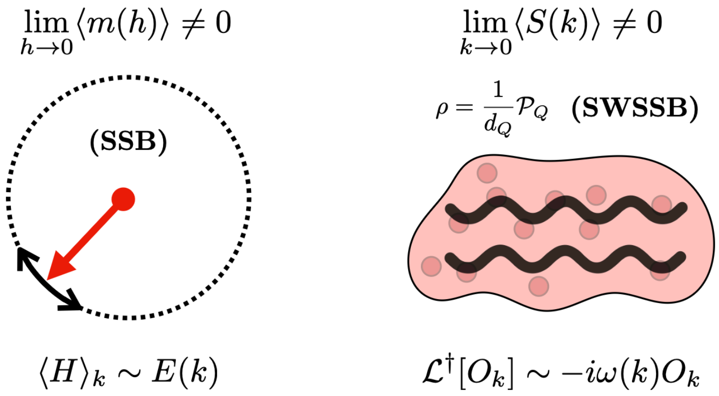
Hydrodynamics as Nambu-Goldstone modes:
Lindbladian evolution with conservation law

Jong Yeon Lee, UIUC

YITP 6/3/2026



Makinde, Feldmeier, **JYL**, arXiv:2304.13028
JYL, arXiv:2605.05288
JYL, upcoming work (soon!)

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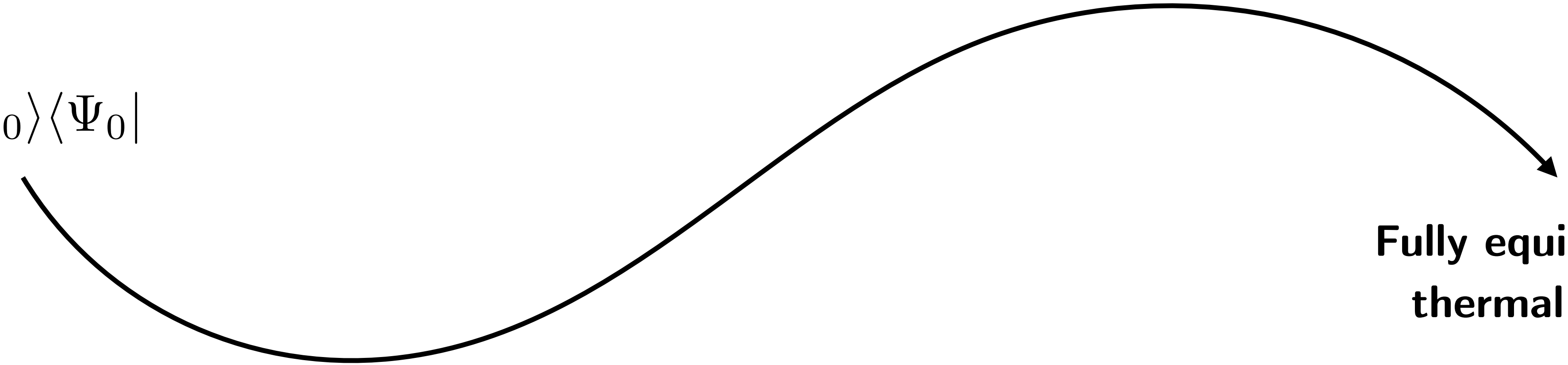
Equilibrium physics



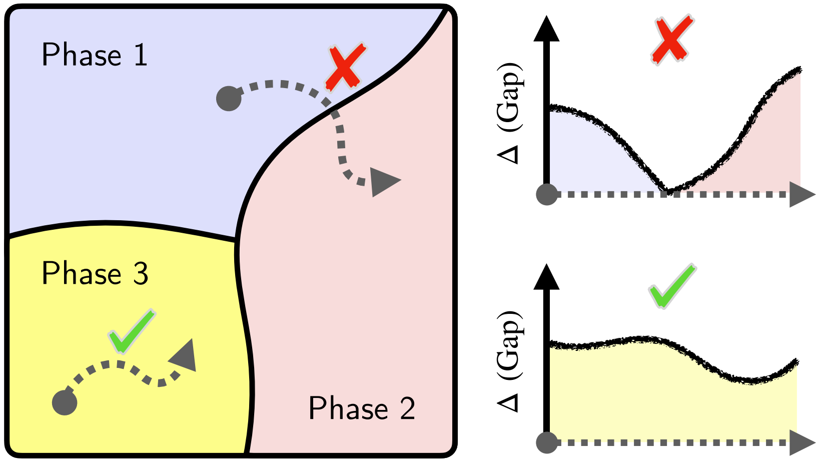
$|\Psi_0\rangle\langle\Psi_0|$

Fully equilibrated thermal state

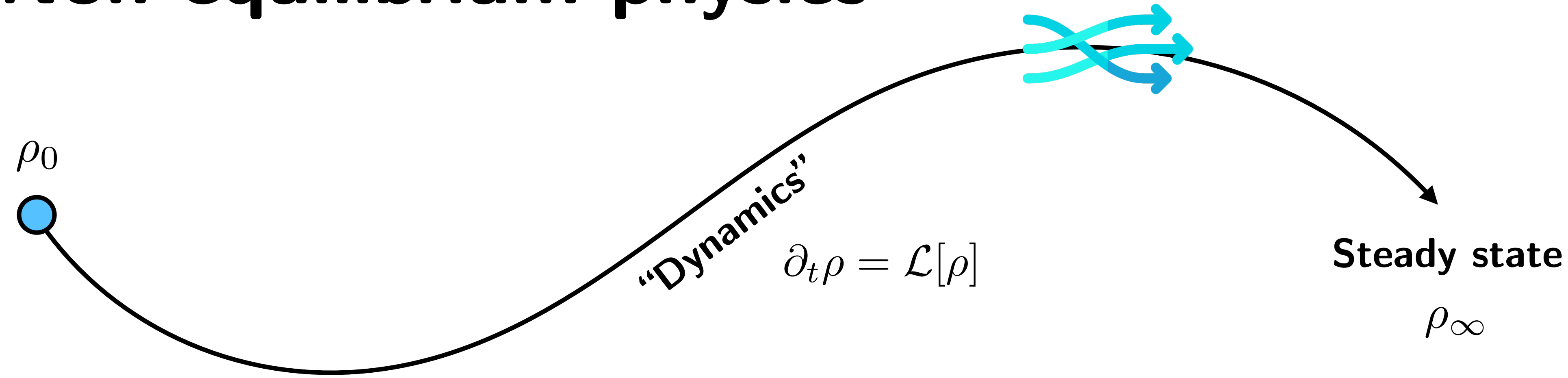
$\rho \propto e^{-\beta H}$

Ground state of **local and gapped Hamiltonian**

$\Rightarrow$ Quantum Phase of Matter



Non-equilibrium physics



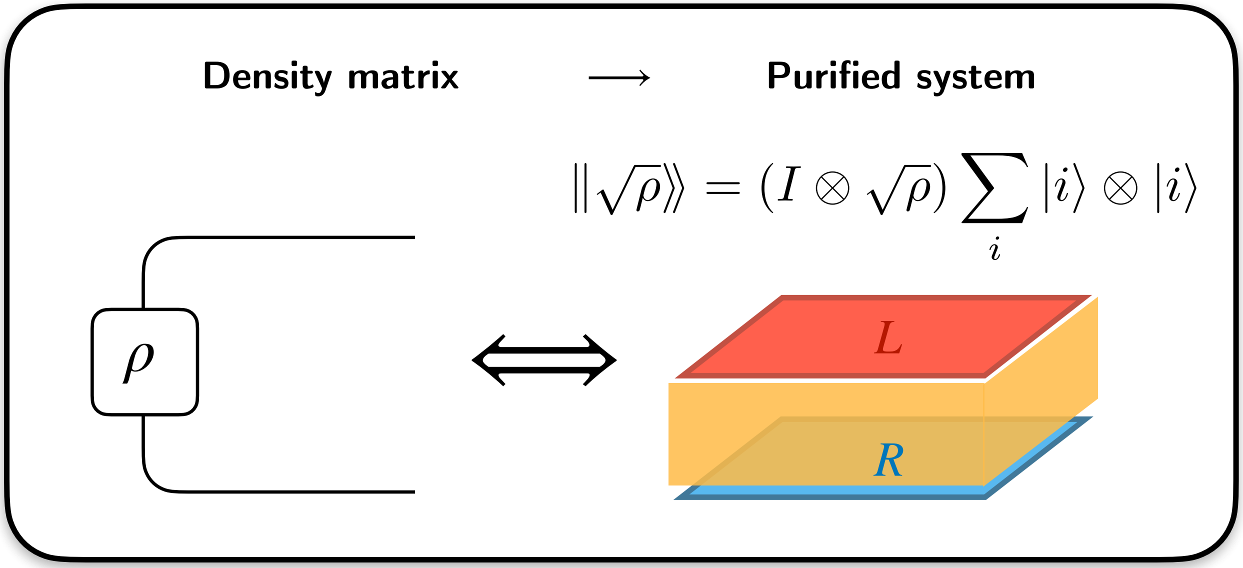
Q : Can there be interesting "phase transition" at certain time t_c?

A : Novel phase transition under symmetric stochastic evolution!

Canonical Purification

Uhlmann, Reports on Mathematical Physics 9 (1976)

- A given mixed state ρ is **subsystem** of an enlarged pure state $|\Psi\rangle$
- There is a **canonical** choice for such an enlargement



Density matrix ⟶ **Purified system**

$$\|\sqrt{\rho}\rangle\!\rangle = (I \otimes \sqrt{\rho}) \sum_i |i\rangle \otimes |i\rangle$$

$$\mathrm{tr}_R \, \|\sqrt{\rho}\rangle\!\rangle\langle\!\langle\sqrt{\rho}\| = \rho$$

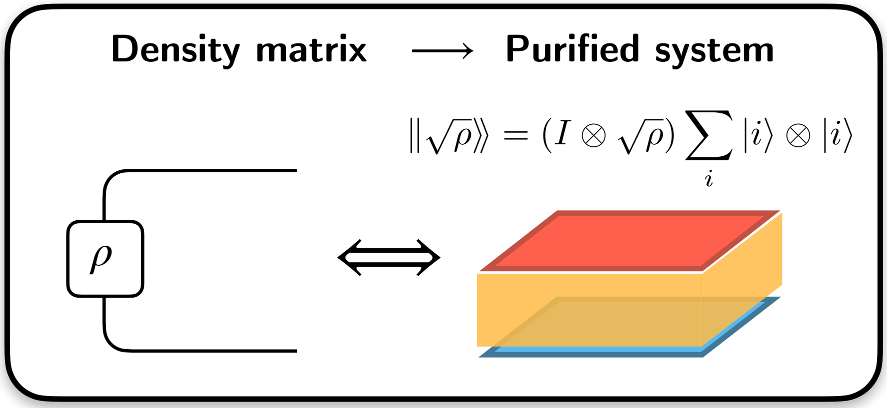
$$\langle\!\langle\sqrt{\rho}\|\sqrt{\rho}\rangle\!\rangle = \mathrm{tr}\,\rho^{1/2}\rho^{1/2} = 1$$

Symmetry in Open System

Buca, Prosen (2012)
Albert, Jiang (2014)

- **Wavefunction level** symmetry $\rho \propto U_g\rho \propto \rho U_g^\dagger$ for $g \in G$ (Strong symmetry)
- **Ensemble level** symmetry $\rho = U_g\rho U_g^\dagger$ for $g \in G$ (Weak symmetry)

$$\rho_0 = \begin{pmatrix} \rho_{11} & \rho_{12} & \cdots \\ \rho_{21} & \cdots & \\ \vdots & & \end{pmatrix}$$



Density matrix $\longrightarrow$ **Purified system**

$$\|\sqrt{\rho}\rangle\!\rangle = (I \otimes \sqrt{\rho}) \sum_i |i\rangle \otimes |i\rangle$$

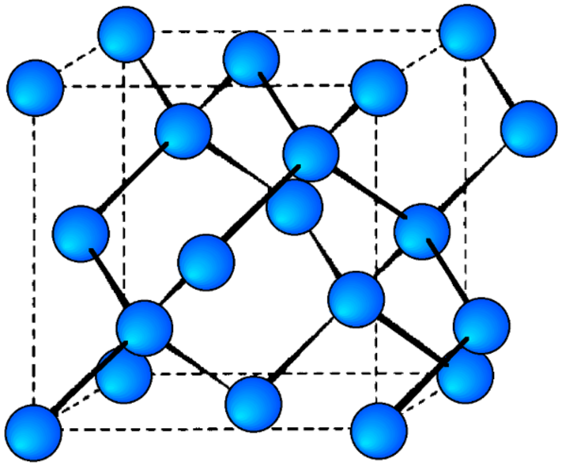
$$\|\sqrt{\rho_0}\rangle\!\rangle = \begin{pmatrix} \sqrt{\rho}_{11} \\ \sqrt{\rho}_{12} \\ \vdots \\ \sqrt{\rho}_{21} \\ \vdots \end{pmatrix}$$

$$U_g\rho = \rho U_g^\dagger = \rho \quad \Rightarrow \quad (U_g \otimes I)\|\sqrt{\rho}\rangle\!\rangle = (I \otimes U_g^*)\|\sqrt{\rho}\rangle\!\rangle = \|\sqrt{\rho}\rangle\!\rangle$$ **"Doubled" symmetry**

$$U_g\rho U_g^\dagger = \rho \quad \Rightarrow \quad (U_g \otimes U_g^*)\|\sqrt{\rho}\rangle\!\rangle = \|\sqrt{\rho}\rangle\!\rangle$$ **"Diagonal" symmetry**

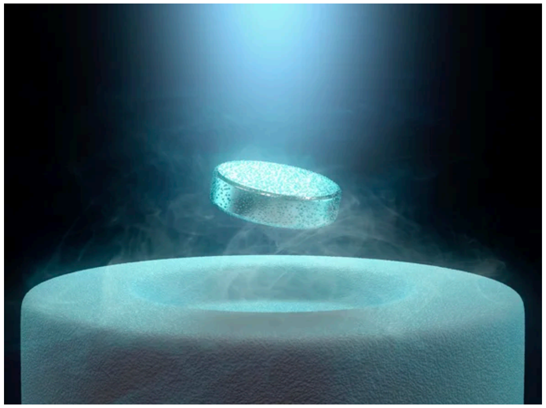
Spontaneous Symmetry Breaking (SSB)

Symmetry breaking provides the basic way to classify different phases



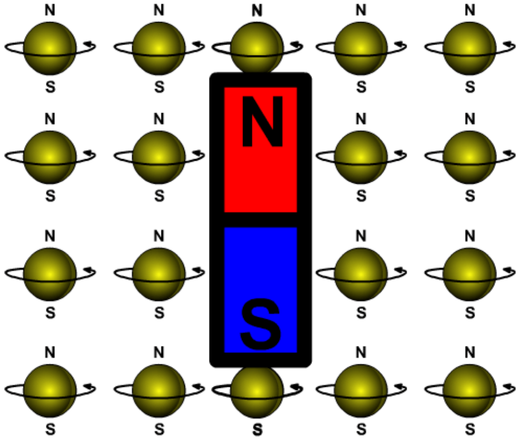
Lattice

$\mathbb{R}^3$ **translation**



Superconductor

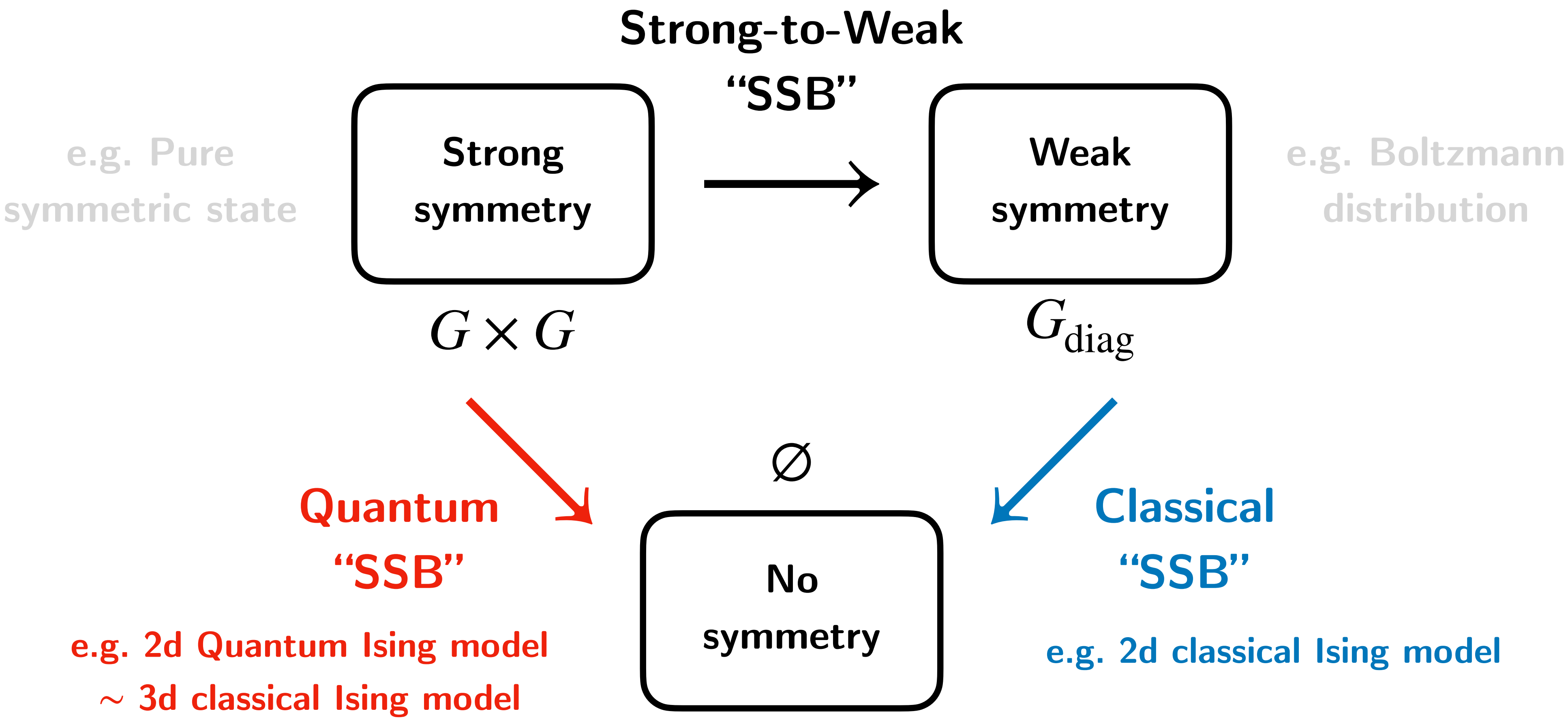
$U(1)$ **phase rotation**



Magnet

$SO(3)$ **spin rotation**

New Hierarchy of SSB



e.g. Pure symmetric state

Strong symmetry

$G \times G$

Strong-to-Weak "SSB"

Weak symmetry

G_{diag}

e.g. Boltzmann distribution

Quantum "SSB"

e.g. 2d Quantum Ising model

$\sim$ 3d classical Ising model

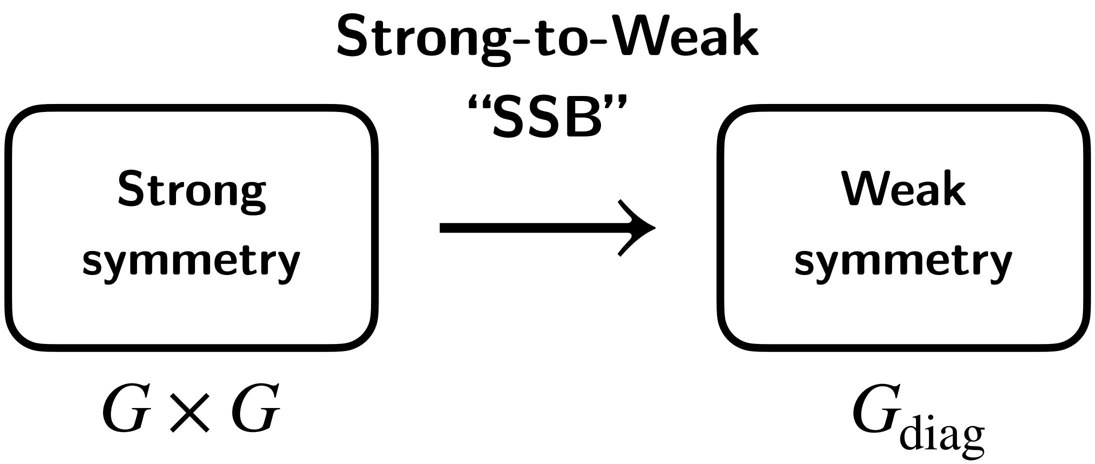
$\varnothing$

No symmetry

Classical "SSB"

e.g. 2d classical Ising model

Strong-to-Weak SSB



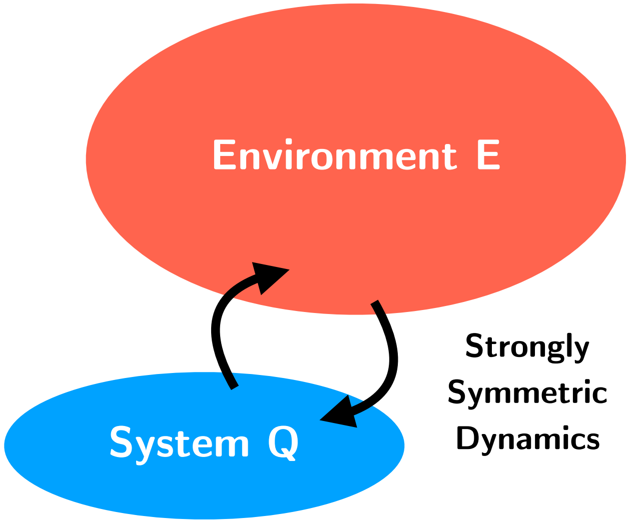
Strong-to-Weak “SSB”

Strong symmetry

$G \times G$

Weak symmetry

G_{diag}



- Evolve under strong-symmetric evolution
- At $t > t_c$, transition detected by the Information-theoretic quantity!

Q. What quantities can detect this **intrinsic transition** of the density matrix?

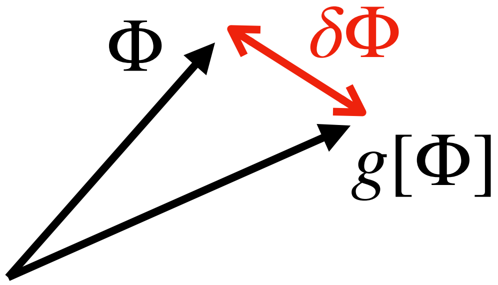
JYL, Jian, Xu, arXiv:2301.05238

Detecting SSB

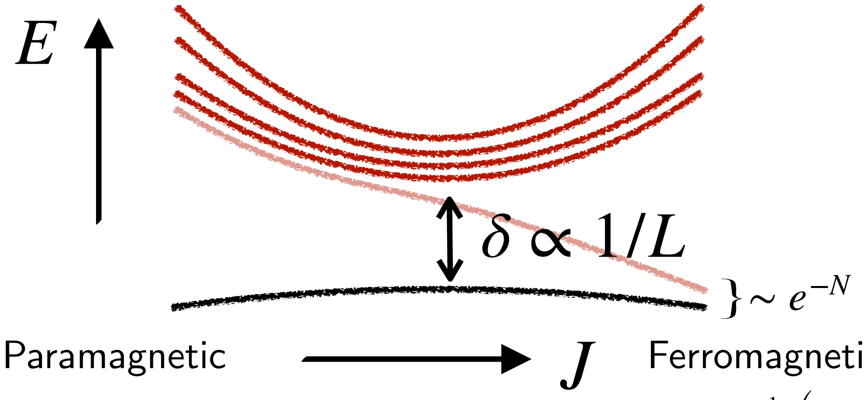
- Hamiltonian H
- Symmetry group G, generated by $Q = \sum Q_i$ with $[H, Q] = 0$
- Define order parameter $\mathcal{O}_i = [iQ, \Phi_i]$ for some Φ_i (transforms non-trivially)
- Apply an **infinitesimal** symmetry breaking field h : $H(h) = H - h\mathcal{O}$

Order Parameter

$$m_0 := \lim_{h \to 0^+} \lim_{N \to \infty} \frac{\langle \mathcal{O} \rangle_h}{N}$$



Detecting SSB



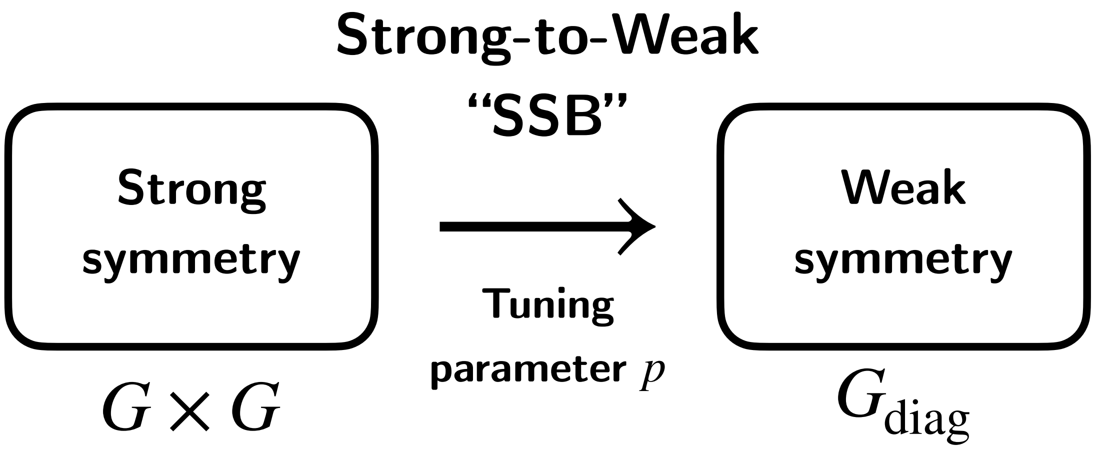
$|\phi_0\rangle = |++\cdots\rangle$

$|\phi_\pm\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\uparrow\cdots\rangle \pm |\downarrow\downarrow\cdots\rangle\Big) \;\Rightarrow \langle\phi_\pm|Z_i|\phi_\pm\rangle = 0$

$$H = J\sum_i Z_i Z_{i+1} + \sum_i X_i$$
$$+h\sum_i Z_i$$

- Symmetry group $G \simeq \mathbb{Z}_2$, generated by $g = \prod_i X_i$ ⇒ Order parameter Z_i
- Ground state never breaks the symmetry; “Cat”-like superposition
- **Only in the limit** $h \to 0^+$, symmetry breaking can be properly diagnosed

Hierarchy of SSB

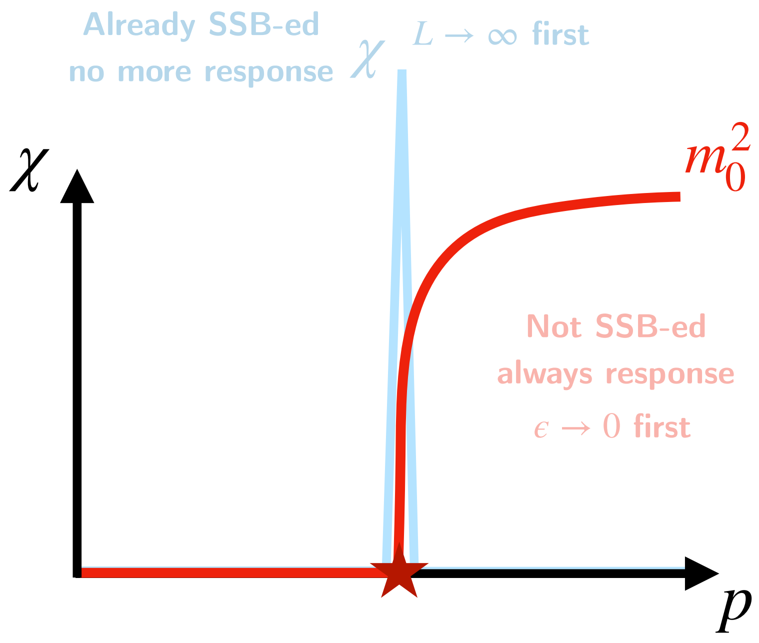


Strong-to-Weak "SSB"

Strong symmetry → Weak symmetry

Tuning parameter p

$G \times G$ → G_{diag}



- We **perturb** by an infinitesimal channel that **breaks** strong symmetry $\mathscr{E}_\epsilon$: $\rho' = \mathscr{E}_\epsilon[\rho]$
- The quantum relative entropy **detects** the difference between ρ and ρ'

A. Singularity in 'information-susceptibility' **defines** the intrinsic transition

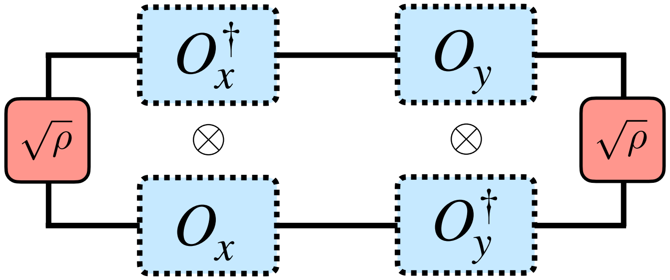
Diagnostics of SWSSB

- **JYL**, Jian, Xu, Quantum criticality under decoherence or weak measurement arXiv:2301.05238
- Lessa, Ma, Zhang, Bi, Cheng, Wang, Strong-to-weak spontaneous symmetry breaking in mixed quantum states arXiv:2405.03639
- Liu et. al., Diagnosing Strong-to-Weak Symmetry Breaking via Wightman Correlators arXiv:2410.09327
- Weinstein, Efficient Detection of Strong-To-Weak Spontaneous Symmetry Breaking via the Rényi-1 Correlator, arXiv:2410.23512

- There are many proposals.
- Most intuitive one is Renyi-1 correlator defined against **purified** state.
- This correlator detects **long-range order** of field that breaks strong symmetry, which is neutral against weak symmetry.

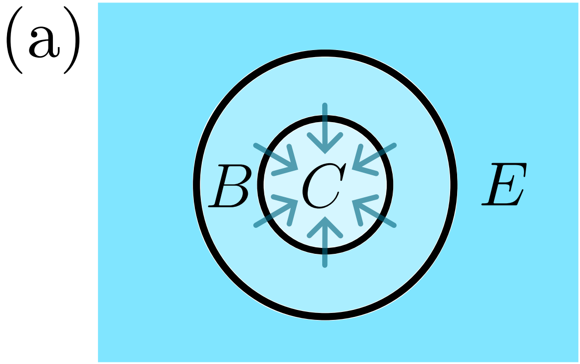
$$R_{xy} := \mathrm{tr}\left(\sqrt{\rho}T_{xy}\sqrt{\rho}T^{\dagger}_{xy}\right) = \langle\!\langle \sqrt{\rho} \| T_{xy} \otimes T^{*}_{xy} \| \sqrt{\rho} \rangle\!\rangle$$

$$T_{xy} = O_x^{\dagger} O_y$$



Info-theoretic Perspective

Yang, Shi, **JYL** arXiv:2506.04221



$$I(C : E\,|\,B) > 0$$

Prevents "Strong-to-Weak Spontaneous Symmetry Breaking"

1-form: **JYL**, You, Xu, arXiv:2210.16323

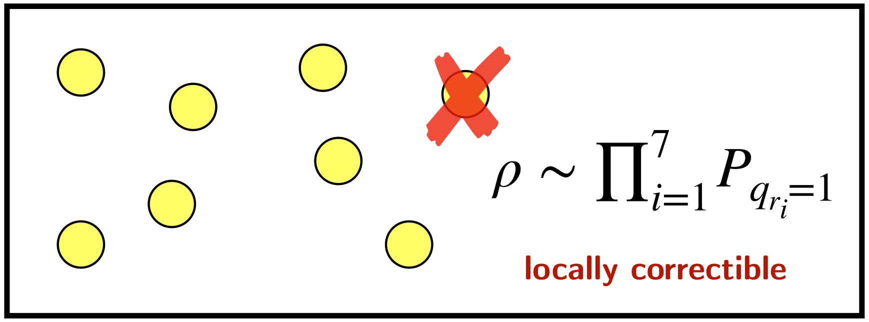
$\mathbb{Z}_2$: **JYL**, Jian, Xu, arXiv:2301.05238

$U(1)$: Ogunnaike, Feldmeier, **JYL**, arXiv:2304.13028

$\mathbb{Z}_2^F$: Kim, Altman, **JYL**, arXiv:2410.24225

See also Sala, Gopalakrishnan, Oshikawa, You, arXiv:2405.02402
Lessa, Ma, Zhang, Bi, Meng, Wang, arXiv:2405.03639

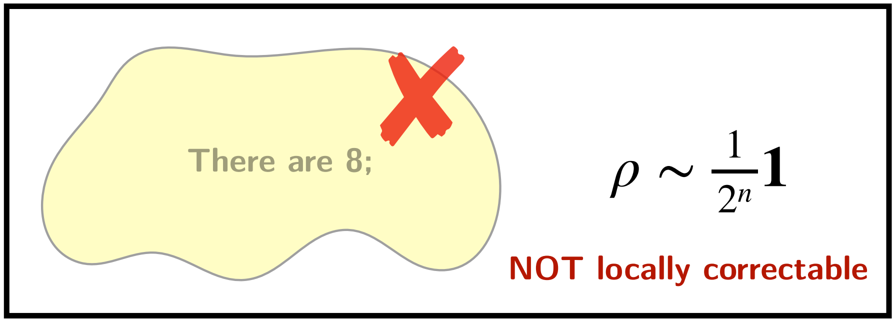
Scenario 1: We know charges at each site well



$$\rho \sim \prod_{i=1}^{7} P_{q_{r_i}=1}$$

locally correctible

Scenario 2: Charges are scrambled; we only know total

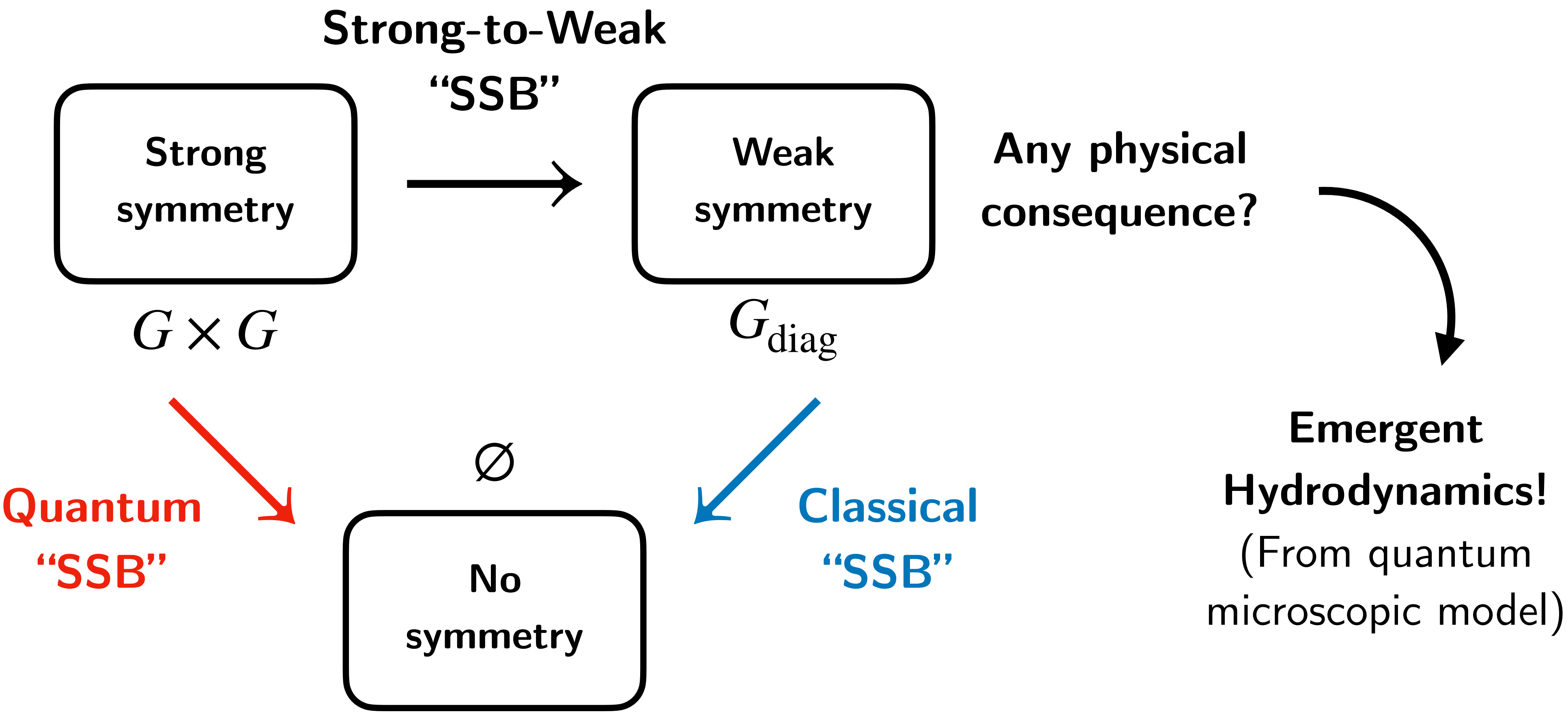


$$\rho \sim \frac{1}{2^n}\mathbf{1}$$

NOT locally correctable

"New form" of "Spontaneous Symmetry Breaking" — **precisely excluded by the axiom**

Physical consequence



Strong-to-Weak
“SSB”

Strong symmetry

$G \times G$

Weak symmetry

G_{diag}

Any physical consequence?

$\varnothing$

Quantum “SSB”

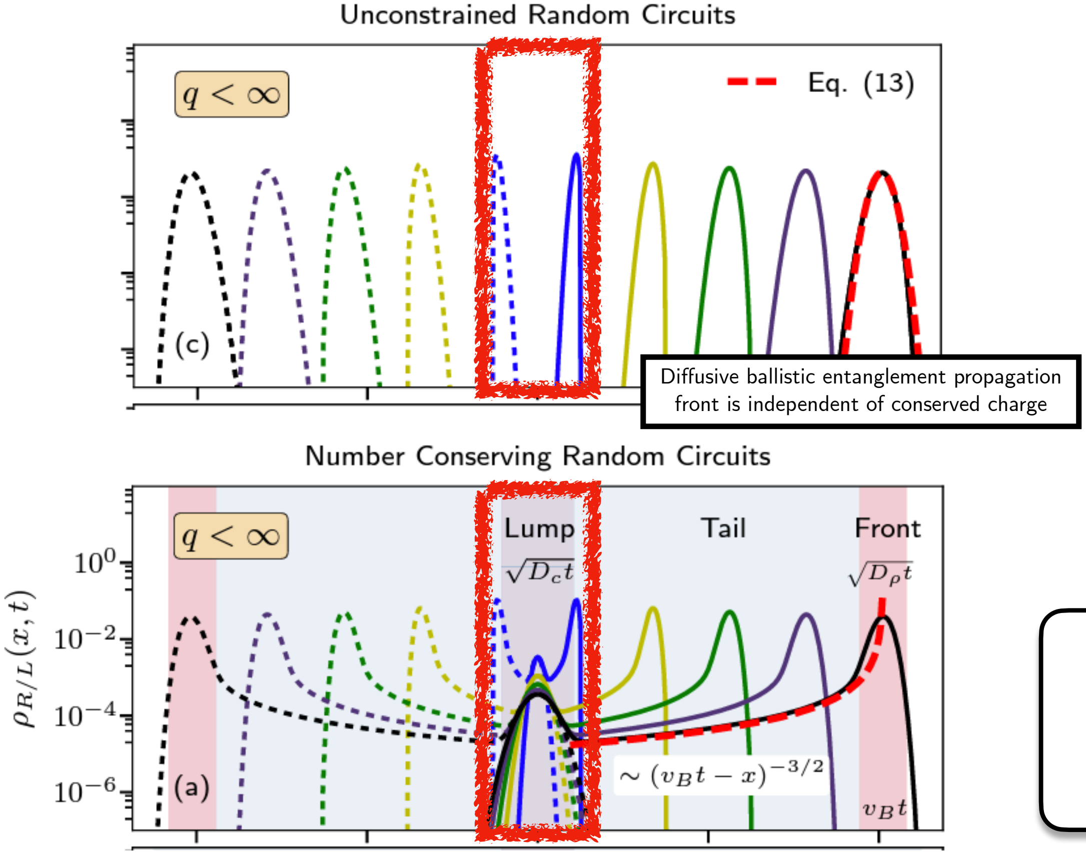
No symmetry

Classical “SSB”

Emergent Hydrodynamics!
(From quantum microscopic model)

(1) U(1) Symmetry and Diffusion

Charge Transport



Diffusive ballistic entanglement propagation front is independent of conserved charge

The operator as a whole spreads ballistically, while its conserved projection forms a diffusive hydrodynamic lump.

Khemani, Vishwanath, Huse (2018)

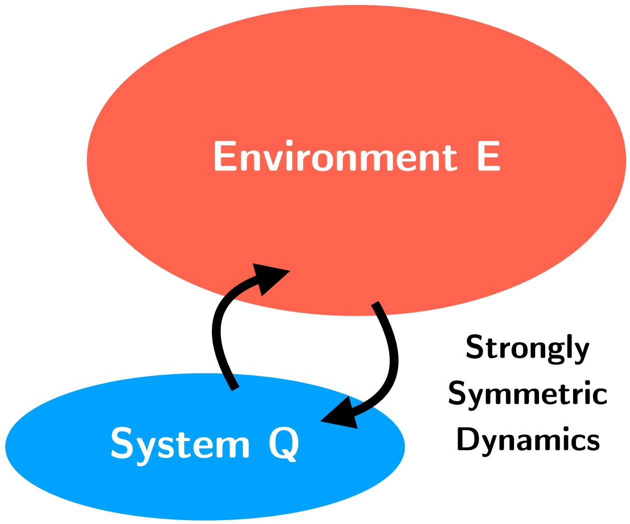
Conserved Charges

= Slow Charge Dynamics

(Diffusive at the center)

Q: How to understand this slow dynamics?

Physical consequence



Consider a symmetry group G

Decoherence channel with symmetric noise $\{L_i\}$

$=$ Time evolution under symmetric **Lindbladian**

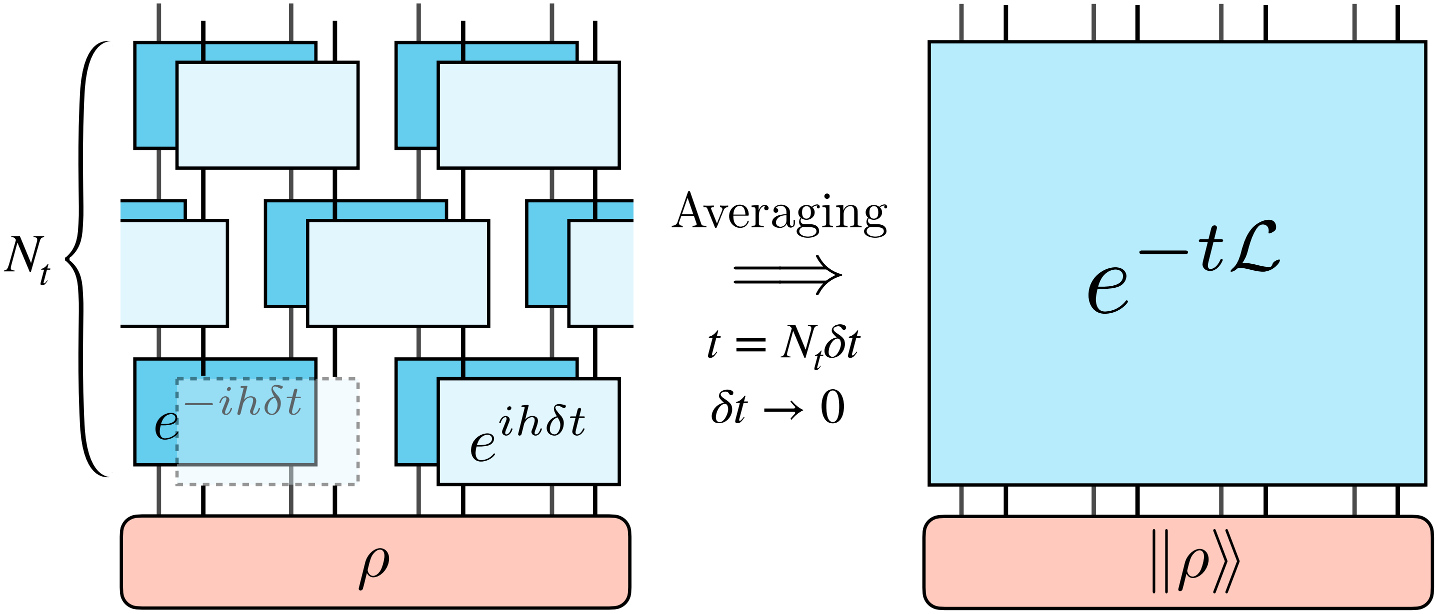
$$\partial_t \rho = -\frac{i}{\hbar}[H,\rho] + \sum_a \gamma_a \left(L_a \rho L_a^\dagger - \frac{1}{2}\{L_a^\dagger L_a, \rho\} \right)$$

Strong noise channel $=$ Long-time evolution ↦ Strong-to-Weak SSB

Dynamical consequence of the strong-to-weak SSB? ***Emergent Hydrodynamics***

Ogunnaike, Feldmeier, **JYL**, arXiv:2304.13028

Lindbladian approach



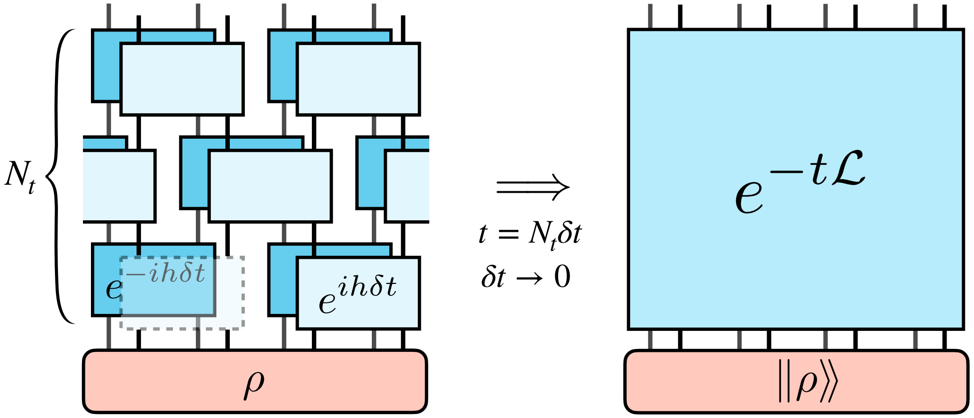
$$\rho(t+\delta) := e^{-iH_t\delta}\rho(t)e^{iH_t\delta}$$

$$H_t = \sum_i h_i \boxed{dB_{i,t}} \longrightarrow$$

Brownian random variable

$$h_i \sim b_i^\dagger b_{i+1} + \mathrm{h.c.} + \text{Interaction} + \ldots$$

Lindbladian approach



Averaging over random variables
⇒ Lindbladian evolution

$$\mathbb{E}[\partial_t \rho] = -\frac{1}{2}\sum_i (h_i^2 \rho - 2h_i \rho h_i + \rho h_i^2) = -\mathcal{L}[\rho]$$

Choi Isomorphism: $\|\rho(t)\rangle\!\rangle = e^{-t\hat{\mathcal{H}}_{\mathcal{L}}}\|\rho_0\rangle\!\rangle$ "Imaginary Schrodinger's Equation"

$$\hat{\mathcal{H}}_{\mathcal{L}} = \frac{1}{2}\sum_i \left| h_i^T \otimes \mathbb{I} - \mathbb{I} \otimes h_i \right|^2 =: \frac{1}{2}\sum_{\boldsymbol{x},\lambda} \mathcal{O}^\dagger_{\boldsymbol{x},\lambda}\mathcal{O}_{\boldsymbol{x},\lambda}.$$

Operator Dynamics

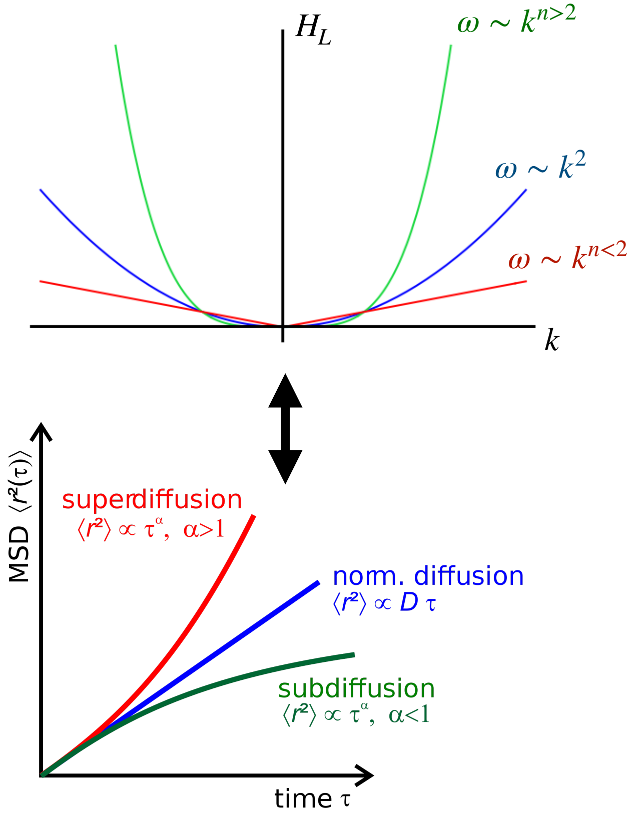
- Assuming translation invariance of H_L

$$\mathbb{E}\langle O_{\boldsymbol{y}}(0)O_{\boldsymbol{x}}(t)\rangle_\rho = \frac{1}{D}\langle\!\langle O_{\boldsymbol{y}}(0)\| e^{-t\hat{\mathcal{H}}_{\mathcal{L}}}\|O_{\boldsymbol{x}}(0)\rangle\!\rangle$$

$$= \frac{1}{D}\sum_{\boldsymbol{k},\nu} e^{-tE_{\boldsymbol{k},\nu}} e^{i\boldsymbol{k}\cdot(\boldsymbol{y}-\boldsymbol{x})}\left|\langle\!\langle \boldsymbol{k},\nu\|O_{\boldsymbol{x}}\rangle\!\rangle\right|^2$$

$$\underset{t\to\infty}{\sim} \int_k e^{-tk^n}\,\mathrm{d}^d\boldsymbol{k} \sim \frac{1}{t^{d/n}} \longrightarrow$$

Diffusion Exponent



Where does the (d=2) gapless modes come from?

Low energy physics: Nambu-Goldstone

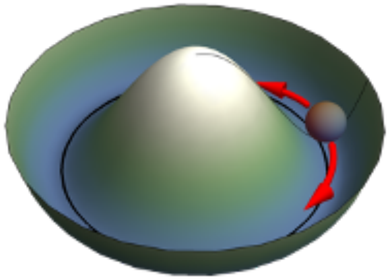
- Doubled symmetries: $\mathrm{U}(1)_L$ and $\mathrm{U}(1)_R$
- Lindbladian takes an identity on total charge sector-m as stationary states

Frustration-free!

- Exact ground states: $(2SN+1)$-fold degenerate $\quad \|\mathbb{I}\rangle\rangle = \sum_{m=0}^{2SL+1} \|\mathbb{P}_m\rangle\rangle$
- Each ground state has a definite $\mathrm{U}(1)_{\mathrm{off}}$ charge of $2m$

$$\|\theta\rangle\rangle := \sum_m e^{i\theta m} \|P_m\rangle\rangle \quad \Rightarrow \quad e^{i\alpha Q_{\mathrm{off}}} \|\theta\rangle\rangle = \|\theta+\alpha\rangle\rangle$$

Spontaneously breaks $\mathrm{U}(1)_{\mathrm{off}}$ symmetry



- Nambu-Goldstone mode is expected! — but k^2 dispersion?

Single-mode approximation

Feynman (1953) Girvin-Macdonald-Platzman (1986) …

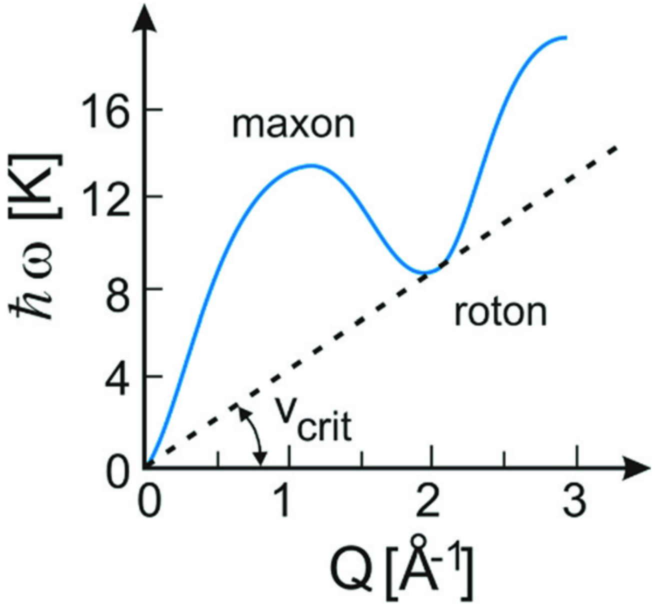
$$|\psi_k\rangle = \frac{1}{L^{d/2}}\rho_k|\phi_0\rangle = \frac{1}{L^{d/2}}\sum_x e^{ik\cdot x}\rho_x|\phi_0\rangle$$

Superfluid He-4: $E_k = \frac{f(k)}{s(k)} \sim \frac{k^2}{k} = k$

$$E_k \approx \frac{\langle\psi_k|H|\psi_k\rangle}{\langle\psi_k|\psi_k\rangle} = \frac{f(k)}{s(k)}$$

$$f(k) = \frac{1}{2L^d}\langle\phi_0|[\rho_k^\dagger,[H,\rho_k]]|\phi_0\rangle$$

$$s(k) = \langle\psi_k|\psi_k\rangle = \frac{1}{L^d}\langle\phi_0|\rho_k^\dagger\rho_k|\phi_0\rangle$$



Single-mode approximation

Feynman (1953) Girvin-Macdonald-Platzman (1986) …

$$\boxed{\langle\!\langle m_{\boldsymbol{k}} \| \hat{\mathcal{H}}_{\mathcal{L}} \| m_{\boldsymbol{k}} \rangle\!\rangle = \sum_{\boldsymbol{x},\lambda} \langle\!\langle m \| [\mathcal{O}_{\boldsymbol{x},\lambda}, \hat{\rho}_{\boldsymbol{k}}]^\dagger [\mathcal{O}_{\boldsymbol{x},\lambda}, \hat{\rho}_{\boldsymbol{k}}] \| m \rangle\!\rangle,}$$

$$[\mathcal{O}_{\boldsymbol{x},\lambda}, \hat{\rho}_{\boldsymbol{k}}] = e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \sum_{\boldsymbol{y}\in\mathcal{N}_{\boldsymbol{x}}} \sum_{n=1}^{\infty} [\mathcal{O}_{\boldsymbol{x},\lambda}, \frac{[i\boldsymbol{k}\cdot(\boldsymbol{y}-\boldsymbol{x})]^n}{n!} \hat{\rho}_{\boldsymbol{y}}] \sim k^a$$

$(a-1)$-th multipole conservation

$$\boxed{\langle\!\langle m_{\boldsymbol{k}} \| m_{\boldsymbol{k}} \rangle\!\rangle \sim \mathrm{Const}}$$

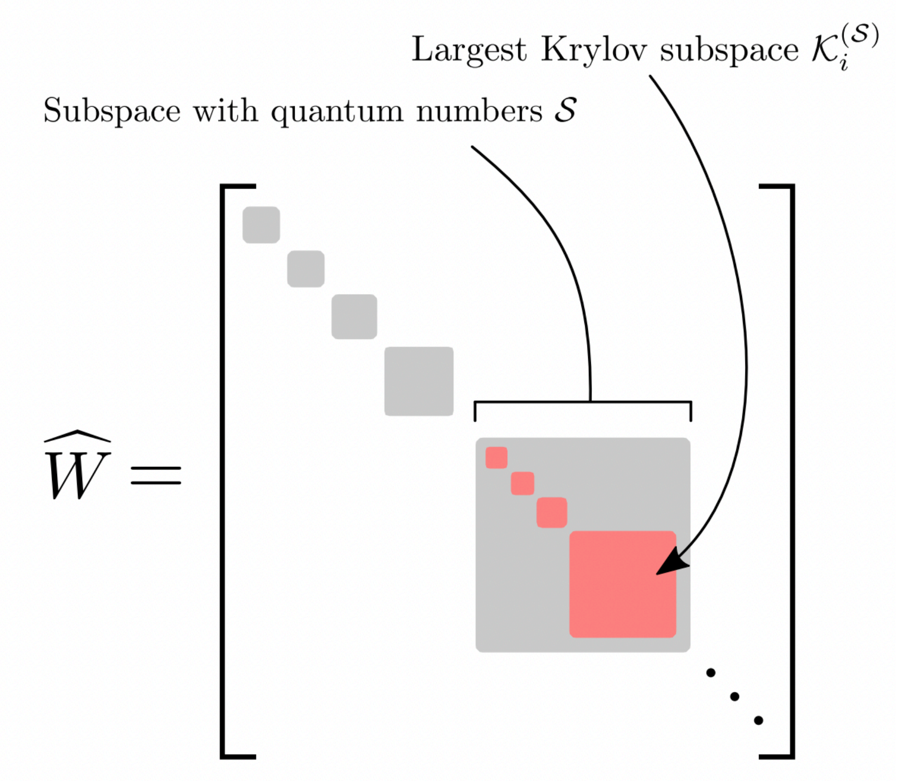
Our ground state is "structure-less"

$$\Longrightarrow E_{\boldsymbol{k}} \leq \frac{\langle\!\langle m_{\boldsymbol{k}} \| H \| m_{\boldsymbol{k}} \rangle\!\rangle}{\langle\!\langle m_{\boldsymbol{k}} \| m_{\boldsymbol{k}} \rangle\!\rangle} \sim \boldsymbol{k}^{2a}$$

Any interesting prediction?

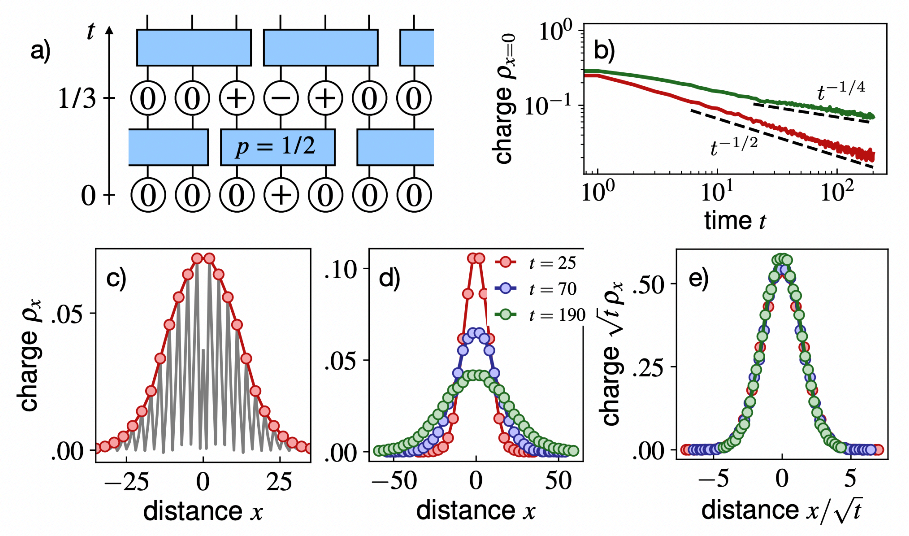
Hilbert space Fragmentation

- Each charge sector can decompose into smaller "Krylov" sectors.
- Projector $P_m = \sum_i K_{m,i}$
- One can consider excitation with respect to Choi-state of Krylov-projector $\| K_{m,i} \rangle\rangle$
- **Structural factor** $\langle\langle K_{m,i} \| \rho_k \rho_{-k} \| K_{m,i} \rangle\rangle \sim k^a$ **is now nontrivial, making diffusion faster!**

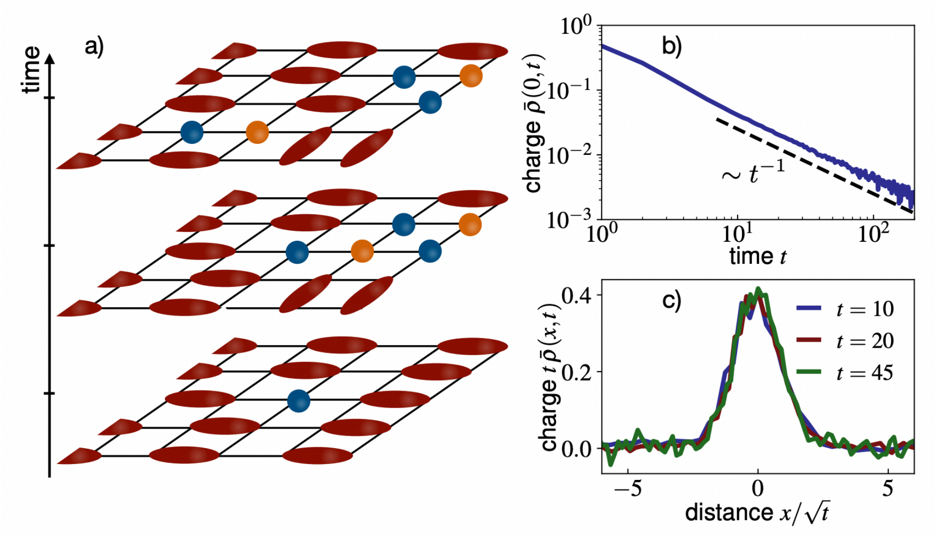


Hilbert space Fragmentation

Expected to have $\langle O(0)O(t)\rangle \sim \dfrac{1}{t^{d/z}}$ with $z = 4$ for dipole conserving system



1d Dipole-Conserving System



2d Dipole-Conserving System

Sectors with $\langle\langle K_{m,i} \| K_{m,i}\rangle\rangle \sim k^2$ exhibits $z = 2$ instead!

Additional remark

- Charge transport dynamics ⇔ softness of Goldstone mode of Strong-to-Weak SSB
- Unifying description for
  - Multipole-conserving system (see also 2304.12342 Gliozzi, May-Mann, Hughes, Tomasi)
  - Long-range interacting system → In this case, k^z with $z < 2$, **giving super-diffusive** behavior
  - **Constrained dynamics within Krylov sector** (structural factor would vanish with k) : exhibiting "diffusive" instead of "subdhiffusive" behavior with dipole conservation

- Provides a generic effective Hamiltonian with SSB of continuous symmetry G. This Hamiltonian is **frustration-free.** Generic route to circumvent Mermin-Wagner theorem

Watanabe, Katsura, **JYL** arXiv:2310.16881 (PRL)

(2) SWSSB and charge scrambling

Several developments

- Akyuz et. al. *The schwinger-keldysh coset construction*, arXiv:2306.17232
- Moudgalya, Motrunic, Symmetries as Ground States of Local Superoperators: Hydrodynamic Implications, arXiv:2309.15167
- Delacretaz, *A bound on thermalization from diffusive fluctuations* arXiv:2310.16948
- Huang et. al., *Hydrodynamics as the effective field theory of strong-to-weak spontaneous symmetry breaking* arXiv:2407.08760

- Schwinger-Keldysh action naturally has "doubled" symmetry as well.
- Assuming SWSSB allows one to write down effective field theory easily.
- Hydrodynamical mode (particularly diffusion) appears to be **Goldstone boson** of broken doubled symmetry (down to diagonal one). Often **susceptibility** χ is referred to as the **order parameter** of the SWSSB in the effective field theory language.
- But... is it really right?

Discrepancy

Hydrodynamics

- Observables are **linear** in the density matrix $\mathrm{tr}\rho O$
- Dynamical statement; defined with respect to an underlying dynamics.
- In principle by measuring correlation decays one can always identify it easily.

SWSSB

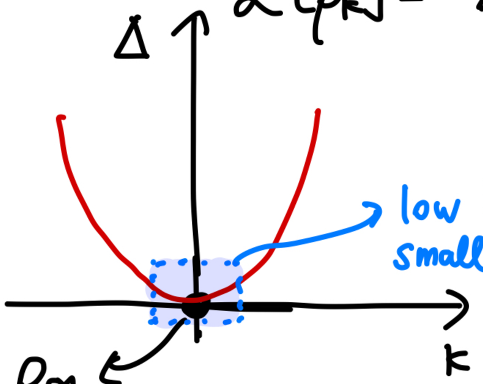
- Observables are **nonlinear** in the density matrix.
- Static statement; defined with respect to a given density matrix with **definite charge.**
- Identifying SWSSB is in principle exponentially hard

Feng, Cheng, Ippoliti, Phys. Rev. Lett. 135 (2025)

General picture

Ogunnaike, Feldmeier, **JYL**, arXiv:2304.13028

$\mathcal{L}[\rho_k^i] = -\Delta_k^i \rho_k$



low k
small Δ
⇒ hydro regime.

steady state ρ_∞

SWSSB $R(r\to\infty) \neq 0$

non-zero charge structure factor $S(k\to 0) \neq 0$

Enforce non-trivial low "Energy" spectrum

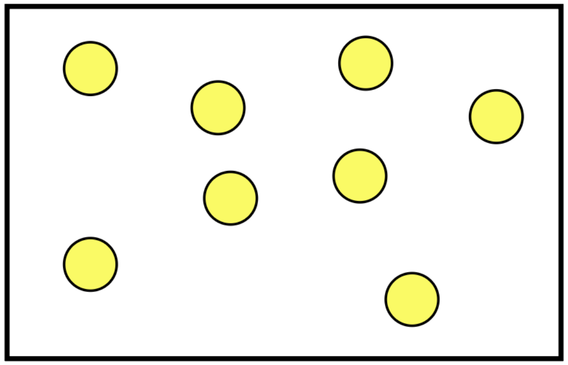
$\Delta_k \sim \frac{ck^2}{S(k)}$

⇒ $S(0)$ is important*

Finite structural factor in $k \to 0$ = **extensive** subsystem fluctuation
How can we guarantee such a behavior just by SWSSB?

Intuitive Picture

(a) **Each charge is localized**

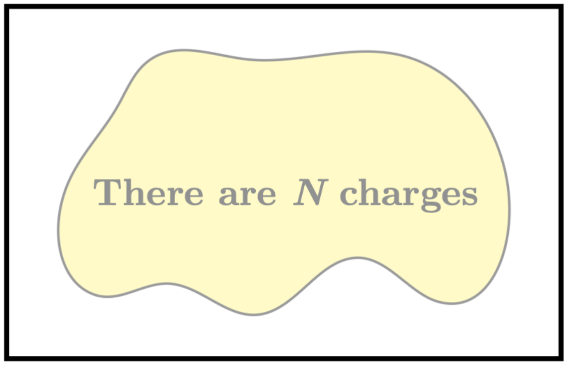


$\rho \sim \prod_i \mathcal{P}_{q_{r_i}=1}$ **locally correctable**

$\Downarrow$

O(1) subsystem charge variance

(b) **Charges are scrambled**



$\rho \sim \mathcal{P}_{q_{\text{tot}}=N}$ **Not locally correctable**

$\Downarrow$

Extensive subsystem charge variance

Main Result

JYL, arXiv:2605.05288

SWSSB

- Long-ranged Renyi-1 correlator for some charged operator, i.e.,

$$\lim_{r\to\infty} R_r = R_\infty > 0$$

Fast approaching R_r

- Renyi-1 correlator approaches to its asymptotic value rapidly

$$R_r \sim R_\infty + c/r^\alpha, \quad \alpha > d-1$$

$$\Downarrow$$

Extensive subsystem charge variance $\mathrm{Var}(Q_V) \sim |V|$

Static structural factor stays finite $\lim_{k\to 0} S(k) > 0$

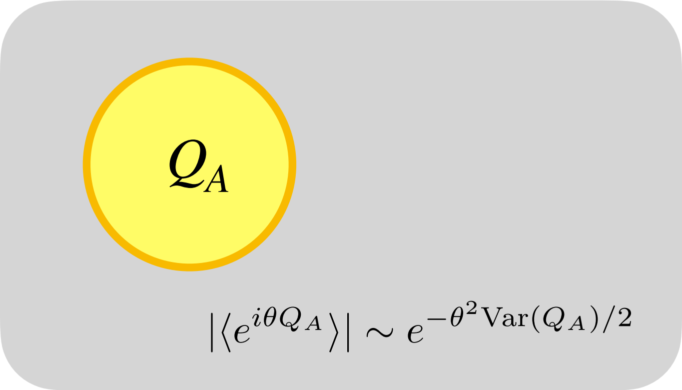
Main Proof Idea

JYL, arXiv:2605.05288

- **Robertson inequality**: Long-ranged Renyi-1 correlator implies $\mathrm{Var}(Q_V)\mathrm{Var}(Y_V) \geq |V|^2$
  - Here Y_V is the magnitude of an real-part of the order parameter
- Decompose a given SWSSB density matrix into **convex sum** of superselection sectors (since they are orthogonal against local perturbations)
  - For each superselection sector (with definite phase variable φ), variance of the order parameter scales extensively $\mathrm{Var}(Y_V) \leq |V|$ **as long as Renyi-1 correlator decays fast enough, i.e., power-law tail has exponent larger than** $d-1$
- Obtain the desired result

SWSSB and Fluctuations

JYL, arXiv:2605.05288



Theorem 2 (SWSSB and lower bound on charge variance)**.** *Consider a strongly* U(1) *symmetric state ρ with a long-ranged Rényi-1 correlator of the charge transfer operator. If the Rényi-1 correlator converges to its asymptotic value faster than $1/r^d$, where d is the spatial dimension, then the subsystem charge variance scales extensively, i.e.,*

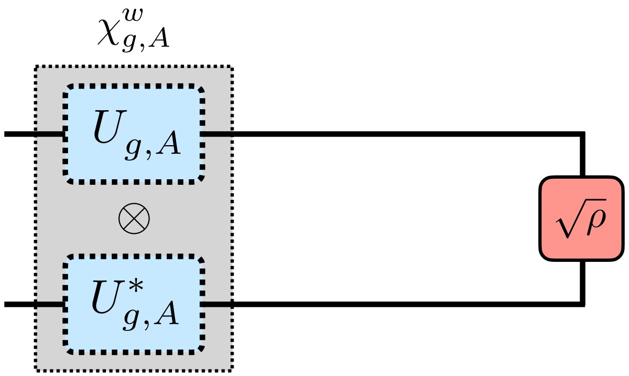
$$\mathrm{Var}(Q_A) \geq c|A|. \tag{8}$$

for some $c > 0$. This also implies that the charge structure factor in the $k \to 0$ limit is nonzero.

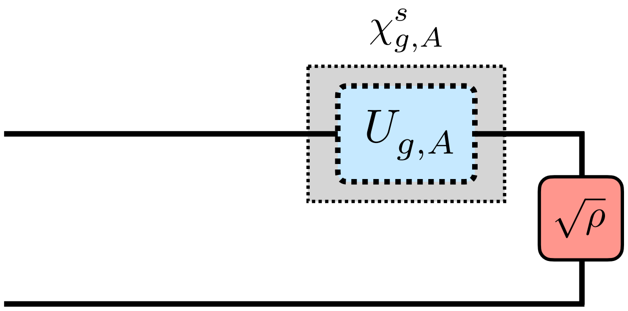
State	Long-range diagnostic	$\mathrm{Var}(Q_A)$	Charge coherence	
Uniform ensemble	$C(r) = 0$ $R(r) = \nu(1-\nu)$	extensive	$I_{\mathrm{WY}} = 0$	SWSSB state with scrambled charge yet no ordinary LRO.
Dicke state	$C(r) = \nu(1-\nu)$ $R(r) = \nu^2(1-\nu)^2$	extensive	$I_{\mathrm{WY}} = \mathrm{Var}(Q_A)$	Extensive charge variance is compatible with genuine symmetry breaking.
Superfluid / XY ferromagnet	Long-ranged $C(r)$ & $R(r)$ with $1/r^{d-1}$ tail.	subextensive	$I_{\mathrm{WY}} = \mathrm{Var}(Q_A)$	Ordinary SSB example motivating the dephased construction below.
Dephased superfluid	$C(r) = 0$, while $R(r) \sim R_\infty + a/r^{d-1}$.	subextensive	$I_{\mathrm{WY}} = 0$	SWSSB counterexample showing that fast Rényi convergence is needed.
Sparse projector ρ_Ω	no long-ranged $R(r)$; no SWSSB.	extensive	$I_{\mathrm{WY}} = 0$	Counterexample showing that the converse of Theorem 2 is false.

Disorder operator (With Caution)

JYL, arXiv:2605.05288



Nonlinear
Weak twist



Linear
Strong twist

$$\chi^w_{g,A} := \langle\!\langle \sqrt{\rho} \| U_{g,A} \otimes U^*_{g,A} \| \sqrt{\rho} \rangle\!\rangle = \mathrm{tr}\left(\sqrt{\rho} U_{g,A} \sqrt{\rho} U^\dagger_{g,A}\right),$$
$$\chi^s_{g,A} := \langle\!\langle \sqrt{\rho} \| U_{g,A} \otimes \mathrm{Id} \| \sqrt{\rho} \rangle\!\rangle = \mathrm{tr}\left(\rho U_{g,A}\right).$$

Second derivative of χ^w gives rise to Wigner-Yanase Information

$$I_{\mathrm{WY}}(\rho, O) := -\frac{1}{2}\mathrm{tr}\left([O, \sqrt{\rho}]^2\right)$$

Strong disorder simply corresponds to generating function of charge moments

Unlike Transverse-field Ising model physics,
there is no order-disorder incompatibility!

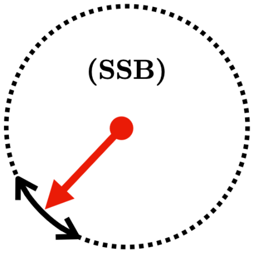
Levin, arXiv:1903.09028

Summary

- Hydrodynamics is seemingly related to this **strong-to-weak SSB.**
- At the effective field theory level, this approach provides pretty nice organizing principle.
- However, **rigorous connection** requires further assumptions.

Things I couldn't talk due to lack of time — **Hydrodynamical Nambu-Goldstone Thm**

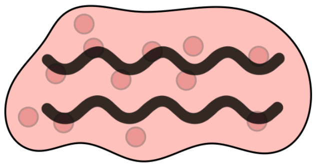
$$\lim_{h\to 0}\langle m(h)\rangle \neq 0$$



$$\langle H\rangle_k \sim E(k)$$

$$\lim_{k\to 0}\langle S(k)\rangle \neq 0$$

$$\rho = \frac{1}{d_Q}\mathcal{P}_Q \quad \textbf{(SWSSB)}$$



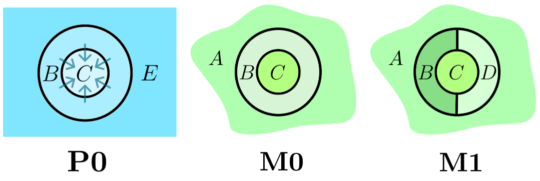
$$\mathcal{L}^\dagger[O_k] \sim -i\omega(k)O_k$$

- Providing Nambu-Goldstone theorem **rigorously** for a given lattice Hamiltonian's ground state (or thermal state) requires several assumptions. One can easily **circumvent** by violating assumptions.
- Similarly, this hand-wavy idea of hydro=goldstone requires some refinement to be **proven** for lattice level Lindbladian evolution. In fact, there are several important assumptions to be added.

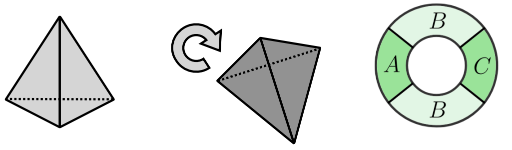
Other works

TH Yang, B Shi, **JYL** arXiv:2506.04221
A Vijay, **JYL** arXiv:2512.22121

Gap analog: “Fixed points conditions”



Characterization for “Different fixed points”

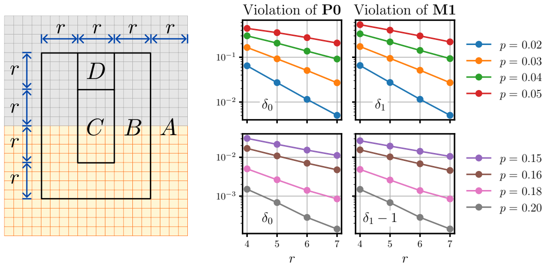


Memory Capacity

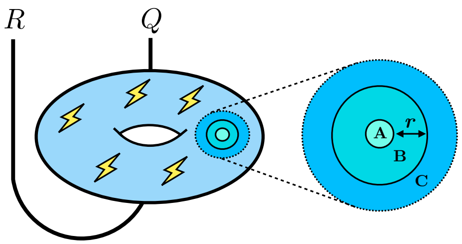
Automorphism Actions

Topological Entropy

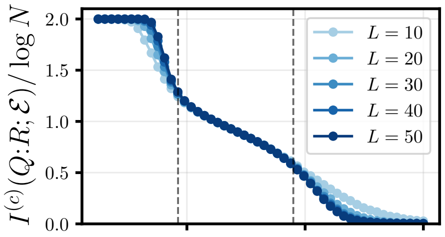
Equivalence Relation “Coarse-grained axioms”



Gapless analog: “Information critical phase”



Fractional value for “Decodable Information”



QLRO phase for SWSSB

Krammers-Wannier dual model