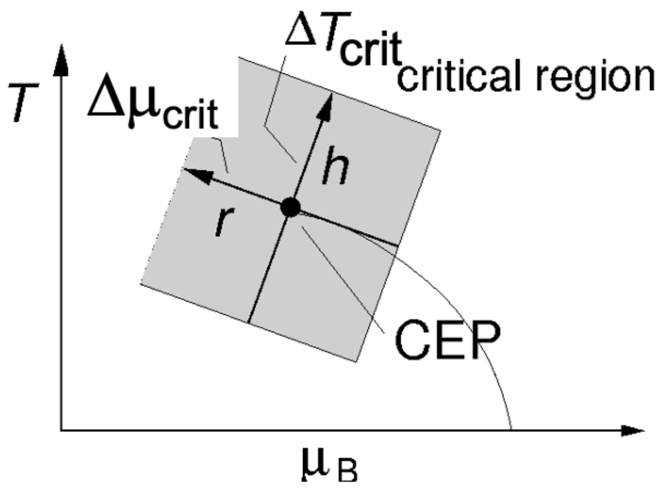
$$S_{\rm real}(T,\mu_B) = \frac{1}{2}\{1-\tanh[S_c(T,\mu_B)]\}S_H(T,\mu_B) + \frac{1}{2}\{1+\tanh[S_c(T,\mu_B)]\}S_Q(T,\mu_B)$$

- $S_H(T,\mu_B)$: Hadron Phase (excluded volume model)
  $S_Q(T,\mu_B)$: QGP phase

- Dimensionless parameter: S_c

$$S_c(T,\mu_B) = s_c\sqrt{(\Delta T_{\rm crit})^2 + (\Delta\mu_{\rm crit})^2} \times D$$

Critical domain



- Choice of parameters: $\Delta T_{\rm crit}, \Delta\mu_{\rm crit}, D$

Thermodynamical inequalities

$$\left(\frac{\partial S}{\partial T}\right)\bigg|_{V,N} \geq 0, \quad \left(\frac{\partial P}{\partial V}\right)\bigg|_{T,N} \geq 0, \quad \left(\frac{\partial \mu}{\partial N}\right)\bigg|_{T,V} \geq 0$$

Critical exponent near CEP keeps correctly.

Thermodynamical Quantities

CN, Asakawa, Phys.Rev.C71:044904(2005)

$$S_{\rm real}(T,\mu_B)=\frac{1}{2}\{1-\tanh[S_c(T,\mu_B)]\}S_H(T,\mu_B)+\frac{1}{2}\{1+\tanh[S_c(T,\mu_B)]\}S_Q(T,\mu_B)$$

- **Baryon number density**

$$n_B(T,\mu_B)=\frac{\partial P}{\partial \mu_B}=\int_0^T\frac{\partial s}{\partial \mu_B}(T',\mu_B)dT'+n_B(0,\mu_B)$$
$$+\left|\frac{\partial T_C(\mu_B)}{\partial \mu_C}\right|\left(S_Q(T_C,\mu_B(T_C))-S_H(T_C,\mu_B(T_C))\right)$$

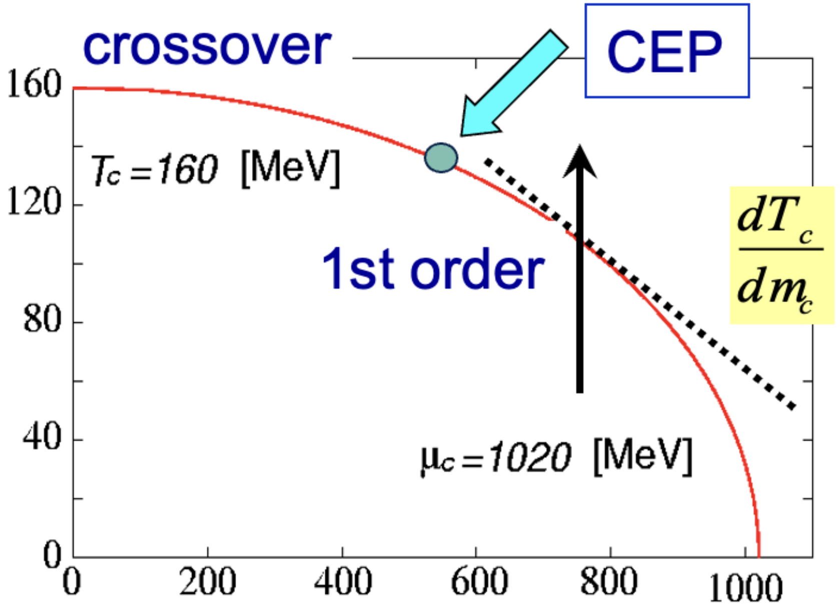
1st order

- **Pressure**

$$P(T,\mu_B)=\int_0^T S_{\rm real}(T',\mu_B)dT'+P(0,\mu_B)$$

- **Energy Density**

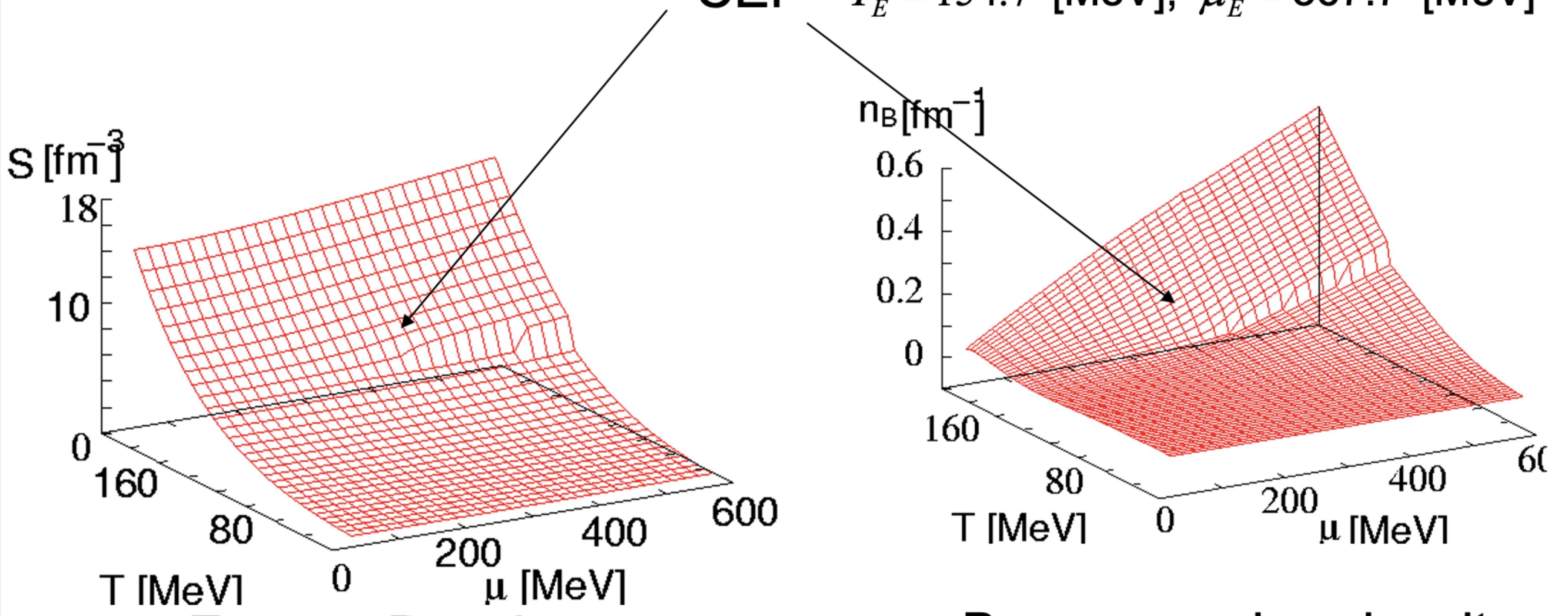
$$\varepsilon=TS_{\rm real}-P-\mu_B n_B$$



Equation of State

CN, Asakawa, Phys.Rev.C71:044904(2005)

CEP $T_E = 154.7$ [MeV], $\mu_E = 367.7$ [MeV]



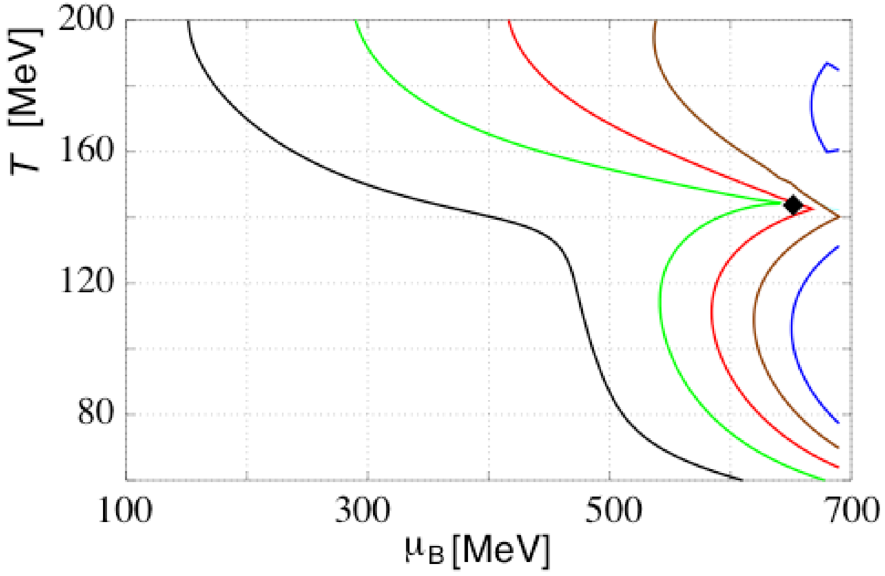
Entropy Density

Baryon number density

A Critical Point as a Hydrodynamic Attractor

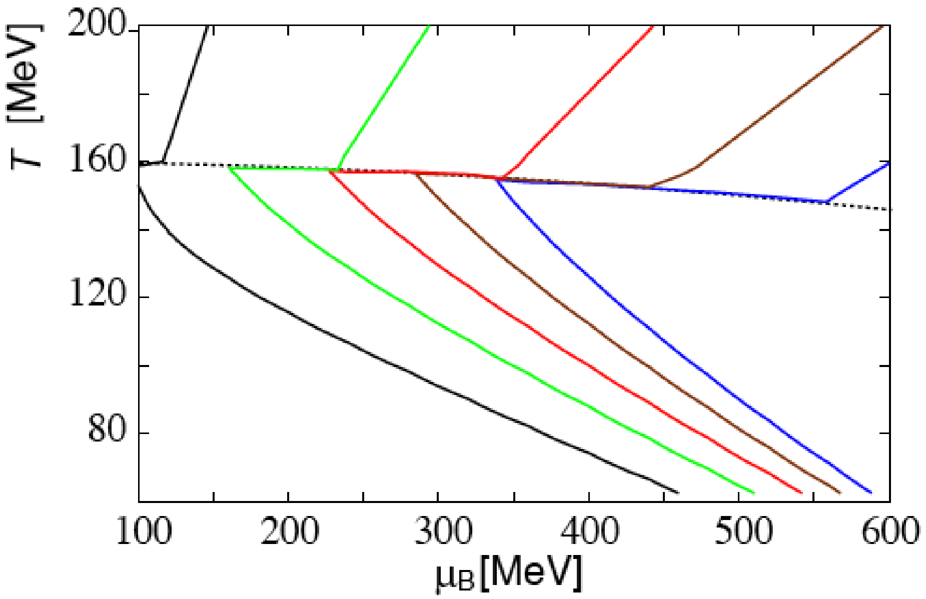
CN, Asakawa, Phys.Rev.C71:044904(2005)

With QCD critical point



Focused

Bag Model +
Excluded Volume Approximation
(No Critical Point)



Not Focused

Open Questions

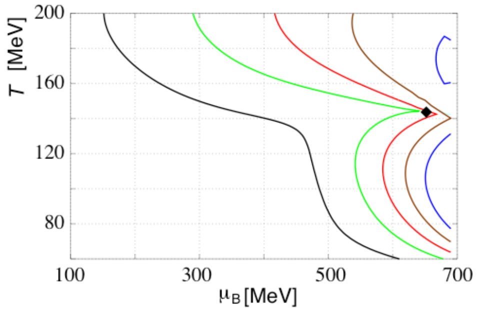
- How realistic is the EoS ?
  - Critical EoS was largely phenomenological.
- Is focusing robust?
  - Generic consequence or model artifact?
- Why does focusing occur?
  - Thermodynamic origin unclear.
- What are the quantitative consequence?
  - Realistic hydrodynamic simulation

Understanding was still incomplete.

Development Since 2005

2005 2010 2020 2024 2025

Focusing



Phenomenological EoS

BES-I, BES-II
Experimental hints

Search for critical signatures

Modern Critical EoS

BEST EoS
Parotto et al.
PRC101 (2020)

MUSES EoS

Constraint form Lattice EoS

Nonmonotonic Specific Entropy
Pradeep, Sogabe, Stephanov, Yee, PRC109,9064905 (2024)

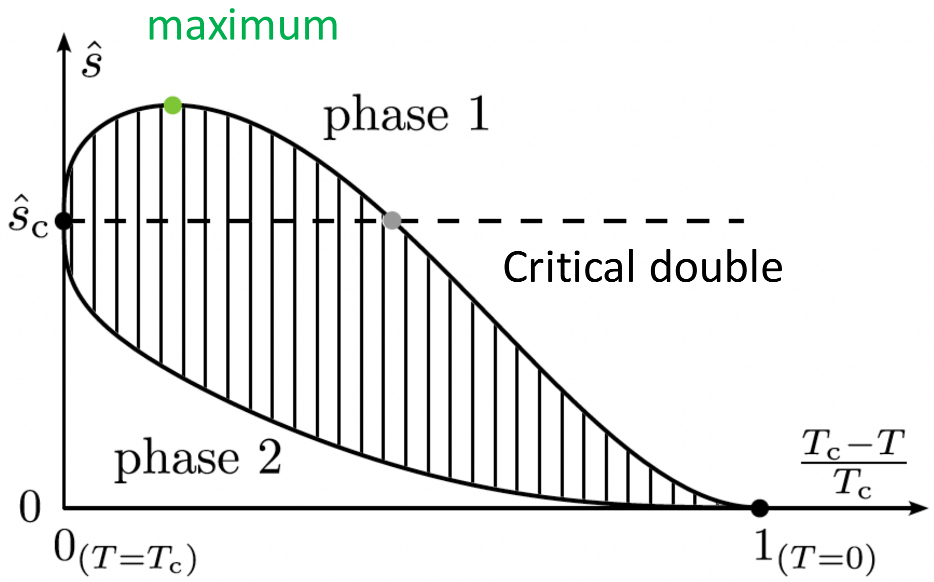
Quantitative consequences
based on hydrodynamic simulation

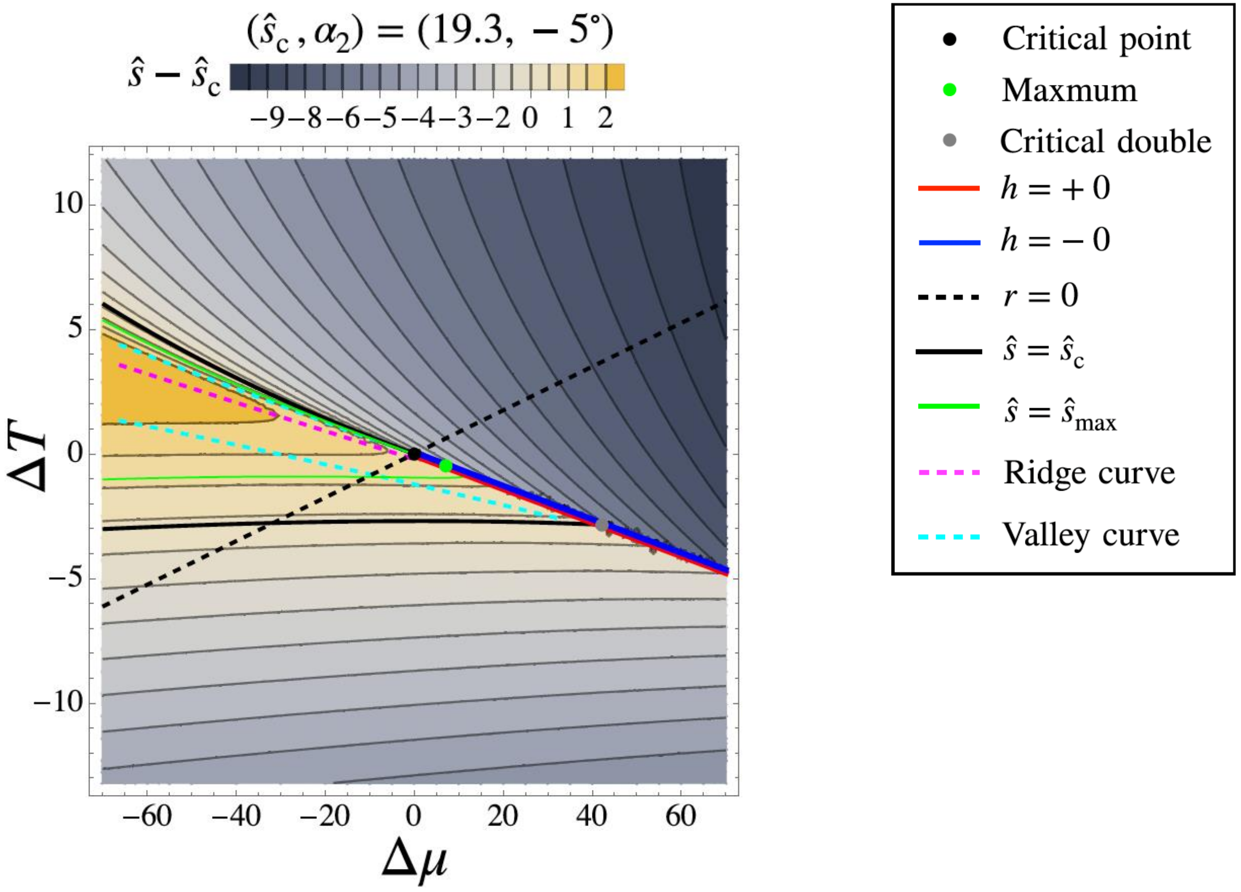
A New Perspective

Pradeep, Sogabe, Stephanov, Yee, PRC109,9064905 (2024)

- Nonmonotonic Specific Entropy Near the QCD Critical Point

Specific entropy $\hat{s} = \frac{s}{n_B}$

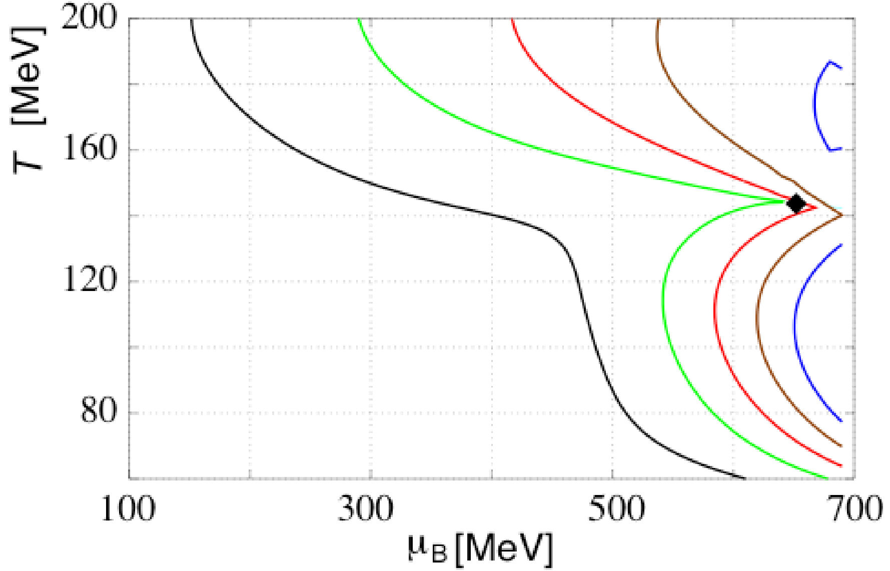




(d)
Ridge structure

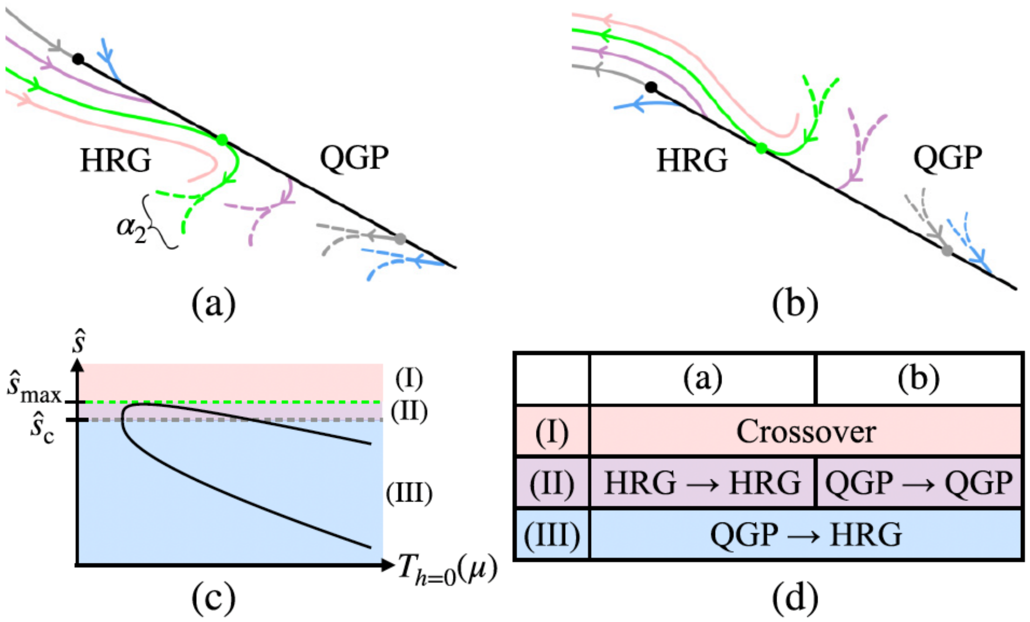
A New Perspective

2005



CN, Asakawa, Phys.Rev.C71:044904(2005)

2024



	(a)	(b)
(I)	Crossover	
(II)	HRG → HRG	QGP → QGP
(III)	QGP → HRG	

Pradeep, Sogabe, Stephanov, Yee, PRC109, 064905 (2024)

Isentropic trajectories are distorted and focused by the ridge.

Three classes of trajectories appear:

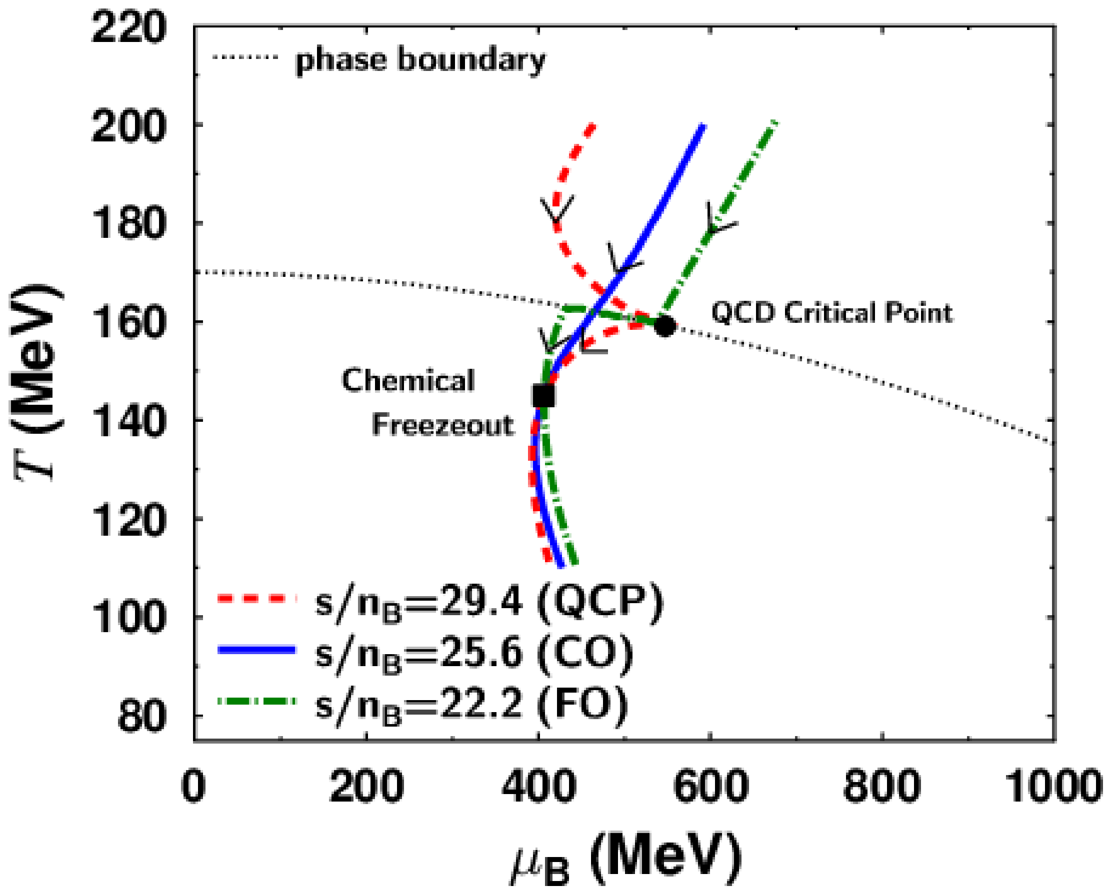
- crossover trajectories,
- trajectories entering and exiting the coexistence region on the same side,
- trajectories crossing from one phase to the other.

Hydrodynamic Consequences of Focusing

Focusing may affect

- Expansion history
- Time spent near the critical region
- Freezeout trajectory
- Hadrochemistry

Example: p/pbar ratio

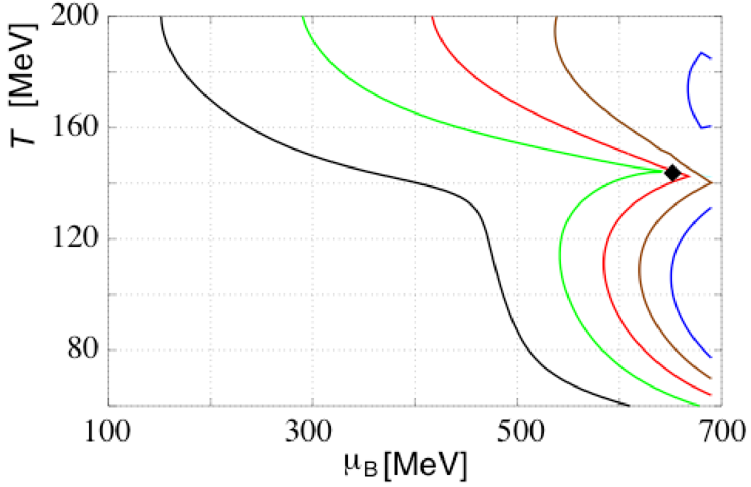


Asakawa, Bass, Müller, C.N.,PRL101,122302(2008)

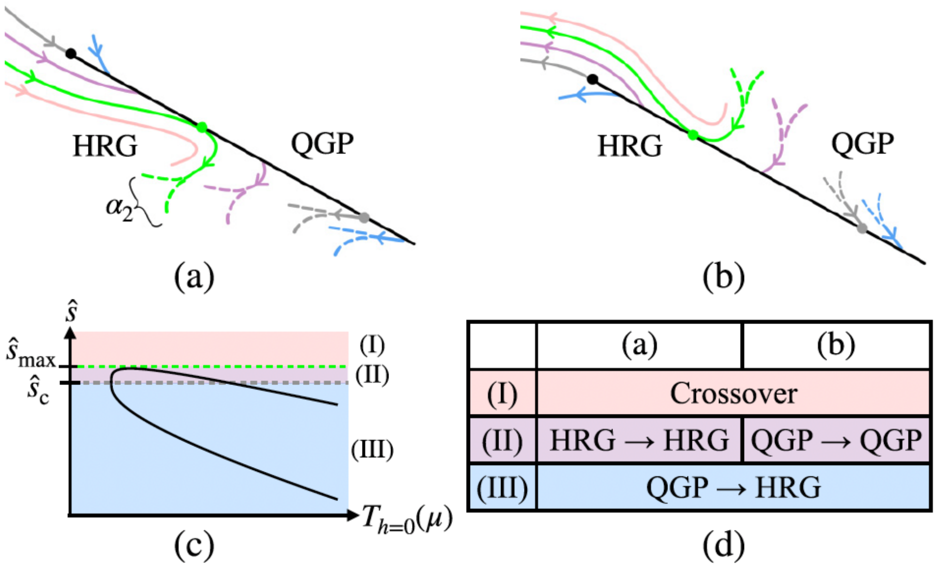
Criticality can modify the bulk evolution even without critical fluctuations.

Revisiting the Original Questions

2005



2024



	(a)	(b)
(I)	Crossover	
(II)	HRG → HRG	QGP → QGP
(III)	QGP → HRG	

(d)

Work in progress with Sogabe

Questions for realistic hydrodynamics

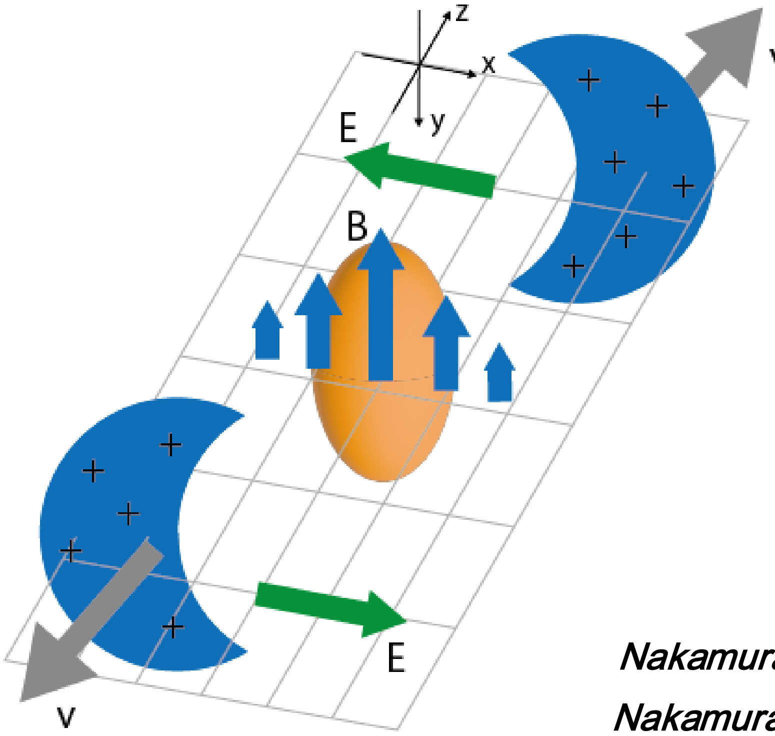
- How large is the focusing effect?
- Which collision energies are most affected?
- How does the expansion history change?
- Can freezeout conditions be modified?
- Are there observable consequences beyond fluctuations?

Relativistic Resistive Magnetohydrodynamics

Beyond conventional hydrodynamics

Electromagnetic Fields in Heavy Ion Collisions ?

- Strong Electromagnetic fields and expansion? Observables?
  - Au + Au ($\sqrt{s_{NN}} = 200$ GeV) : 10^{14} T ~10 m_π^2
  - Pb + Pb ($\sqrt{s_{NN}} = 2.76$ TeV) : 10^{15} T



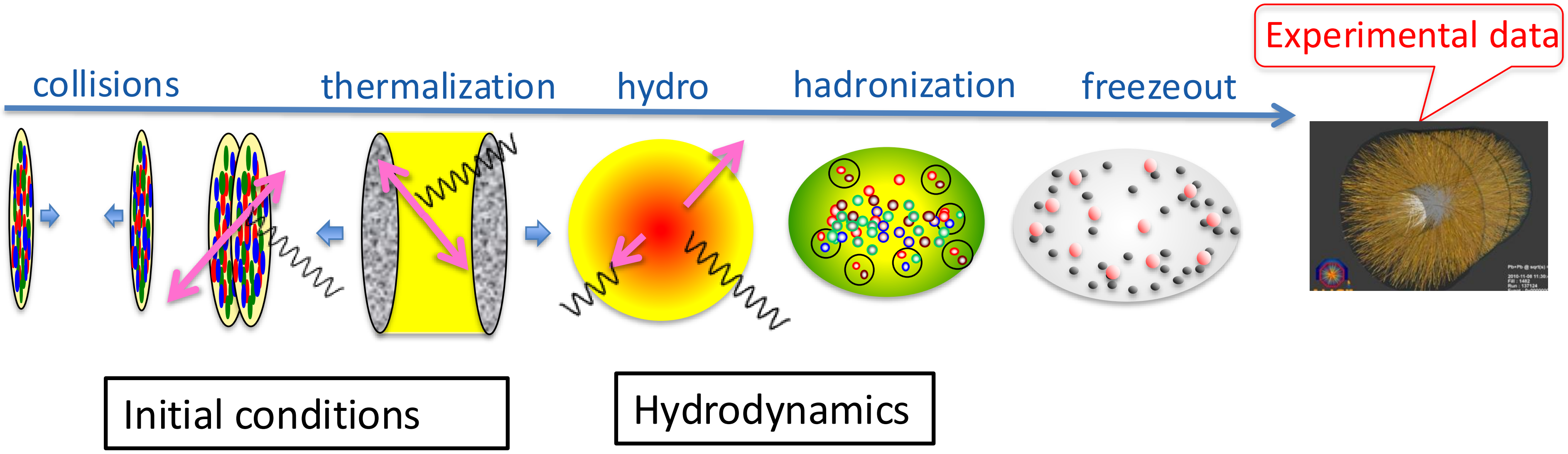
Lifetime of EM fields
Space-time evolutions?

Electric conductivity of QGP?

Nakamura, Miyoshi, C. N. and Takahashi, Phys. Rev. C 107, (2023) 014901
Nakamura, Miyoshi, C. N. and Takahashi, Eur.Phys.J.C 83 (2023) 3, 229.
Nakamura, Miyoshi, C. N. and Takahashi, Phys. Rev. C 107 (2023) 3, 034912
Benoit, Miyoshi, C. N., and Takahashi, Phys. Rev. C 112, 024911(2025)

Relativistic Resistive Magneto-Hydrodynamics (RRMHD)

Nakamura, Miyoshi, C. N. and Takahashi, PRC107, no.1, 014901 (2023)



Glauber model
+approximate solutions of Maxwell eq.

Hydrodynamic eq. + Maxwell eq. + Ohm's law

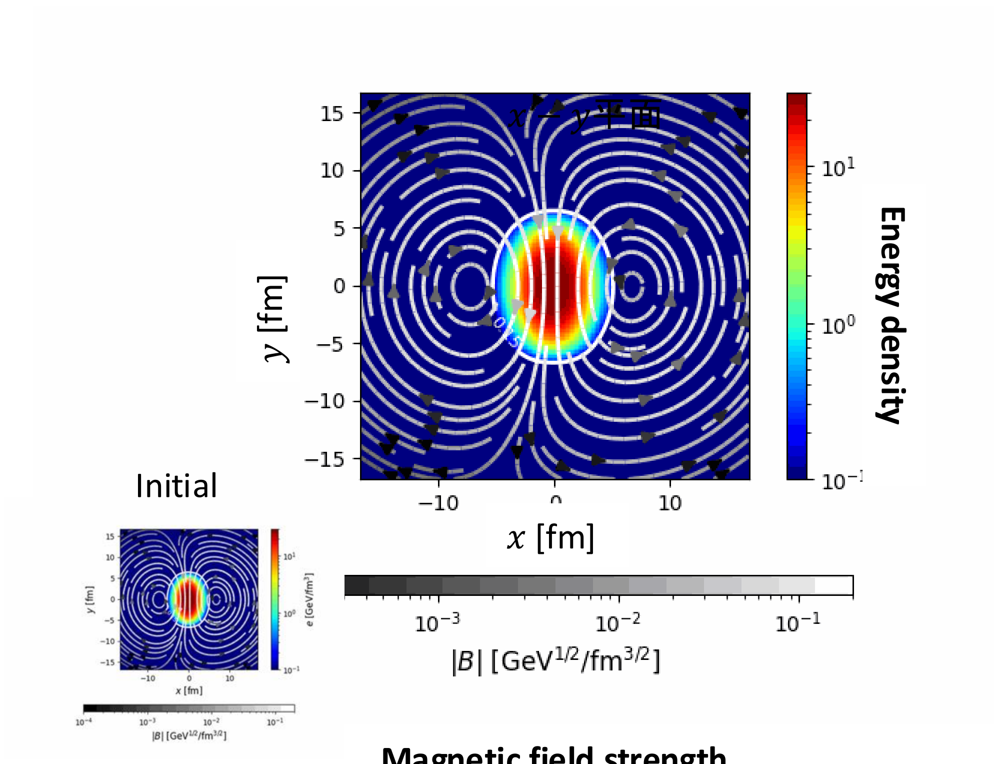
$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \qquad J^\mu = \sigma e^\mu$$

Space-time Evolution

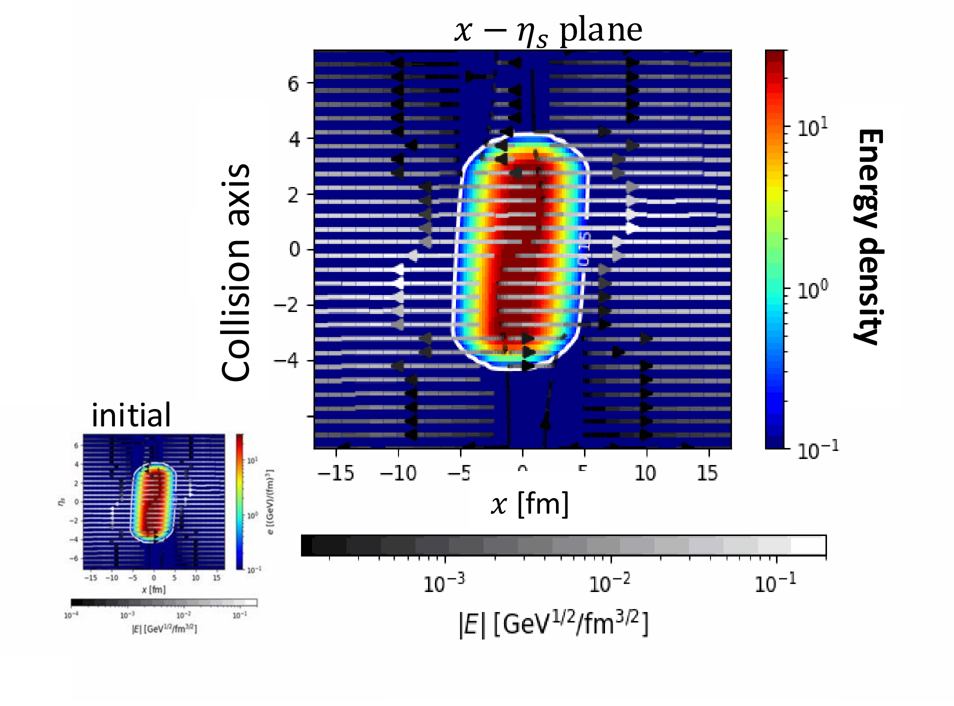
Nakamura, Miyoshi, C. N. and Takahashi, PRC 107, no.1, 014901 (2023)

Au+Au collision system

First calculation in HIC with RRMHD code



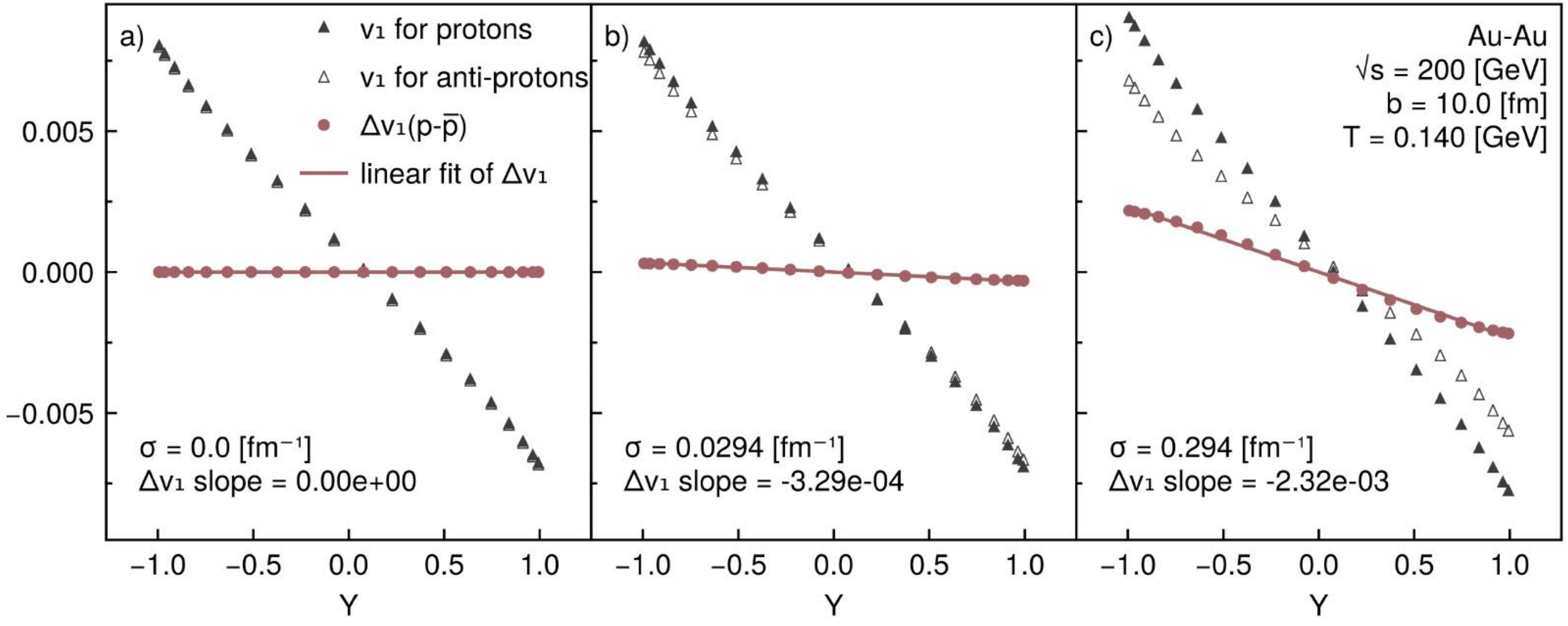
Magnetic field strength



Electric field strength

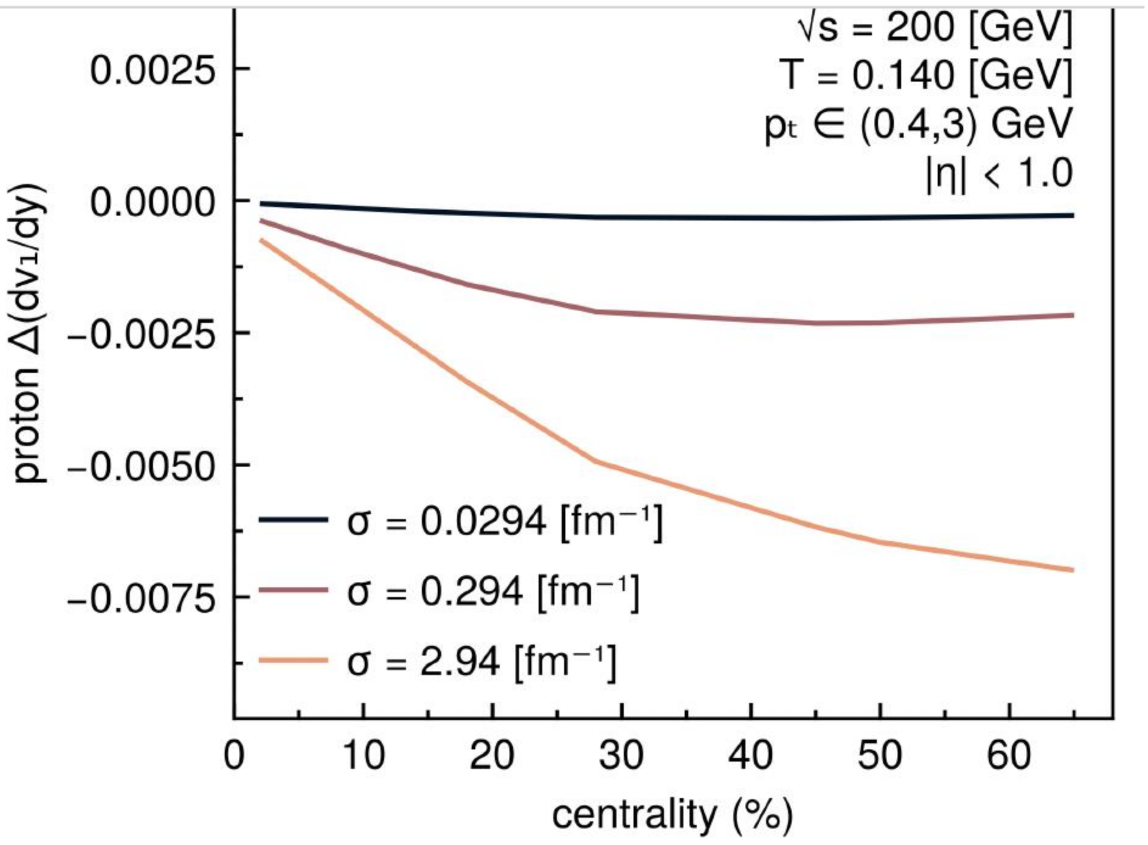
Analysis of Heavy Ion Collisions

Charge Dependent Directed Flow

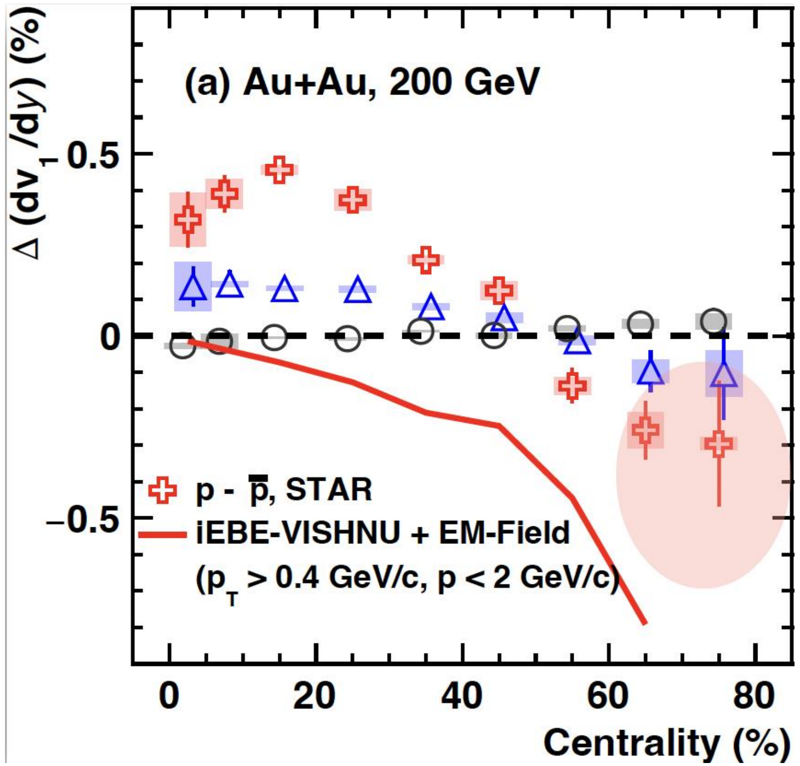


Benoit, Miyoshi, C. N., and Takahashi, Phys. Rev. C 112, 024911(2025)

Charge Dependent Flow



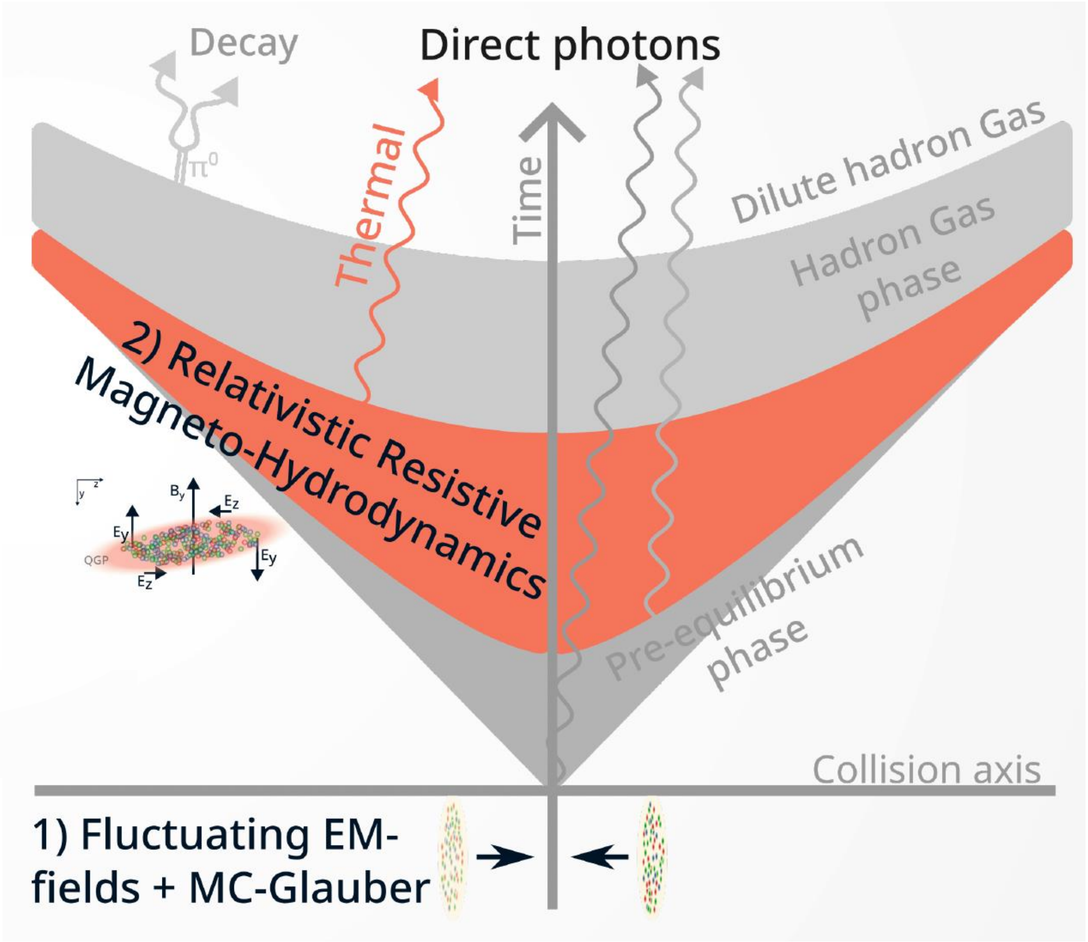
Benoit, Miyoshi, C. N., and Takahashi, Phys. Rev. C 112, 024911(2025)



Caveat:

- No baryon current
- Initial condition
- EoS
- Final state interactions

Photon

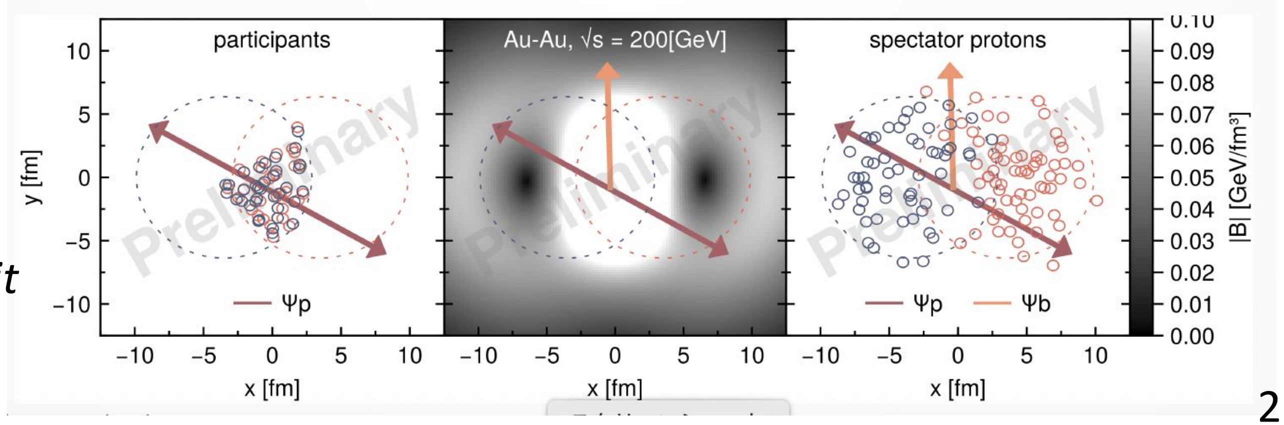


- A photon carries information from the early stages through to the final stage
- It is affected by EM field.

⬇

- P_T spectra and elliptic flow of photon
- Including event by event fluctuations in initial conditions
- Consider the direction of the lab frame EM fields

By courtesy of Benoit



Electromagnetic Dissipation for QGP Photon

- Electromagnetic fields inside QGP
  - The fluid + EM field contributions from hydrodynamics
  - All of those values can be calculated self-consistently using relativistic resistive magneto-hydrodynamics (RRHMD)

Temperature and four velocity

conductivity

$$\delta f^{(1)}_{a,EM}(X,k) = -\frac{-f_{a,eq}(1-f_{a,eq})}{T\chi_{el}k^{\mu}u_{\mu}} e\sigma Q_a e^{\mu} k_{\mu}$$

Electric susceptibility of QGP

$$\chi_{a,el} = -\frac{1}{3}\int \frac{d\vec{p}}{(2\pi)^3 E_p}(p^{\sigma}p^{\nu}\Delta_{\sigma\nu})\frac{-f_{a,eq}(1-f_{a,eq})}{p^{\mu}u_{\mu}}$$

Spacetime dependent EM fields in QGP medium

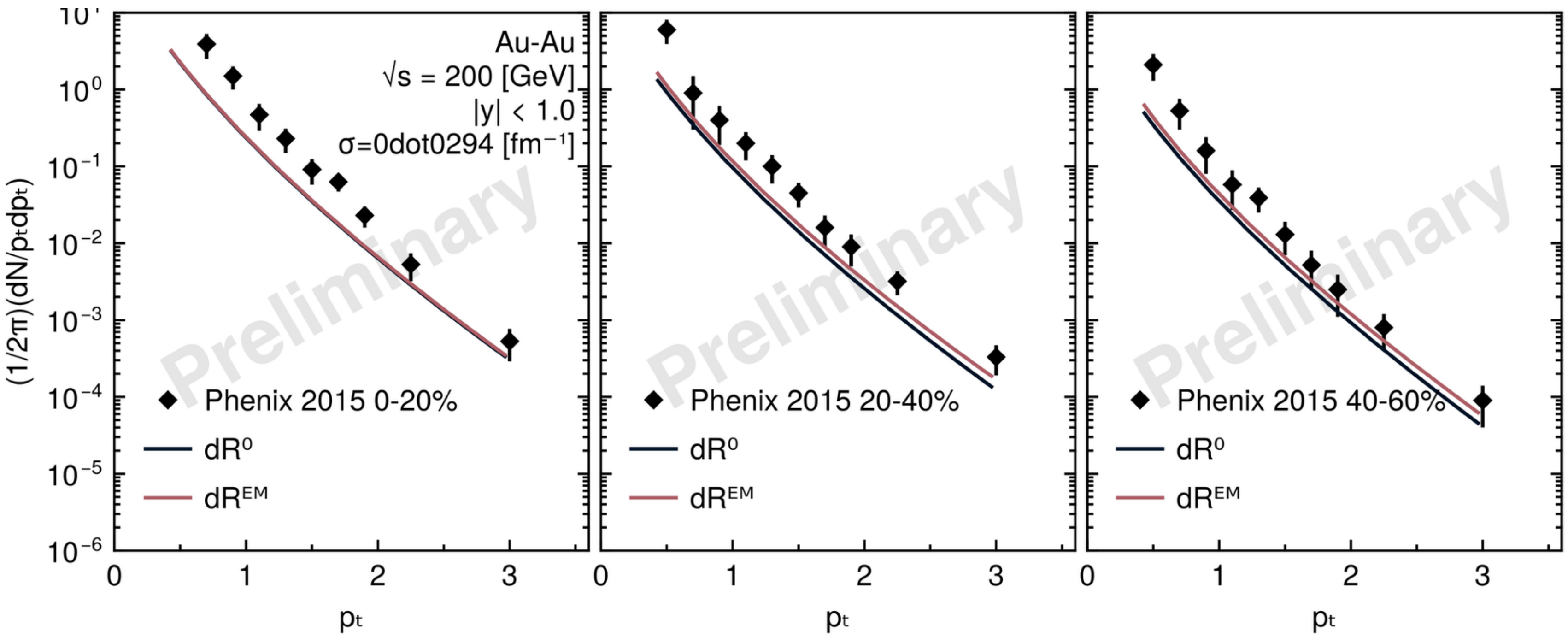
$$e^{\mu} = (\gamma v_k E^k, \gamma E^i + \gamma\epsilon^{ijk}v_j B_k)$$

P_T Spectra of Photon

- Au+Au $\sqrt{s_{\mathrm{NN}}} = 200$ GeV
  - Set normalization of initial conditions from charge π rapidity distribution

σ=0.0294 fm^{-1}

T~ 220 MeV (lattice QCD)



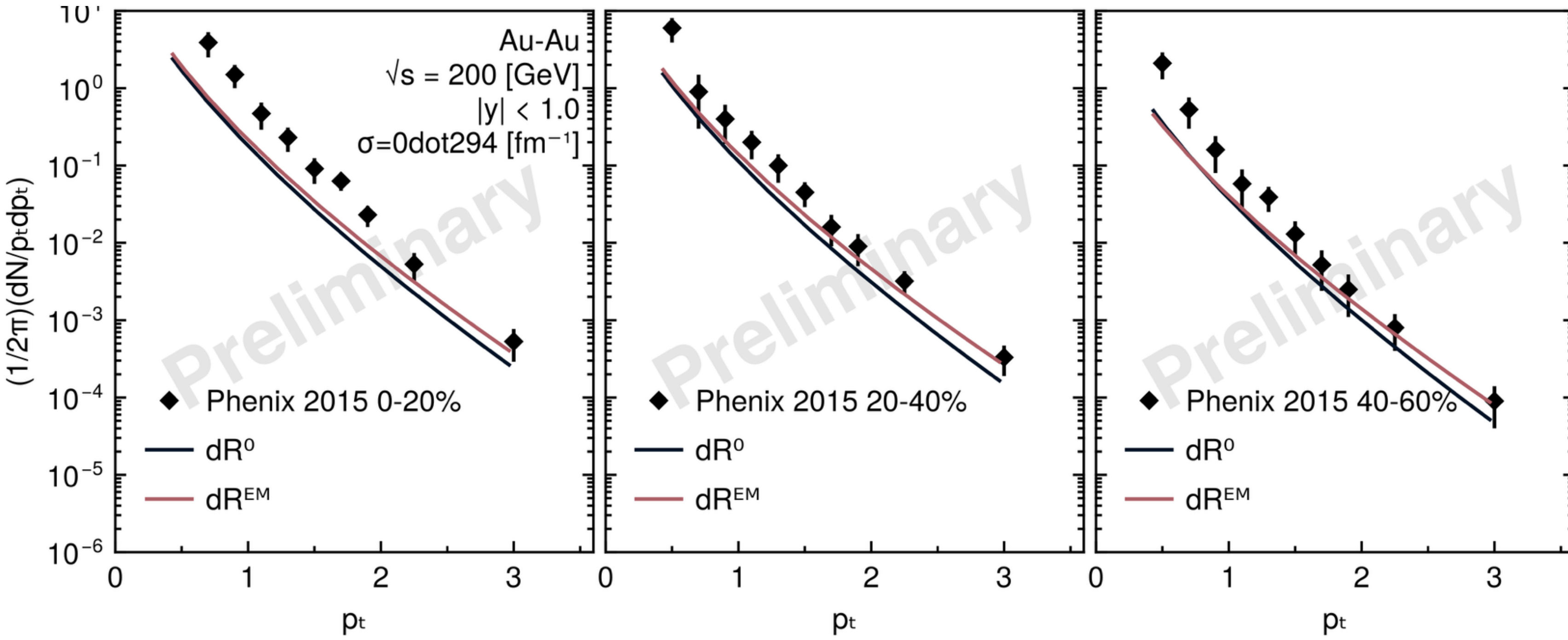
- EM fields create mild increase in photon yield
- Larger pt (~ 3 GeV) has greater increase

P_T Spectra of Photon

- Au+Au $\sqrt{s_{\mathrm{NN}}} = 200$ GeV
  – Set normalization of initial conditions from charge π rapidity distribution

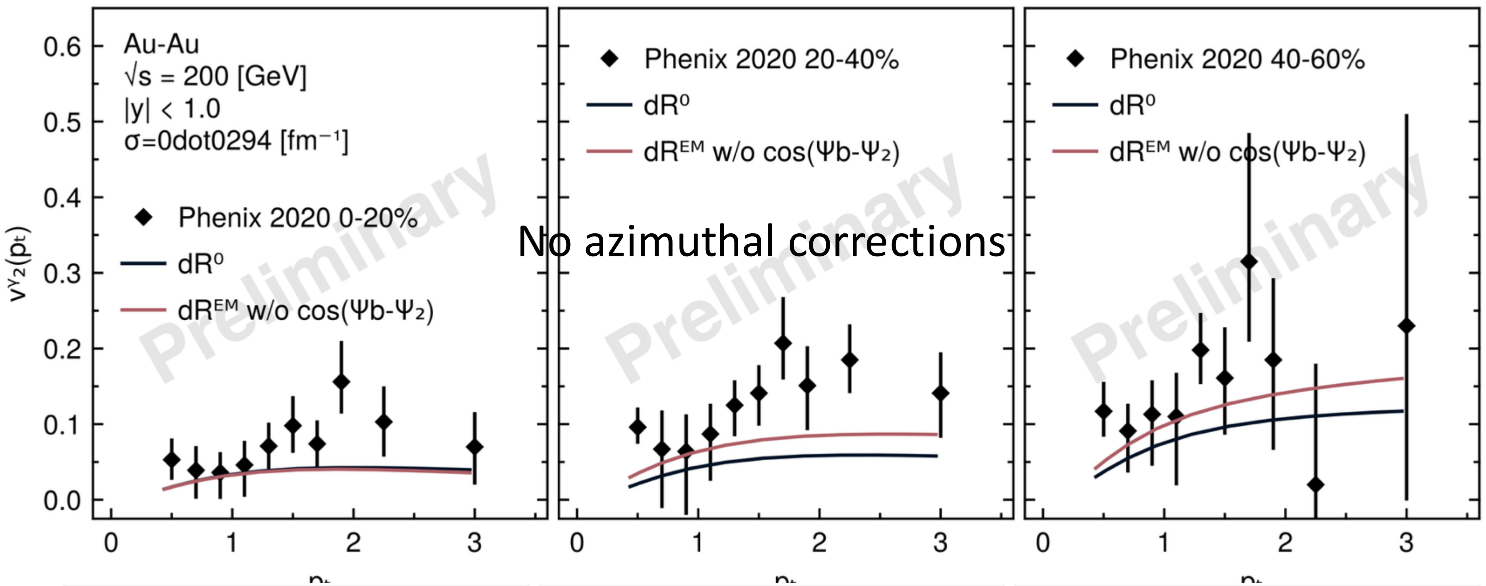
σ=0.294 fm^{-1}

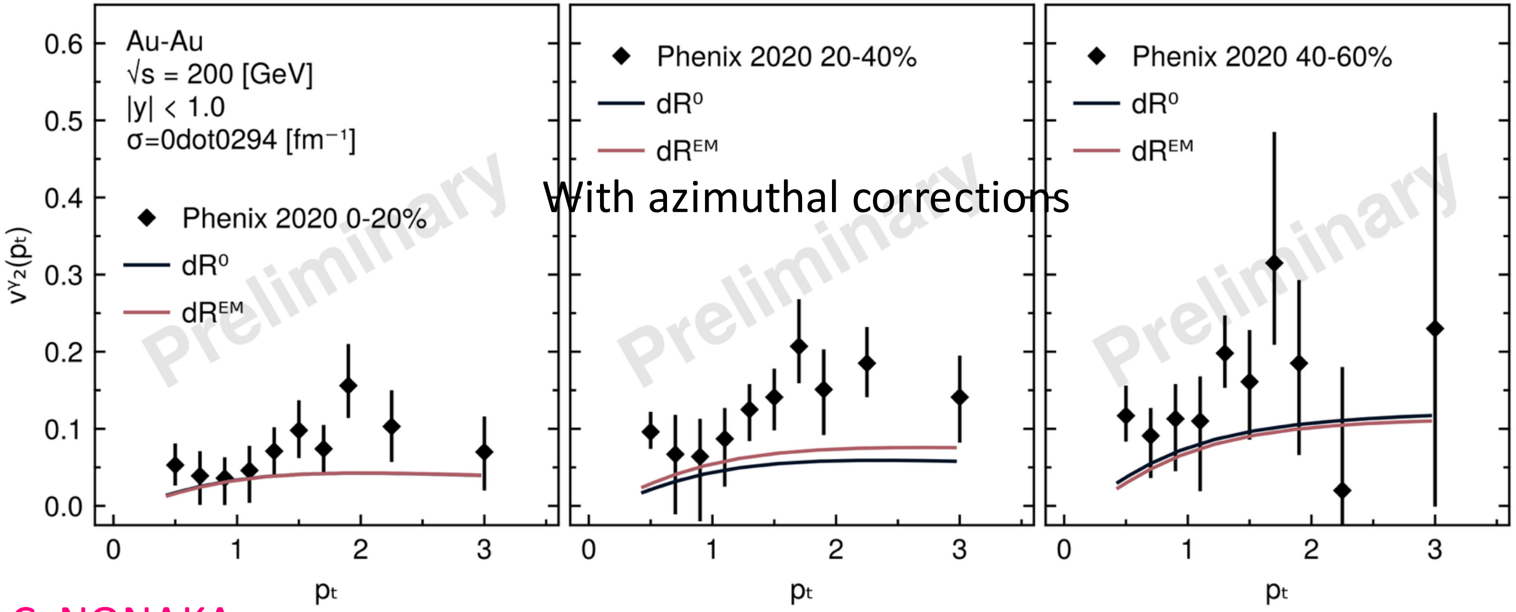
T~ 340 MeV (lattice QCD)



- EM fields create mild increase in photon yield
- Larger pt (~ 3 GeV) has greater increase
- O(10) larger QGP conductivity σ increases the higher Pt yield

Elliptic Flow of Photon





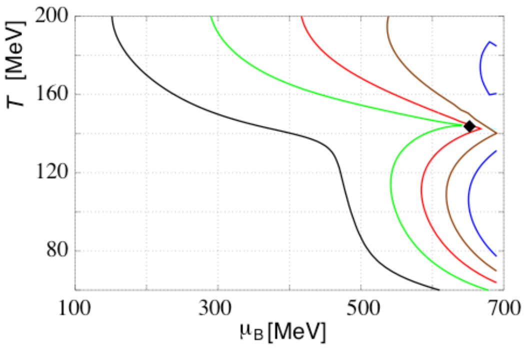
Including the azimuthal fluctuations

- The elliptical flow is not always increased by EM fields (depends on centrality)
- Suggests the geometry of the EM can be probed, but effects become smaller.

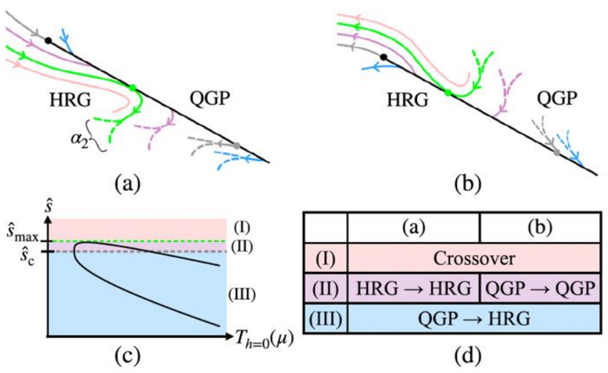
Photons can carry a local snapshot of the EM fields (Magnetometer)
possibility to constrain field lifetime
(the electric conductivity σ of the QGP)

Summary

- Criticality has two faces: fluctuations and hydrodynamic response
- A critical point can focus hydrodynamic trajectories
- Recent work suggests that focusing originates from a universal entropy-ridge structure near the critical point.
- The next challenge is to quantify these effects in realistic hydrodynamic simulations.



CN, Asakawa, Phys.Rev.C71(2005)



Pradeep, Sogabe, Stephanov, Yee, PRC109(2024)

Beyond conventional hydrodynamics:
Relativistic resistive
magnetohydrodynamics