Tunable many-body burst in isolated quantum systems

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Outline

1. Introduction: burst and thermalization
2. Methods for creating a burst
3. Burst at short times: numerical results
4. Burst at long times: analytical results
5. Discussion and outlook

Outline

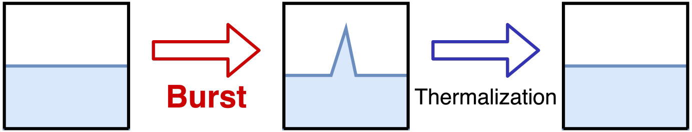
Burst phenomenon

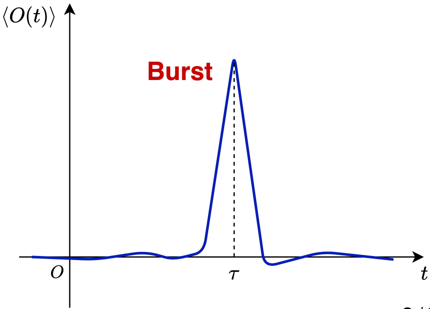
Isolated quantum many-body systems

- Fundamental setup for studying thermalization
- Realization by ultracold atoms, trapped ions, etc.

"Burst": A temporal deviation of an expectation value of an observable from its thermal equilibrium value

- Is a burst compatible with thermalization?





Eigenstate thermalization hypothesis (ETH)

L: system size, Δ_O: spectral width of the observable O

ETH (for O) J. M. Deutsch, Phys. Rev. A **43**, 2046 (1991); M. Srednicki, Phys. Rev. E **50**, 888 (1994).

Every energy eigenstate $|E_n\rangle$ in a microcanonical energy shell $[E-\delta E, E+\delta E]$ with $\delta E = o(L)$ satisfies

$$\Delta_O^{-1} |\langle E_m|O|E_n\rangle - \delta_{mn} O_{\mathrm{mc}}(E)| = o(L^0),$$

where $O_{\mathrm{mc}}(E)$ is the microcanonical expectation value of O.

- Off-diagonal elements ($m \neq n$): ensure the eventual approach to a steady value.
- Diagonal elements ($m = n$): ensure steady value $\simeq$ thermal equilibrium value.

$\rightarrow$ The ETH ensures thermalization from any initial state with an $o(L)$ energy fluctuation.

The ETH is believed to be an underlying mechanism for thermalization of nonintegrable systems.

M. Rigol, V. Dunjko, and M. Olshanii, Nature **452**, 854 (2008).

Burst compatible with the ETH L. Knipschild and J. Gemmer, Phys. Rev. E **101**, 062205 (2020).

Construct matrix elements of O in the energy eigenbasis as

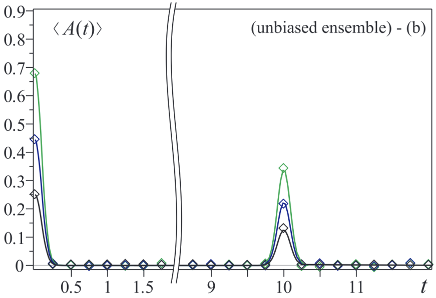
$$\langle E_m|O|E_n\rangle = C_1 D^{-1/2}\sqrt{\tilde{f}(E_m - E_n)}R_{mn}. \leftarrow \text{satisfies the ETH}$$

C_1: constant, D: Hilbert space dimension, R_{mn}: random numbers with zero mean and unit variance

Then, for the vast majority of initial states ρ_0, $\langle O(t)\rangle \simeq \langle O(0)\rangle f(t)$ holds with $f(0) = 1$.

$\rightarrow$ Pick $f(t)$ to be a burst-like function, and we see that **a burst is compatible with the ETH**.

However, such O must be specially designed, so is generally nonlocal.



[33–35]. Apart from that the ETH may miss some correlations that are in fact present in the matrices representing physical observables in physical systems [36]. These correlations may possibly rule out strange dynamics. Furthermore arguments are viable that are based on the condition of Hamiltonians being local [37]. Helpful insights may also come from clarifying

Burst via quantum recurrence

There are other theoretical methods to create a burst, i.e., quantum recurrence and time reversal.

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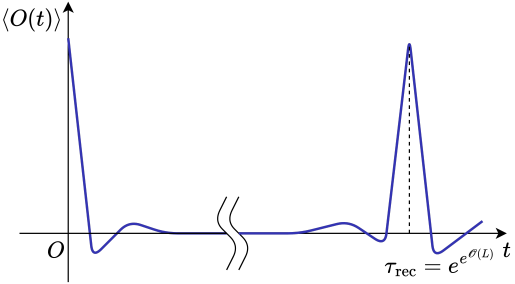
Quantum recurrence theorem P. Bocchieri and A. Loinger, Phys. Rev. **107**, 337 (1957).

Let H be a time-independent Hamiltonian with a discrete spectrum and $|\psi_t\rangle := e^{-iHt}|\psi_0\rangle$.
For any $\epsilon,\ t_0 > 0$ and any initial state $|\psi_0\rangle$, there exists a time $\tau_{\mathrm{rec}} > t_0$ s.t. $\||\psi_{\tau_{\mathrm{rec}}}\rangle - |\psi_0\rangle\| < \epsilon$.

However, τ_{rec} is typically double exponential in L. $\rightarrow$ It requires an extremely long time.

A. Peres, Phys. Rev. Lett. **49**, 1118 (1982).



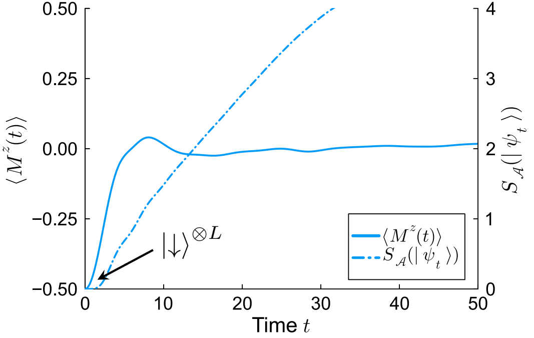
Burst via time reversal

Consider $|\psi_0\rangle = e^{iH\tau}|\Psi_{\mathrm{GS}}\rangle$ as a new initial state. $|\Psi_{\mathrm{GS}}\rangle$: ground state of O

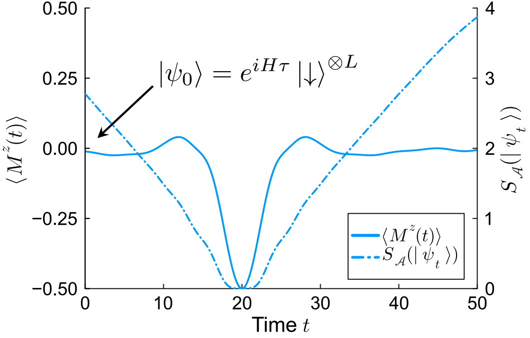
$\rightarrow$ At $t=\tau$, $|\psi_\tau\rangle = e^{-iH\tau}|\psi_0\rangle = |\Psi_{\mathrm{GS}}\rangle$.

However, $|\psi_0\rangle$ is generally highly entangled, and thus hard to prepare in experiments.

Thermalization ($M^z = L^{-1}\sum_i S_i^z$)



Burst via time reversal



Our study

We investigate a burst satisfying the conditions below, which align with typical experimental setups:

- Hamiltonian H: local
- Observable O: local
- Initial state $|\psi_0\rangle$: low-entangled
- Burst time τ: finite and not too long

Is such a burst possible?

	H	O	$\lvert\psi_0\rangle$	τ
Knipschild & Gemmer	$\triangle$	$\times$	$\checkmark$	$\checkmark$
Recurrence	$\checkmark$	$\checkmark$	$\checkmark$	$\times$
Time reversal	$\checkmark$	$\checkmark$	$\times$	$\checkmark$
Our study	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$

$\checkmark$: satisfied, $\times$: not satisfied, $\triangle$: partially satisfied

Our study

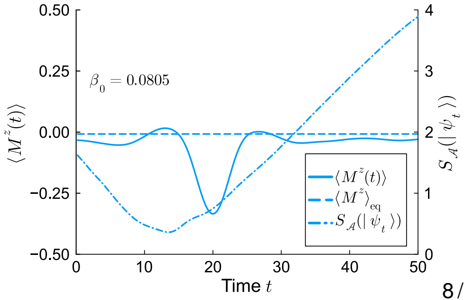
We investigate a burst satisfying the conditions below, which align with typical experimental setups:

- Hamiltonian H: local
- Observable O: local
- Initial state $|\psi_0\rangle$: low-entangled
- Burst time τ: finite and not too long

Is such a burst possible?
$\to$ **Yes**, for short times.

	H	O	$\vert\psi_0\rangle$	τ
Knipschild & Gemmer	$\triangle$	$\times$	✓	✓
Recurrence	✓	✓	✓	$\times$
Time reversal	✓	✓	$\times$	✓
Our study	✓	✓	✓	✓

✓: satisfied, $\times$: not satisfied, $\triangle$: partially satisfied



Outline

Setup

Hamiltonian: spin-1/2 mixed-field Ising chain

$$H = \sum_{i=1}^{L-1} J_z S_i^z S_{i+1}^z + \sum_{i=1}^{L} (h_x S_i^x + h_z S_i^z) \quad \underline{\text{local}}$$

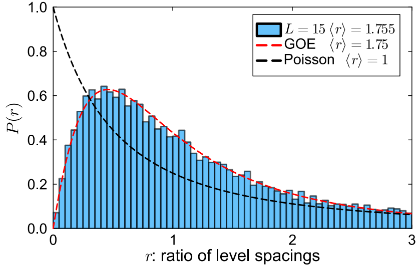
$$(J_z, 2h_x, 2h_z) = (1, 0.9045, 0.8090)$$

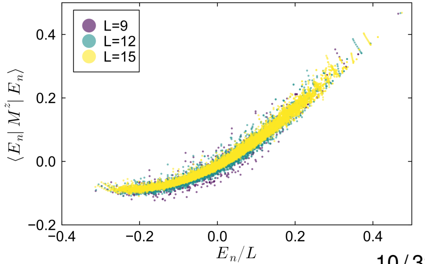
Nonintegrability and the ETH of this model have been verified numerically and analytically.

H. Kim and D. A. Huse, Phys. Rev. Lett. **111**, 127205 (2013);
H. Kim, T. N. Ikeda, and D. A. Huse, Phys. Rev. E **90**, 052105 (2014);
Y. Chiba, Phys. Rev. B **109**, 035123 (2024).

Observable: average magnetization along the z-axis

$$M^z = \frac{1}{L} \sum_{i=1}^{L} S_i^z \quad \underline{\text{local}}$$





MPSs: experimentally accessible low-entangled states

Matrix product states (MPSs): an efficient representation of low-entangled states

$$|\psi\rangle = \sum_{\{\sigma_i\}} A_1^{\sigma_1} A_2^{\sigma_2} \cdots A_L^{\sigma_L} |\sigma_1 \sigma_2 \cdots \sigma_L\rangle \quad \text{(open boundary condition)}$$

σ_i: local state at site i (e.g., $\uparrow, \downarrow$ for spin-1/2)
$A_i^{\sigma_i}$: $D_i \times D_{i+1}$ matrix with $D_1 = D_{L+1} = 1$
$\chi := \max_i D_i$: **bond dimension**
For any bipartition, entanglement entropy $S_{\mathcal{A}}(|\psi\rangle) \leq \ln \chi$.

- An MPS with an $\mathcal{O}(1)$ bond dimension is the ground state of an $\mathcal{O}(1)$-local Hamiltonian.
  D. Perez-Garcia et al., Quantum Inf. Comput. **7**, 401 (2007).
- An MPS with a small bond dimension can approximately be prepared by a shallow quantum circuit. M. S. Rudolph et al., Quantum Sci. Technol. **9**, 015012 (2024).

$\rightarrow$ MPSs are experimentally accessible.

Method 1: time reversal and truncation

- We fix the bond dimension χ, which can be determined by experimental capability, and search for an MPS $|\psi_0\rangle$ that creates a burst at a designated time $t = \tau$.

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Method 1:

1. Compute $e^{iH\tau} |\Psi_{\mathrm{GS}}\rangle$, where $|\Psi_{\mathrm{GS}}\rangle$ is the ground state of O.
2. Obtain an MPS by truncating $e^{iH\tau} |\Psi_{\mathrm{GS}}\rangle$ to the bond dimension χ.

Method 1 relies on a heuristic truncation.
$\rightarrow$ It generally creates a burst smaller than that obtained by a direct optimization of an expectation value.

Method 2: DMRG search

Density matrix renormalization group (DMRG): S. R. White, Phys. Rev. Lett. **69**, 2863 (1992).
A variational algorithm to obtain an MPS that approximates the ground state of a (usually local) Hamiltonian

Method 2:

1. Compute the following cost function:

$$H_{\mathrm{DMRG}} = \underbrace{O(\tau)}_{\text{minimize expectation value}} + \underbrace{\lambda_L (H - \langle H \rangle_\beta)^2}_{\text{suppress energy fluctuation}} .$$

   $O(t) := e^{iHt} O e^{-iHt}$, $\langle \bullet \rangle_\beta := \mathrm{Tr}\left[\bullet\, e^{-\beta H}\right] / \mathrm{Tr}\left[e^{-\beta H}\right]$
   β: target inverse temperature, λ_L (≥ 0): penalty weight

2. Using the DMRG algorithm, obtain an MPS with the bond dimension $\leq \chi$ that minimizes the cost function.

Comparison of two methods

Method 1 (compute $e^{iH\tau} |\Psi_{\mathrm{GS}}\rangle$ and truncate it to χ)

- is simple and requires less computational cost.

$\rightarrow$ is applied to infinite systems.

Method 2 (DMRG search for minimizing $H_{\mathrm{DMRG}} = O(\tau) + \lambda_L (H - \langle H \rangle_\beta)^2$)

- generally creates a larger burst when $\lambda_L = 0$.
- can tune the target energy and energy fluctuation by β and λ_L.

$\rightarrow$ is applied to the finite-size scaling analysis.

Outline

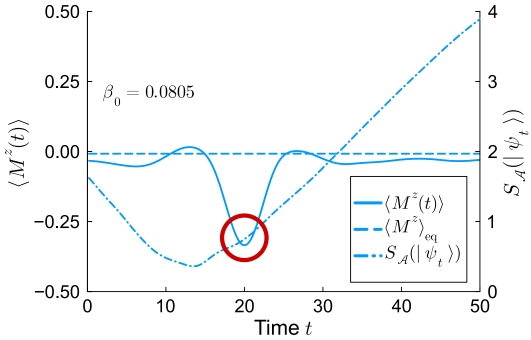
Burst from low-entangled states (Method 2)

$H_{\mathrm{DMRG}} = M^z(\tau) + \lambda_L(H - \langle H\rangle_\beta)^2$
β_0: inverse temperature satisfying $\langle H\rangle_{\beta_0} = \langle\psi_0|H|\psi_0\rangle$
$\langle M^z\rangle_{\mathrm{eq}} := \langle M^z\rangle_{\beta_0}$

A burst is observed at $t = \tau$.

We have successfully tuned the energy shell from which $|\psi_0\rangle$ is selected.

- $|{\downarrow}\rangle^{\otimes L}$ (ground state of M^z):
  $\beta_0 = -0.2582$, $\langle M^z\rangle_{\mathrm{eq}} = 0.0295$
- $|\psi_0\rangle$ (our initial state):
  $\beta_0 = 0.0805$, $\langle M^z\rangle_{\mathrm{eq}} = -0.0078$
  $\to$ close to the target $\beta = 0.1$



$L = 40$, $\tau = 20$, $\chi = 10$, $\beta = 0.1$, $\lambda_L = 72/L^2$

Decrease in entanglement entropy (Method 2)

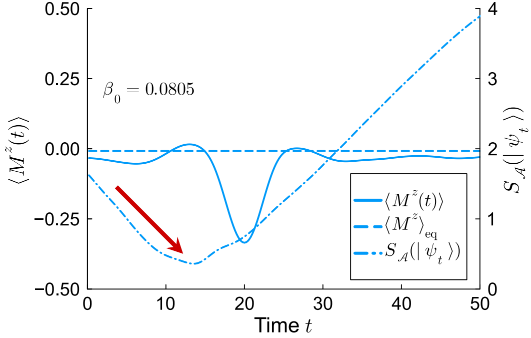
The burst is accompanied by a temporal decrease in entanglement entropy $S_{\mathcal{A}}(|\psi_t\rangle)$.

- Local information is persistent.
- A state stays out of equilibrium.

$\because$ The reduced density operator of the subsystem remains far from the Gibbs state.

cf. A typical growth of entanglement is positive and linear in time.

H. Kim and D. A. Huse, Phys. Rev. Lett. **111**, 127205 (2013).



$L = 40,\ \tau = 20,\ \chi = 10,\ \beta = 0.1,\ \lambda_L = 72/L^2$

Correlation of the initial state (Method 2)

A low-entangled state is not necessarily easy to prepare when it has a long-range correlation.
e.g. GHZ state ($\chi = 2$)

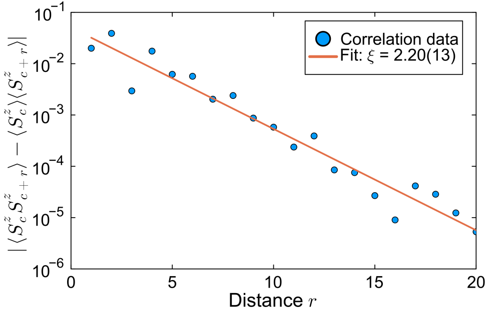
Correlation length ξ:
$|\langle S_i^z S_j^z \rangle - \langle S_i^z \rangle \langle S_j^z \rangle| \sim \exp(-|i-j|/\xi)$

The obtained initial state $|\psi_0\rangle$ has
a short correlation length $\xi = 2.20(13)$.
$\rightarrow$ Robust against local perturbations

cf. The ground state of the antiferromagnetic spin-1 Heisenberg chain: $\xi = 6.03(1)$

S. R. White and D. A. Huse, Phys. Rev. B **48**, 3844 (1993)

Experimental realization: P. Sompet et al., Nature **606**, 484 (2022).

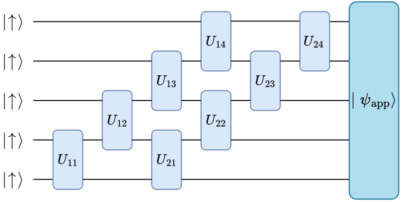


$L = 40$, $\tau = 20$, $\chi = 10$, $\beta = 0.1$, $\lambda_L = 72/L^2$,
$c = \lceil L/2 \rceil = 20$

Burst from a quantum circuit (Method 2)

We approximate the obtained MPS $|\psi_0\rangle$ by a state $|\psi_{\mathrm{app}}\rangle$ constructed by a quantum circuit with $K = \mathcal{O}(1)$ layers.

M. S. Rudolph et al., Quantum Sci. Technol. **9**, 015012 (2024).

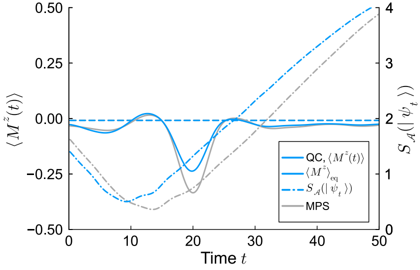


$L = 5$, $K = 2$ layers

We can still observe a burst from $|\psi_{\mathrm{app}}\rangle$ accompanied by a negative entanglement growth.

However, compared to the burst from $|\psi_0\rangle$,

- the burst is smaller.
- the decrease in entanglement is less pronounced.



$L = 40$, $K = 5$ layers

Time dependence of a burst (Method 2)

We fix the bond dimension χ.

For each (L, τ), we compute the burst amplitude $\langle O \rangle_{\text{eq}} - \langle O(\tau) \rangle$.

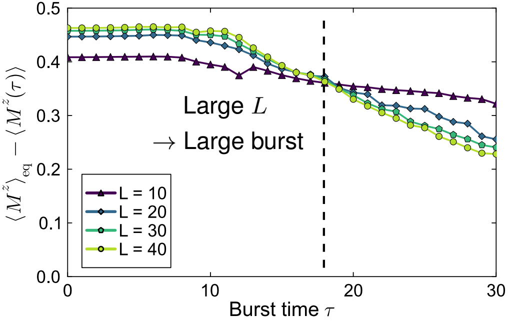
The burst amplitude remains large with increasing L. (especially for $\tau \lesssim 18$)

$\rightarrow$ **A large burst occurs at a designated short time even for a large system.**

The burst amplitude decays gradually with τ.

$\because$ Scrambling due to the unitary $e^{-iH\tau}$ becomes stronger.

T. Zhou and D. J. Luitz, Phys. Rev. B **95**, 094206 (2017).



$\chi = 10,\ \beta = 0.1,\ \lambda_L = 72/L^2$

Time dependence of a burst in the thermodynamic limit (Method 1)

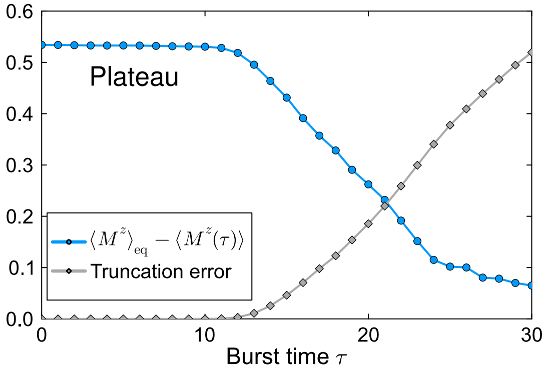
Recap. of Method 1:
Compute $e^{iH\tau}\,|{\downarrow}\rangle^{\otimes L}$ and truncate it to χ.

A burst is realized at short times even in the thermodynamic limit.

Plateau behavior for $\tau \lesssim 11$:
$e^{iH\tau}\,|{\downarrow}\rangle^{\otimes L}$ can be well approximated by an MPS with $\chi = 10$.

The burst amplitude decays gradually with τ.

Behavior similar to the finite-size analysis



$L \to \infty$, $\chi = 10$

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Time evolution by a local random circuit

Now, we provide a general, analytical argument for the time dependence of a burst.

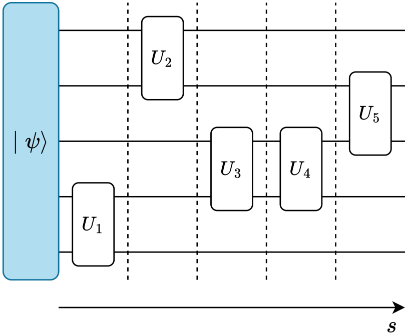
- We consider the time evolution by a **local random quantum circuit**.

F. G. S. L. Brandão, A. W. Harrow, and M. Horodecki, Commun. Math. Phys. **346**, 397 (2016).

d: local Hilbert space dimension (e.g., $d = 2$ for a qubit), s: number of gates

Random unitary U following a measure $\nu_{d,L}^{*s}$:
In the i-th step $(i = 1, 2, \ldots, s)$,

1. Randomly choose a pair of neighboring sites.
2. Apply a 2-site unitary U_i drawn from the Haar measure on the pair.



Evaluation of a burst amplitude

$\mathcal{M}_\chi$: set of MPSs with the bond dimension $\leq \chi$
The probability of a burst $\geq \Delta_O a$:

$$P_{\chi,a} := \mathrm{Pr}_{U\sim\nu_{d,L}^{*s}}\left[\sup_{|\psi\rangle\in\mathcal{M}_\chi}\left|\langle\psi|U^\dagger O U|\psi\rangle - \frac{\mathrm{Tr}[O]}{d^L}\right| \geq \Delta_O a\right].$$

Its logarithm is approximately upper bounded as

$$\ln P_{\chi,a} \lesssim -m\ln\left(md^L\right) + 2(dL\chi^2+m)\ln\frac{dL\chi^2+m}{a},$$
$$\mathbb{Z}_{\geq 0} \ni m \leq \frac{k}{2},\ k = \left\lfloor\left(\frac{s}{CL^2d^2\ln d}\right)^{1/11}\right\rfloor.$$

C ($\simeq 2.1\times 10^6$): constant

Evaluation of a burst amplitude (cont'd)

$$\ln P_{\chi,a} \lesssim -m\ln\left(md^L\right) + 2(dL\chi^2+m)\ln\frac{dL\chi^2+m}{a}$$

$$\mathbb{Z}_{\geq 0} \ni m \leq \frac{k}{2},\ k = \left\lfloor \left(\frac{s}{CL^2d^2\ln d}\right)^{1/11} \right\rfloor$$

We set $s \propto L\tau$ to mimic the time evolution. $\rightarrow k \simeq C'(\tau/L)^{1/11}$ for a large τ

τ	m	Leading term of RHS	Upper bound of $P_{\chi,a}$
$\tau \ll L(\ln L)^{11}$	$\lesssim \frac{C'}{2}\left(\frac{\tau}{L}\right)^{1/11}$	$2dL\chi^2\ln L$	No decay
$L(\ln L)^{11} \ll \tau \ll Ld^{11L}$	$\simeq \frac{C'}{2}\left(\frac{\tau}{L}\right)^{1/11}$	$-\frac{C'}{2}L^{10/11}\tau^{1/11}\ln d$	Exp. decay with $L^{10/11}\tau^{1/11}$
$Ld^{11L} \ll \tau$	$\simeq \frac{d^La^2}{e}$	$-\frac{d^La^2}{e}$	Double-exp. decay with L

$\rightarrow$ **A large burst is probabilistically rare at long times.**

Proof ideas

Concentration of measure for a local random circuit

R. A. Low, Proc. Math. Phys. Eng. Sci. **465**, 3289 (2009);

F. G. S. L. Brandão, A. W. Harrow, and M. Horodecki, Commun. Math. Phys. **346**, 397 (2016).

For a fixed state $|\psi\rangle$, the following holds:

$$\mathrm{Pr}_{U\sim\nu_{d,L}^{*s}}\left[\left|\langle\psi|U^\dagger O U|\psi\rangle - \frac{\mathrm{Tr}[O]}{d^L}\right| \geq \Delta_O a\right] \leq 2\left(\frac{m}{d^L a^2}\right)^m,\ \mathbb{Z}_{\geq 0} \ni m \leq \frac{k}{2},\ k = \left\lfloor\left(\frac{s}{CL^2 d^2 \ln d}\right)^{1/11}\right\rfloor.$$

ϵ-net of MPSs

There exists an ϵ-net $\mathcal{N}_{\chi,\epsilon} \subset \mathcal{M}_\chi$ of MPSs that satisfies $|\mathcal{N}_{\chi,\epsilon}| \leq \left(3L\sqrt{\chi}/\epsilon\right)^{2dL\chi^2}$, that is, for any $|\psi\rangle \in \mathcal{M}_\chi$, there exists $|\psi'\rangle \in \mathcal{N}_{\chi,\epsilon}$ s.t. $\||\psi\rangle - |\psi'\rangle\| \leq \epsilon$.

By employing the union bound over $\mathcal{N}_{\chi,\epsilon}$, we obtain the result.

Outline

Summary of results

- We have provided numerical methods to systematically construct MPSs that create a burst.
- A burst of a local observable from a low-entangled initial state can be realized for a nonintegrable local Hamiltonian at a short time.
  $\rightarrow$ Realizable in experiments
- A burst is large at short times but decays gradually with time, eventually becoming rare at long times.

Implication for thermalization

Even if a local observable

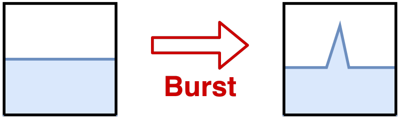
- satisfies the ETH,
- and initially relaxes near its thermal equilibrium value,

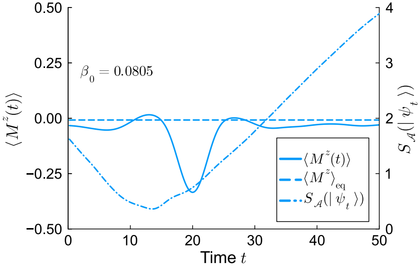
it can subsequently exhibit

- a large deviation from the equilibrium value,
- accompanied by a temporal decrease in entanglement entropy.

We have shown that such anomalous behavior

- arises from low-entangled states,
- and can be constructed systematically.





Implication for thermalization (cont’d)

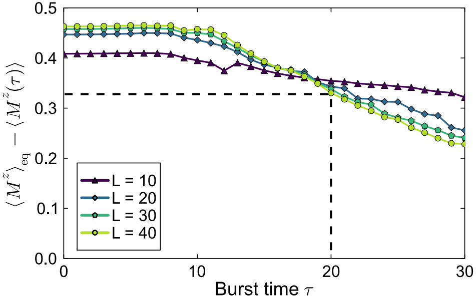
Many previous studies argue that a local observable spends “most of the time” near its thermal equilibrium.

A. J. Short and T. Farrelly, New J. Phys. **14**, 013063 (2011);

T. Mori et al., J. Phys. B At. Mol. Opt. Phys. **51**, 112001 (2018).

On the other hand, we investigate the fundamental limits of a deviation from equilibrium at a specific time.

e.g. **No** MPS with $\chi = 10$ around $\beta = 0.1$ can create a burst larger than 0.35 at $t = 20$ ($L = 40$).



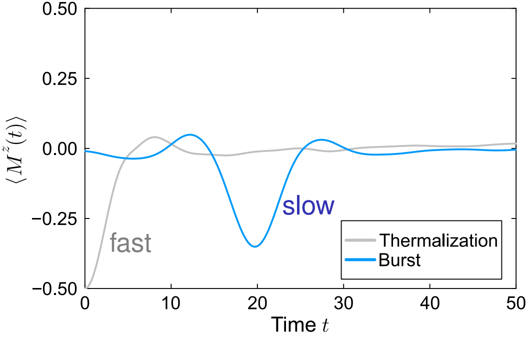
$\chi = 10,\ \beta = 0.1,\ \lambda_L = 72/L^2$

Implication for quantum Mpemba effect

Mpemba effect:
A system initially away from equilibrium relaxes faster than one that is closer to equilibrium.

E. B. Mpemba and D. G. Osborne, Phys. Educ. **4**, 172 (1969).

A burst can be interpreted as the latter case of the quantum Mpemba effect:
An initially near-equilibrium state later exhibits a large deviation, thereby thermalizing slowly.



Outlook

- Dependence of a burst on the system
  Relation to integrability, long-range interactions, etc.

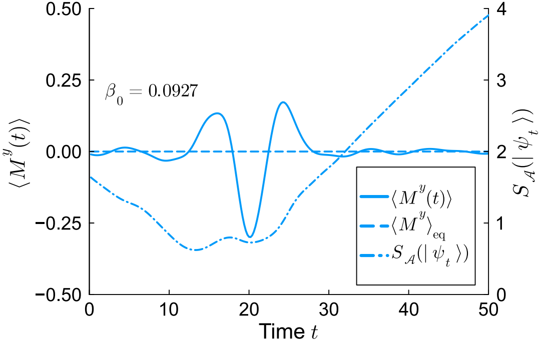
- Method 2 can be extended to find nonequilibrium trajectories other than a burst by modifying the cost function.
  $\rightarrow$ What other anomalous behaviors can be realized from low-entangled states?

- Application to quantum sensors
  One could estimate system parameters by comparing theoretical predictions with experiemental data of a burst.

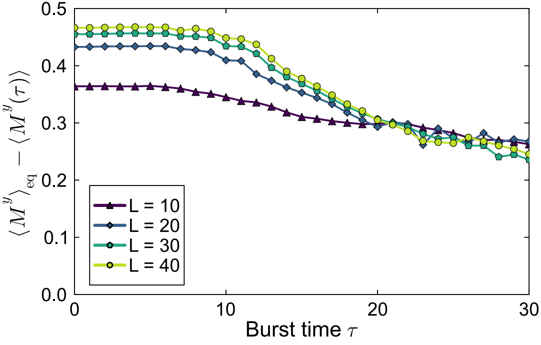
arXiv:2602.09665

Burst of M^y

$$H = \sum_{i=1}^{L-1} J_z S_i^z S_{i+1}^z + \sum_{i=1}^{L} (h_x S_i^x + h_z S_i^z), \quad M^y = L^{-1} \sum_{i=1}^{L} S_i^y$$



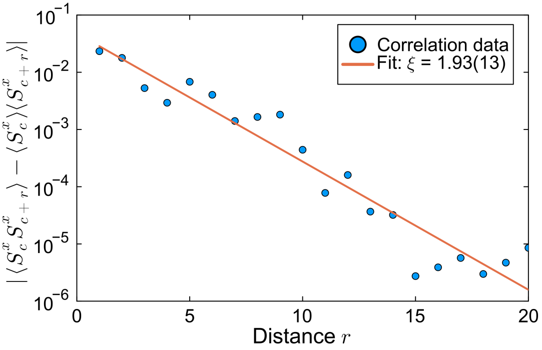
$L = 40$, $\tau = 20$, $\chi = 10$, $\beta = 0.1$, $\lambda_L = 72/L^2$



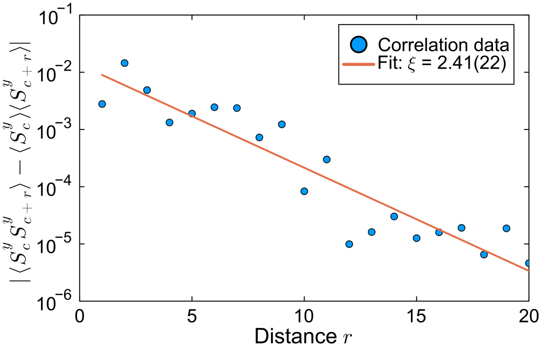
$\chi = 10$, $\beta = 0.1$, $\lambda_L = 72/L^2$

Correlation of the initial state along other axes

Correlation length ξ: $|\langle S_i^\sigma S_j^\sigma \rangle - \langle S_i^\sigma \rangle \langle S_j^\sigma \rangle| \sim \exp(-|i-j|/\xi)$, $\sigma = x, y, z$



$\sigma = x$, $\xi = 1.93(13)$



$\sigma = y$, $\xi = 2.41(22)$

Burst in integrable systems

$H = \sum_{i=1}^{L-1} J_z S_i^z S_{i+1}^z + \sum_{i=1}^{L} h_x S_i^x, \quad M^z = L^{-1} \sum_{i=1}^{L} S_i^z$