Mixed-State Entanglement in a Minimal Model of Quantum Chaos

Tanay Pathak

(Department of Physics, Kyoto University)

Based on : arXiv:2603.14292 [quant-ph]

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- Exactly solvable many-body dynamics from space-time duality, B. Bertini, P. Clayes, T. Prosen, Rev. Mod. Phys. 98, 025001, 2026.

REVIEWS OF MODERN PHYSICS, VOLUME 81, APRIL–JUNE 2009

Quantum entanglement

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(Published 17 June 2009)

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Exactly solvable quantum many-body dynamics from space-time duality

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(published 15 April 2026; corrected 17 April 2026)

Quantum chaos

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Berry-Tabor Conjecture

Absence of eigenvalue correlations.

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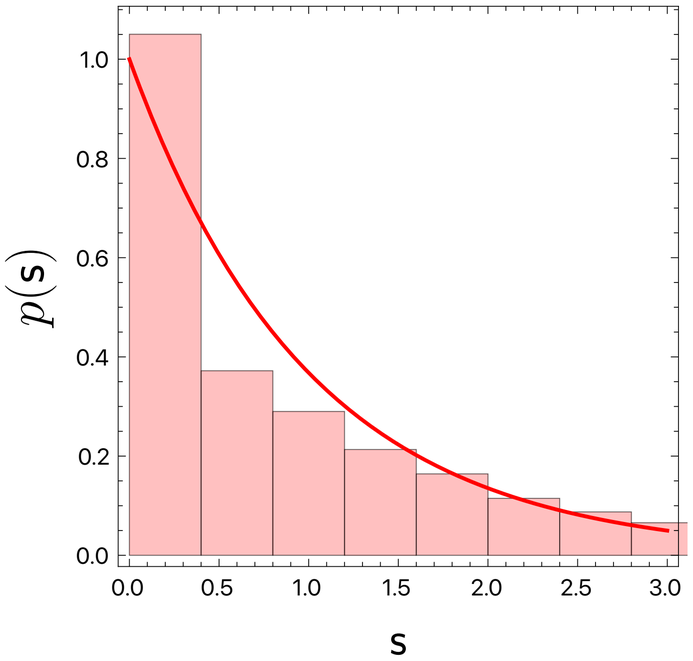

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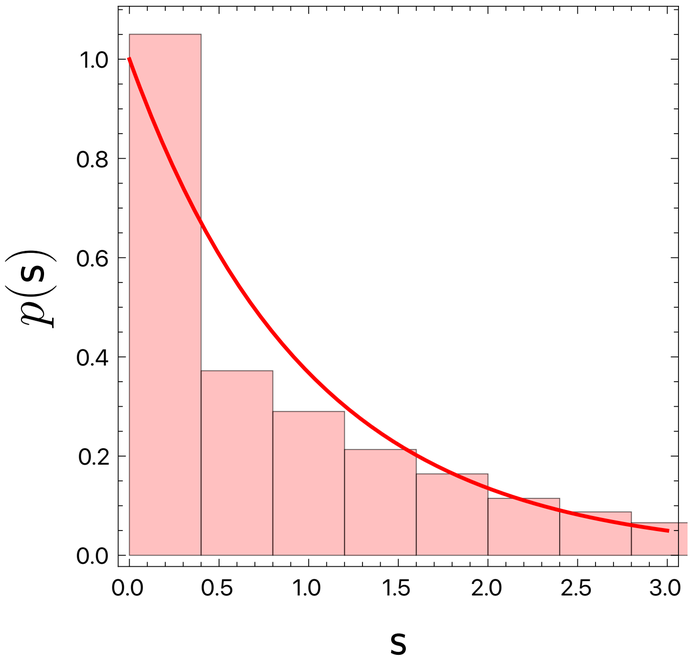

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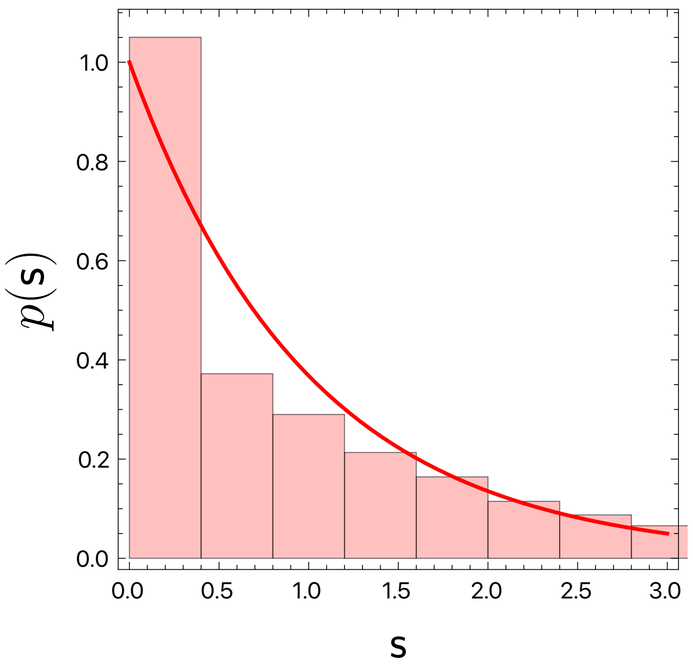

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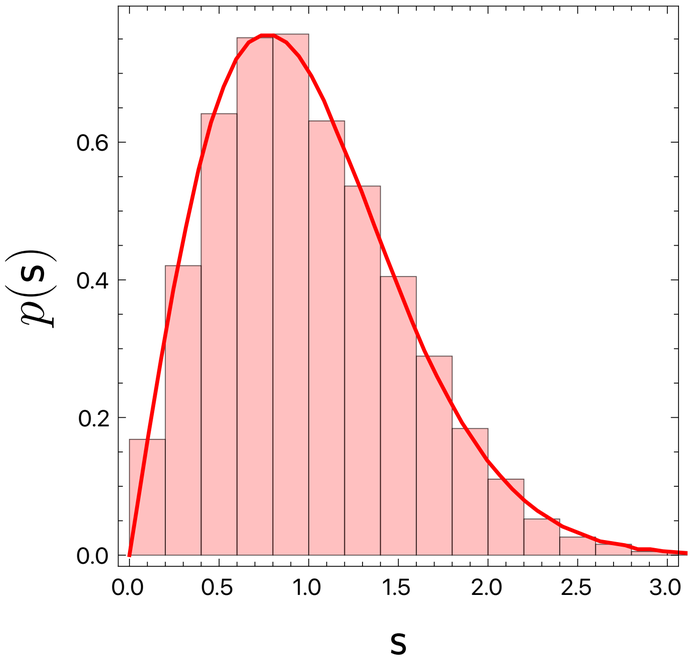

A minimal model of quantum chaos

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$$H = H_I + H_K \sum_{\tau=-\infty}^{\infty} \delta(t-\tau)$$

$$H_I = J \sum_{i=1}^{L} \sigma_i^z \sigma_{i+1}^z + \sum_{i=1}^{L} h_i \sigma_i^z, \quad H_K = \sum_{i=1}^{L} b\, \sigma_i^x \,.$$

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Total Floquet operator: $U[\mathbf{h}] = U_K U_I,$ with $U_I = e^{-iH_I}$, $U_K = e^{-iH_K}$

A discrete space time duality

M. Akila, D. Waltner, B. Gutkin, T. Guhr, arXiv:1602.07130.

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K is complex.

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$Z(N,T) = \mathrm{Tr}(\tilde{U}_T^N)$ where $\tilde{U}_T$ is the dual unitary operator.

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$\tilde{U}_T$ is non-unitary.

At $|J| = |b| = \frac{\pi}{4}$, $\tilde{U}_T$ becomes unitary .

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Consic

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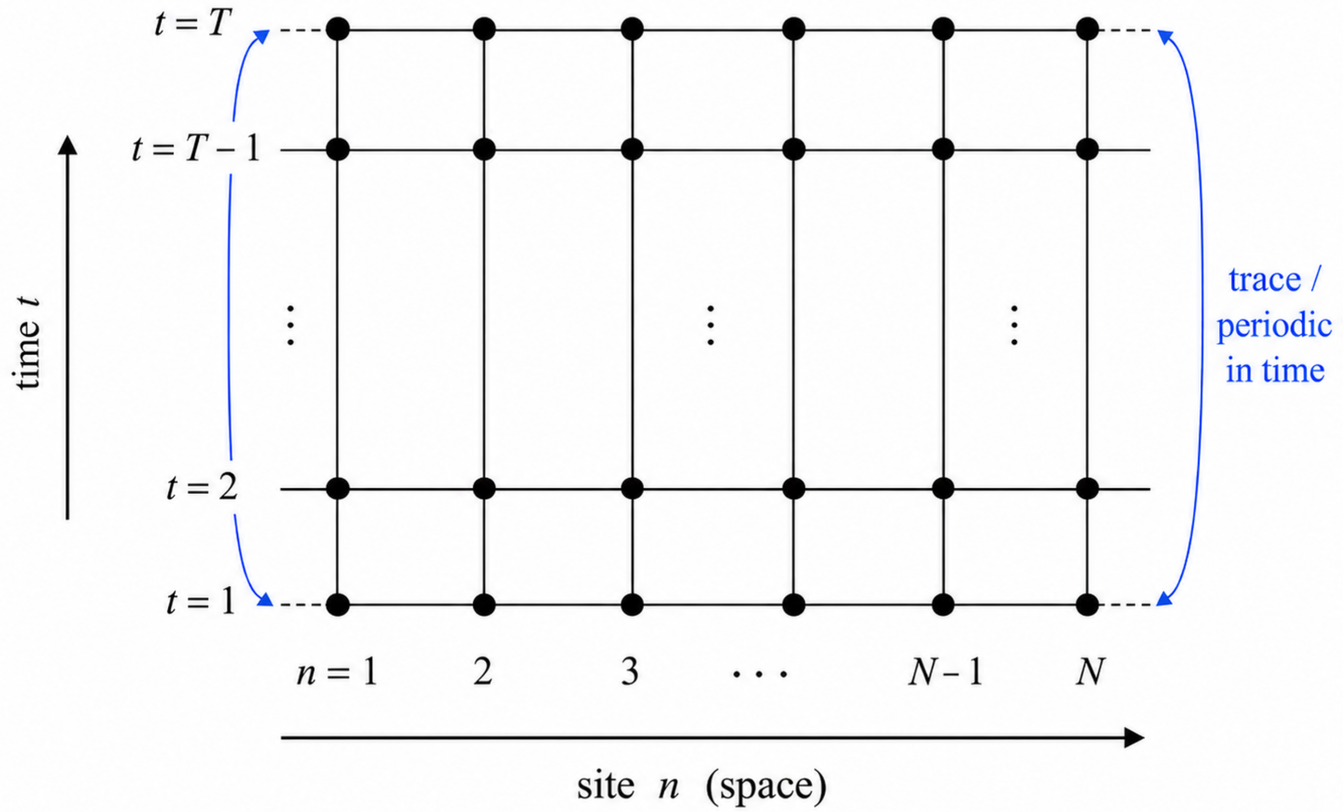

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Dual unitary

B. Bertini, P. Clayes, T. Prosen:
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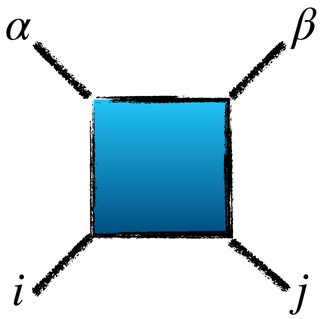

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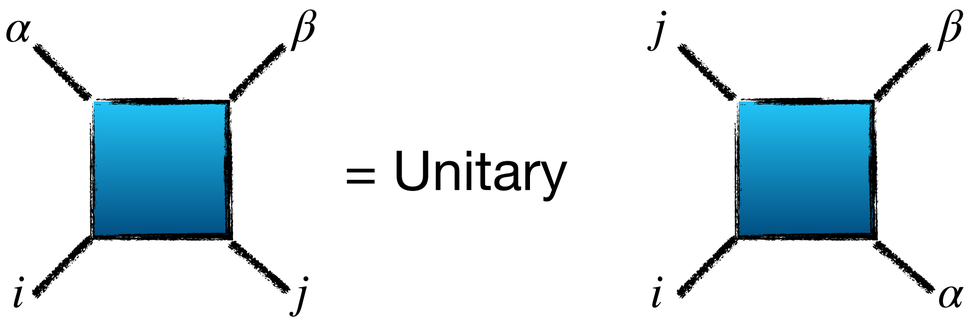

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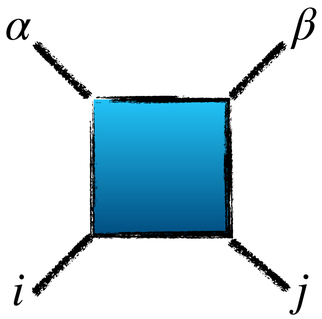

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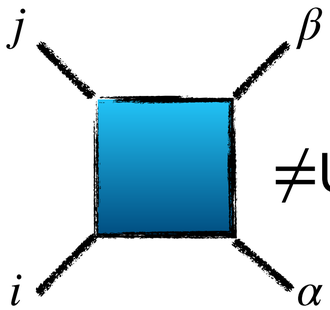

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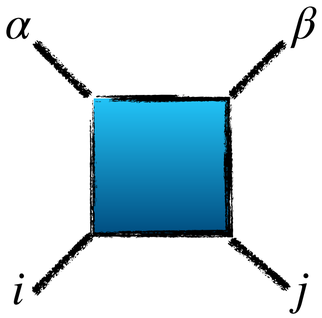

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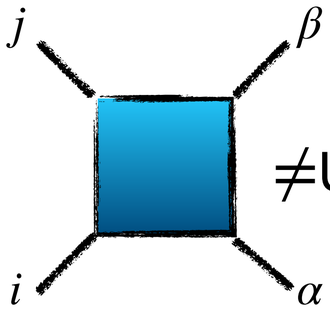

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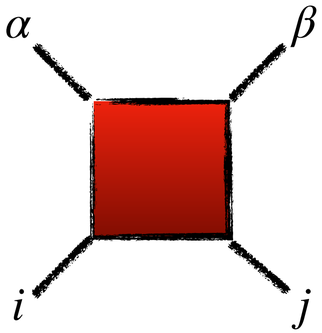

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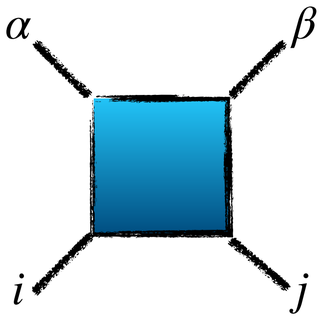

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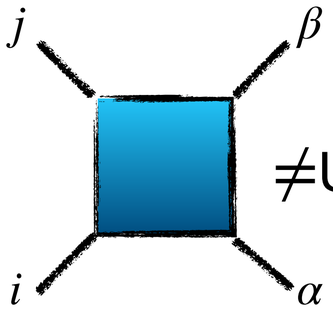

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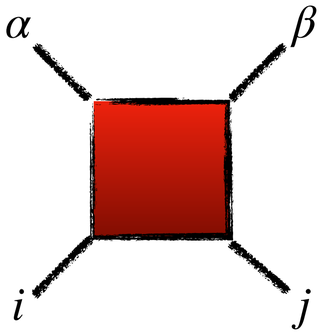

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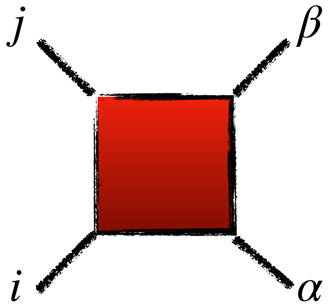

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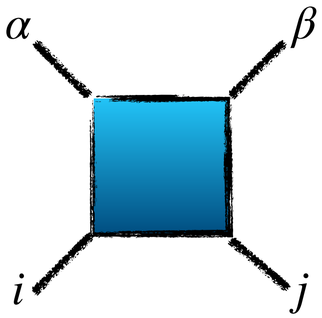

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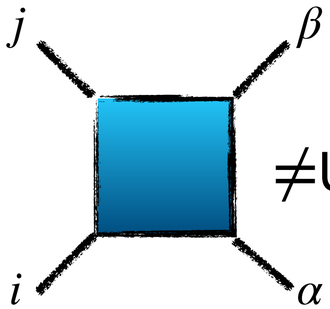

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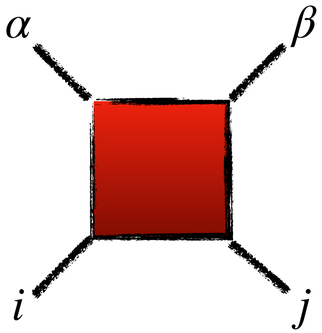

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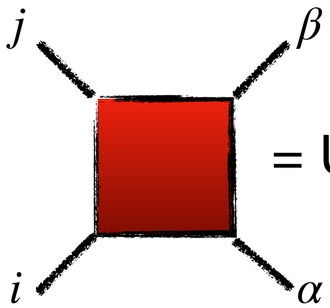

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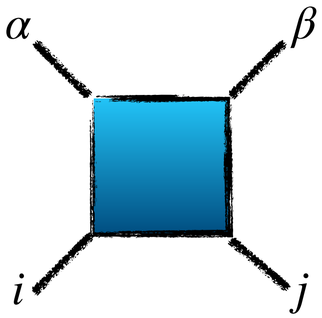

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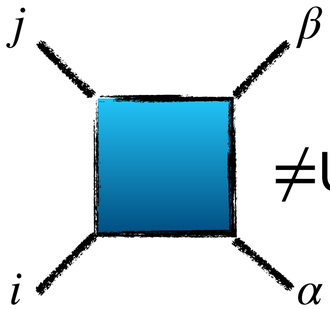

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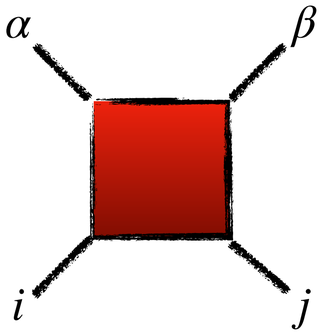

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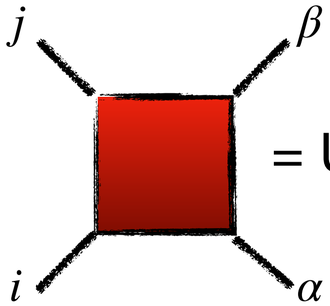

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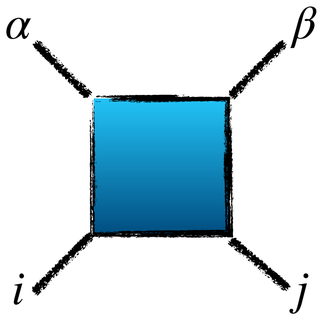

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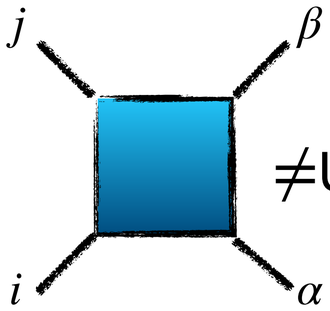

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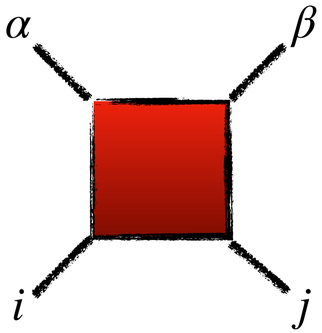

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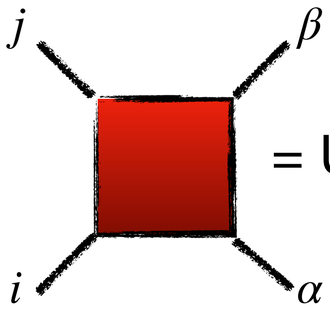

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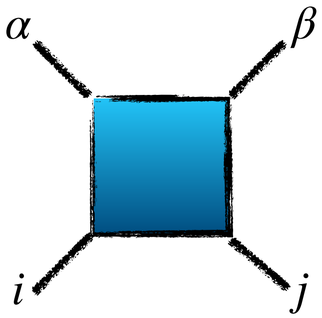

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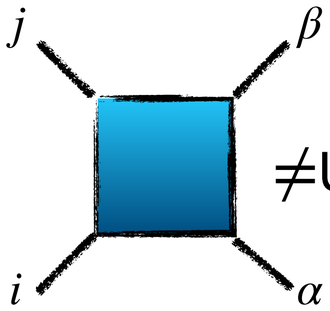

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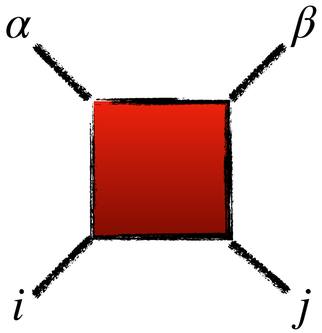

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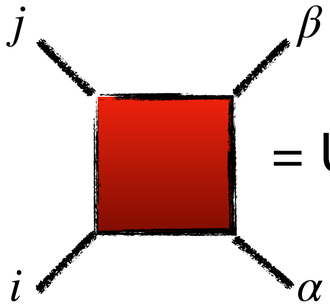

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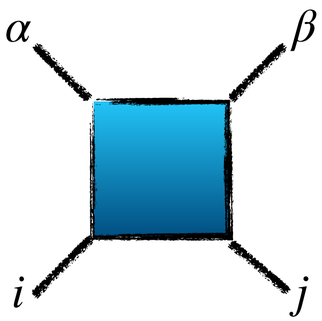

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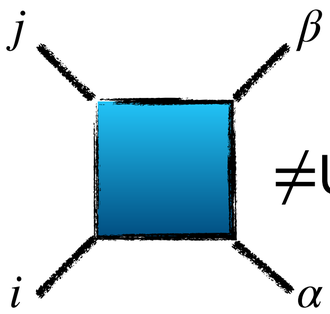

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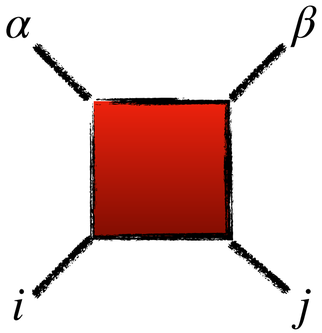

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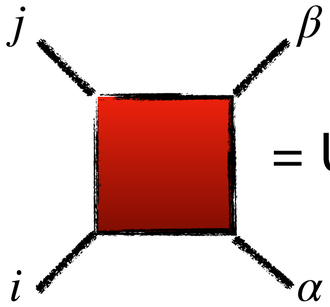

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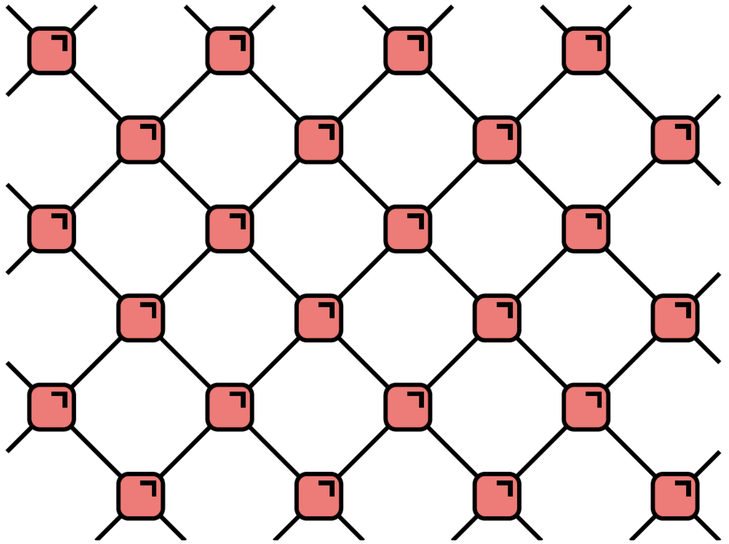

Dual unitary

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- Correlation functions.

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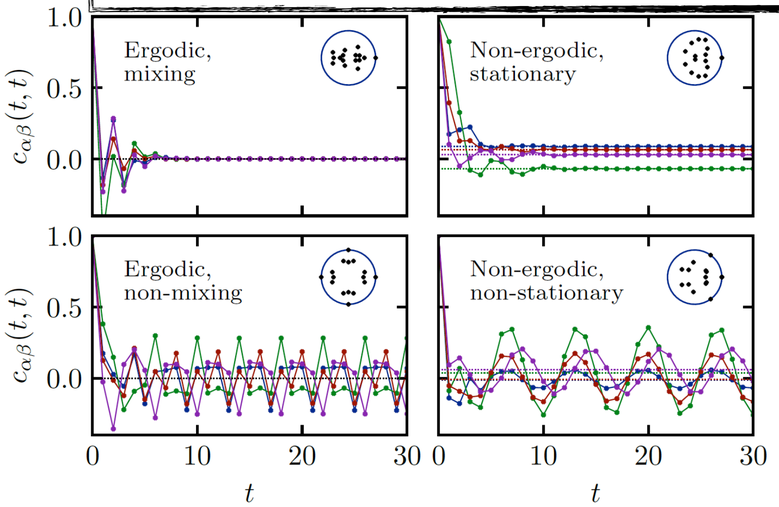

Dual unitary

- Correlation functions.
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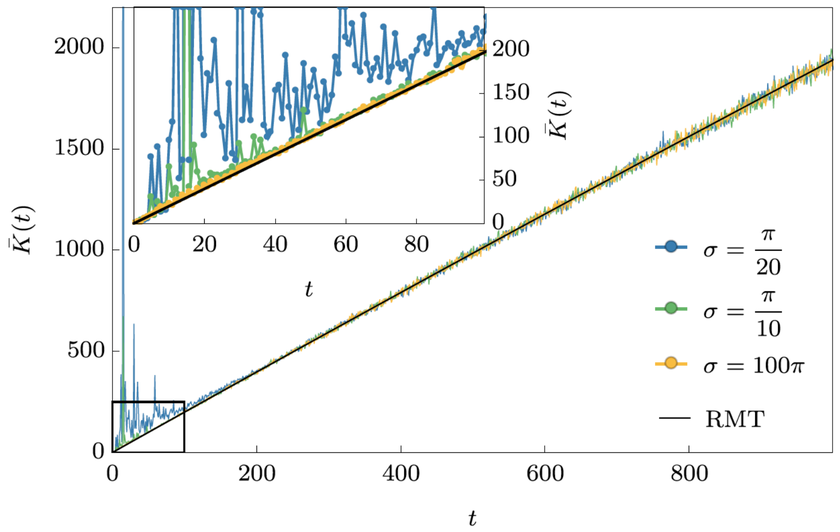

B. Bertini, P. Kos, T. Prosen,
PRL **121**, 264101,**2018**.

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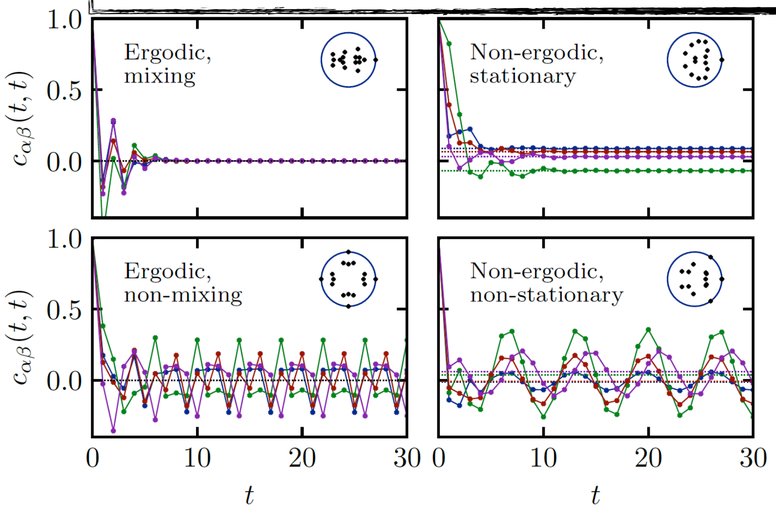

Dual unitary

- Correlation functions.
- Spectral form factor.
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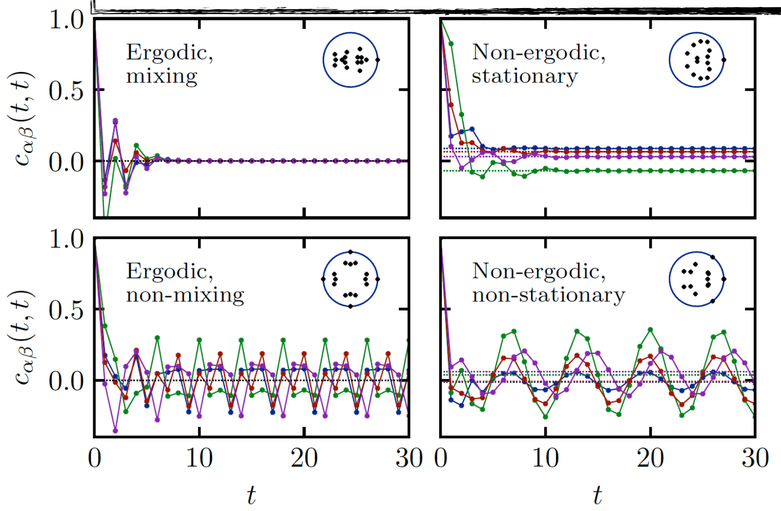

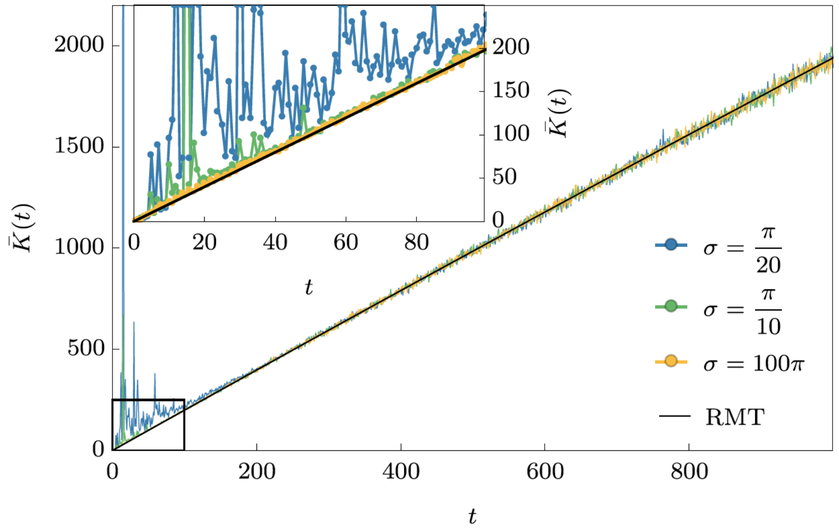

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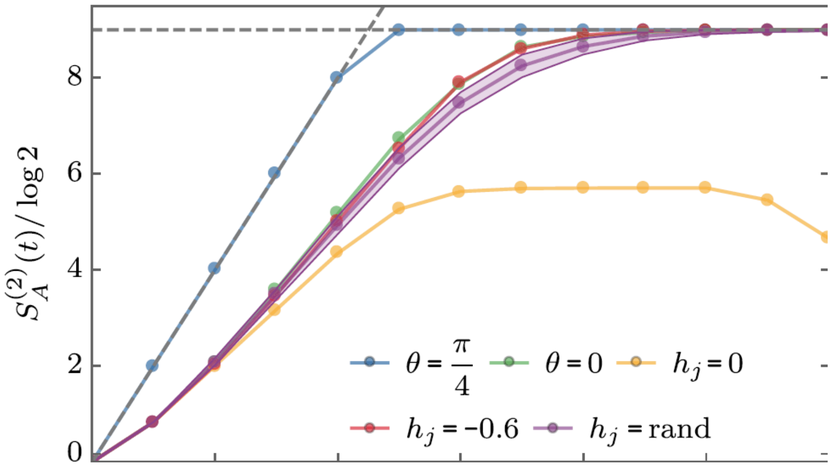

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Pure state Entanglement

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Horodecki[4]: RMP, Vol.81, 865, 2009

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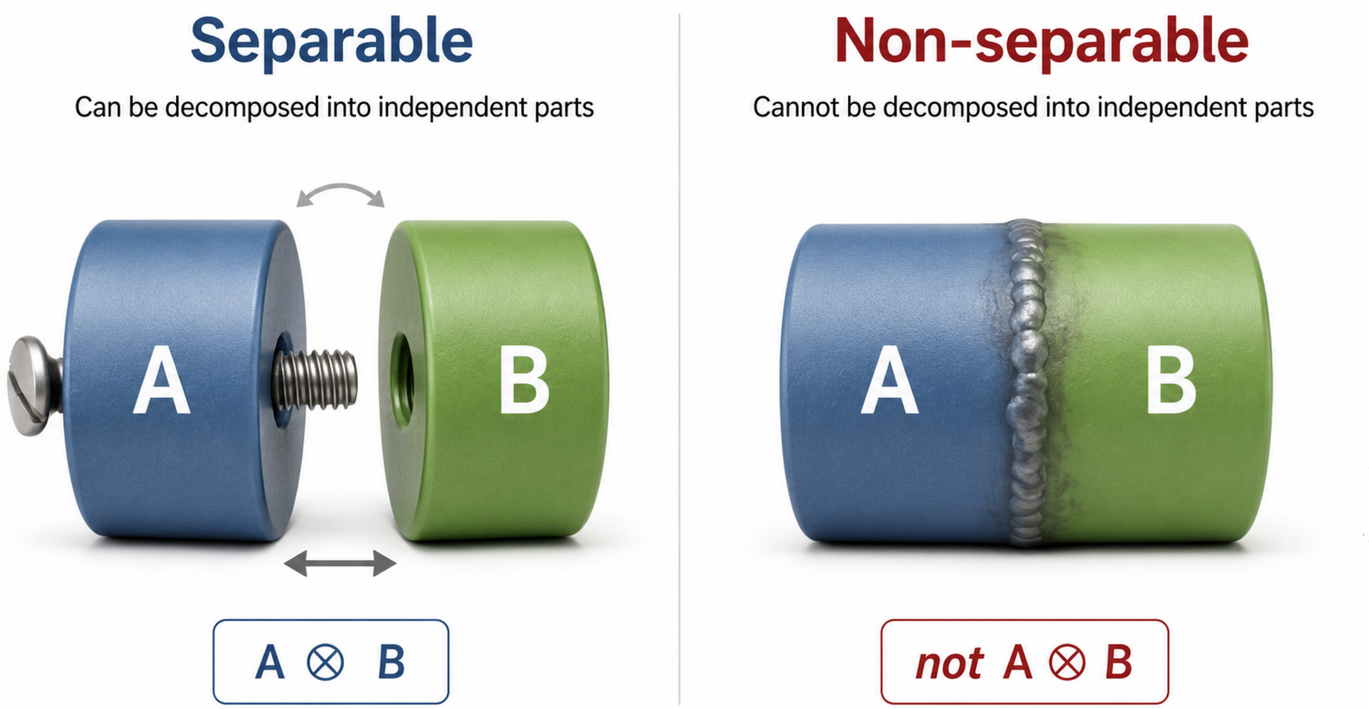

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Setup

TP arXiv:2603.14292

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For $2t < L_A, L_B$ $\quad \boxed{\mathcal{E}(t) = t\ln(2)}$

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TP arXiv:2603.14292

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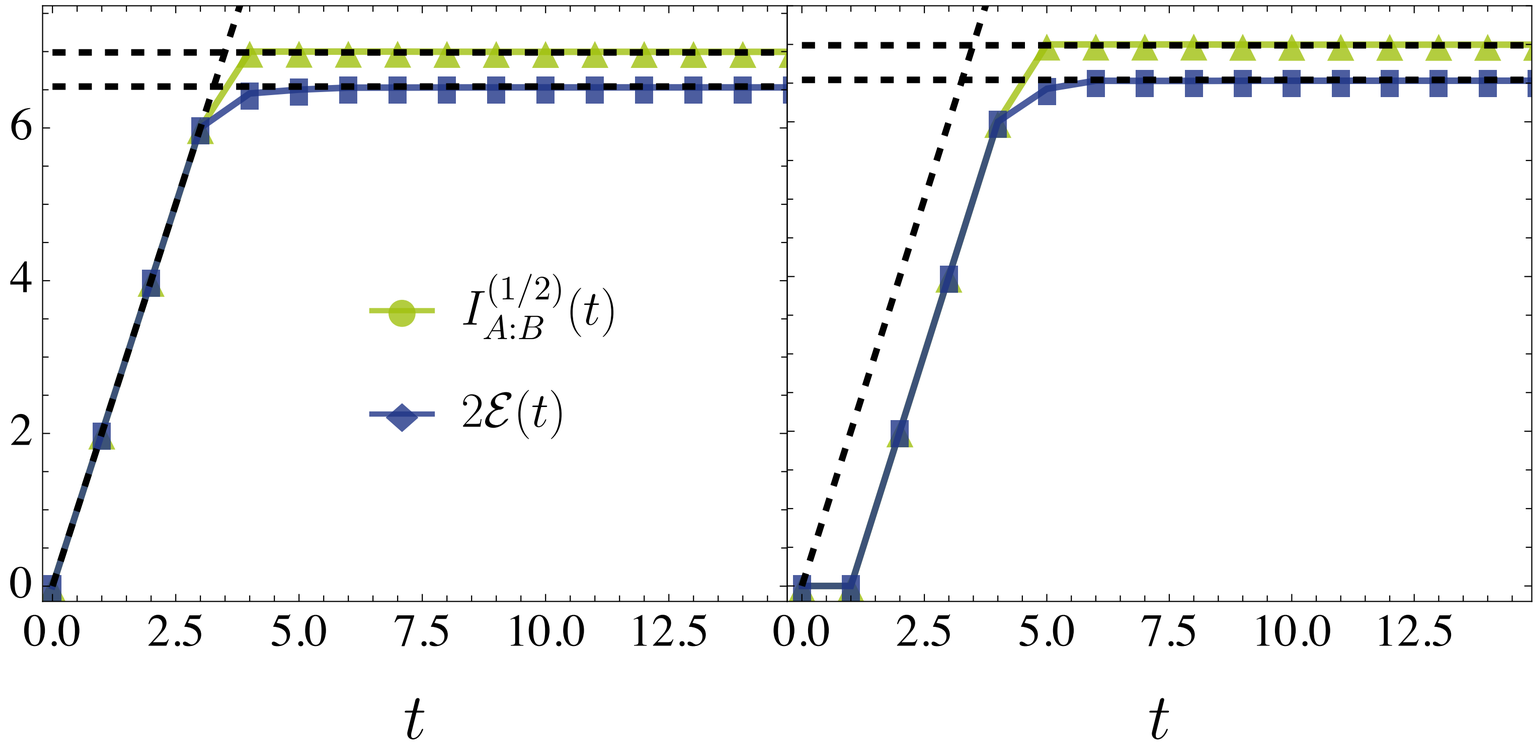
At early times we thus have $$2\mathscr{E}(t) = I^{(\alpha)}_{A:B}(t)$$

Solvable states

TP arXiv:2603.14292

$$L = 21, \quad L_A = L_B = L_C = 7$$



Generic states

TP arXiv:2603.14292

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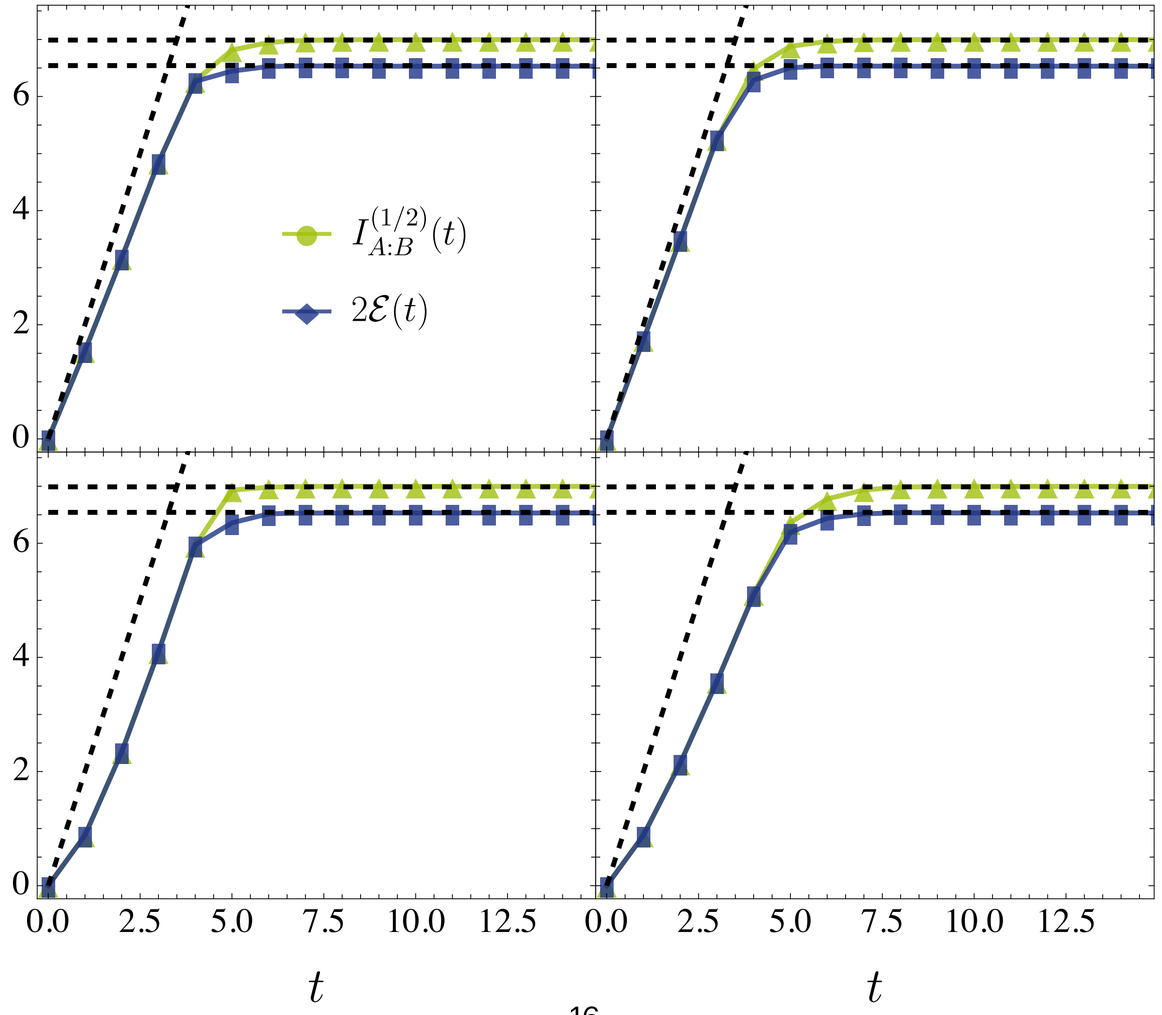

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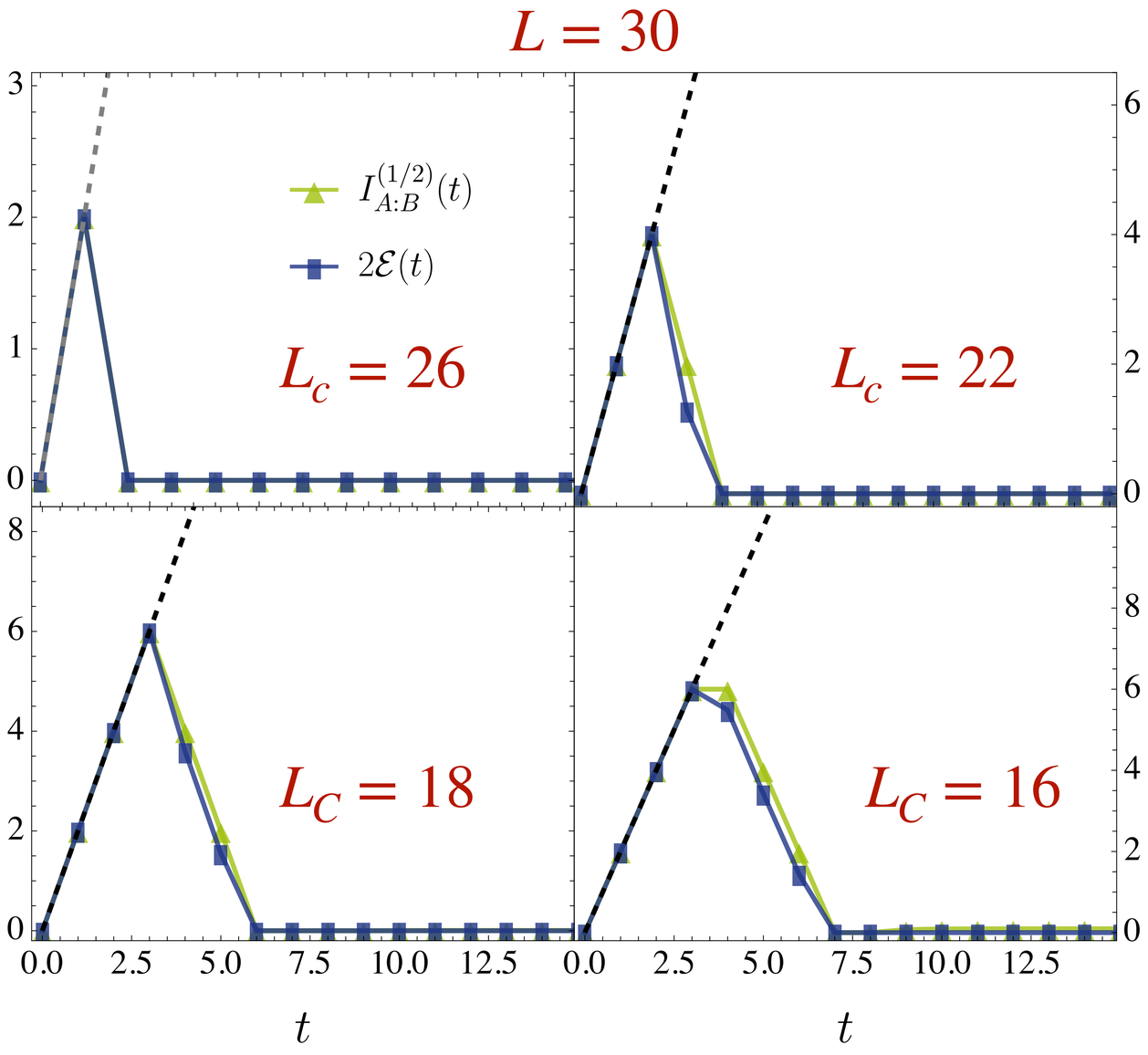
Solvable state: $L_C \gg L_A, L_B$



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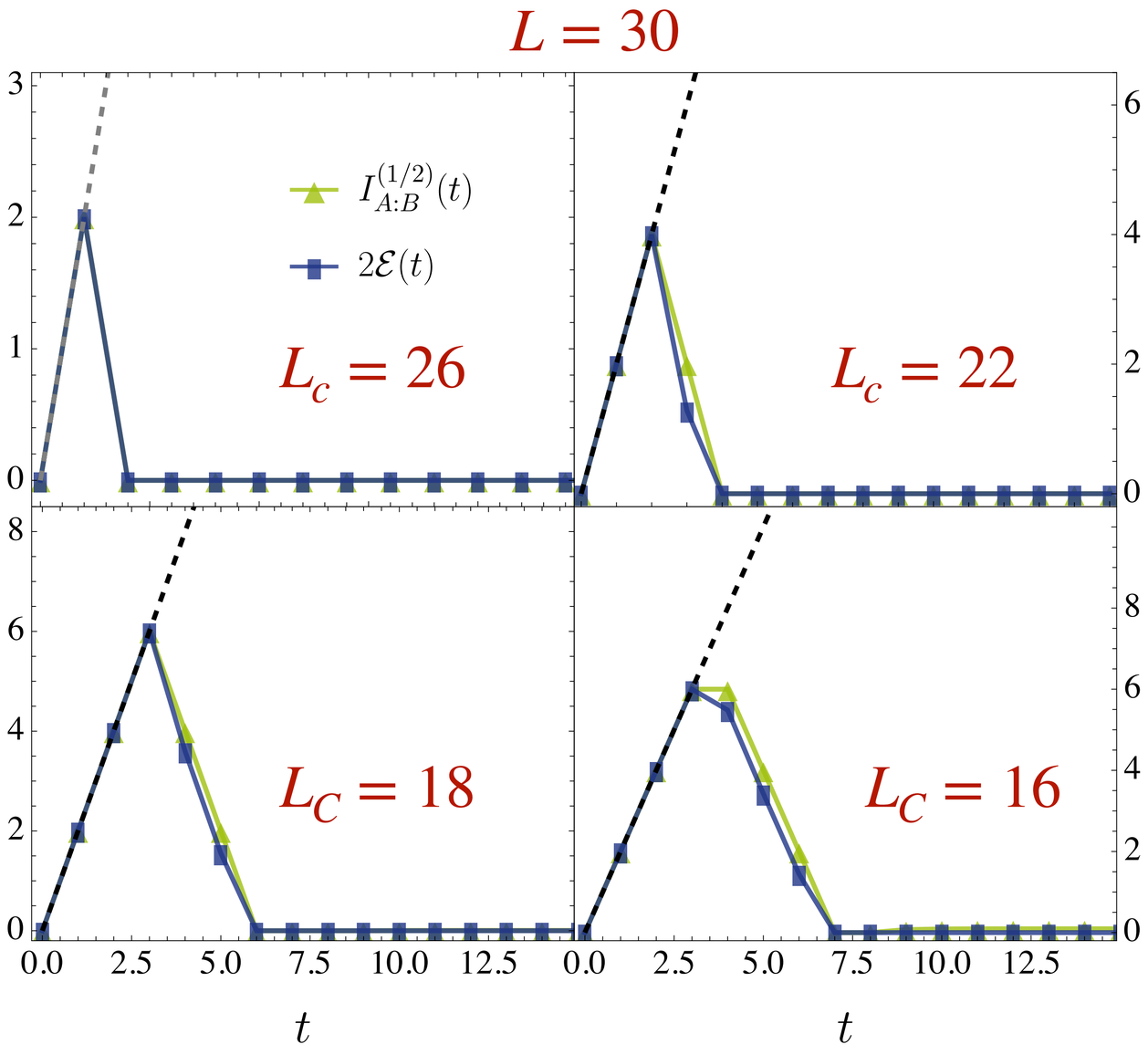

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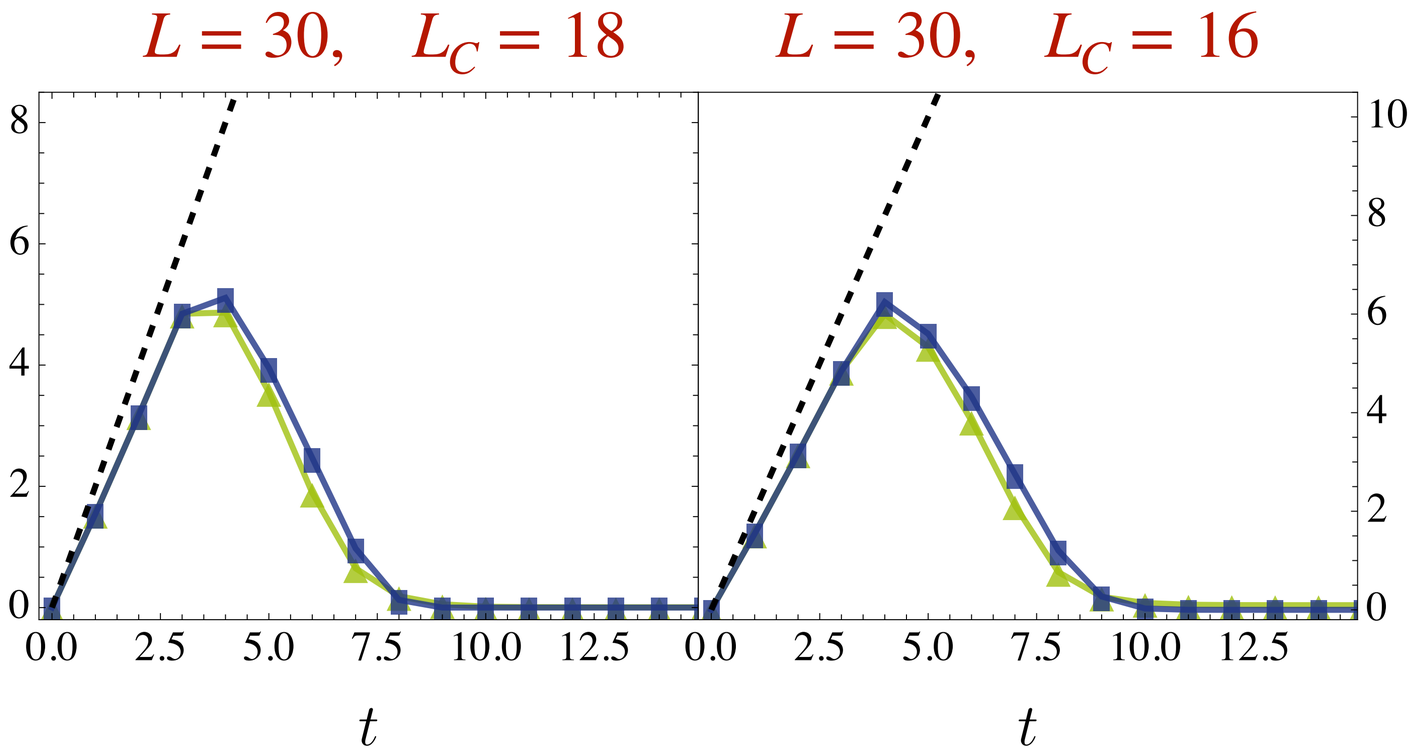

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Conjecture:

$$2\mathscr{E}(t) = I^{(\alpha)}_{A:B}(t),$$

hold for generic states at all times t. Specifically, for $2t \gg L_A, L_B, L_C$, we have

$$2\mathscr{E}(t) = I^{(\alpha)}_{A:B}(t) \begin{cases} = 0, & L_A, L_B \ll L_C, \\ \neq 0, & L_A = L_B = L_C, \end{cases}$$

Summary

- We derive the relation $2\mathscr{E}(t) = I^{(\alpha)}_{A:B}(t)$ for the self dual kicked field Ising model.
- A plausible conjecture for generic states and for all times
- We can further show that at early times $2\mathscr{E}(t) = I^{(\alpha)}_{A:B}(t) = \frac{2}{3}\mathscr{E}^{(O)}(t)$ at early times.
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