Exact many-body dynamics in quantum circuits via space-time duality

Bruno Bertini

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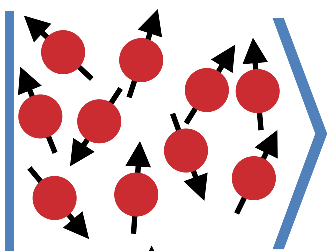
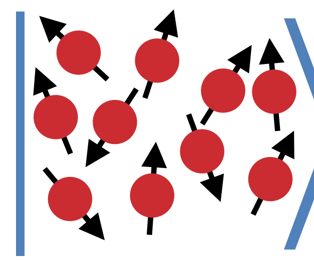
YITP

5 June 2026

Can we describe interacting many-particle quantum systems out of equilibrium?

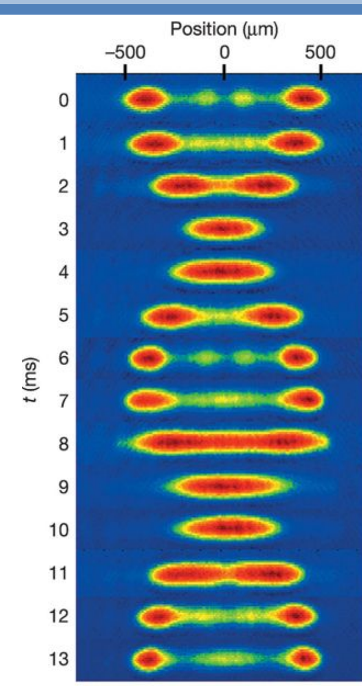
Quantum Quench

$|\Psi_0\rangle =$  $\mapsto$ $|\Psi_t\rangle = e^{-iHt}$ 

GS of some local H_0

different local Hamiltonian

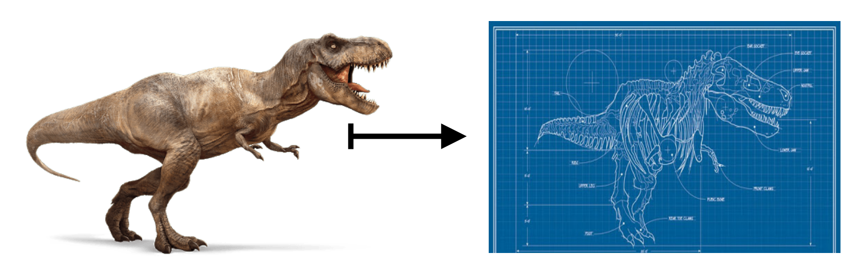
Experimentally Relevant



Kinoshita, Wenger, and Weiss, Nature **400**, 900 (2006).

Simple and well defined

Displays the key physics



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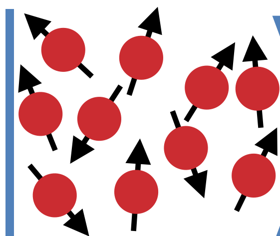
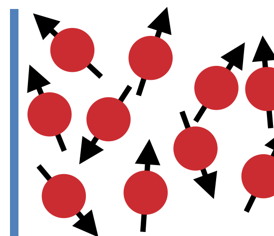
$$\left|\Psi_0\right\rangle = \Big|$$  $$\Big\rangle \mapsto \left|\Psi_t\right\rangle = e^{-iHt}\Big|$$  $$\Big\rangle$$

Questions

1. How does **equilibrium physics** emerge from **coherent dynamics**?

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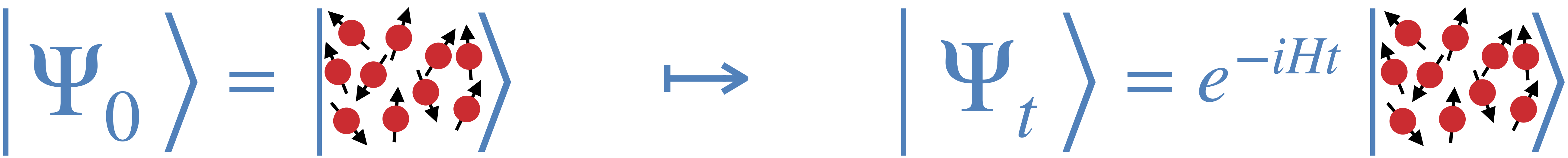

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Hard!

1. No exact method in the presence of interactions (e.g. integrability does not help)

2. Exact numerical approaches are limited (by system size or entanglement growth)

3. No control over the approximations

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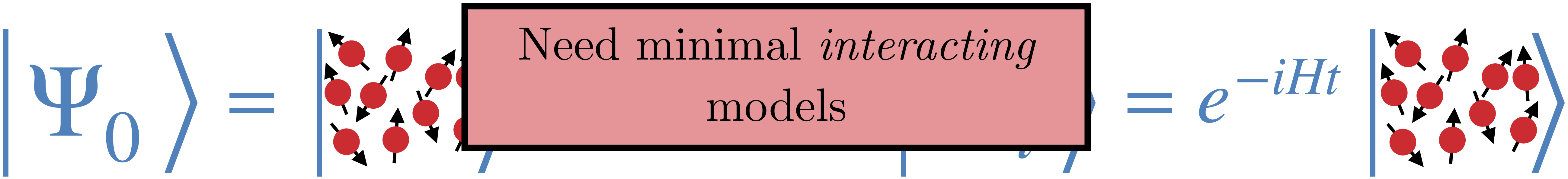

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Local Quantum Circuits

◆ Consider a chain of qudits (d states) prepared in a simple initial state

$$|\Psi_0\rangle = \bigotimes_{j=1}^{2L} |\psi_j\rangle \xrightarrow{\text{diagram}} \langle i|\psi_j\rangle = \quad i = 1,\ldots,d$$

i

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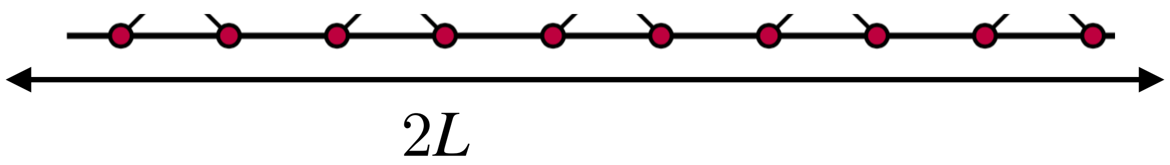

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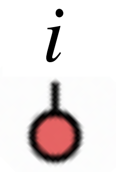

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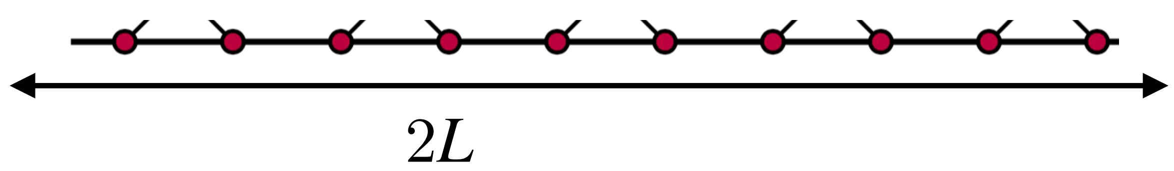

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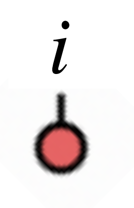
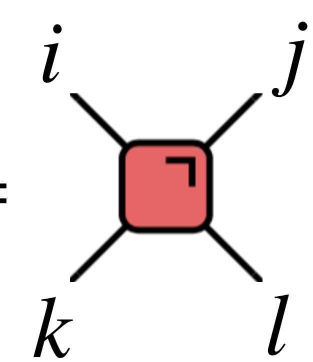


$$i = 1, \ldots, d$$

◆ Evolve them by applying unitary gates only connecting nearest neighbours

$$\mathbb{U} = \Pi U^{\otimes L} \Pi^{-1} U^{\otimes L} \quad \xrightarrow{\text{diagram}} \quad \langle i, j | U | k, l \rangle =$$



$UU^\dagger = I \qquad \Pi =$ one-site shift

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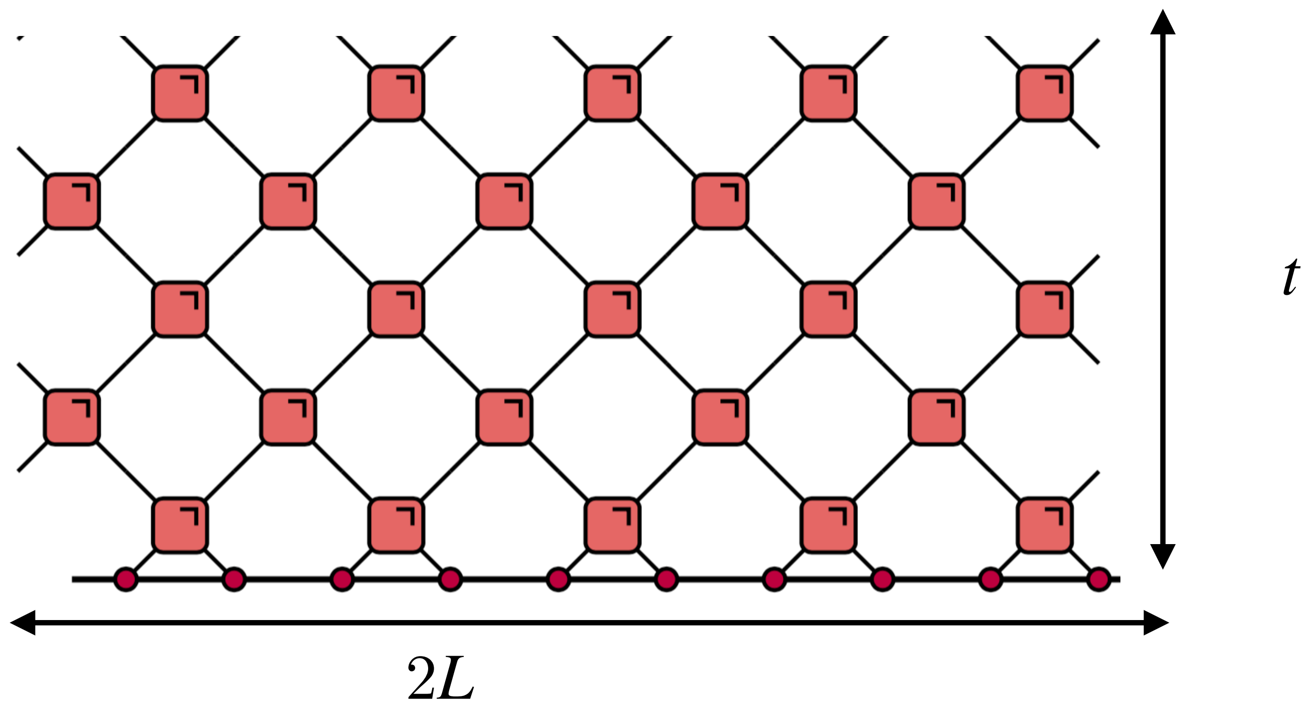

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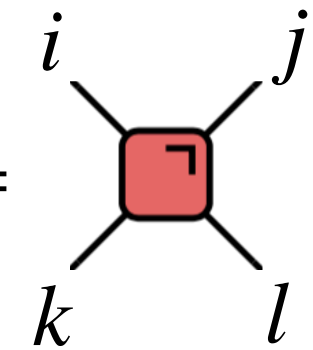

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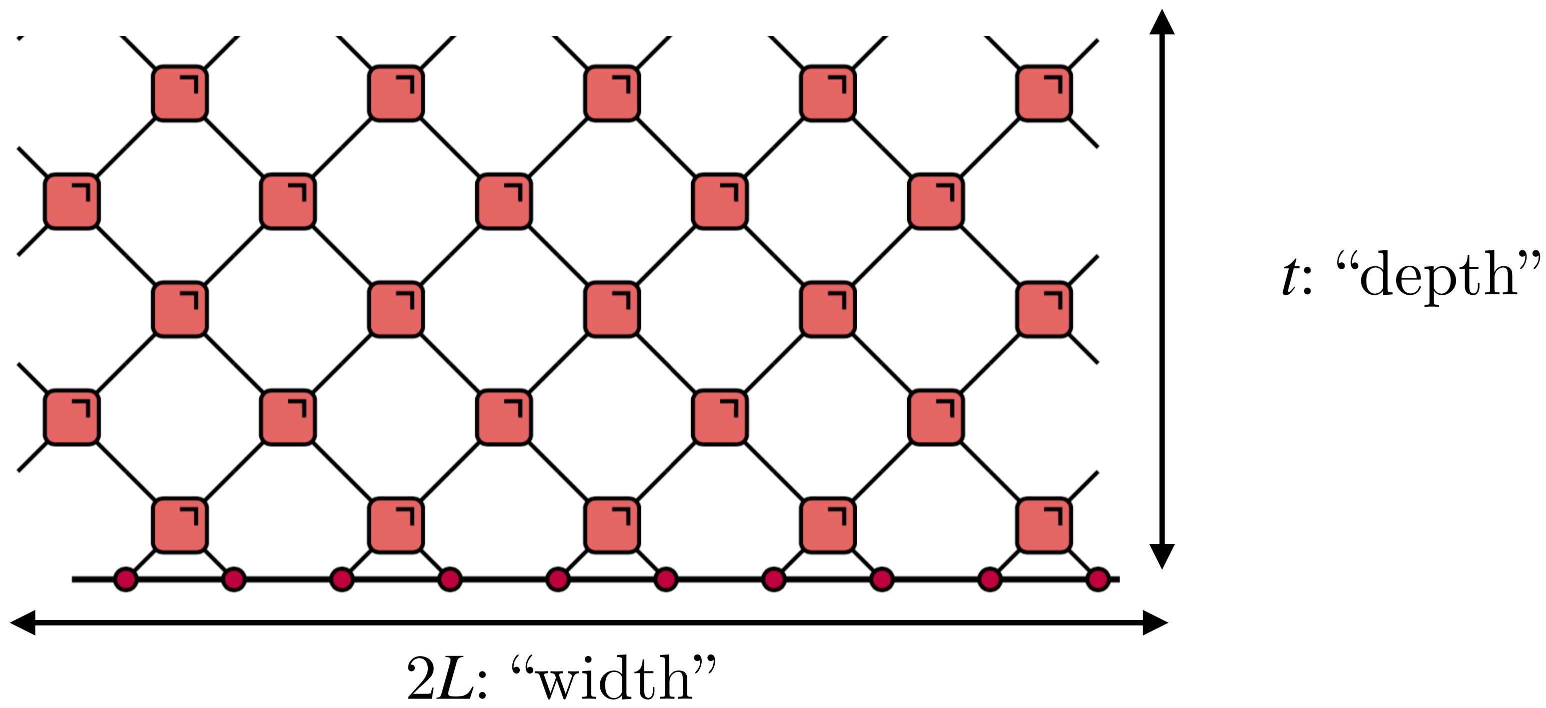

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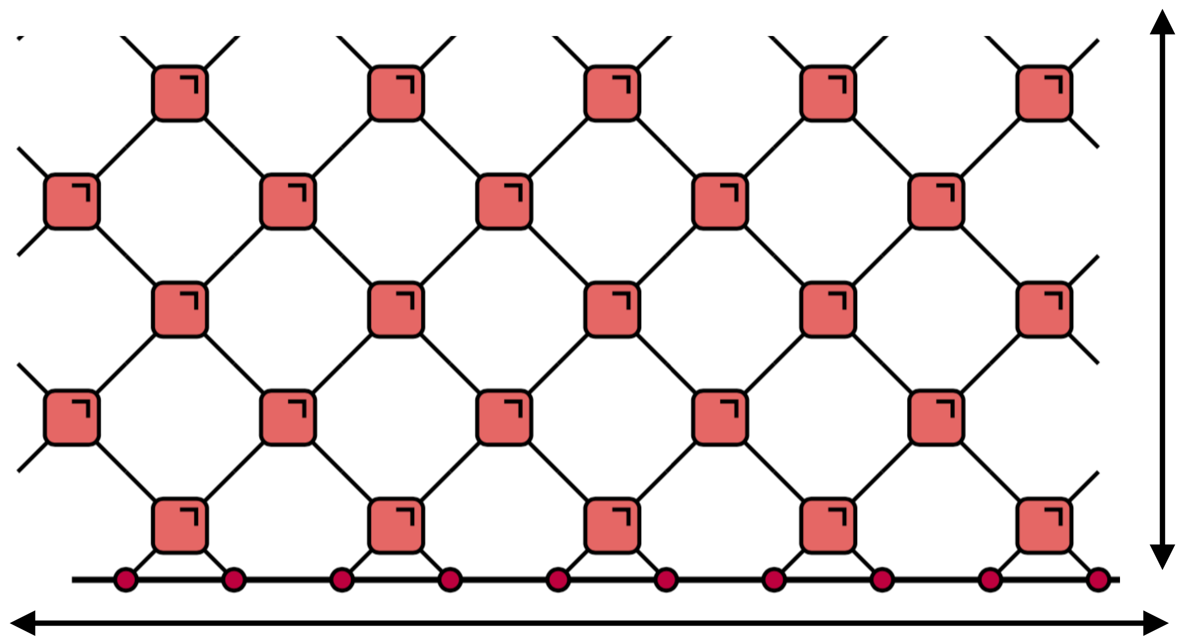
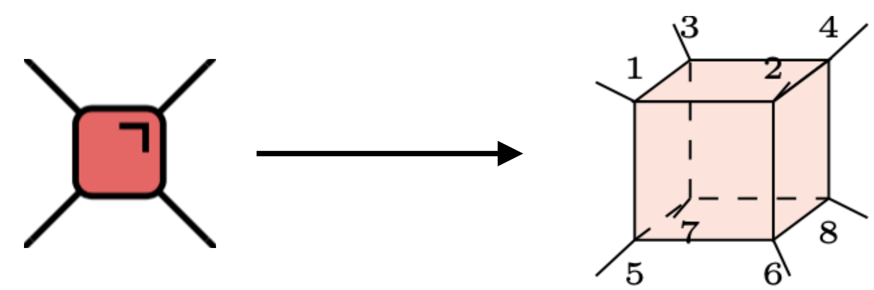


t: “depth”

$2L$: “width”

◆ Defined in any spatial dimension



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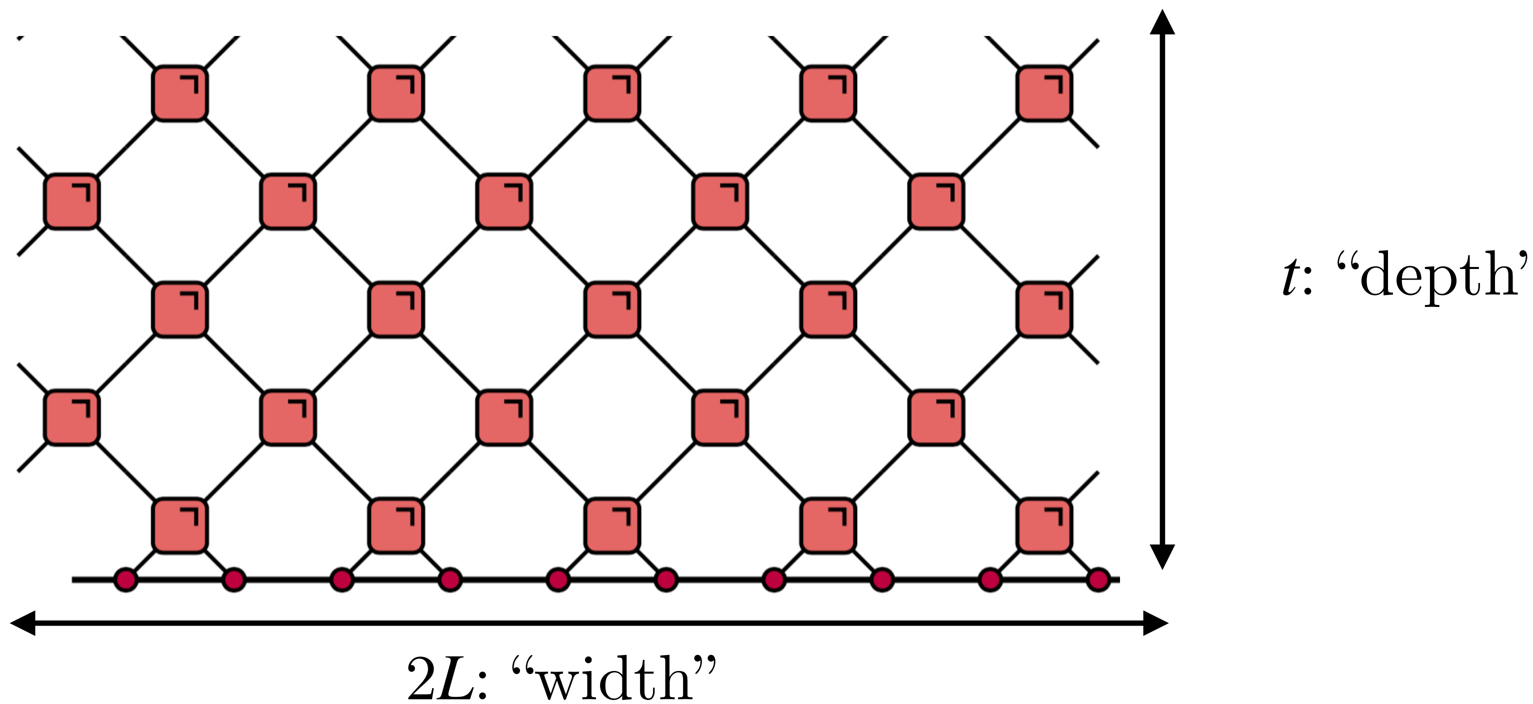

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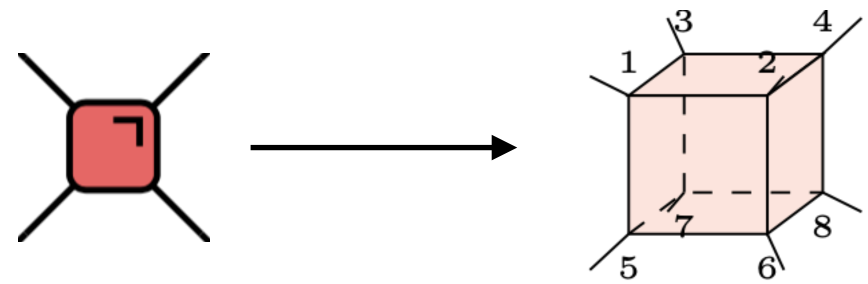

◆ Strict causality

Local Quantum Circuits

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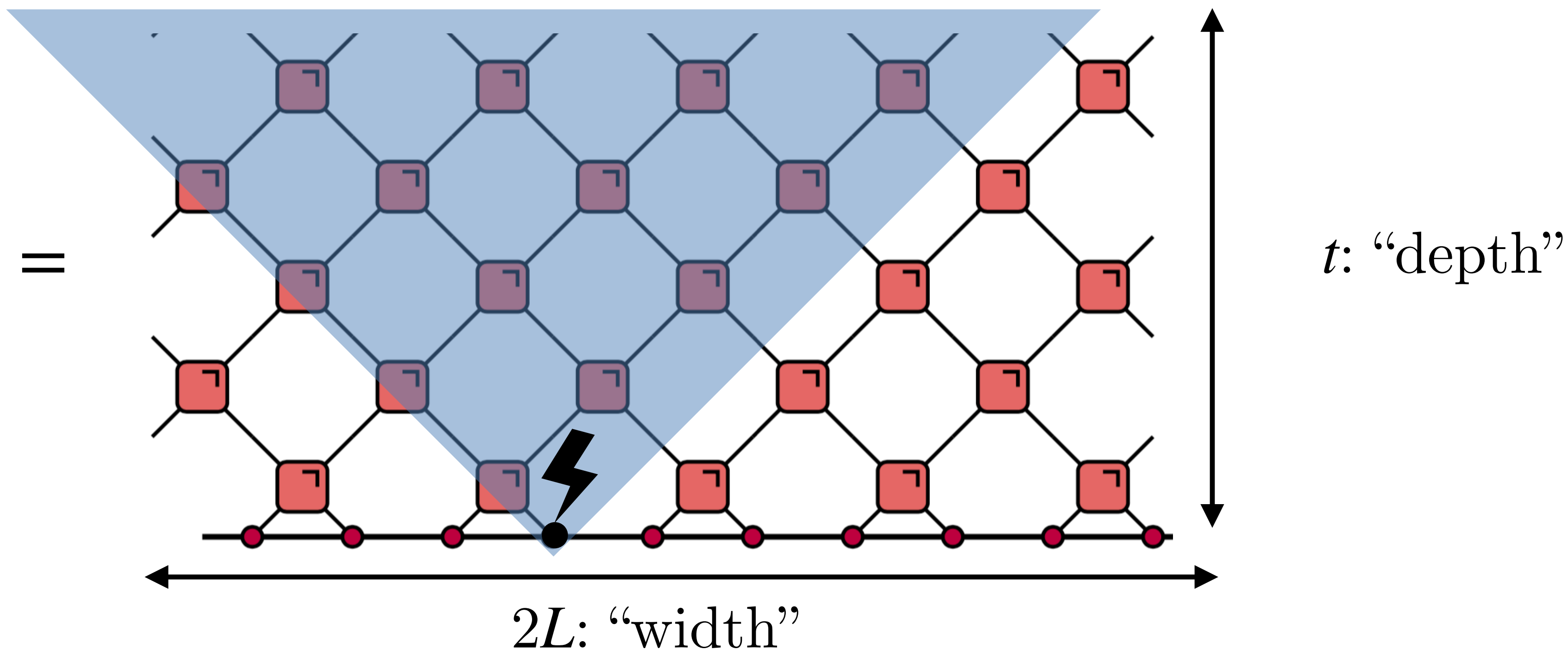

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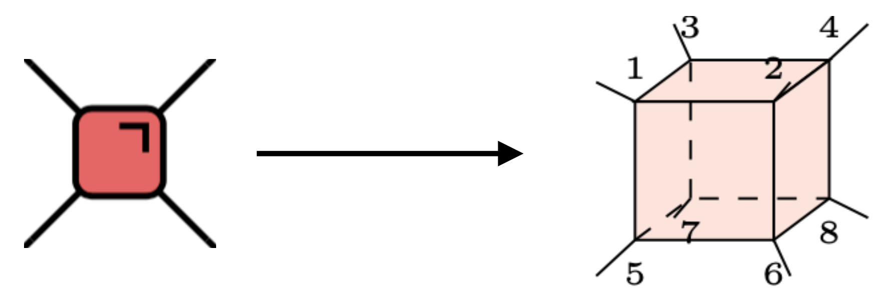

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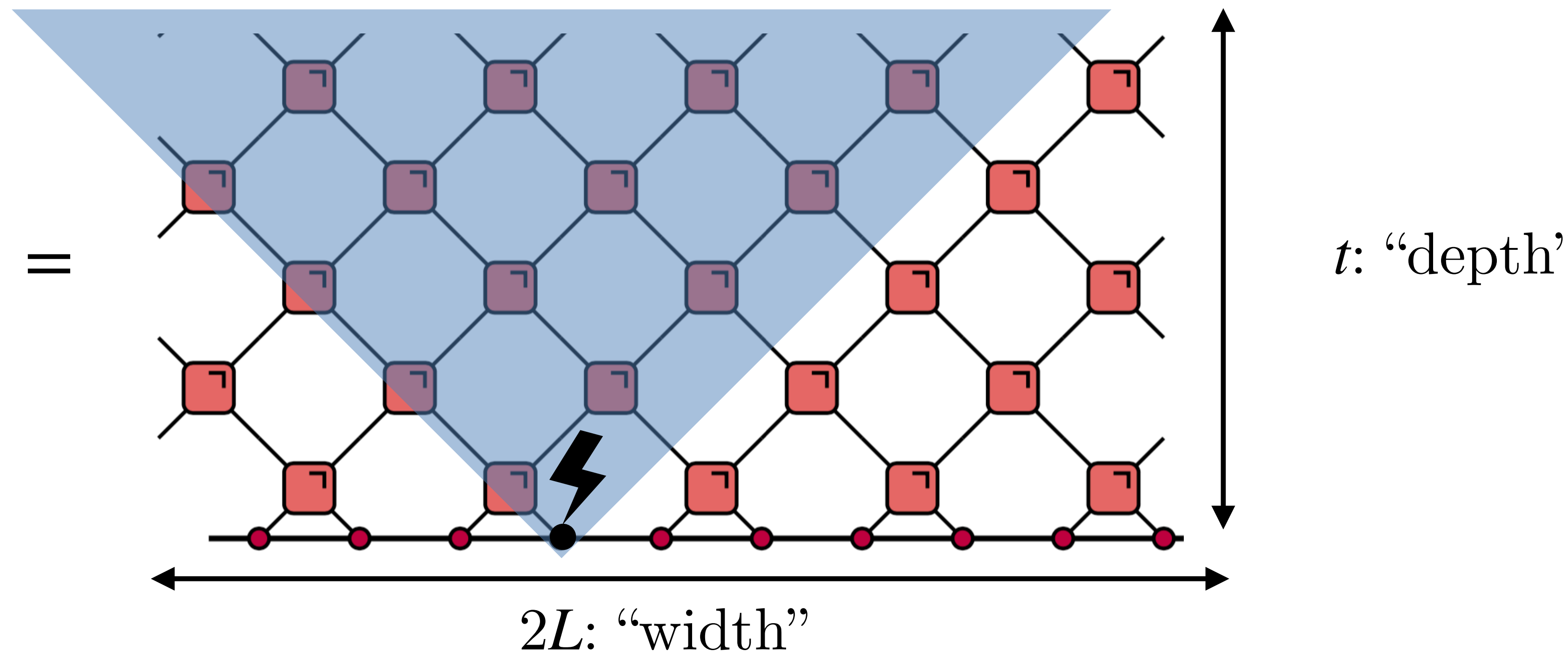

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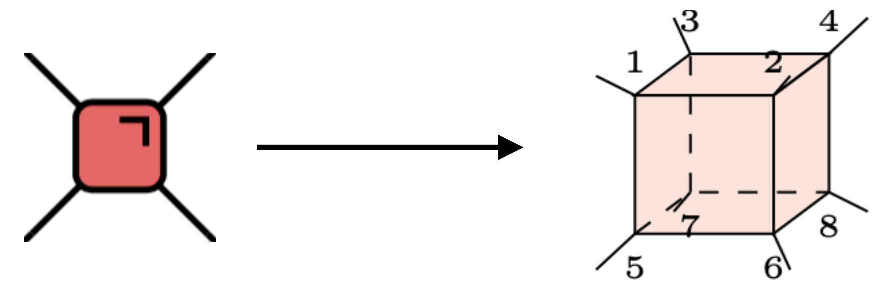
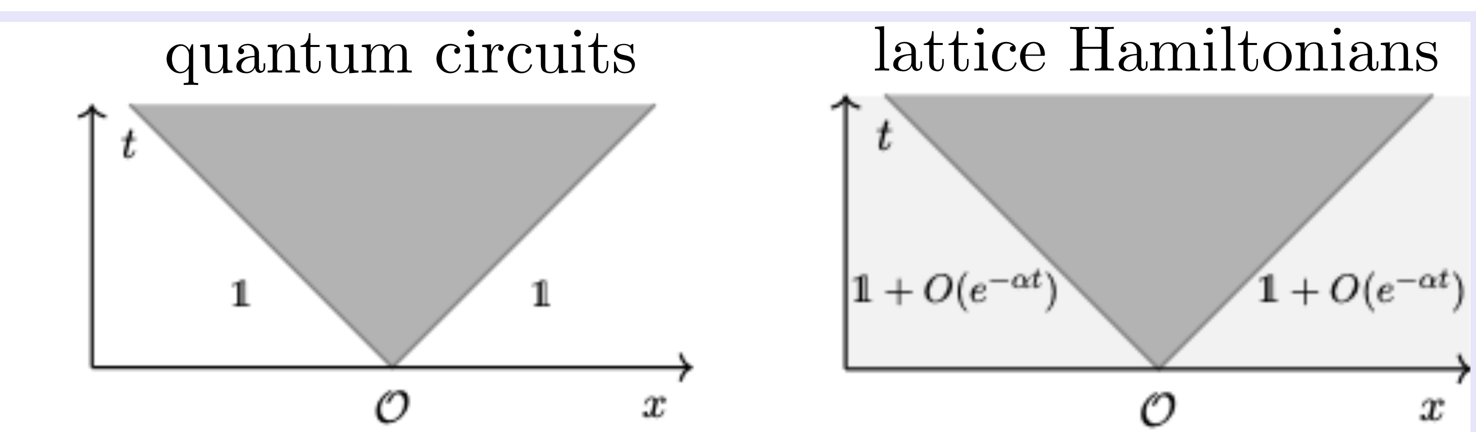


◆ Strict causality

- similar to lattice Hamiltonians

$$\|[\mathcal{O}_A(t), \mathcal{O}_B(0)]\| \leq D \exp[-\frac{(\ell - vt)}{\xi}]$$

Lieb and Robinson, CMP, **28**(3), 251–257 (1972).



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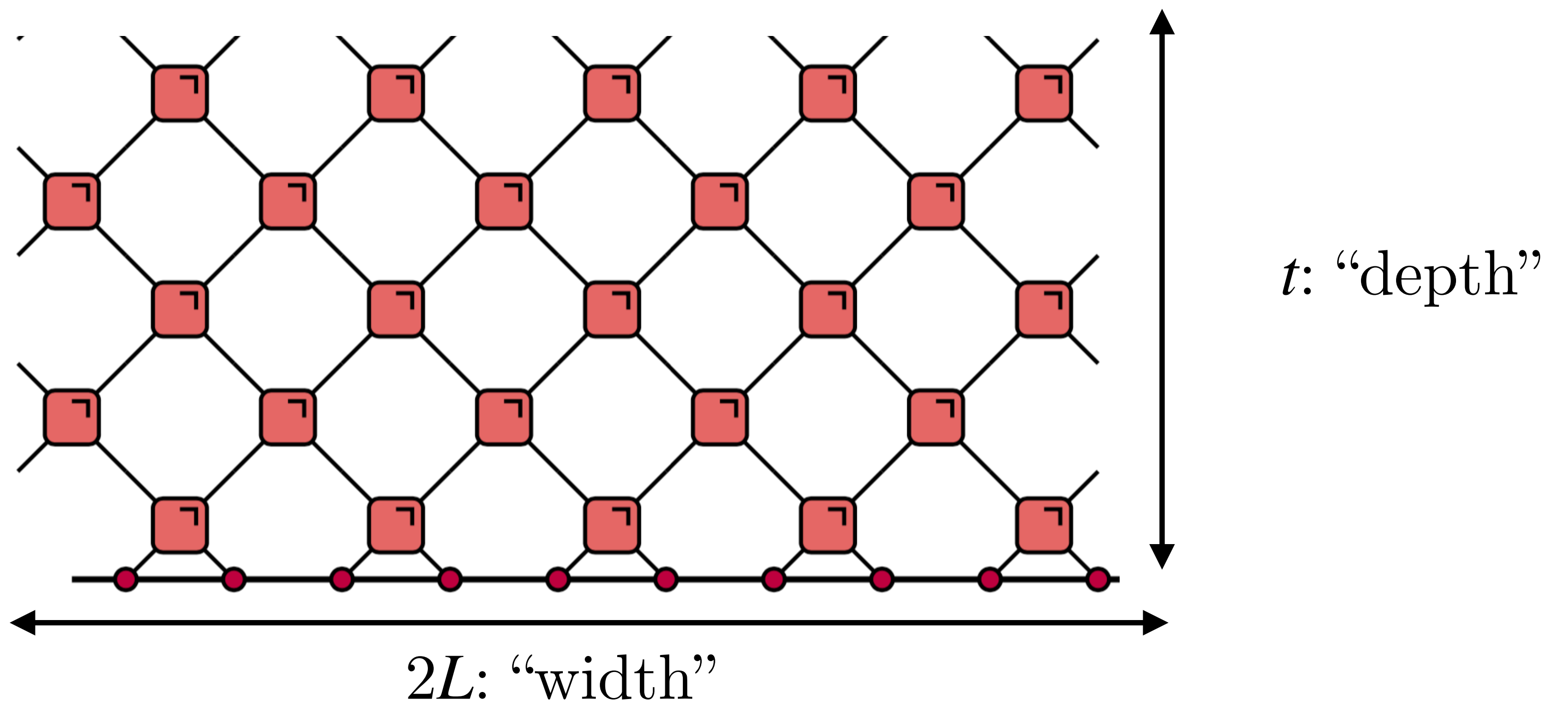

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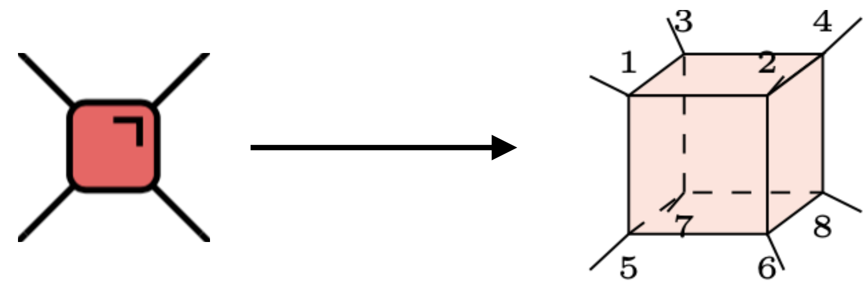

- Strict causality

- Approximate arbitrary the dynamics of lattice Hamiltonians

Suzuki, PLA, **146**(6), 319-323 (1990);
Trotter, PAMS, **10**(4), 545–551 (1954);
Osborne, PRL **97**, 157202 (2006).

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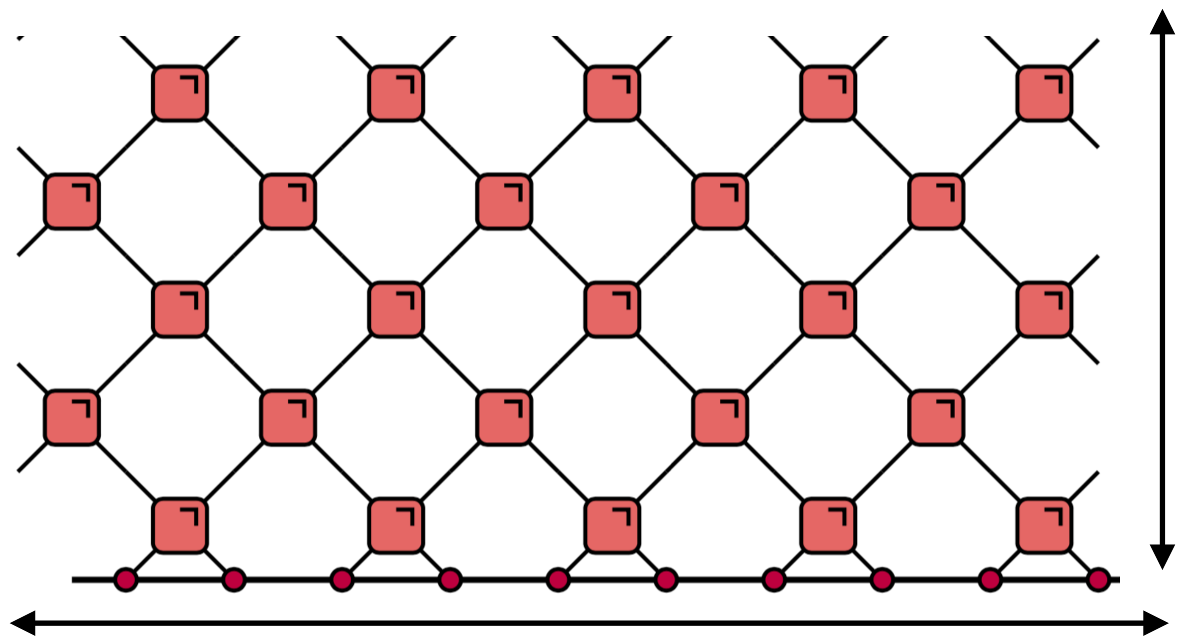

t: “depth”

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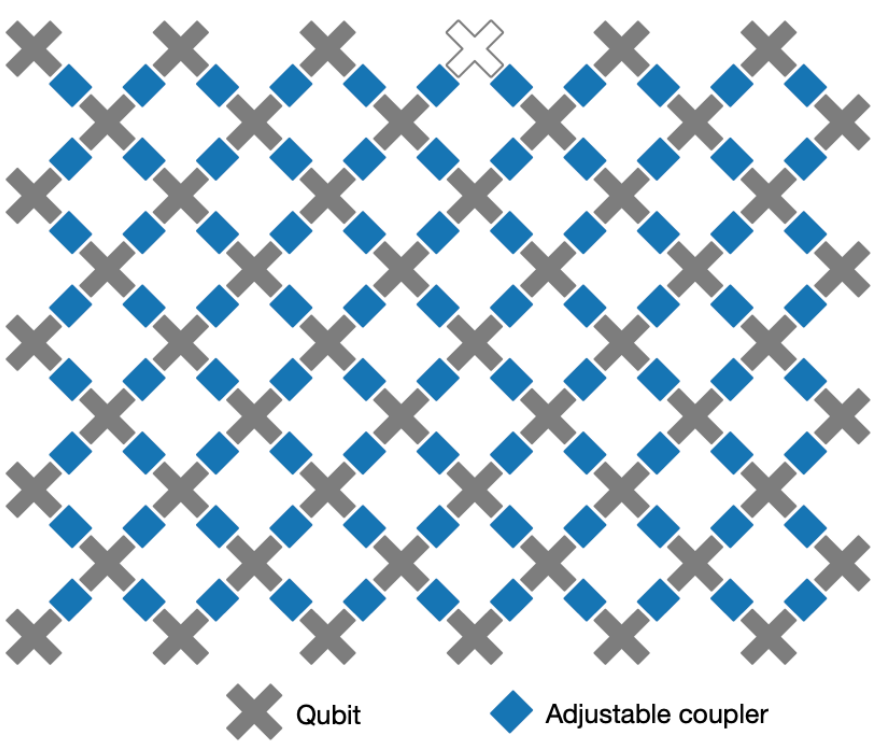
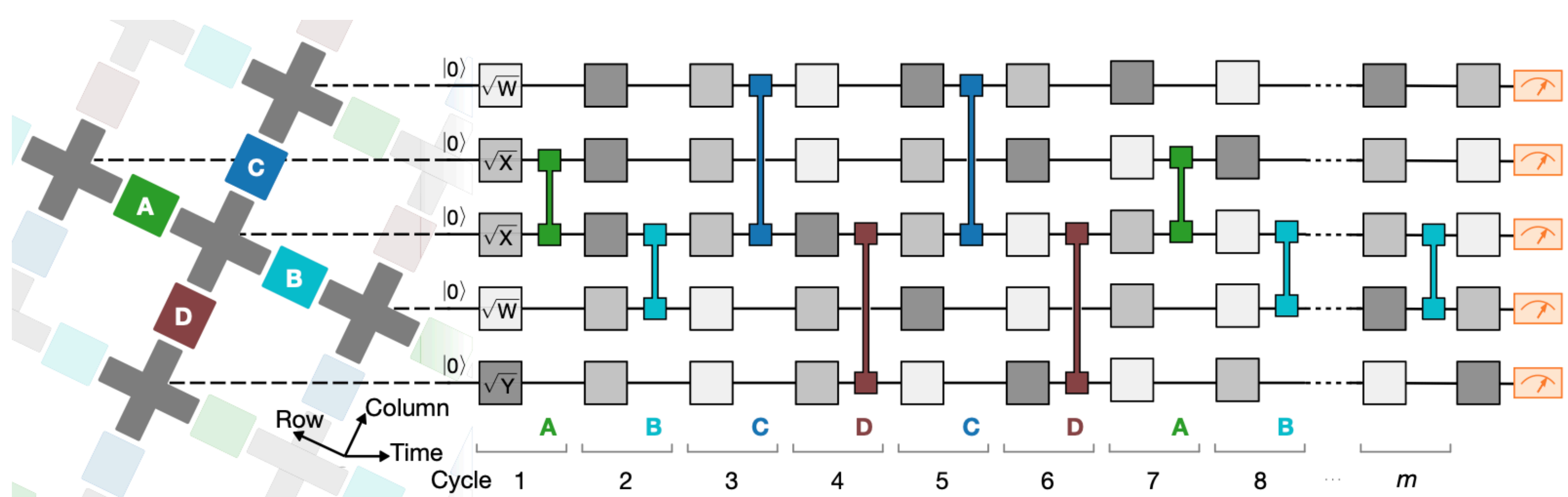
- Model the architecture of real (digital) quantum simulators





Arute, et al. Nature **574**, 505 (2019).

Local Quantum Circuits

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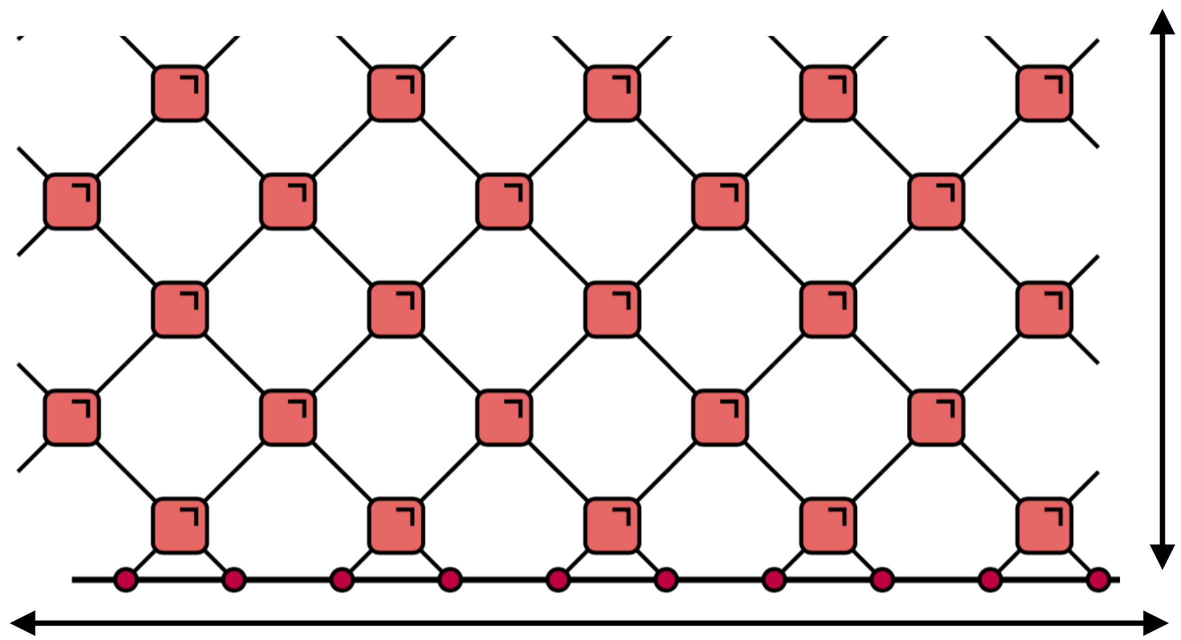

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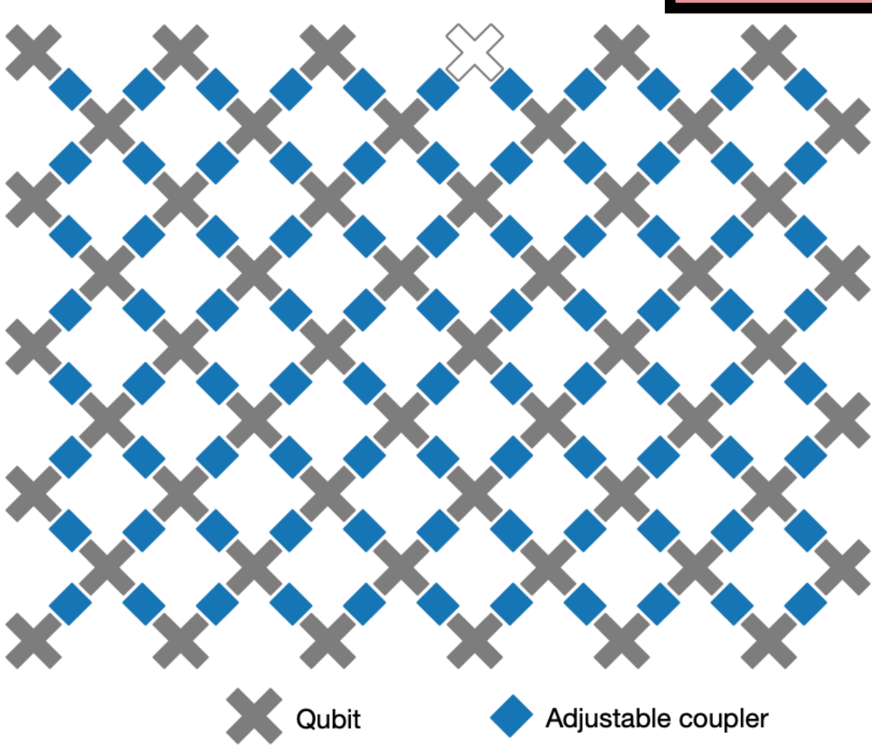
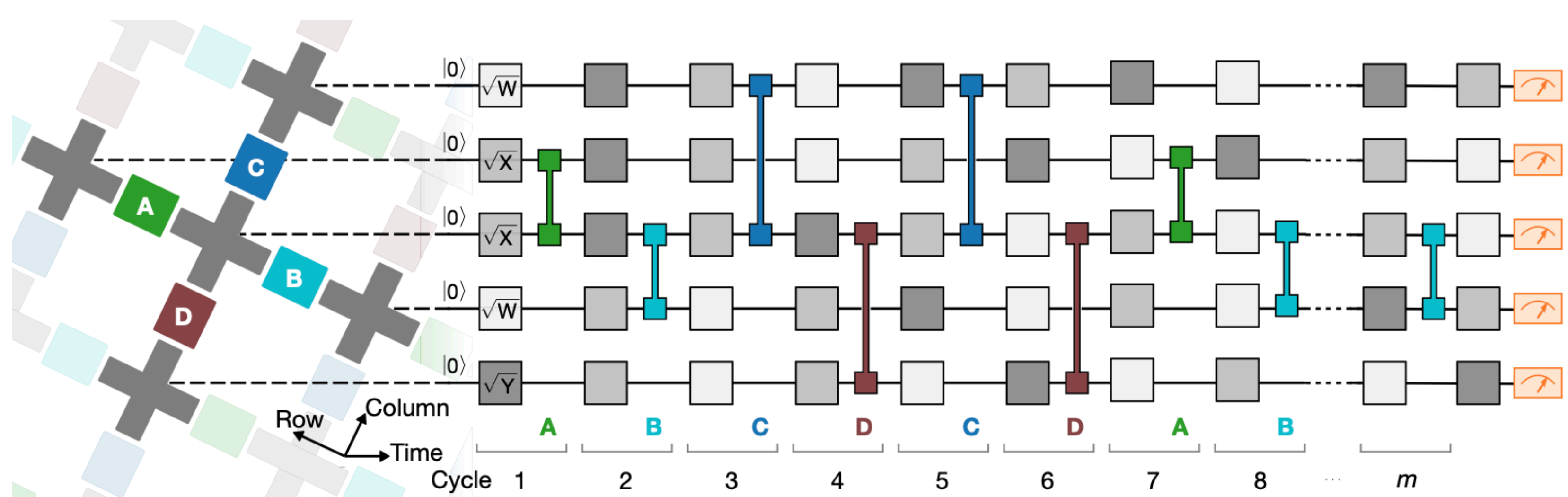
- Model the architecture o

Treatable dynamics?





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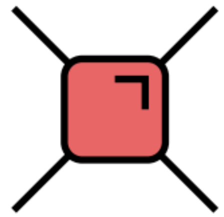
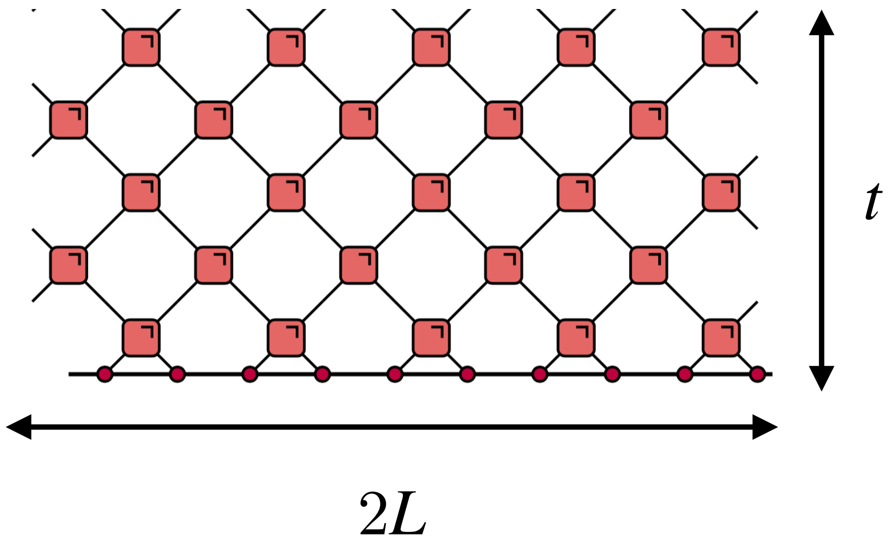
Solvable Quantum Circuits

1) Introduce randomness





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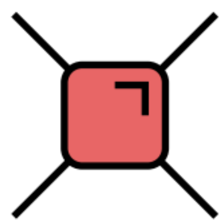

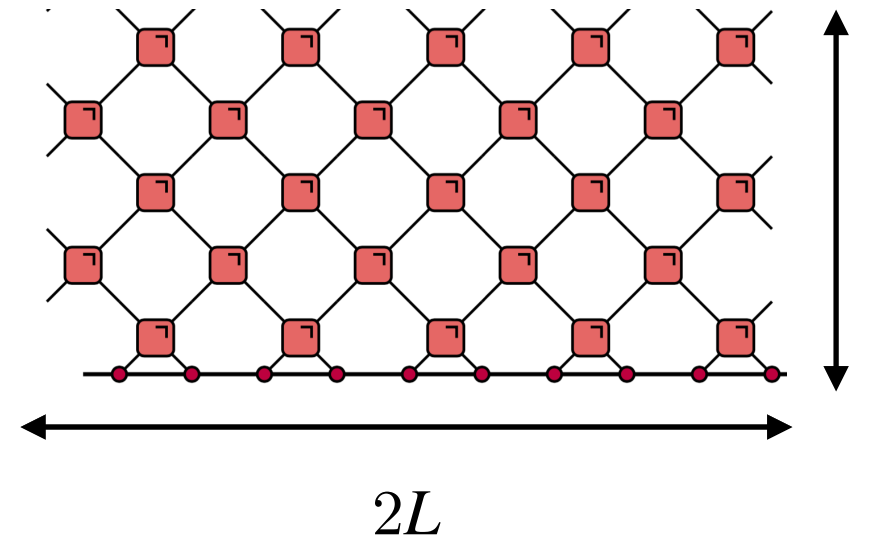

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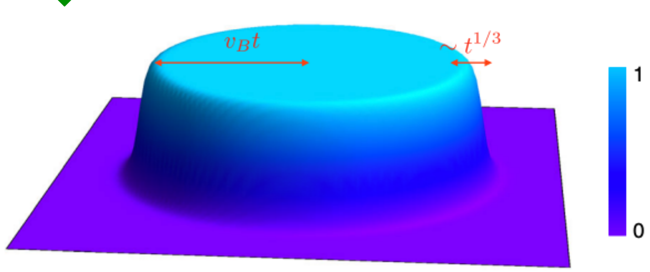

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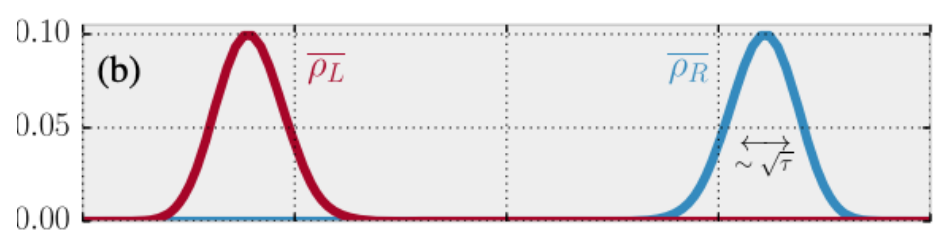
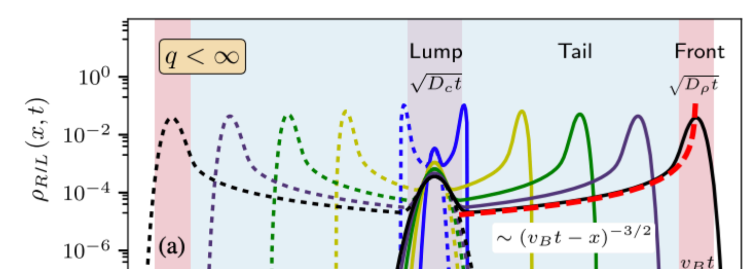


von Keyserlingk, Rakovszky, Pollmann, and Sondhi, PRX **8**, 021013 (2018)



Khemani, Vishwanath, and Huse, PRX **8**, 031057 (2018)

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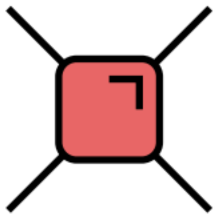

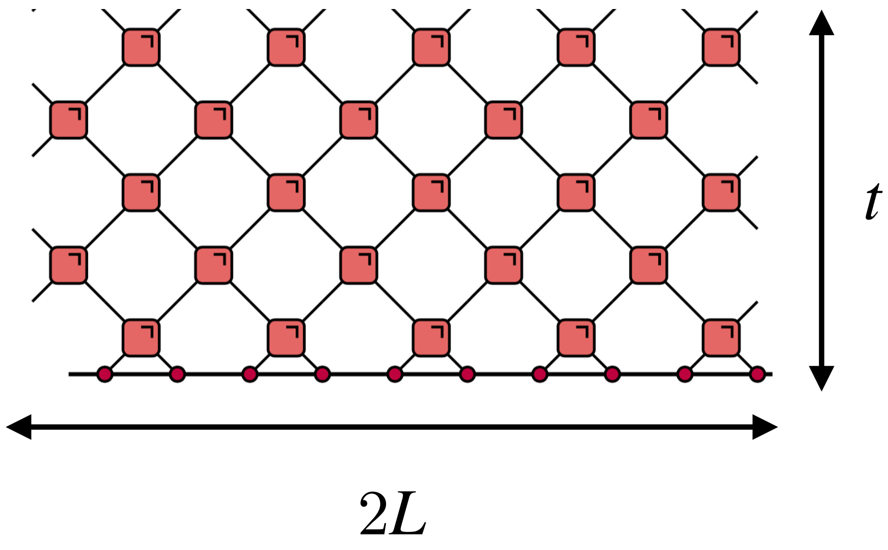

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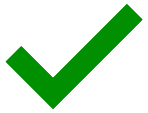

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Randomness is introduced *ad hoc*, making the system effectively open

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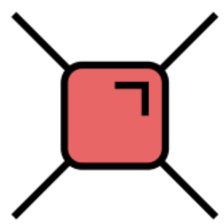

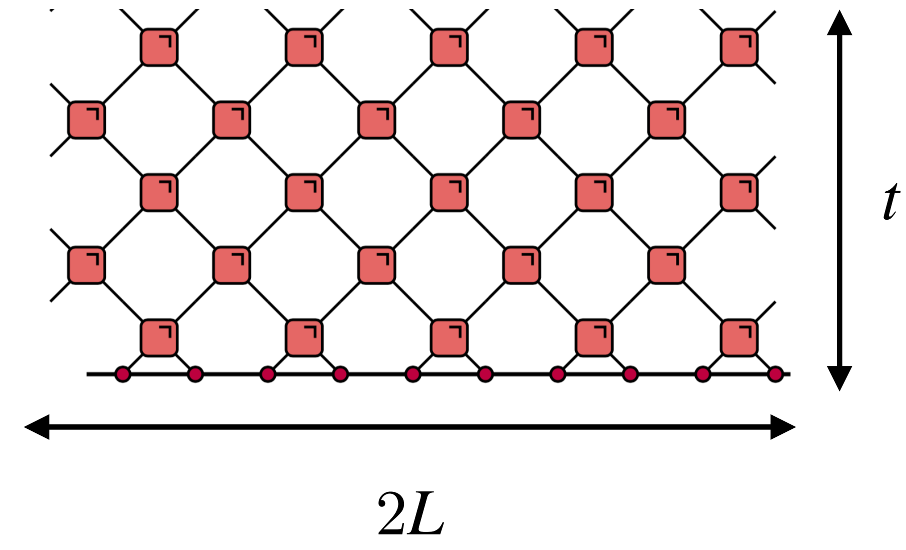

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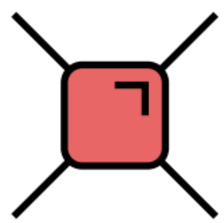

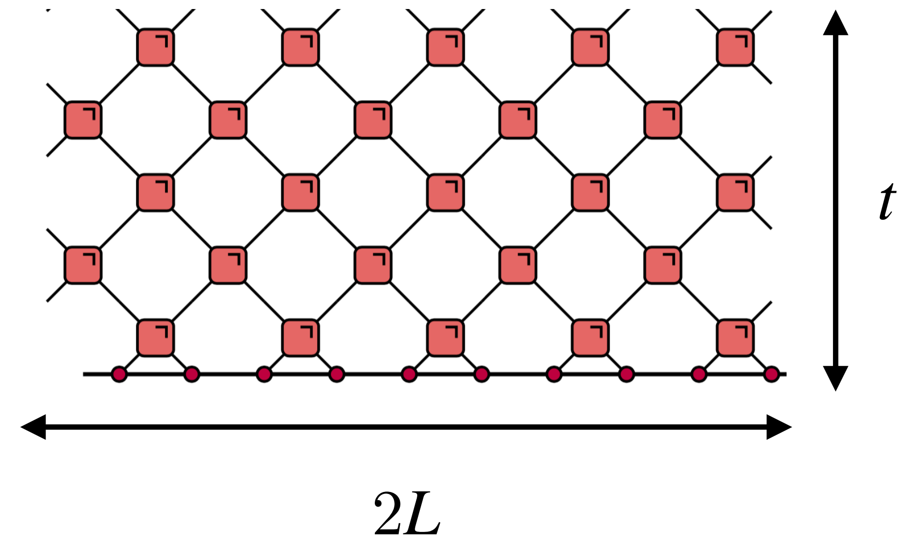

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Harder

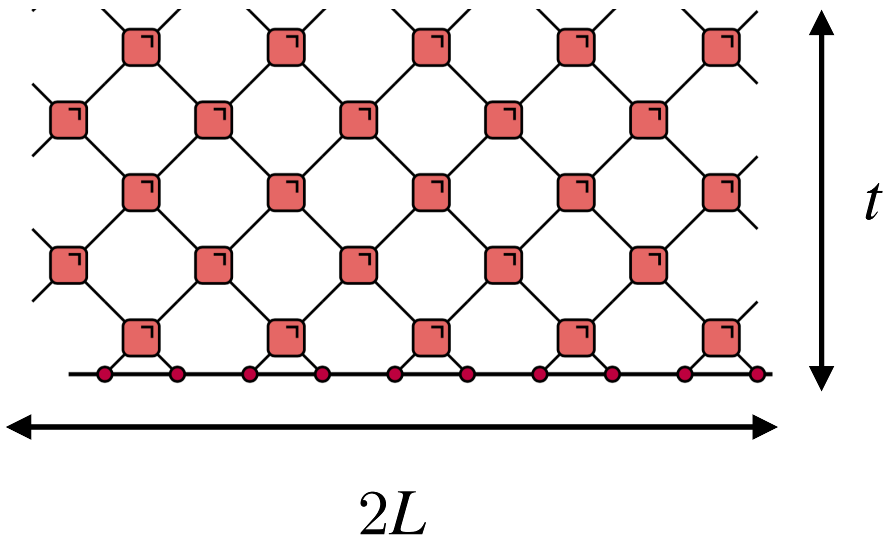
✓ Useful to understand qualitatively *entanglement growth* and *operator spreading*

✗ Randomness is introduced *ad hoc*, making the system effectively open

✗ Making the noise more correlated makes the system harder to treat

Solvable Quantum Circuits

2) Impose conditions on the gates



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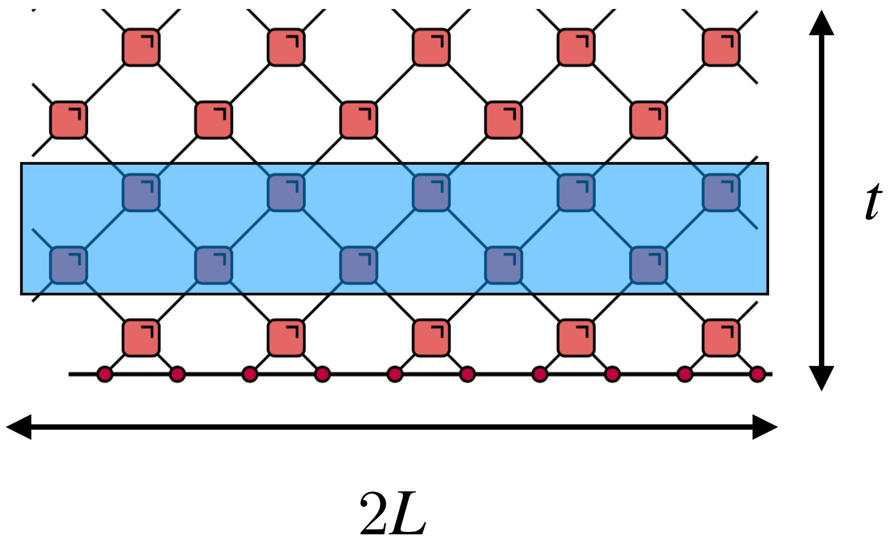

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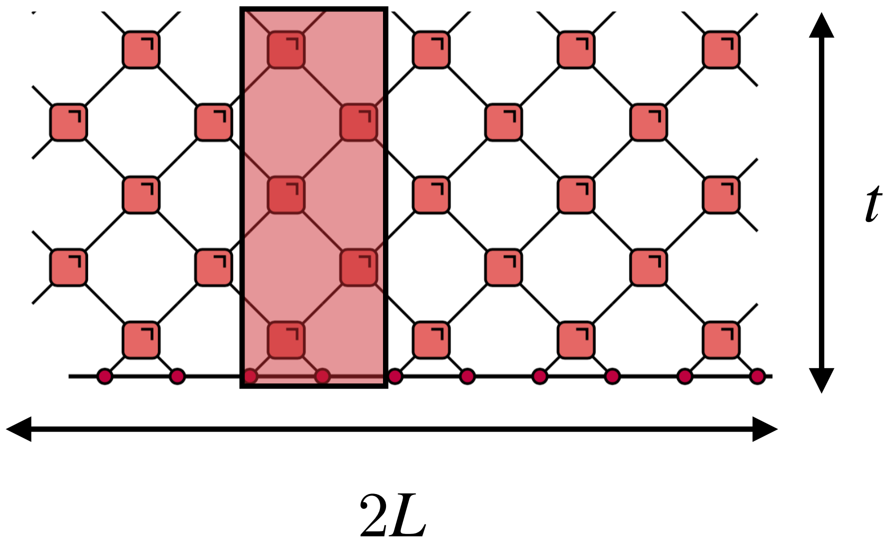

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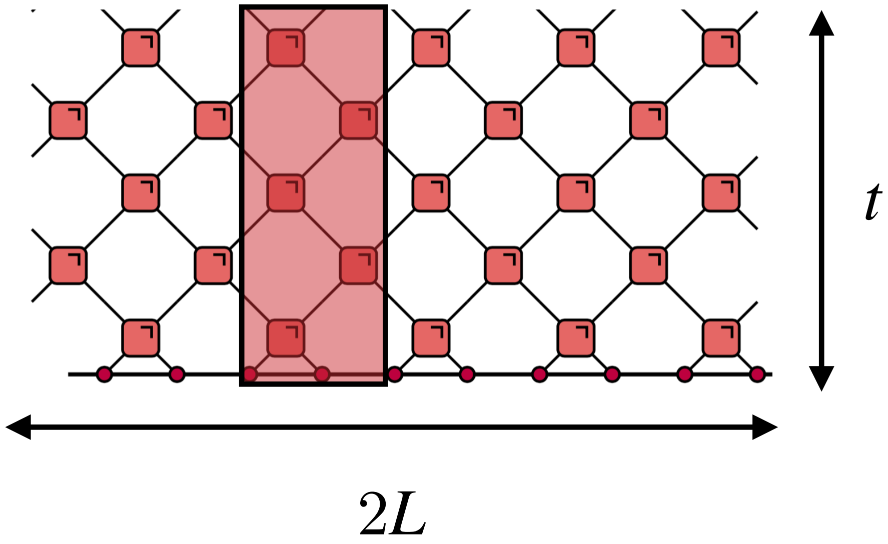

Impose it generates unitary dynamics also in space

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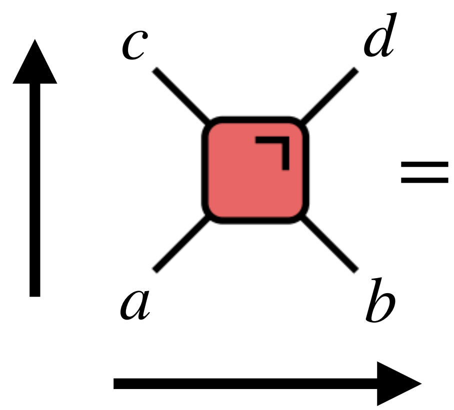
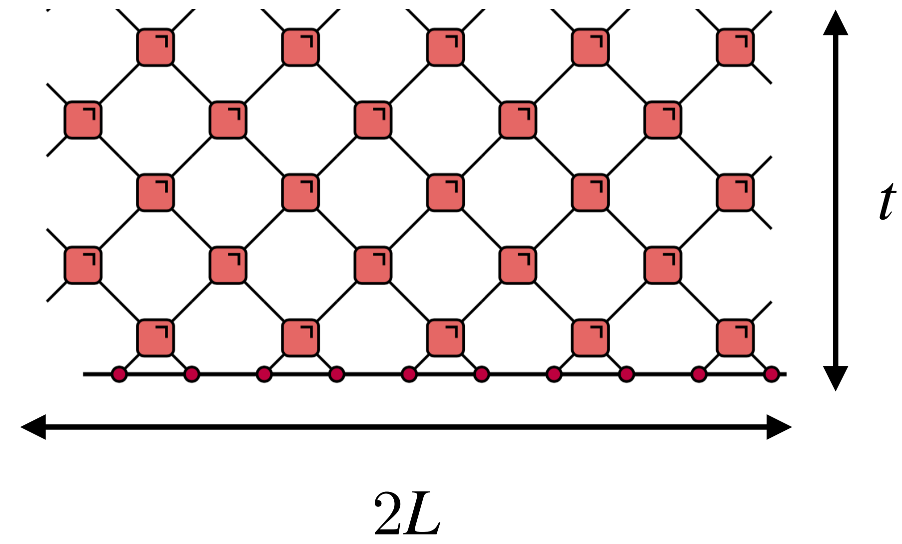


$$= \langle ca | \tilde{U} | db \rangle$$



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BB, Kos, and Prosen, PRL **123**, 210601 (2019);
Gopalakrishnan and Lamacraft, PRB **100**, 064309 (2019)

$$UU^\dagger = U^\dagger U = I$$
$$\tilde{U}\tilde{U}^\dagger = \tilde{U}^\dagger \tilde{U} = I$$

Dual-Unitary

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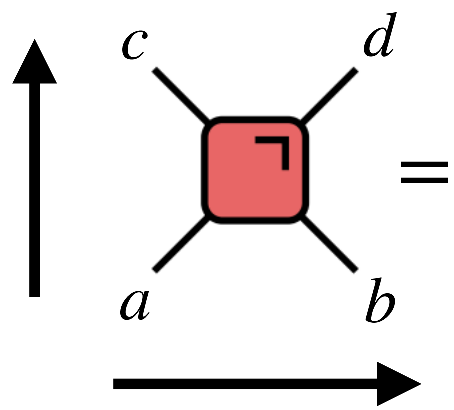

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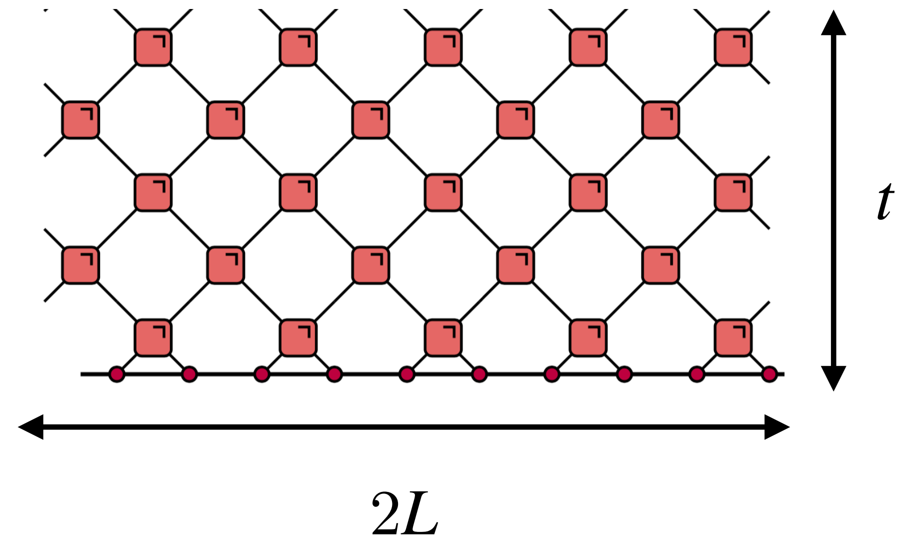


Impose it generates unitary dynamics also in space

BB, Kos, and Prosen, PRL **123**, 210601 (2019);
Gopalakrishnan and Lamacraft, PRB **100**, 064309 (2019)

$$UU^\dagger = U^\dagger U = I$$
$$\tilde{U}\tilde{U}^\dagger = \tilde{U}^\dagger \tilde{U} = I$$

Dual-Unitary

- Can be defined for any d:
  - $d = 2$: full parameterisation (12 parameters)

    BB, Kos, and Prosen, PRL **123**, 210601 (2019)
  - $d > 2$: partial parameterisations ($\propto d^3$ parameters)

    Prosen, Chaos **31**, 093101 (2021)

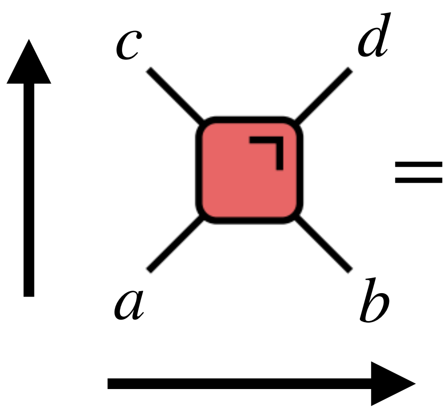
- Same idea also applies in higher D and different geometries

Jonay, Khemani, and Ippoliti, PRR **3**, 043046 (2021); Kasim and Prosen, 2023 JPA **56** 025003

Solvable Quantum Circuits

2) Impose conditions on the gates

$$\langle ab | U | cd \rangle =$$



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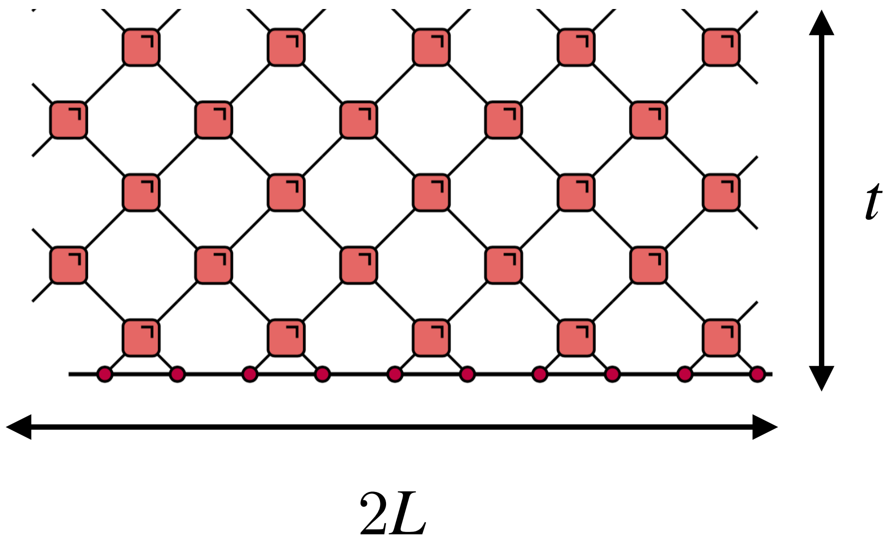

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Dual-Unitary

- Examples

$$U =$$

SWAP

$$\mathbb{U} = e^{i\frac{\pi}{4}\sum_j \left\{\sigma_j^z \sigma_{j+1}^z + h_j \sigma_j^z\right\}} e^{i\frac{\pi}{4}\sum_j \sigma_j^x}$$

Self Dual Kicked Ising

Solvable Quantum Circuits

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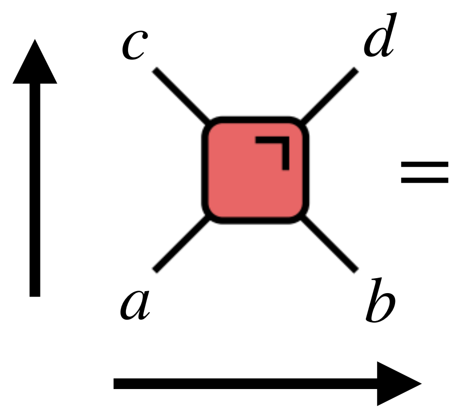
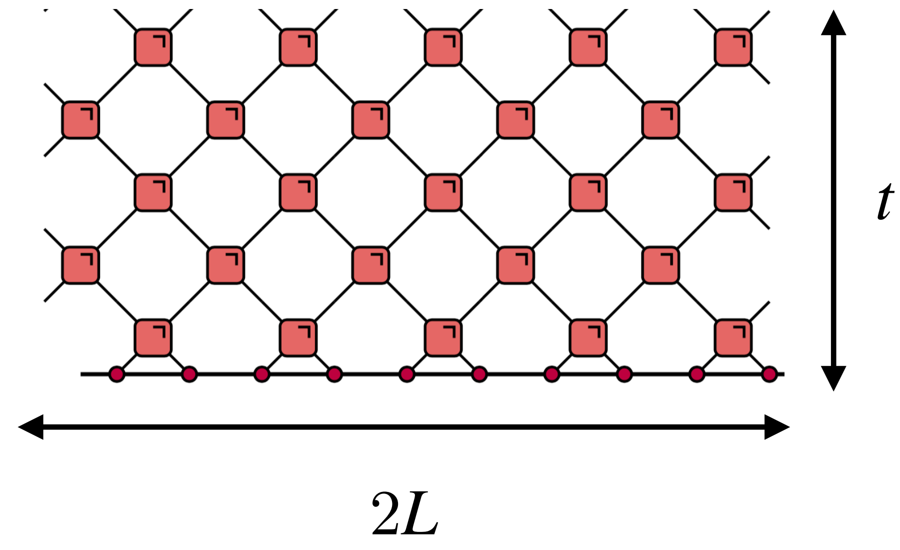
$$\langle ab | U | cd \rangle =$$  $$= \langle ca | \tilde{U} | db \rangle$$



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Dual-Unitary

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$$U =$$  *SWAP*

Provably Quantum Chaotic! (COE spectral correlations)

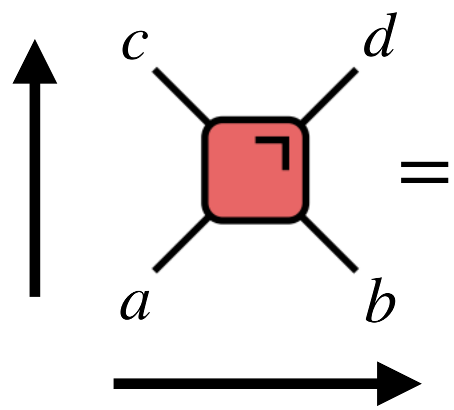
BB, Kos, and Prosen, PRL **121**, 264101 (2018)

$$\mathbb{U} = e^{i\frac{\pi}{4}\sum_j \left\{ \sigma_j^z \sigma_{j+1}^z + h_j \sigma_j^z \right\}} e^{i\frac{\pi}{4}\sum_j \sigma_j^x}$$ *Self Dual Kicked Ising*

Solvable Quantum Circuits

2) Impose conditions on the gates

$$\langle ab|U|cd\rangle =$$



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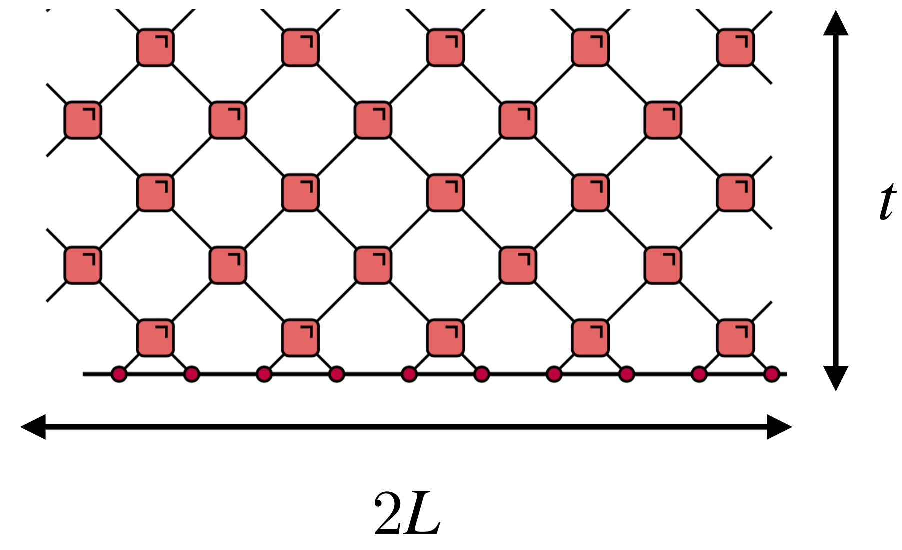

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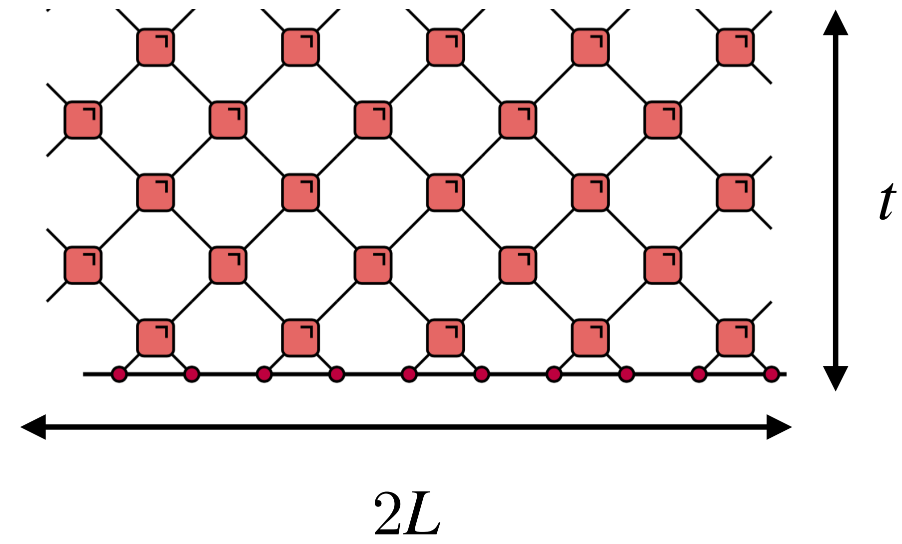
Self Dual Kicked Ising

$$\lim_{L\to\infty} \langle |\mathrm{tr}[\mathbb{U}^t]|^2 \rangle = 2t$$

Solvable Quantum Circuits



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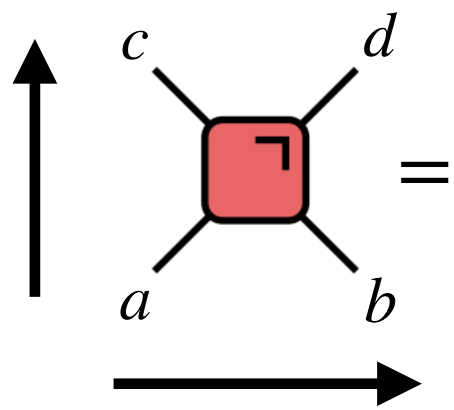

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Dual-Unitary

- Examples

spectral correlations)

L **121**, 264101 (2018)

$$|\mathrm{tr}[\mathbb{U}^t]|^2 \rangle = 2t$$

*Almost all dual-unitary circuits are provably quantum chaotic**

$$\lim_{L\to\infty} \langle |\mathrm{tr}[\mathbb{U}^t]|^2 \rangle = t$$

BB, Kos, and Prosen, CMP **387,** 597 (2021)

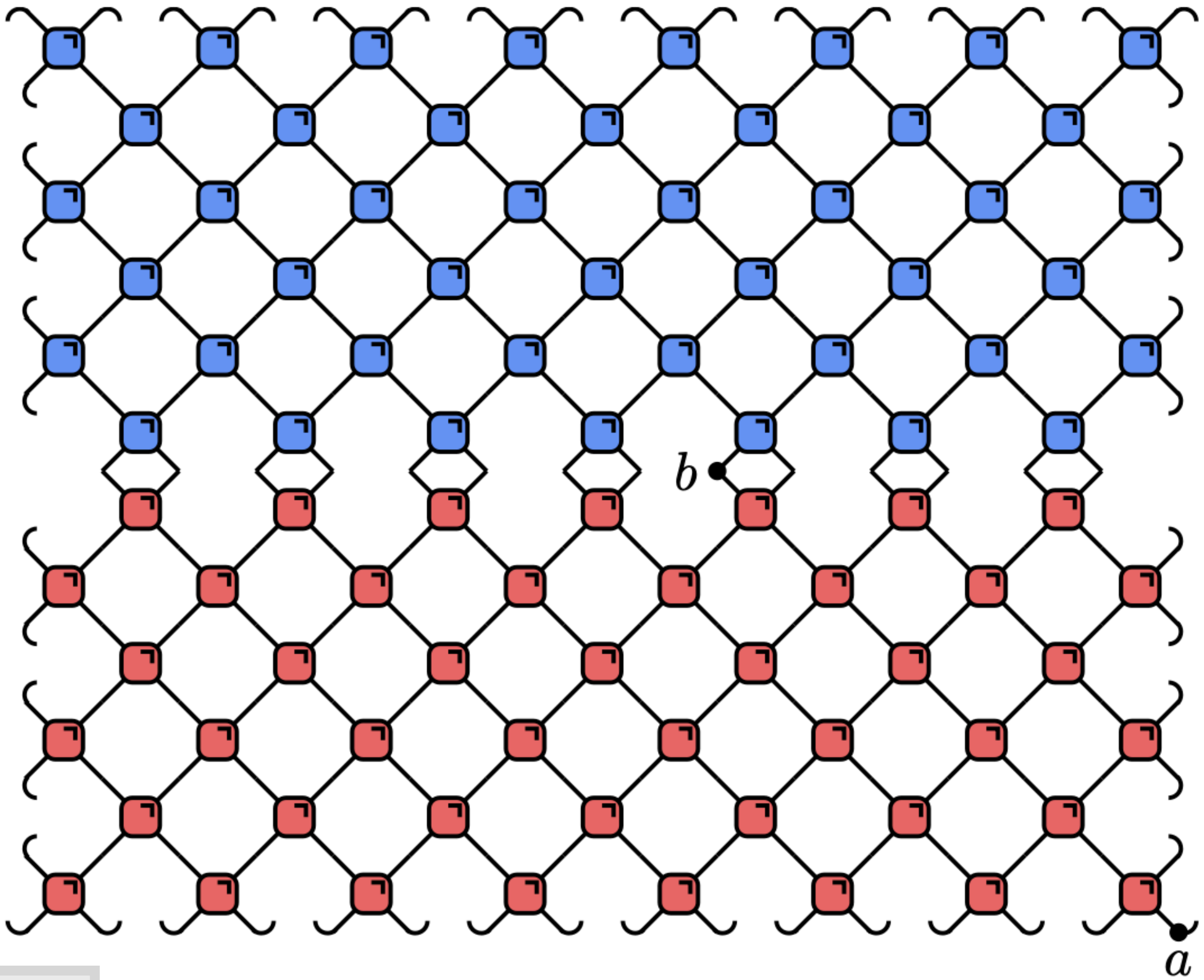
Dynamics of dual unitary circuits

Some dynamical properties directly determined by diagrammatic calculus

$$UU^\dagger = U^\dagger U = I$$

$$\tilde{U}\tilde{U}^\dagger = \tilde{U}^\dagger \tilde{U} = I$$

$$\frac{1}{d^{2L}}\mathrm{tr}[a_{0,0}b_{x,t}] = \frac{1}{d^{2L}}$$



BB, Kos, and Prosen, PRL **123**, 210601 (2019)

Dynamics of dual unitary circuits

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Causal light cone in time

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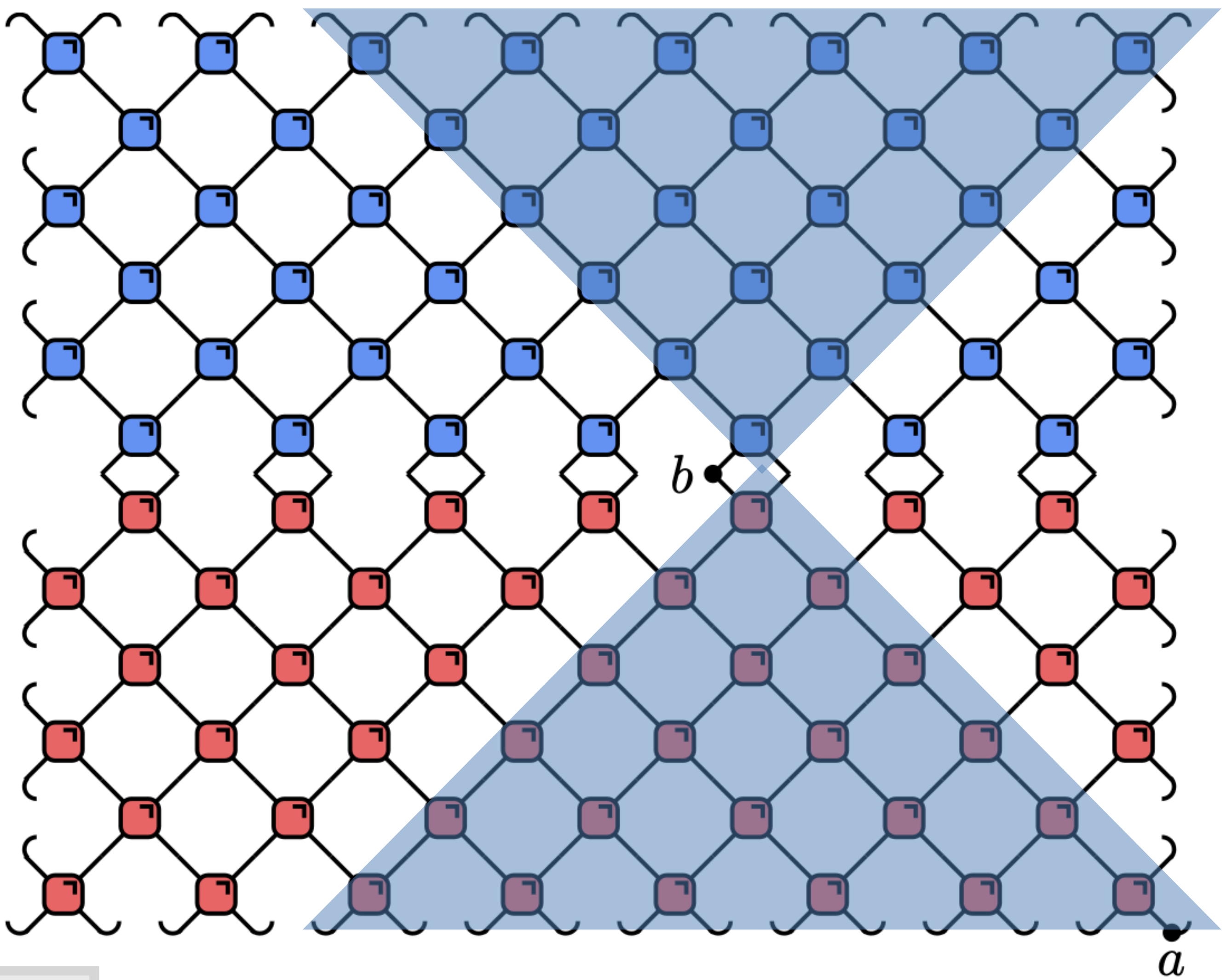

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Causal light cone in space

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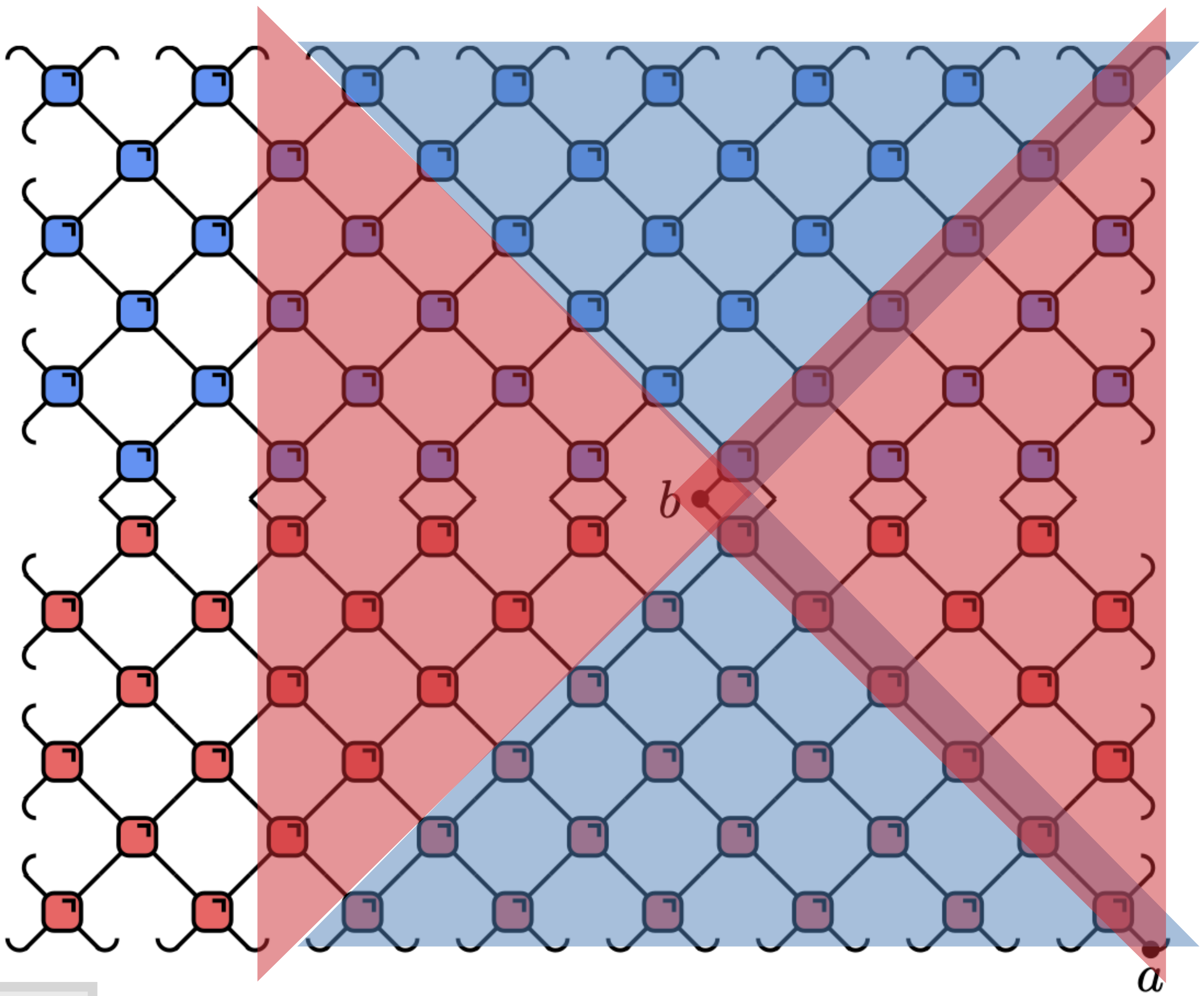

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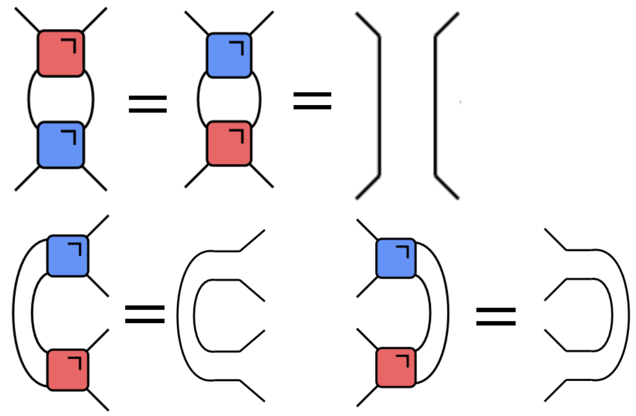

Causal light cone in time

Causal light cone in space

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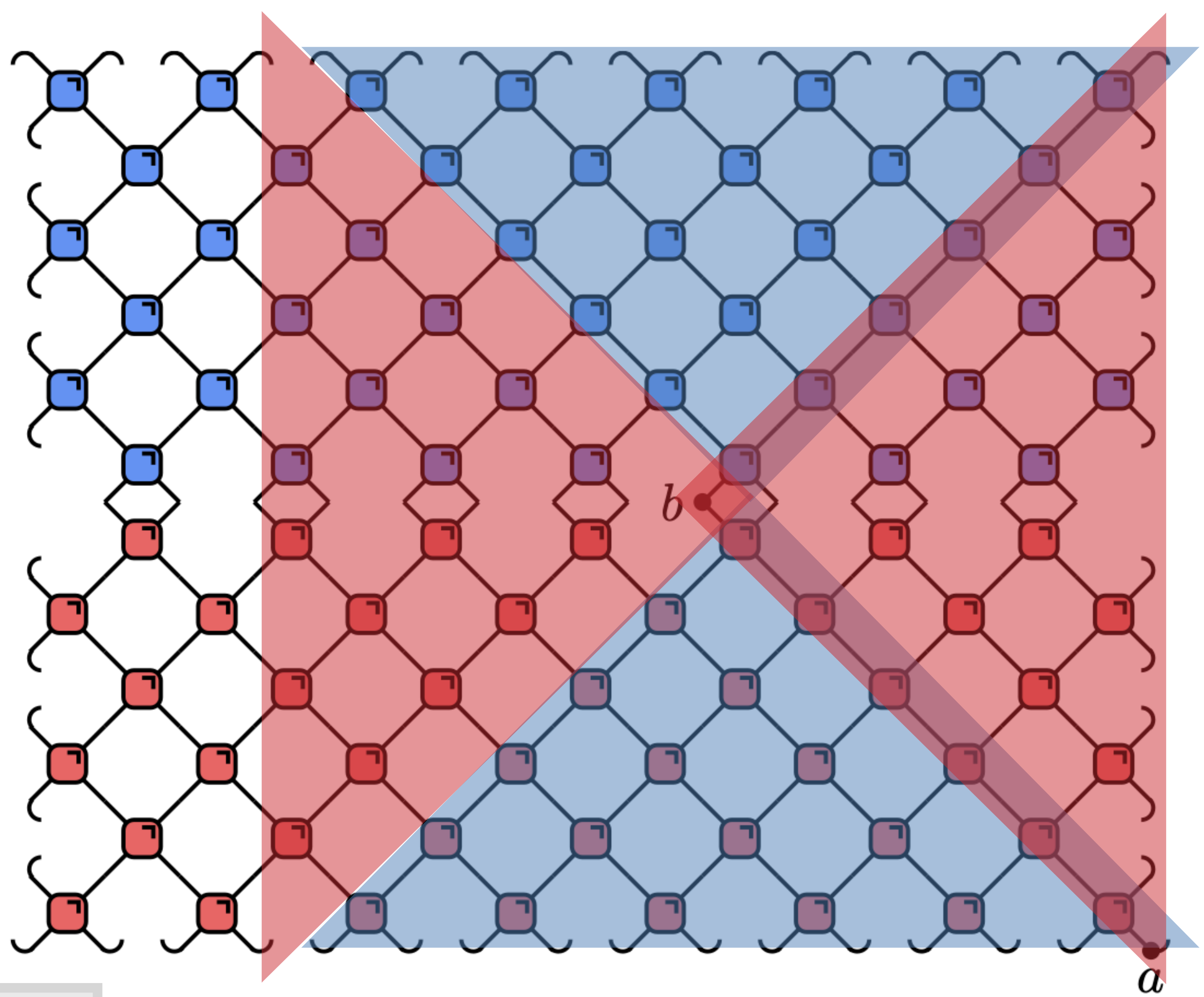

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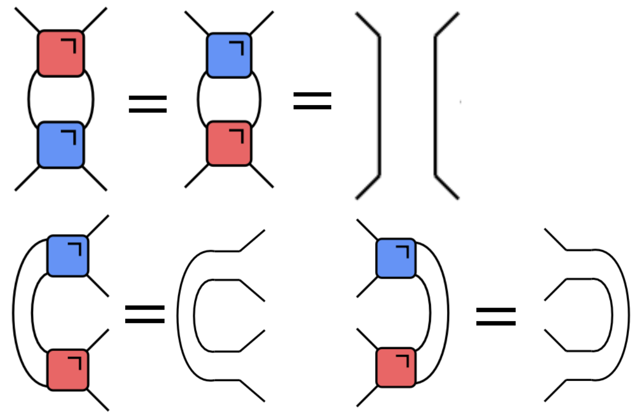

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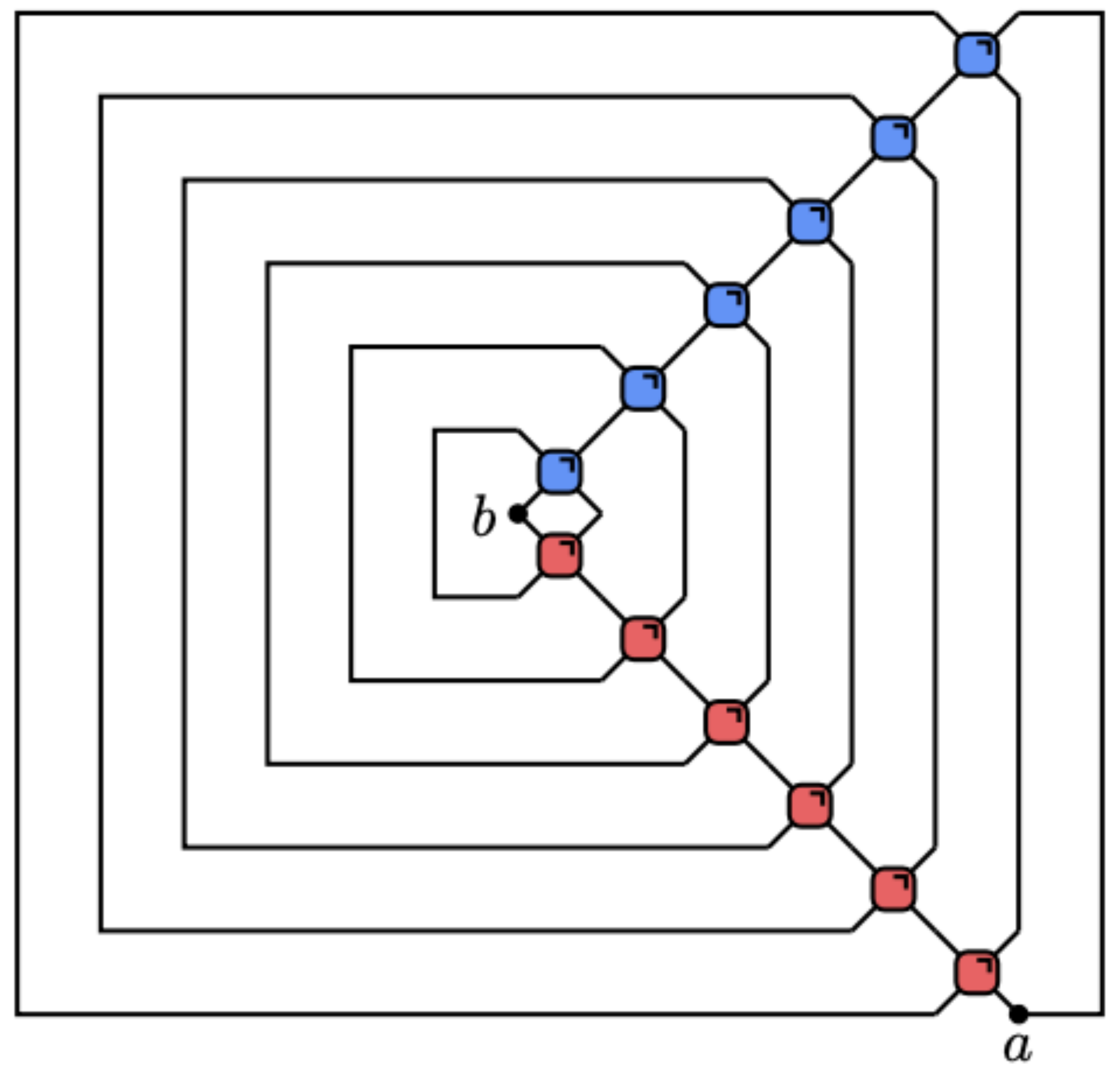
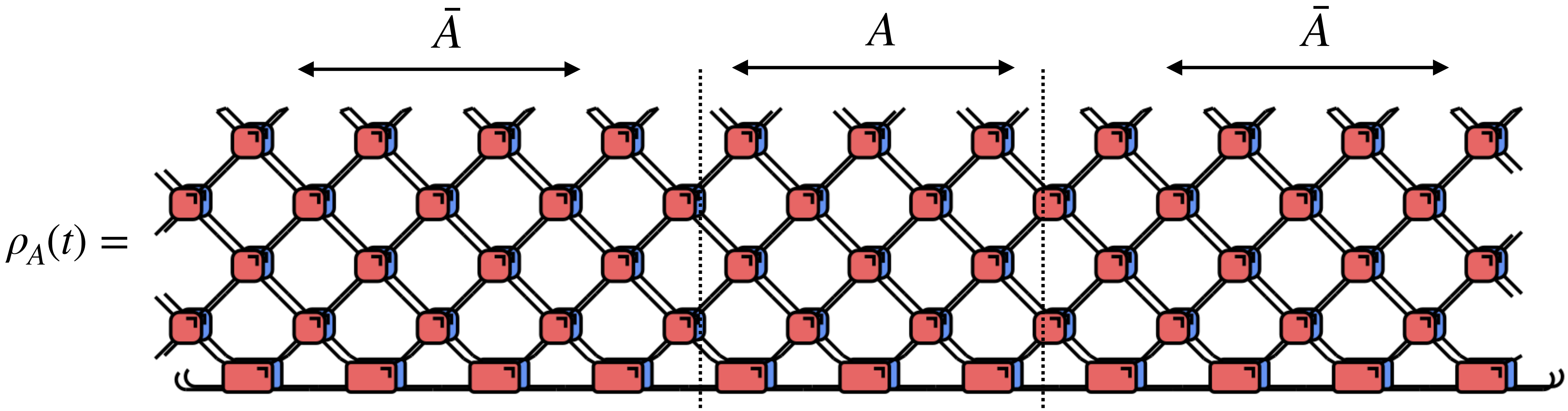


BB, Kos, and Prosen, PRL **123**, 210601 (2019)

Dynamics of dual unitary circuits

Quantum Quench



Look at *Rényi entropies*

$$S_A^{(\alpha)}(t) = \frac{1}{1-\alpha} \log \mathrm{tr}[\rho_A(t)^\alpha] \qquad \alpha \in \mathbb{R}$$

$$\lim_{\alpha \to 1} S_A^{(\alpha)}(t) = -\,\mathrm{tr}[\rho_A(t) \log \rho_A(t)]$$

Dynamics of dual unitary circuits

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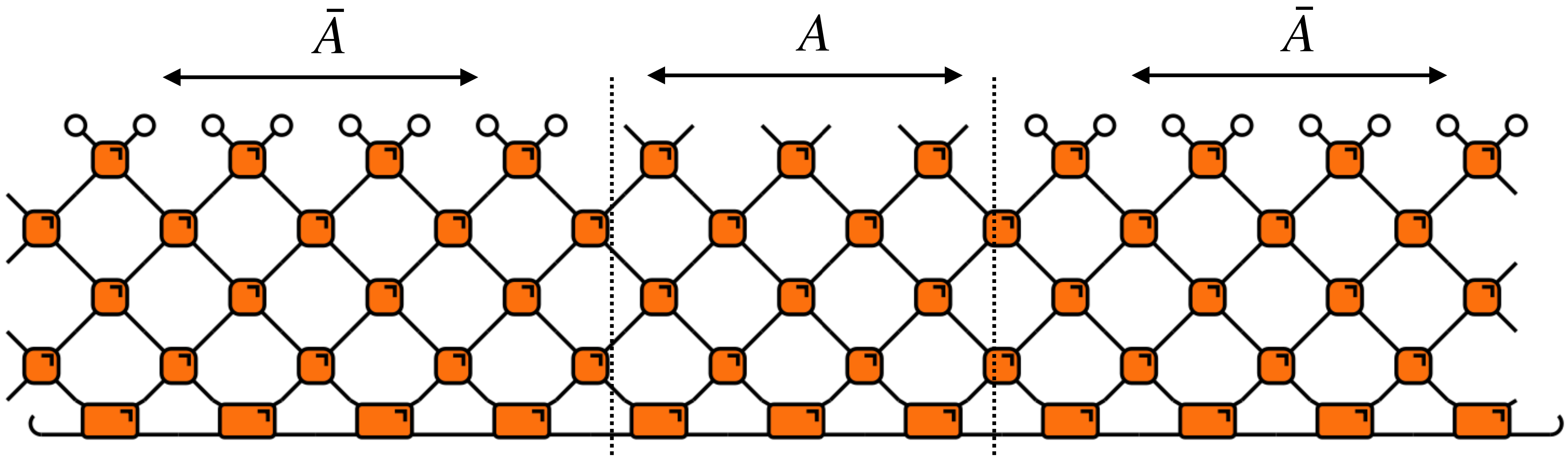
$\rho_A(t) =$ 

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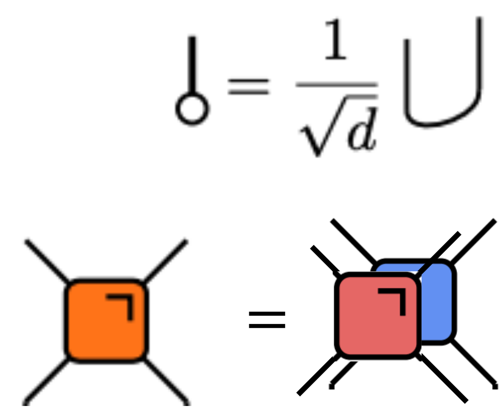

$= U \otimes U^*$

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Dynamics of dual unitary circuits

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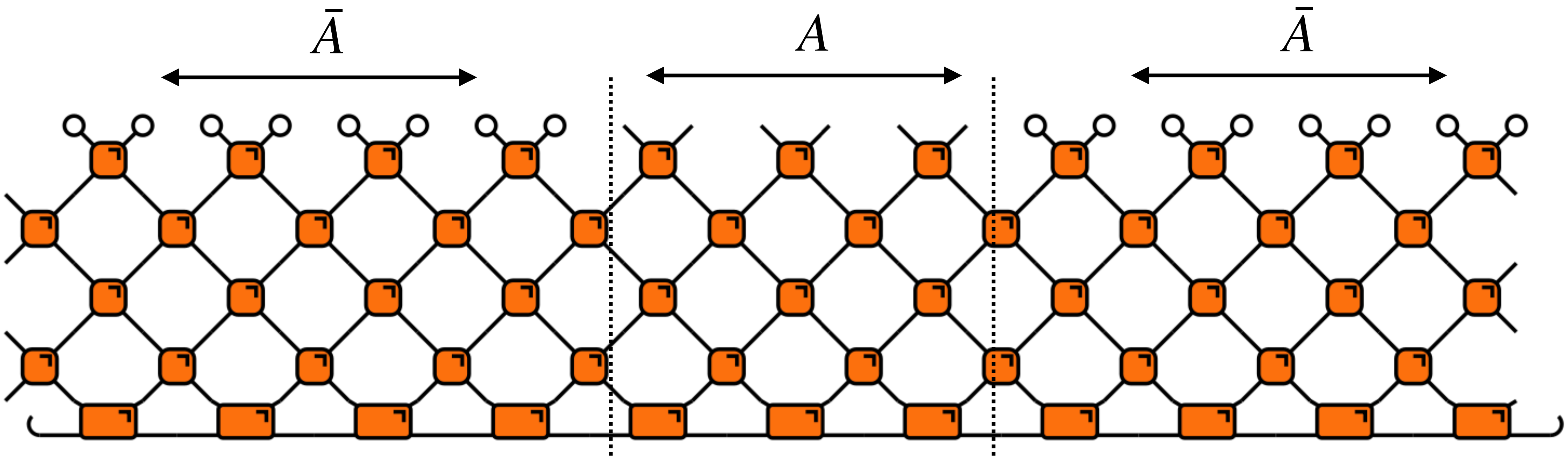

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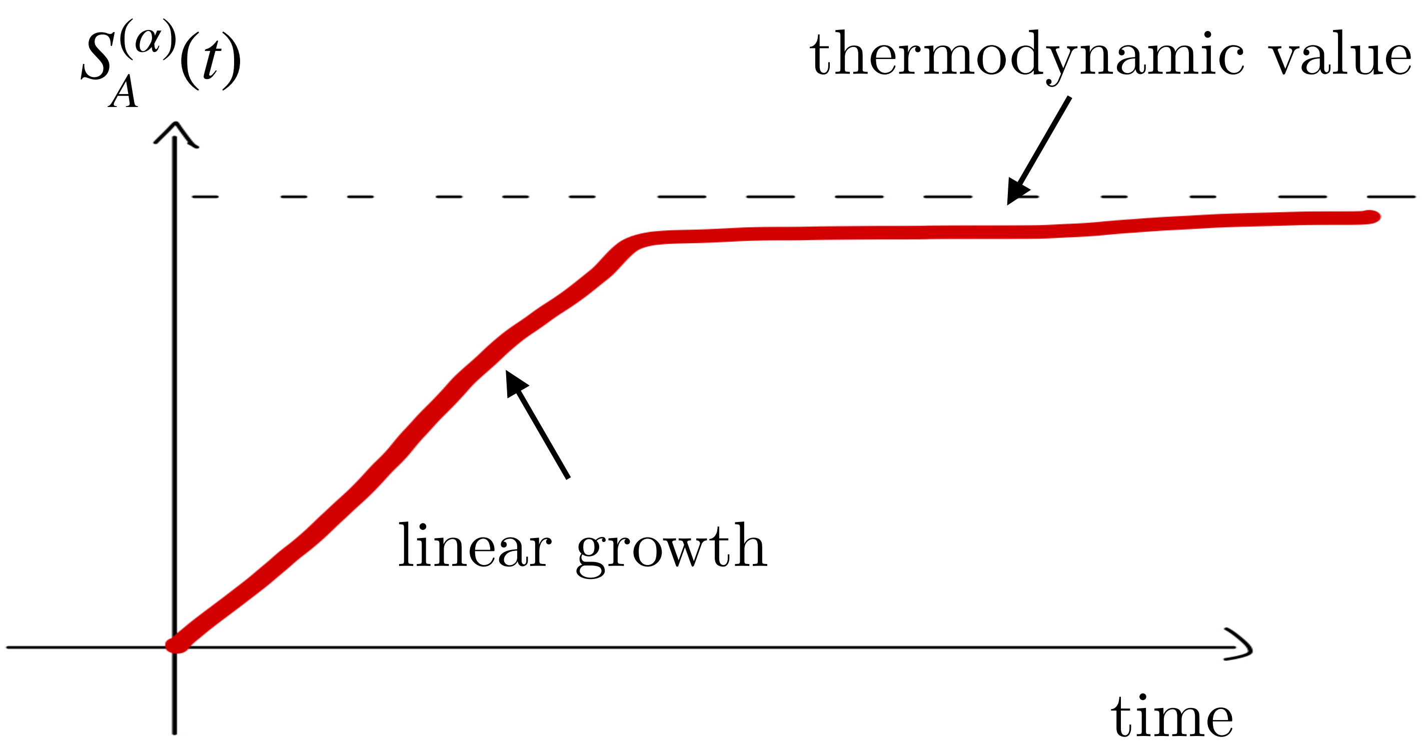
Universal evolution in clean systems



Dynamics of dual unitary circuits

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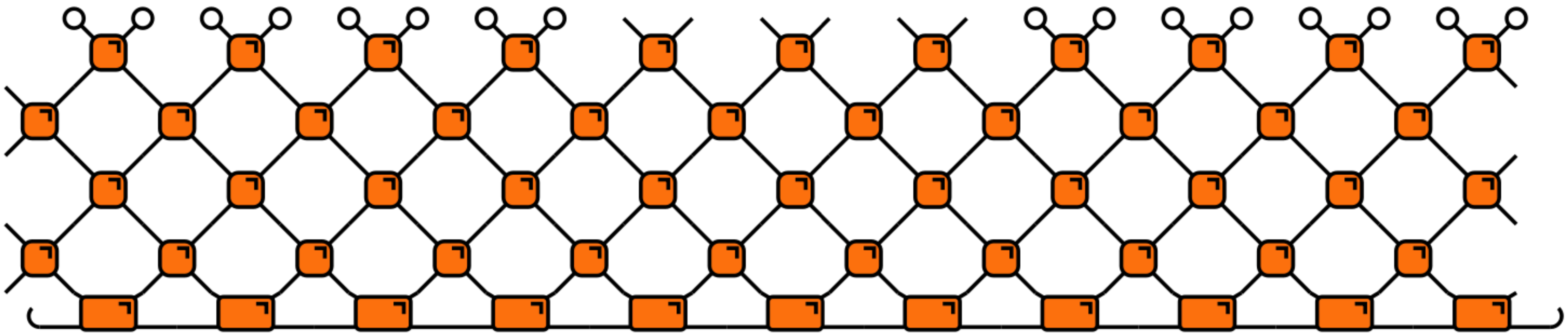
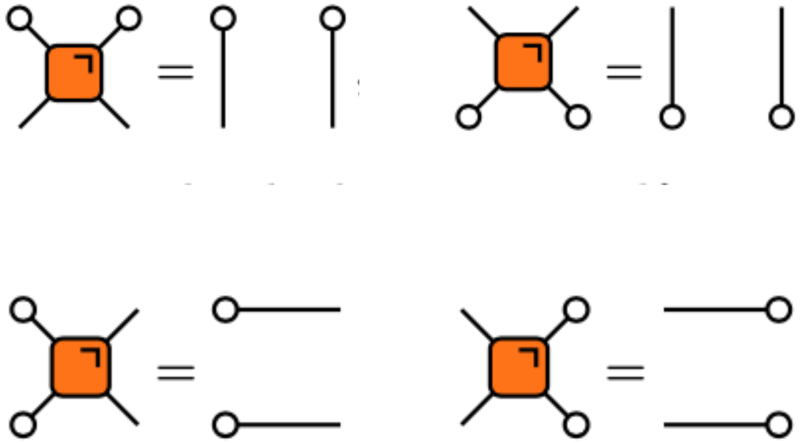
$\rho_A(t) =$ 



BB, Kos, and Prosen, PRX **9**, 021033 (2019)

Piroli, BB, Cirac, and Prosen, PRB **101**, 094304 (2020)

Dynamics of dual unitary circuits

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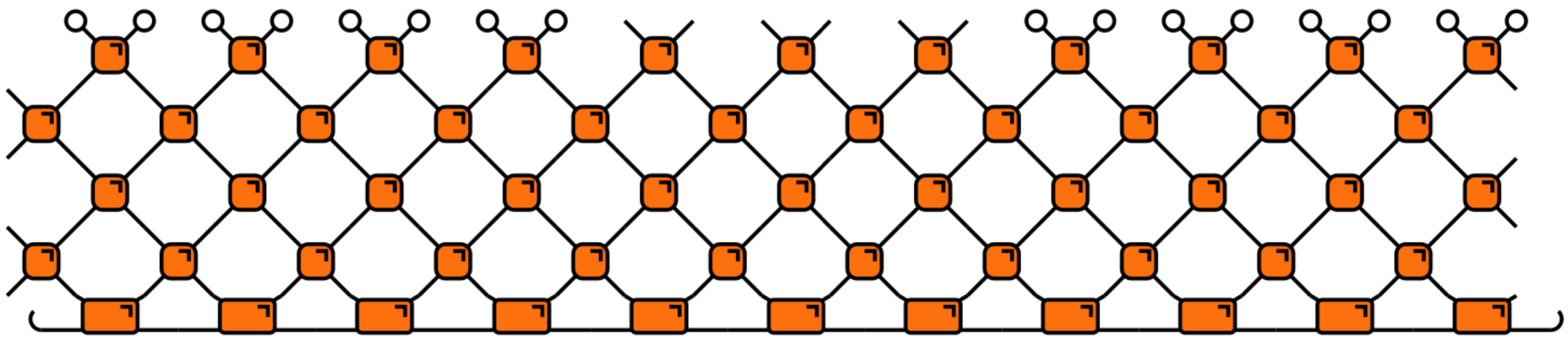
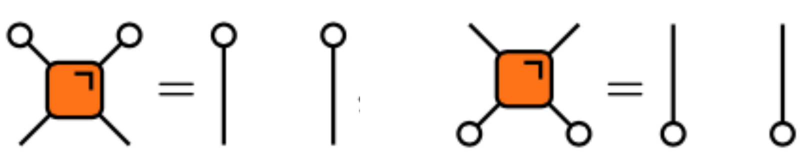
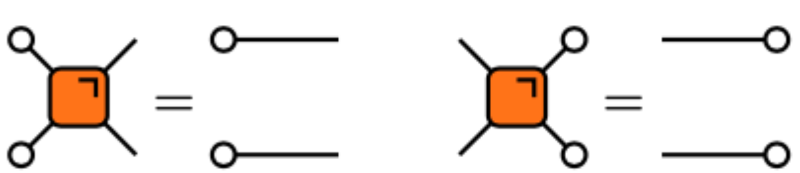
$\rho_A(t) =$ 





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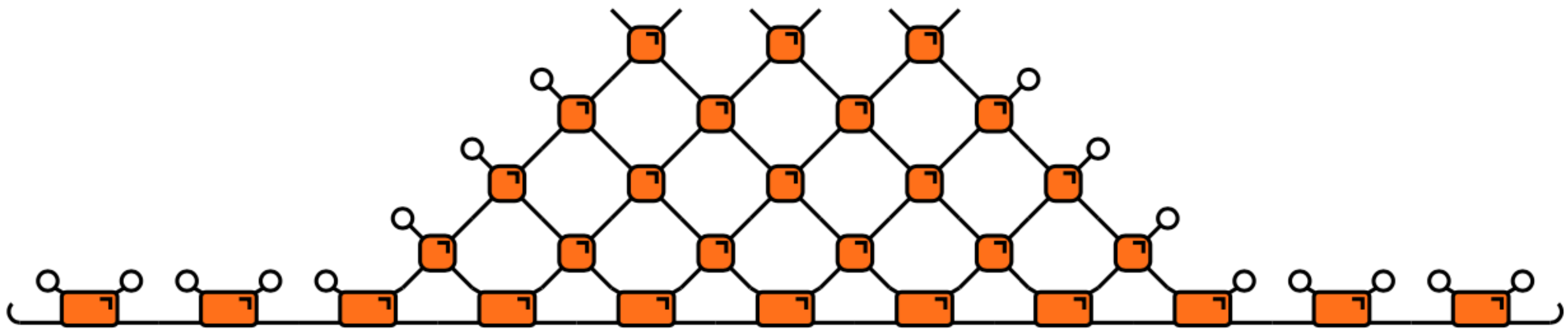
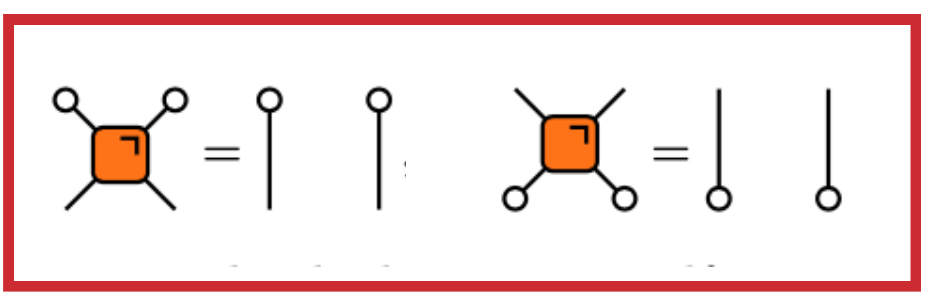
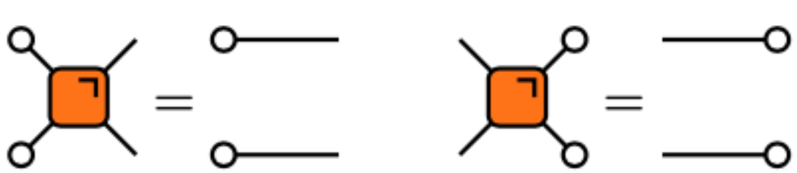
$\rho_A(t) =$ 





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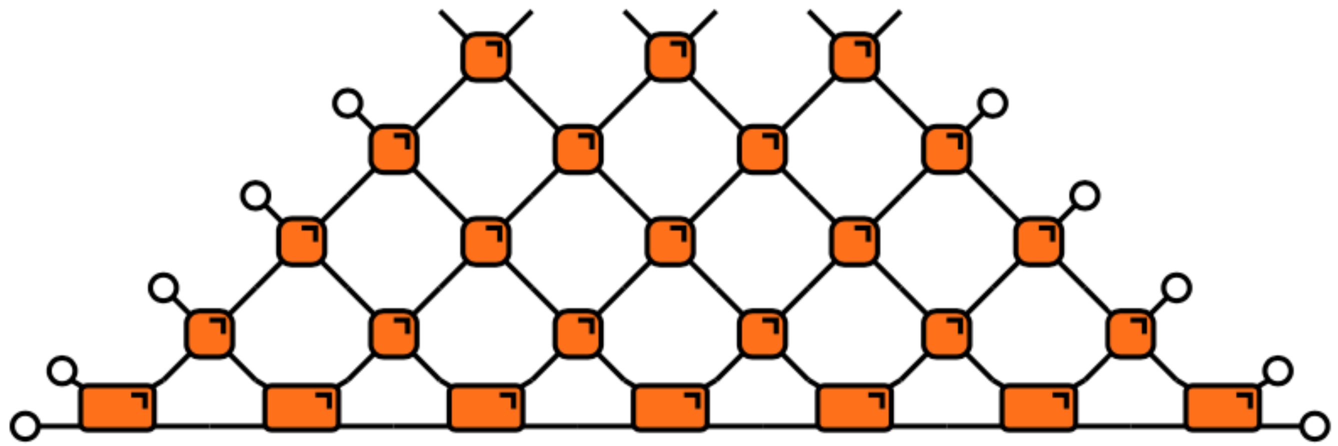
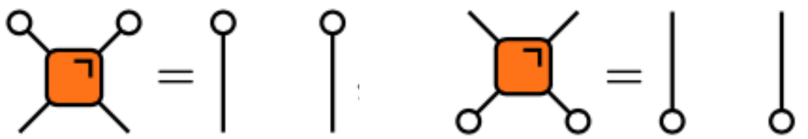
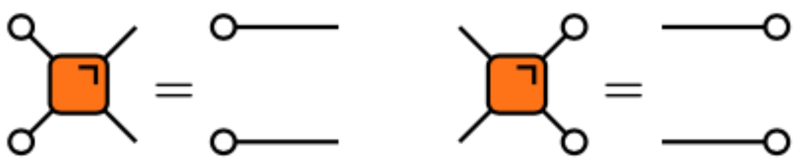
$\rho_A(t) =$ 





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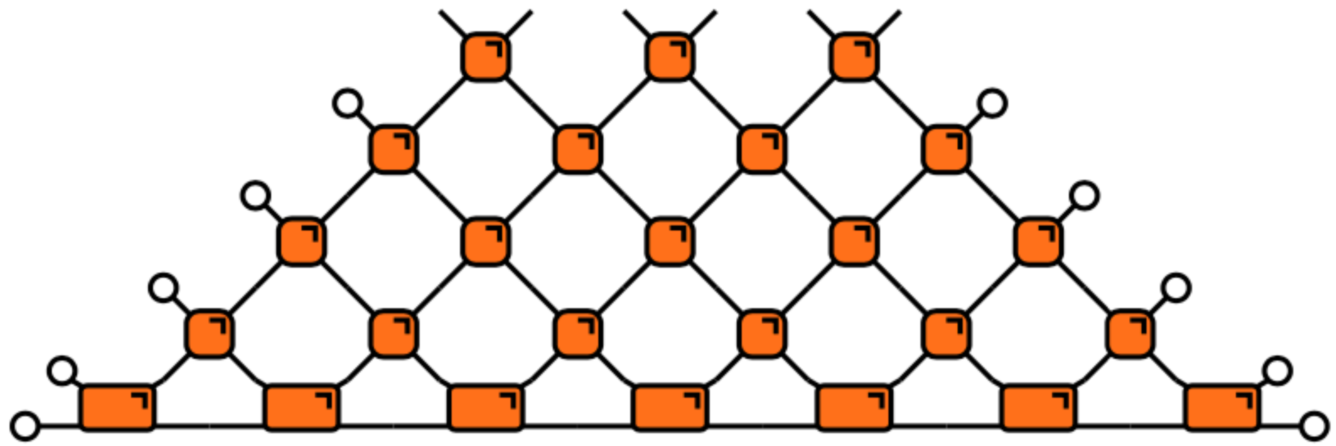
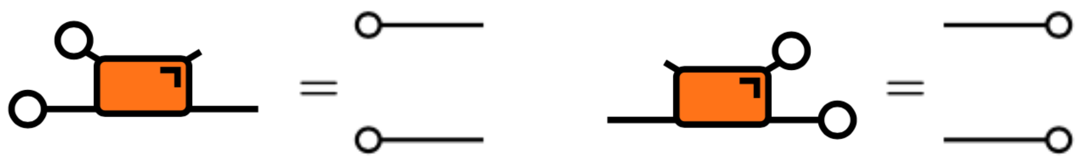
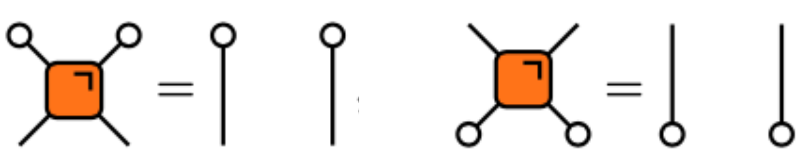
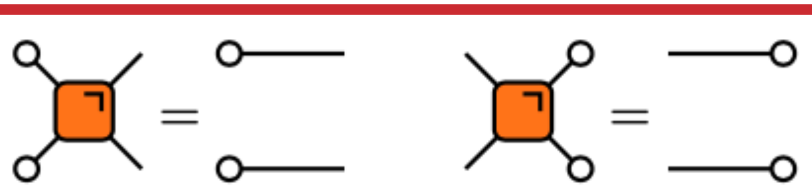
$\rho_A(t) =$ 



Solvability Condition





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Dynamics of dual unitary circuits

Quantum Quench

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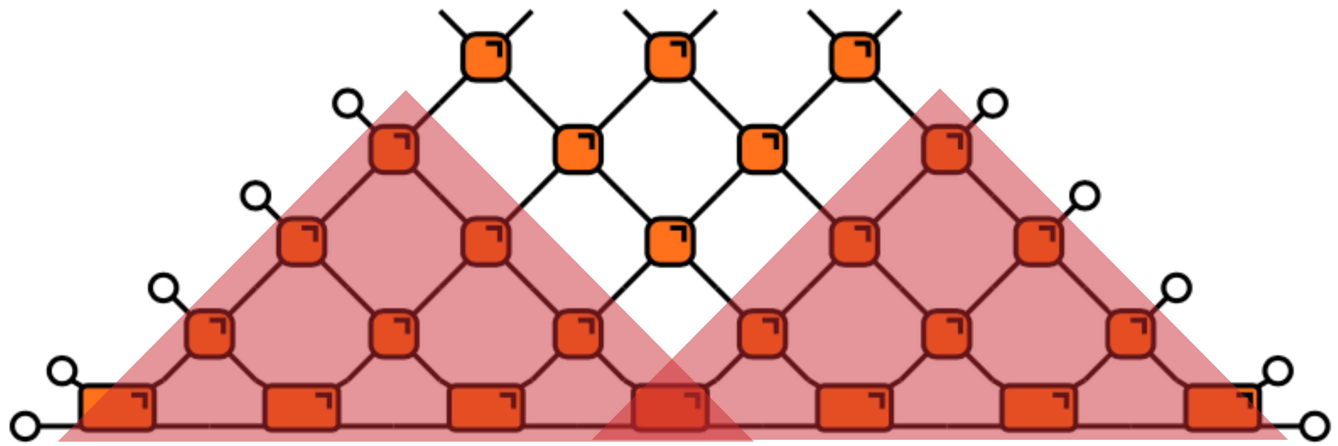

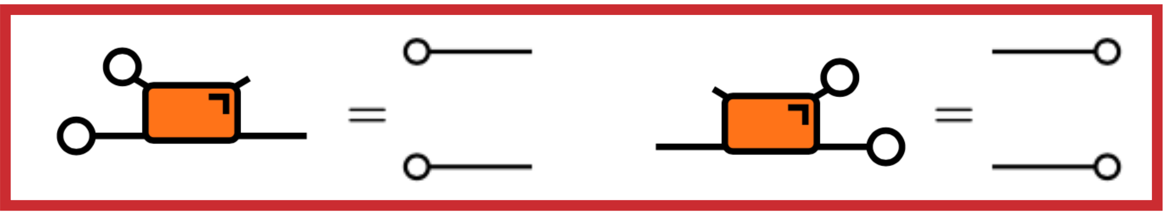

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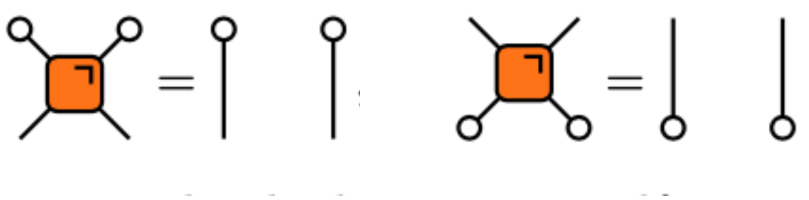

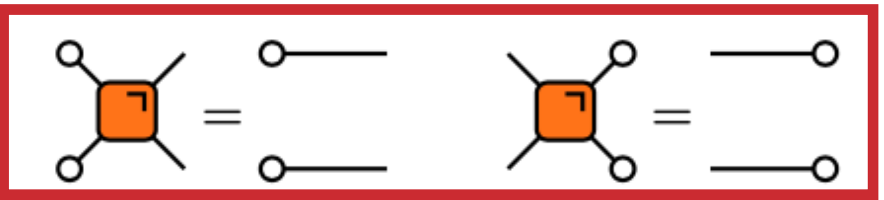

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Piroli, BB, Cirac, and Prosen, PRB **101**, 094304 (2020)

Dynamics of dual unitary circuits

Quantum Quench

$$\rho_A(t) = \begin{cases} \frac{1}{d^{|A|}} I_A & t \geq |A|/4 \\ \frac{1}{d^{4t}} W(I_{2t} \otimes \mathrm{tr}_{A \setminus 4t}[\rho(0)] \otimes I_{2t}) W^\dagger & t < |A|/4 \end{cases}$$

$$WW^\dagger = I$$

$$S_A^{(n)}(t) = \min(4t, |A|) \log d + O(1)$$

BB, Kos, and Prosen, PRX **9**, 021033 (2019)

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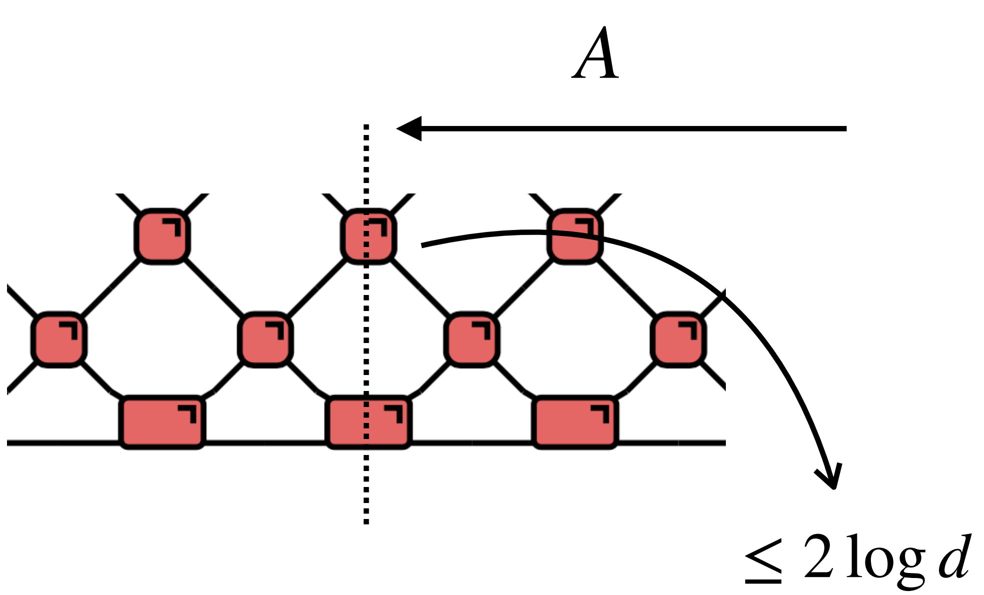

- No n-dependence

- Maximal entanglement growth at each step

$$v_E^{(n)} = \lim_{t\to\infty} \lim_{A\to\infty} \frac{S_A^{(n)}(t)}{4t \log d} = 1 \overset{!}{=} v_B$$

BB, Kos, and Prosen, PRX **9**, 021033 (2019)

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Dynamics of dual unitary circuits

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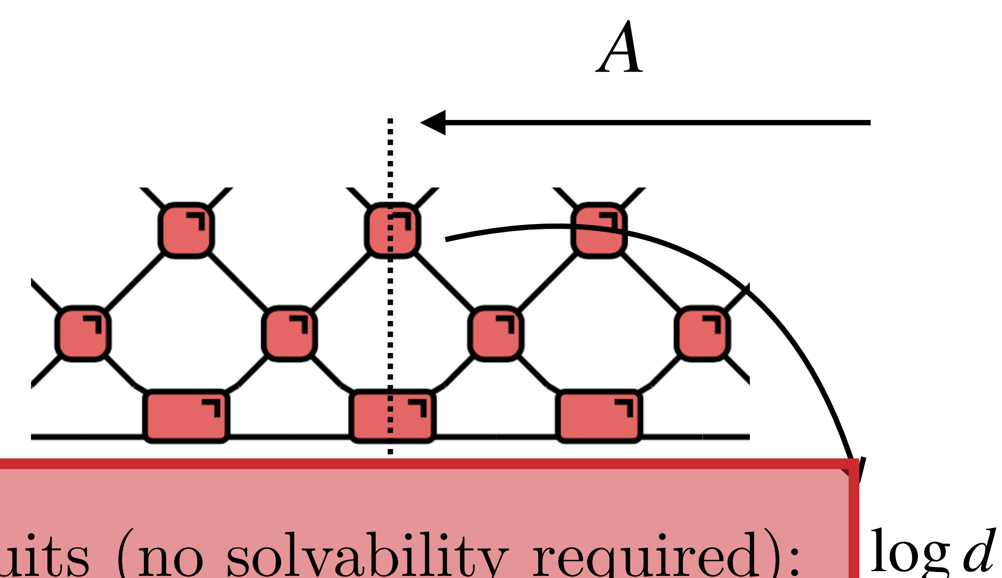

For any initial state and generic DU circuits (no solvability required):

$v_E^{(1)} = 1 \quad \Leftrightarrow \quad$ dual unitarity

$\Rightarrow$ Zhou and Harrow, PRB **106**, L201104 (2022).

$\Leftarrow$ Foligno and BB, PRB **107**, 174311 (2023)

Dual unitary circuits

Many more applications in:

Quantum many-body Chaos, **Quantum Dynamics**, **Quantum Information**, and **Quantum Computation**

BB, Claeys, and Prosen, RMP **98**, 025001 (2026).

Dual unitary circuits

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Quantum many-body Chaos, **Quantum Dynamics**, **Quantum Information**, and **Quantum Computation**

BB, Claeys, and Prosen, RMP **98**, 025001 (2026).

- Benchmarking Quantum Computers

  - Current quantum computers *large* but *noisy* ("NISQ" devices)

  - Many different *error mitigation* schemes

Google's *"Willow"*

Dual unitary circuits

Many more applications in:

Quantum many-body Chaos, **Quantum Dynamics**, **Quantum Information**, and **Quantum Computation**

BB, Claeys, and Prosen, RMP **98**, 025001 (2026).

Google's *"Willow"*

- Benchmarking Quantum Computers

  - Current quantum computers *large* but *noisy* ("NISQ" devices)

  - Many different *error mitigation* schemes

Use exact results in DU circuits to test them

OTOCs	Mi et al., Science **374**, 6574 (2021)	(Google)
Quantum Quench	Chertkov et al., Nature Physics **18**, 1074–1079 (2022)	(Quantinuum)
	⋮	
	Fischer et al., Nature Physics **22**, 302-307 (2026)	(IBM)

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◆ Benchmarking Quantum Computers

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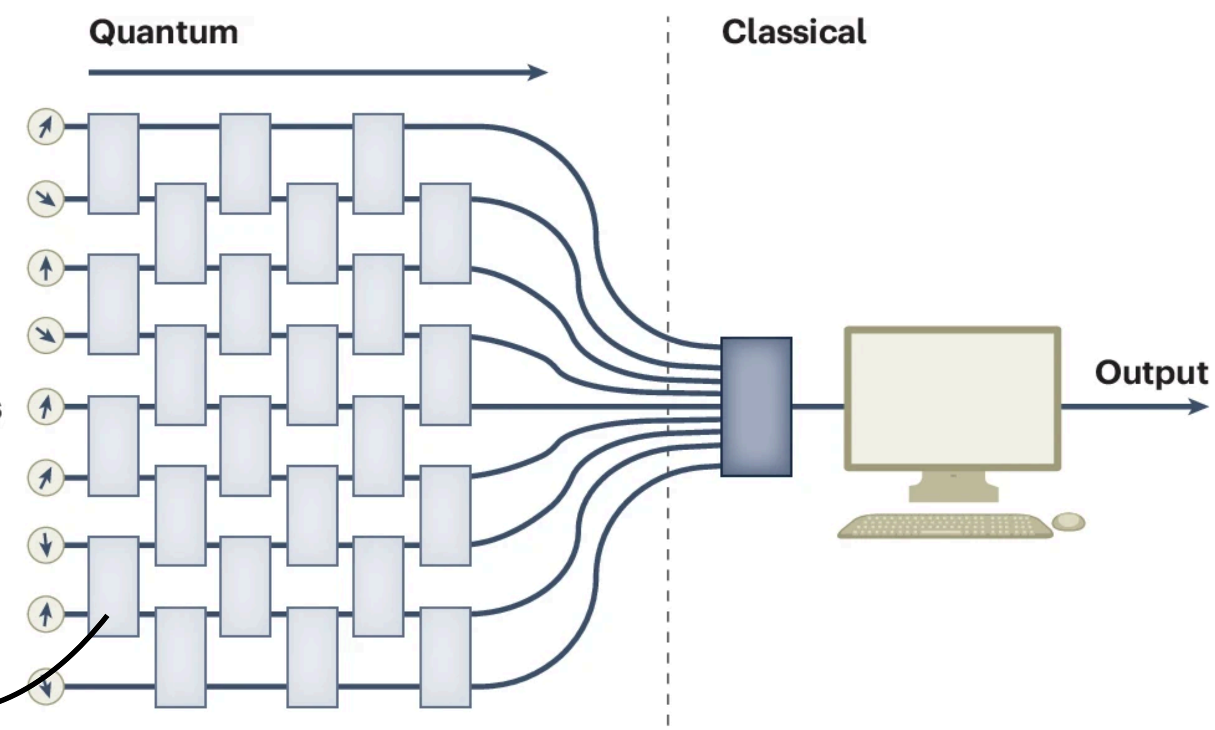
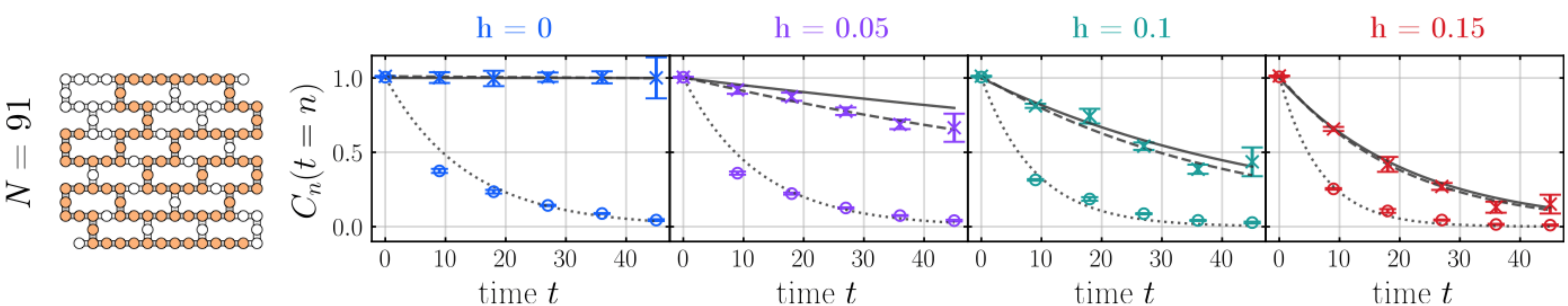
$$e^{i\frac{\pi}{4}\sum_j \left\{\sigma_j^z\sigma_{j+1}^z + h_j\sigma_j^z\right\}} e^{i\frac{\pi}{4}\sum_j \sigma_j^x}$$

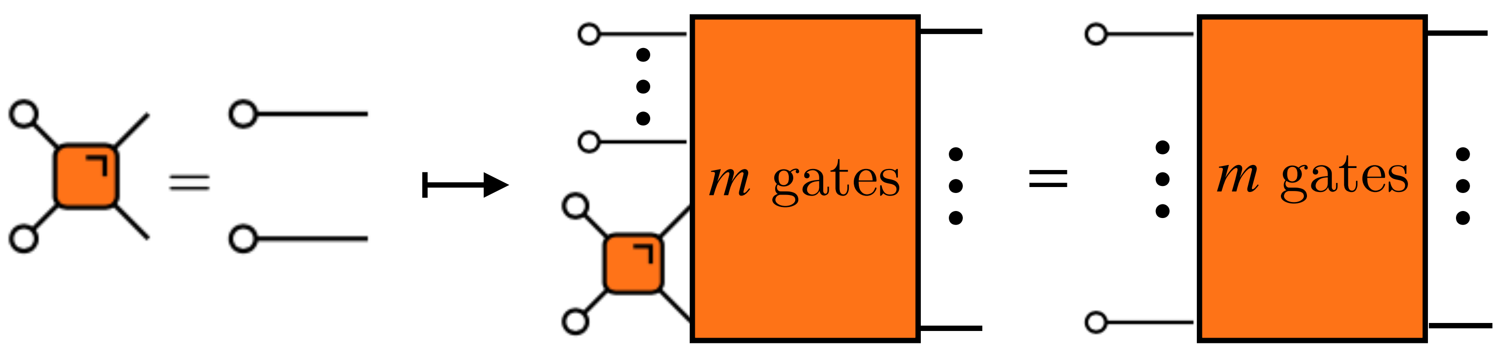




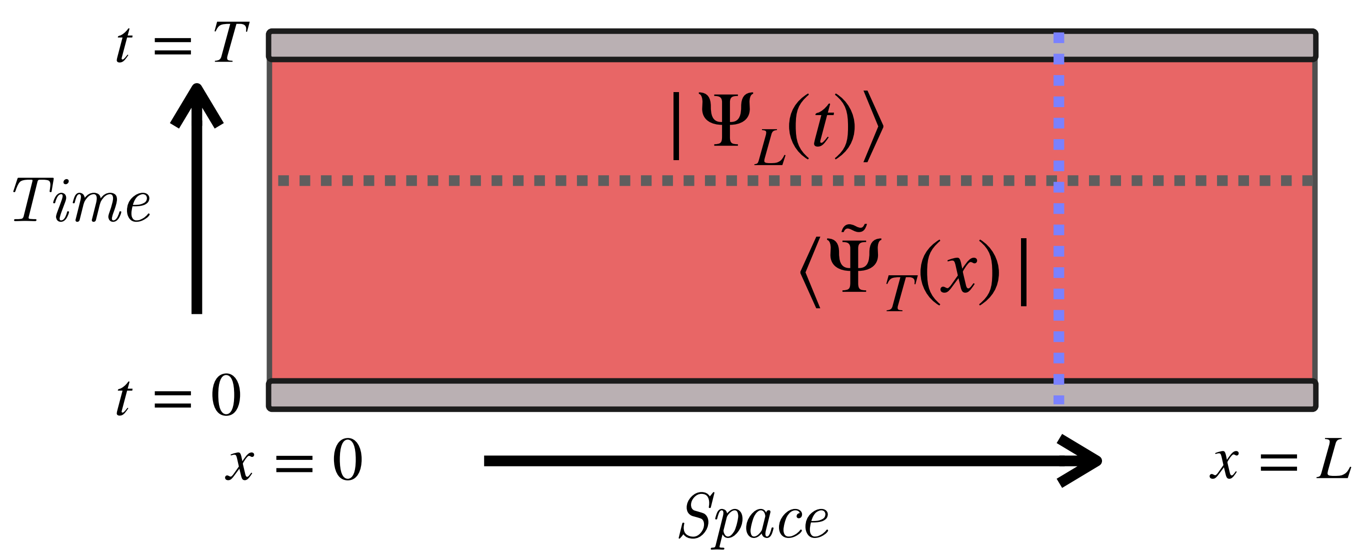
Beyond dual unitarity?

Beyond dual unitarity?

- Systematic "weakening" of the dual-unitary condition



- "Space-time duality" in generic systems



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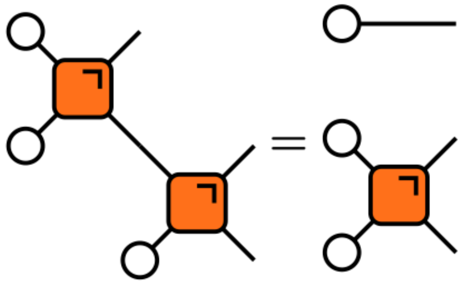
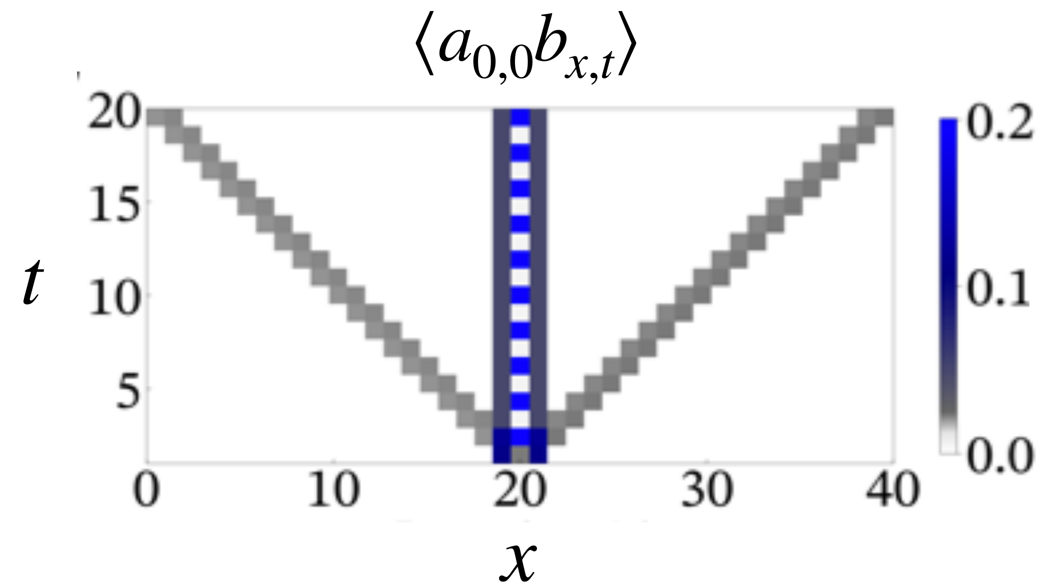


"DU2"

Yu, Wang, and Kos, Quantum **8**, 1260 (2024)



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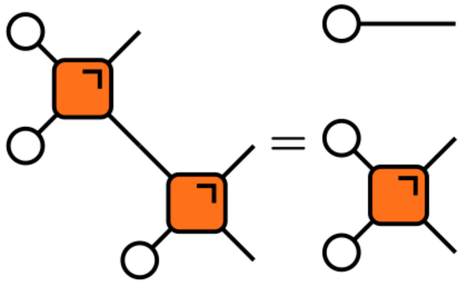

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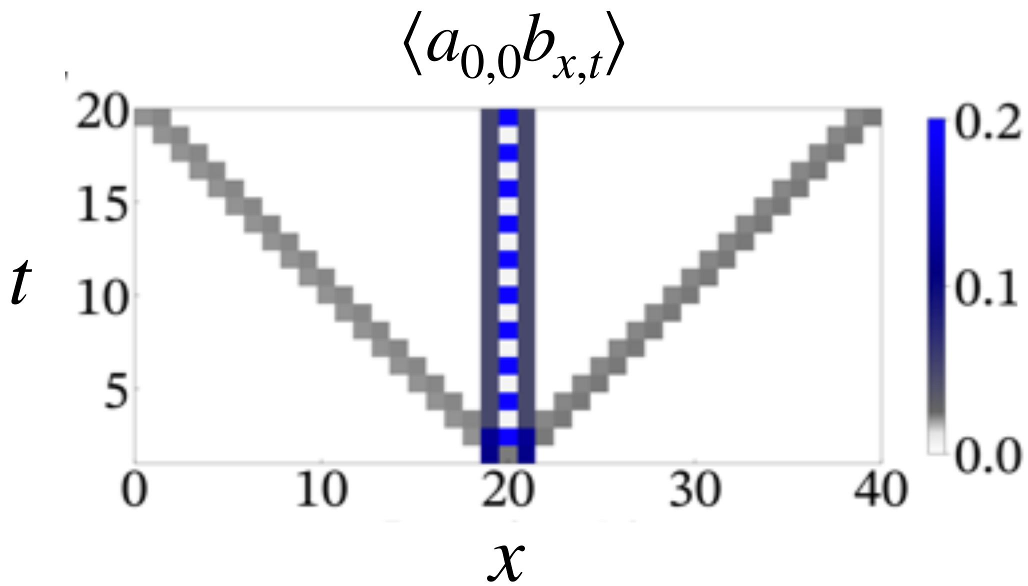

Yu, Wang, and Kos, Quantum **8**, 1260 (2024)

1. Operators still spread at the maximal velocity

2. The entanglement velocity (from compatible states) can still be computed exactly

$$v_E^{(n)} = \frac{\log(\Lambda)}{2\log(d)} \qquad \Lambda = 1, \ldots, d^2$$

- *n-independent*
- *sub-maximal*

Foligno, Kos, and BB, PRL **132**, 250402 (2024)

Beyond dual unitarity?

Pickering and BB, arXiv:2602.24276 (2026)

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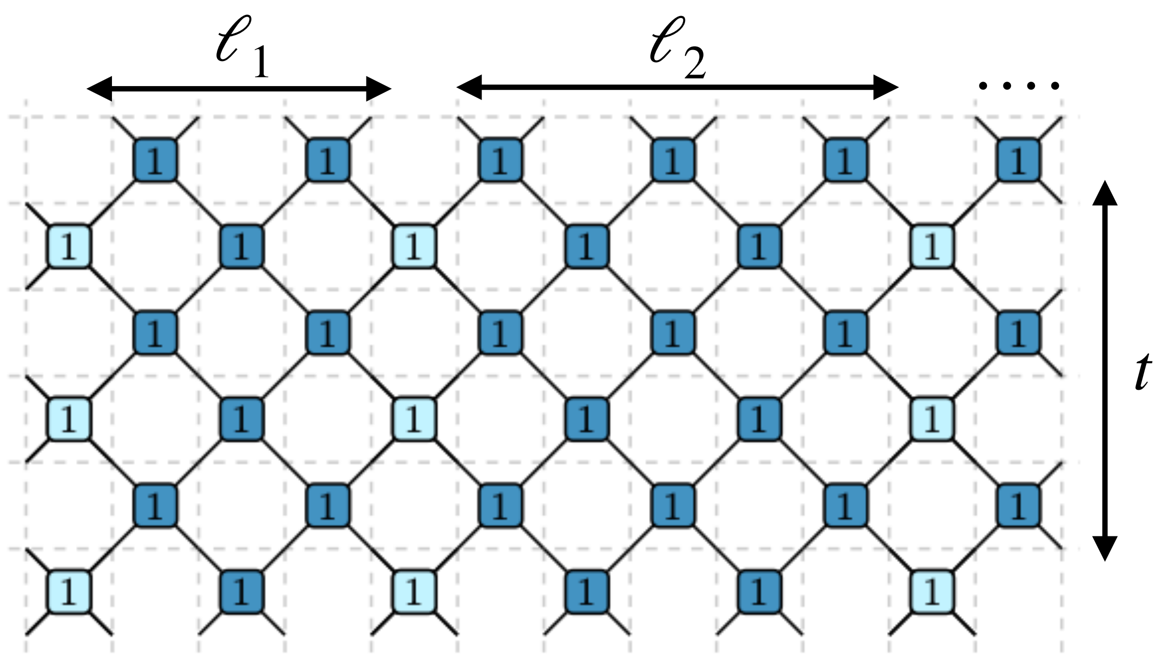
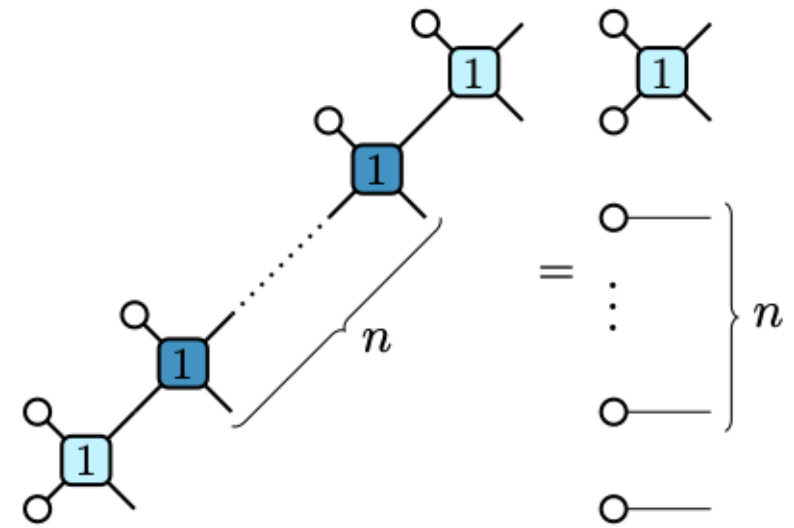
$\mathbb{U}^t =$ 



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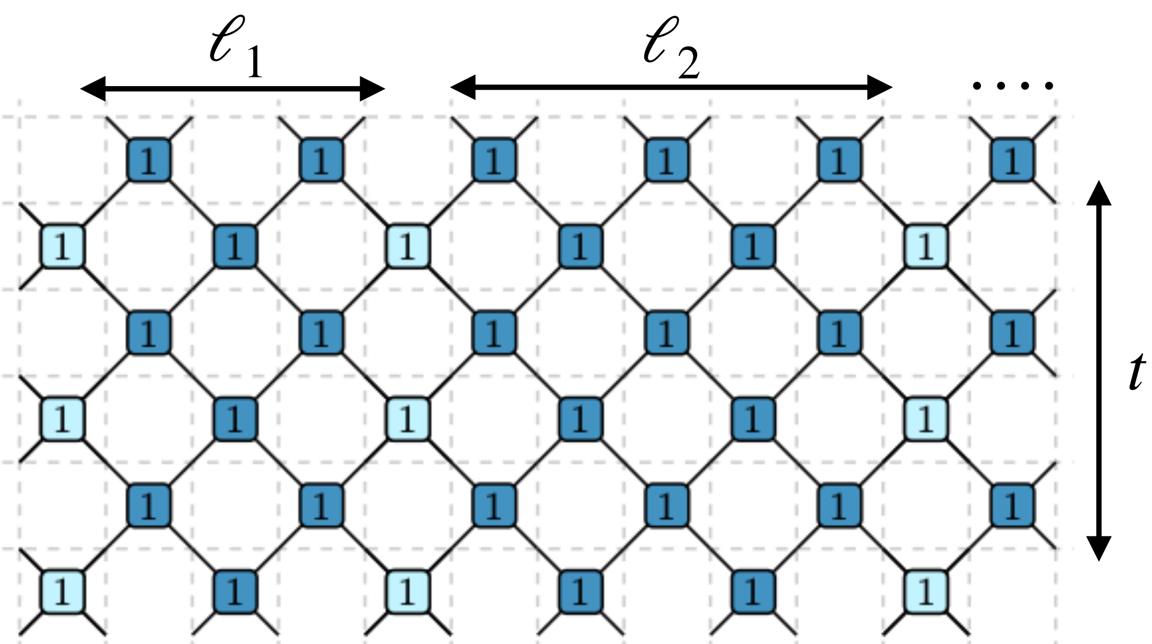
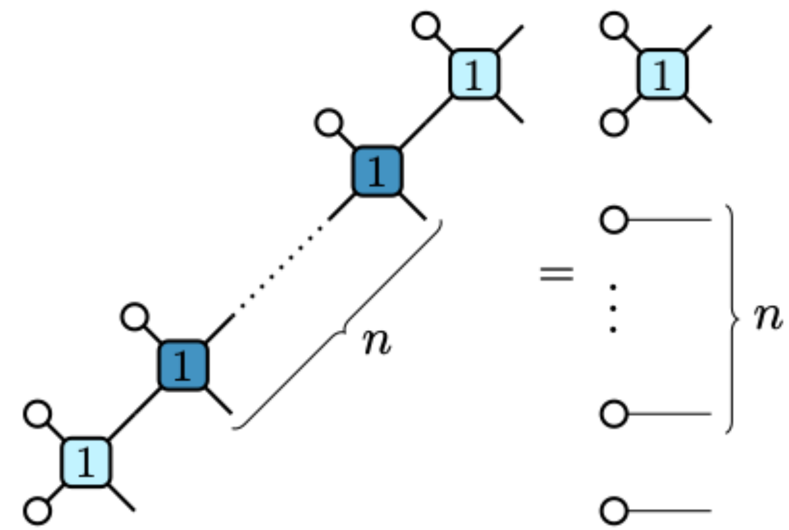
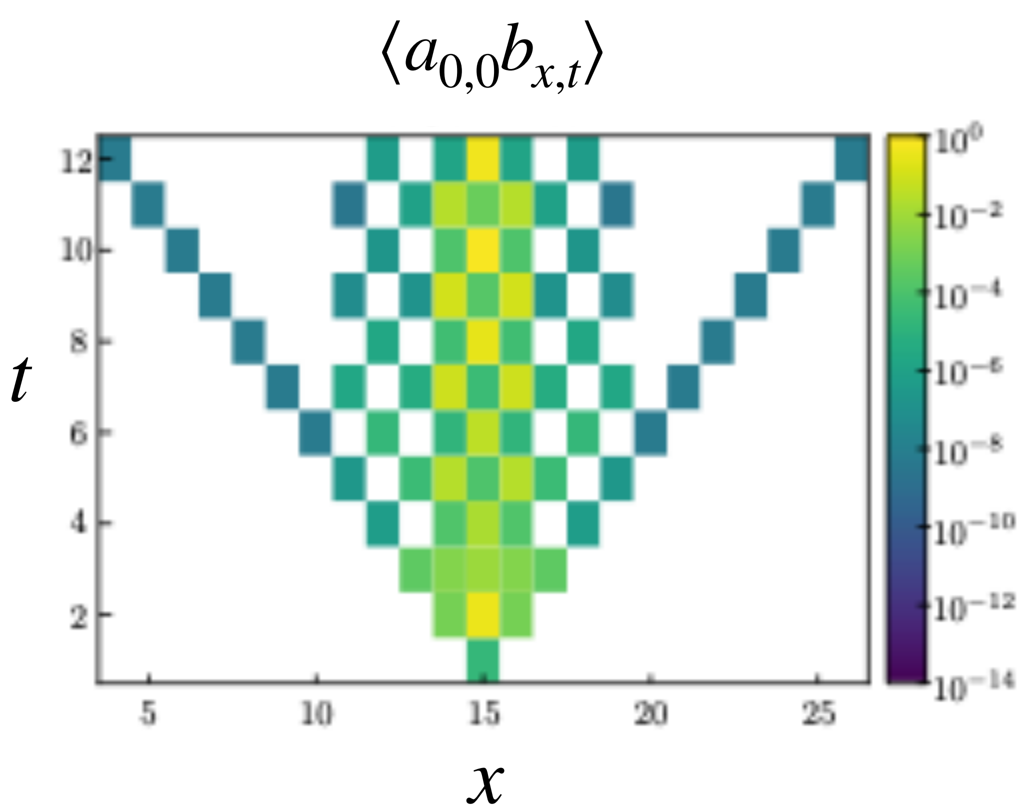
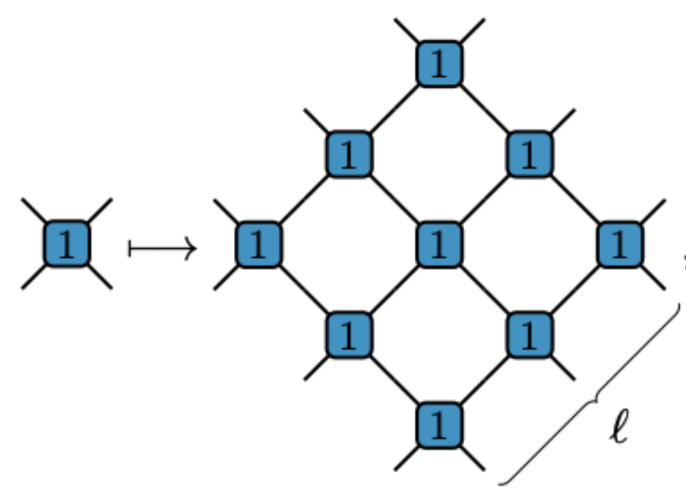
$\mathbb{U}^t =$ 





- Only solvable for $t \gg \max(\ell_j)$

- For $\ell_j = \ell$ flow to DU2 under



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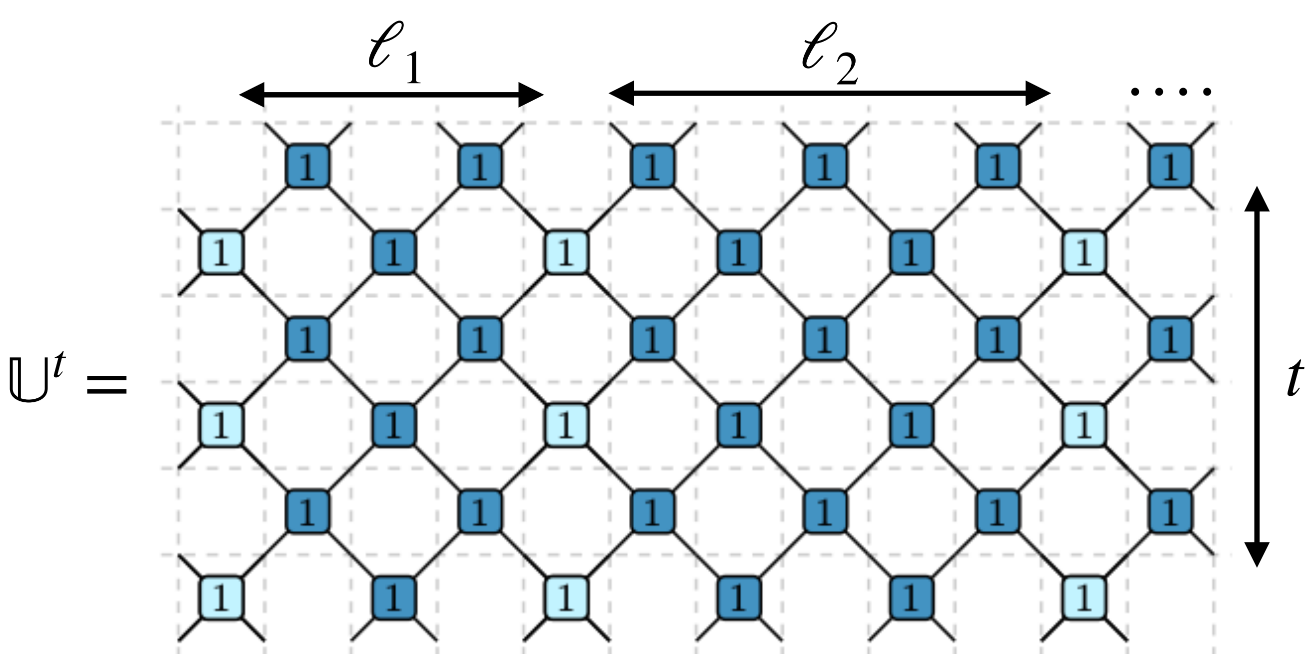

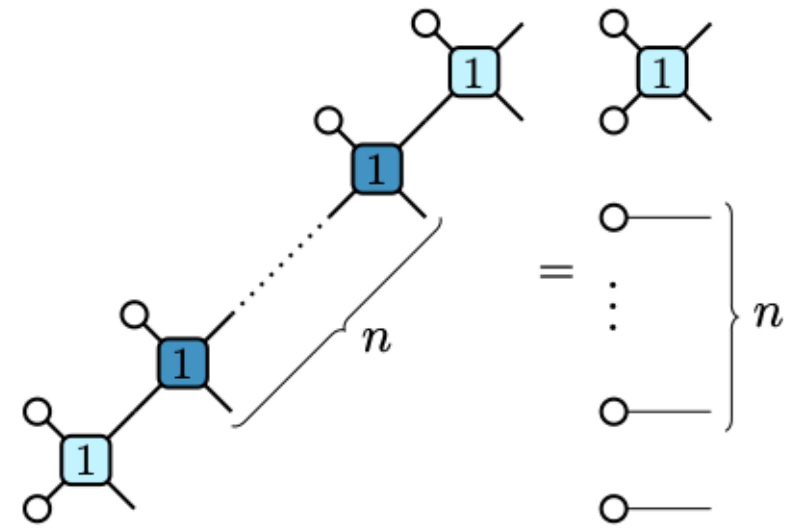

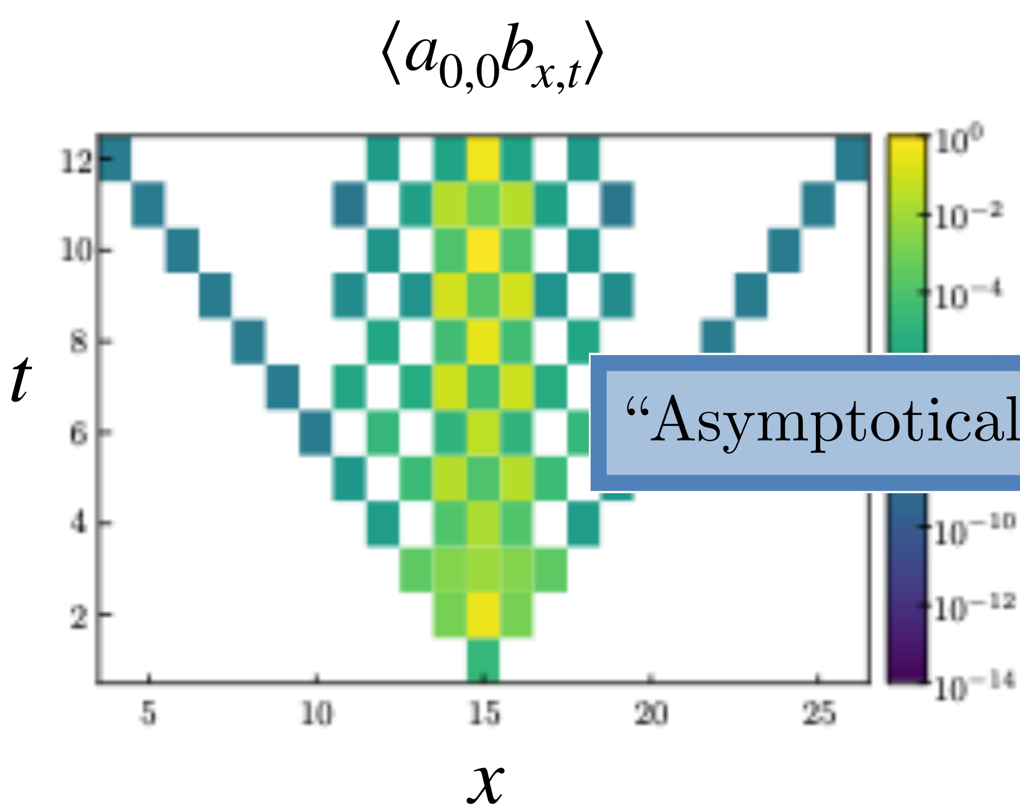

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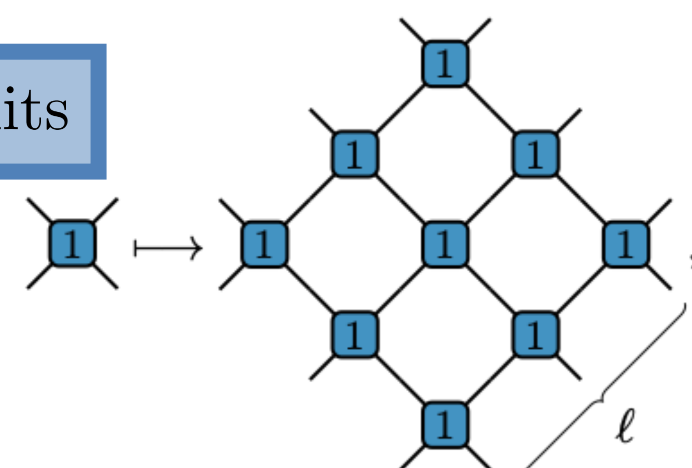

"Asymptotically solvable" circuits

Beyond dual unitarity?

- "Space-time duality" in generic systems

BB, Klobas, Alba, Lagnese, and Calabrese, PRX **12**, 031016 (2022).

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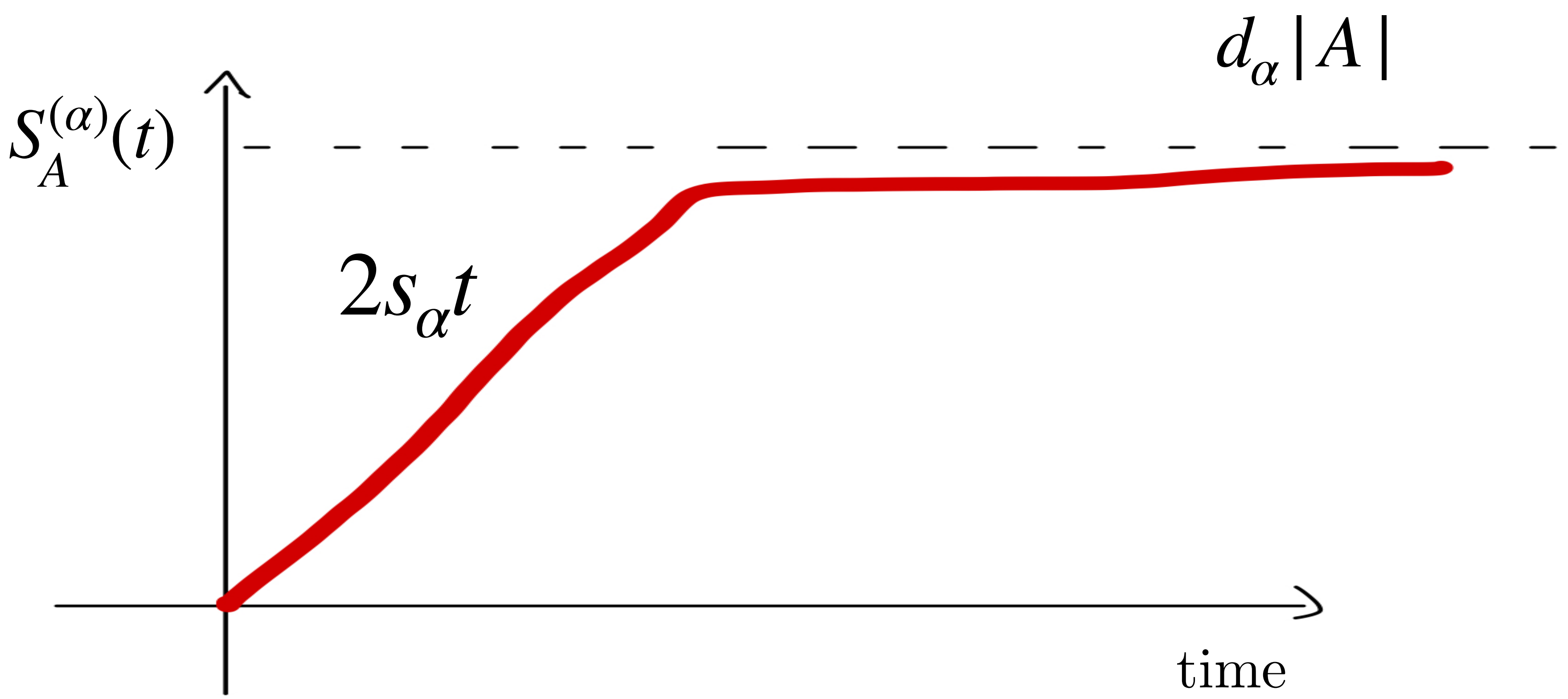

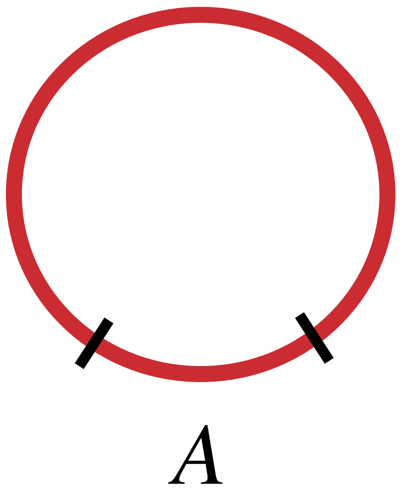

$$d_\alpha = \lim_{|A|\to\infty} \lim_{t\to\infty} \left(\lim_{L\to\infty} \frac{S_A^{(\alpha)}}{|A|} \right)$$

$$s_\alpha = \lim_{t\to\infty} \lim_{|A|\to\infty} \left(\lim_{L\to\infty} \frac{S_A^{(\alpha)}}{2t} \right)$$

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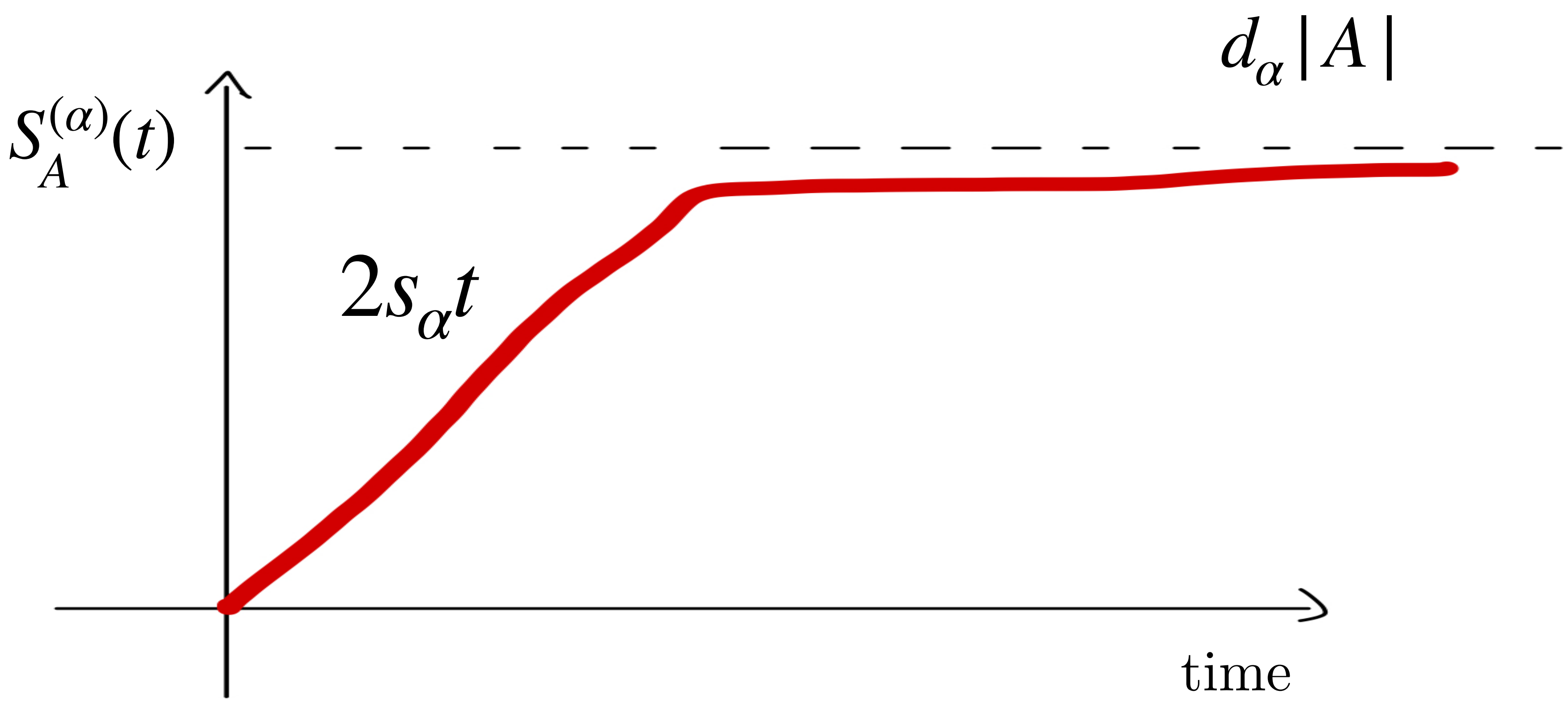

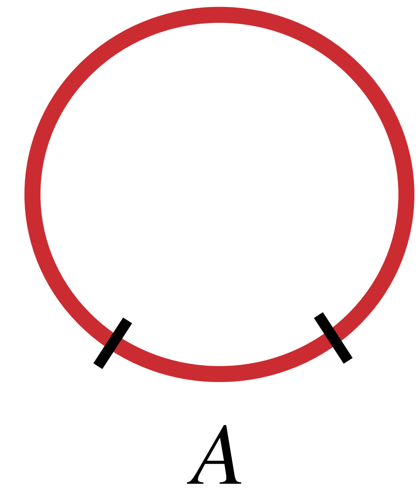

$$d_\alpha = \lim_{|A|\to\infty} \lim_{t\to\infty} \left(\lim_{L\to\infty} \frac{S_A^{(\alpha)}}{|A|} \right)$$

$$= \frac{1}{1-\alpha} \lim_{|A|\to\infty} \frac{1}{A} \ln \mathrm{tr}[\rho_{\mathrm{st},A}^{\alpha}]$$

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Local interactions

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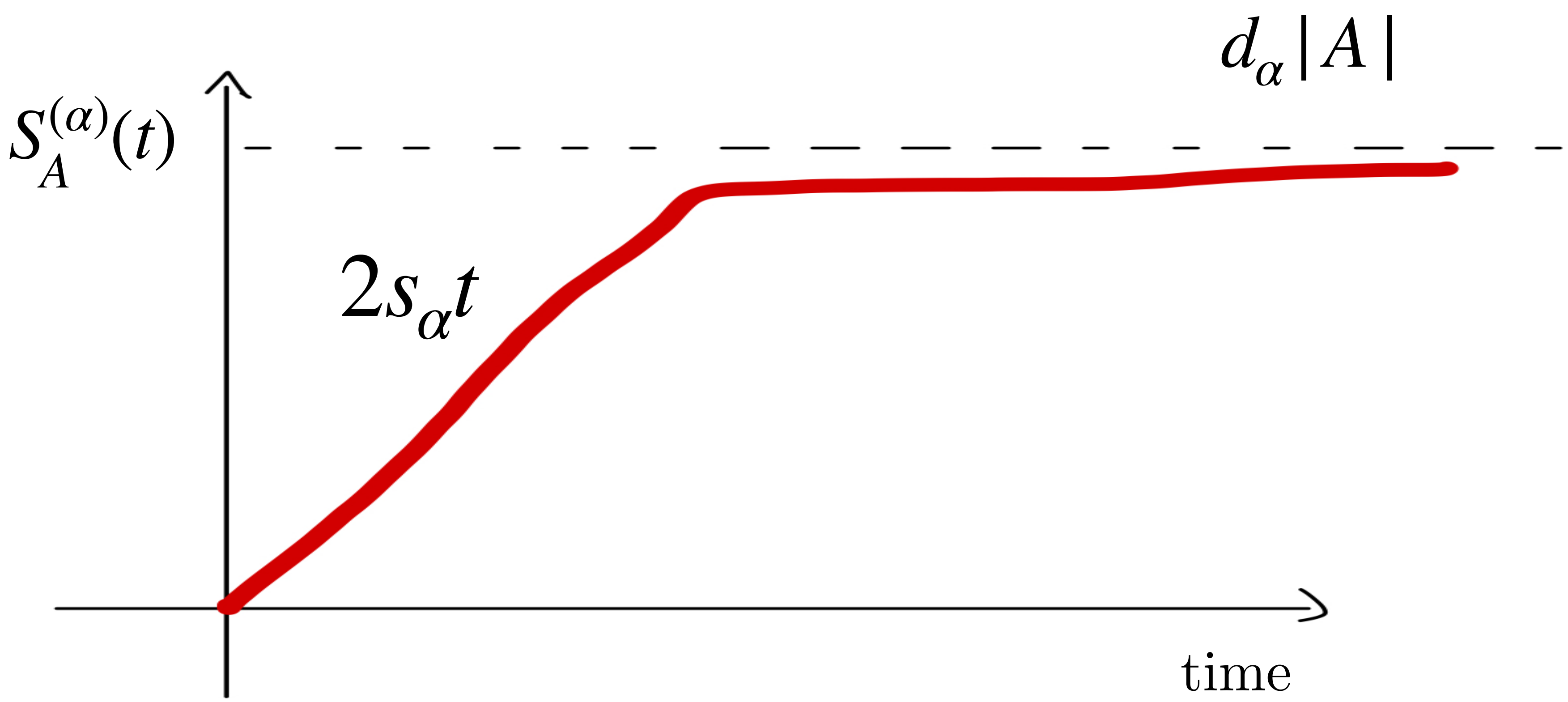

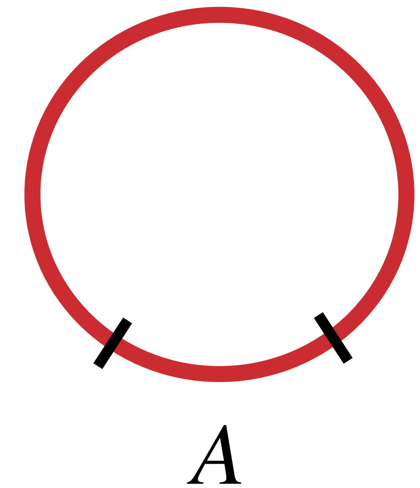

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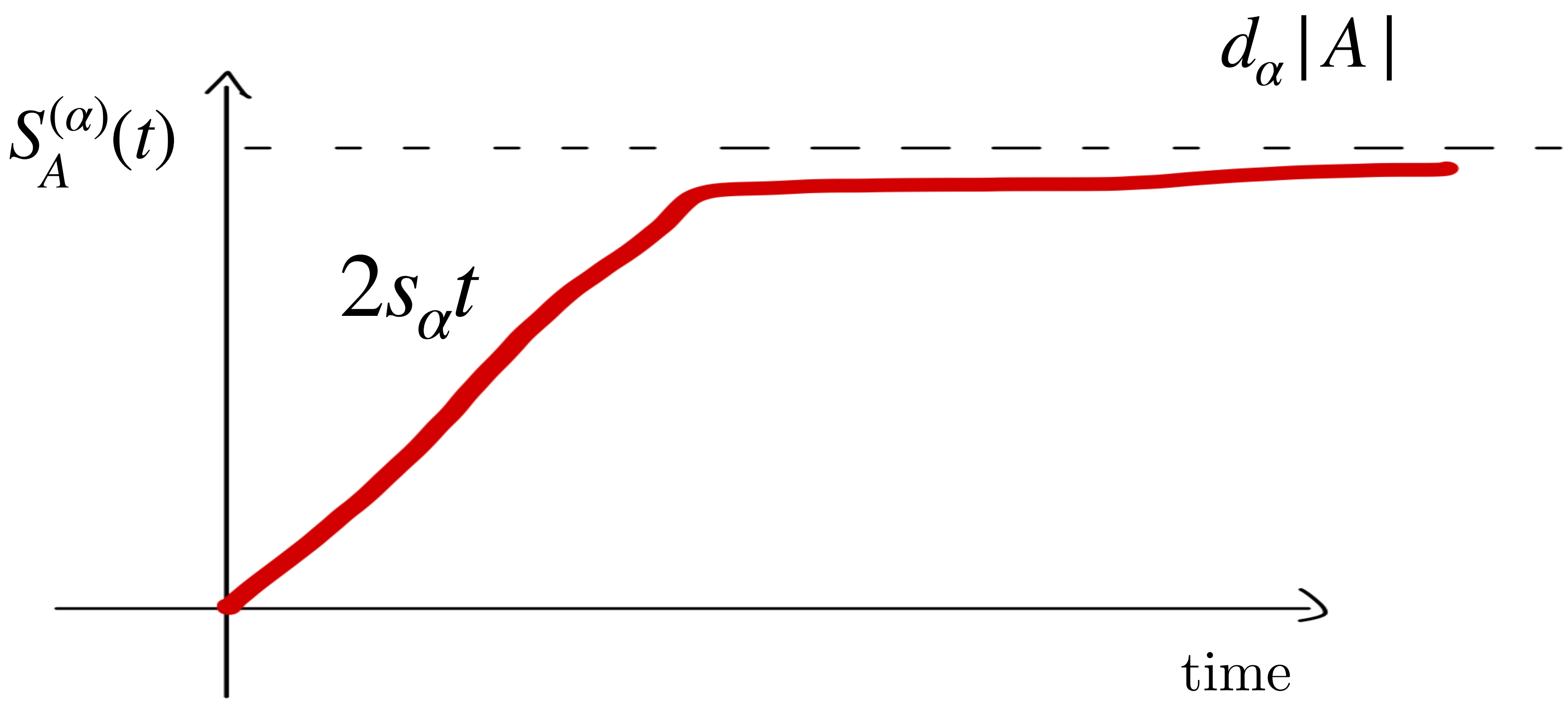

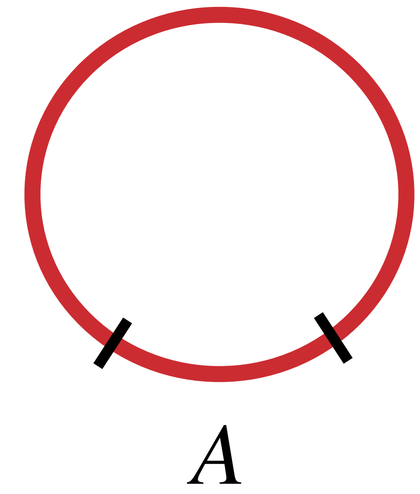

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Compute d_α and perform the analytic continuation $(|A|, t) \mapsto (t, |A|)$

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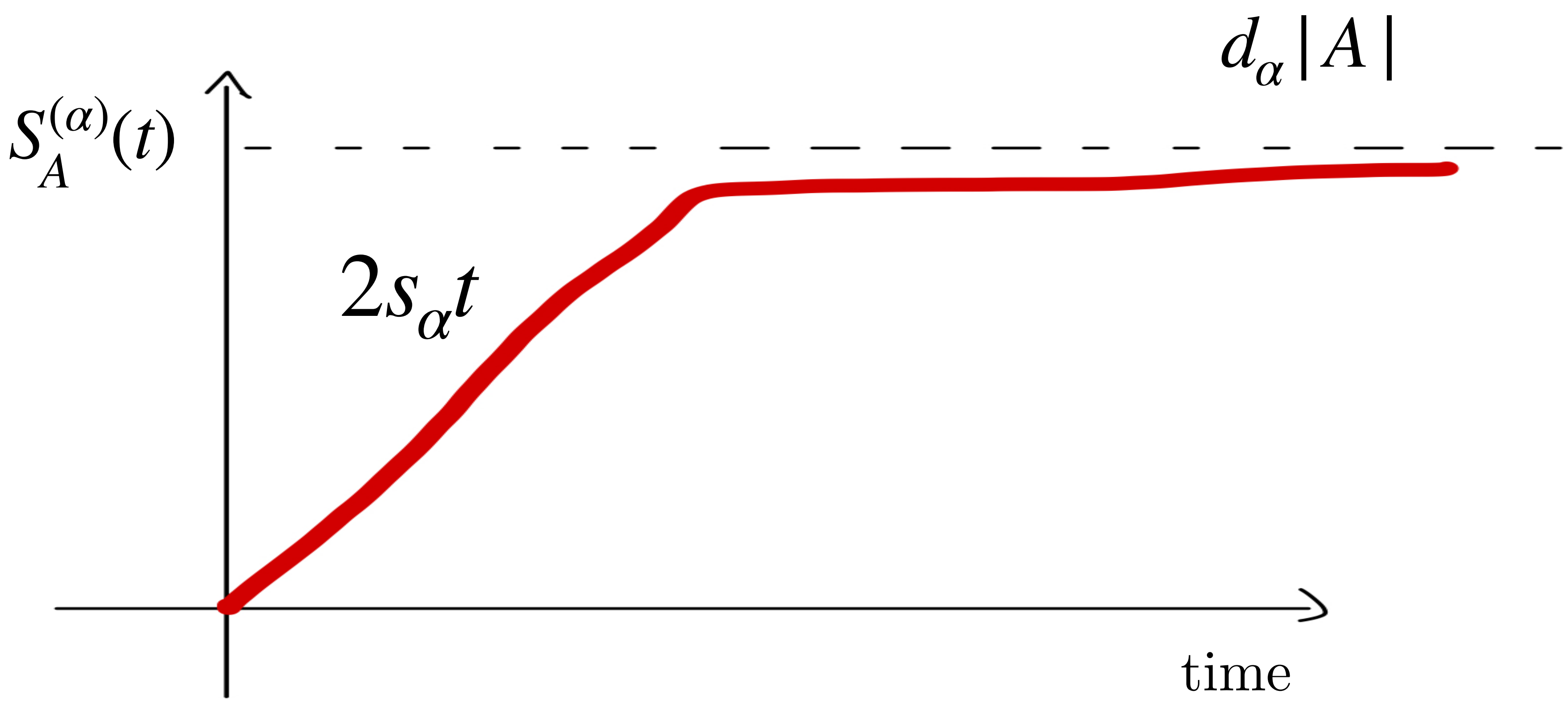

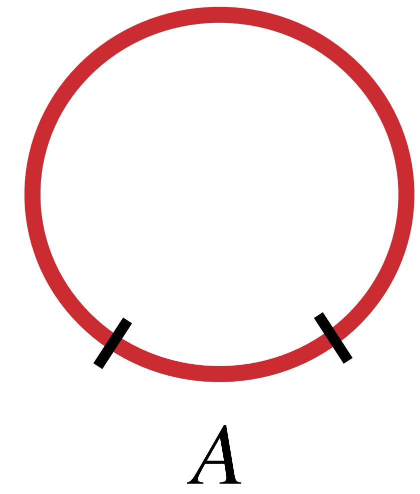

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It can be done, e.g., in interacting integrable models!

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$$H = \frac{J}{4}\sum_{j=1}^{L}\left[\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y + \Delta(\sigma_j^z\sigma_{j+1}^z - 1)\right]$$

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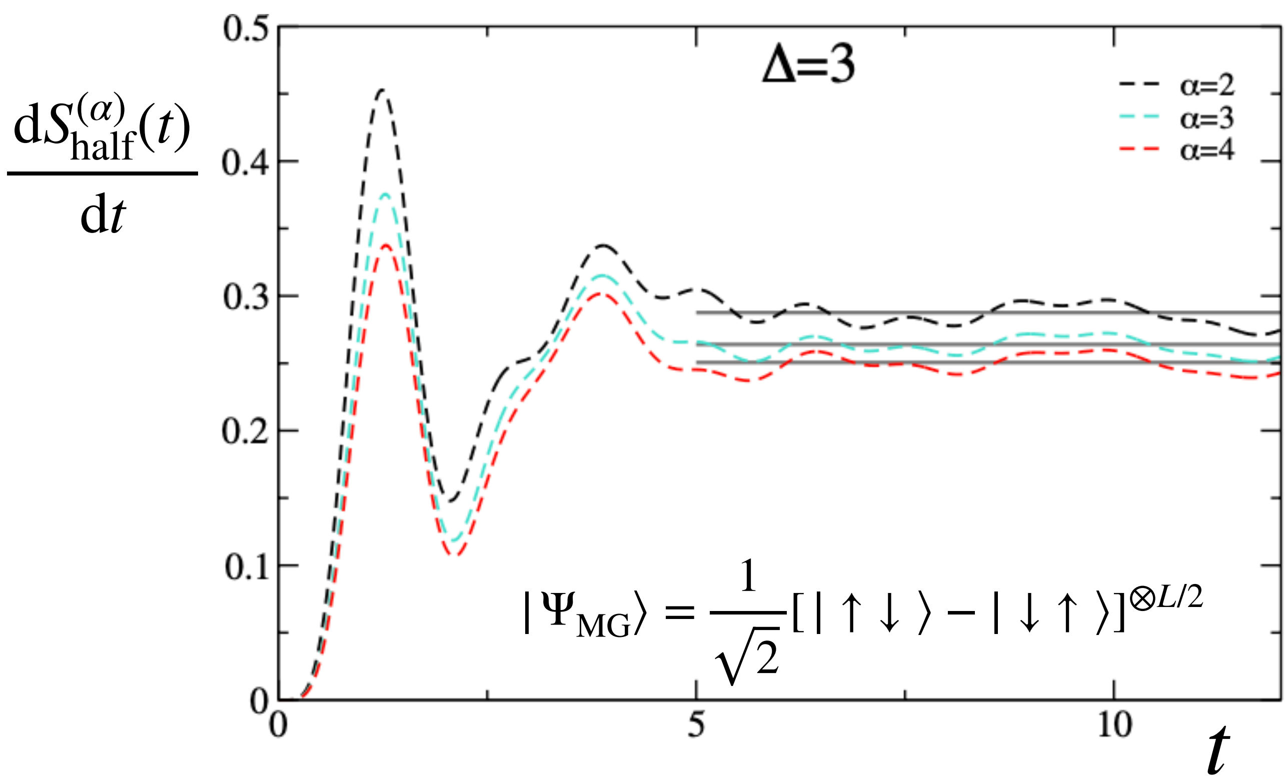

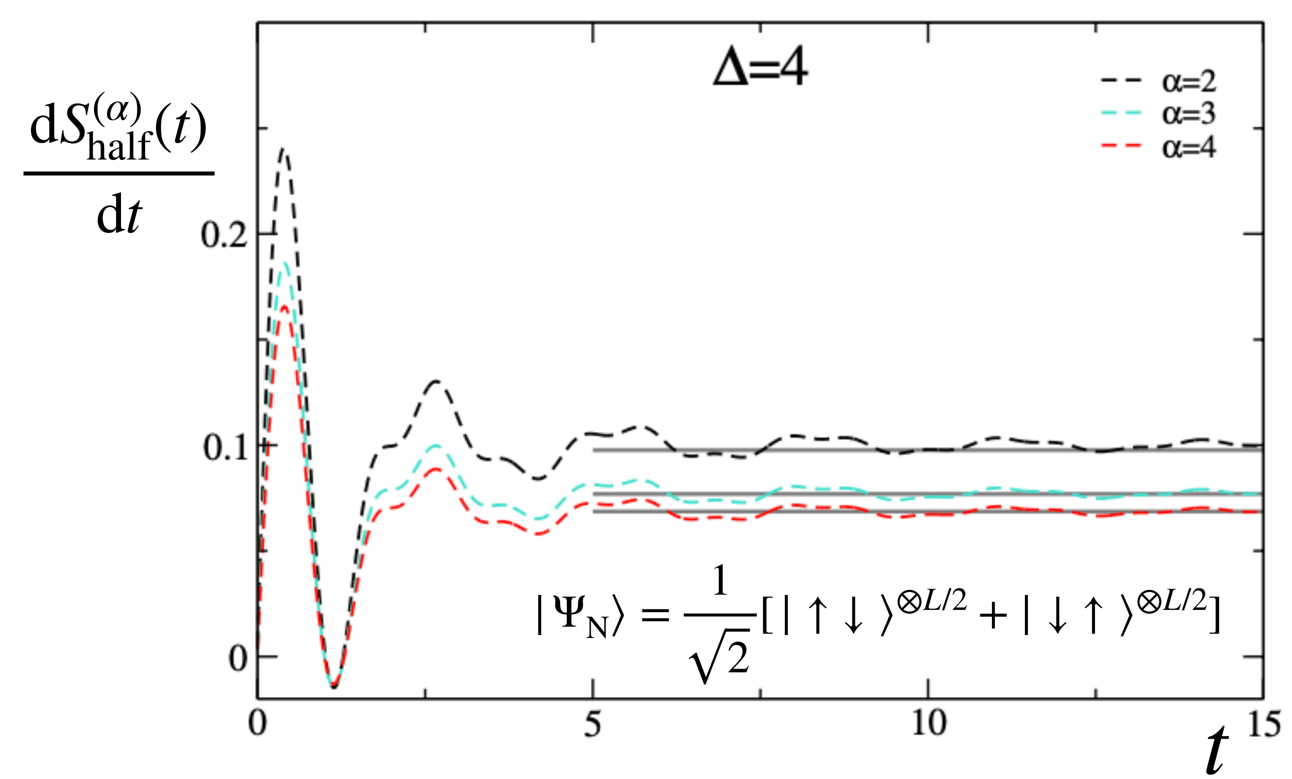

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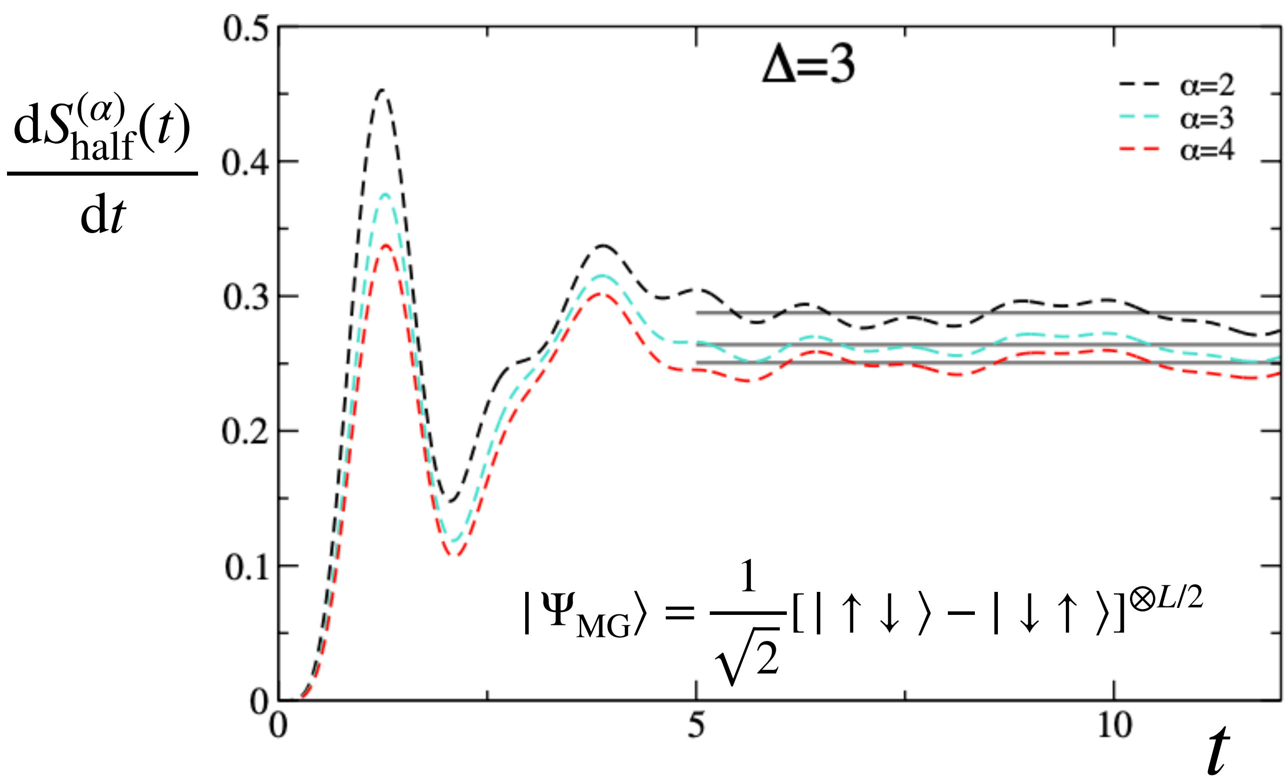
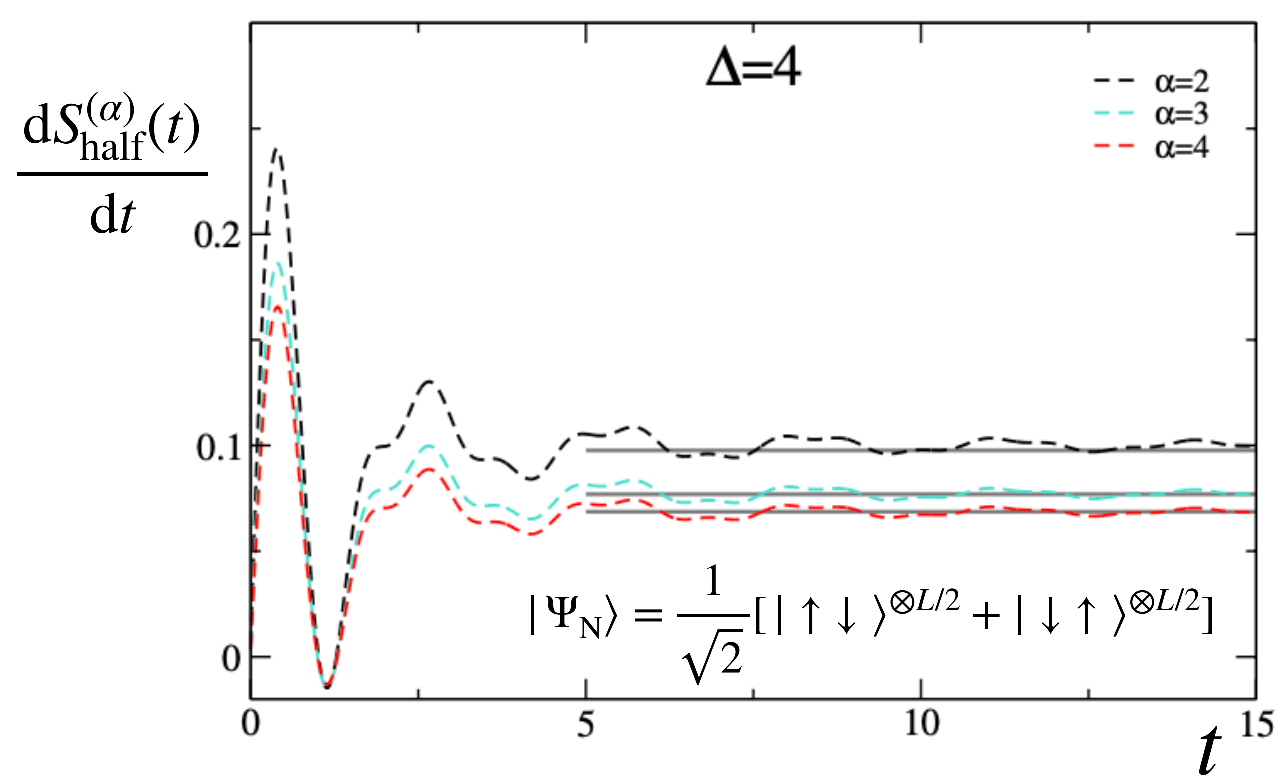
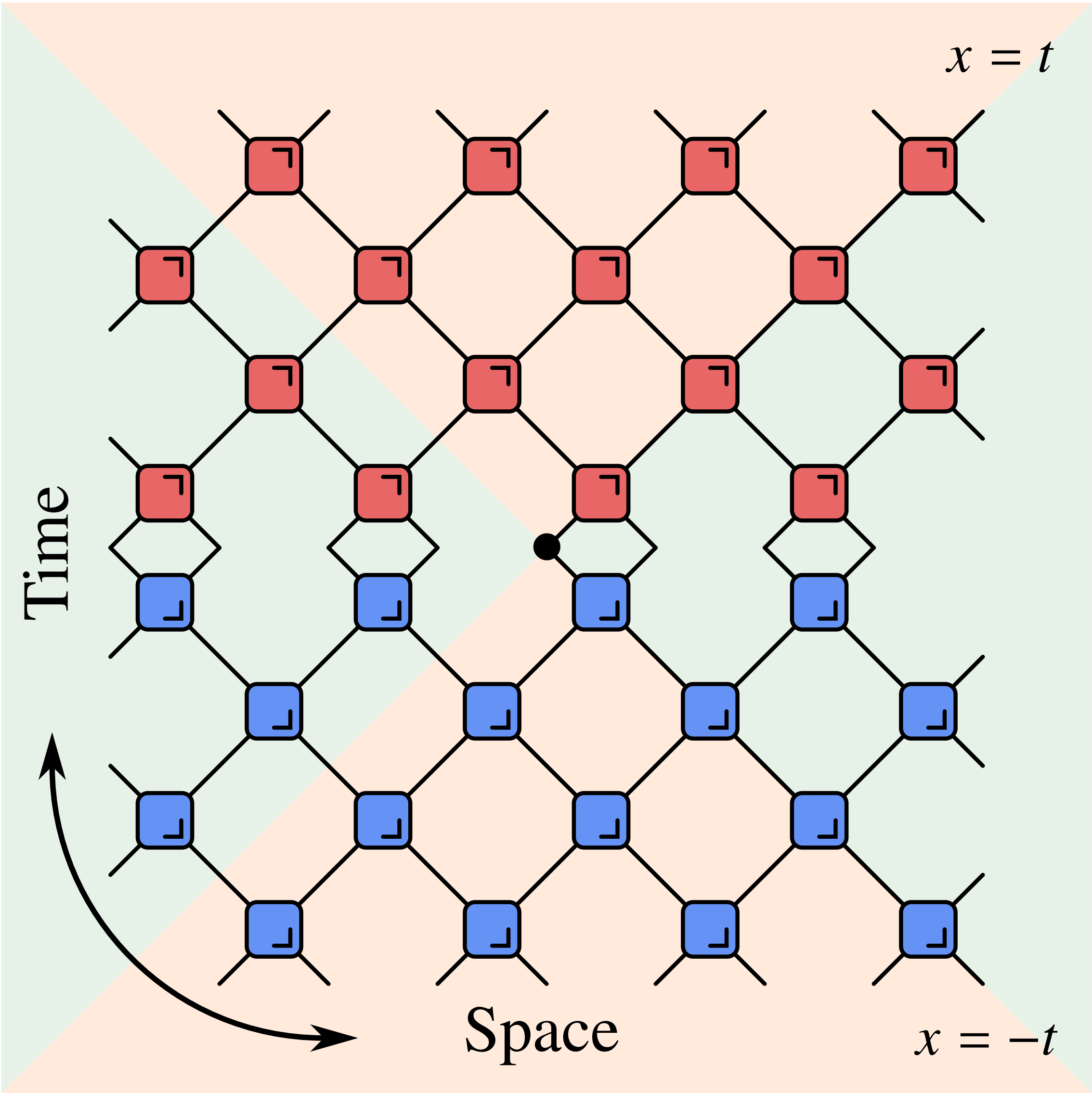




Same idea applies to other dynamical quantities!

Summary

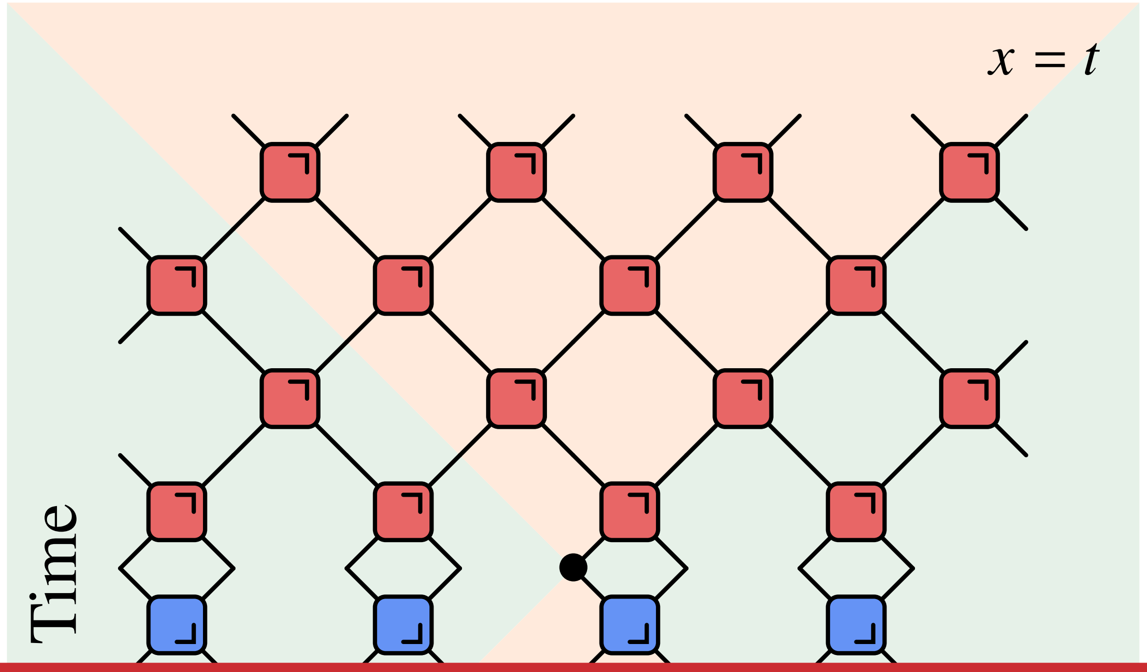


1. Understand quantum many-body dynamics by solving interacting quantum circuits

2. Define solvable yet “chaotic” minimal models by imposing a duality symmetry

3. Relaxing space-time duality symmetry allows us to go beyond exact solutions

Summary



Thank you !

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