Hydrodynamic Behavior in Ising Field Theory from Real-Time Lattice Simulation

Xiaojun Yao

InQubator for Quantum Simulation

University of Washington

arXiv: 2606.09984

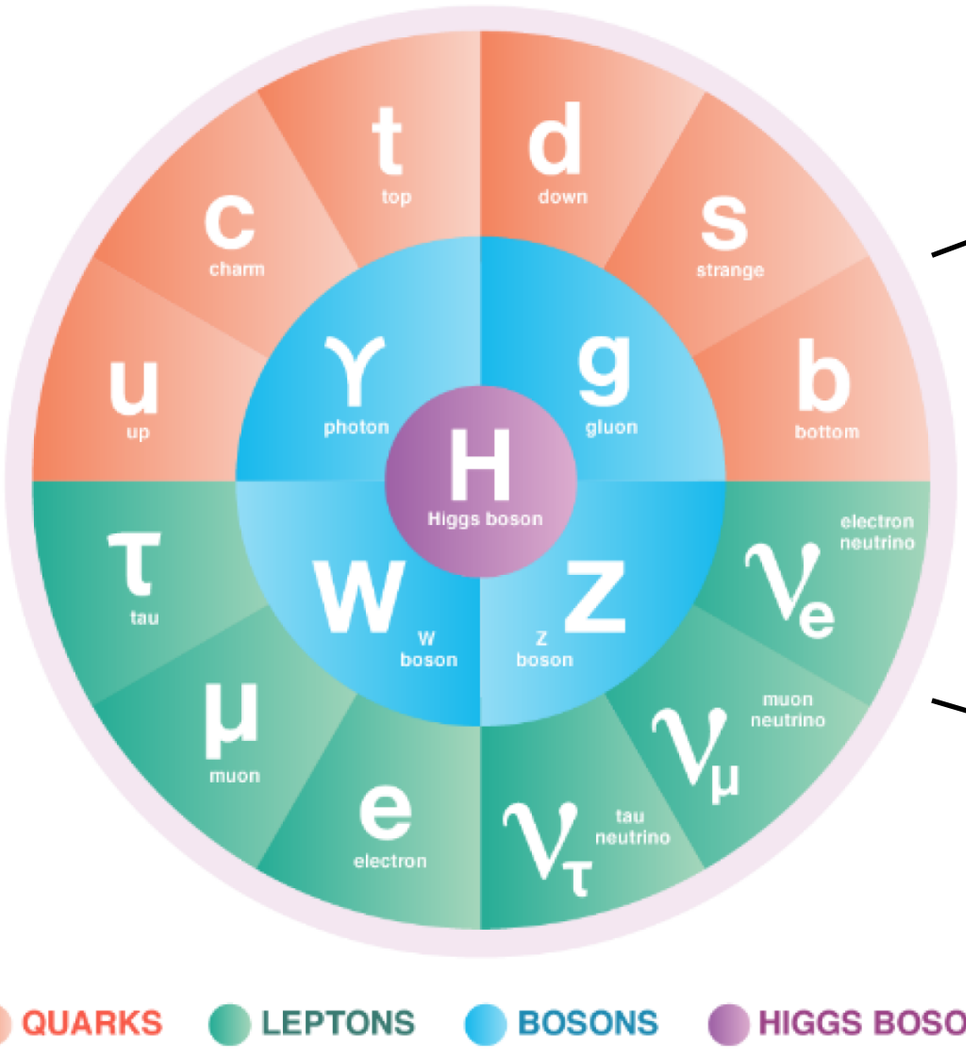
Frontiers of Quantum Computing and Classical Tensor Networks for Gauge Theories

Yukawa Institute for Theoretical Physics, Kyoto University

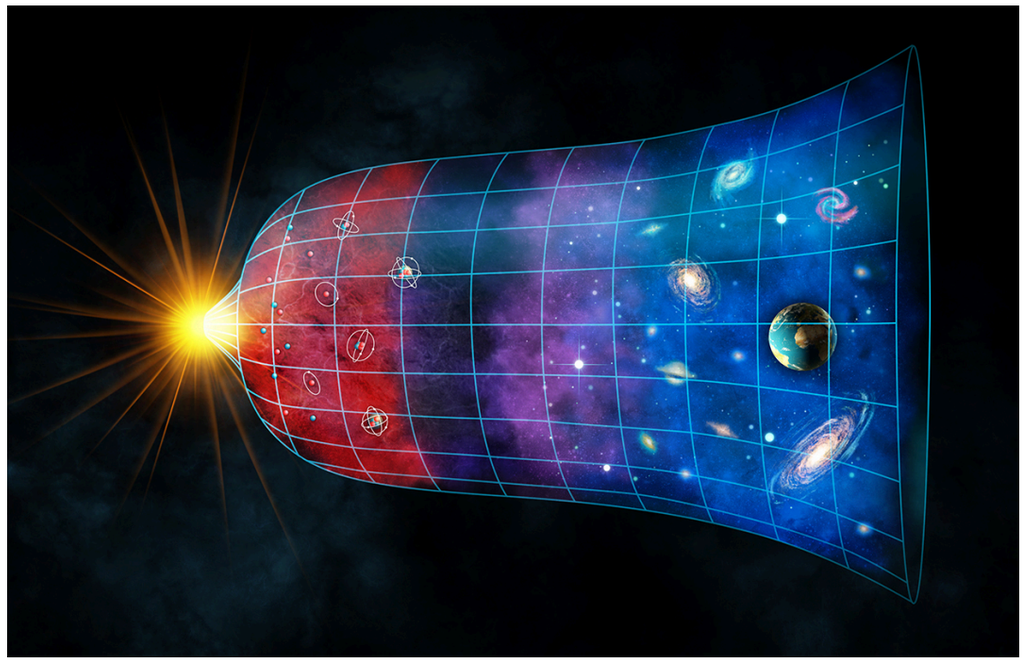
November 16, 2026

Standard Model

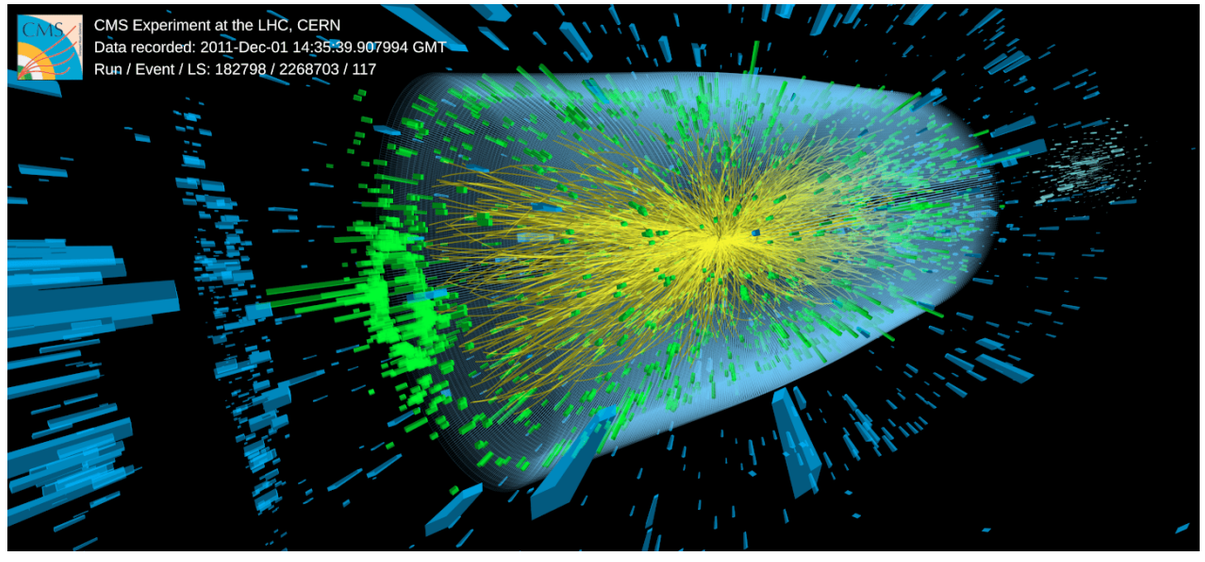
- **EM, weak, strong forces formulated as gauge theories**



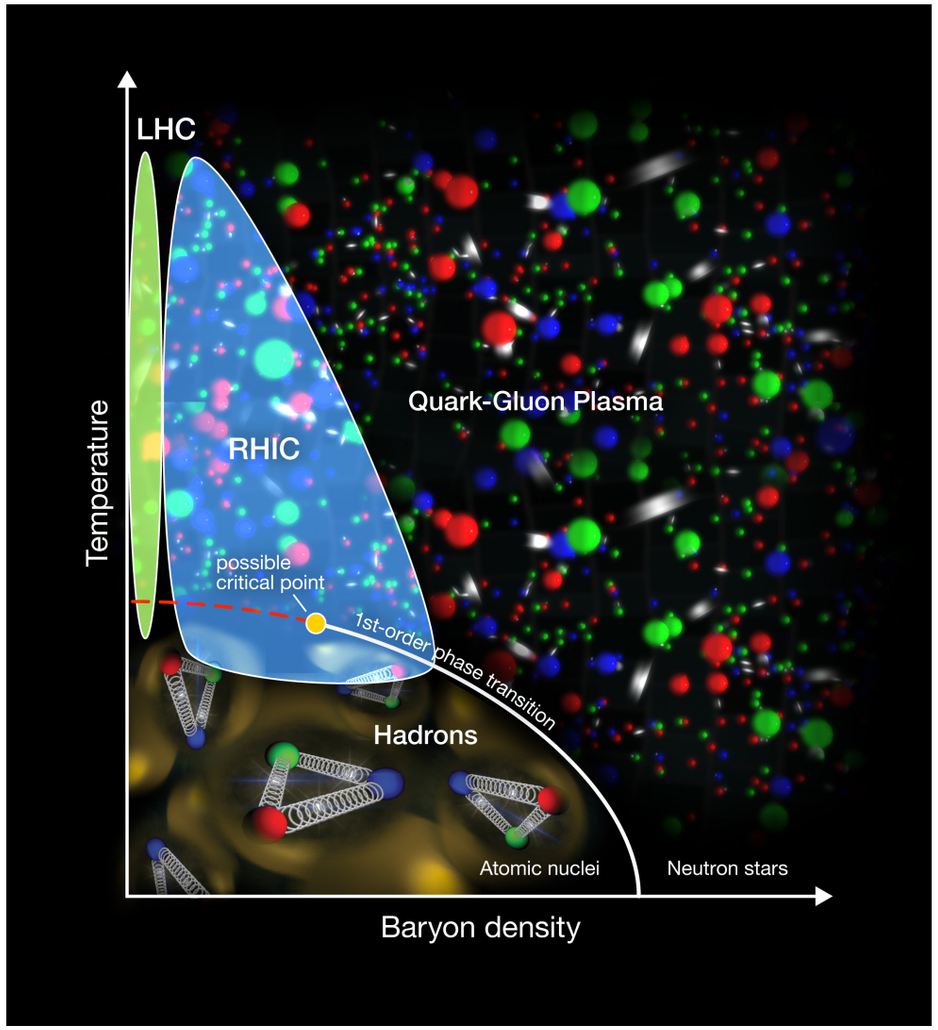
https://www.energy.gov/science/doe-explainsthe-standard-model-particle-physics

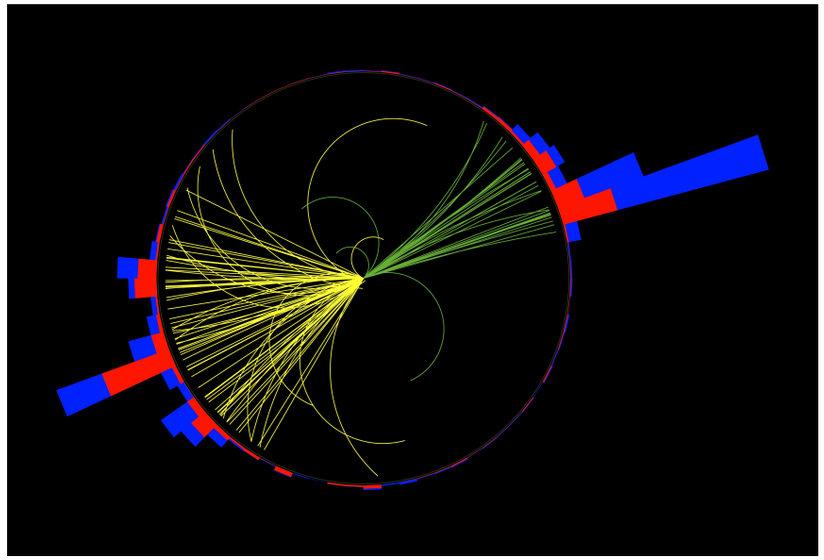


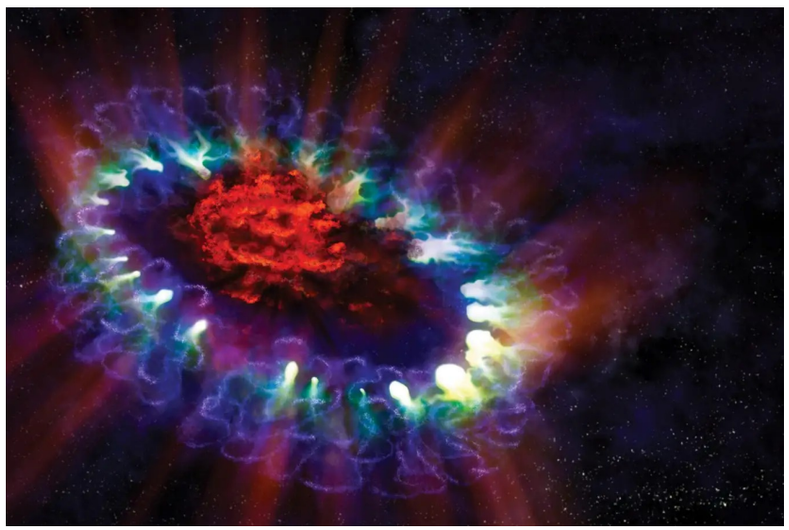
https://www.skyatnightmagazine.com/space-science/expansion-universe

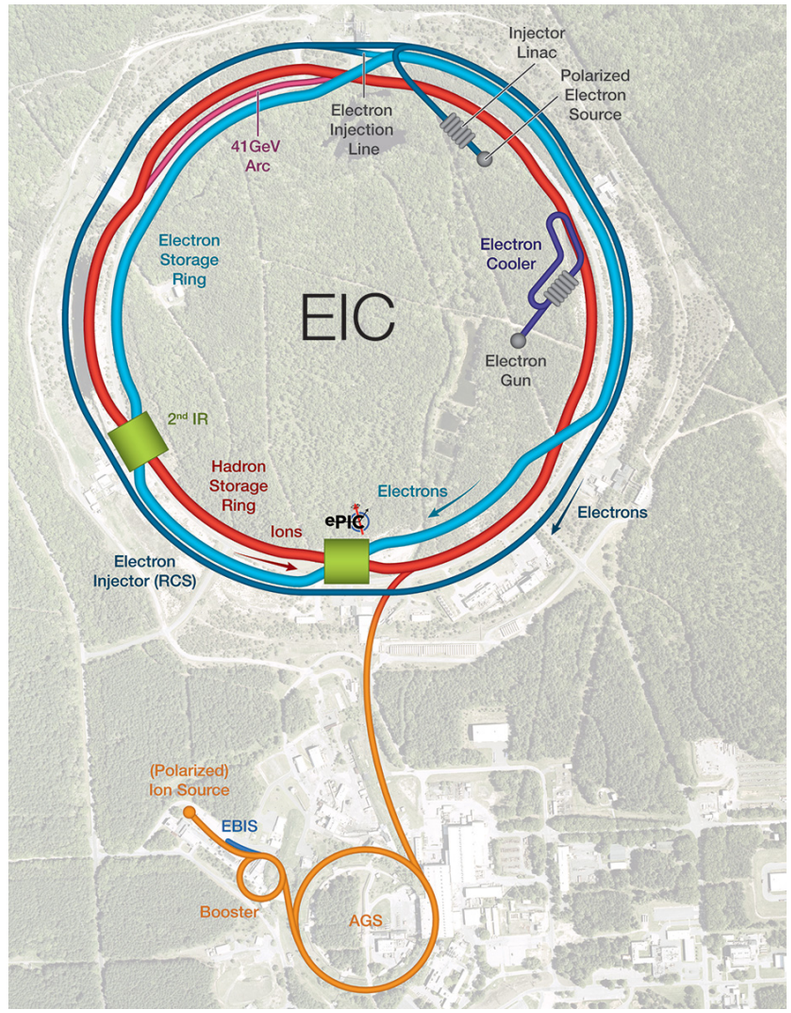


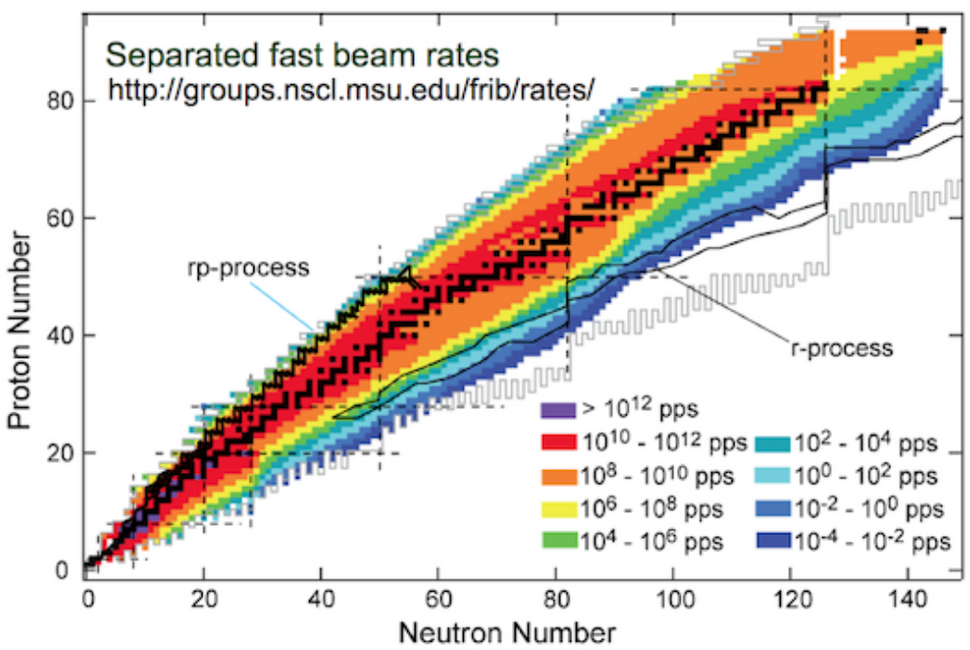
Nuclear and Particle Physics

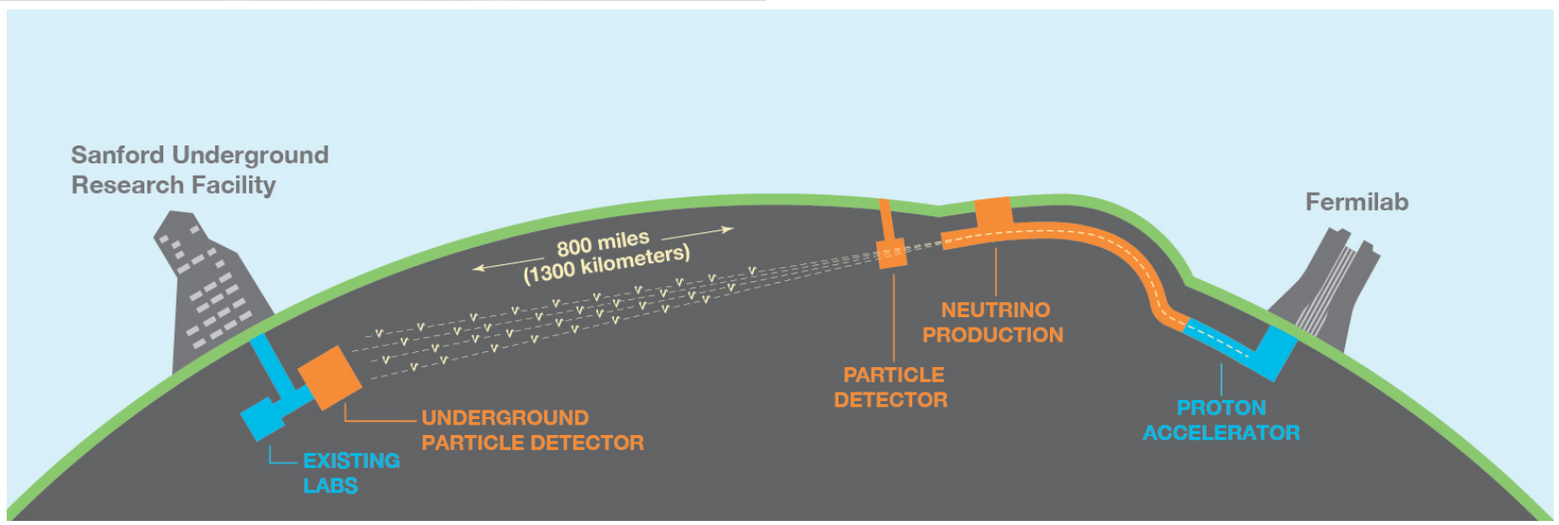








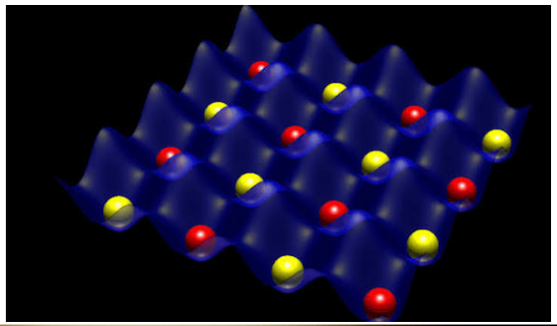


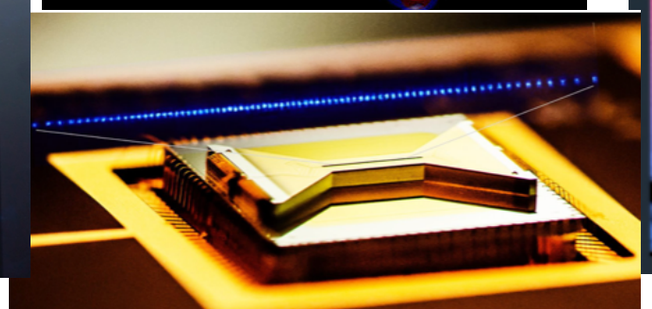


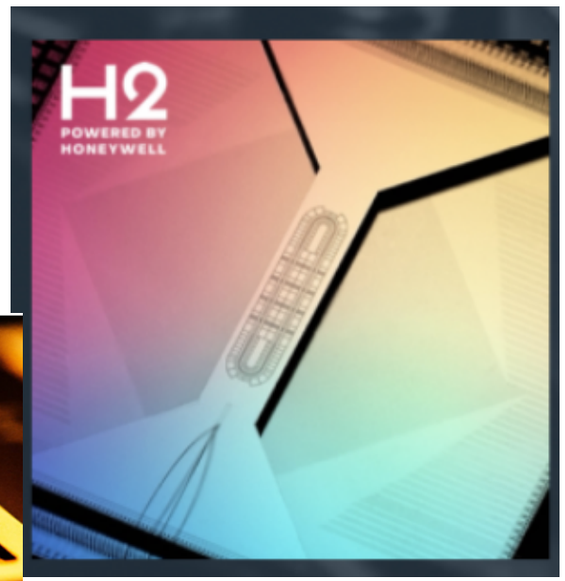
Why Quantum Computing/Tensor Networks?

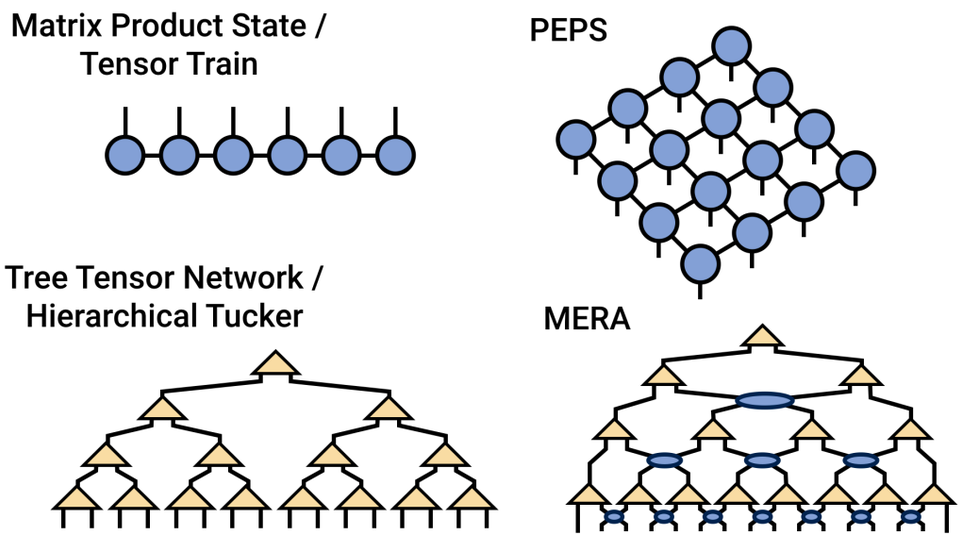
- **QCD is nonperturbative at hadronic scale:** need nonperturbative input for precision comparison with experimental data
- **Euclidean lattice QCD very successful:** hadron mass and structure e.g. PDF/TMD, muon g-2, phase transition, heavy quark diffusion coefficient
- **But faces sign problems for real-time and finite baryon density:** QCD critical point, real-time non-equilibrium dynamics, viscosity
- **Hamiltonian lattice gauge theory (LGT):** known for 50 yrs, but not practical due to Hilbert space exponential growth
- **Using QC/TN to simulate Hamiltonian LGT attracts a lot of interest:** string breaking, phase structure, scattering, thermalization







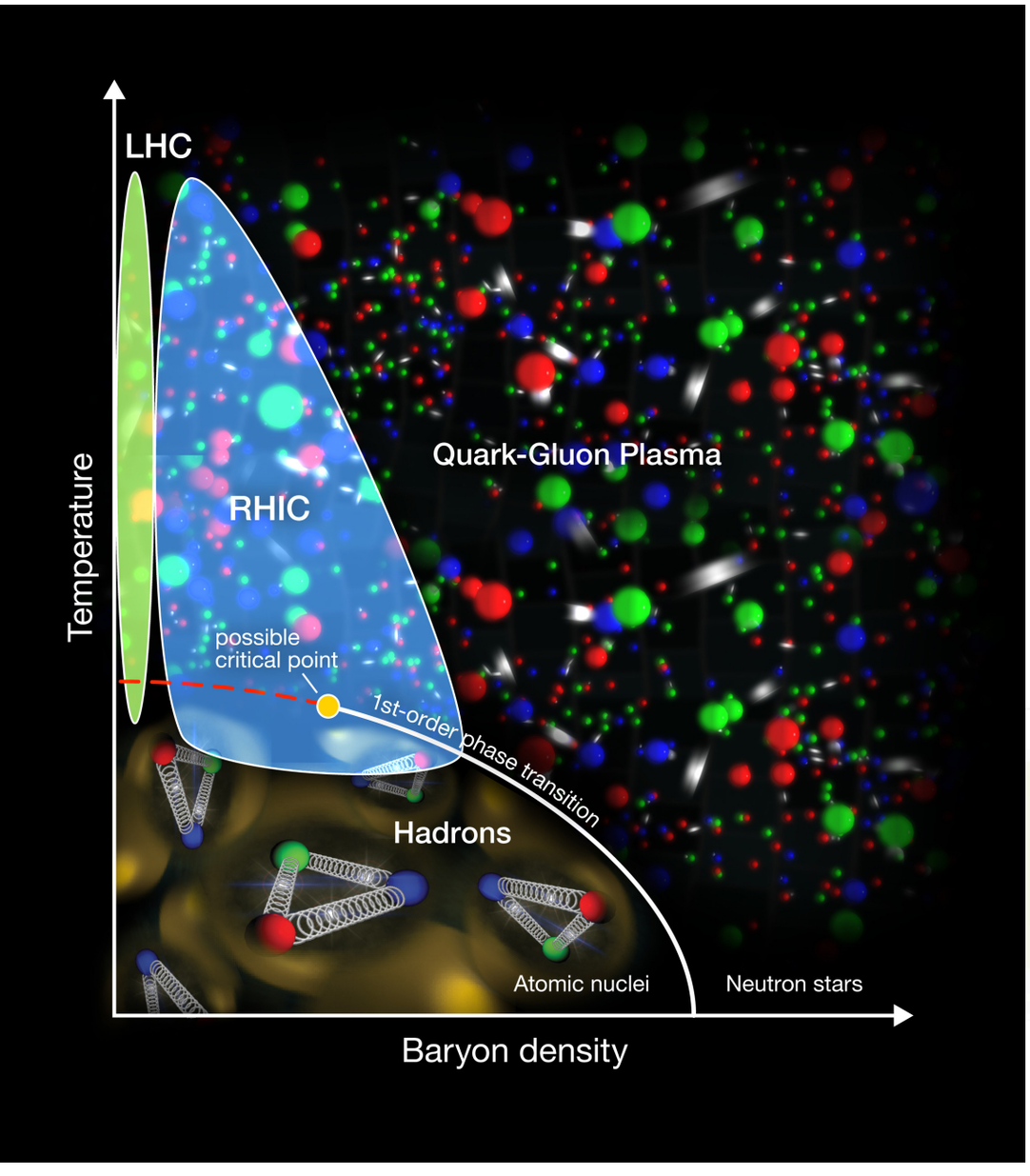


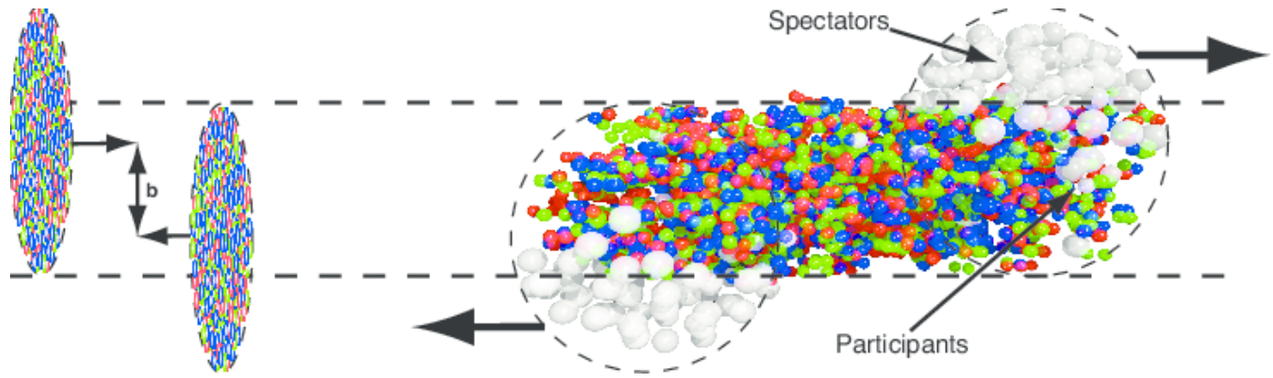


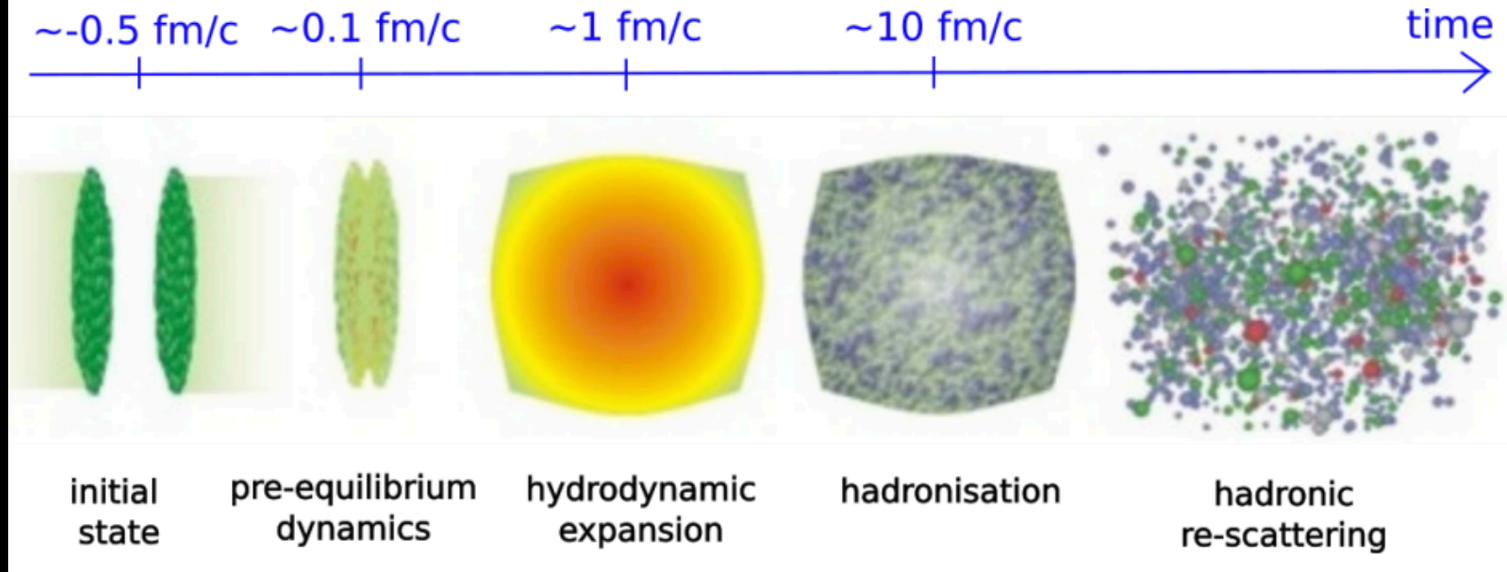
A Nuclear Physics Problem that Real-Time Lattice Simulation May Help to Solve

Motivation: Heavy Ion Collisions

- **Study deconfined phase of nuclear matter: quark-gluon plasma (QGP)**



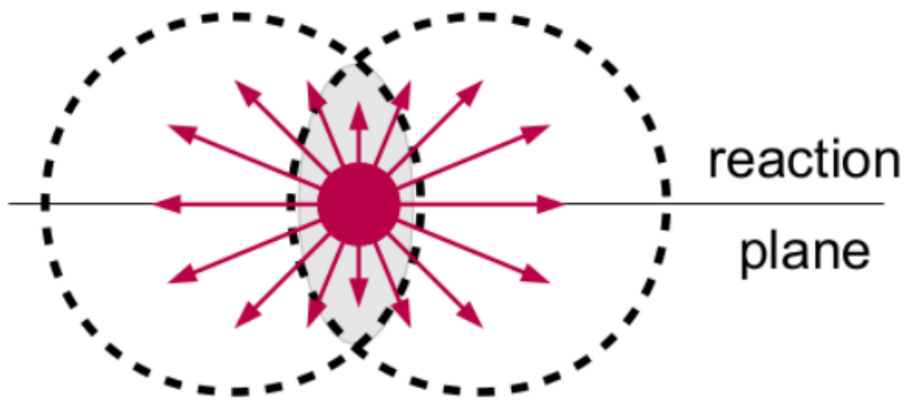




Credit: Steffen Bass

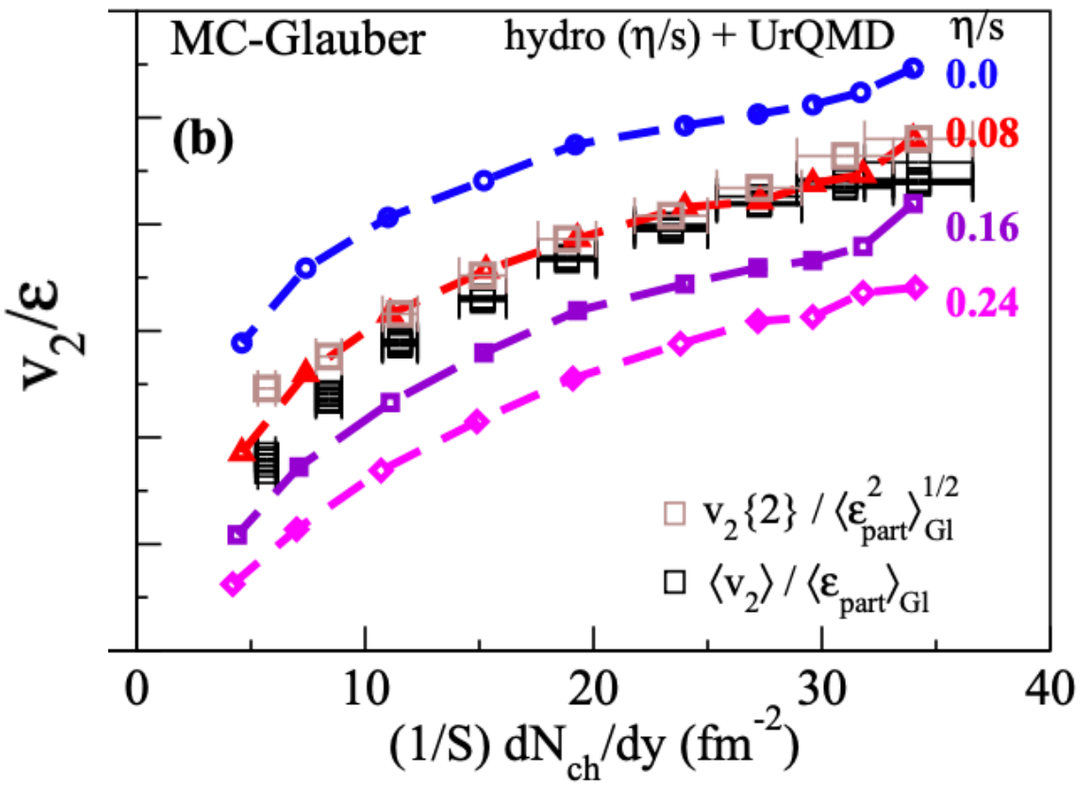
Motivation: Hydrodynamic Description of Flow

- **Hydrodynamics analysis confronted w/ experimental data favors a very small shear viscosity and rapid local thermalization**

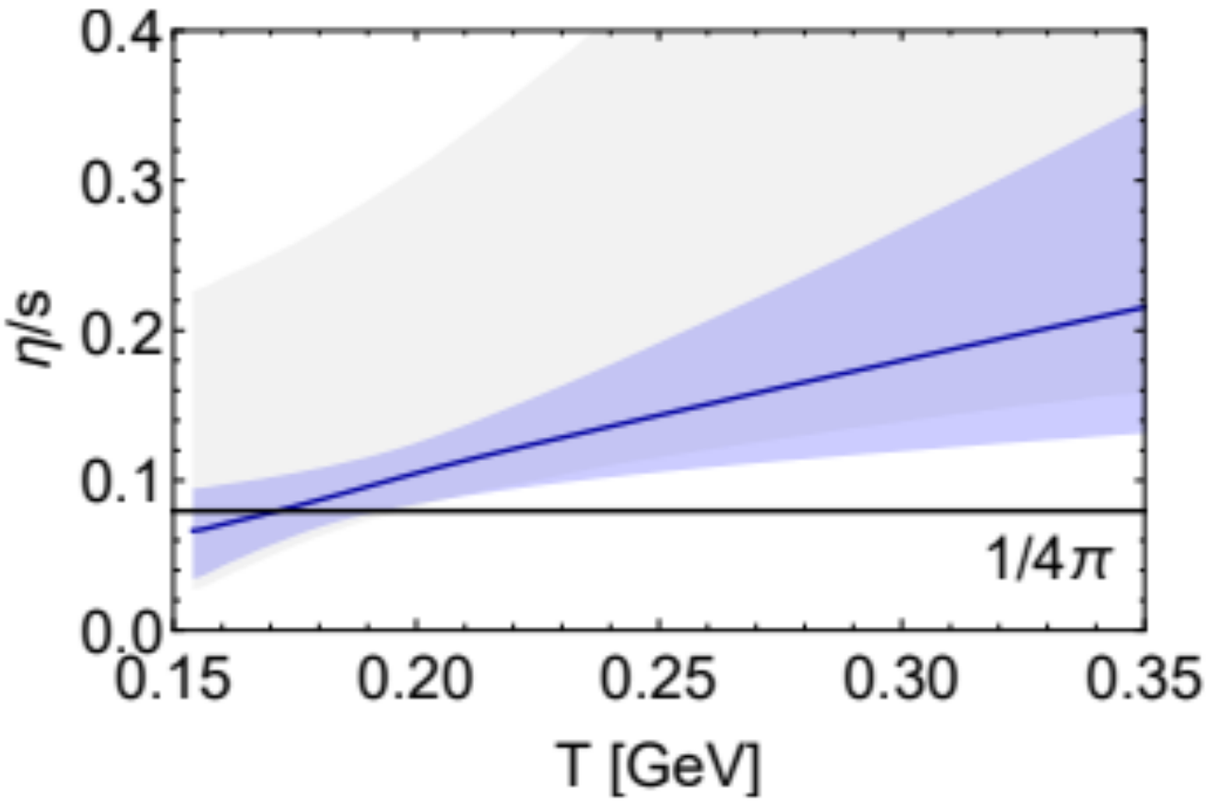


Particle momentum distribution in azimuthal plane

$$\rho(\phi) = \frac{1}{2\pi}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos(n\phi)\right]$$



Song, Bass, Heinz, Hirano, Shen, 1011.2783

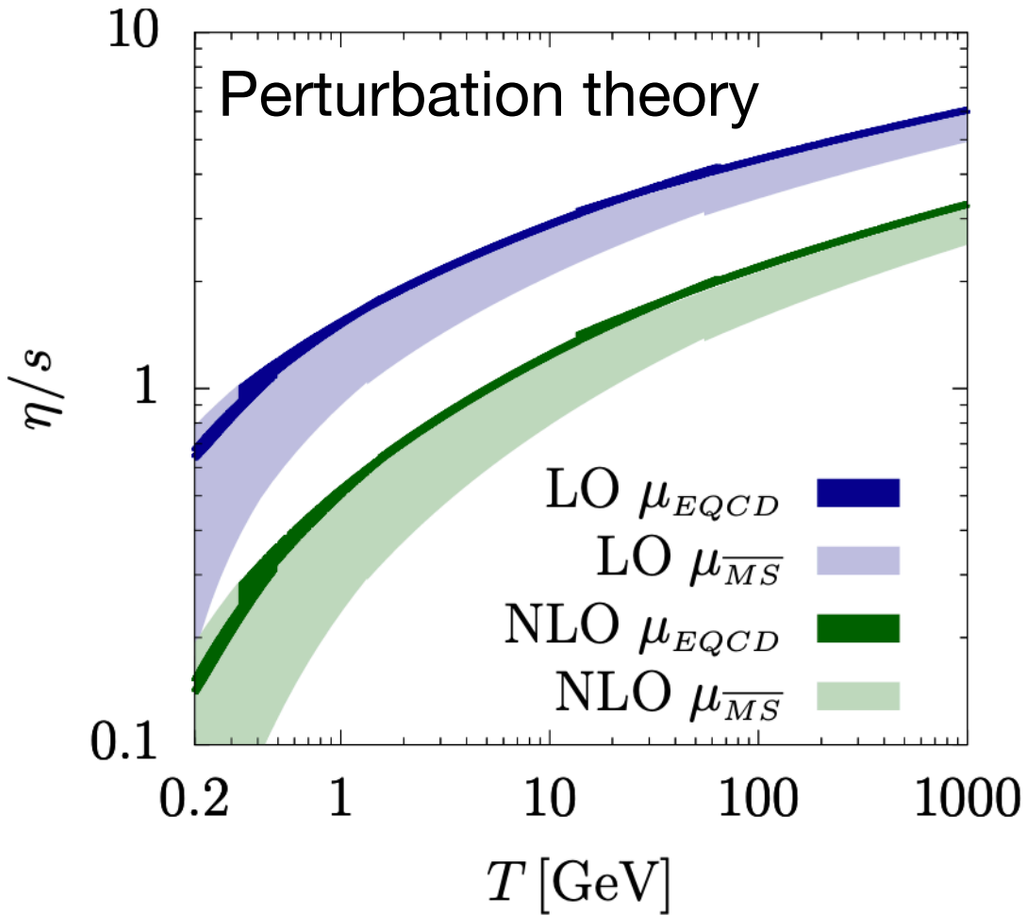


Nijs, van der Schee, Gursoy, Snellings, 2010.15130

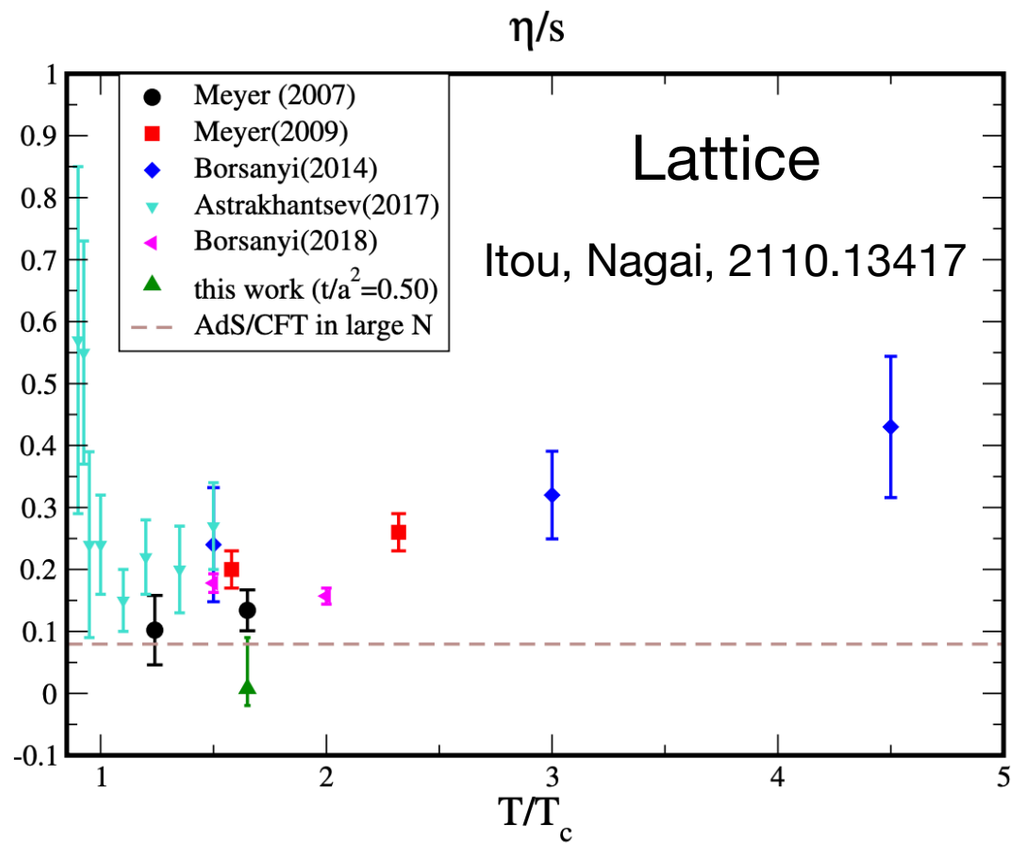
Open Questions in Hydrodynamic Description

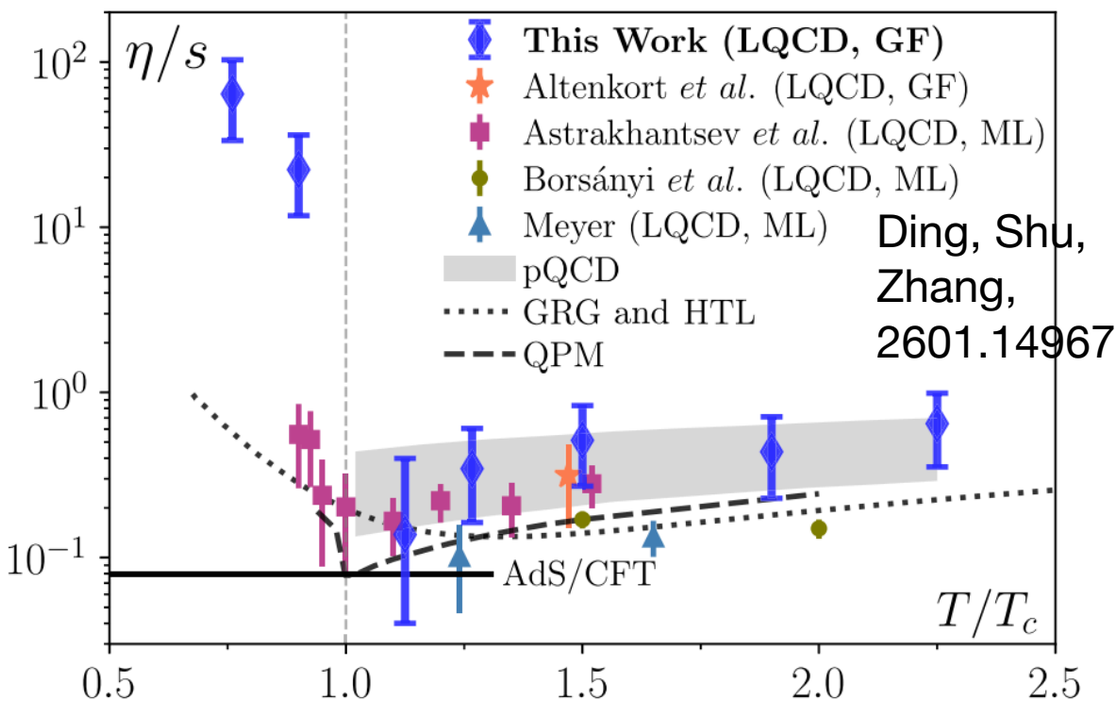
- **How nuclear matter hydrodynamizes (approximate *local* thermalization) rapidly (less than 1 fm/c) in heavy ion collisions?**

- **No reliable calculations of shear viscosity**



Ghiglieri, Moore, Teaney, 1802.09535





- **Goal: test hydro paradigm, understand hydrodynamization, and calculate transport coefficients for QCD**

Take 1+1D Ising Field Theory as Example

Study Hydrodynamics from Real-Time Lattice Simulation

1+1D Ising Field Theory

- **Defining action**

$$S_{\mathrm{IFT}} = S_{\mathrm{ICFT}} - \tau \int \mathrm{d}^2x\, \epsilon(x) - h \int \mathrm{d}^2x\, \sigma(x)$$

$$[\tau] = 1, \;\; [h] = 15/8 \qquad \xi = \frac{\tau}{|h|^{8/15}}$$

Delfino, Grinza, Mussardo, hep-th/0507133

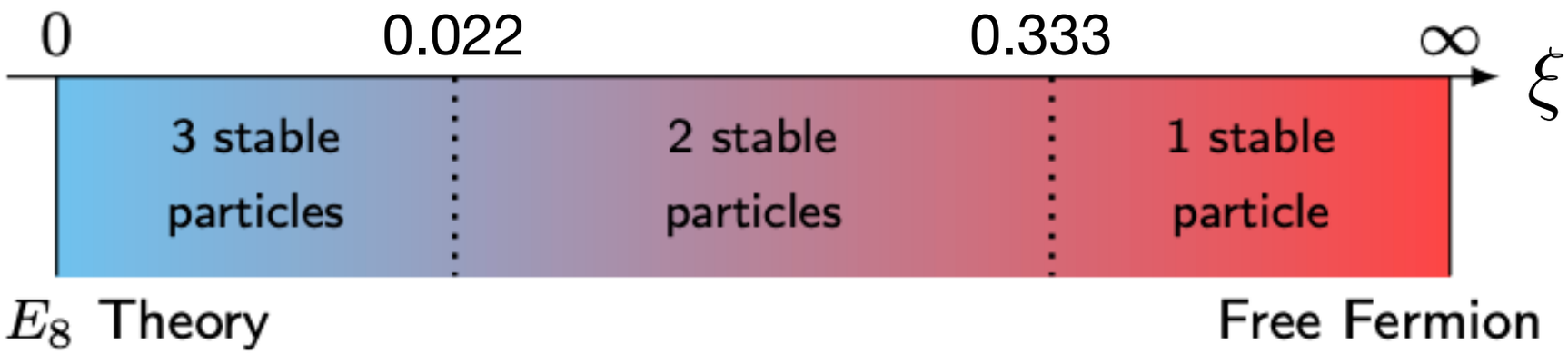


Image: Jha, Milsted, Neuenfeld, Preskill, Vieira, 2411.13645

- **Lattice formulation**

$$H = \sum_{i=0}^{L-1} \left(J\sigma_i^z \sigma_{i+1}^z + h_x \sigma_i^x + h_z \sigma_i^z \right)$$

$$J = -1 \qquad h_x = -1 - \xi_{\mathrm{lat}} |h_z|^{8/15} \qquad h_z \to 0^- \qquad |h_z|^{8/15} L \to \infty$$

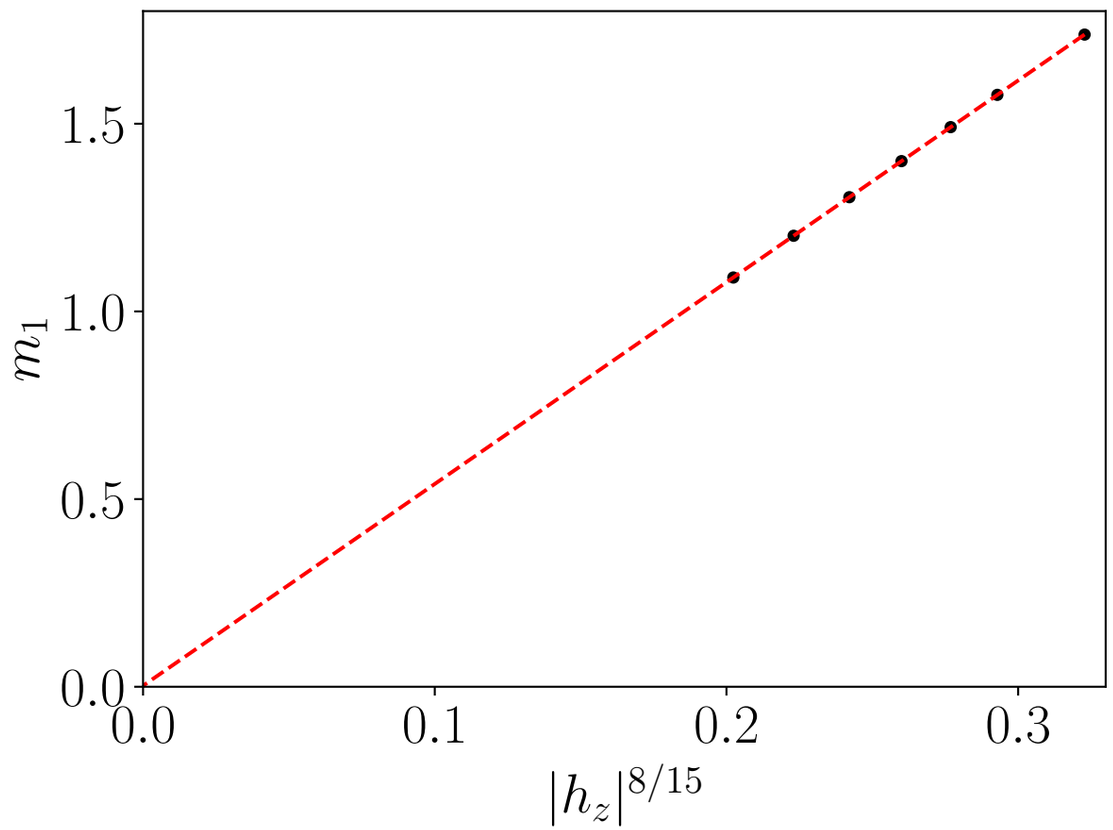
Continuum Limit

- **On line of constant physics**

$$m_1 = a\, m_1^{\rm phy}$$

$$a \propto |h_z|^{8/15}$$

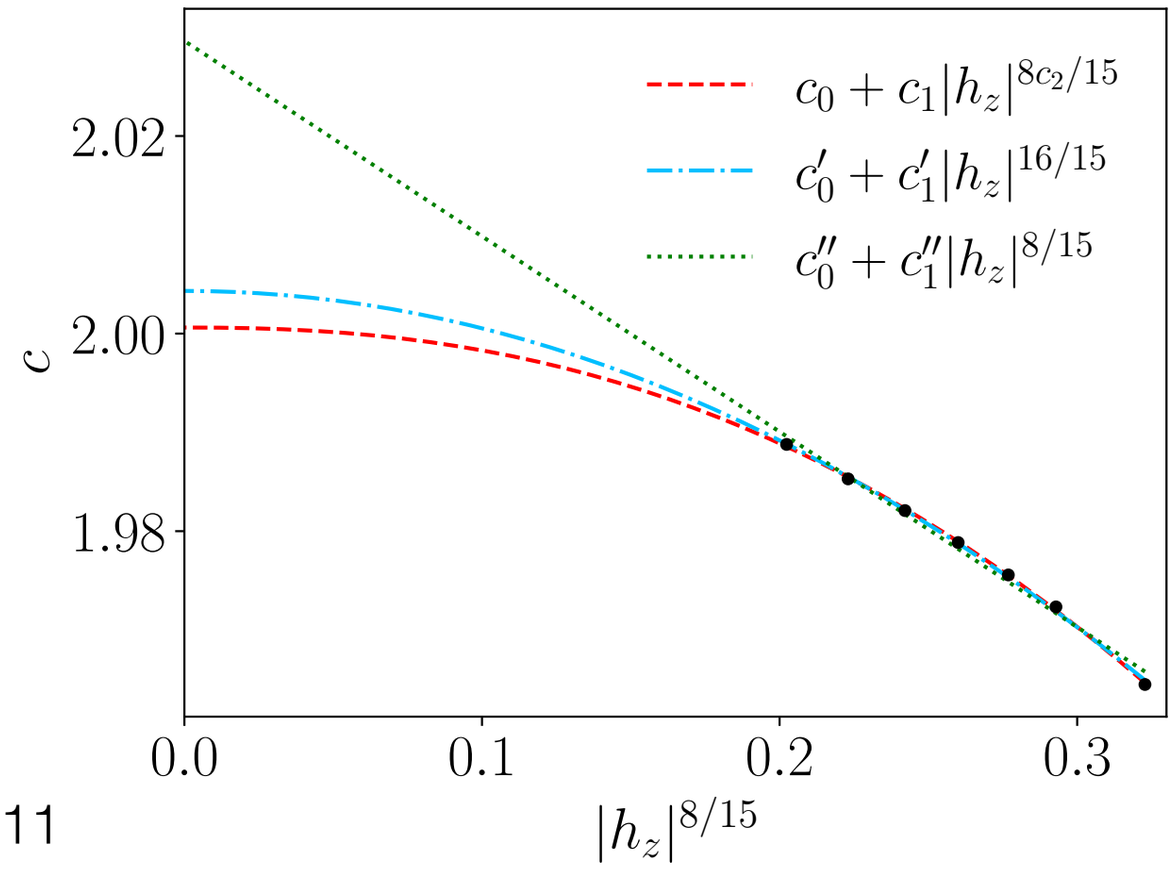
$$\frac{\mathrm{d}}{\mathrm{d}\ln a} \ln |h_z| = \frac{15}{8}$$



- **Speed of light**

$$[E_1(k)]^2 = m_1^2 + 4c^2 \sin^2(k/2)$$

If observable computed from the ratio of energy and momentum to the kth power, need multiplying $(1/c)^k$ or $(1/2)^k$



Correlation of Stress Energy Tensor and Hydro

- **Symmetric correlation function**

$$G_s^{\mu\nu}(t,x) = \mathrm{Tr}[\{T^{\mu\nu}(t,x), T^{\mu\nu}(0,0)\}\rho_T]$$

- **From non-equilibrium dynamics**

$$\rho(t=0) = \rho_T + T^{\mu\nu}(0)\rho_T + \rho_T T^{\mu\nu}(0)$$

$$\delta\langle T^{\mu\nu}(x)\rangle(t) = \mathrm{Tr}[T^{\mu\nu}(x)\rho(t)] - \mathrm{Tr}[T^{\mu\nu}(x)\rho_T] = G_s^{\mu\nu}(t,x)$$

- **Hydro description at late time** $\nabla_\mu T^{\mu\nu} = 0$

Linear expansion $T^{\mu\nu} = T_0^{\mu\nu} + \delta T^{\mu\nu}$

$$T_0^{\mu\nu} = \begin{pmatrix} \varepsilon_0 & 0 \\ 0 & P_0 \end{pmatrix} \qquad \delta T^{\mu\nu} = \begin{pmatrix} \delta\varepsilon & (\varepsilon_0 + P_0)\delta u^x \\ (\varepsilon_0 + P_0)\delta u^x & \delta P - \zeta\partial_x \delta u^x \end{pmatrix}$$

$$\partial_t \delta\varepsilon + \partial_x g^x = 0 \qquad \delta\varepsilon(t,x) = \delta\langle T^{00}(x)\rangle(t)$$

$$\partial_t g^x + c_s^2 \partial_x \delta\varepsilon - \gamma_\zeta \partial_x^2 g^x = 0 \qquad g^x(t,x) = \delta\langle T^{01}(x)\rangle(t)$$

Hydro in 1+1D ?

- **Solution to linearized hydro**

$$g^x(t,k) = g^x(t=0,k)\cos(c_s k t)e^{-\frac{\gamma_\zeta k^2 t}{2}} - ic_s\delta\varepsilon(t=0,k)\sin(c_s k t)e^{-\frac{\gamma_\zeta k^2 t}{2}}$$

$$\gamma_\zeta = \frac{\zeta}{\varepsilon_0 + P_0} \qquad\qquad c_s \gg \gamma_\zeta k$$

- **Divergence in bulk viscosity due to fluctuations**

$$\zeta = \frac{1}{4T}\lim_{\omega\to 0} G_S^{11}(\omega, k=0) = \frac{1}{4T}\int \mathrm{d}t \int \mathrm{d}x \langle\{\delta T^{11}(t,x), \delta T^{11}(0,0)\}\rangle_T$$

With quadratic terms in stress:

$$\zeta \propto \lim_{\omega\to 0}\int \mathrm{d}t\, e^{i\omega t}\int \mathrm{d}k\, e^{-\gamma_\zeta k^2 t} = \lim_{\omega\to 0}\int \mathrm{d}t\, e^{i\omega t}\frac{1}{\sqrt{\gamma_\zeta t}}$$

- **No hydrodynamics in 1+1D ?**

Let’s compute using real-time lattice simulation

Stress Energy Tensor on Lattice

- **Energy density**

$$T^{00}(0,i) \to H_i = \frac{J}{2}\left(\sigma^z_{i-1}\sigma^z_i + \sigma^z_i\sigma^z_{i+1}\right) + h_x\sigma^x_i + h_z\sigma^z_i$$

- **Energy flux/momentum density**

$$\partial_t H_i(t) = e^{iHt}(i[H_{i+1}, H_i] - i[H_i, H_{i-1}])\, e^{-iHt}$$

$$\partial_t T^{00}(t,x) = -\partial_x T^{10}(t,x)$$

$$T^{10}(0,i) \to J_i = -i[H_{i+1}, H_i] = Jh_x(\sigma^z_{i+1}\sigma^y_i - \sigma^z_i\sigma^y_{i+1})$$

- **Recover momentum conservation**

$$[H, \sum_{i=0}^{L-1} J_i] = -2iJh_xh_z \sum_{i=0}^{L-1}(\sigma^z_{i+1}\sigma^x_i - \sigma^z_i\sigma^x_{i+1})$$

$$\begin{matrix} h_z \to 0^- \\ |h_z|^{8/15}L \to \infty \end{matrix} \longrightarrow \; || \, [H, J_{\text{tot}}] \, ||_2 \leq 4|Jh_xh_z|L \to 0$$

Umklapp Processes on Lattice

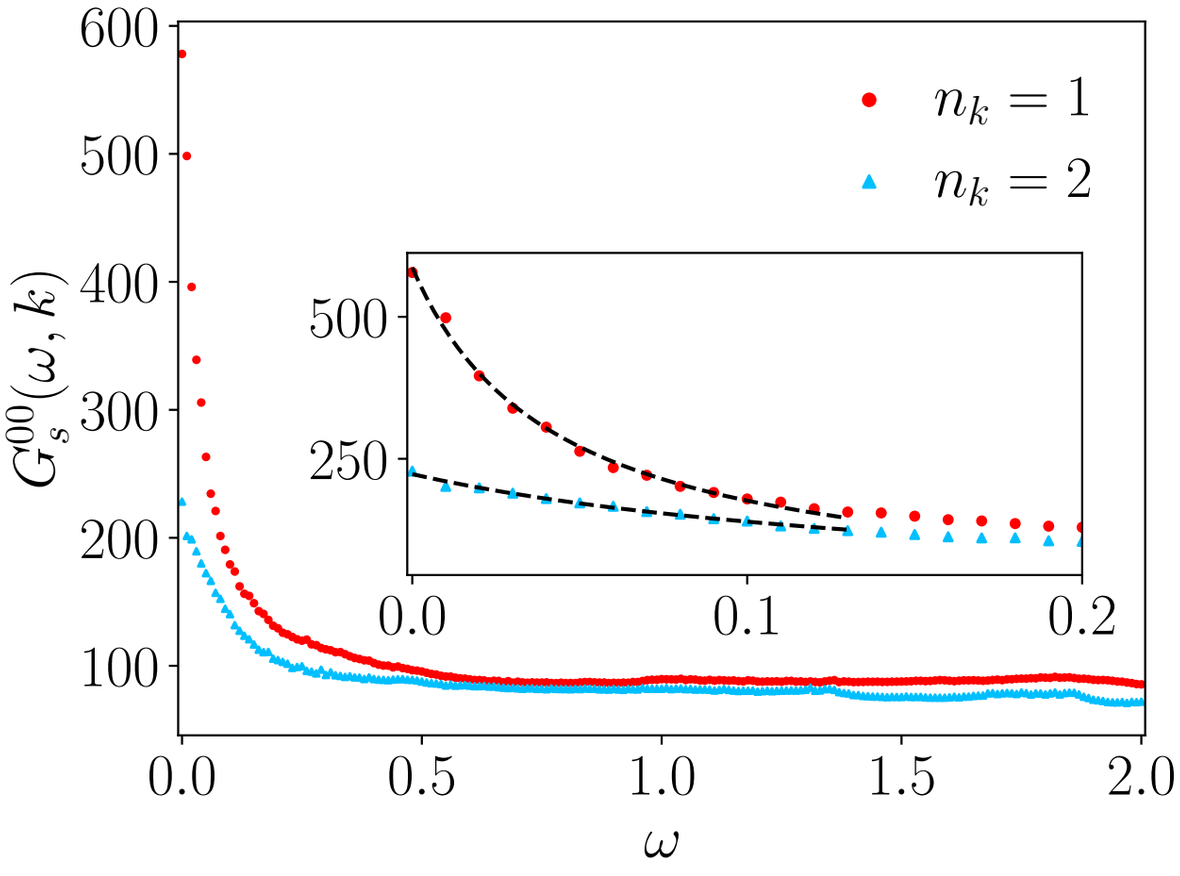
- **Umklapp processes** $k_1 + k_2 = k_1' + k_2' + n_b \frac{2\pi}{a}$

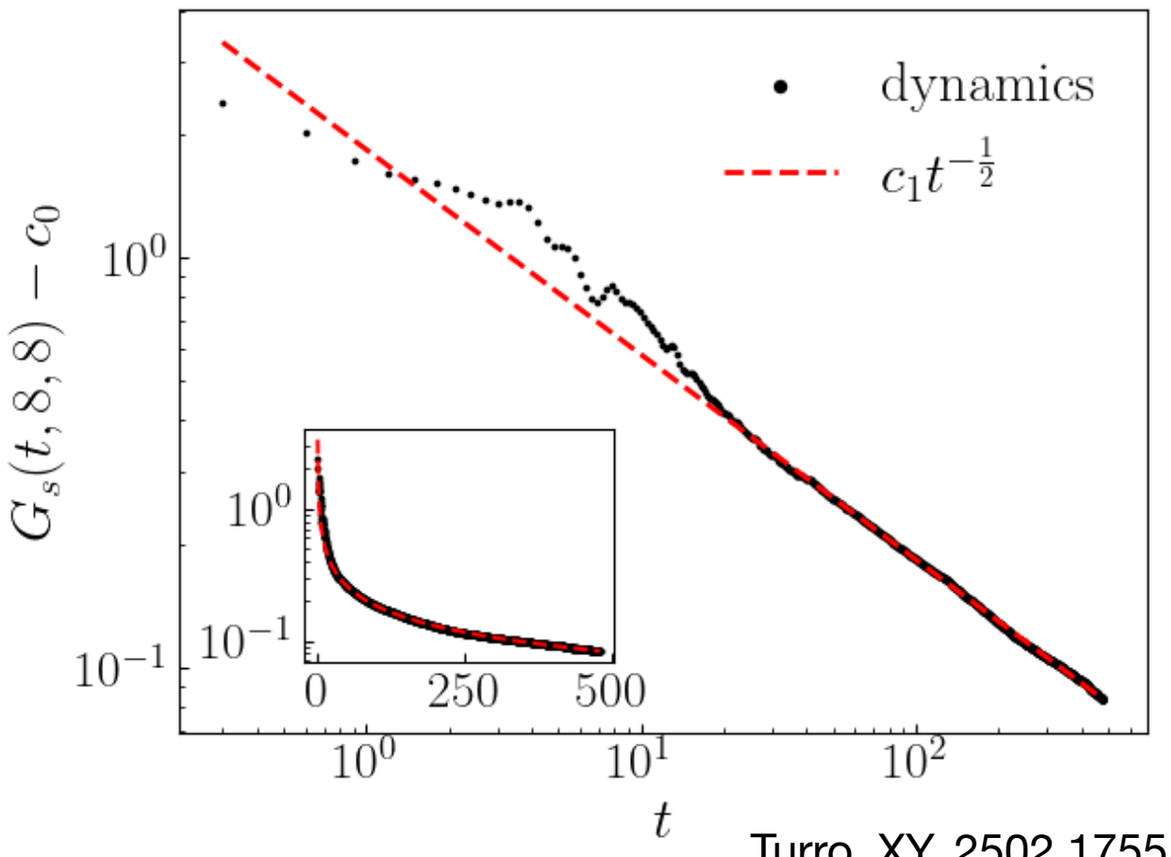
  $n_b \neq 0$: break continuum momentum conservation

- **Sound modes —> diffusive mode**

$$\partial_t \delta\varepsilon - D_\varepsilon \partial_x^2 \delta\varepsilon = 0$$

Arnold, Yaffe, hep-ph/9709449

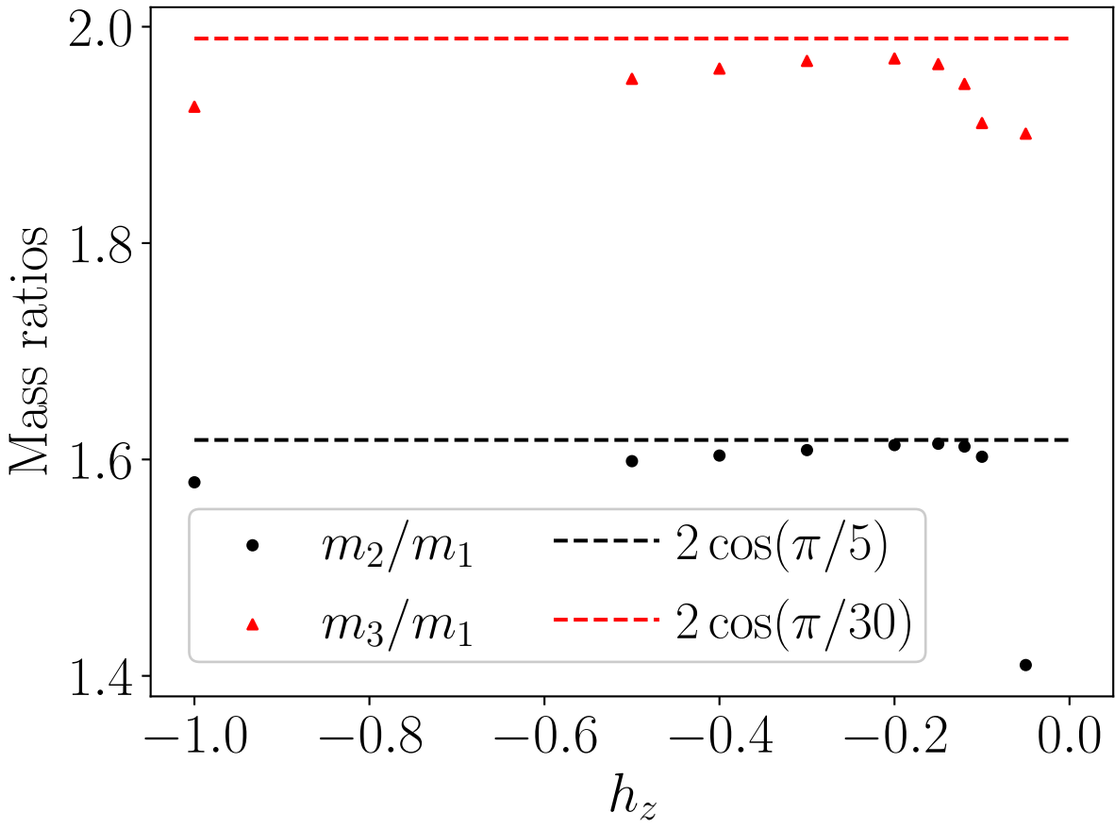


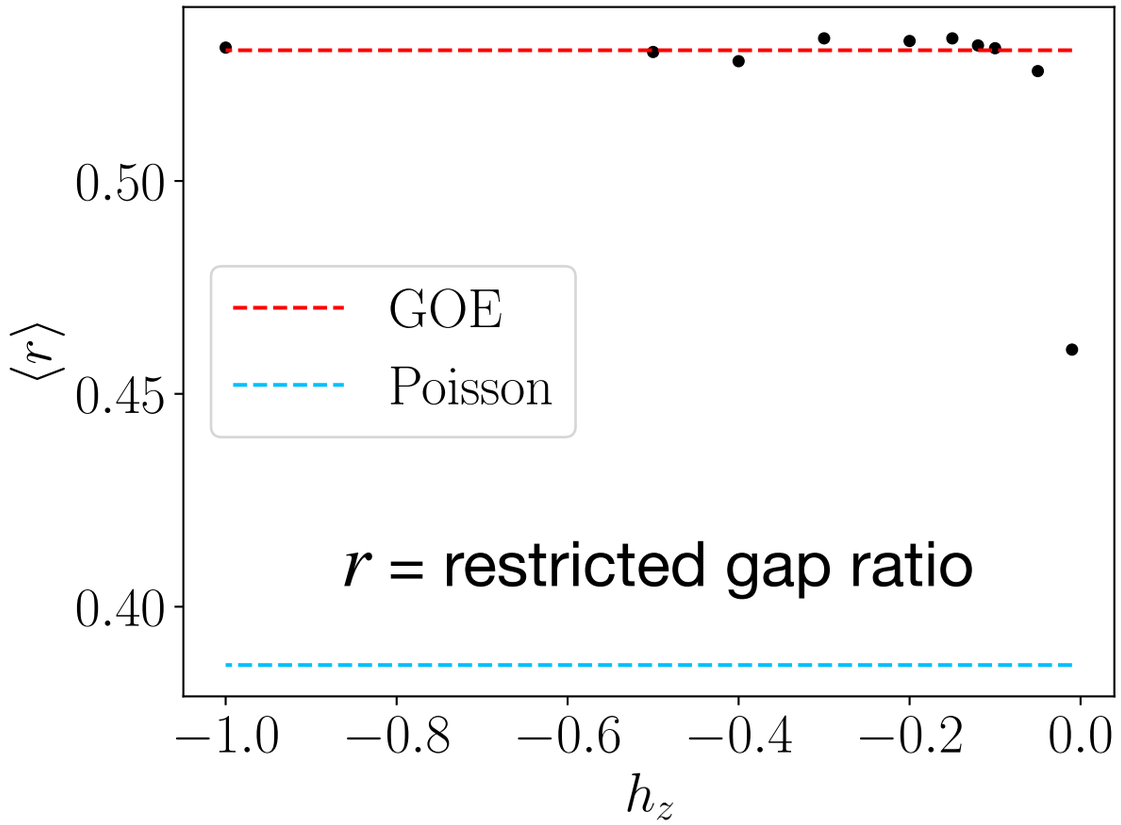


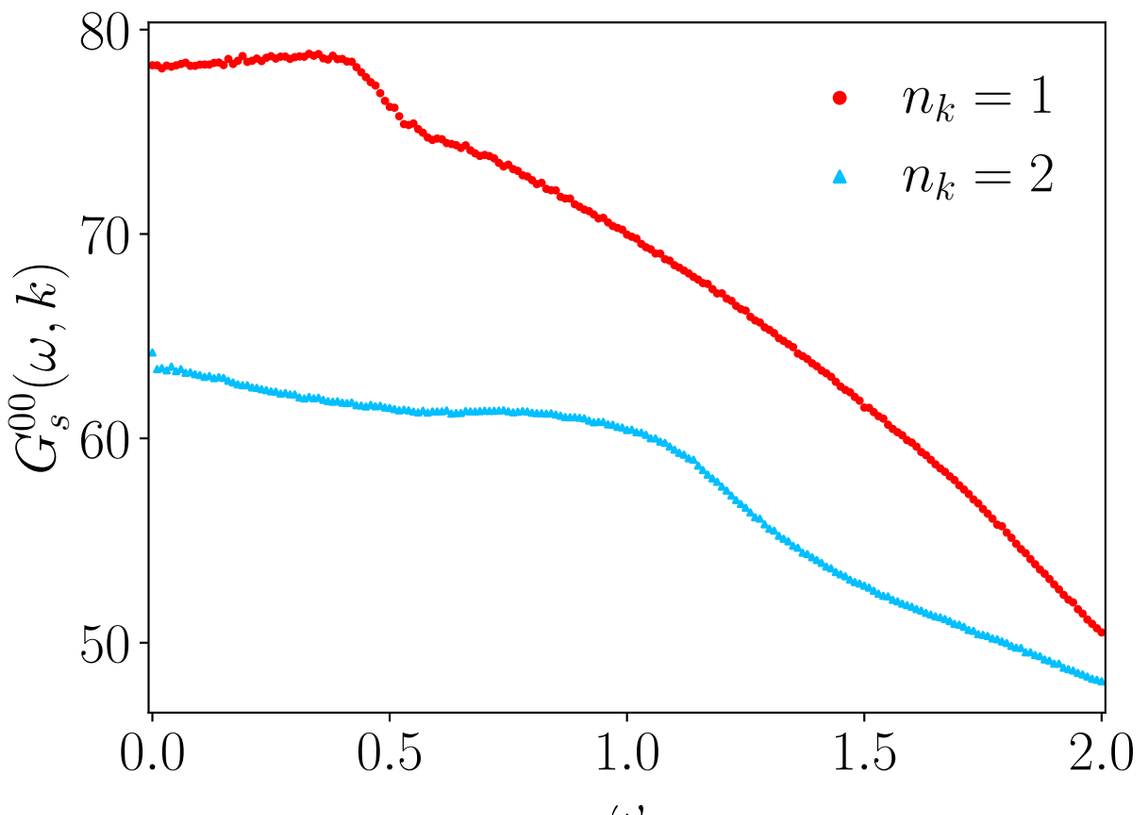
Turro, XY, 2502.17551

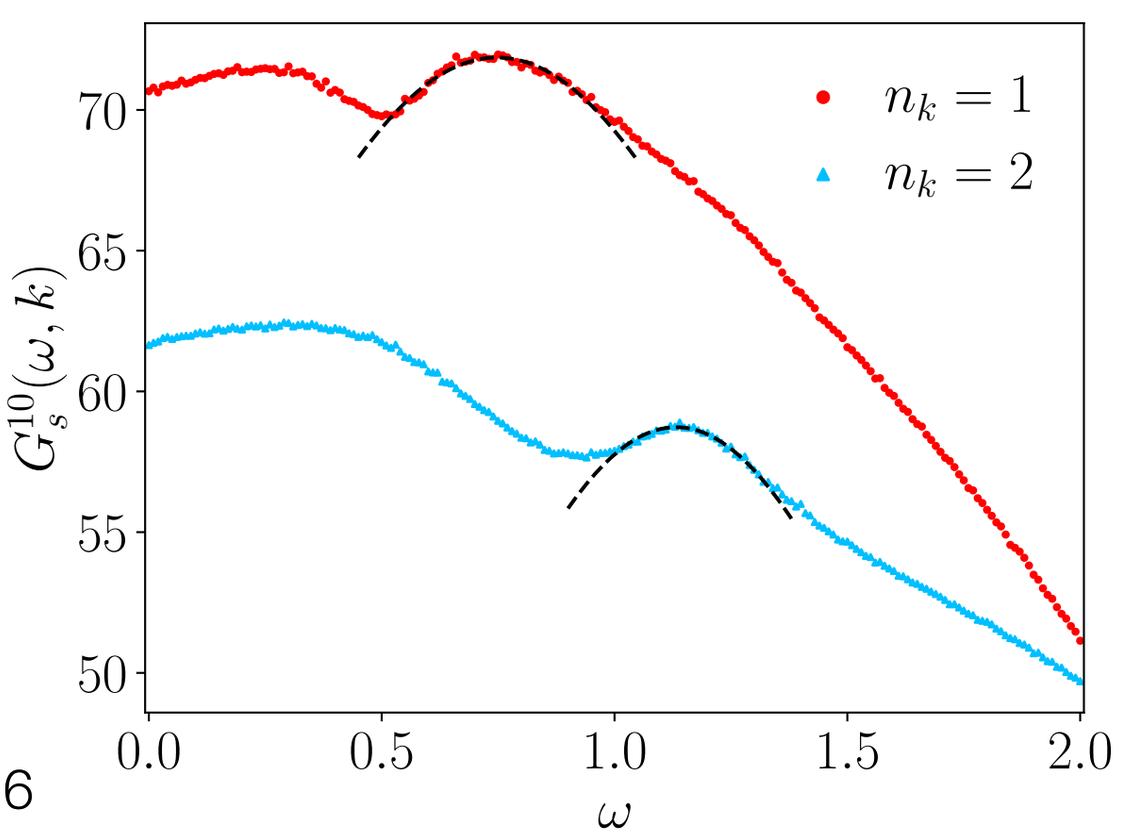
- **Expect suppression in continuum limit**

Scaling Region of Lattice Theory ($L = 20$ PBC)





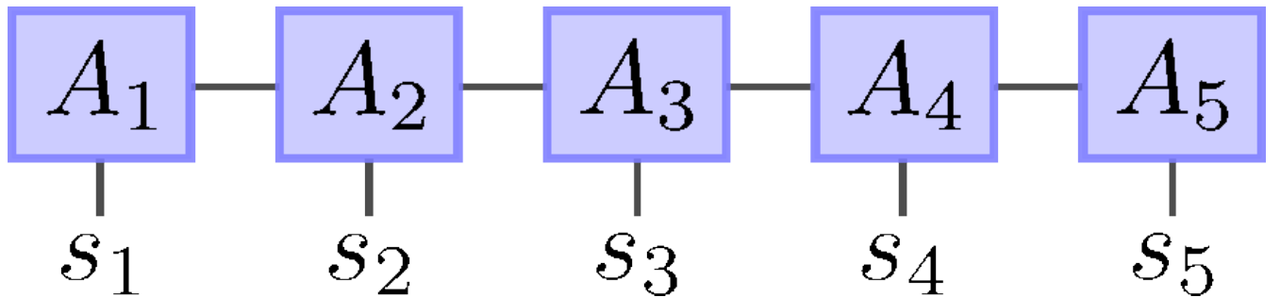




MPS Calculation

- **Matrix product state**

$$|\psi\rangle = \psi_{s_1 s_2 \ldots s_N} |s_1 s_2 \ldots s_N\rangle$$



$$|\psi\rangle \to A_1^{(s_1)} A_2^{(s_2)} \cdots A_N^{(s_N)} |s_1 s_2 \ldots s_N\rangle$$

Cost: $d^N \to N \cdot d \cdot D^2$ Bond dimension upper limit: D, cutoff: χ

- **Symmetric correlation function from MPS**

Thermal state preparation: imaginary time evolution, TEBD $I \to e^{-\beta T}$

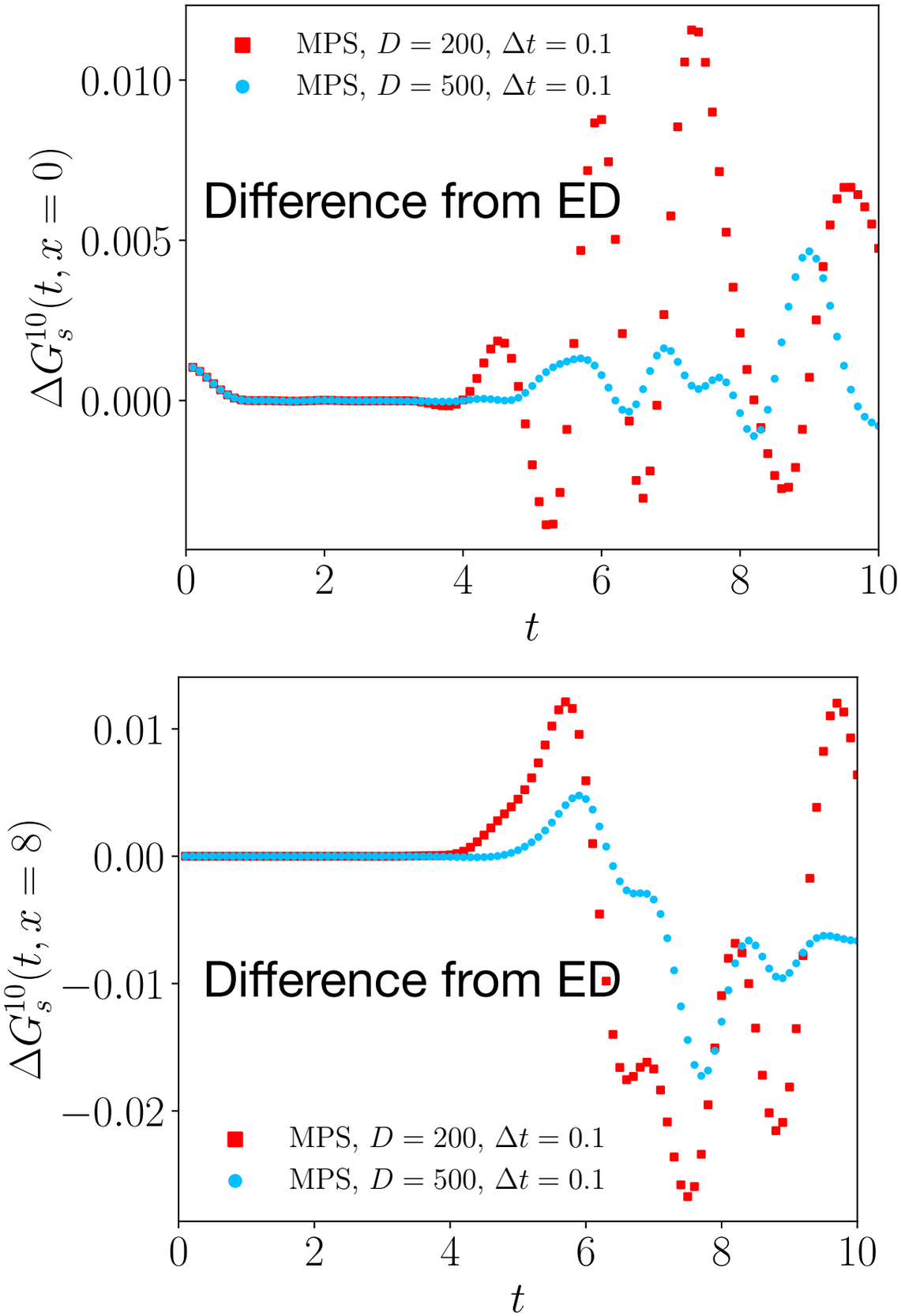
Apply perturbation $e^{\Delta T^{\mu\nu}(0)}$ $\Delta = \pm 0.01$

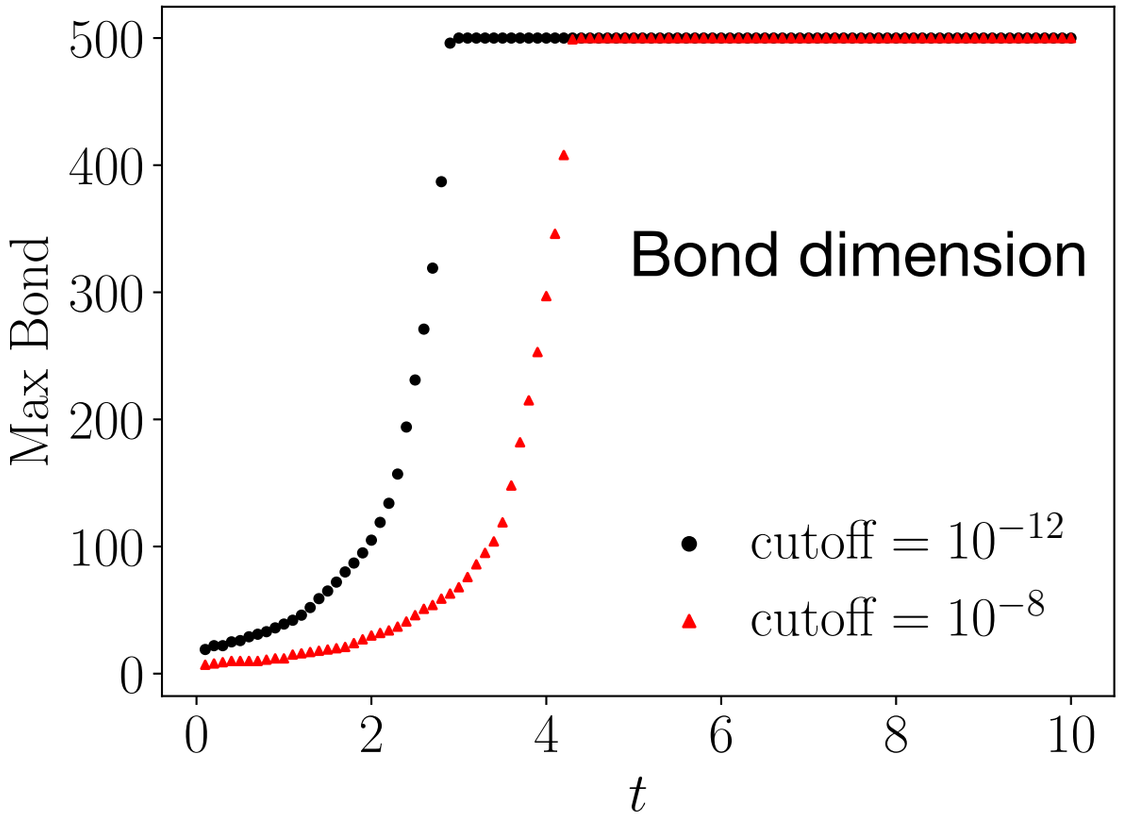
Time evolution: TDVP $e^{-iHt} e^{\Delta T^{\mu\nu}(0)} \rho_T e^{\Delta T^{\mu\nu}(0)} e^{iHt}$

Measure $T^{\mu\nu}(x)$ at time t

$$\langle T^{10}(x)\rangle_{\Delta}(t) - \langle T^{10}(x)\rangle_{-\Delta}(t) = 2\Delta G_s^{10}(t,x) + O(\Delta^3)$$

MPS Calculation v.s. Exact Diag on $L = 16$ PBC

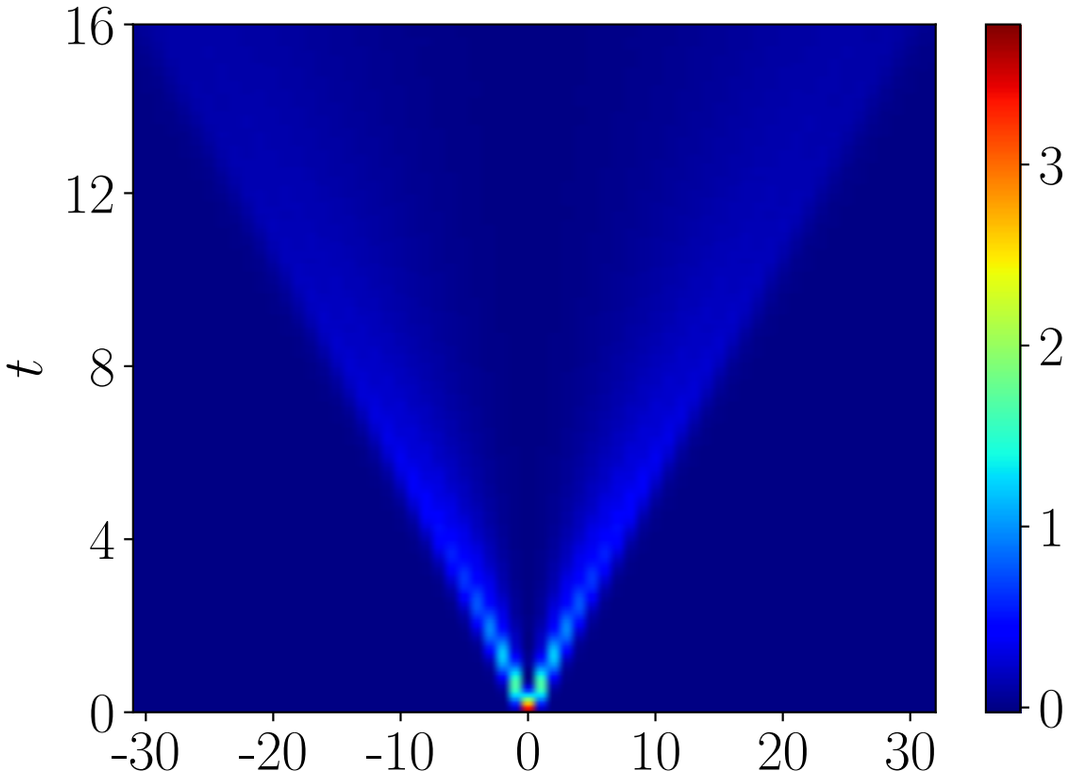


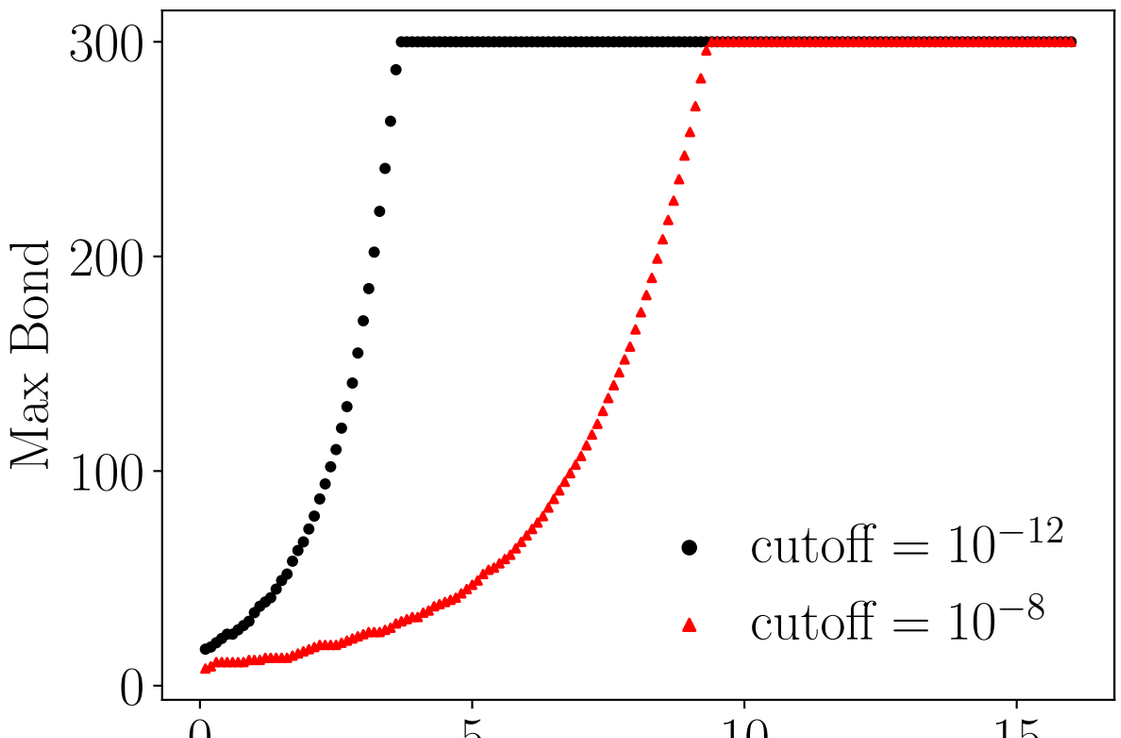


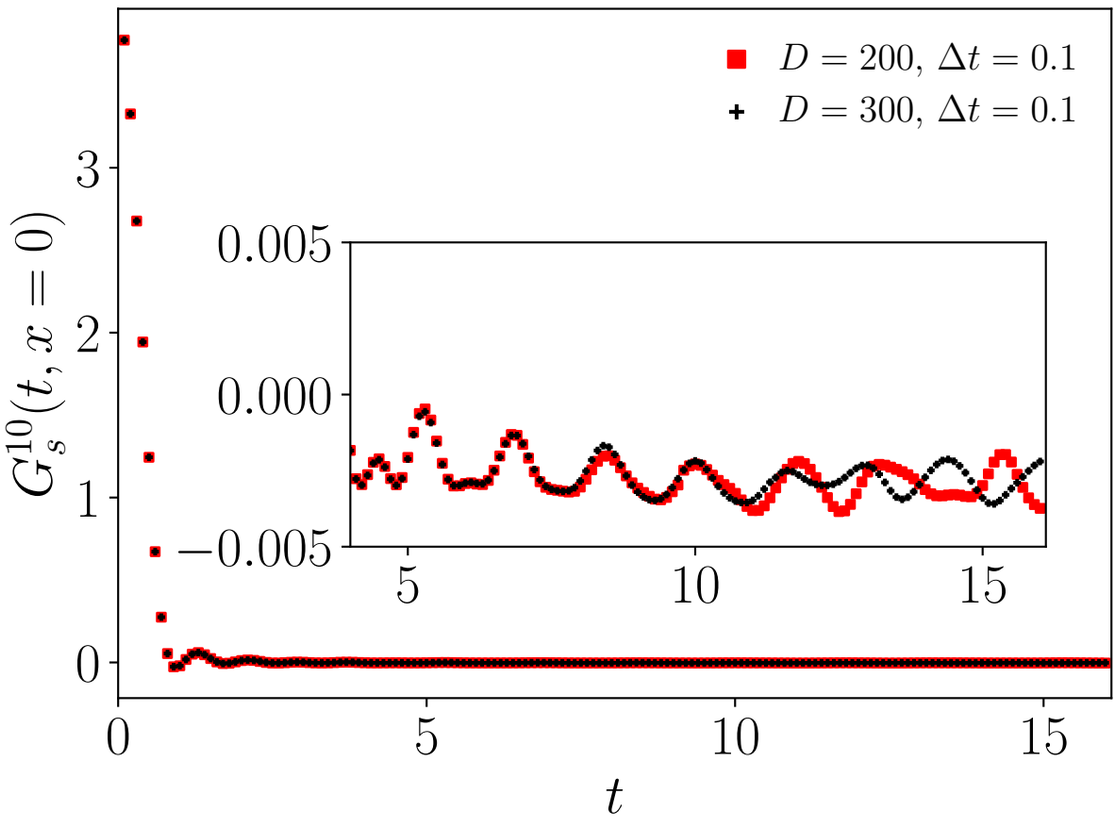
Perturbation crosses boundary (periodic) at $t \approx 5$

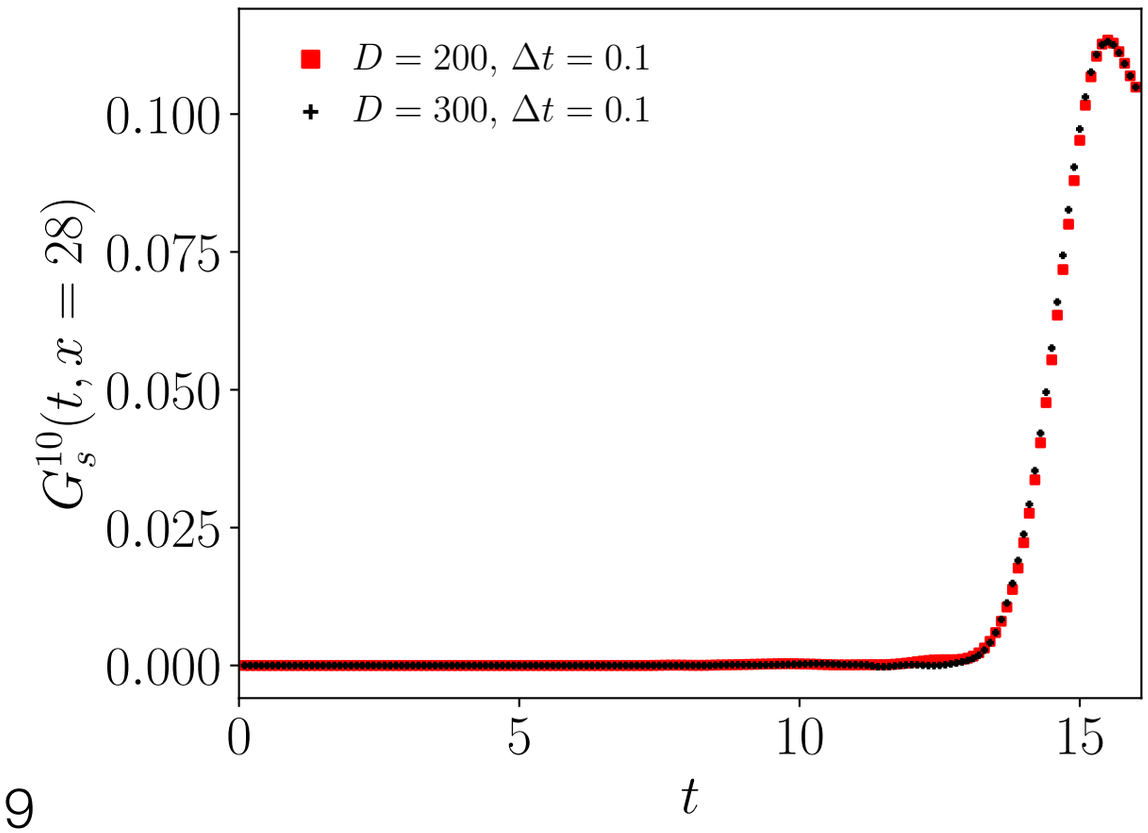
Before this, accuracy is reasonable

MPS Calculation $h_z = -0.08, T/m_1 \approx 7.14$

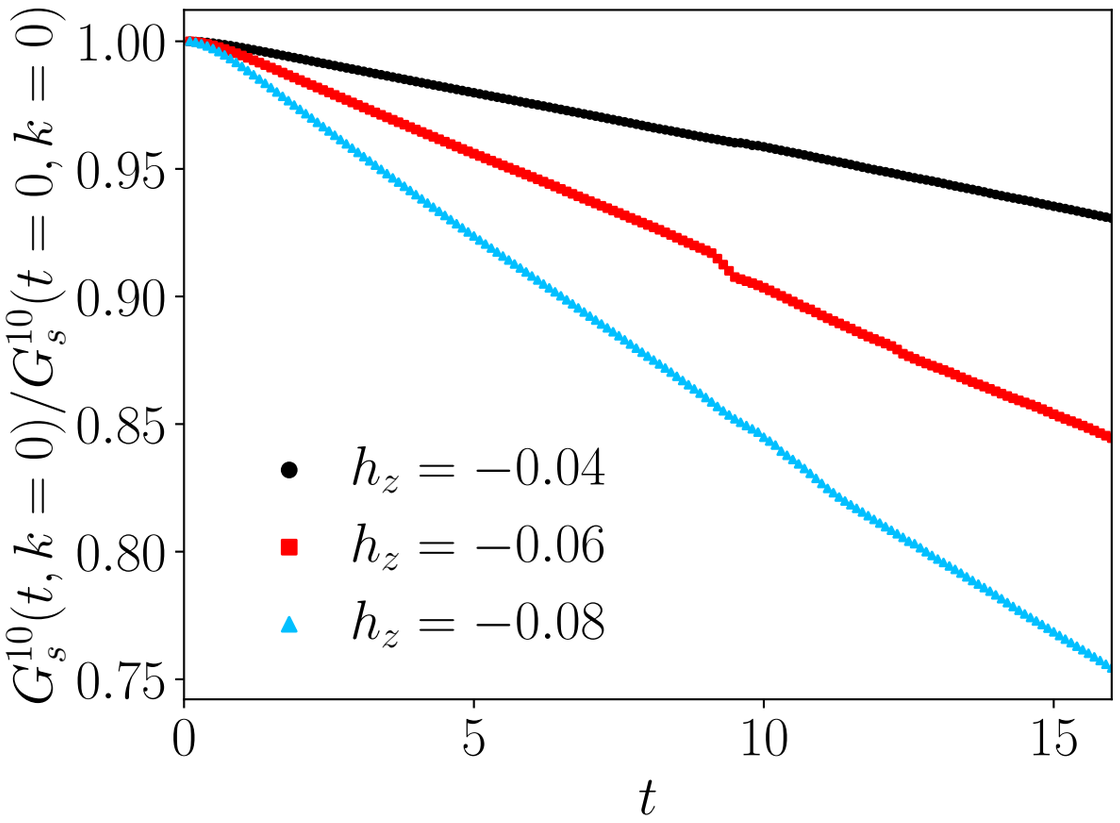


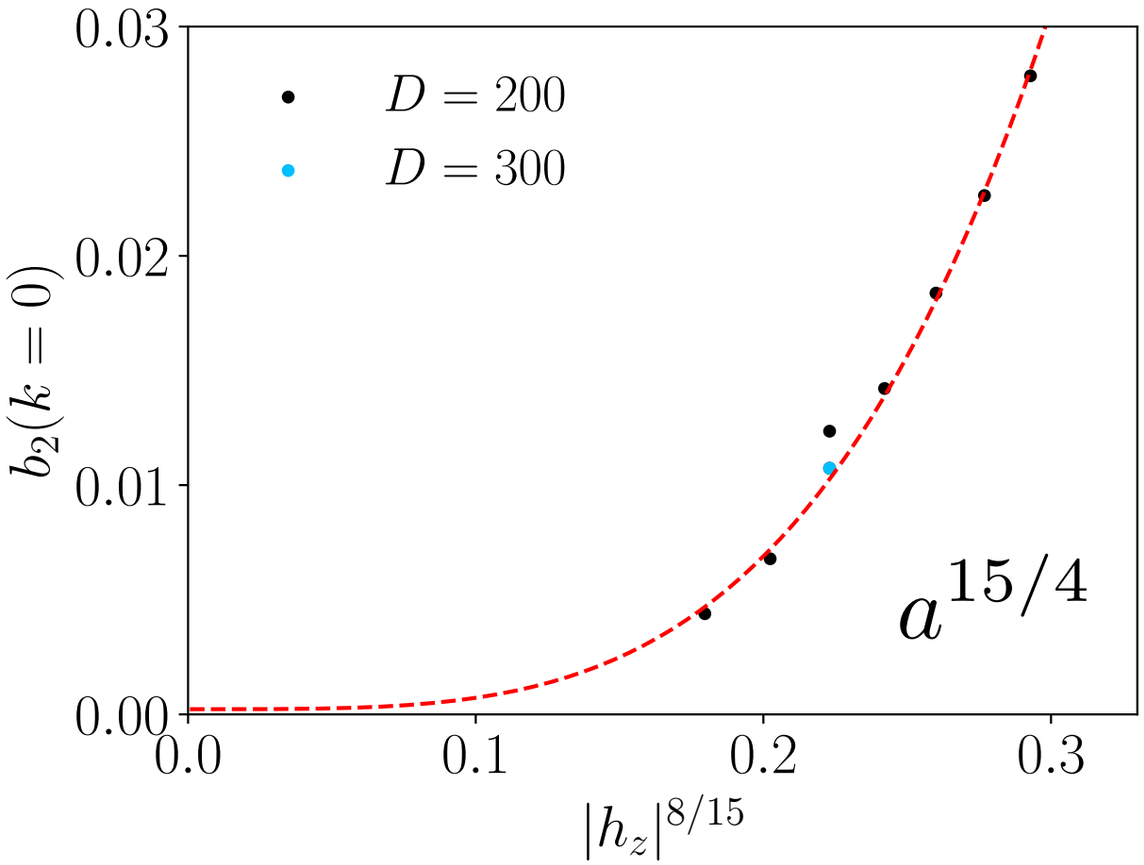






Recover Continuum Momentum Conservation





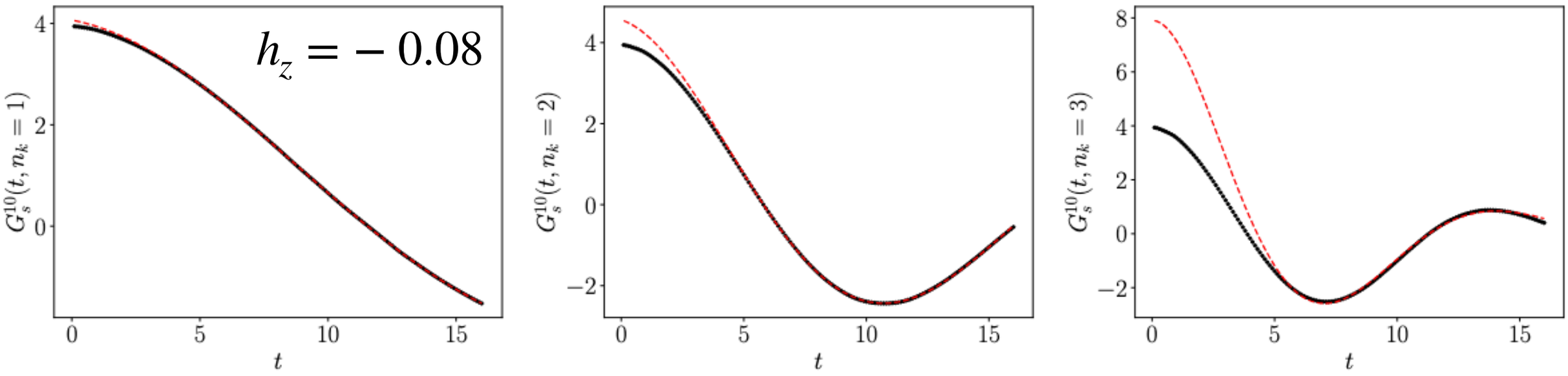
$$|| \, [H, J_{\text{tot}}] \, ||_2 \leq 4|Jh_x h_z|L \to 0$$

$$J_i = Jh_x(\sigma_{i+1}^z \sigma_i^y - \sigma_i^z \sigma_{i+1}^y)$$

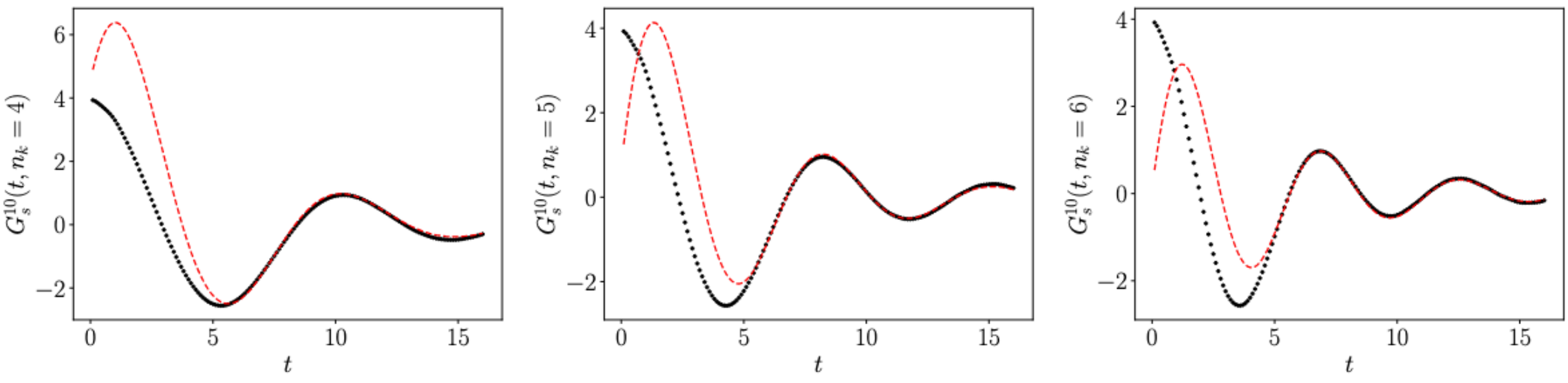
Invariant under overall spin flip, leading correction h_z^2

Fitting Sound Modes

$$G_s^{10}(t,k) = \sum_{x=0}^{L-1} e^{ikx} G_s^{10}(t,x) \qquad b_0 \cos(b_1 t + b_3) e^{-b_2 t}$$



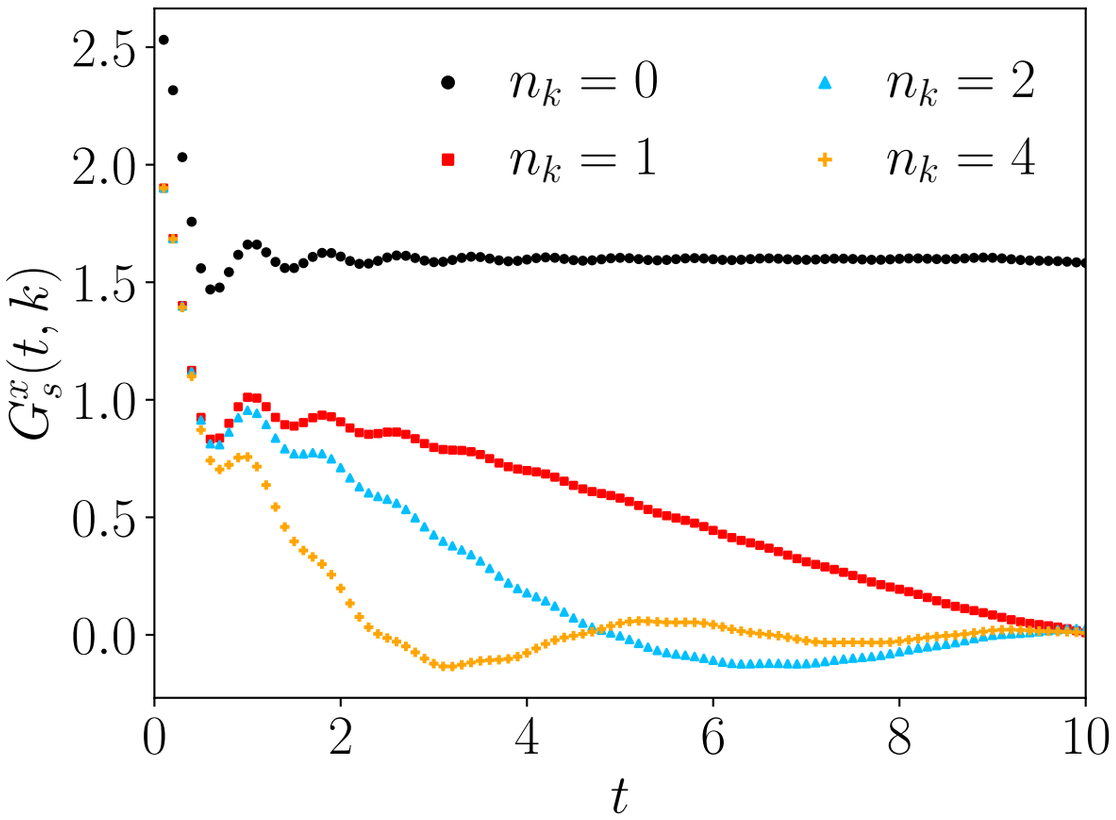
(a) $n_k = 1$. (b) $n_k = 2$. (c) $n_k = 3$.



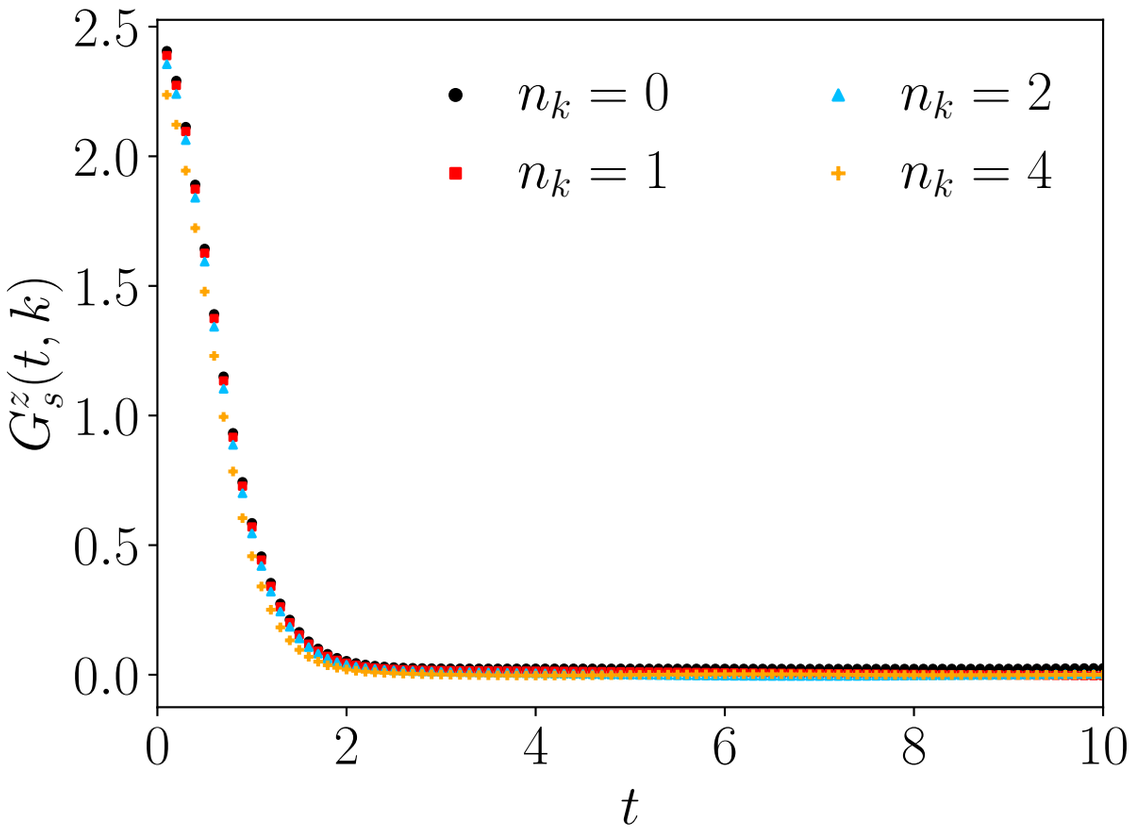
(d) $n_k = 4$. (e) $n_k = 5$. (f) $n_k = 6$.

Not Genuine Spin Dynamics on Lattice

$$\langle\{X(t,x),X(0,0)\}\rangle_T$$



$$\langle\{Z(t,x),Z(0,0)\}\rangle_T$$

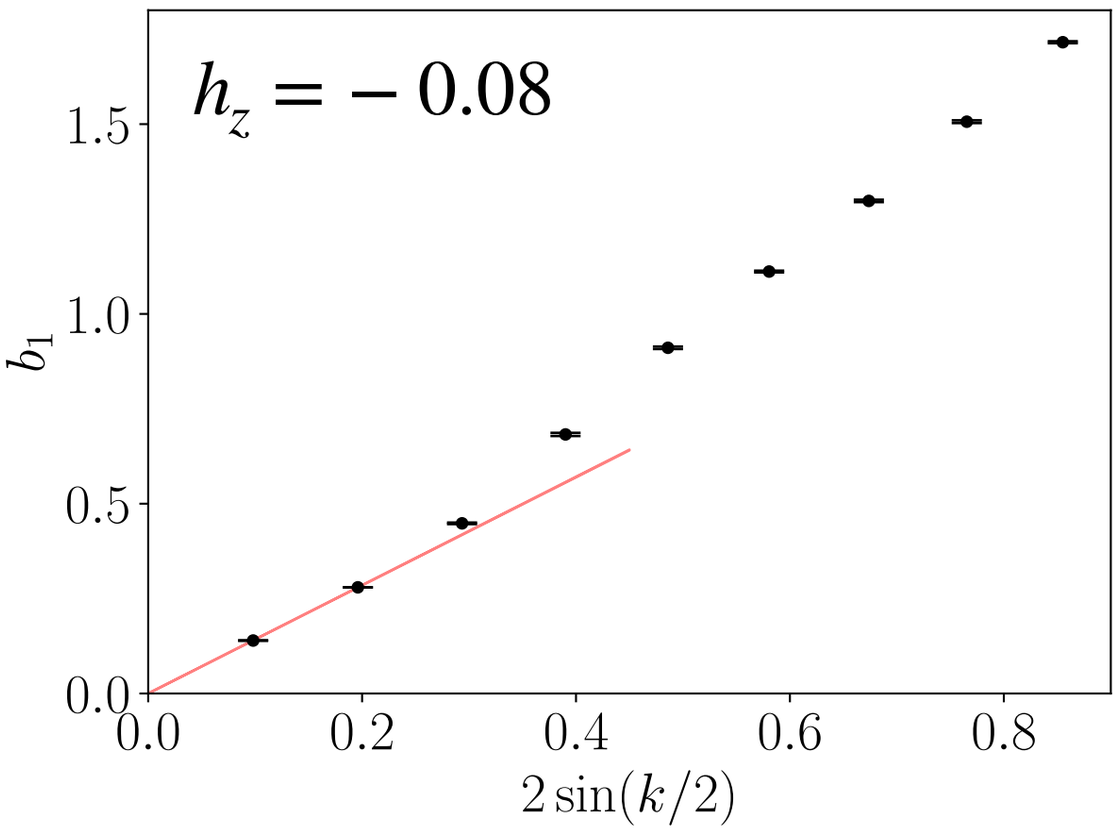


What we observed on previous slide ≠ genuine spin dynamics on the lattice

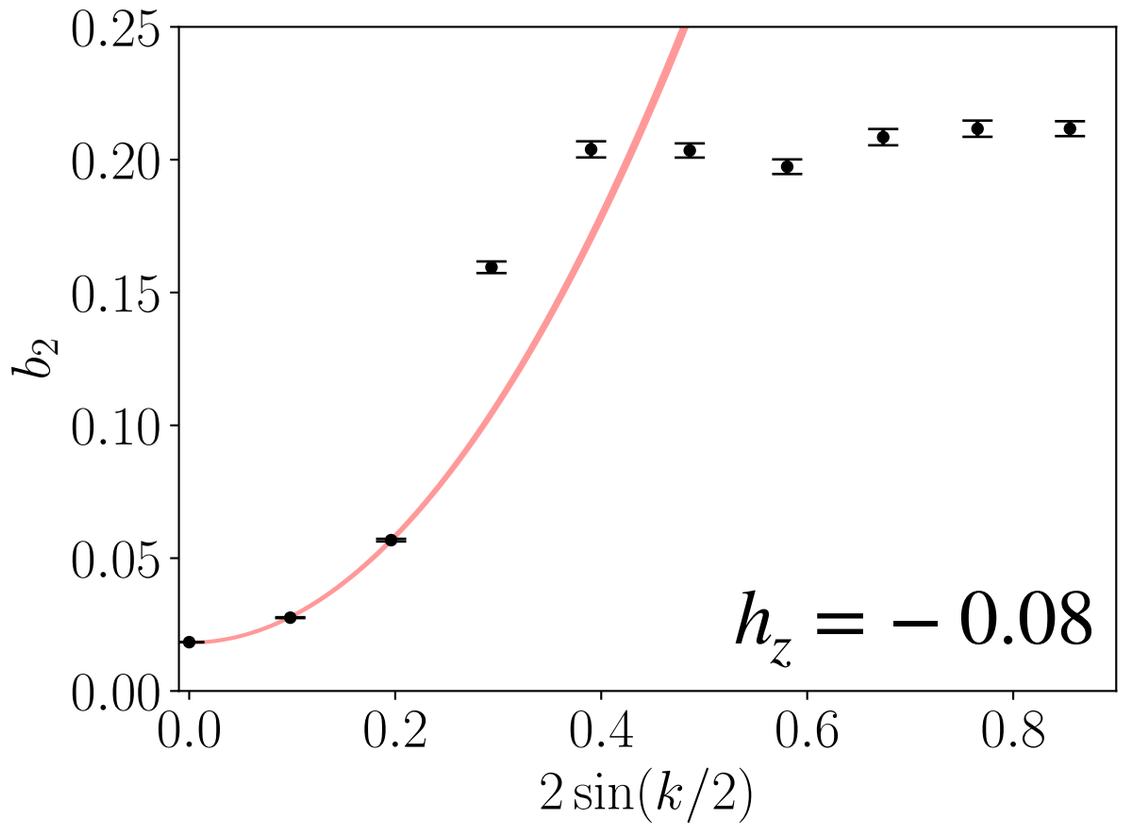
Check Scalings of Sound Modes

$$b_0 \cos(b_1 t + b_3) e^{-b_2 t}$$

$$b_1 = c_s \, 2\sin(k/2)$$

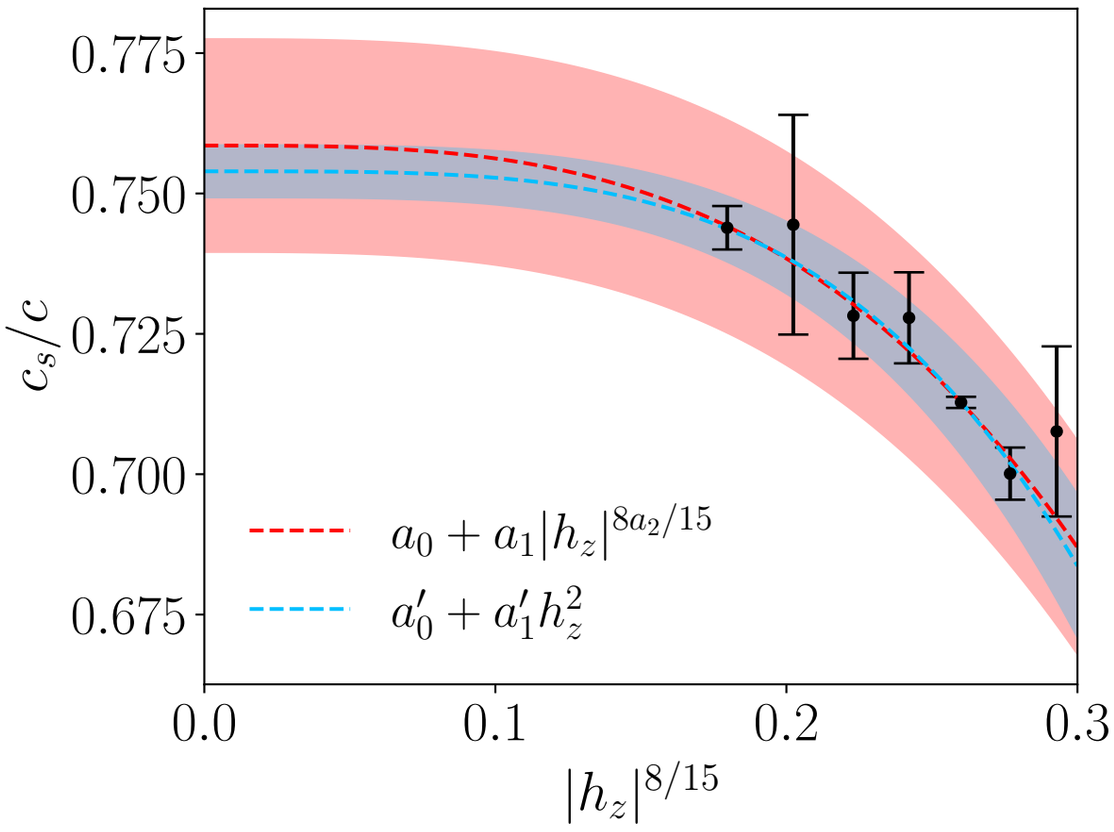


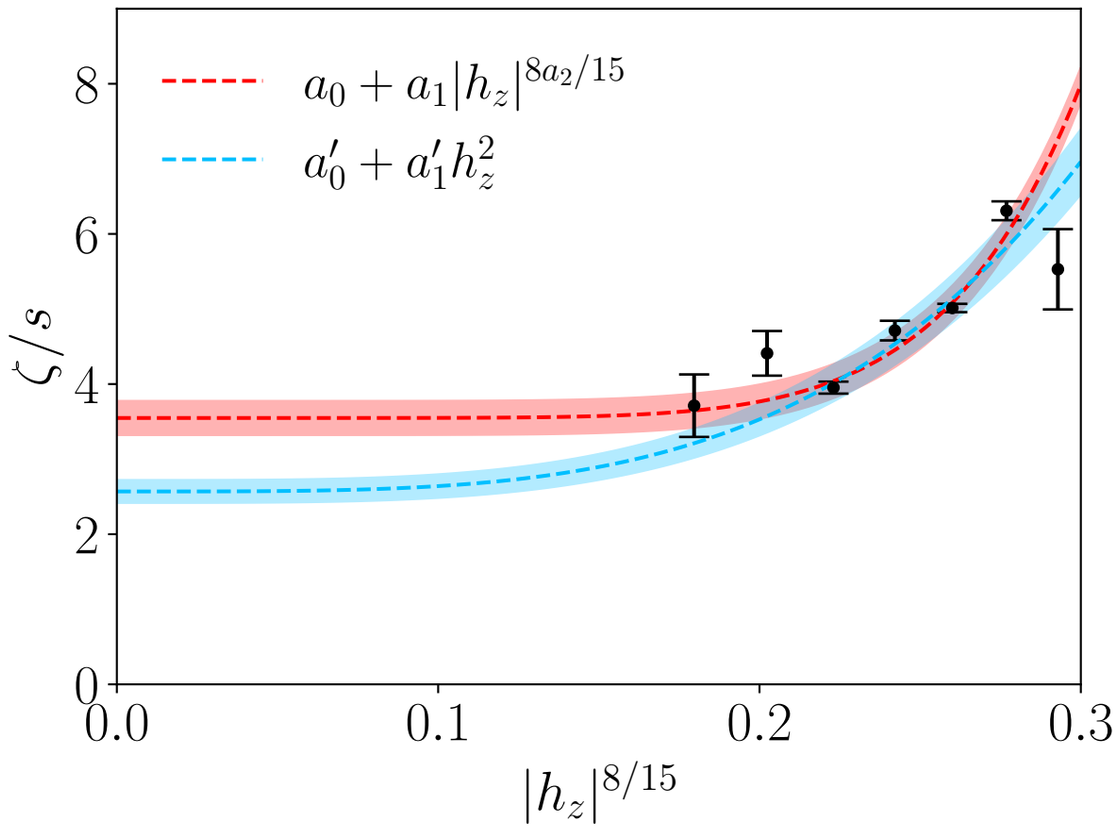
$$b_2 = \Gamma_0 + 2\tilde{\gamma}_\zeta \sin^2(k/2)$$



$$\mathrm{Kn} \equiv \frac{\tilde{\gamma}_\zeta \sin(k/2)}{c_s} \approx 0.0688,\ 0.138,\ 0.206$$

Continuum Limit





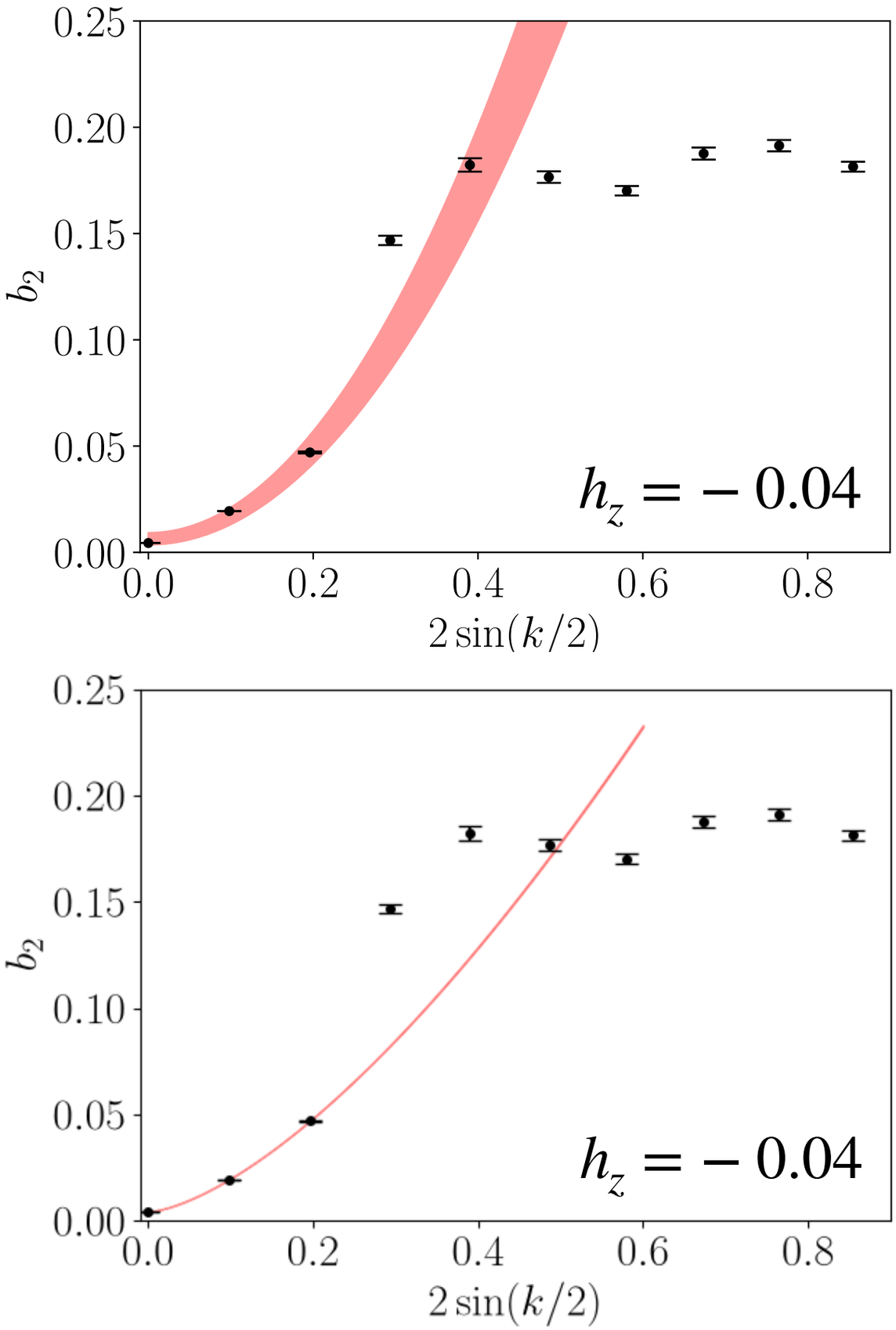
Parameters	c_s/c	ζ/s
a_0	0.759 ± 0.019	3.548 ± 0.225
a_1	-3.099 ± 7.365	$(3.361 \pm 5.984) \times 10^4$
a_2	3.130 ± 2.061	7.419 ± 1.433
a_0'	0.754 ± 0.005	2.569 ± 0.150
a_1'	-6.420 ± 0.734	401.257 ± 25.598

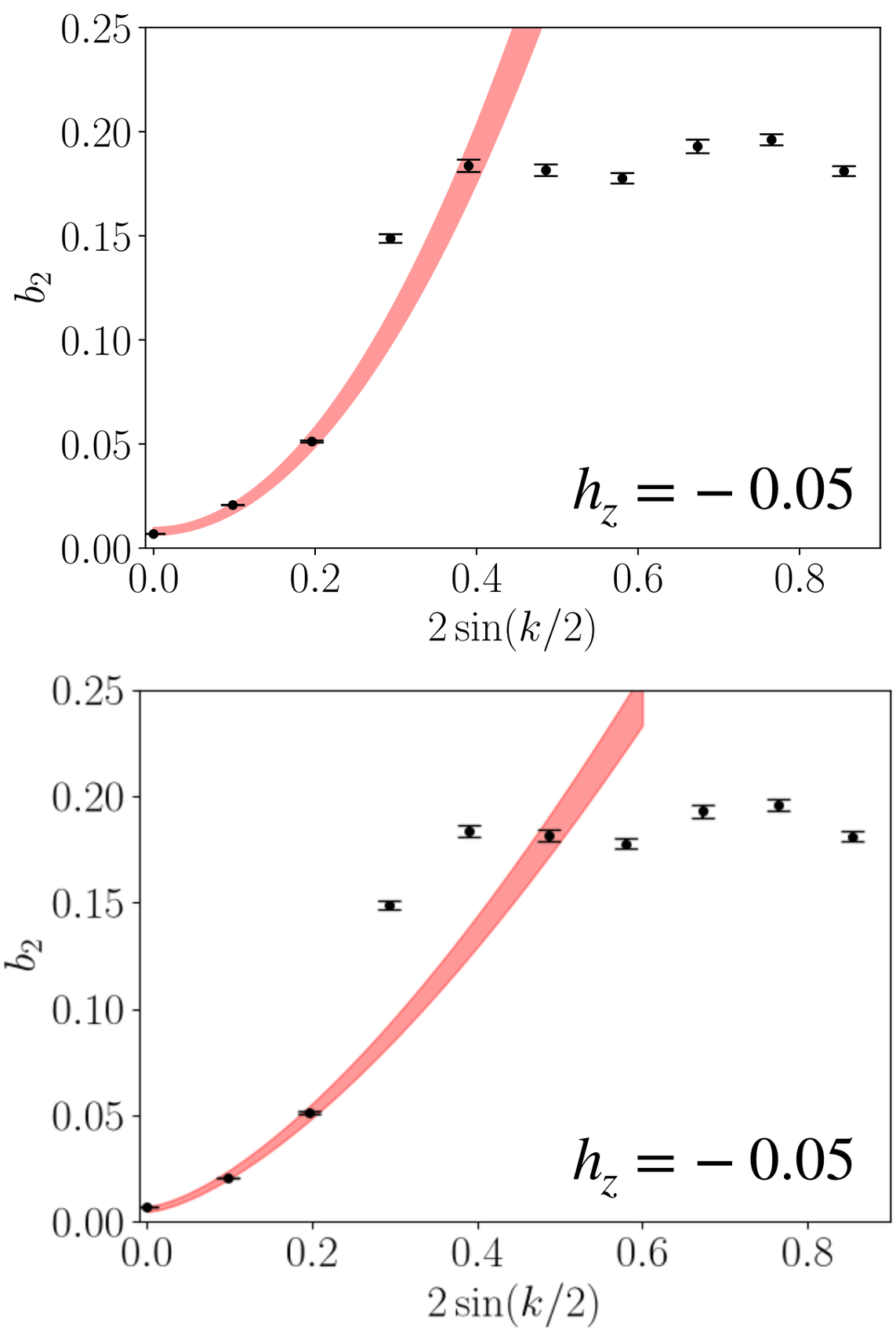
c_s close to thermal estimate

Large ζ/s, weakly-coupled scalar gas?

Big uncertainty: hint of hydro breakdown?

$k^{3/2}$ Scaling in Damping Rate?





Summary

- Viscosity key transport property of QGP

- Real-time lattice computation for hydrodynamization and viscosity

- 1+1D Ising field theory: MPS, hydrodynamization (before reaching boundary), sound modes (scaling of damping?)

- Ultimate goal: QCD; near-term: U(1), SU(2), pure gauge, lower dimensions, etc