The era of synthetic quantum matter

… progress driven by hardware development of devices.

… scope of numerical methods limited.

… a portfolio of novel analytical concepts — linear algebraic, tensor networks, discrete stat-mech…

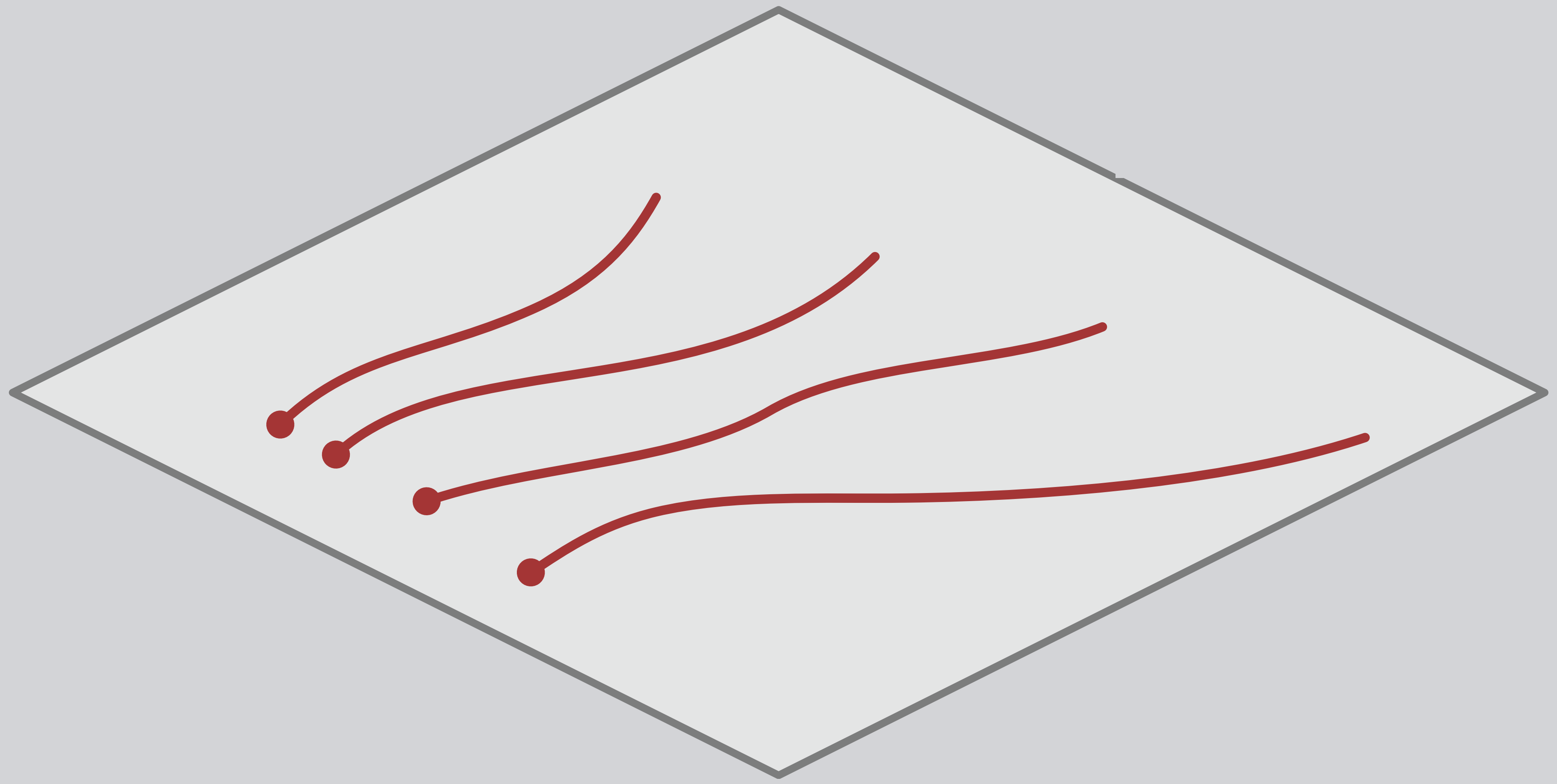
Q: Why is there so little QFT in the physics of entangled quantum matter?

Functional integration
RG Methods
CFT
Emergence & collective modes
Variational approaches
Large deviation & instanton calculus
⋮

Q1: Are there conceptual reasons? Or …

Q2: "Just" methodological roadblocks?

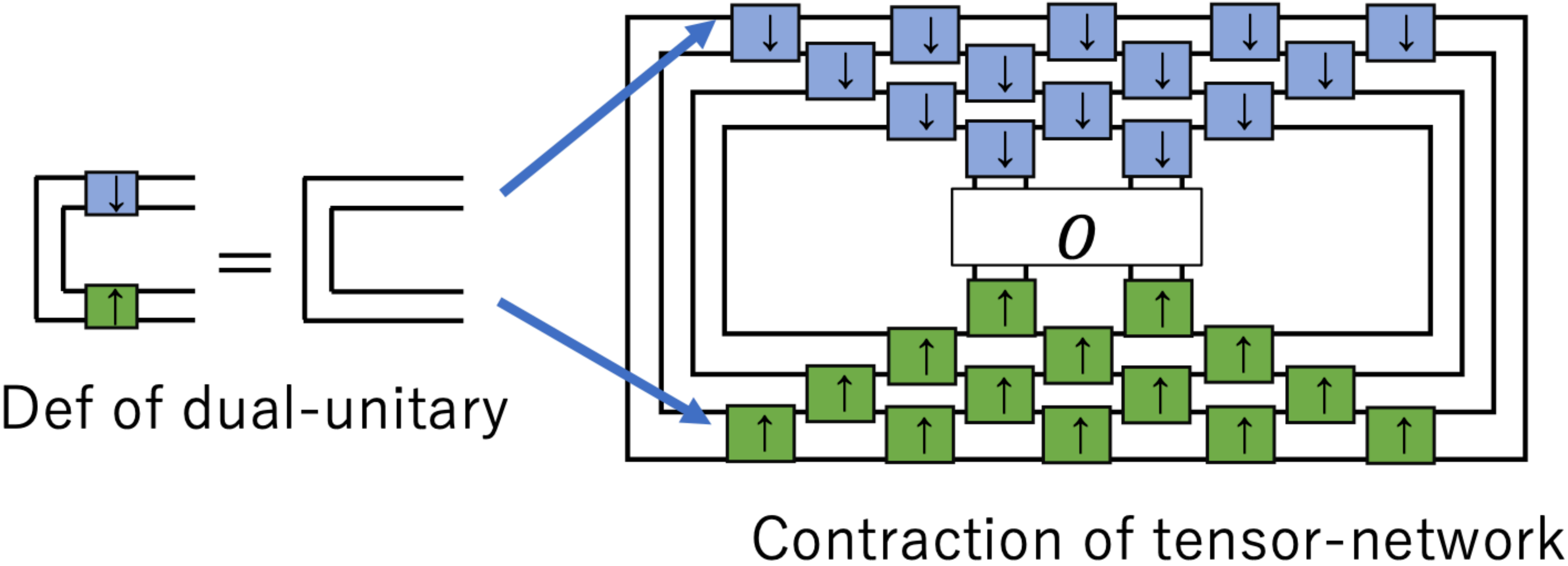
Q3: And what would QFT be good for anyway?



Example: circuit dynamics

stat. mech. models,
tensor networks
Weingarten calculus, …

Ludwig, Chalker, Nahum, Garratt, De Luca, Chan, Pollmann, Vasseur, Potter, …



Suzuki *et al.* 21

Example: circuit dynamics

stat. mech. models,
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Weingarten calculus, …

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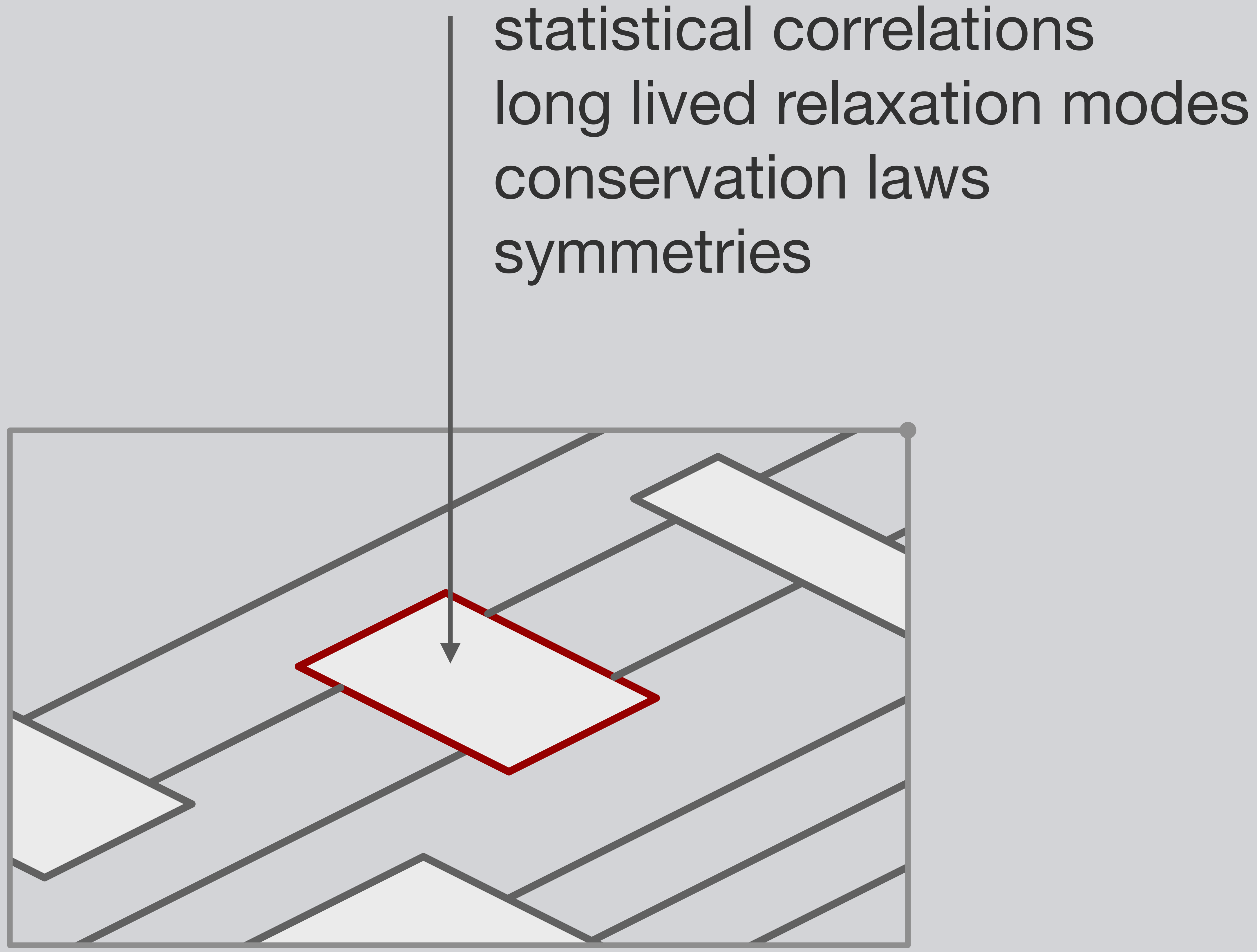
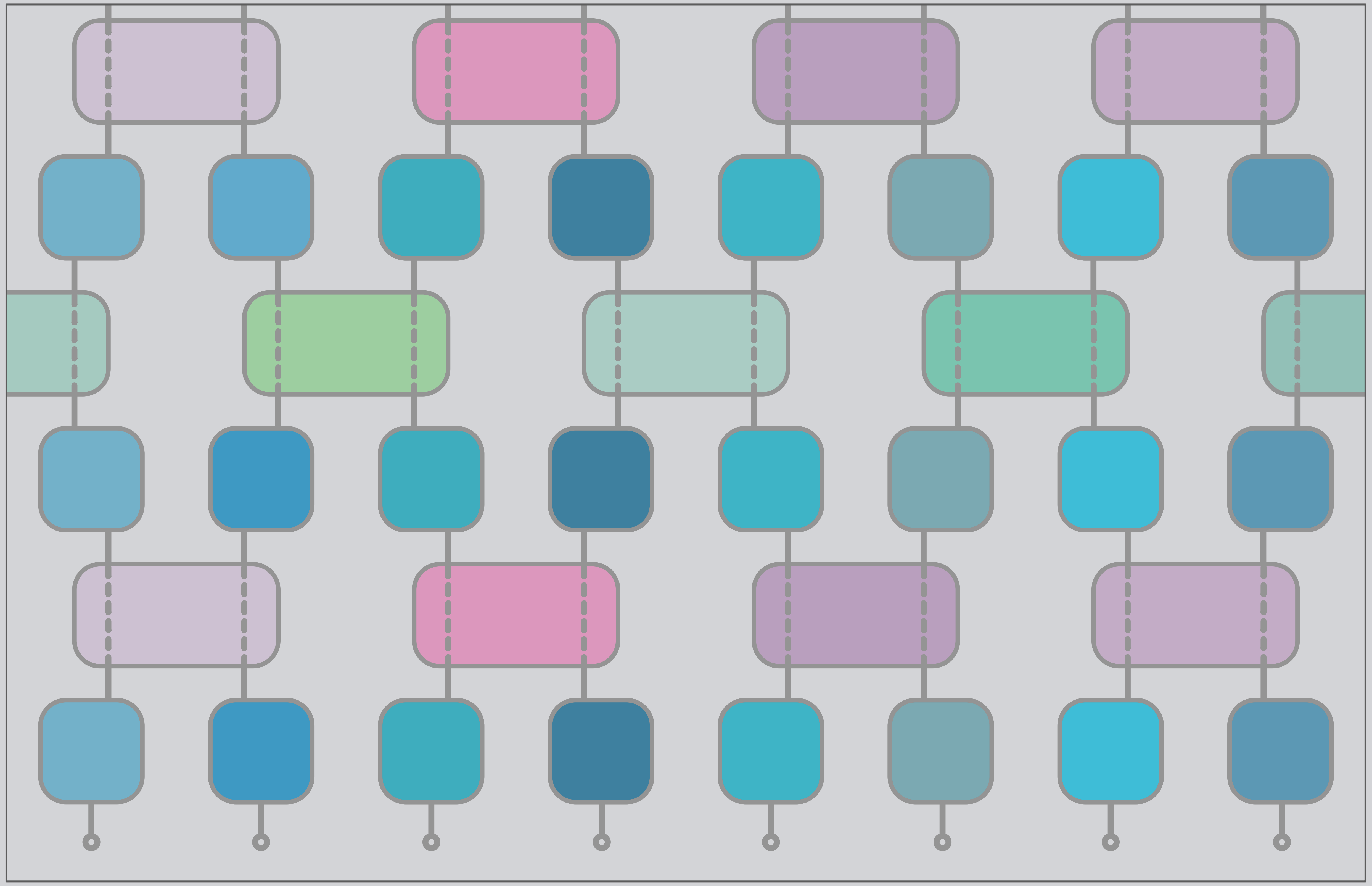
Here: develop path integral approach to

- resolve non-perturbative* evolution
- access wider families of system classes



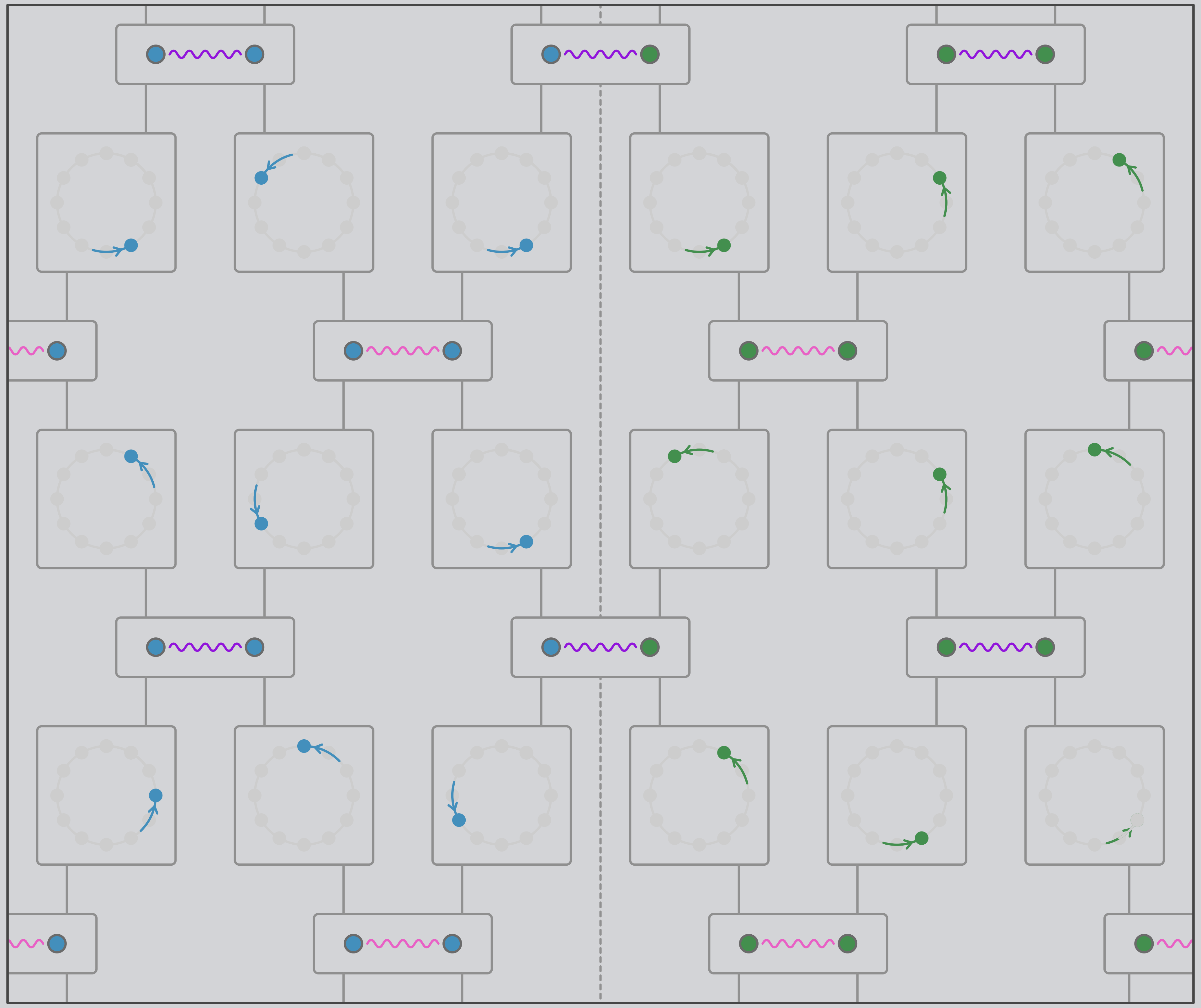
*in circuit dimensions, D.

This talk: Discuss approach for two case studies



Coupled unitary circuits:

Non-perturbative dynamics



Coupled quantum rotors:

Proxies of non-integrability in quantum hardware

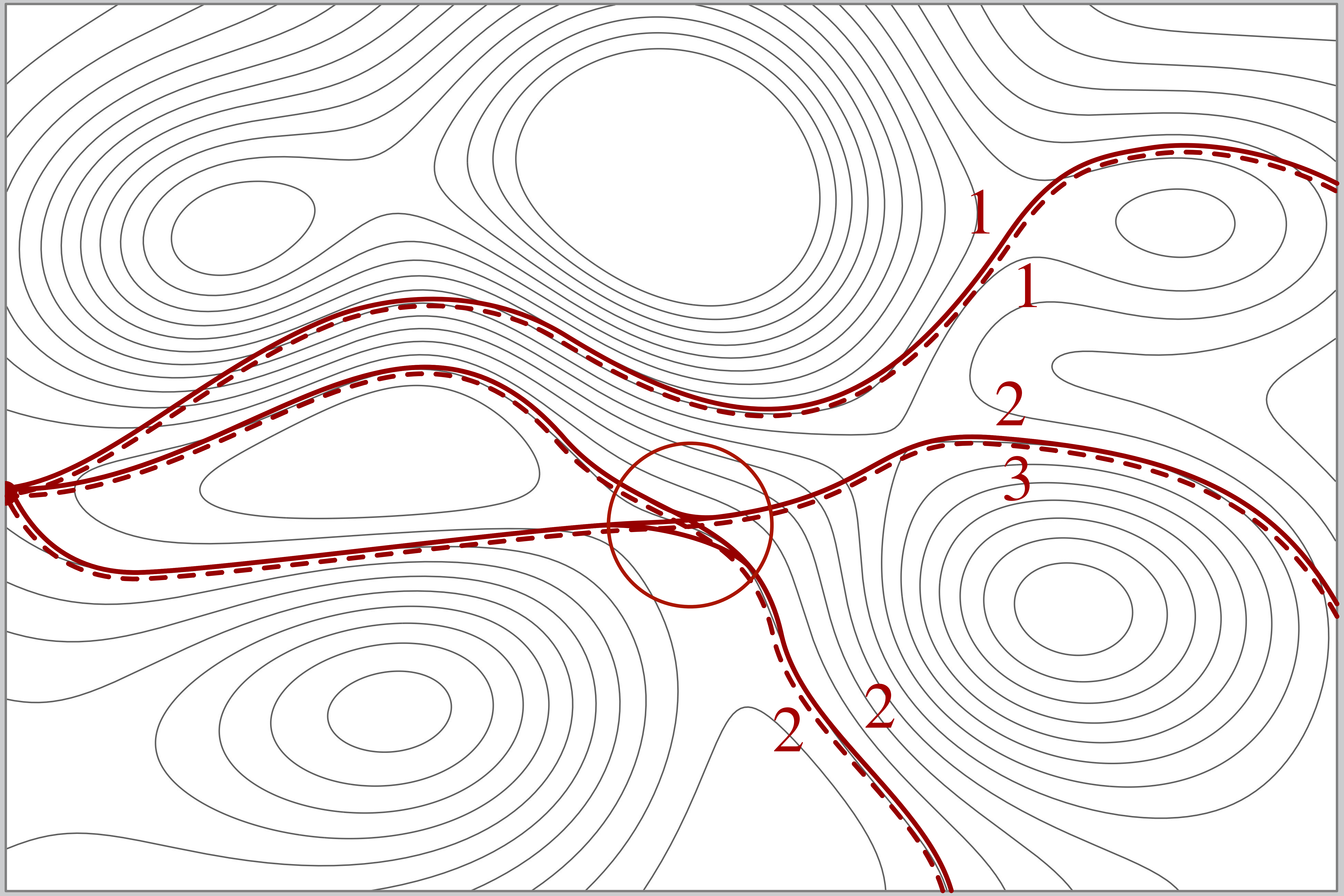
Field integral approach to many-body quantum chaos

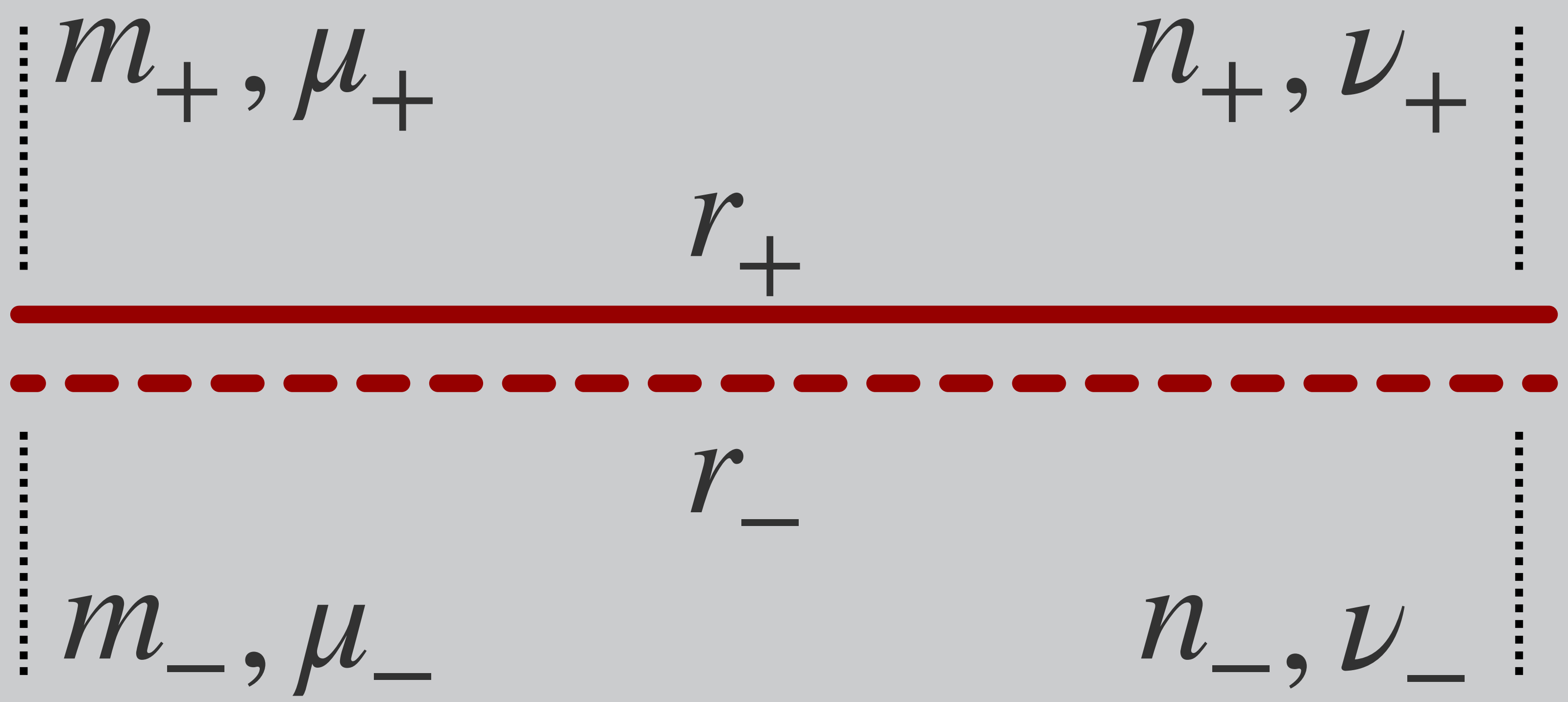
- … is based on two principles: 1) chaos as a symmetry breaking phenomenon, combined with 2) a semiclassical understanding of its Goldstone modes.

- Intermediate time scales: Causal symmetry spontaneously broken

- late time scales: symmetry restoration

Goldstone modes of causal symmetry breaking



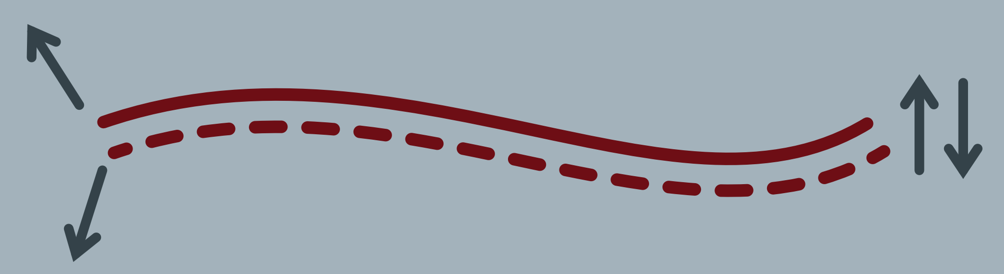


Phase coherent pair amplitudes depending on …

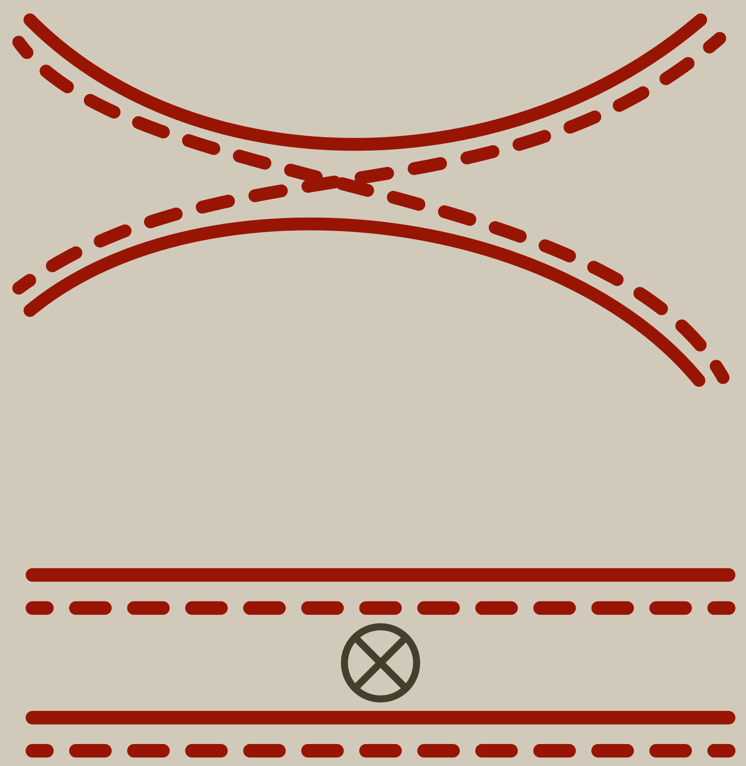
- (discrete) time, $m_\pm$
- Hilbert space labels $\mu_\pm$
- replicas $r_\pm$

Modes $B^{r_+ r_-}_{(m_+\mu_+)(n_-,\mu_-)}$ d.o.f. of QFT

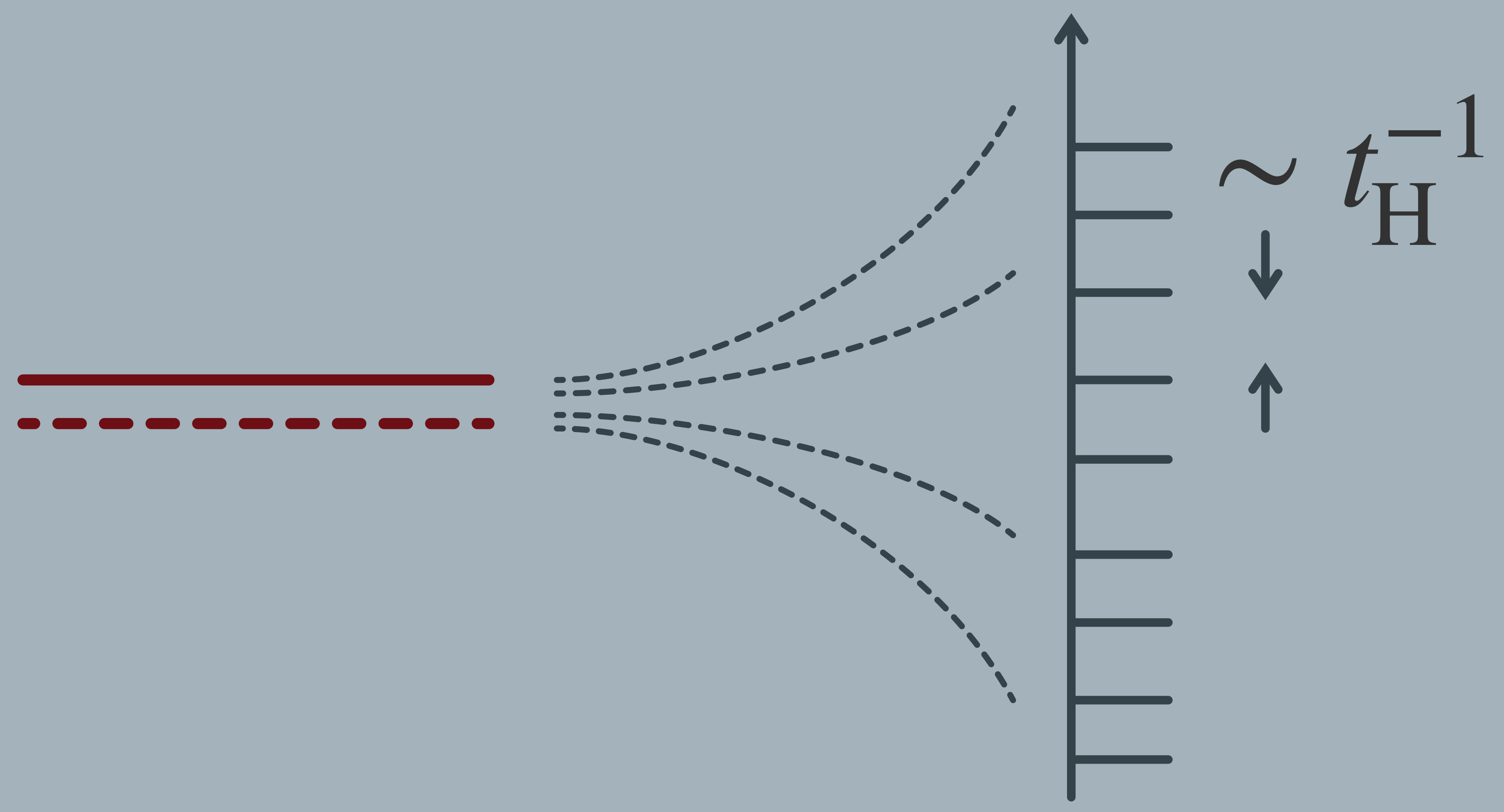
Hierarchy of B-modes



pre-ergodic: relaxation of slow system variables



ergodic: $B = \{B^{r_+ r_-}_{m_+ m_-}\}$ become Goldstone modes.
Replica correlations and mode clustering



deep quantum: nonperturbatively strong Goldstone mode fluctuations restoring unitarity/causality.

Field Thoery

zero-dimensional σ-model

'hydrodynamic' mode theory

t_{erg}

t_{H}

time

Solution strategy

1. Start from exact discrete time Grassmann field integral representation
2. Pass to effective B-variable formulation by generalized Hubbard-Stratonovich transformation (aka Luttinger-Ward construction)
3. Do B-integral in large D limit

Field theory approach to circuit dynamics

Kyoto, Jun 26

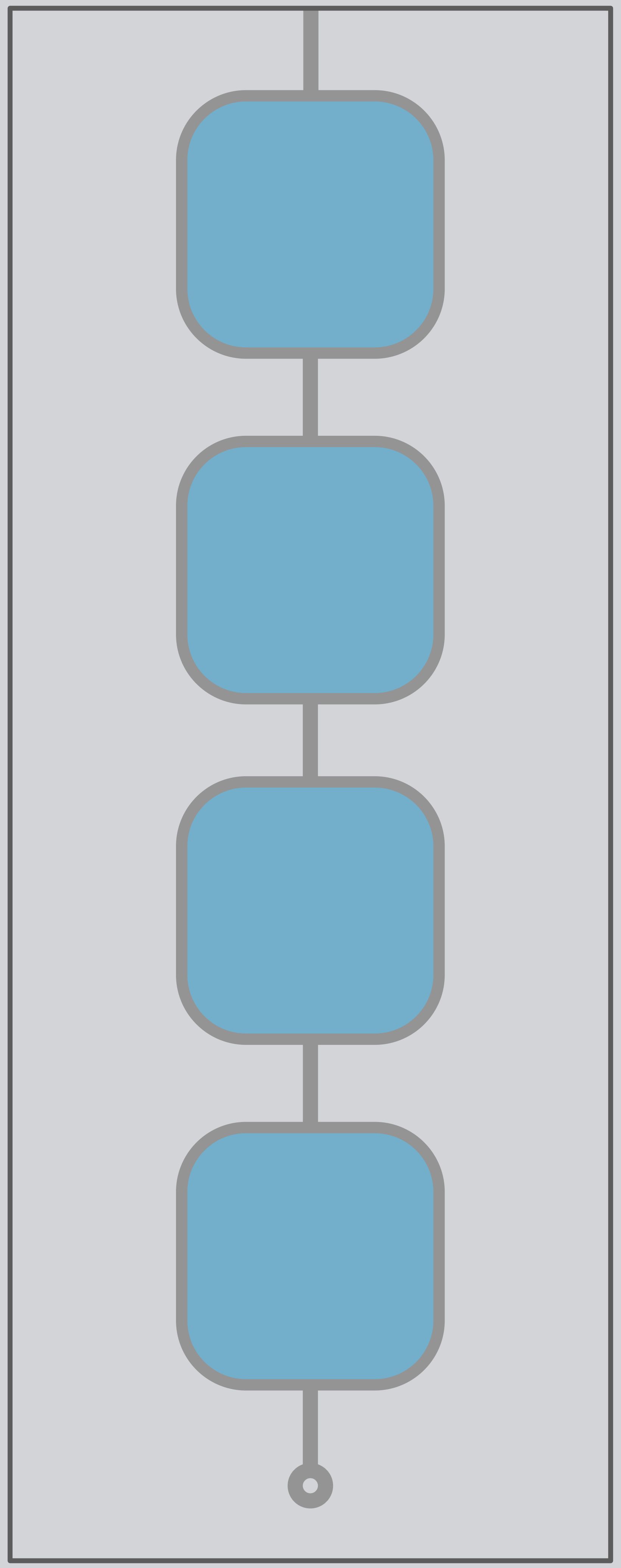
Alexander Altland, University of Cologne
Tobias Micklitz, CBPF Rio de Janeiro
Kim Kun Woo, Chung-Ang University
Felix Dusel, Frank Pollmann, TU Munich

1. Haar random circuit
2. $\mathrm{Haar}^{\otimes 2}$
3. Brickwork thermalization
4. Rotor entanglement

arXiv:2404.10919, PRR 24
arXiv:2509.06028, PRR 26
unpublished

Haar circuit

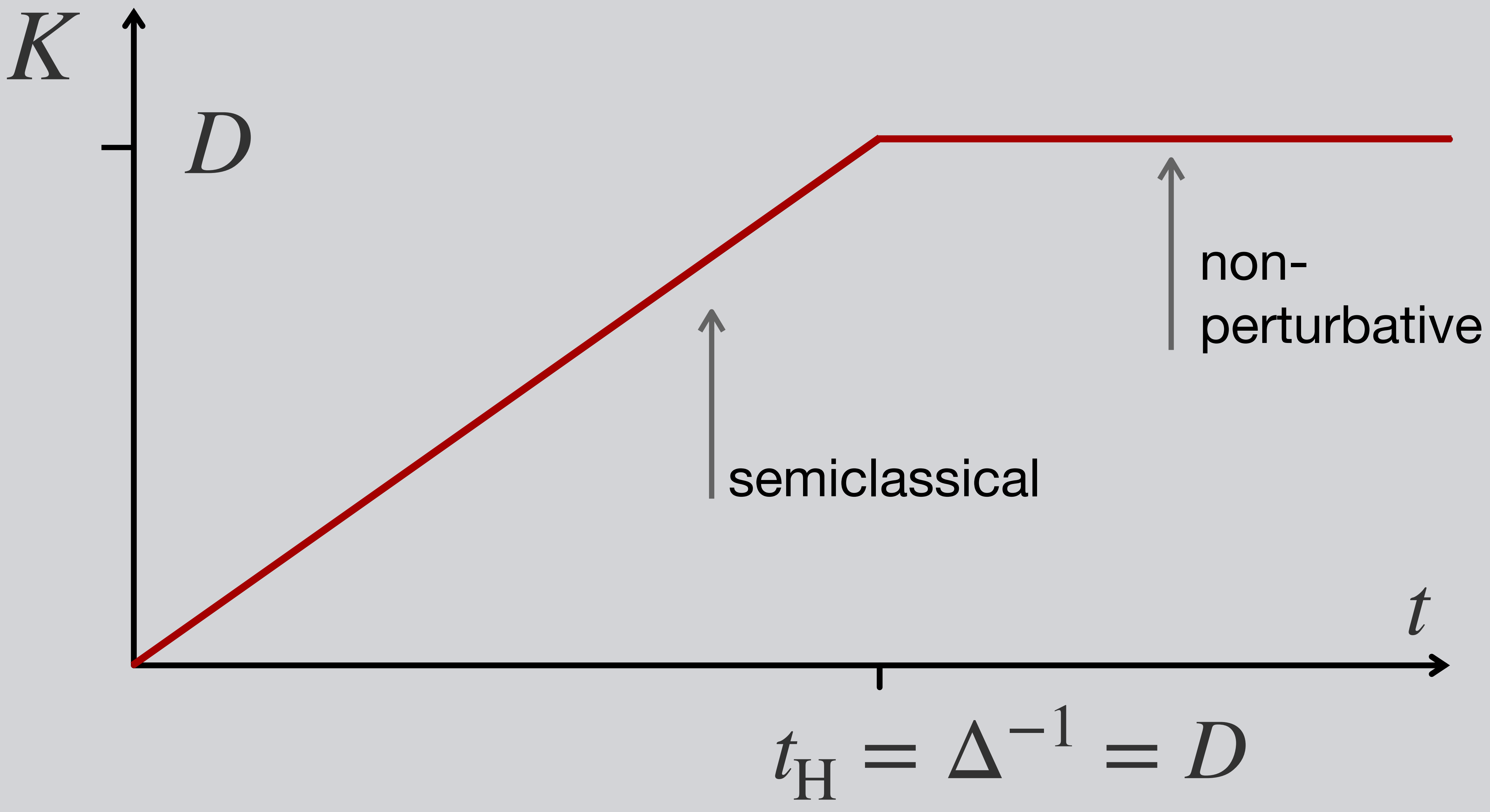
Single Haar circuit



Haar distributed unitary
U of dimension D

Consider discrete time evolution U^t witnessed by spectral form factor

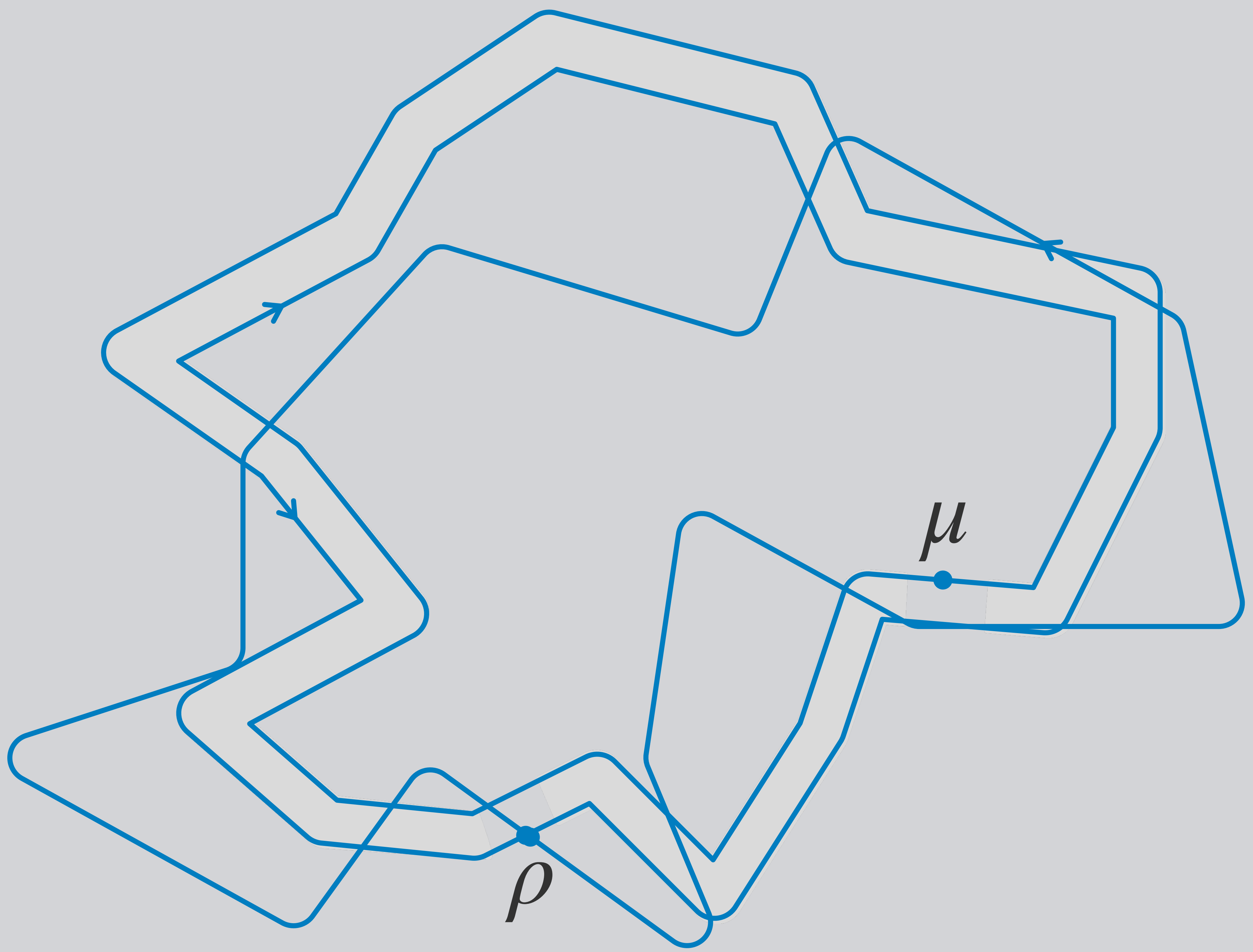
$$K(t) = \left\langle \mathrm{tr}\left(U^t\right)\ \mathrm{tr}\left(U^{\dagger t}\right)\right\rangle$$



Semiclassical analysis

$$\mathrm{tr}\left(U^t\right) = \sum_{\mu} \left\langle \mu \left| U^t \right| \mu \right\rangle$$

$$\mathrm{tr}\left(U^{\dagger t}\right) = \sum_{\rho} \left\langle \rho \left| U^{\dagger t} \right| \rho \right\rangle$$



Semiclassical analysis

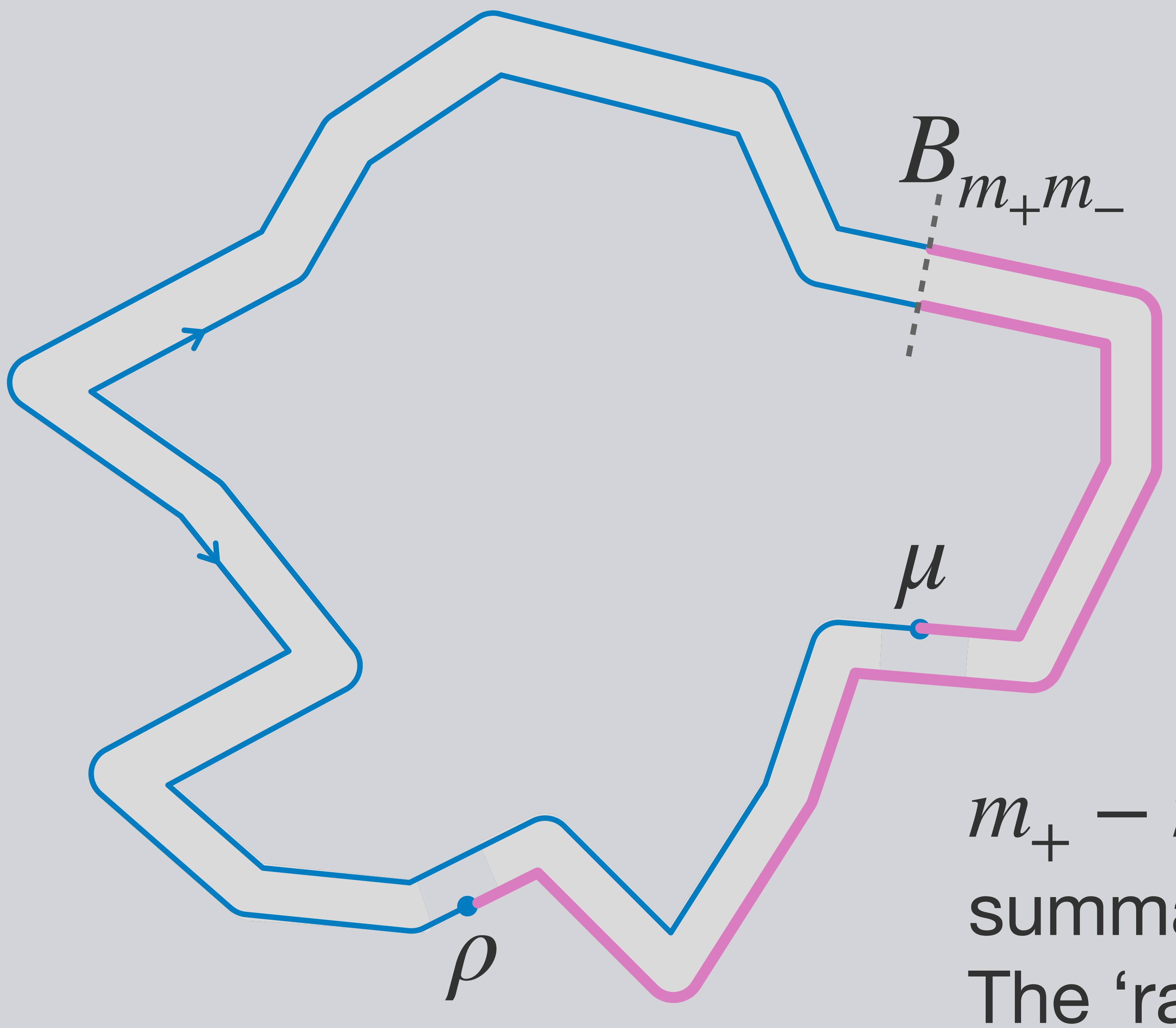
$$\mathrm{tr}\left(U^t\right) = \sum_\mu \left\langle \mu \left| U^t \right| \mu \right\rangle$$

$$\mathrm{tr}\left(U^{\dagger t}\right) = \sum_\rho \left\langle \rho \left| U^{\dagger t} \right| \rho \right\rangle$$

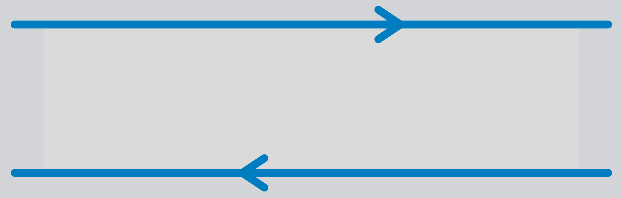
$$B_{nm} \sim \sum_\nu \psi^+_{m_+,\nu} \bar{\psi}^-_{m_-,\nu}$$

Hilbert space

discrete time



$m_+ - m_- \equiv \tau$ free summation coordinate. The ‘ramp’

$$\Pi_{n,m} = \; n$$  $$m$$

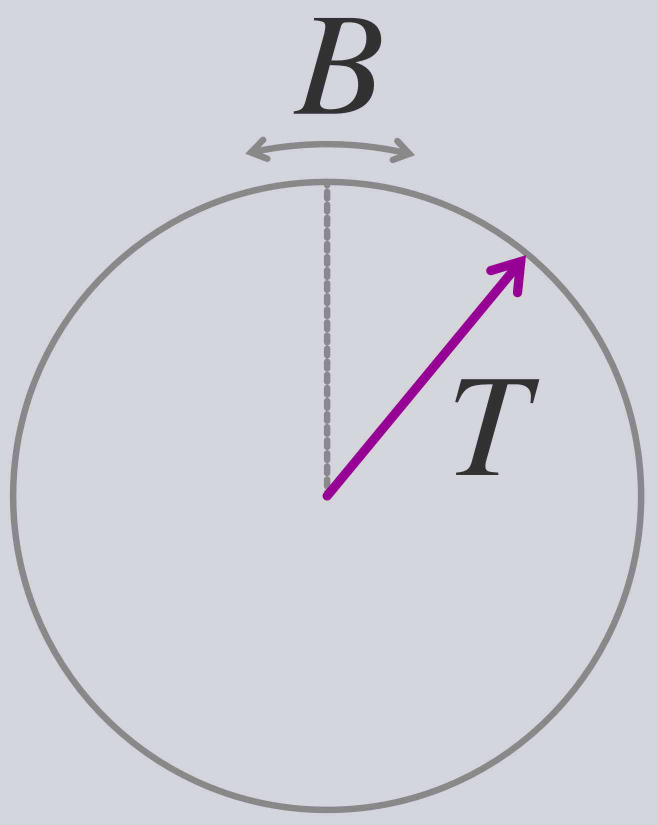
$$= \frac{1}{D}\delta_{\Delta n,\Delta m}\Theta(\bar{n} - \bar{m})$$

$$\Delta n = n_+ - n_-$$

$$\bar{n} \equiv \frac{n_+ + n_-}{2}$$

Nonlinear theory

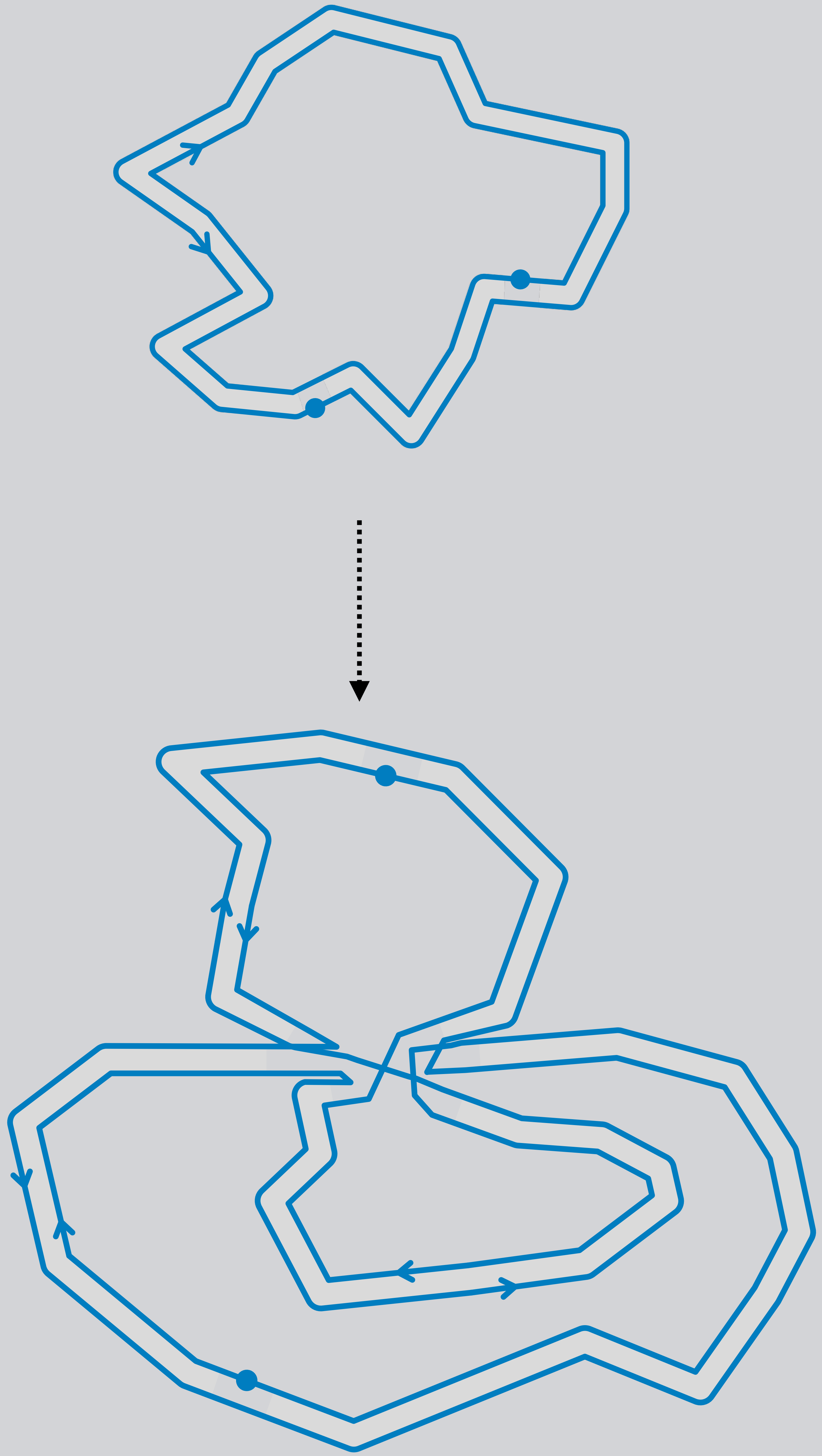
$$S[B] = D\,\mathrm{tr}(B^\dagger d^{*}B) + \ldots$$



$$T = \exp\begin{pmatrix} & B \\ -B^\dagger & \end{pmatrix}$$

$$S[T] = \frac{D}{2}\,\mathrm{tr}(T\tau_3 dT^{-1})$$

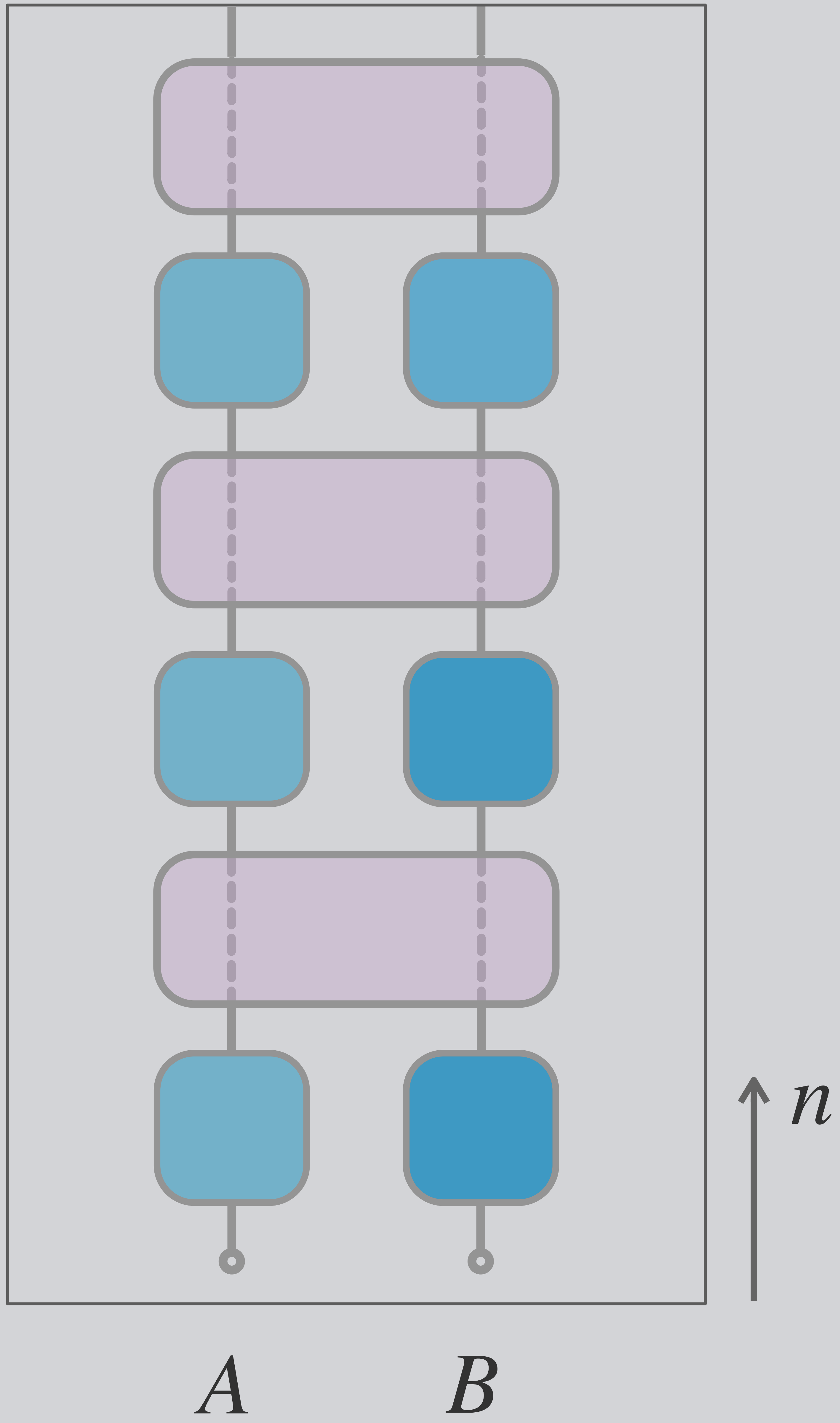
Larkin 68
Wegner 79
Efetov 81 …



$^{*}(dX)_{nm} = X_{n+1m+1} - X_{nm}$

$(\text{Haar circuit})^{\otimes 2}$

Warmup: unitary two-qudit dynamics

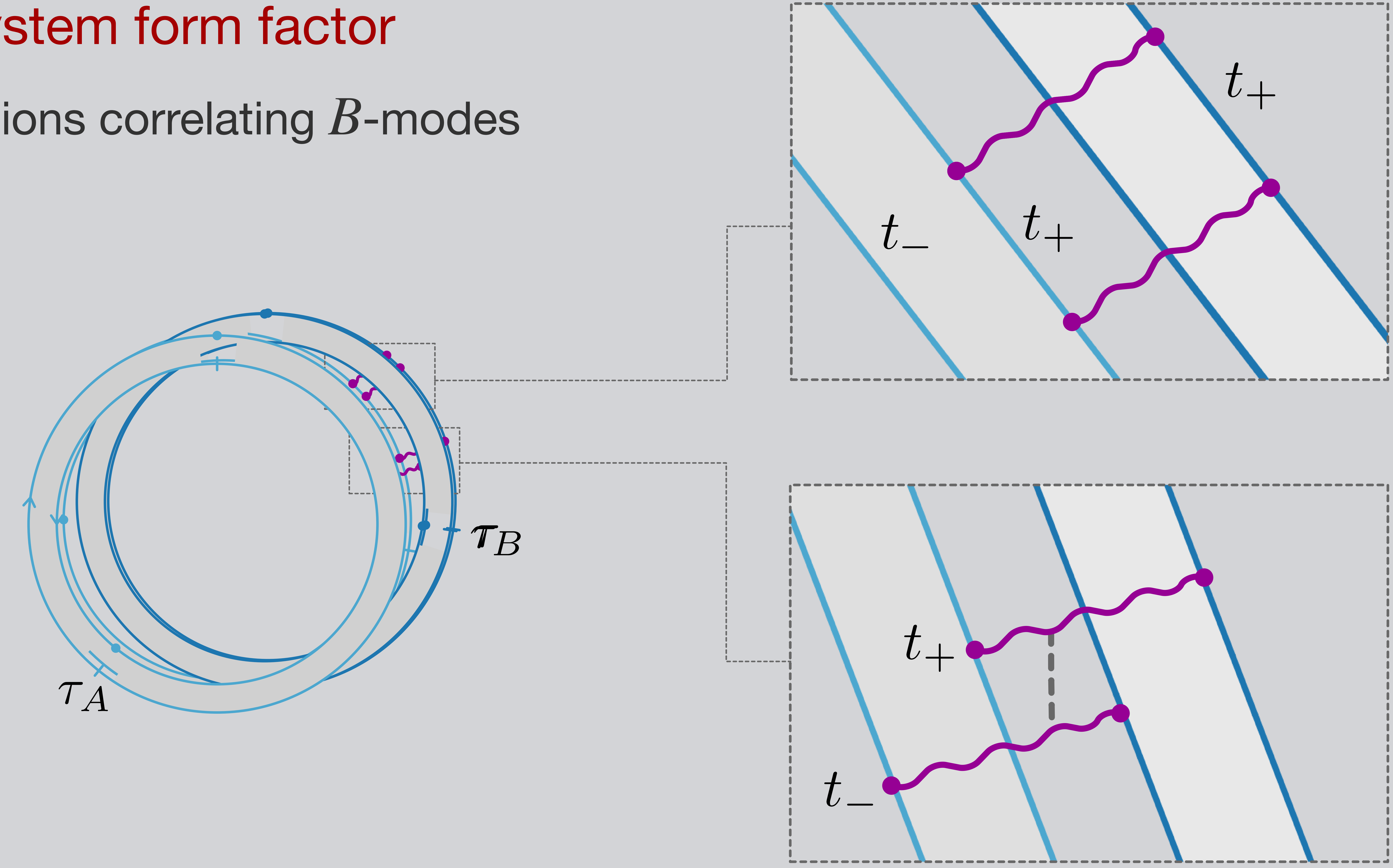


$: \; U = \{U_{\mu\rho}\}$

$: \; \exp(-iHt), \qquad \left| \left\langle H_{\mu\nu,\rho\sigma} \right\rangle \right| = \frac{\Lambda^2}{2D^2}$

Two system form factor

Interactions correlating B-modes



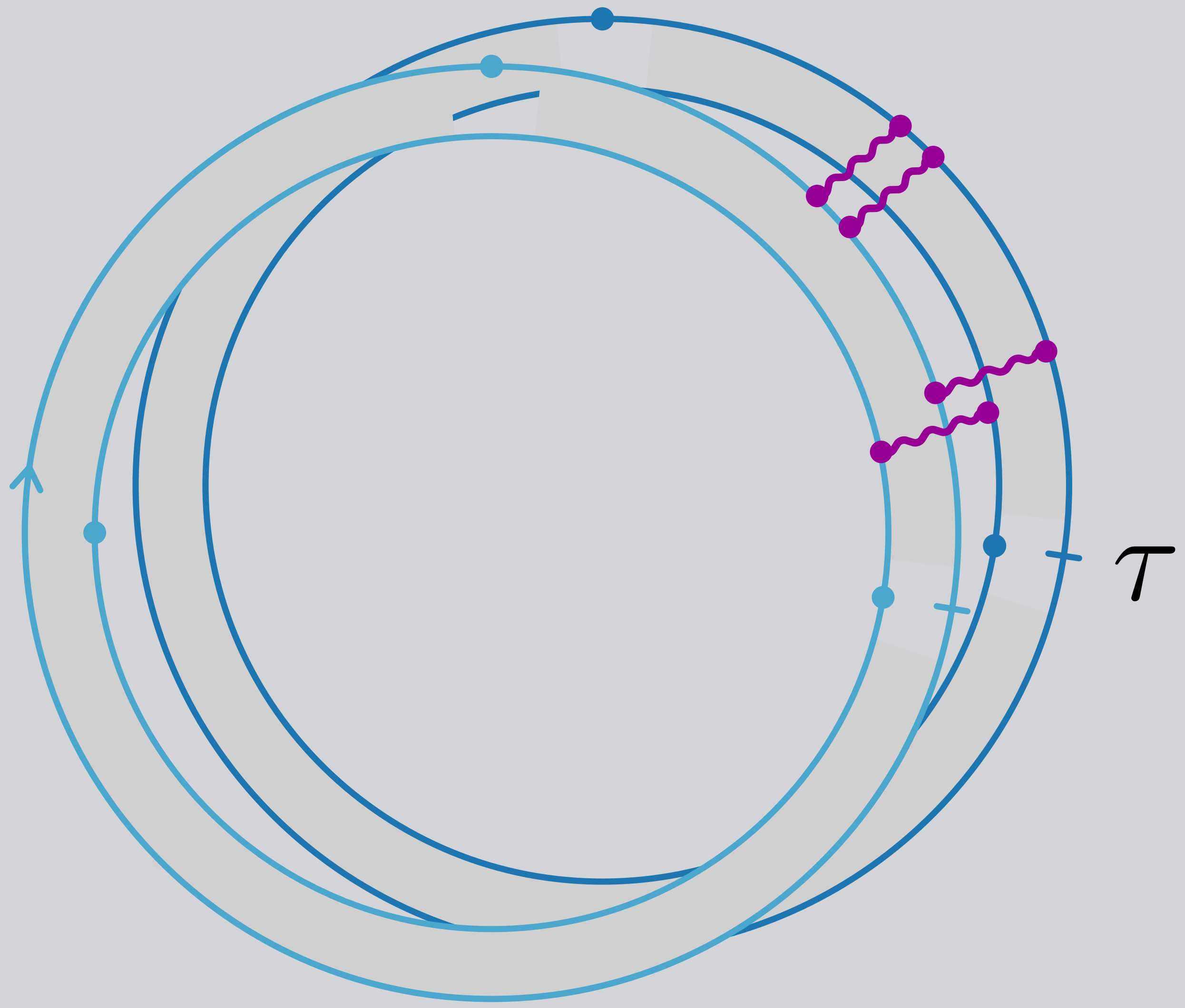
Dynamically synchronized, $\tau_A = \tau_B$ leads to $K(t) = t^2$

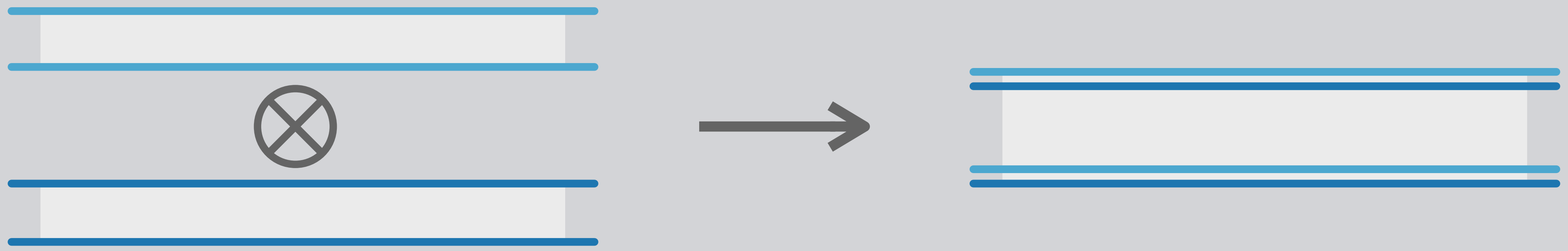
Absence of coupling: two zero-modes τ_A, τ_B, amplitudes escape decoherence

Asymptotic form factor $K(t) = t$.

Two system form factor

Interactions correlating B-modes



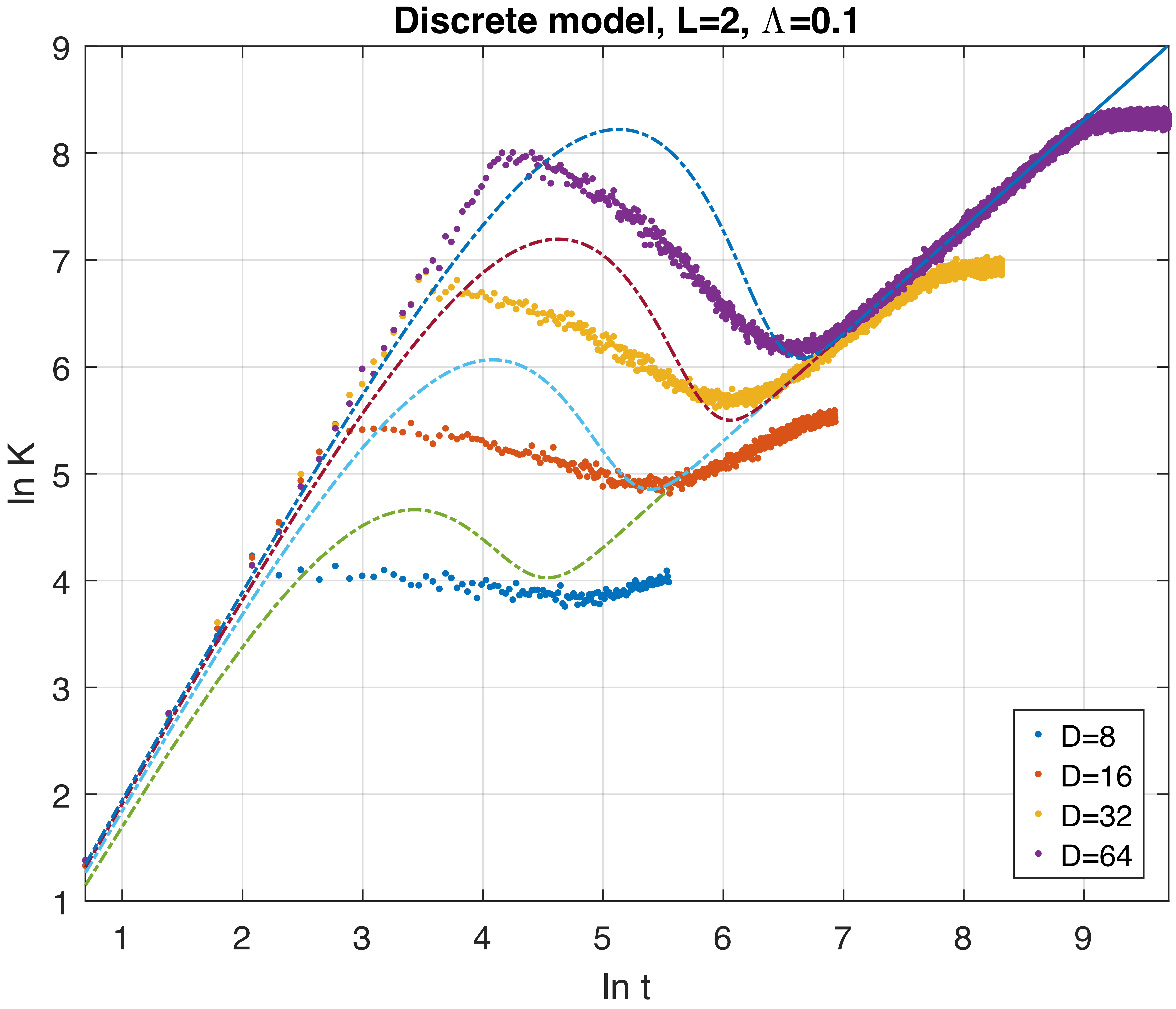


$$K(t) = \left(1 + (t-1)\, e^{-\frac{\Gamma t}{2}}\right)^2 + (t-1)\left(1 - e^{-\frac{\Gamma t}{2}}\right)^2$$

Chan *et al.* 18

Essentially: $$K(t) \sim \begin{cases} t^2, & \Gamma t \ll 1 \\ t, & \Gamma t \gg 1 \end{cases}$$

Numerical validation



Nonperturbative extension

Semiclassical form factor

$$K(t) = \left(1 + (t-1)\, e^{-\frac{\Gamma t}{2}}\right)^2 + (t-1)\left(1 - e^{-\frac{\Gamma t}{2}}\right)^2$$

Chan *et al.* 18

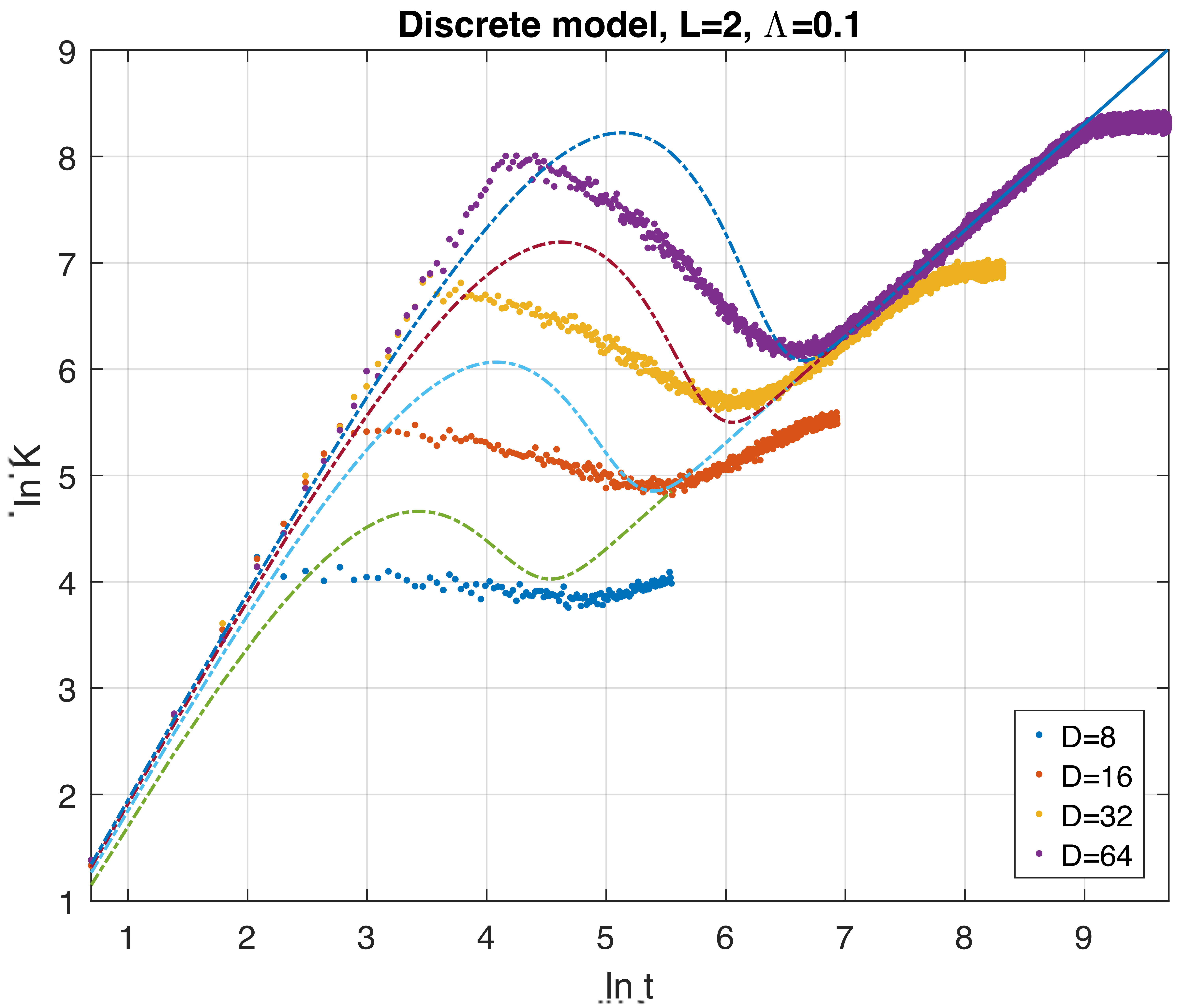
Inclusion of non-perturbative effects (relevant for moderate values of D)

$$K(t) = \left(1 + K_1(t)e^{-\frac{\Gamma t}{2}}\right)^2 + K_2(t)\left(1 - e^{-\frac{\Gamma t}{2}}\right)^2$$

aa, Micklitz *et al.* 24

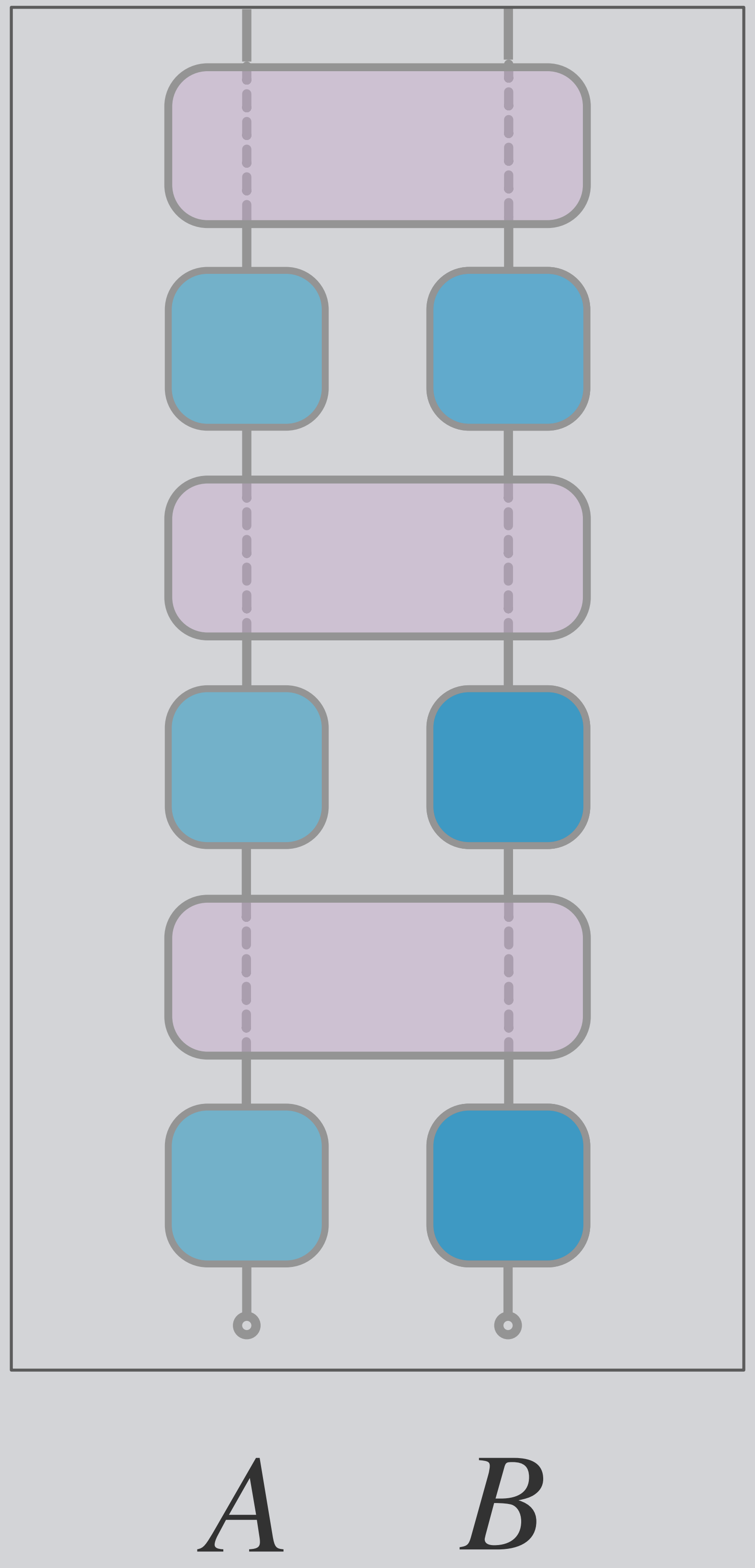
$$K_n(t) = t - \Theta(t - D^n)(t - D^n)$$

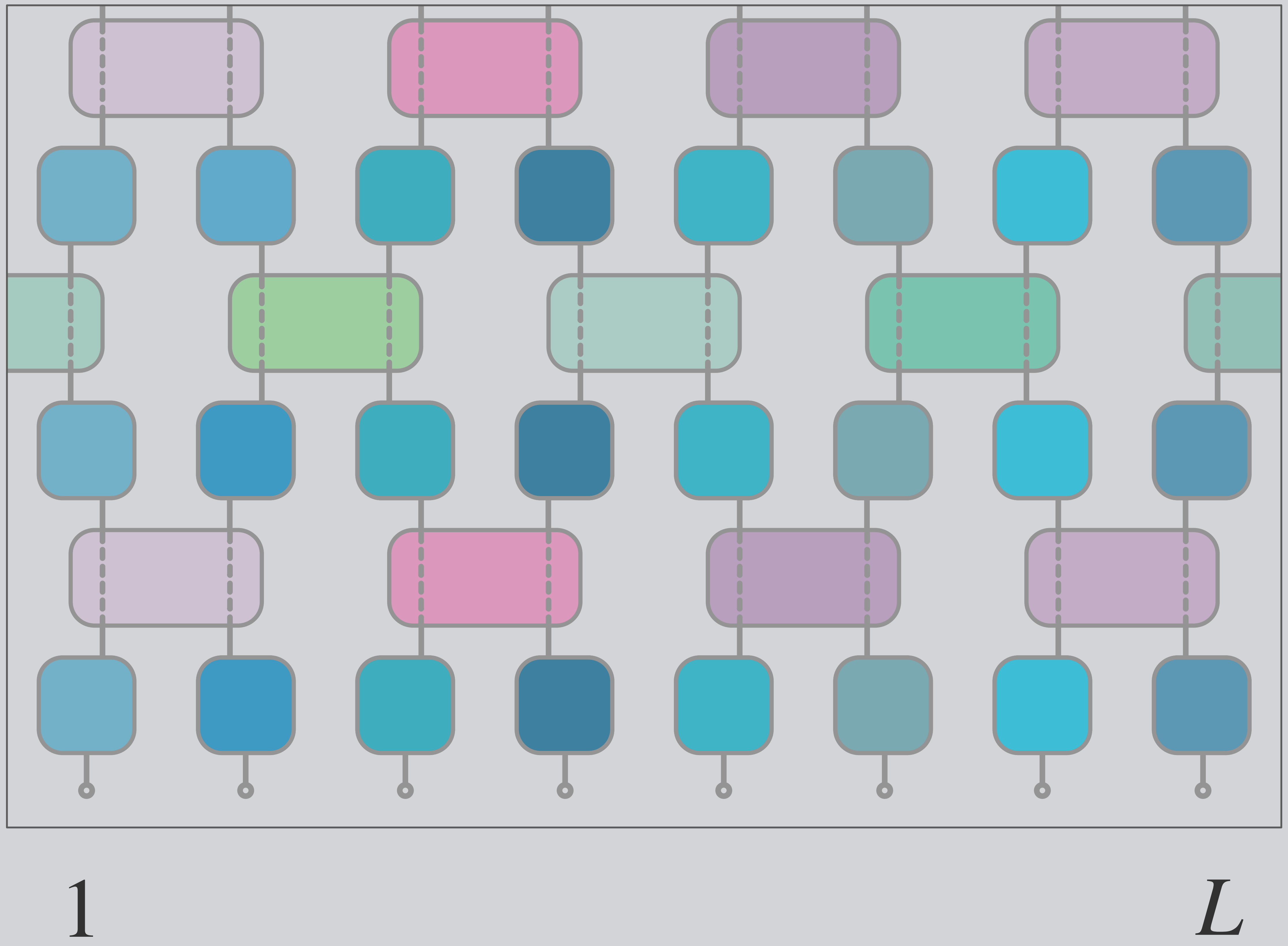
Numerical validation



Brickwork thermalization

L-qudit chain





Form factor results

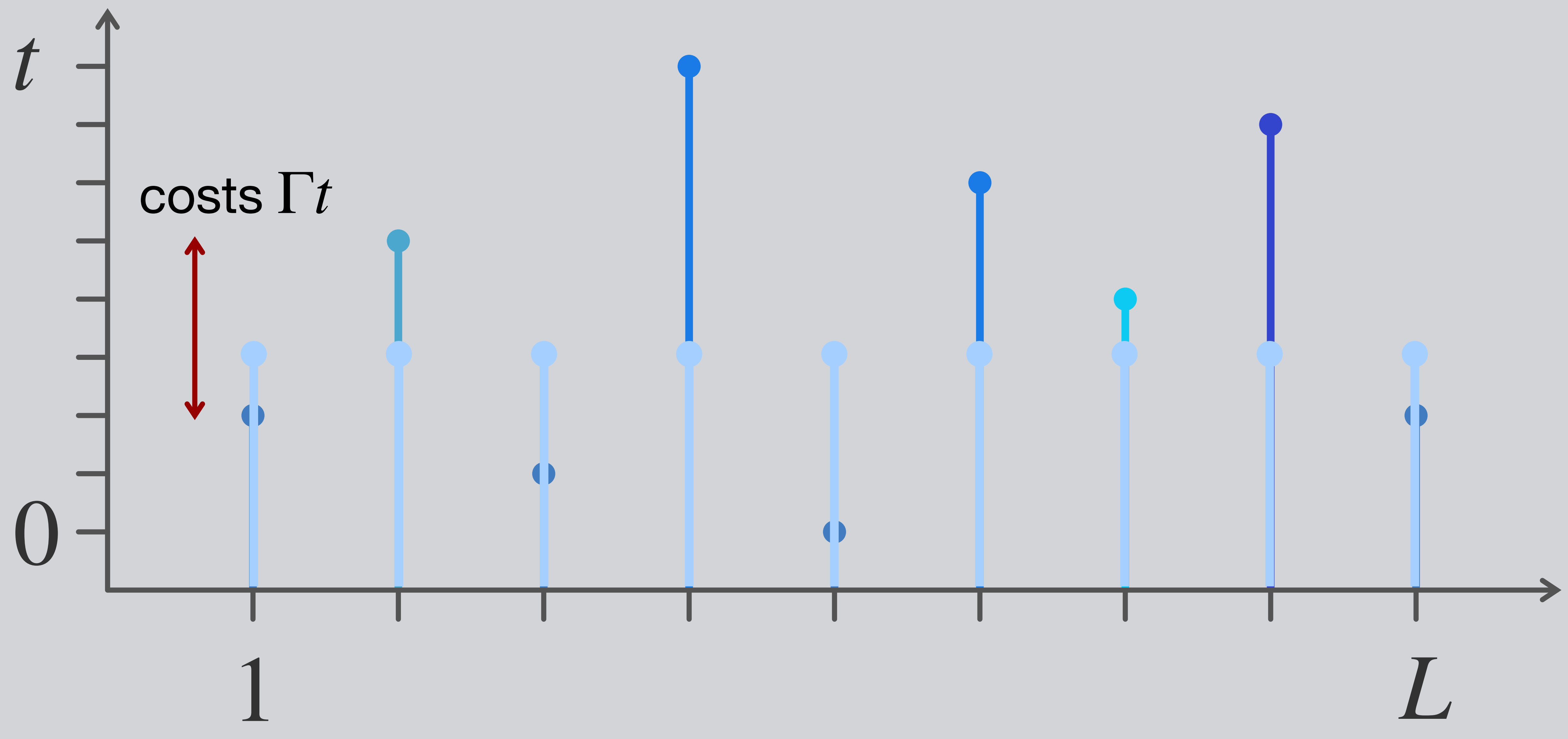
$$K(t) \simeq \left(1 + te^{-\frac{\Gamma t}{2}}\right)^L + t\left(1 - e^{-\frac{\Gamma t}{2}}\right)^L$$

Partition sum of t-state Potts model

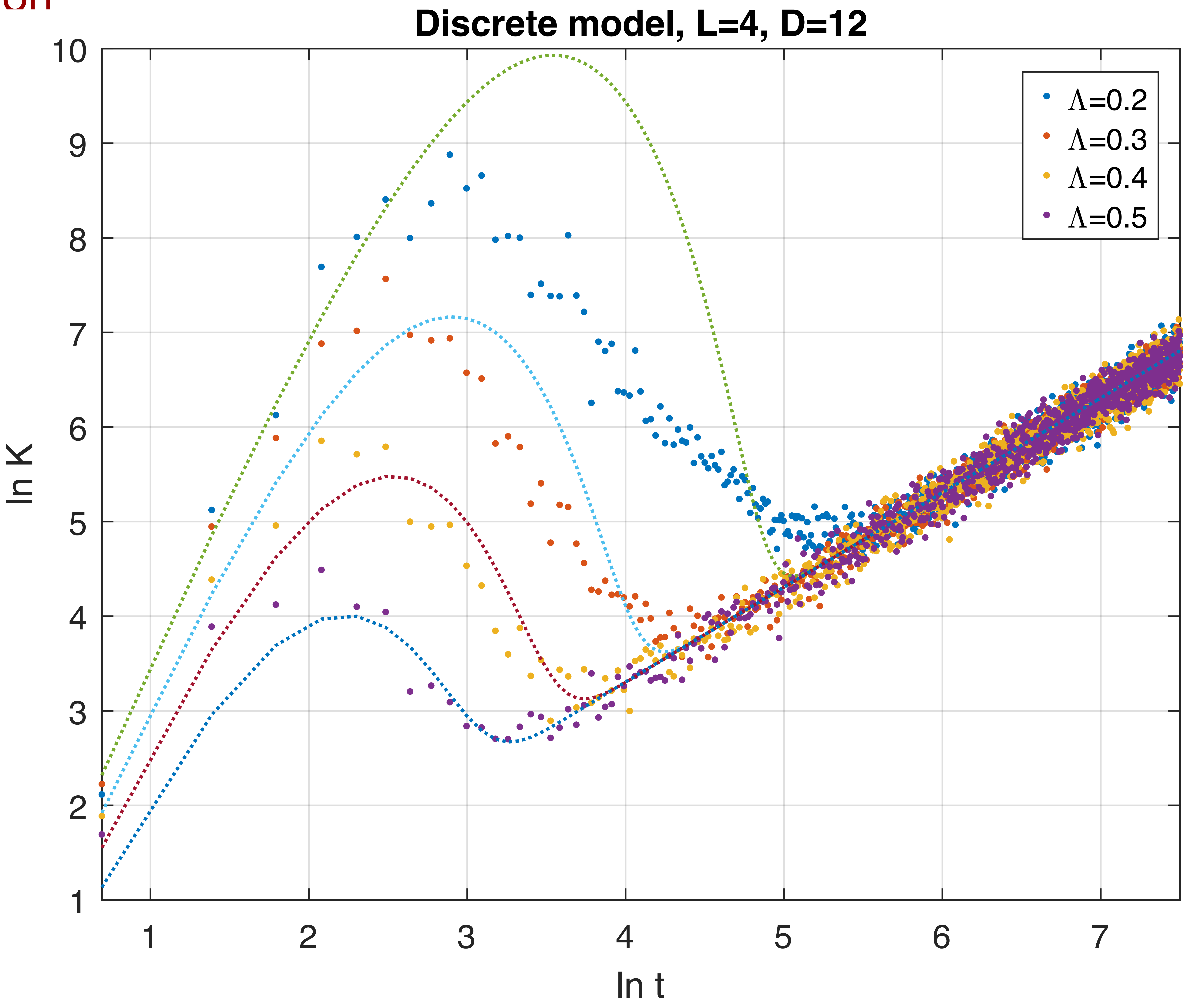
$$K(t) = \sum_{\{S_l\}} e^{-\frac{\Gamma t}{2}\sum_{\langle j,l\rangle}\delta_{S_j,S_l}}$$

$$S_l \in \mathbb{Z}_t$$

Chan *et al.* 18



Numerical validation



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Chan *et al.* 18

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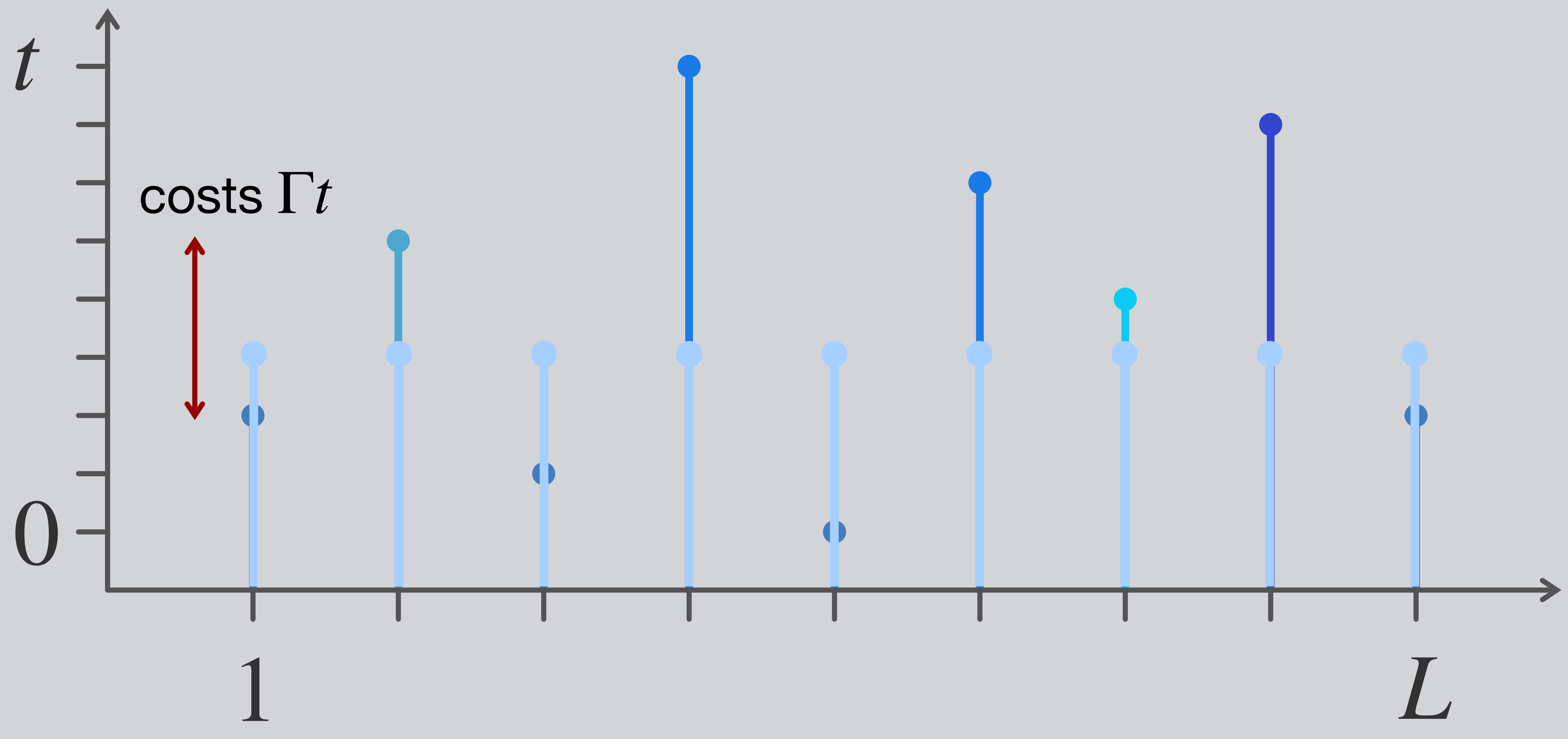
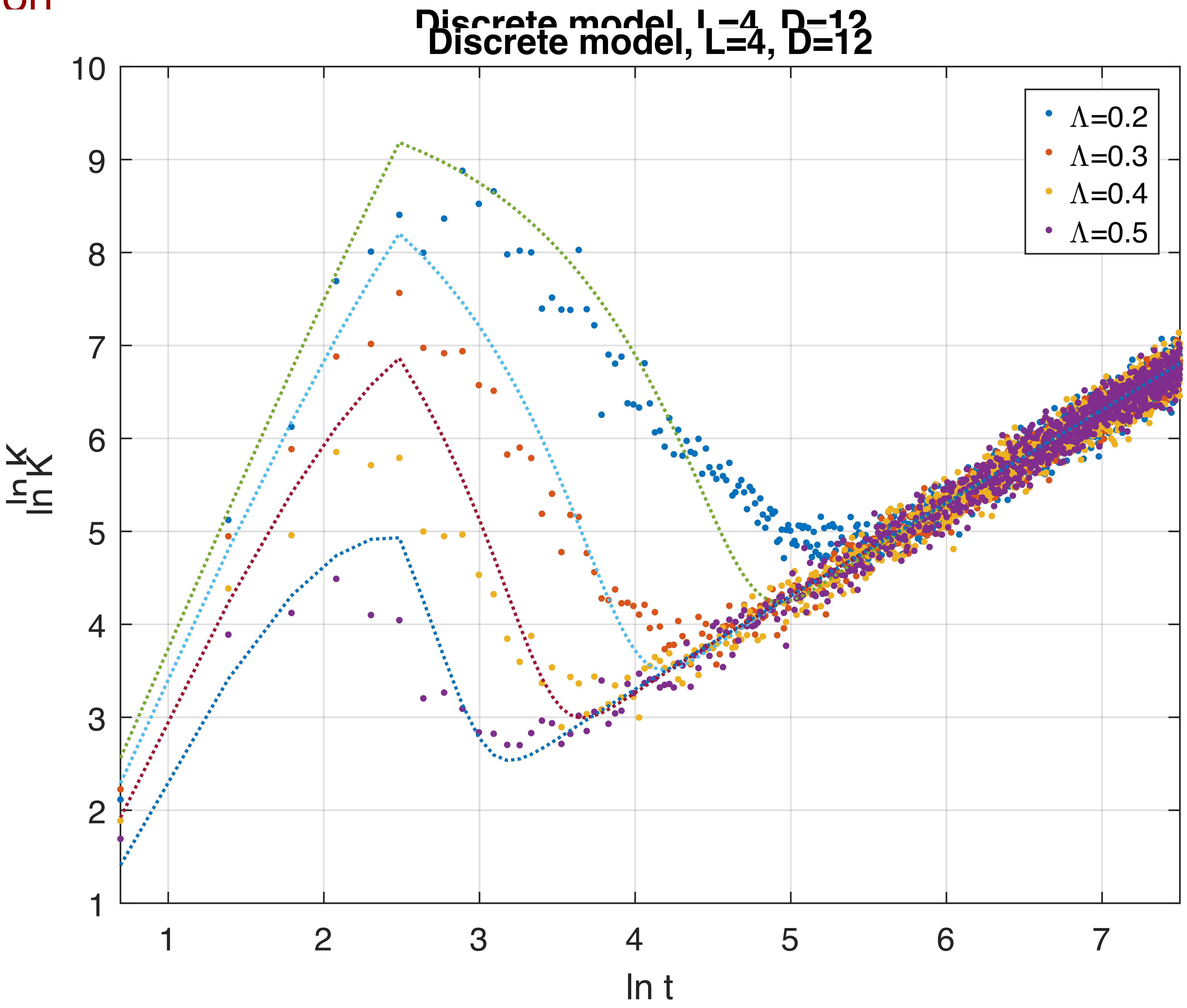


Non-perturbative generalization

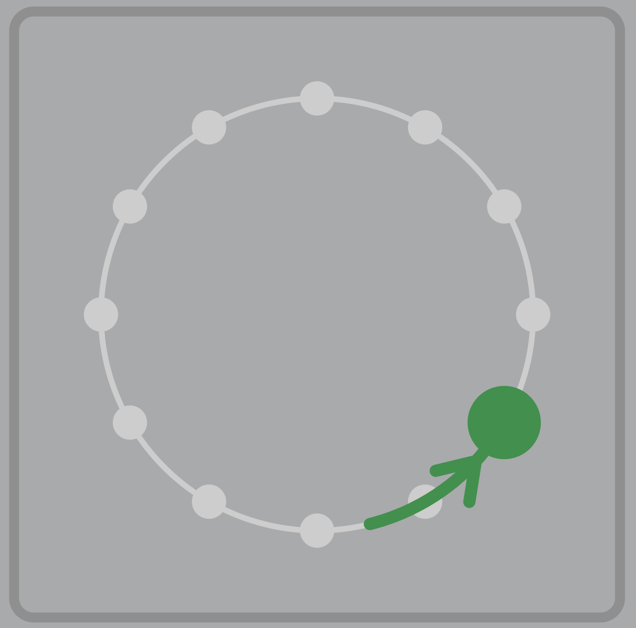
$$K(t) = \left(1 + K_1(t) e^{-\frac{\Gamma t}{2}}\right)^L + K_L(t)\left(1 - e^{-\frac{\Gamma t}{2}}\right)^L$$

Numerical validation



Coupled rotors

Quantum rotor model



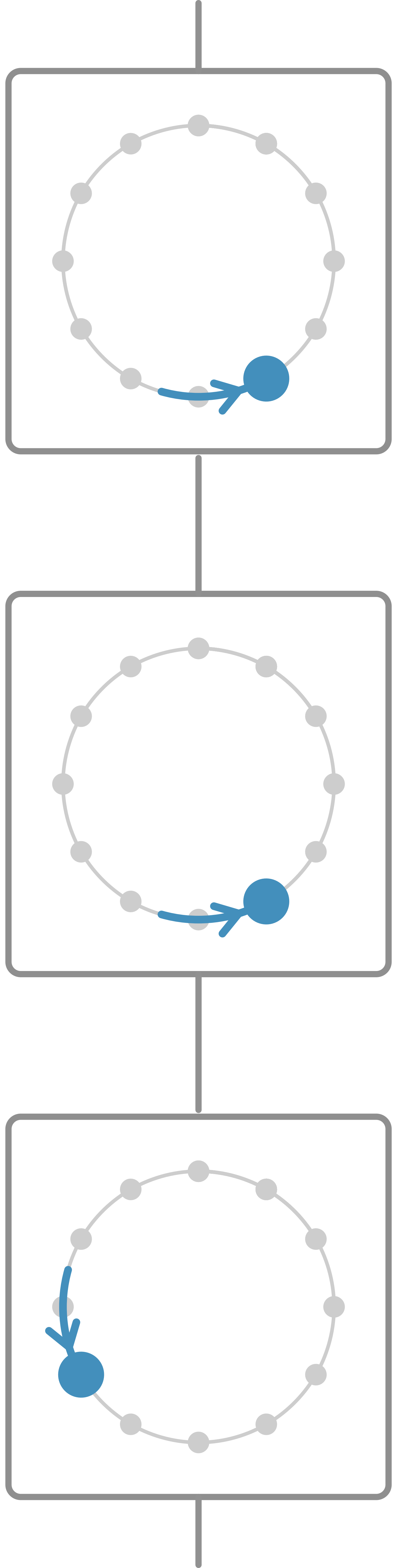
Quantum rotor

- N-dimensional Hilbert space
- $XZ = ZXe^{\frac{2\pi i}{N}}$ – generalized Pauli algebra

$$U = VW$$

$$W = \exp\left(-\frac{iN}{16\pi}(X - X^\dagger)^2\right)$$

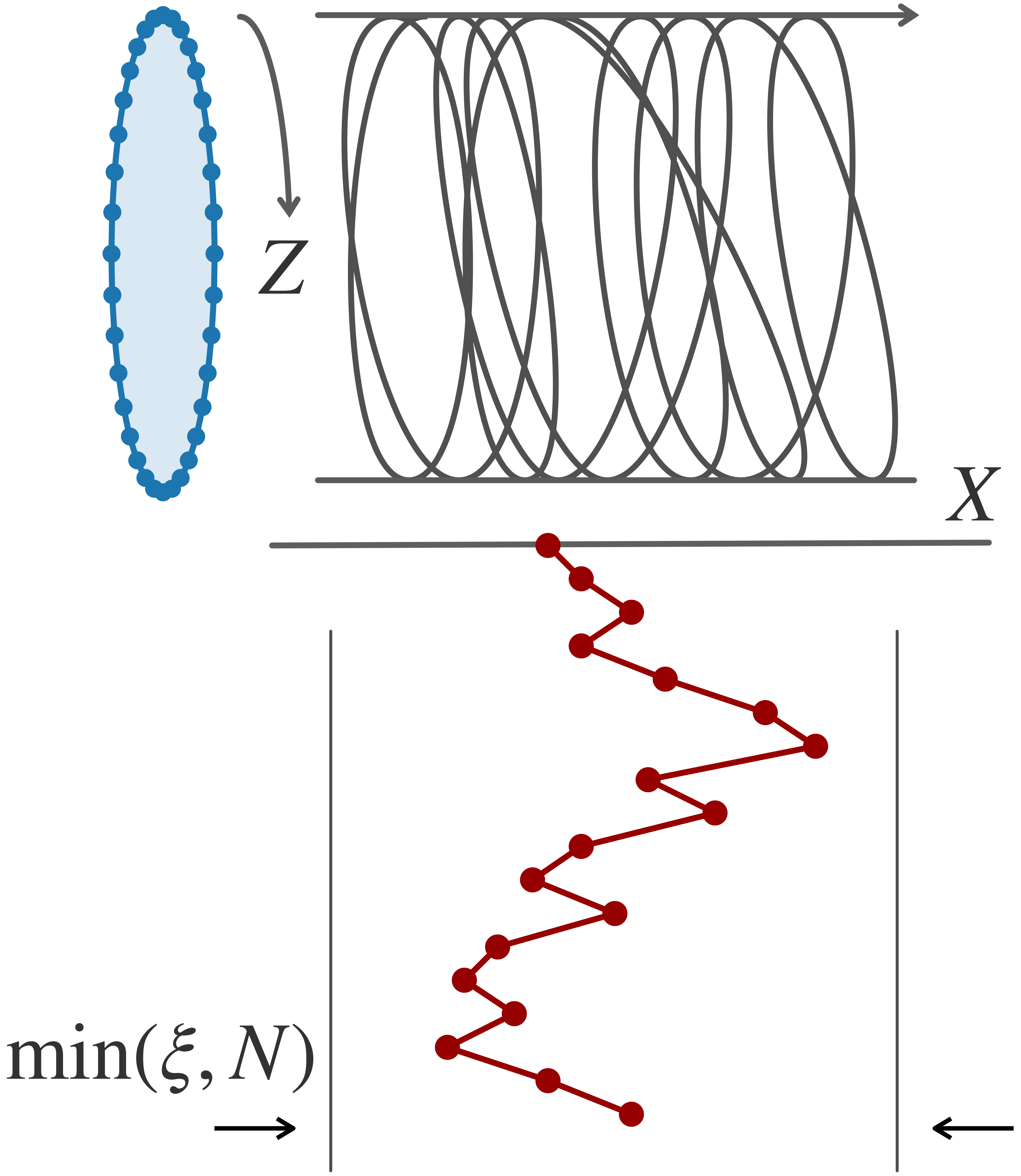
$$V = \exp\left(\frac{i\alpha N}{4\pi}(Z + Z^\dagger)\right)$$

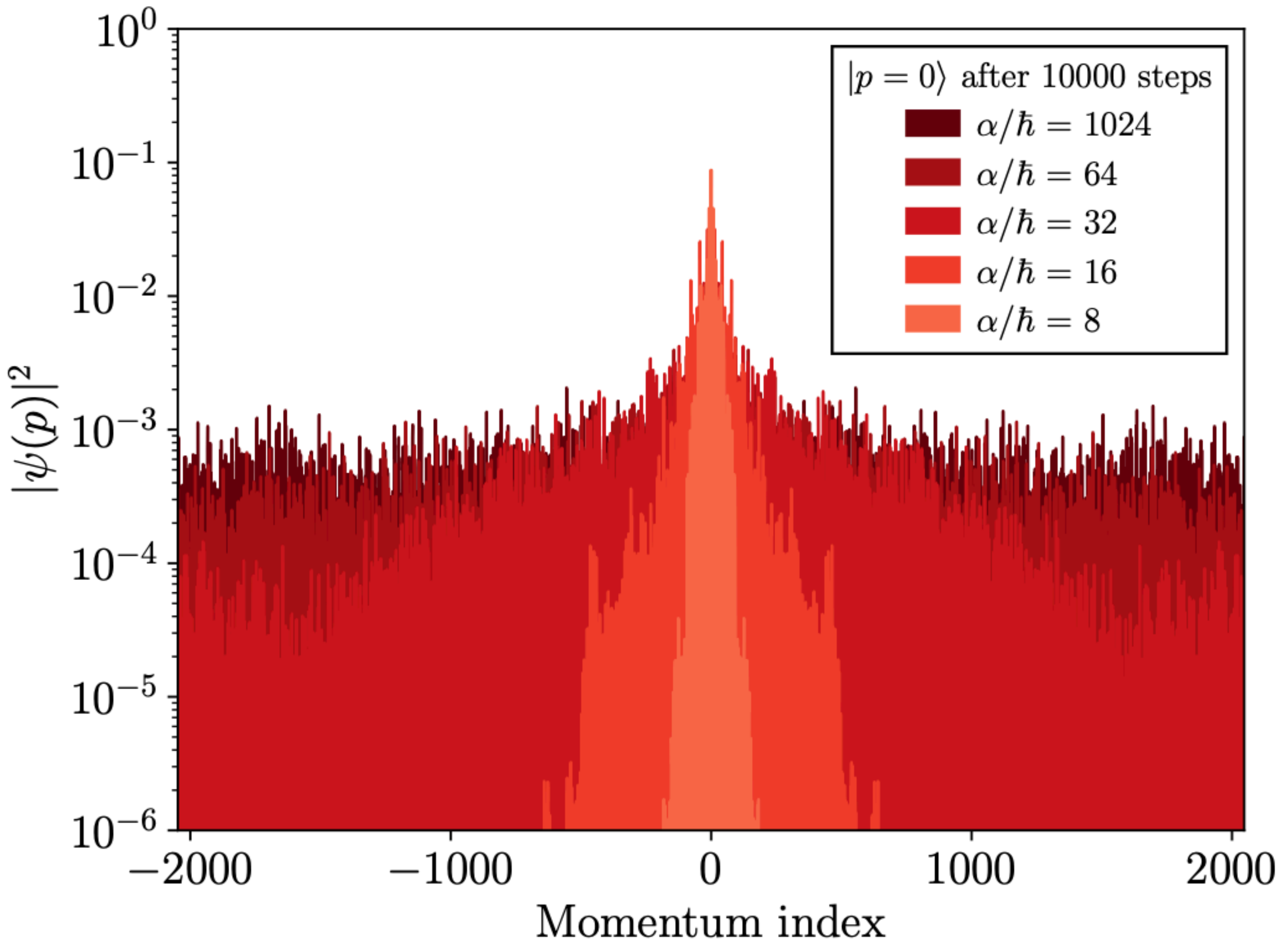


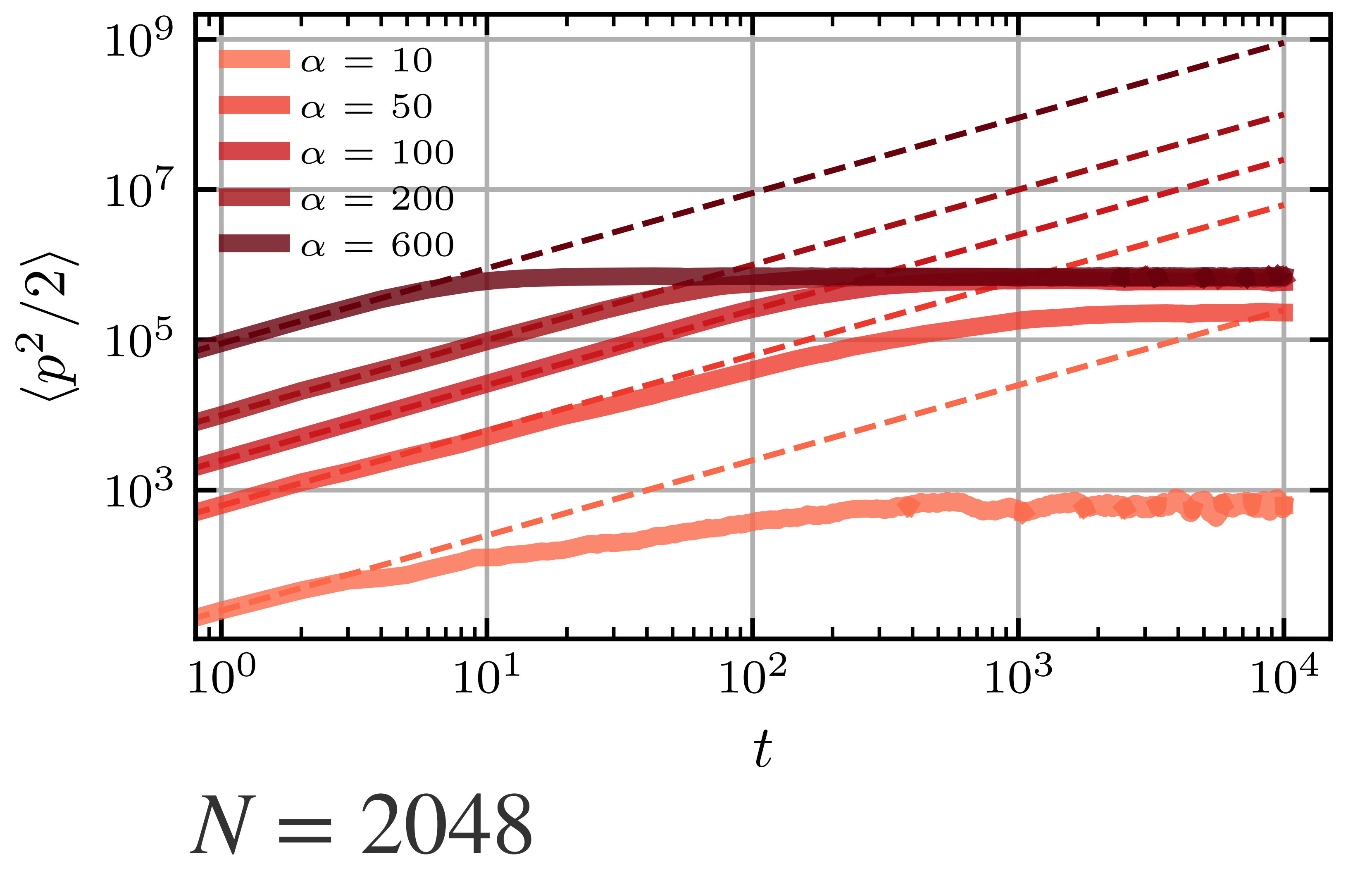
Single rotor evolution

Chaotic diffusive dynamics in X-eigenspace — no disorder.

Casati *et al.* 79







Single rotor field theory

Quantitatively represented by

diffusion coefficient

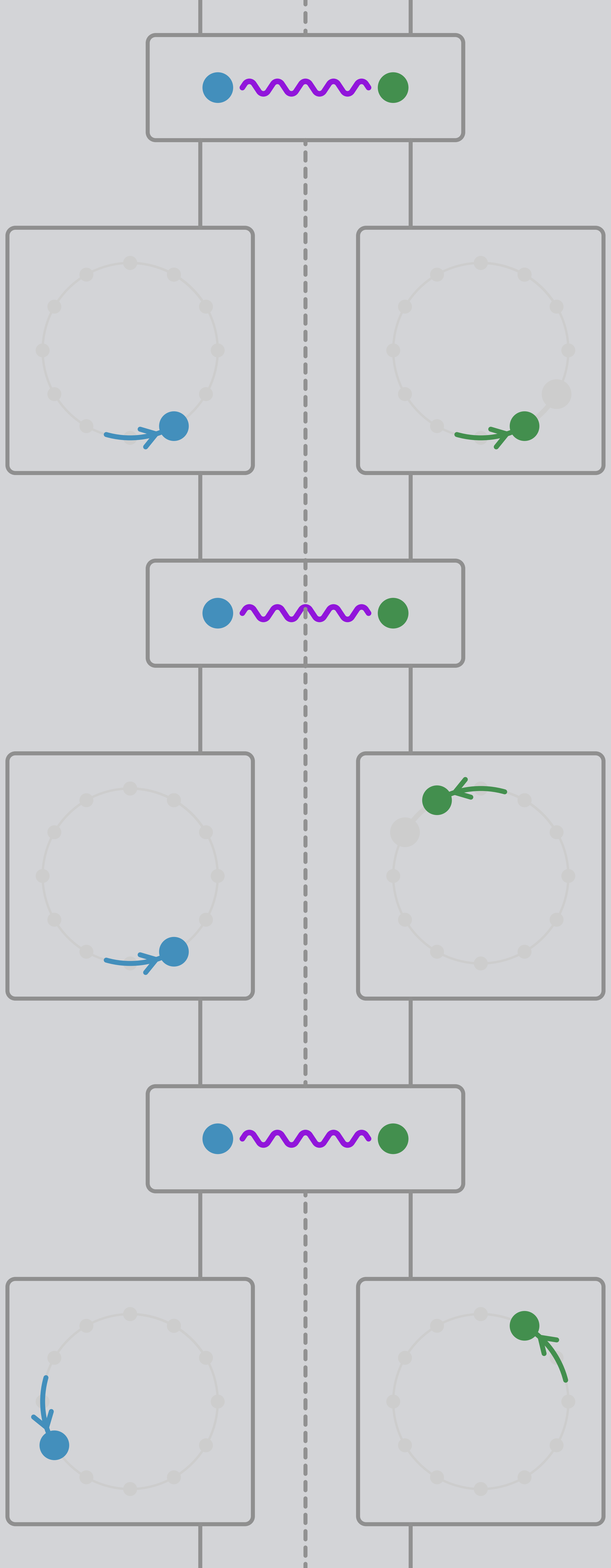
$$S[Q] = -\frac{D_\alpha}{16}\int_0^N dx\ \mathrm{str}\Big(\partial_x Q \partial_x Q - 2i\omega\, Q\, \sigma^3_{\mathrm{AR}}\Big)$$

AA-Tian, 2010

X-coordinate

Fourier-time

Describes diffusion ($N < \xi \sim D_\alpha$) or dynamical localization (($N > \xi$)



Two rotor evolution

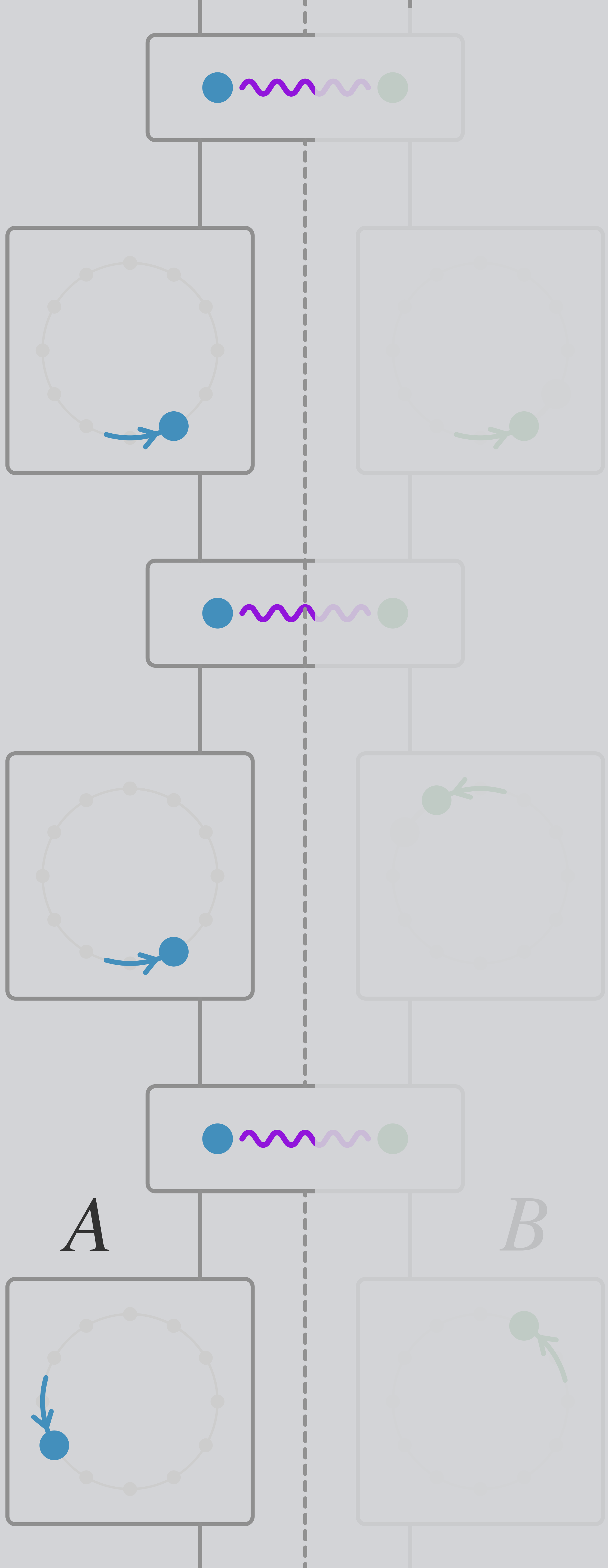
- coupling described by $V = \exp(-iX)$

$$X = \frac{\beta N}{4\pi}(Z_A \otimes Z_B^\dagger + Z_A^\dagger \otimes Z_B)$$

- semiclassically: two-body diffusion governed by diffusion constants

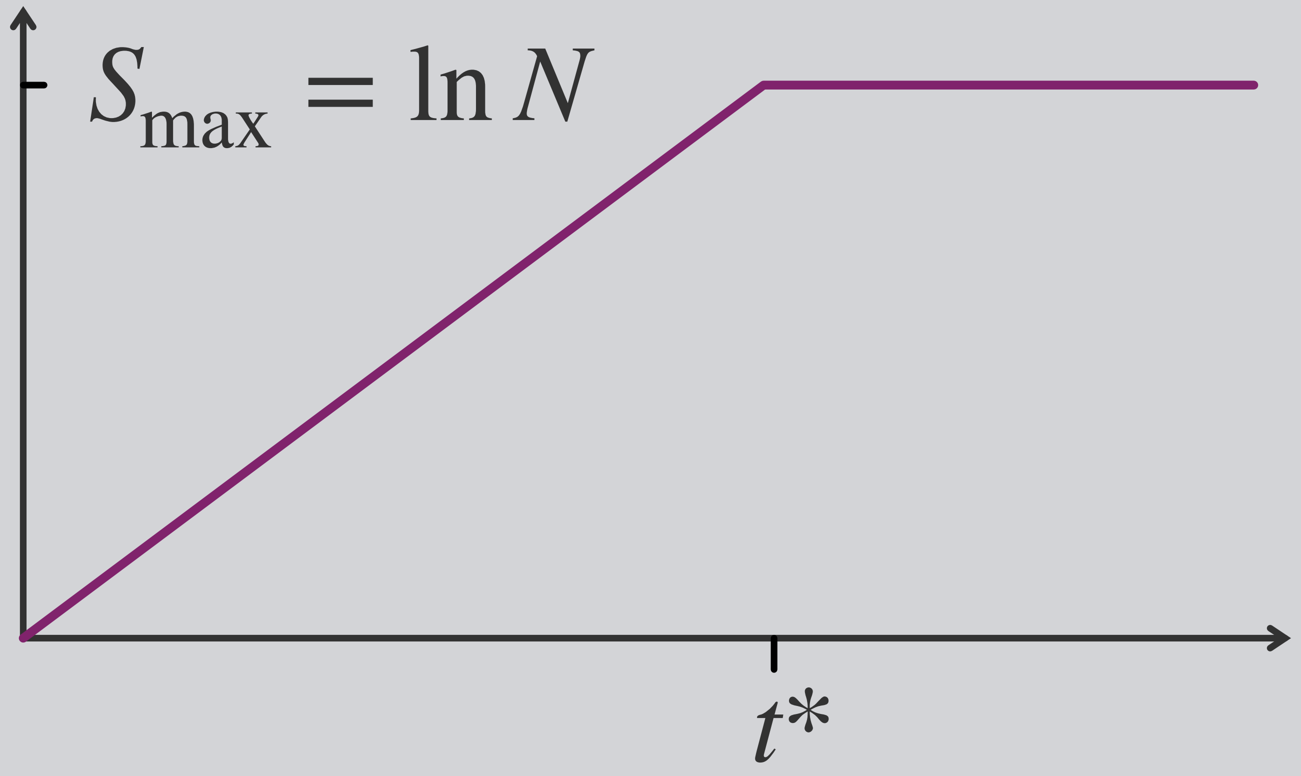
$$D = D_\alpha + D_\beta = \left(\frac{\alpha N}{2\pi}\right)^2 + \left(\frac{\beta N}{2\pi}\right)^2$$

Q: how does rotor dynamics interfere with entanglement generation



Two rotor entanglement

Consider entanglement entropy $S_A = \mathrm{tr}(\rho_A \ln \rho_A)$. Standard lore:



Classical perspective: particle initialized at known state will diffusively explore phase volume:

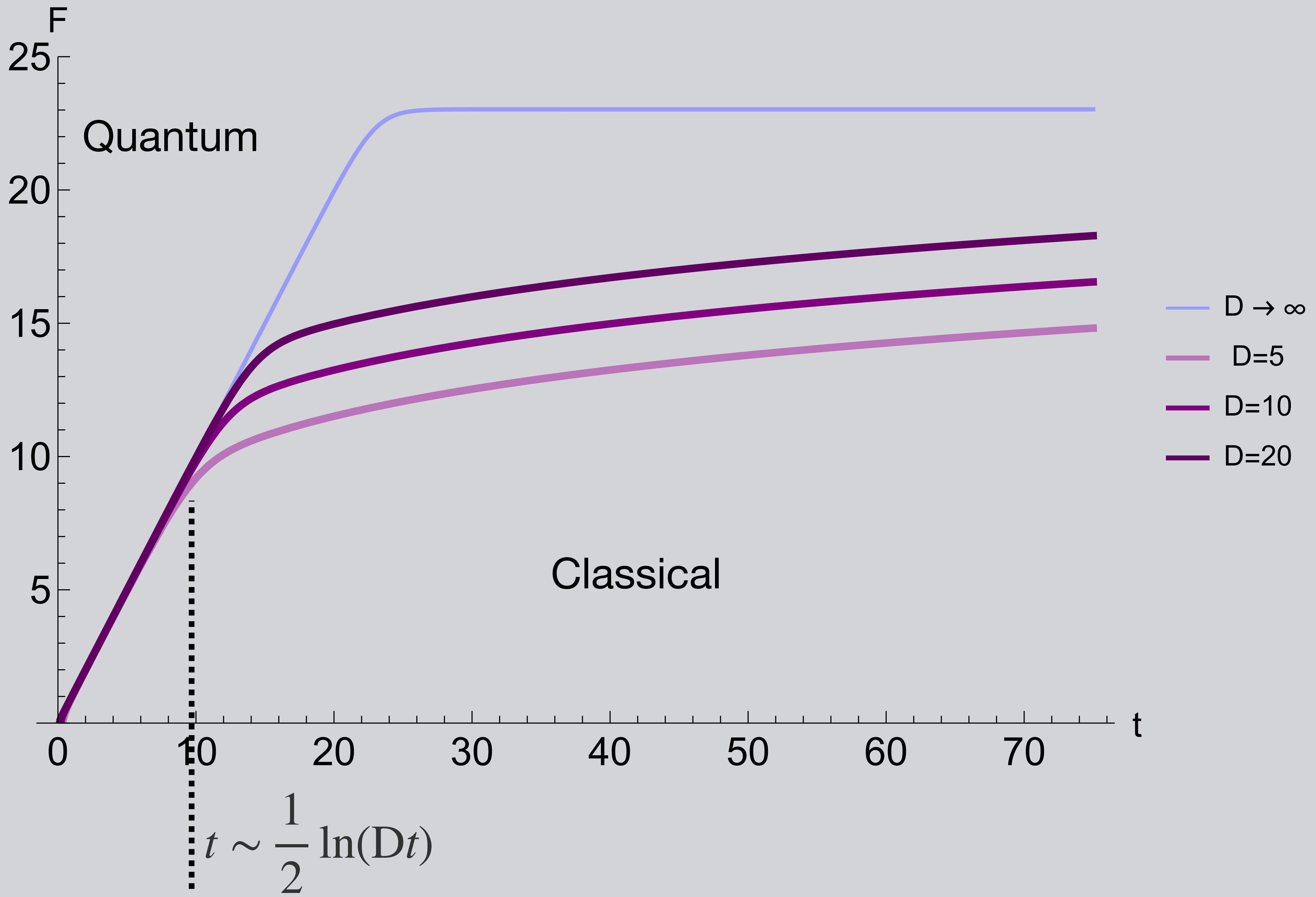
$$S_A(t) \sim \begin{cases} \ln(D_\alpha t) & , t \ll t_{\mathrm{erg}} \equiv N^2/D_\alpha \\ S_{\max} & , t \gg t_{\mathrm{erg}} \end{cases}$$

Generalization of single rotor field theory to tensor structure

Consider second Rényi entropy:

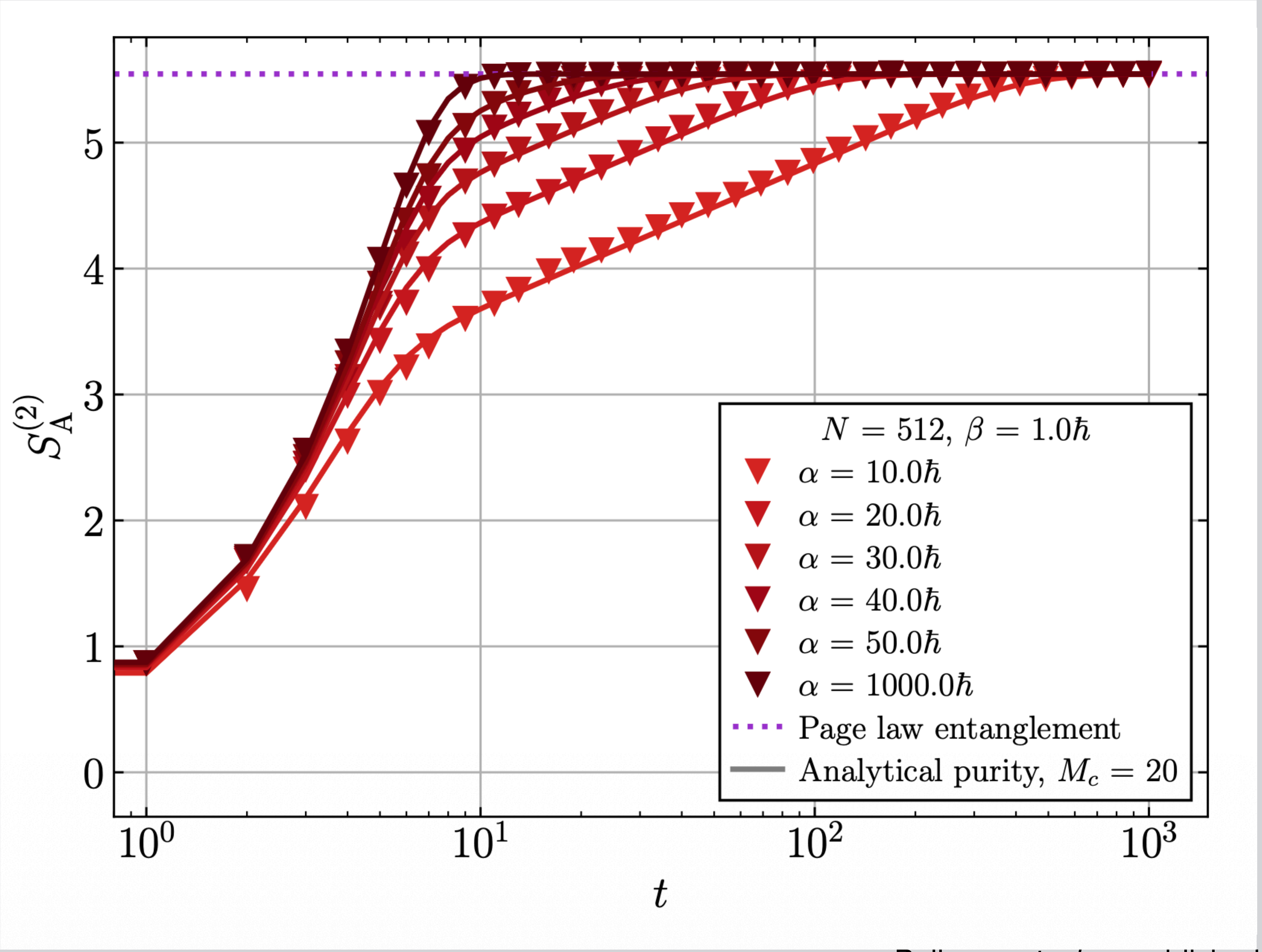
$$S_2 = -\ln(\mathrm{tr}_A(\rho_A^2))$$

$$S_2 = -\ln\left(e^{-4D_\beta t} + \frac{2}{N}\sum_{\phi} e^{-2Dt\phi^2} + (\times - \text{over})\right)$$



cf. Bäcker *et al.*, 2020

Numerical validation



aa, Pollman, *et. al.* unpublished

Summary

Discussed path integral for chaotic circuit dynamics. Approach …

… less straightforward than many effective formulations.

… tailored to resolving pre-ergodic phenomena in complex systems, and

… non-perturbative aspects of late time evolution.