Cohesive induced hysteresis and breakdown of marginal stability in jammed granular materials

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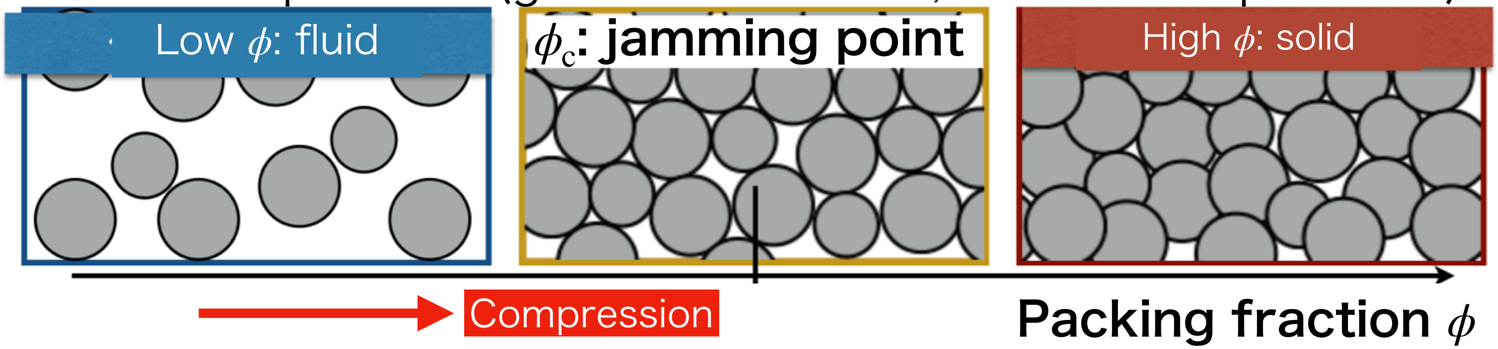
Jamming in frictionless repulsive grains

▸ Jamming: emergence of rigidity

Athermal particles (granular materials, colloidal suspensions)



Linear repulsive force

Pressure: $p \propto (\phi - \phi_c) \geq 0$

Shear modulus:

$G \propto (\phi - \phi_c)^{1/2} \geq 0$

Coordination number:

$Z - Z_{iso} \sim (\phi - \phi_c)^{1/2}$, ($Z_{iso} = 2D$)

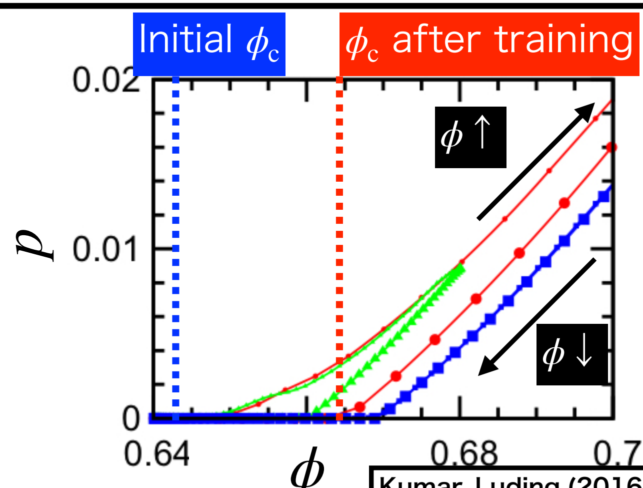
▸ Hysteresis in ϕ_c: protocol dependence

- **Annealing process**

  Chaudhuri et al., PRL (2010), Ozawa, et al., PRL (2012)
- **Mechanical training**

  Kumar, Luding, Granular Matter(2016), Kawasaki, Miyazaki, PRL (2024)

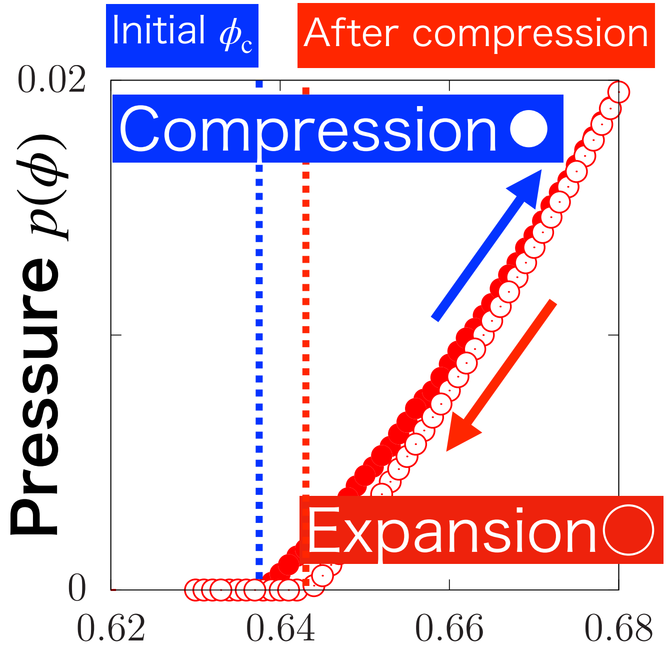
→ p, G, and $Z - Z_{iso}$ at a fixed ϕ exhibit Hysteresis.

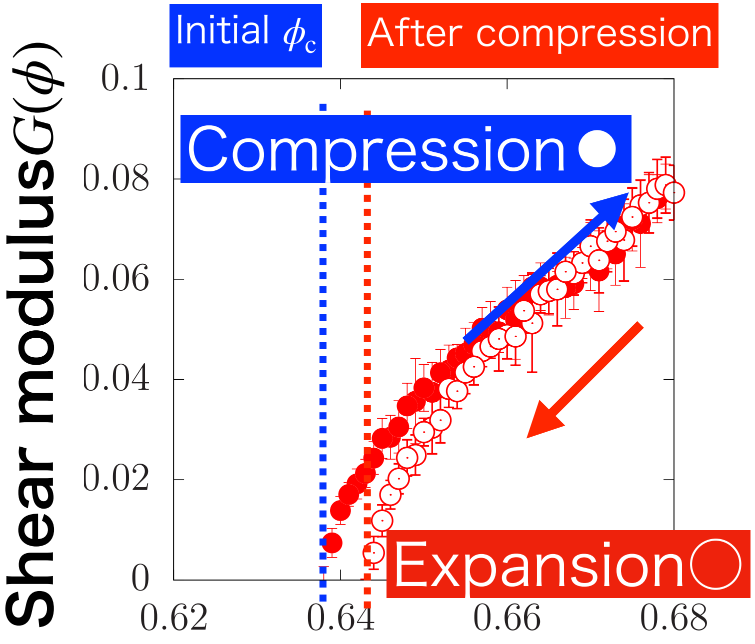


Kumar, Luding,(2016)

Hysteresis of repulsive grains

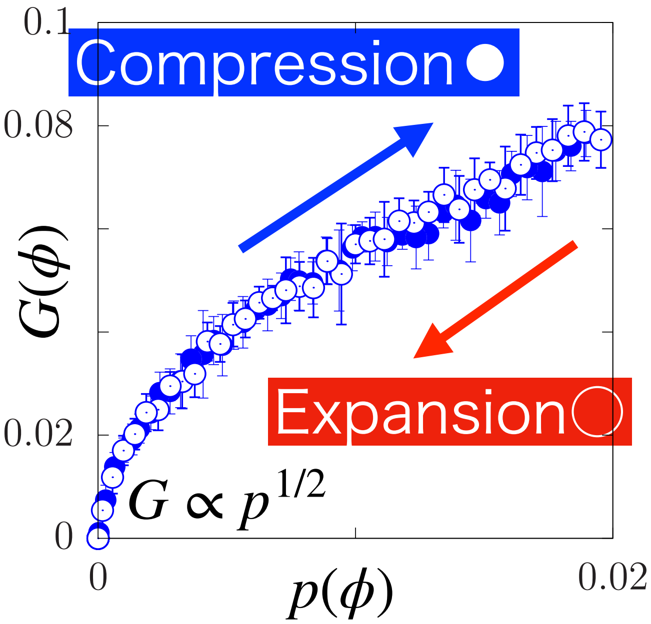
▸ Hysteresis by mechanical training





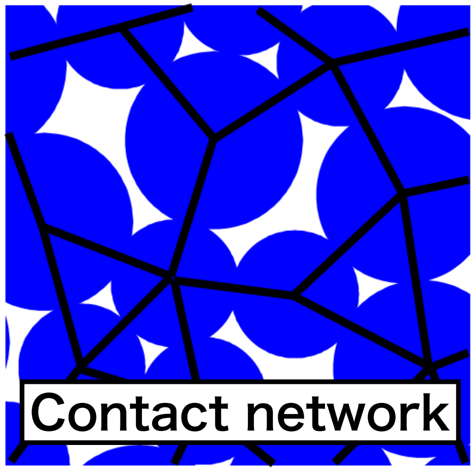
Kumar, Luding, Granular Matter(2016), Kawasaki, Miyazaki, PRL (2024)

▸ Scaling using p



Using p as a state variable removes hysteresis.

Theoretical explanation for repulsive grains



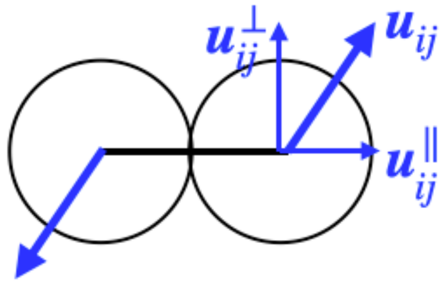
▸ **Hessian $\boldsymbol{H}$ (linear repulsive force F_{ij})**

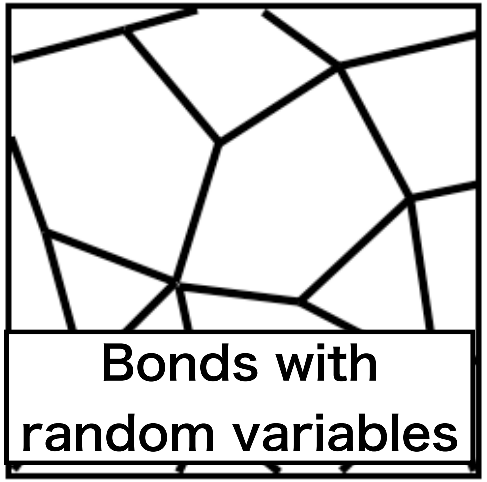
Energy variation:

$$\Delta E \simeq \sum_{(i,j)} \frac{1}{2} \left\{ k \left(u_{ij}^{\parallel}\right)^2 - \frac{F_{ij}}{r_{ij}} \left(u_{ij}^{\perp}\right)^2 \right\}$$

$$= \boldsymbol{u} \cdot \boldsymbol{H} \cdot \boldsymbol{u}$$

Relative displacement $\boldsymbol{u}_{ij}$





▸ **Effective medium theory, random matrix**

$\boldsymbol{H}$ → Random matrix $H^{\mathrm{R}}(Z,p)$

Mean field analysis

Marginal stability condition ([Minimum eigen-value of H^{R}] $= 0$)

→ $Z - Z_{\mathrm{iso}} \propto p^{1/2}$, $G \propto Z - Z_{\mathrm{iso}}$ → $G \propto p^{1/2}$

H. Mizuno and A. Ikeda, arXiv 2407.15323,
H. Ikeda and M. Shimada, PRE (2022)

Jammed cohesive grains?

- **Cohesiveness**
  - Wetting
  - Electricity
  - van der Waals force

General in granular materials

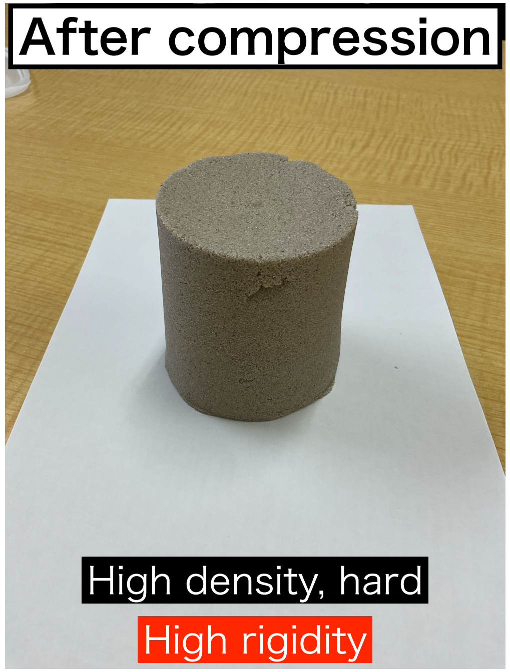
Toy of cohesive grains


Pouring them into a cup

No compression


Low density, soft
No rigidity

After compression


High density, hard
High rigidity

Cohesive grains

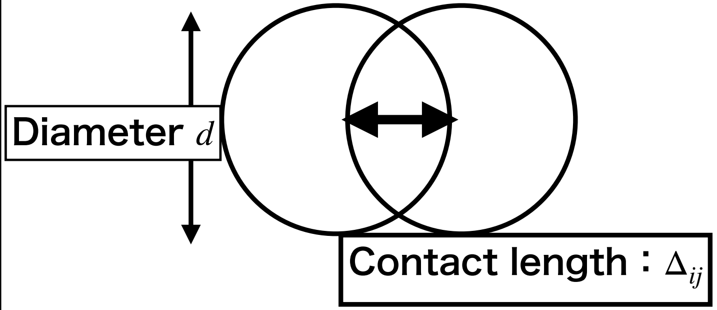
- **Initial compression→ the shear modulus G exhibits hysteresis.**
- **Even for $p = 0$ (at the surface of columns), $G > 0$.**

Different behavior from repulsive grains?

Contents

Setup: 3D DEM simulations (cohesive grains)

3D mono-disperse system



Interaction:

Static force for ij grains: F_{ij}
(Repulsion+Cohesion)

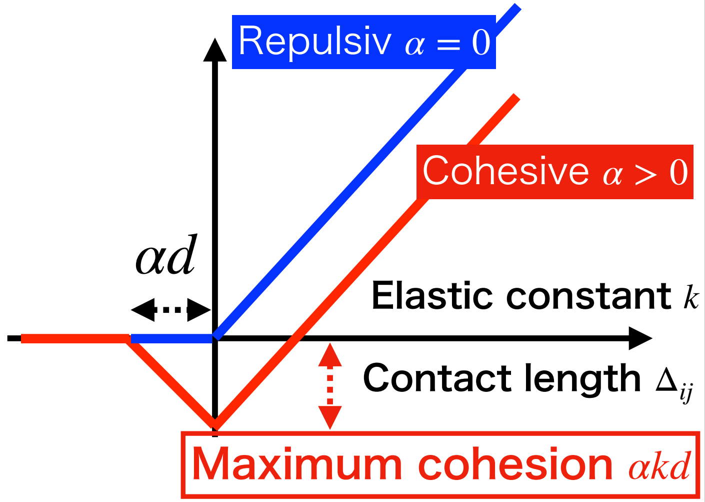
Dissipative force $F_{ij}^{(d)}$

Time evolution:

$$m\ddot{\boldsymbol{r}}_i = \sum_j \left(F_{ij} + F_{ij}^{(d)}\right)\boldsymbol{n}_{ij}$$

Position $\boldsymbol{r}_i$, normal vector $\boldsymbol{n}_{ij}$

Interaction force F_{ij}

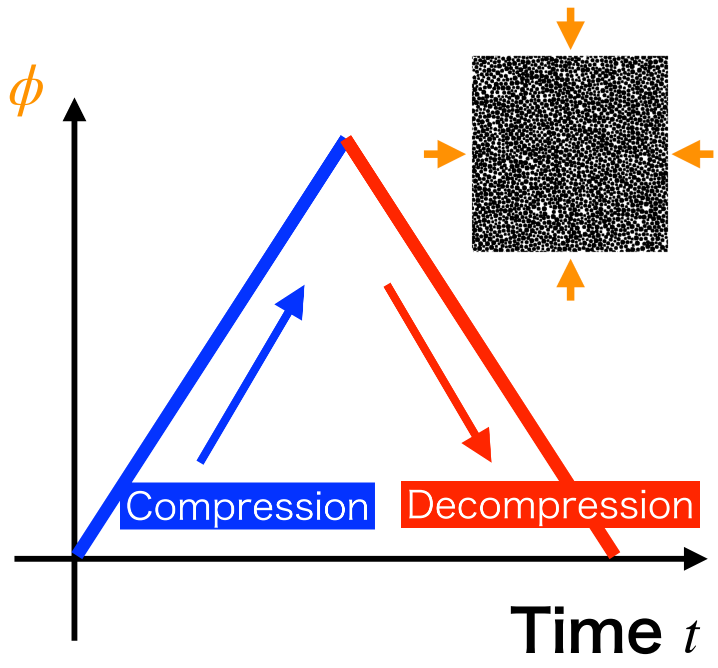


Cohesiveness α

No hysteresis

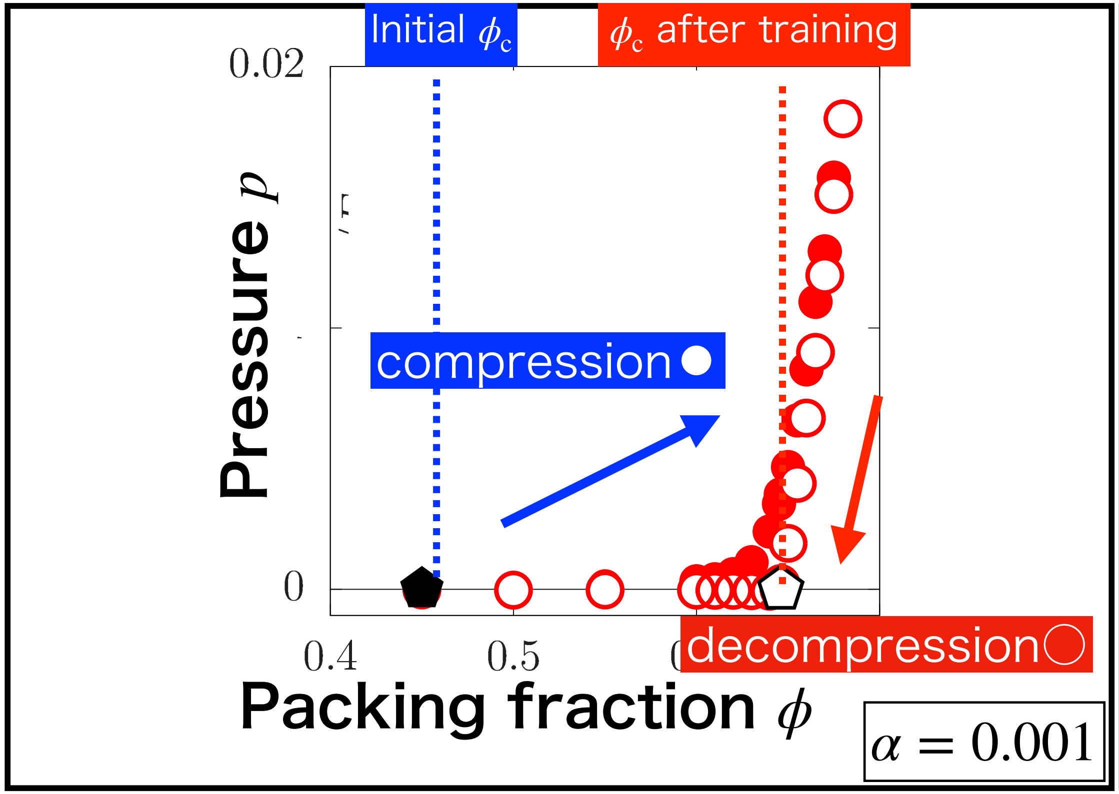
Koeze, et al., (2018)

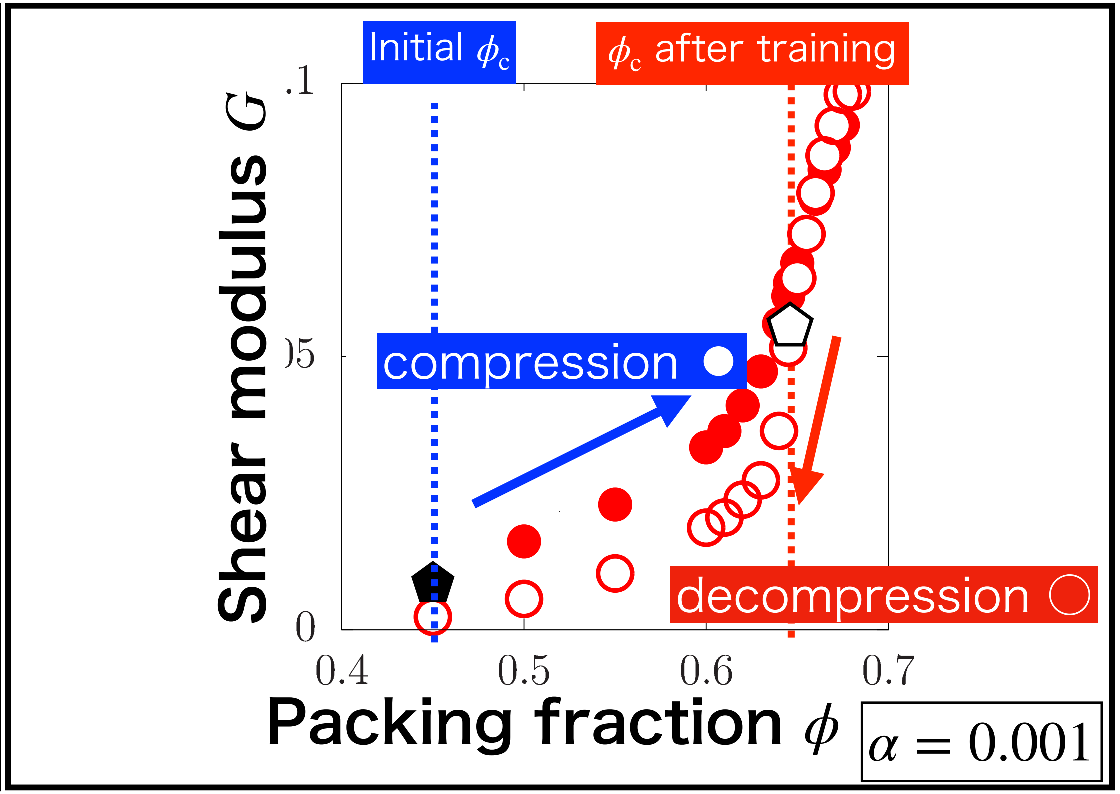
Cyclic compression



- Repulsive ($\alpha = 0$), Cohesive ($\alpha > 0$)
- p, G, and Z measured during compression and decompression.

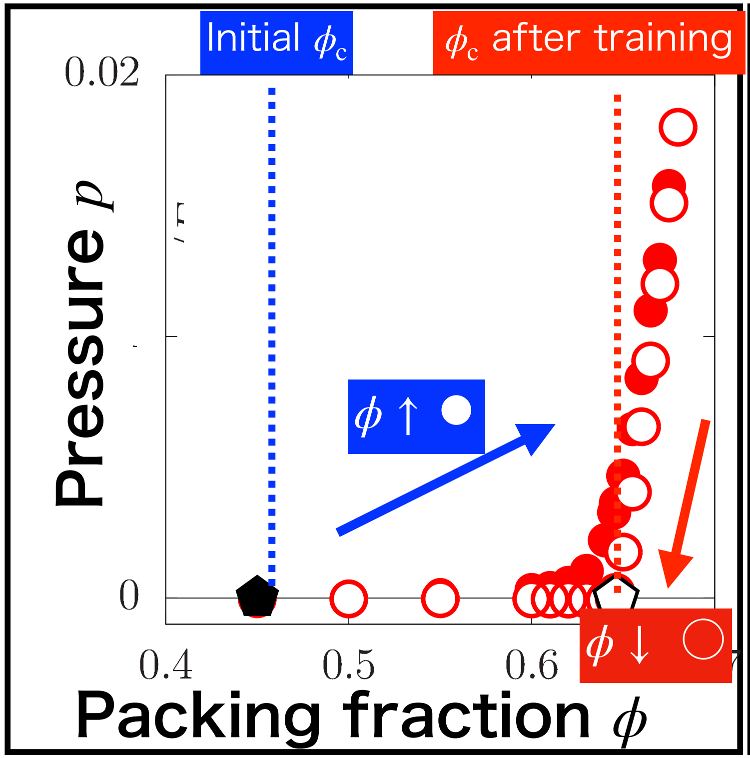
Hysteresis in cohesive grains

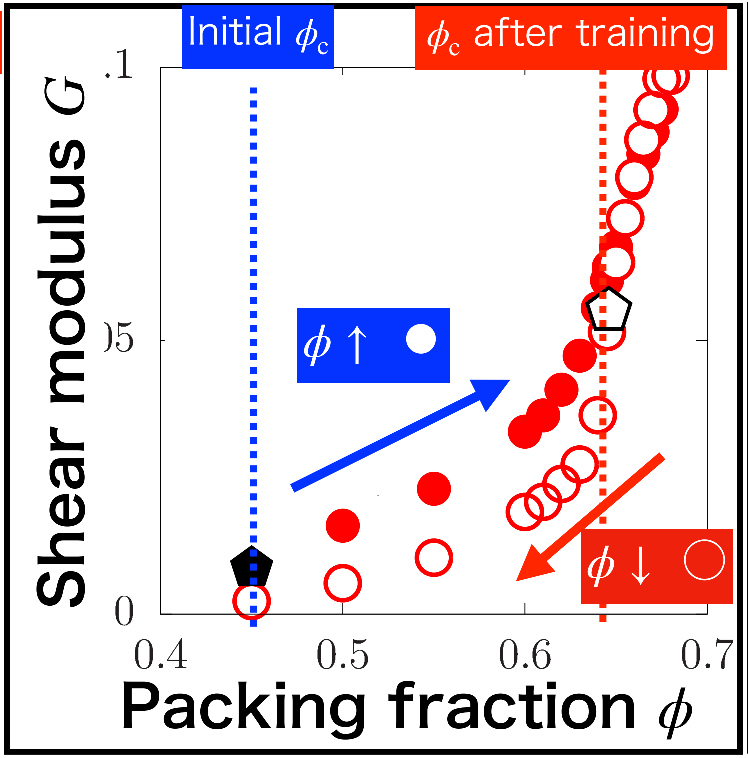


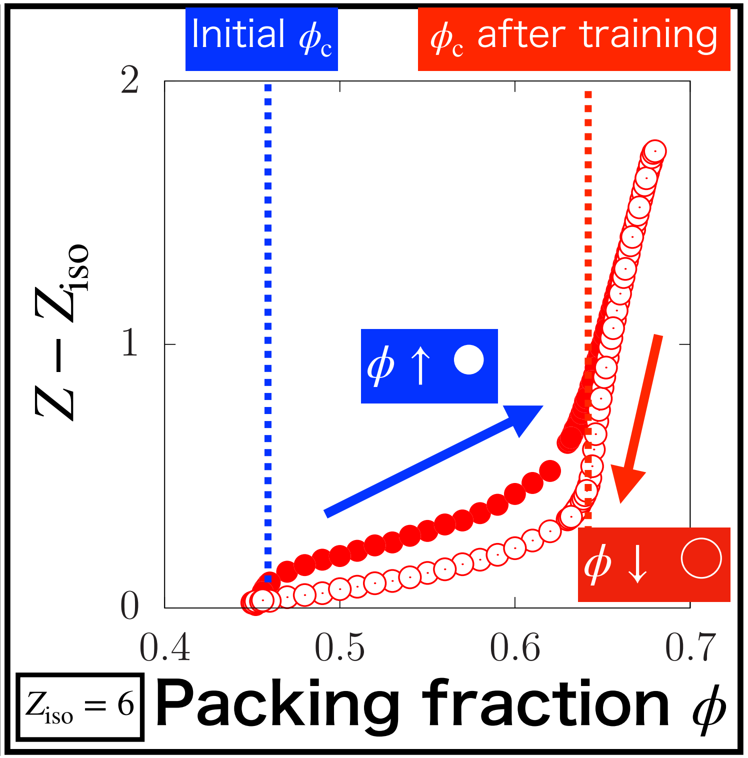


- Hysteresis in p (change in ϕ_c).
- ϕ_c : p vanishes
- Hysteresis in G
- $G > 0$ even for $\phi < \phi_c$

Hysteresis in cohesive particles





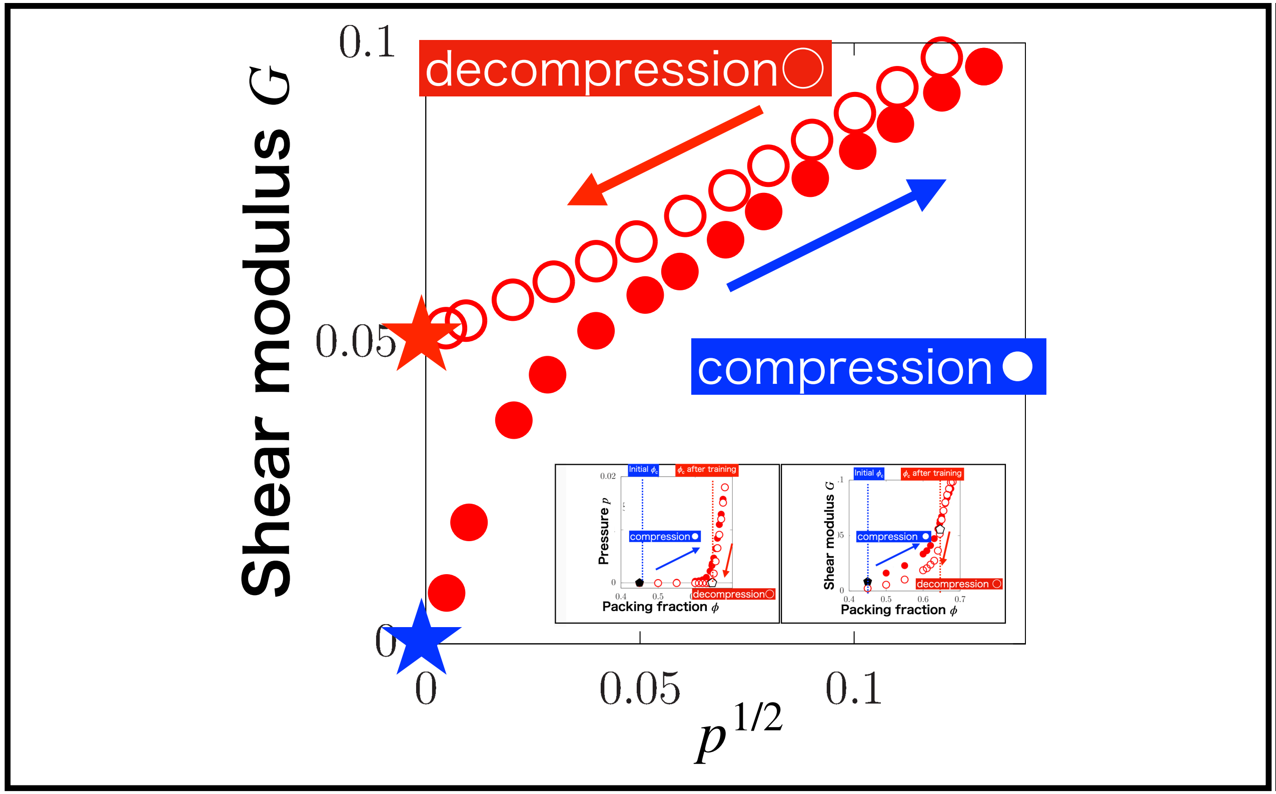


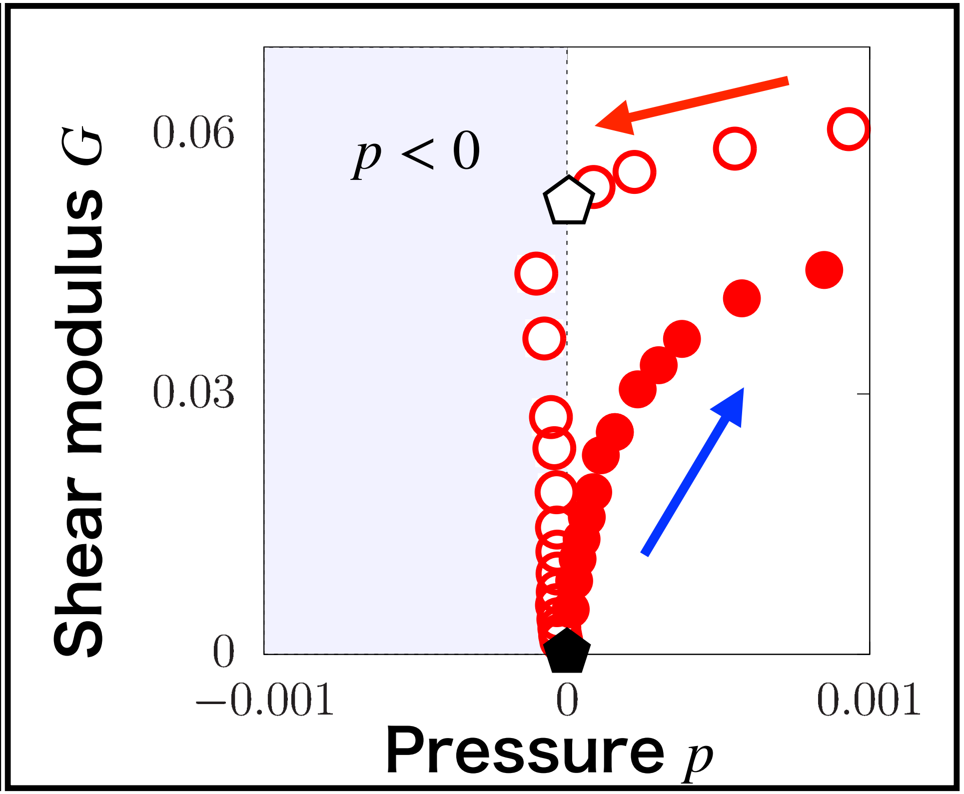
- Hysteresis in p
- ϕ_c : p vanishes
- Hysteresis in ϕ_c

- Hysteresis in G
- $G > 0$ even for $\phi < \phi_c$

- Hysteresis in Z
- $Z - Z_{iso} > 0$ for $\phi < \phi_c$

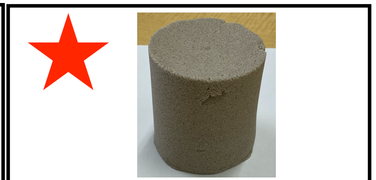
Hysteresis in $G(p)$ and rigidity for $p < 0$





- ~~$G \propto p^{1/2}$~~
- **Hysteresis**
- $G > 0$ **for** $p \to 0$.

Collapse at $p = 0$.



No collapse at $p = 0$.

- $G > 0$ **even for** $p < 0$.

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