Hydrodynamic Attractor in Ultracold Atoms

KF & T. Enss, Phys. Rev. Lett. **133**, 173402 (2024)

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

STRUCTURES CLUSTER OF EXCELLENCE

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科研費 KAKENHI

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Quantum thermalization, Hydrodynamics and Gravity

June 2026

Tilman Enss

Plan of this talk

1. What is hydrodynamic attractor?

Review papers:

P. Romatschke, JHEP **12** 079 (2017);
W. Florkowski, M. P. Heller, & M. Spaliński, Rep. Progr. Phys. **81**, 046001 (2018);
P. Romatschke & U. Romatschke, Cambridge University Press, (2019);
J. Berges, M. P. Heller, A. Mazeliauskas, & R. Venugopalan, Rev. Mod. Phys. **93**, 035003 (2021);
A. Soloviev, Eur. Phys. J. C **82**, 319 (2022).

☑ Michel's lecture

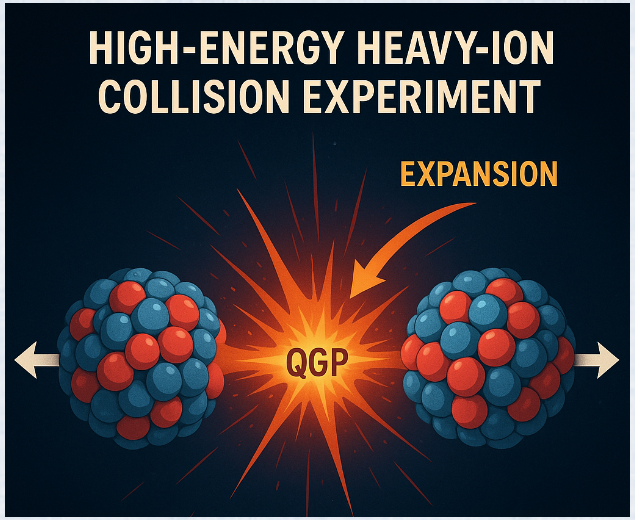
2. Phenomenological model & Toy model

3. Our proposal for realizing the hydrodynamic attractor in ultracold atoms

KF & Y. Nishida, PRA **98**, 063634 (2018).
KF & T. Enss, PRL **133**, 173402 (2024)

4. Summary

Historical background: heavy-ion collision



Created by ChatGPT.

Q: What kind of relaxation does the created plasma exhibit?

A: Hydrodynamics works immediately after the collision.

To successfully reproduce observables,
hydrodynamic simulations have to be applied at surprisingly early time,
when the system is expected to have
large gradients and strong pressure anisotropy.

→ **"Unreasonable" effectiveness of hydrodynamics**

Hydrodynamics far from local equilibrium ??

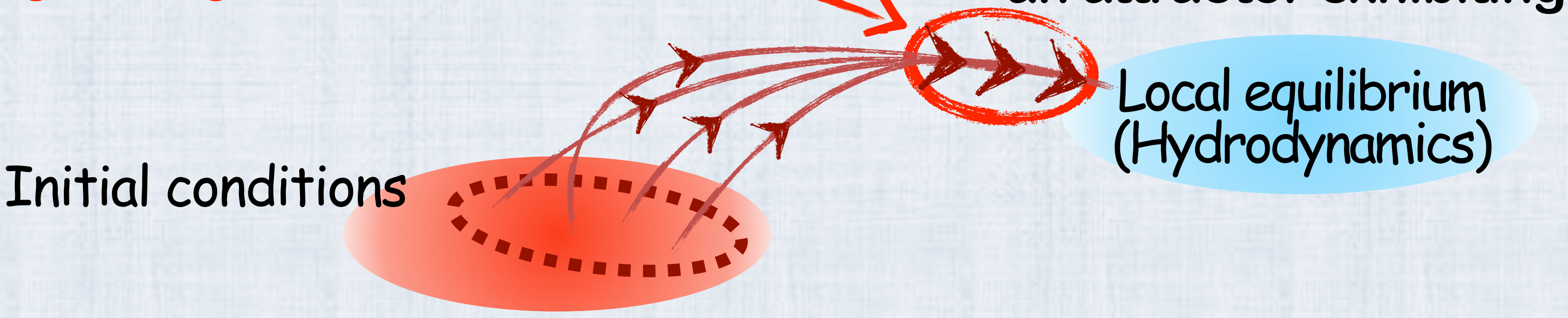
✓ Hydrodynamic attractor is the key idea for solving this puzzle.

Although this concept was originally motivated by heavy-ion collisions,
it should be significant as a general problem regarding
how non-equilibrium relaxation dynamics can be reduced to hydrodynamics.

What is hydrodynamic attractor?

✓ Hydrodynamic Attractor

an attractor exhibiting “hydrodynamic” behavior



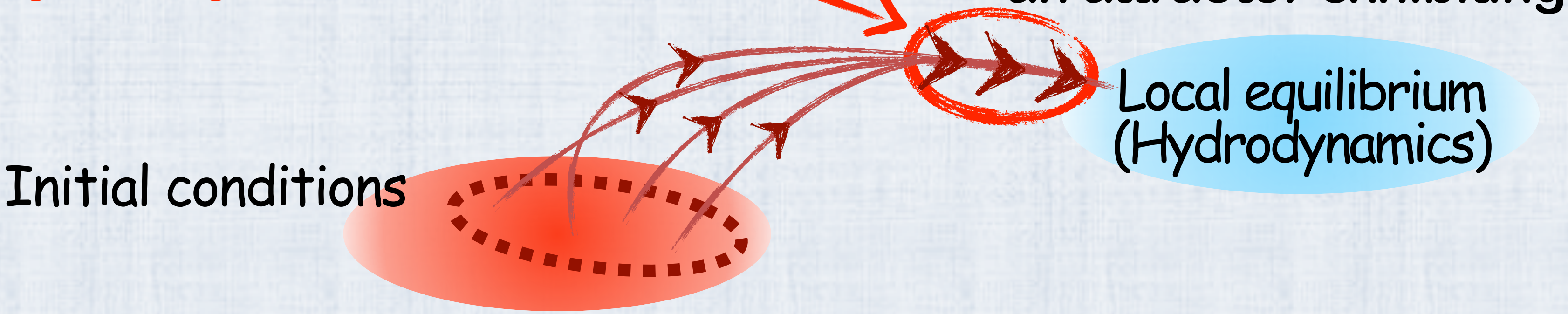
- The initial-condition dependence has been lost, but it is not in local thermal equilibrium.
- Emergence from a restricted class of non-equilibrium states, such as heavy-ion collision, etc.

Hydrodynamic attractor describes **hydrodynamization** before **local thermalization**.

What is hydrodynamic attractor?

✓ **Hydrodynamic Attractor**

an attractor exhibiting "hydrodynamic" behavior



Hydrodynamic attractor describes **hydrodynamization** before **local thermalization**.

Hydrodynamization:

The charge fluxes are expressed in terms of the charge densities and their derivatives.

e.g. energy-momentum tensor $T^{\mu\nu}(x) = T^{\mu\nu}[\beta(x), \vec{p}(x), \mu(x)]$

Local thermalization:

The system is described in terms of a local Gibbs state at the state level.

$$\rho(x) \simeq \rho_{\text{Gibbs}}[\beta(x), \vec{p}(x), \mu(x)]$$

Hydrodynamization & Local thermalization

Hydrodynamic attractor describes **hydrodynamization** before **local thermalization**.

In the absence of background flows, these two concepts are considered identical.

Obvious : **local thermalization** $\Longrightarrow$ **hydrodynamization**

With the use of gradient expansion, we can find the standard hydrodynamic equations.

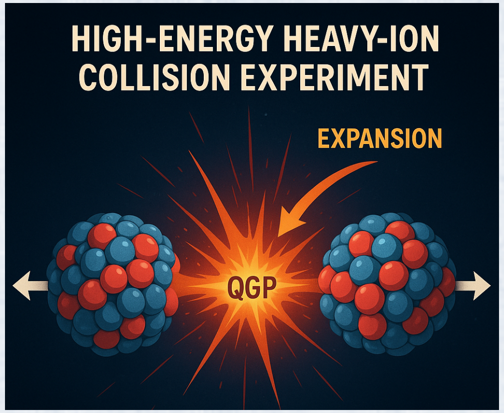
☑ **Masaru's talk**

In the presence of background flows, these two concepts requires distinction.

☑ **Michel's lecture**

E.g. relaxation process under expansion

(In the context of heavy-ion collisions, Bjorken flow, Gubser flow)



In the **hydrodynamization** stage,

it is not necessarily assumed that the gradients are small.

$$T^{\mu\nu}(x) = T^{\mu\nu}[\beta(x), \vec{p}(x), \mu(x)]$$

▸ The explicit functional form is unspecified.

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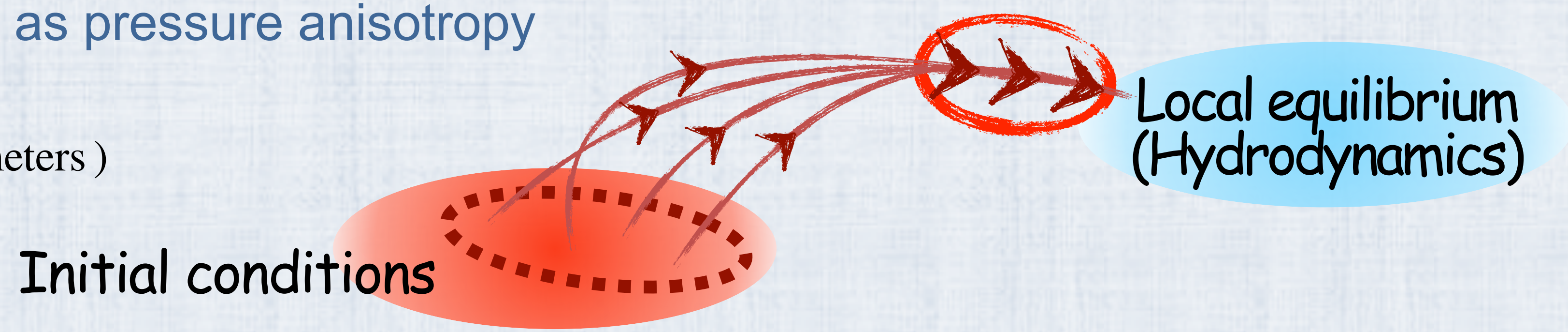
4. Summary

Hydrodynamic relaxation model

We focus on quantities $\pi_a(t)$ that vanishes at local equilibrium.

such as pressure anisotropy

- $\pi_a = 0$ at local equilibrium
  ($a = 1,2,\ldots,\#$ of relevant parameters)



We effectively describe their time evolution
under a background flow that ultimately leads to local equilibrium.

such as expansions in heavy-ion collisions

Minimal effective model : describing the competition between relaxation and driving

$$\tau_R \partial_t \pi_a(t) + \pi_a(t) = f_a(t, \pi)$$

τ_R : Relaxation time

$\pi_a(t)$: Quantity that vanish at local equilibrium $f_a(t, \pi)$: Driving force originating from a background flow

The solutions of this model explicitly demonstrate the trajectories shown in the schematic figure.

Hydrodynamic relaxation model

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- Similar to a MIS-type equation in the context of relativistic hydrodynamics Müller (1967); Israel (1976); Israel & Stewart (1979)

Navier-Stokes hydro. :

$$\pi_{ij}(t, \vec{x}) = -\eta \sigma_{ij}$$

Shear stress

π_{ij} : traceless part of the stress tensor

MIS formulation : hydrodynamics extended to include the relaxation time τ_R

$$\tau_R \partial_t \pi_{ij}(t, \vec{x}) + \pi_{ij}(t, \vec{x}) = -\eta \sigma_{ij}$$

At long time $t \gg \tau_R$, the standard Navier-Stokes relation $\pi_{ij} = -\eta \sigma_{ij}$ is recovered.

One way to get the above effective model is to simplify such MIS-type equations.

More microscopic approaches: kinetic theory, holography, etc.

Hydrodynamic relaxation model

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- Similar to a MIS-type equation in the context of relativistic hydrodynamics Müller (1967); Israel (1976); Israel & Stewart (1979)

- $f_a(t, \pi)$ possesses complexity due to microscopic structures such as conservation laws and the temperature dependence of transport coefficients and so on.

- Such an effective model is derived for heavy-ion collisions from the extended hydrodynamic model under Bjorken flow M. P. Heller and M. Spaliński, PRL **115**, 072501 (2015).

$$\tau \frac{d}{d\tau} E(\tau) = -\frac{4}{3} E(\tau) + \phi(\tau) \qquad \tau_\pi \frac{d}{d\tau} \phi(\tau) = \frac{4}{3} \frac{\eta}{\tau} - \frac{4}{3} \frac{\tau_\pi}{\tau} \phi(\tau) - \phi(\tau)$$

$E(\tau)$: Energy density $\phi(\tau) = -\pi^y{}_y$: Shear stress (τ : proper time)

Hydrodynamic relaxation model

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  from the extended hydrodynamic model under Bjorken flow

- In the absence of background flow, the system naively approaches local equilibrium ($\pi = 0$).

$$f = 0 \implies \pi(t) = \pi_0 e^{-t/\tau_R}$$

Exponential decay of $\pi(t)$

Attractor behavior in a nutshell

As a hyper-simplified toy example,

we roughly approximate the background fluid expansion as $f(t,\pi) \propto t^{-\gamma}$.

$$\tau_R \partial_t \pi(t) + \pi(t) = f_{\rm power}(t) \quad \text{with} \quad f_{\rm power}(t) = \pi_0 \times \left(t_0/t\right)^{\gamma} \quad \text{Initial condition : } \pi(t_{\rm ini}) = \pi_{\rm ini}$$

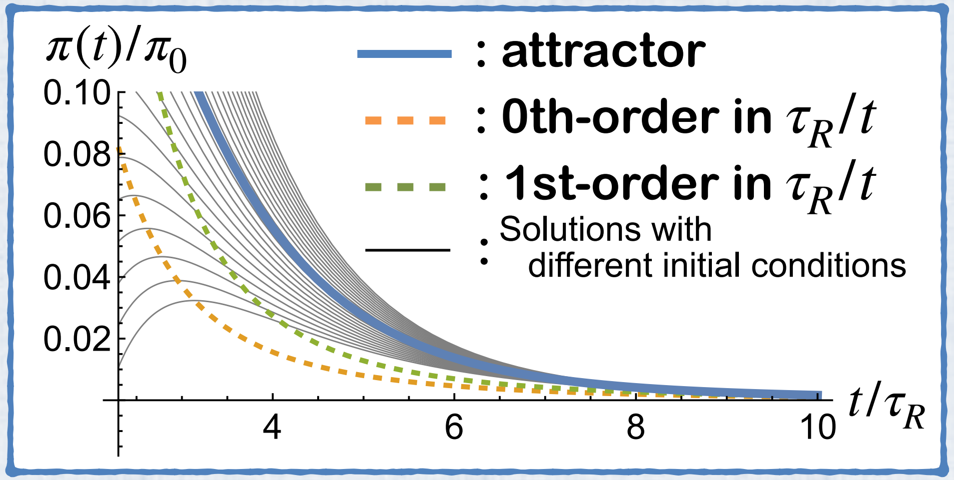
What we expect in this example

- $f_{\rm power}(t)$ vanishes at long time → The system eventually approaches local equilibrium, $\pi = 0$.

  ✓ **How do the solutions approach local equilibrium?**

- At long time $t \gg \tau_R$, the solution should be well described by an expansion in τ_R/t.

  analogue of the derivative expansion w.r.t $\tau_R \partial_t$ underlying hydrodynamics



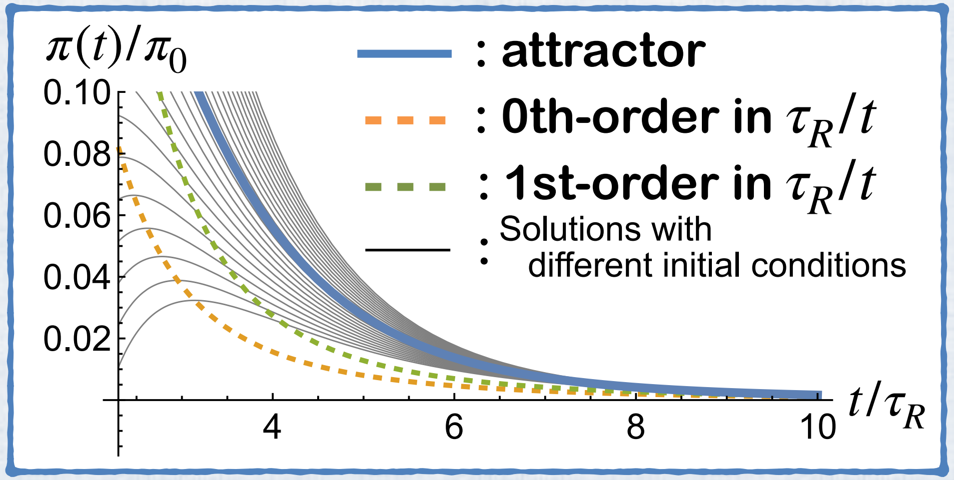
Expanded solutions (i.e., hydrodynamic prediction)

0th-order : $\pi(t) \sim f_{\rm power}(t)$ 1st-order : $\pi(t) \sim f_{\rm power}(t)\left[1 + \gamma \frac{\tau_R}{t}\right]$

Solutions converge to an attractor before converging to the hydrodynamic predictions.

Attractor behavior in a nutshell

$$\tau_R \partial_t \pi(t) + \pi(t) = f_{\rm power}(t) \quad \text{with} \quad f_{\rm power}(t) = \pi_0 \times (t_0/t)^\gamma \qquad \text{Initial condition : } \pi(t_{\rm ini}) = \pi_{\rm ini}$$



Solutions converge to an attractor before converging to the hydrodynamic predictions.

You might ask

Can we see the attractor if we push the expansion to sufficiently high order?

No!! **The expansion is asymptotic.**

▸ **Analytic solution**

$$\pi(t) = \pi_{\rm ini} e^{-(t-t_{\rm ini})/\tau_{\rm relax}} + \pi_{\rm att}(t)$$

Non-hydro. mode

$$\pi_{\rm att}(t) = \int_{t_{\rm ini}}^{t} \frac{ds}{\tau_R} f_{\rm power}(s) e^{-(t-s)/\tau_R}$$

Expansion with respect to $\tau_{\rm relax}/t$.

$$\pi_{\rm att}(t) \sim f_{\rm power}(t) \left[1 + \gamma \frac{\tau_R}{t} + \gamma(\gamma+1)\left(\frac{\tau_R}{t}\right)^2 + \cdots \right]$$ **: Divergent series**

n th-order coefficient $\propto (n+\gamma)!$ → **factorial divergent (Borel summable)**

Lessons from the toy model

What the toy model captures

$f(t, \pi) \sim t^{-\gamma}$

- **Hydrodynamization** before the low-order gradient expansion becomes reliable.

  Disappearance of initial condition dependence in the toy model

  $\pi_{\rm att}(t)$ **is referred to as** **the hydrodynamic attractor,** when $f_a(t, \pi)$ is **internally** provided.

- Asymptotic nature of the derivative expansion

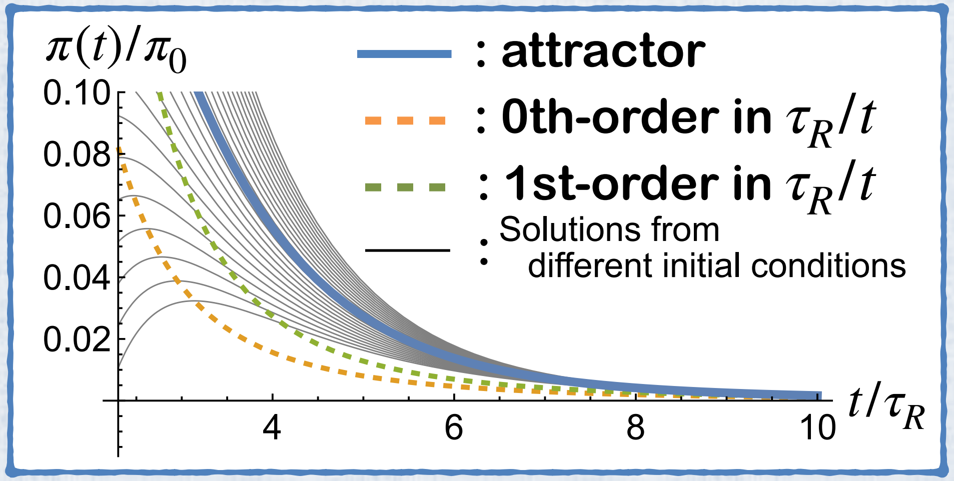
  In a heavy-ion collision model, hydrodynamic gradient expansion is asymptotic.

  → **Its Borel sum leads to the attractor.**

  M. P. Heller and M. Spaliński, PRL **115**, 072501 (2015). "Hydrodynamics Beyond the Gradient Expansion: Resurgence and Resummation"

Minimal effective model

$$\tau_R \partial_t \pi_a(t) + \pi_a(t) = f_a(t, \pi)$$



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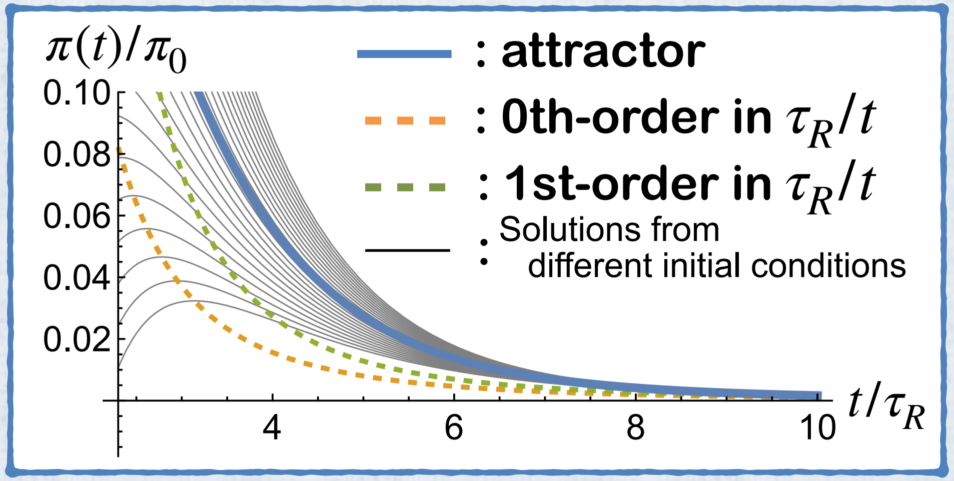

What the toy model misses

- The complexity of $f_a(t, \pi)$ and its resulting rich structure

  e.g. fixed-point structure

  Local equilibrium $\pi = 0$ can be regarded as a trivial fixed point when $f = 0$.
  Any other transient fixed points?

- The origin of the effective model
  - How to derive this model from microscopic theories?
  - How many degrees of freedom are required? (Identification of the slow manifold)

Plan of this talk

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A. Soloviev, Eur. Phys. J. C **82**, 319 (2022).

2. Phenomenological model & Toy model

Lesson : A certain class of $f_a(t,\pi)$ leads to hydrodynamic attractors.

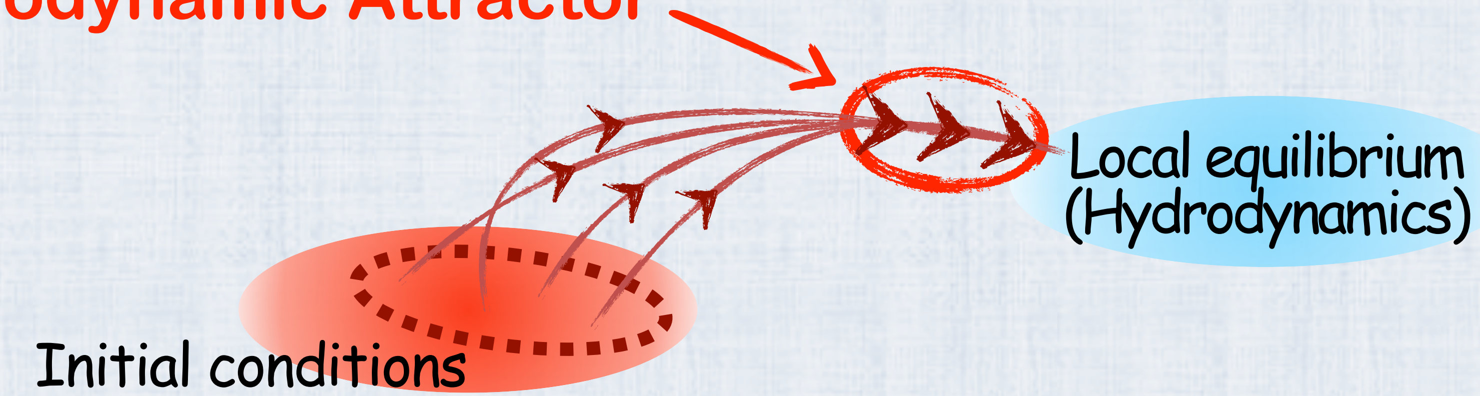
3. Our proposal for realizing the hydrodynamic attractor in ultracold atoms

KF & Y. Nishida, PRA **98**, 063634 (2018).
KF & T. Enss, Phys. Rev. Lett. **133**, 173402 (2024)

4. Summary

Observation in real-time dynamics

✓ Hydrodynamic Attractor



Can we observe this attractor behavior in real-time dynamics?

😢 In heavy-ion collisions, we can only access the final-state particle momenta as observables.

👍 **Ultracold atom experiments offer well-controlled time-resolved measurements.**

➡ **Proposal of "a fluid expansion" leading to the attractor, realizable in ultracold atoms**

Our key idea: KF & T. Enss Phys. Rev. Lett. **133**, 173402 (2024)

Realizing phenomena equivalent to fluid expansions by driving the scattering length.

Time-dep. scattering length in hydrodynamics

Ultracold atomic gases :

- Their inter-particle interaction is characterized only by the (*s*-wave) scattering length a.
- The scattering length a can be tuned via Feshbach resonances.
  Its spatiotemporal modulation is also possible.

➡ **Time-dependent scattering length $a(t)$**

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Hydrodynamically, this $a(t)$ results in the same effect as isotropic fluid expansion.

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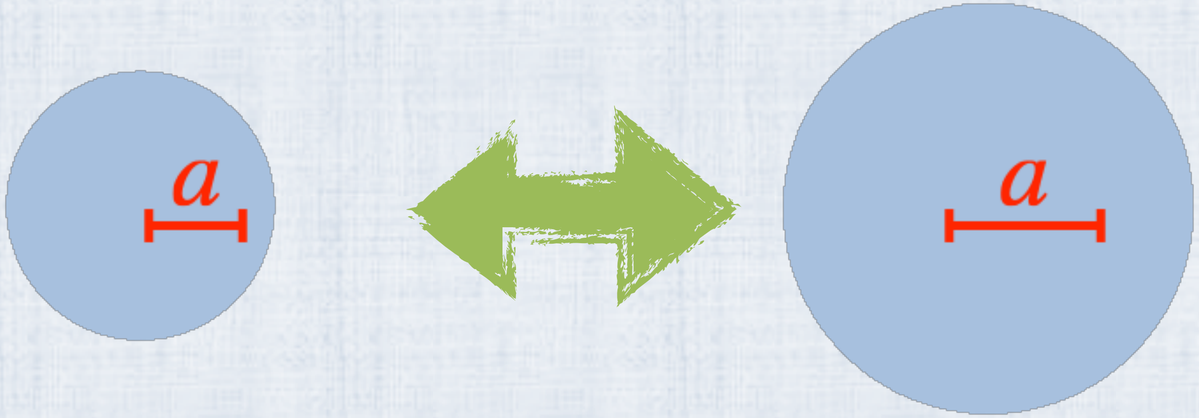

In terms of $\dfrac{\textbf{Fluid Size}}{\textbf{Scattering Length}}$ these two phenomena are equivalent.

Isotropic expansion & contraction = **Shrinking & stretching of the scattering length**

Time-dep. scattering length in hydrodynamics

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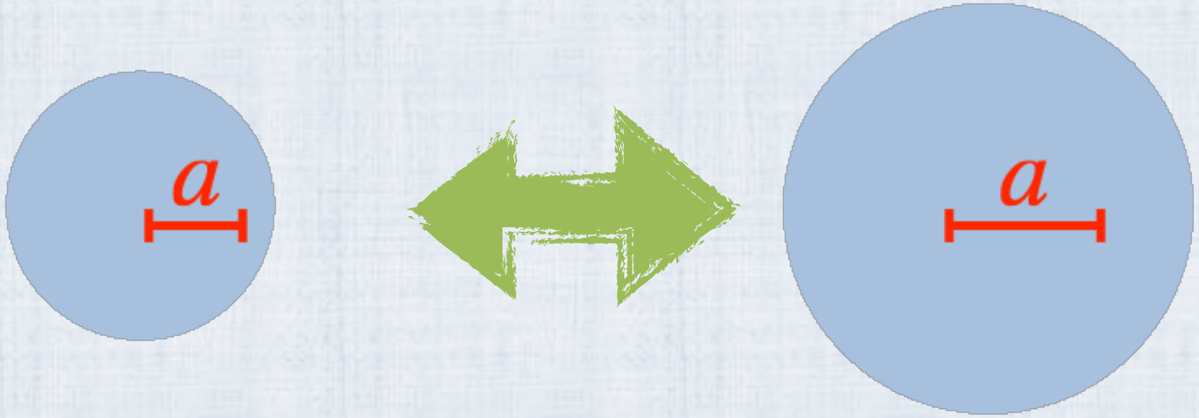


In terms of $\frac{\textbf{Fluid Size}}{\textbf{Scattering Length}}$ these two phenomena are equivalent.

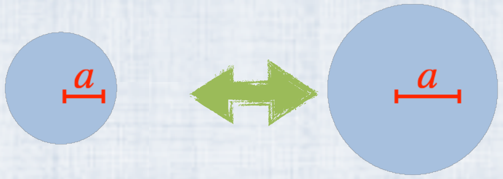
✓ Driving the scattering length allows us to **emulate arbitrary isotropic fluid expansion,** while the system remains uniform and at rest.

Our proposal

KF & T. Enss, Phys. Rev. Lett. **133**, 173402 (2024)

✓ Driving the scattering length allows us to **emulate arbitrary isotropic fluid expansion,** while the system remains uniform and at rest.

The power-law driving of the scattering length
⟶ simulation of **a hyper-simplified toy model**.



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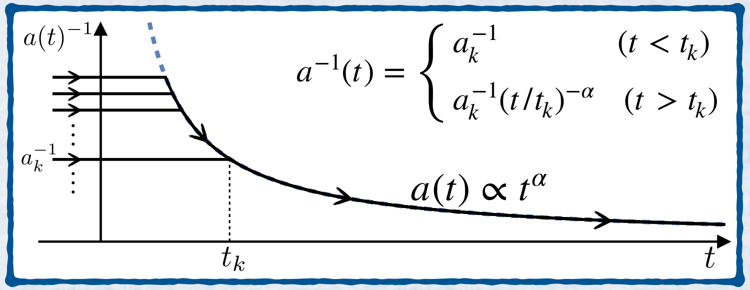
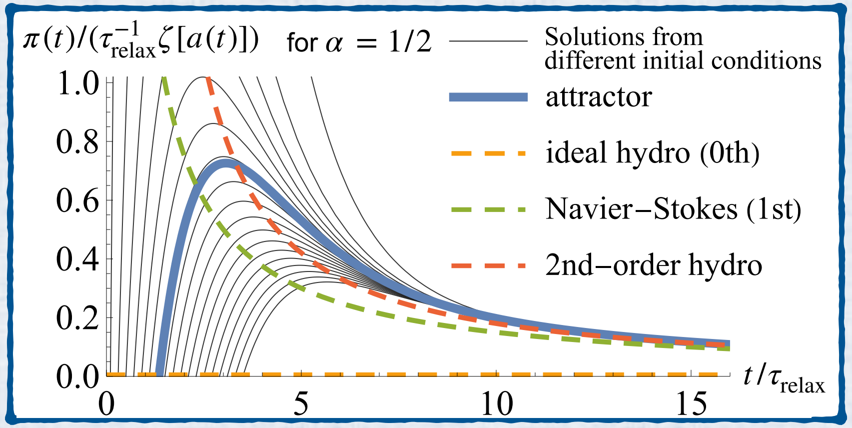




▸ **Visible in experiments** : for example, in a two-component Fermi gas of ^{40}K, the time window $t/\tau_{\rm relax} \sim 5 - 10$ will be visible.

Thywissen group, Science (2014), PRL (2017)

- Contact dynamics was measured with 0.1ms time resolution. ($E_F \sim h \times 20$ kHz, $T/T_F \sim 0.25$ ➞ $\tau_{\rm relax} \sim 0.15$ ms)

✓ The Development of This Research

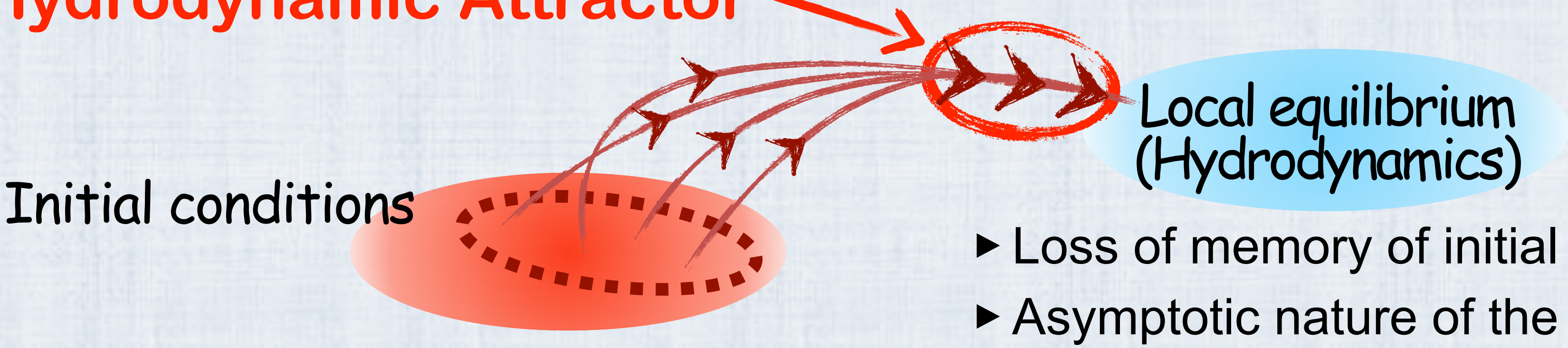
M. P. Heller & C. Werthmann, arXiv:2507.02838 (2025)

- The importance of the complexity of $f(t, \pi)$ under the same power-law driving of the scattering length
- Other driving protocol of the scattering length

A. Mazeliauskas & T. Enss, PRL **136**, 103402 (2026)

Summary

✓ Hydrodynamic Attractor



- Loss of memory of initial conditions
- Asymptotic nature of the derivative expansion

Hydrodynamic attractors are everywhere!!

In heavy-ion collisions, cosmology (Hubble expansion),

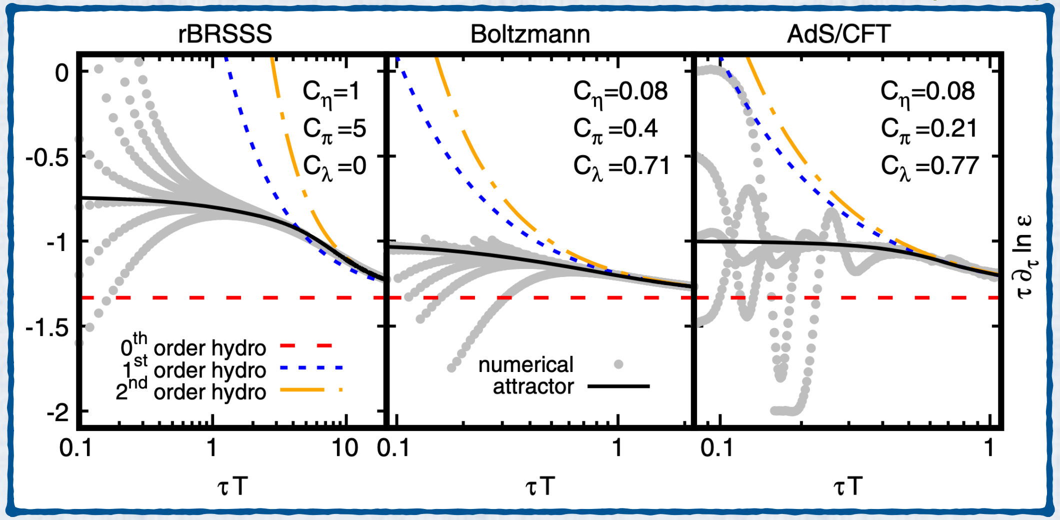
Du, Huang, & Taya, PRD (2021)

And in ultracold atoms.

KF & T. Enss, PRL **133**, 173402 (2024)

from extended hydro., kinetic theory, holography…

P. Romatschke, PRL **120**, 012301 (2018)



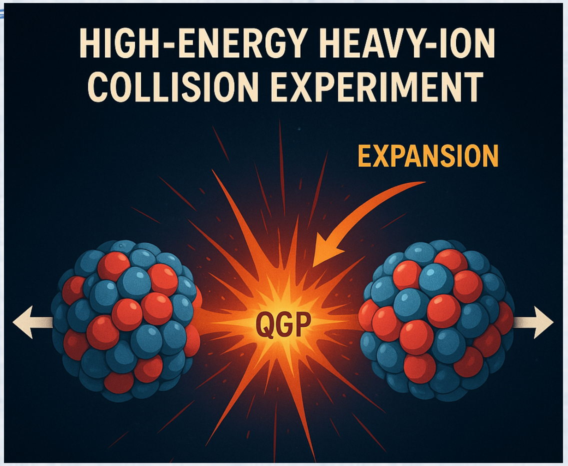
Still many open questions….

- How to get $f_a(t, \pi)$, or the effective model?
- What kind of motion triggers the hydro. attractor?

and so on…

Thank you!!

Effective model under Bjorken expansion



$E(\tau)$: Energy density $\quad \phi(\tau) = -\pi^y{}_y$: Shear stress $\quad$ (τ : proper time)

Assuming conformality

$$\tau \frac{d}{d\tau}E(\tau) = -\frac{4}{3}E(\tau) + \phi(\tau) \qquad \tau_\pi \frac{d}{d\tau}\phi(\tau) = \frac{4}{3}\frac{\eta}{\tau} - \frac{4}{3}\frac{\tau_\pi}{\tau}\phi(\tau) - \phi(\tau)$$

(τ_π : relaxation time)

$$\Rightarrow \quad C_{\tau_\pi} w \left(1 + \frac{\mathscr{A}}{12}\right)\frac{d}{dw}\mathscr{A} + \left(\frac{1}{3}C_{\tau_\pi} + \frac{1}{8}\frac{C_{\lambda_1}}{C_\eta}w\right)\mathscr{A}^2 + \frac{3}{2}w\mathscr{A} - 12C_\eta = 0$$

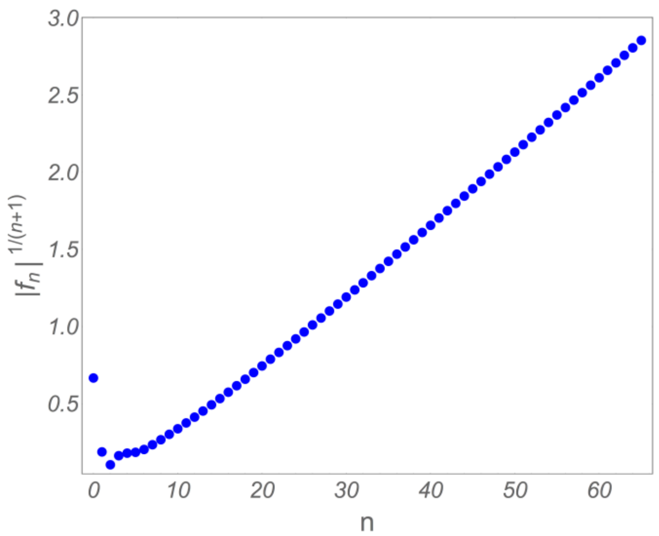
M. P. Heller and M. Spaliński, PRL **115**, 072501 (2015).

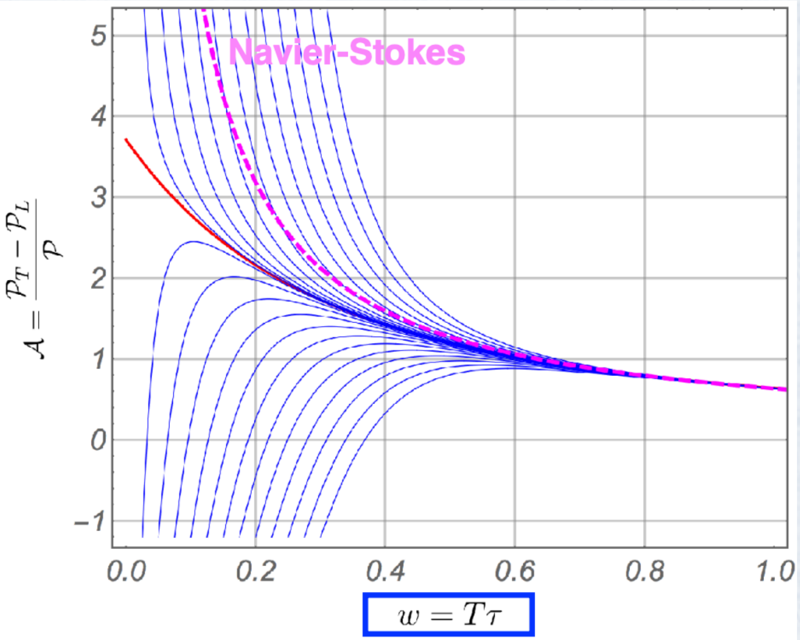
$\mathscr{A}(w)$: Pressure anisotropy

$w = \tau T(\tau)$: dimensionless time

$T(\tau)$: effective temperature defined via $E_{\rm eq}(T(\tau)) = E(\tau)$

✔ The expansion of the solution w.r.t. $1/w$ diverges factorially.





Non-equilibrium dynamics at long time

Hydrodynamics universally describes long-time and long-distance dynamics.

$t \gg \tau_{\rm relax}$ $\qquad$ $x \gg l_{\rm mfp}$

- Local thermal equilibrium assumption

Hydrodynamic equations are constructed based on **the derivative expansion** w.r.t ∂_t and ∂_x.

$\sim \tau_{\rm relax}/t$ $\qquad$ $\sim l_{\rm mfp}/x$

For example,

Stress tensor in hydrodynamics : $T_{ij} = \underbrace{p\delta_{ij} + \rho v_i v_j}_{\text{0th order}} + \underbrace{(\text{viscous terms})}_{\text{1st order}} + \underbrace{\cdots}_{\text{higher order}}$

Dynamics with time-dep. scattering length

Focusing on two-component Fermi gases in the normal phase (uniform, 3-dim)

- Energy density production (Dynamic sweep theorem) S. Tan, Ann. Phys. (2008)

$$\frac{d}{dt}\mathscr{E}(t) = \frac{C(t)}{4\pi m a^2(t)} \frac{d}{dt} a(t)$$

$\mathscr{E}(t)$: Energy density

$C(t)$: Contact density

- Contact density (conjugate quantity to the scattering length)

$$C(t) = C_{\mathrm{eq}}[a(t)] + 12\pi m a(t)\pi(t)$$

Dissipative correction

$\pi(t)$ represents **the pressure deviation** from its equilibrium value.

cf. Pressure relation $P = \frac{2}{3}\mathscr{E} + \frac{C}{12\pi m a}$

- Hydrodynamic relaxation dynamics for $\pi(t)$

$$\tau_{\mathrm{relax}}\partial_t \pi(t) + \pi(t) = -\zeta V_a(t)$$

with

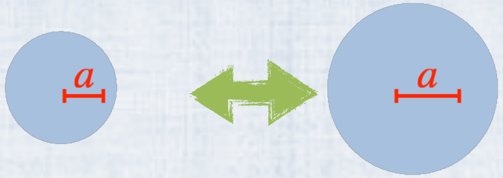
$$V_a(t) = -3\frac{\partial_t a(t)}{a(t)}$$

Bulk viscosity

The bulk viscosity is given by the contact-contact correlation

☑ **Tilman's talk**

Bulk strain rate tensor



In general situation with fluid velocity $\vec{v}(t,\vec{x})$

$$V_a(t,\vec{x}) = \nabla\cdot\vec{v}(t,\vec{x}) - 3\left[\frac{\partial_t a(t,\vec{x})}{a(t,\vec{x})} + \vec{v}(t,\vec{x})\cdot\frac{\nabla a(t,\vec{x})}{a(t,\vec{x})}\right]$$

KF & Y. Nishida, PRA **98**, 063634 (2018).

✓ **The consequence of the equivalence**

Dynamics of the pressure deviation

▸ Hydrodynamic relaxation dynamics for $\pi(t)$

$$\tau_{\text{relax}}\partial_t\pi(t) + \pi(t) = -\zeta V_a(t) \quad \text{with} \quad V_a(t) = -3\frac{\partial_t a(t)}{a(t)}$$

- cf. Muller-Israel-Stewart theory in relativistic hydrodynamics
- One can derive this equation from the linear-response theory with exponential relaxation.

$$\delta C(t) = \int_{-\infty}^{t} dt' \frac{\partial C(t)}{\partial a^{-1}(t')}\delta a^{-1}(t') \quad \text{with} \quad \frac{\partial C(t)}{\partial a^{-1}(t)} \simeq \left(\frac{\partial C}{\partial a^{-1}}\right)_{\text{eq}} e^{-(t-t')/\tau_{\text{relax}}} \quad \Leftrightarrow \quad \zeta(\omega) = \frac{i\chi}{\omega + i\tau_{\text{relax}}^{-1}}$$

(valid for long times)

- **The gradient expansion underlying the hydrodynamics corresponds to expanding the solution $\pi(t)$ with respect to τ_{relax}/t.**

From the equation, $\pi(t) = -\zeta V_a(t) + O(\tau_{\text{relax}}/t)$

Navier-Stokes hydro. Result

▸ Higher-order hydrodynamic corrections can be obtained from the expansion as needed.

Power-law driving for hydro attractor

- Hydrodynamic relaxation dynamics for $\pi(t)$

$$\tau_{\rm relax}\partial_t \pi(t) + \pi(t) = -\zeta V_a(t) \quad \text{with} \quad V_a(t) = -3\frac{\partial_t a(t)}{a(t)}$$

☑ Protocol for driving the scattering length :

Initially push the system out of equilibrium, then let it gradually approach thermal equilibrium

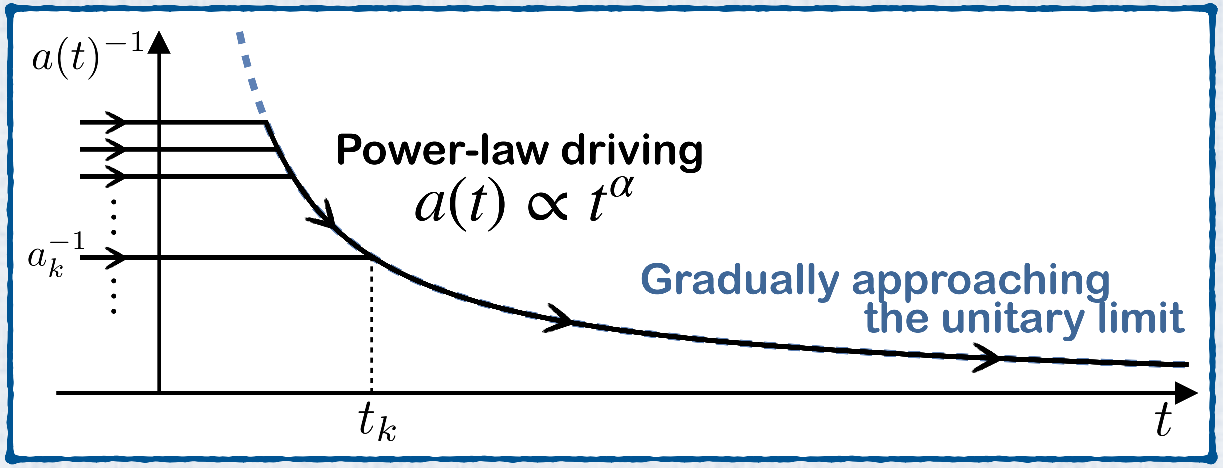
- **Example for two-comp. Fermi gases close to the unitary limit**

$$a^{-1}(t) = \begin{cases} a_k^{-1} & (t < t_k) \\ a_k^{-1}(t/t_k)^{-\alpha} & (t > t_k) \end{cases}$$

Keep the scattering length fixed at a value a_k until $t = t_k$, then start the power-law driving, i.e., $a(t) \propto t^\alpha$.

- The system approaches equilibrium because of $V_a(t) = -3\alpha/t$.

By varying a_k & t_k, various initial states can be realized.



$\tilde{a} := a_k(t_k/\tau_\zeta)^\alpha$ is fixed
so that the driven scattering length follows a single curve.

Resulting attractor behavior

- Hydrodynamic relaxation dynamics for $\pi(t)$

$$\tau_{\rm relax}\partial_t \pi(t) + \pi(t) = -\zeta V_a(t) \quad \text{with} \quad V_a(t) = -3\frac{\partial_t a(t)}{a(t)}$$

Protocol for driving the scattering length :

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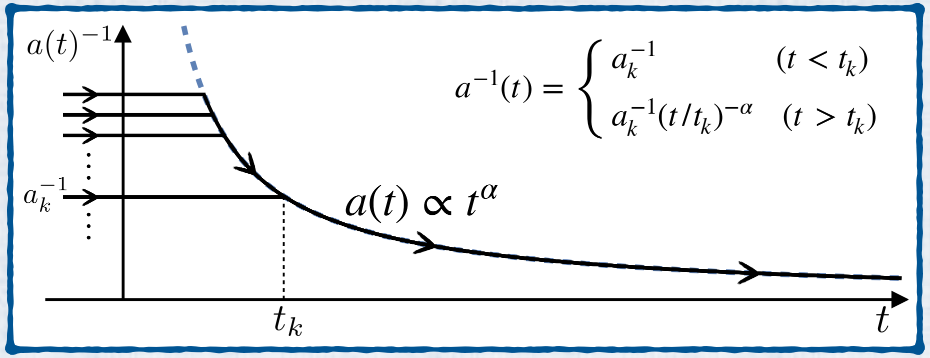
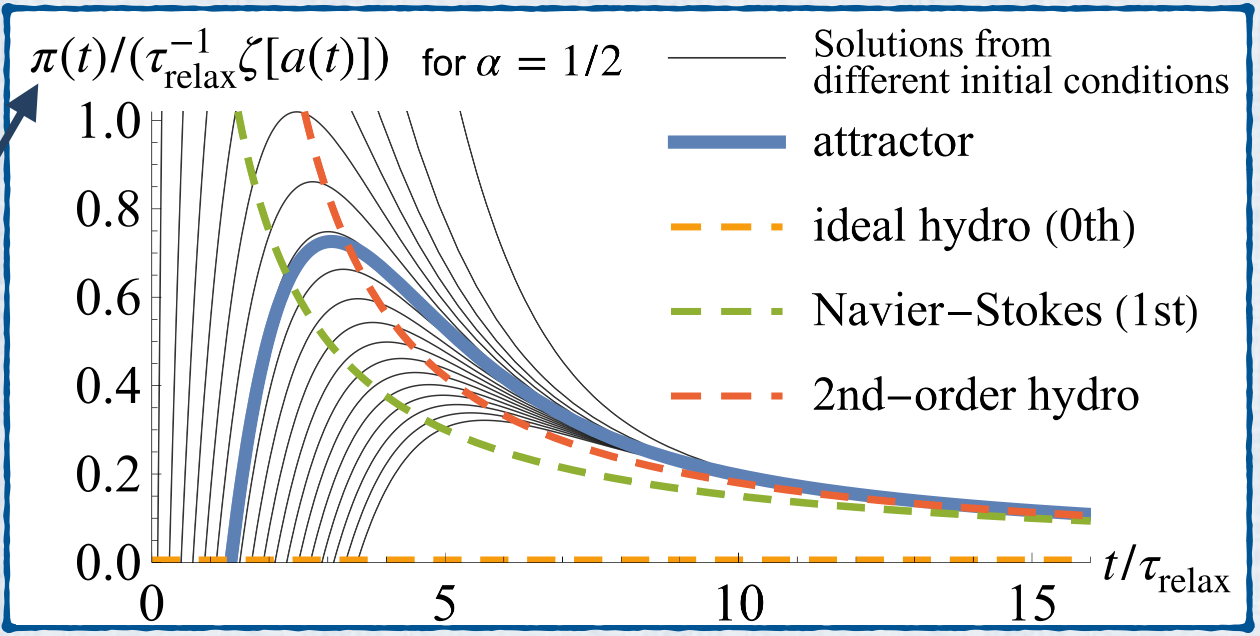
- **Example for two-comp. Fermi gases close to the unitary limit**



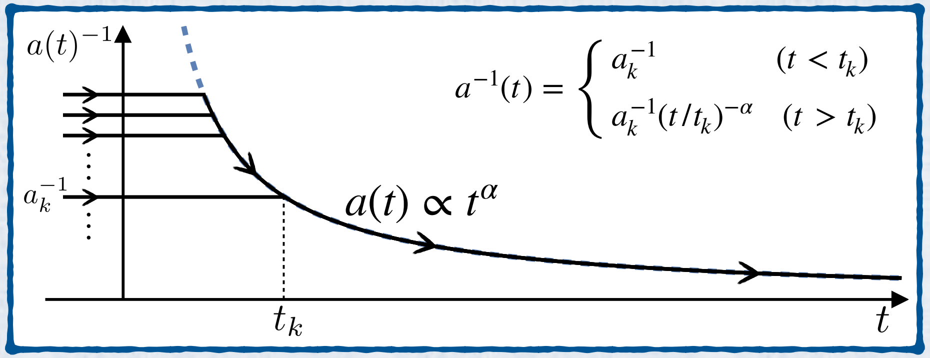
(Assuming $\zeta[a] \propto a^{-2}$ around the unitary limit)

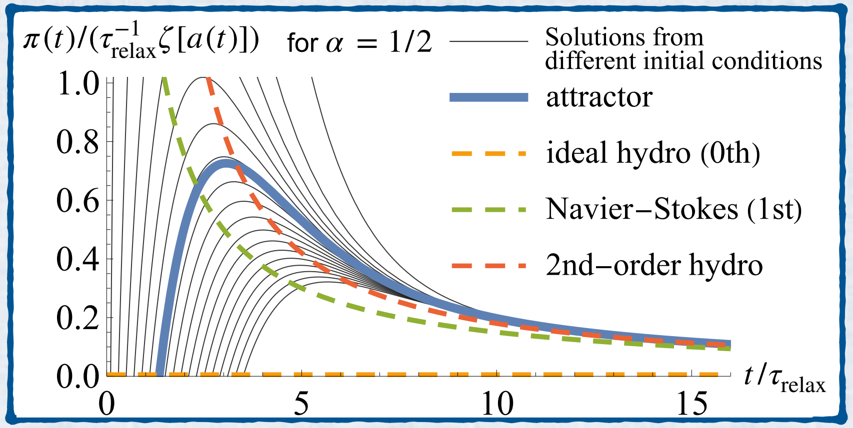


Dimensionless $\pi(t)$

- Solutions first converge to the attractor.
- Afterwards, the attractor approaches the hydrodynamic behavior.

Analytic results : divergence of the expansion





▸ **Analytical solution**

$$\pi(t) = \pi_{\rm ini} e^{-(t-t_{\rm ini})/\tau_{\rm relax}} + \pi_{\rm att}(t)$$

Non-hydrodynamic mode

- Depend on initial conditions a_k & t_k
- Cannot be expanded w.r.t $\tau_{\rm relax}/t$

✓ Attractor solution $\pi_{\rm att}(t) = \dfrac{3\alpha\zeta[\tilde{a}]}{\tau_{\rm relax}}(-1)^{2\alpha+1} e^{-t/\tau_{\rm relax}} \Gamma(-2\alpha, -t/\tau_{\rm relax})$

- Does NOT depend on a_k & t_k separately ➡ **Universal!!**
- Can be expanded with respect to $\tau_{\rm relax}/t$.

$$\pi_{\rm att}(t) \sim (\tau_{\rm relax}/t)^{2\alpha+1}\left[1 + (2\alpha+1)(\tau_{\rm relax}/t) + \cdots\right]$$ **: divergent series**

Navier-Stokes (→ 1) 2nd-order hydro (→ $(2\alpha+1)(\tau_{\rm relax}/t)$)

n th-order coefficient $\propto (n+2\alpha)!$ ⟶ **factorial divergent (Borel summable)**

The gradient expansion for the attractor solution is significantly less accurate.

▸ The attractor solution cannot be obtained even if higher-order fluid corrections are summed up.

▸ Origin of time-scale separation: non-hydro → attractor, attractor →hydro.

Fluid expansion by time-dep. scattering length

Equivalence between isotropic fluid expansion and contraction of scattering length

K.Fujii & Y. Nishida, PRA **98**, 063634 (2018).

In ultracold atomic gases whose inter-particle interaction is characterized only by the *s*-wave scattering length,

Relative size change between the fluid size & the scattering length

✓ We can realize isotropic fluid expansion in a well-controlled manner without moving parts by using $a(t)$.

via Feshbach resonance