Quantum Error Correction & Lattice Gauge theory

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based on collaboration with ref.: 2604.06087 [quant-ph]

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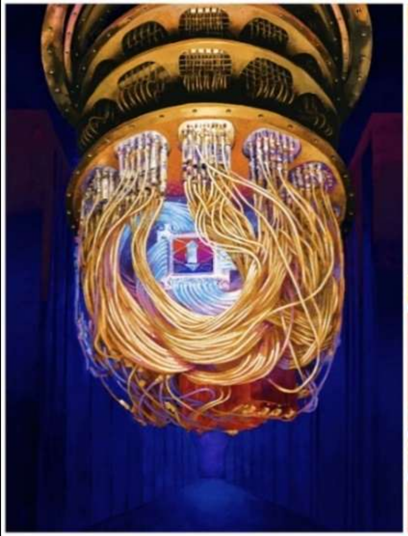
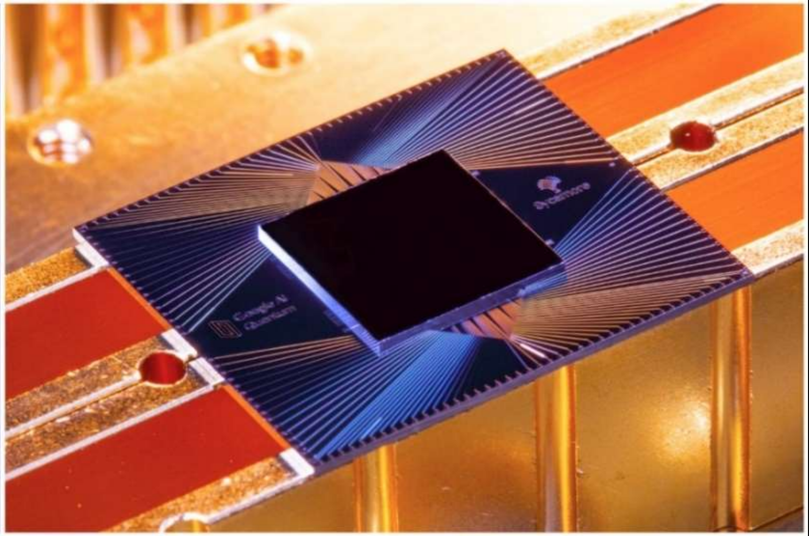
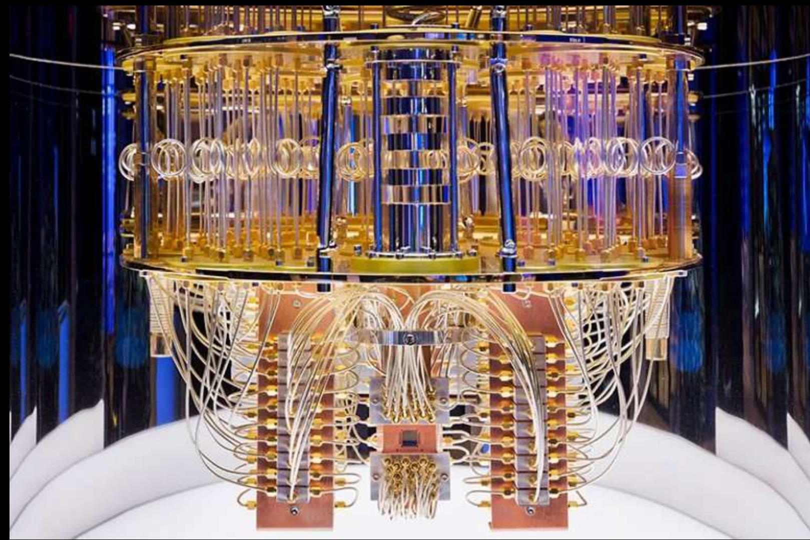
and Philipp A. Hoehn (OIST)

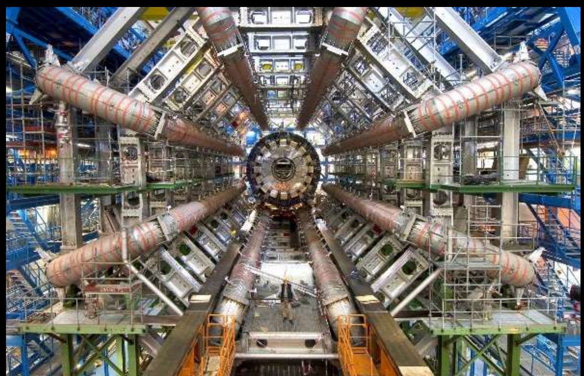
4th Aug. 2026 基研研究会: 量子基礎・量子情報の新展開

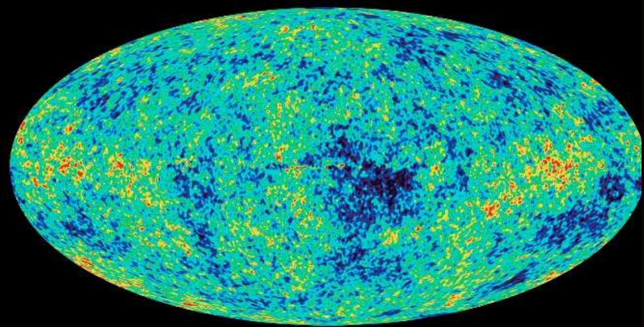
Quantum simulation in future?

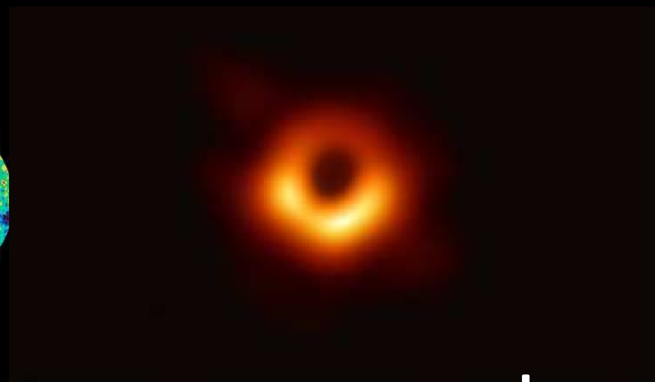












etc…

Quantum simulation is a promising approach if $^{\exists}$much computational resource in future

Challenges:

- to get sufficient # of qubits to implement quantum error correction (QEC)
- to identify efficient ways to put gauge theory on quantum computers

This talk:

Relations between QEC & gauge theory

relations between QEC & gauge theory

Motivations

[Spirit may be similar to Rajput-Roggaro-Wiebe '21, Gustafson-Lamm '23, etc...]

(some points elaborated later)

1. $^\exists$explicit examples

   ex.) Toric code = $\boldsymbol{Z_2}$ lattice gauge theory [Kitaev '97]

2.

3.

4.

relations between QEC & gauge theory

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2. Conceptual similarities:

$$\begin{cases} \text{QEC} = \text{redundant description of logical qubits} \\ \text{Gauge theory} = \text{redundant description of physical states} \end{cases}$$

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3. Nature = Gauge theory & Nature = Quantum computer

   $\Longrightarrow$ Gauge theory may know something on QEC?

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4. $^\exists$proposals on relations among QEC & concepts in HEP

   ex.) Holography, Black hole, CFT, Renormalization group

   [Almheiri-Dong-Harlow ’14, Hayden-Preskill ’07, Dymarsky-Shapere ’20, Kawabata-Nishioka-Okuda ’22, Furuya-Lashkari-Moosa ’21, etc...]

Main results

[Lacambra-Chatwin-Davies-MH-Hoehn '26]

- Generic Abelian lattice gauge theory is interpreted as error correcting code
- constructed two types of codes

  — Gauss law code: stabilizer = Gauss law

  — Vacuum code: stabilizers

  (toric code∈) = Gauss law + matter "charge"

  (restriction to superselection sector of conserved quantity)

- Cases w/ matter can be also interpreted as subsystem codes
- code properties are comprehensively explained by quantum reference frame (briefly here)

Dictionary for Gauss law code (vacuum code)

QEC	Gauge theory
errors	unphysical op. (& excitation)
logical qubits	physical states (+ fixed charge)
"no error conditions" (stabilizer)	Gauss law (& fixed charge)
logical op.	gauge invariant op.
⋮	⋮

Contents

Quantum Error Correction (stabilizer formalism)

1.Encoding

$$|\psi\rangle \in \mathcal{H} \quad \longrightarrow \quad |\psi_E\rangle \in \mathcal{H}_E \qquad (\mathcal{H} \subset \mathcal{H}_E)$$

2. Error detection

Take set of operators $\{O_1, \cdots\}$ s.t.

$$O_i|\psi_E\rangle = |\psi_E\rangle, \quad O_i(\mathrm{error})|\psi_E\rangle \neq (\mathrm{error})|\psi_E\rangle$$

Then find eigenvalues of O_i's using ancillary qubits

3. Error recovery

Act "inverse of error" based on the eigenvalues

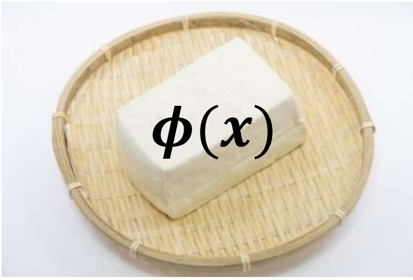
Lattice field theory

Quantum field theory (QFT): difficult to analyze

(even difficult to define)

⇨ Numerical analysis after regularizing it by finite d.o.f. (then taking limit to come back)

Lattice field theory: putting fields on lattice



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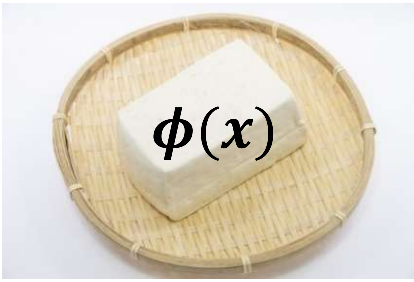


➡

Path integral formalism

$$\int D\phi\, e^{-S[\phi(x)]} \quad \Rightarrow \quad \int d\phi\, e^{-S[\phi(x_n)]}$$

∞-dim. ➡ finite-dim.

Operator formalism (typically)

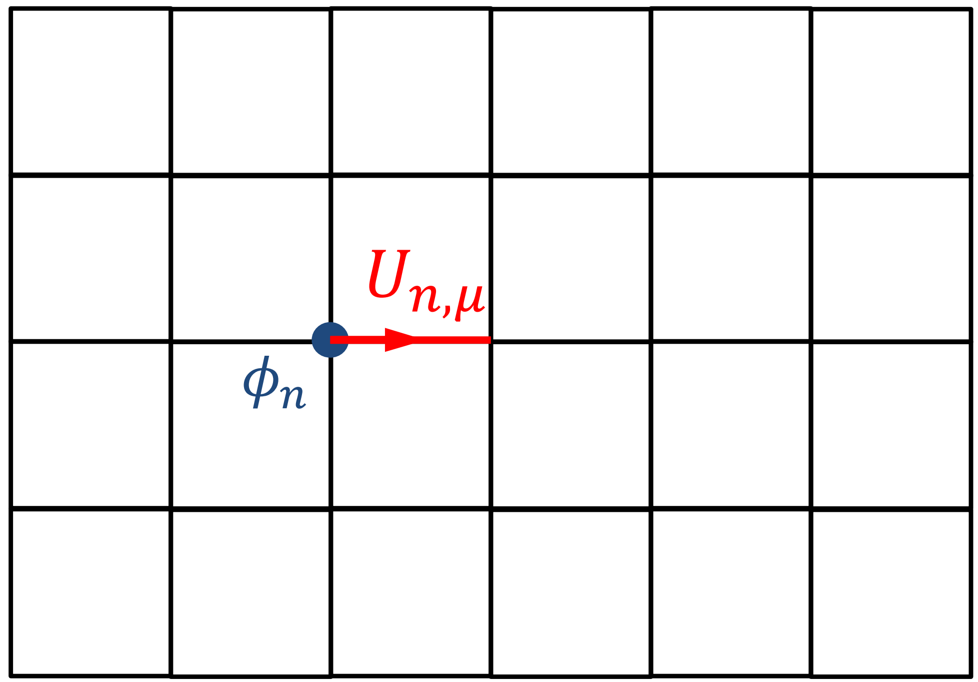
dim($\mathcal{H}$)=(uncountably ∞) ➡ dim($\mathcal{H}$)=(countably ∞) or (finite)

Lattice gauge theory = Gauge theory on lattice

[Wilson '74]

- matter fields on sites
- "gauge fields" on links

$(U \sim e^{iA})$



Physical states are subject to lattice version of Gauss law:

$$"\boldsymbol{\nabla} \cdot \widehat{\boldsymbol{E}}(x)"|\mathrm{phys}\rangle = \hat{\rho}(x)|\mathrm{phys}\rangle$$

Conceptual similarity?

Quantum error correction:

description of logical qubits by more qubits

Ex.) 3-qubit bit flip code

$$c_0|0\rangle + c_1|1\rangle \longrightarrow c_0|000\rangle + c_1|111\rangle$$

Gauge theory:

Conceptual similarity?

Quantum error correction:

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Ex.) 3-qubit bit flip code

$$c_0|0\rangle + c_1|1\rangle \longrightarrow c_0|000\rangle + c_1|111\rangle$$

Gauge theory:

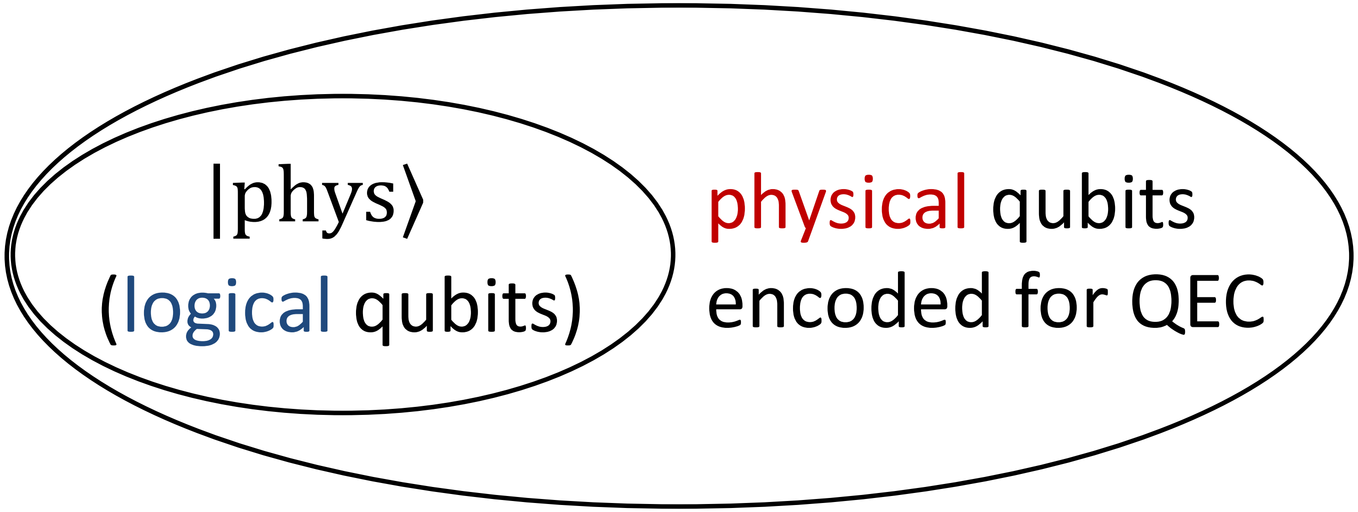
description of physical states by larger state space

Ex.) $U(1)$ gauge theory + matters

$$\boldsymbol{\nabla} \cdot \widehat{\boldsymbol{E}}(x)|\mathrm{phys}\rangle = \hat{\rho}(x)|\mathrm{phys}\rangle$$ *"Gauss law"*

Gauge theory on QC w/ error correction

Case 1: When we solve Gauss law before simulation



Case 2: When we don’t solve Gauss law before simulation

Gauge theory on QC w/ error correction

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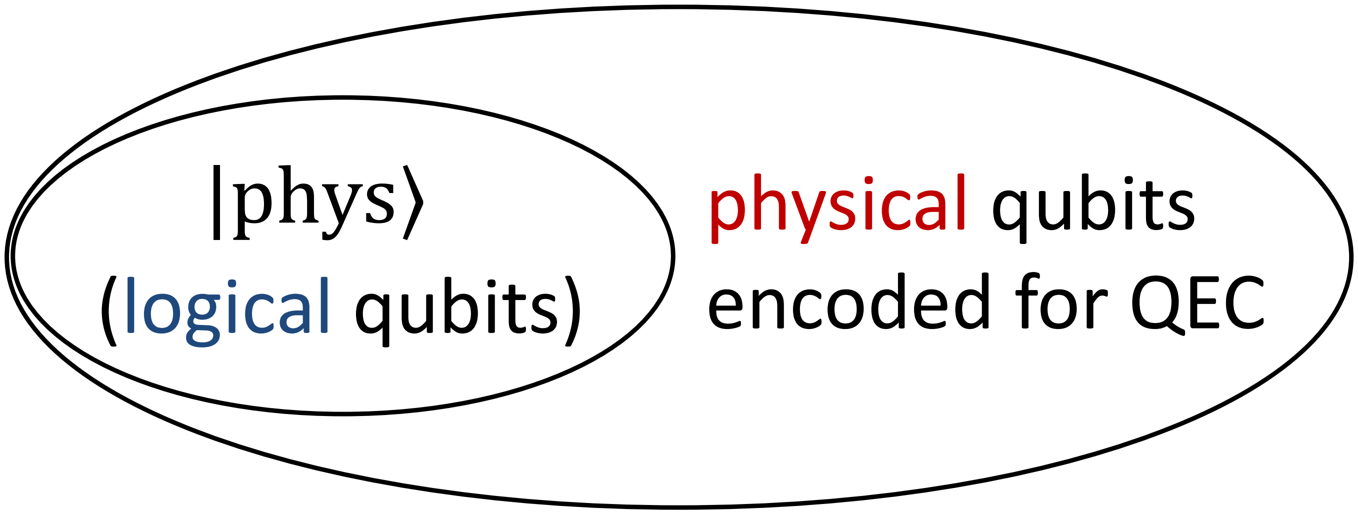

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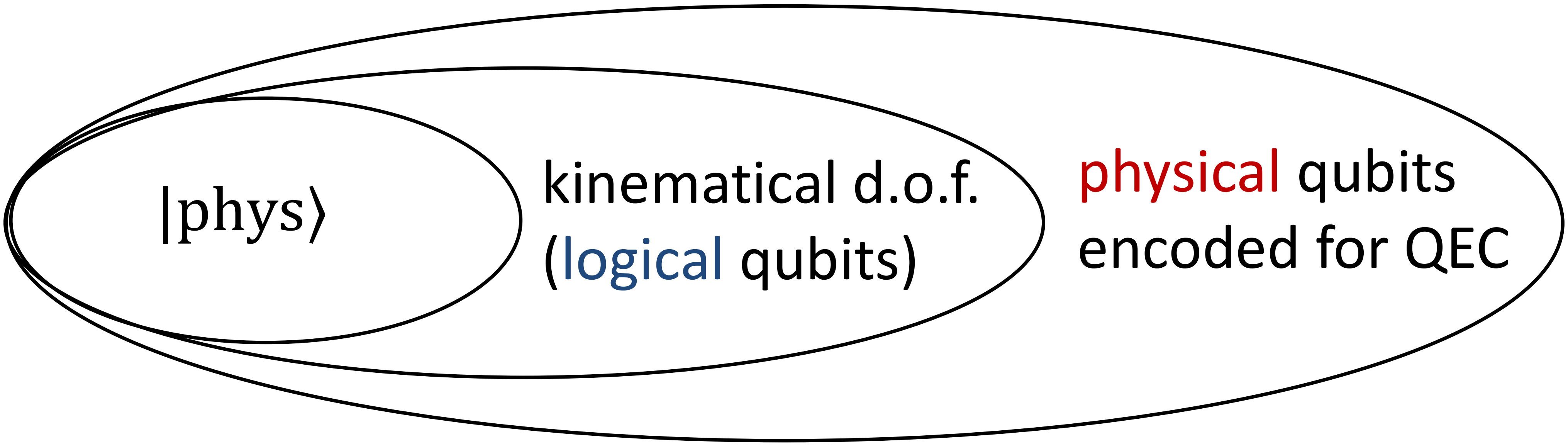


Gauge theory on QC w/ error correction

Case 1: When we solve Gauss law before simulation

$|\text{phys}\rangle$
(logical qubits)

physical qubits
encoded for QEC

redundancy^2 ‼

Case 2: When we don't solve Gauss law before simulation

$|\text{phys}\rangle$

kinematical d.o.f.
(logical qubits)

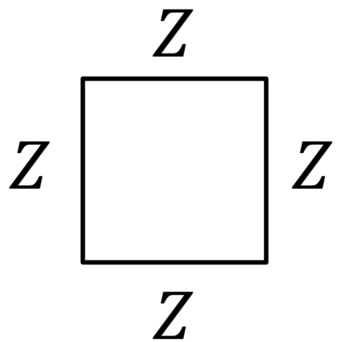
physical qubits
encoded for QEC

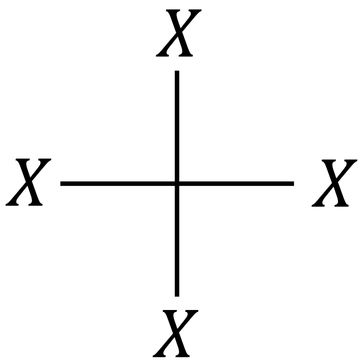
Ex.0) Toric code

[Kitaev '97]

Consider 2d periodic square lattice and put qubits on edges

$$H = -J \sum_{\text{face}} \prod_{e \in \partial(\text{face})} Z_e - J \sum_{\text{vertex}} \prod_{e | \partial e = \text{vertex}} X_e$$





"No error" condition = minimum energy condition:

$$\prod_{e \in \partial(\text{face})} Z_e |\psi_E\rangle = |\psi_E\rangle, \quad \prod_{e | \partial e = \text{v}} X_e |\psi_E\rangle = |\psi_E\rangle$$

$\Rightarrow$ logical op. = products of X, Z along nontrivial cycles

Ex.0) Toric code (cont'd)

$\boldsymbol{Z_2}$ gauge theory on 2d square lattice: $(U \sim e^{iA}, \Pi \sim e^{iE} \in \boldsymbol{Z_2})$

$$H = g^2 \sum_e \Pi_e - J \sum_{\text{face}} \prod_{e \in \partial(\text{face})} U_e$$

<u>Gauss law:</u> $(\Pi_e U_{e'} \Pi_e^\dagger = -\delta_{ee'} U_e)$

$$\prod_{e | \partial e = \text{verte}} \Pi_e \, |\text{phys}\rangle = |\text{phys}\rangle$$

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$$\prod_{e|\partial e = \text{vertex}} \Pi_e \, |\text{phys}\rangle = |\text{phys}\rangle$$

Ground state for $g = 0$:

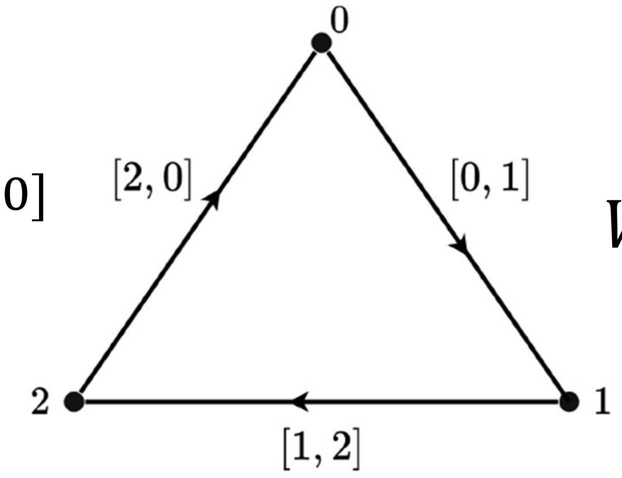
$$\prod\nolimits_{e|\partial e = \text{vert}} U_e \, |\text{ground}\rangle = |\text{ground}\rangle$$

In identification (U-basis)∼(computational basis),
this is the same condition as the toric code

Ex.1) $\boldsymbol{Z_2}$ lattice gauge theory on 3 sites

$(W \sim e^{iA}, \Pi \sim e^{iE} \in \boldsymbol{Z_2})$

$W_{[2,0]}, \Pi_{[2,0]}$



$W_{[0,1]}, \Pi_{[0,1]}$

$W_{[1,2]}, \Pi_{[1,2]}$

Hamiltonian:

$(\Pi_{[m,m+1]} W_{[n,n+1]} \Pi^{\dagger}_{[m,m+1]} = -\delta_{mn} W_{[n,n+1]})$

$$H = -J \sum_{n=0}^{2} \left(\Pi_{[n,+1]} + \Pi^{\dagger}_{[n,n+1]} \right)$$

Gauss law:

$$\Pi_{[n-1,n]} \Pi^{\dagger}_{[n,n+1]} |\mathrm{phys}\rangle = |\mathrm{phys}\rangle \quad (n = 0,1,2)$$

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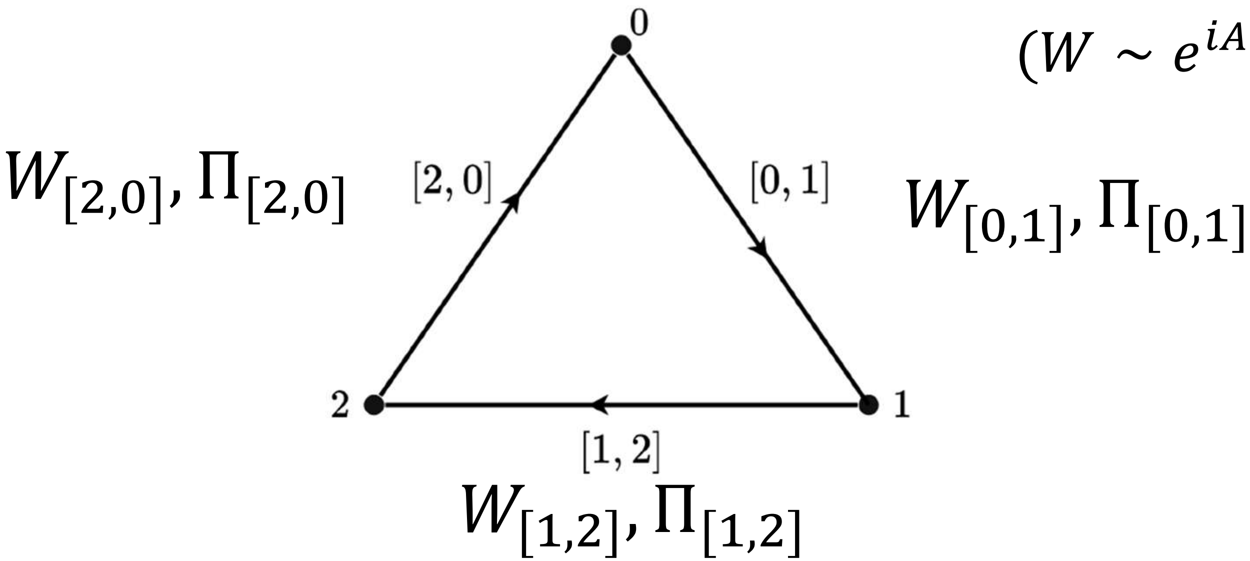


$$\boxed{\Pi_{[n-1,n]}\Pi^{\dagger}_{[n,n+1]}|\mathrm{phys}\rangle = |\mathrm{phys}\rangle}$$

Taking (computational basis) ~ (eigenstate of $\Pi_{[n,n+1]}$)

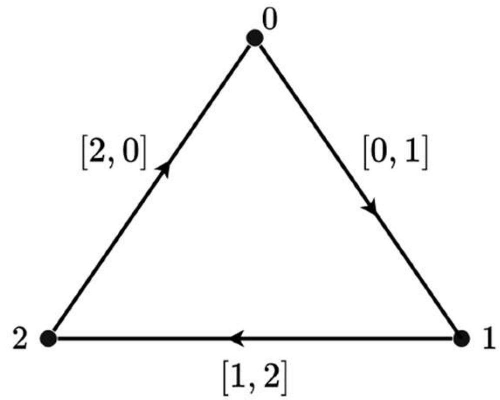
$$\Downarrow$$

$$Z_{[0,1]}Z_{[1,2]}|\mathrm{phys}\rangle = |\mathrm{phys}\rangle\,, \quad Z_{[1,2]}Z_{[2,0]}|\mathrm{phys}\rangle = |\mathrm{phys}\rangle$$

Stabilizer condition in 3-qubit bit flip code!

(correctable error set: $\{X_{[0,1]}, X_{[1,2]}, X_{[2,0]}\}$, logical $X = X_{[0,1]}X_{[1,2]}X_{[2,0]}$)

Quantum reference frame viewpoint



Gauge symmetry generators:

$$G_n := Z_{[n-1,n]} Z_{[n,n+1]} \quad (n = 0,1,2)$$

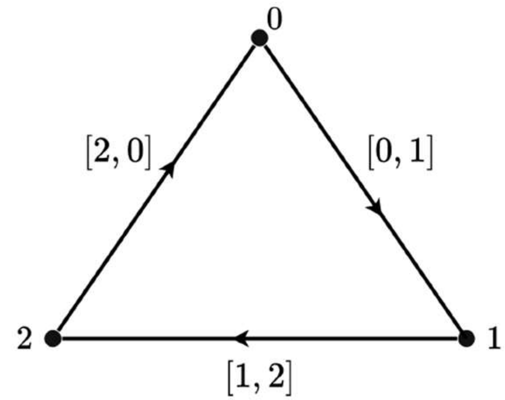
2 independent d.o.f.

→ choose 2 links as a reference frame (say $[0,1]\&[1,2]$)

Splitting into reference frame & system:

$$\mathcal{H}_{\mathrm{kin}} = \mathcal{H}_R \otimes \mathcal{H}_S \quad (R = [0,1]\&[1,2], S = [2,0])$$

Quantum reference frame viewpoint



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Reorganization in terms of frame & gauge inv. (logical) info:

$$\mathcal{H}_{\text{kin}} = \bigoplus_{\boldsymbol{q}} \mathcal{H}_{\boldsymbol{q}}$$

$$\mathcal{H}_{\boldsymbol{q}} = \{\prod_{\ell \in R} (X_\ell)^{q_\ell} |\psi\rangle_L \mid |\psi\rangle_L \in \mathcal{H}_L\}, \quad \mathcal{H}_L = \mathrm{span}\{|000\rangle, |111\rangle\}$$

Quantum reference frame viewpoint (cont'd)

$$\left(\begin{array}{c} \mathcal{H}_{\text{kin}} = \bigoplus_{\boldsymbol{q}} \mathcal{H}_{\boldsymbol{q}} \\ \mathcal{H}_{\boldsymbol{q}} = \{\prod_{\ell \in R} (X_\ell)^{q_\ell} |\psi\rangle_L \mid |\psi\rangle_L \in \mathcal{H}_L\}, \quad \mathcal{H}_L = \text{span}\{|000\rangle, |111\rangle\} \end{array}\right)$$

Introducing R-dependent tensor product:

$$\otimes_R : \mathcal{H}_R \times \mathcal{H}_L \to \mathcal{H}_{\text{kin}} \quad \text{s.t. } |\boldsymbol{q}\rangle_R \otimes_R |\psi\rangle_L := \prod_{\ell \in R} (X_\ell)^{q_\ell} |\psi\rangle_L \in \mathcal{H}_{\boldsymbol{q}}$$

$$\mathcal{H}_{\text{kin}} = \mathcal{H}_R \otimes_R \mathcal{H}_L$$

Quantum reference frame viewpoint (cont'd)

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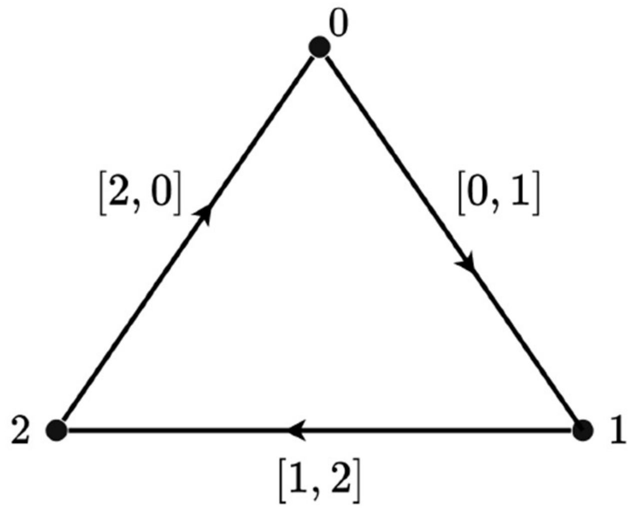
$$\mathcal{H}_{\text{kin}} = \mathcal{H}_R \otimes_R \mathcal{H}_L$$

Single bit flip error: (say $\mathrm{R} = [0,1]\&[1,2]$)

$$\left\{\begin{array}{l} X_{[0,1]}\left|q_{[0,1]}, q_{[1,2]}\right\rangle_R \otimes_R |\psi\rangle_L = \left|q_{[0,1]}+1, q_{[1,2]}\right\rangle_R \otimes_R |\psi\rangle_L \\ X_{[1,2]}\left|q_{[0,1]}, q_{[1,2]}\right\rangle_R \otimes_R |\psi\rangle_L = \left|q_{[0,1]}, q_{[1,2]}+1\right\rangle_R \otimes_R |\psi\rangle_L \\ X_{[2,0]}\left|q_{[0,1]}, q_{[1,2]}\right\rangle_R \otimes_R |\psi\rangle_L = \left|q_{[0,1]}+1, q_{[1,2]}+1\right\rangle_R \otimes_R |\psi\rangle_L \end{array}\right.$$

Correctable errors only change information on frame!

Ex.2) $\boldsymbol{Z_2}$ lattice gauge theory w/ matter



Put qubits on both sites and links
(matter) (gauge)

Gauss law:

$$Z_{[n-1,n]} Z_{[n,n+1]} |\mathrm{phys}\rangle = Z_n |\mathrm{phys}\rangle$$

"Electric field" "Charge"

One can correct an error if it is either of $\{X_n,\ X_{[n,n+1]}\}$

As a stabilizer code, this is

$$[6,3,3]_X \text{ code}$$

Ex.2) Subsystem code interpretation

$$\mathcal{H}_{\mathrm{kin}}^{(6q)} = \mathcal{H}_{\mathrm{Gauss}}^{(3q)} \otimes \mathcal{H}_{\mathrm{red}}^{(3q)}$$

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$$\mathcal{H}^{(6q)}_{\mathrm{kin}} = \mathcal{H}^{(3q)}_{\mathrm{Gauss}} \otimes \mathcal{H}^{(3q)}_{\mathrm{red}}$$

Decompose the gauge invariant Hilbert sp. as

$$\mathcal{H}^{(3q)}_{\mathrm{Gauss}} = \mathcal{H}^{(1q)}_{\mathrm{loop}} \otimes \mathcal{H}^{(2q)}_{\mathrm{dressed}}$$

$$\begin{cases} \mathcal{H}^{(1q)}_{\mathrm{loop}}\text{: spanned by eigenstates of the loop op. } X_{[0,1]}X_{[1,2]}X_{[2,0]} \\ \mathcal{H}^{(2q)}_{\mathrm{dressed}}\text{: spanned by eigenstates of } \{\, X_0X_{[0,1]}X_1,\ X_1X_{[1,2]}X_2\} \end{cases}$$

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Decompose the gauge invariant Hilbert sp. as

$$\mathcal{H}_{\mathrm{Gauss}}^{(3q)} = \mathcal{H}_{\mathrm{loop}}^{(1q)} \otimes \mathcal{H}_{\mathrm{dressed}}^{(2q)}$$

- $\mathcal{H}_{\mathrm{loop}}^{(1q)}$: spanned by eigenstates of the loop op. $X_{[0,1]}X_{[1,2]}X_{[2,0]}$
- $\mathcal{H}_{\mathrm{dressed}}^{(2q)}$: spanned by eigenstates of $\{ X_0 X_{[0,1]} X_1,\ X_1 X_{[1,2]} X_2 \}$

If we want to protect only $\mathcal{H}_{\mathrm{dressed}}^{(2q)}$ & don't care $\mathcal{H}_{\mathrm{loop}}^{(1q)}$,

then correctable errors extended: any products of $X_{[n,n+1]}$'s

Therefore, $\mathcal{H}_{\mathrm{loop}}^{(1q)} \simeq$ "gauge subsystem"

Finally, it becomes $[6,2,1,3]_X$ subsystem code

Comments on general case

- The above analyzes can be extended to general lattice gauge theory w/ abelian gauge group

  (general matter contents, dimensions, types of lattice)

- General argument is complicated but application of quantum reference frame to lattice gauge theory is useful

  [cf. Carrozza-Chatwin-Davies-Hoehn-Mele ’25]

- When gauge group is finite, lattice gauge theory is interpreted as (generalized) CSS codes

- In one dimension, code distance depends on details

- In higher dimensions, code distance is basically 3

Summary & Outlook

Summary

[Lacambra-Chatwin-Davies-MH-Hoehn '26]

- Generic Abelian lattice gauge theory is interpreted as error correcting code
- constructed two types of codes
  - Gauss law code: stabilizer = Gauss law
  - Vacuum code: stabilizers = Gauss law + matter "charge"

    (toric code∈)

    (restriction to superselection sector of conserved quantity)
- Cases w/ matter can be also interpreted as subsystem codes
- code properties are comprehensively explained by quantum reference frame

Outlook: developing QEC from HEP viewpt.

- Relation between gauge theory & QEC (this talk)

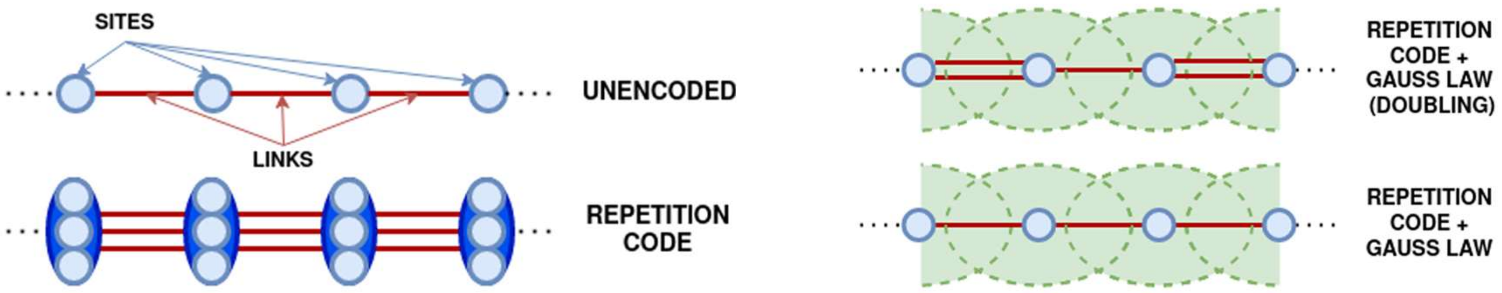
[Lacambra-Chatwin-Davies-MH-Hoehn '26]

- Improve QEC for simulation of gauge theory

[Modi-Nagano-MH-Yoshioka-Bauer '26]

[cf. Rajput-Roggero-Wiebe, Haruna, etc.]

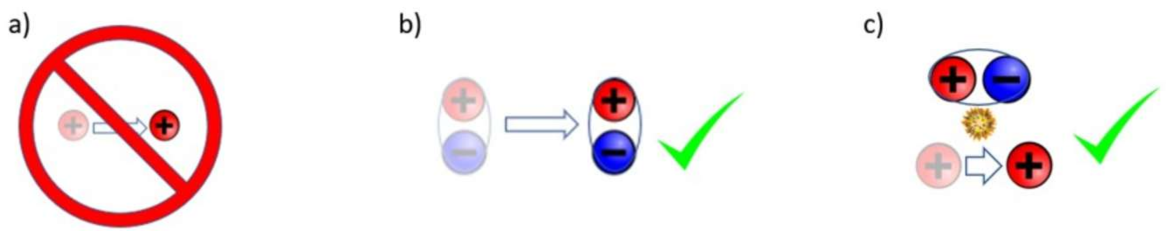
e.g.)



- Systems w/ large ground state degeneracy related to fractons (=particle w/ mobility constraints)

[cf. Haah, Pace-Wen, Ebisu-MH-Nakanishi, Ebisu-MH-Nakanishi-Shimamori, Oishi-Saito-Ebisu etc.]

e.g.)



Thanks!