Yukawa Unification in Renormalizable SU(5) GUT with Froggatt-Nielsen mechanism

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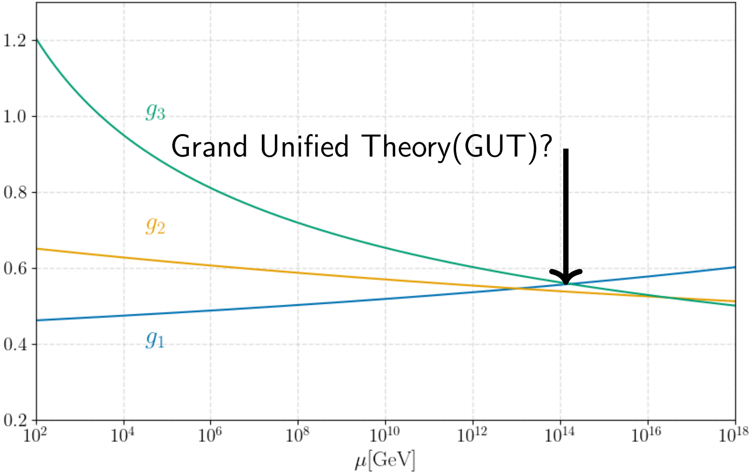
Based on "Fermion Multiplicities at the GUT Scale: A Statistical Study of Unification and Proton Decay"
arXiv:2604.08237 with Akifumi Chitose, Masahiro Ibe and Satoshi Shirai

Gauge Coupling Unification?

Standard Model (SM) : $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$



Minimal SU(5) GUT

- $\mathrm{SU}(5) \supset \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
- **Matter unification**: SM fermions embedded into $(\bar{\mathbf{5}} \oplus \mathbf{10}) \times 3$ chiral representation.

$$\bar{\mathbf{5}} = \begin{pmatrix} \overline{d} \\ \ell \end{pmatrix}, \quad \mathbf{10} = \begin{pmatrix} \overline{u} & q^T \\ -q & \overline{e} \end{pmatrix}$$

- $\mathrm{SU}(5)$ is spontaneously broken to the SM by an adjoint scalar Σ,

$$\langle \Sigma \rangle = V_{GUT} \times \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}$$

Known Issues in Minimal SU(5) GUT

Two classic problems

- **Gauge coupling non-unification**
- **Too short proton lifetime**

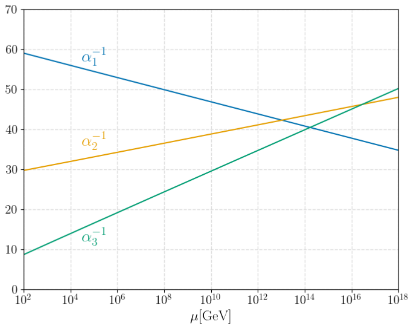
Lifetime estimate:

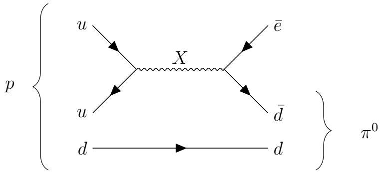
$$\tau(p \to \pi^0 + e^+) \simeq 5 \times 10^{26} \left(\frac{M_X/g_5}{10^{14}\ \mathrm{GeV}}\right)^4 \ \mathrm{year}$$

Current experimental limit (Super-Kamiokande):

$$\tau(p \to \pi^0 + e^+) \gtrsim 2.4 \times 10^{34}\ \mathrm{year}$$

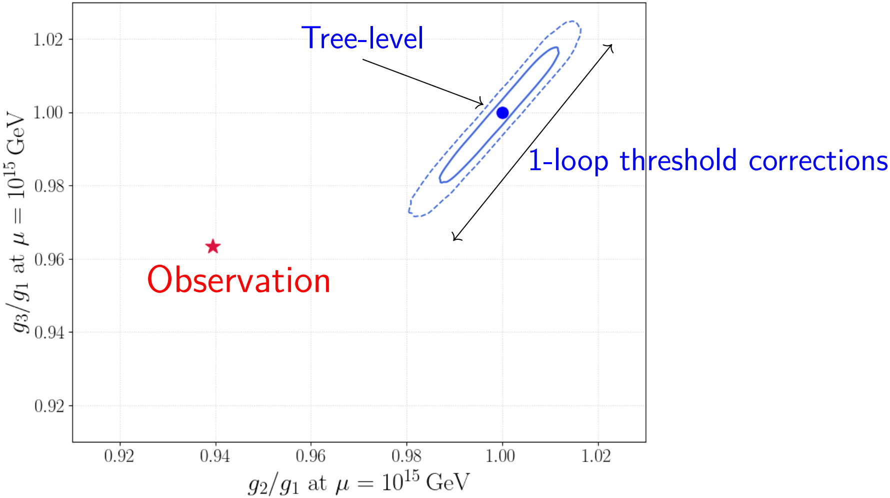
Ref.: A. Takenaka *et al.* (Super-Kamiokande), Phys. Rev. D **102** (2020) 112011





Gauge Coupling Prediction for Minimal SU(5) GUT

Free Parameters: M_X, M_{H_c}, M_Σ, g_5



Is Minimality Really Well-Motivated?

BSM Physics

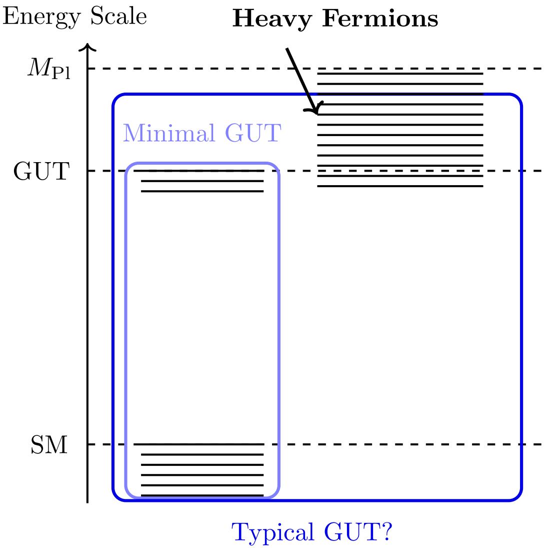
- Quantum Gravity?
- Extra Dimensions?
- Axion?
- Seesaw Mechanism?
- Flavor Models?

GUT may be connected to other BSM physics.

Minimality is not essential.

What Does a Typical GUT Look Like?

- Many additional fermions at the GUT scale?



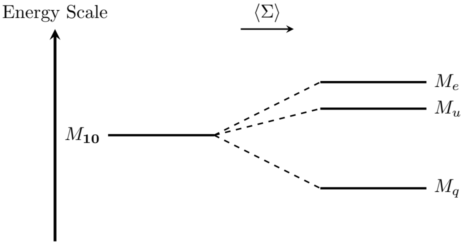
Model Setup

Matter Content (contain three chiral generations)

$$(\overline{\mathbf{5}},\, \mathbf{10}) \times 3 \;+\; (\mathbf{5},\, \overline{\mathbf{5}}) \times n \;+\; (\mathbf{10},\, \overline{\mathbf{10}}) \times n$$

Vector-like fermions have a generally complex mass matrix:

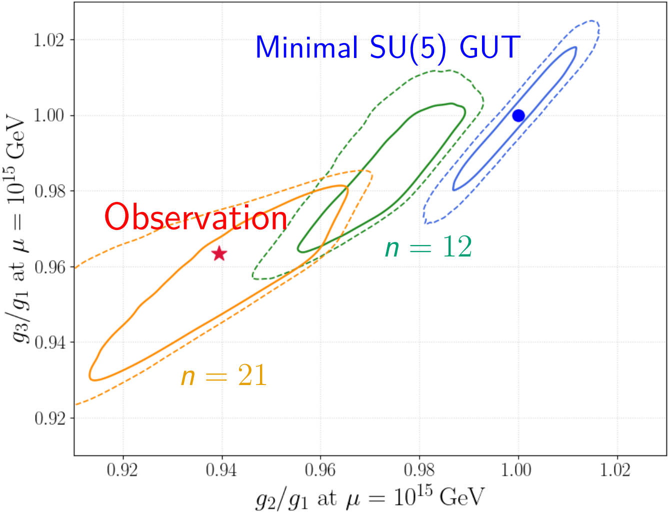
$$\mathcal{L}_M = -\mathbf{10}\,(M_{10} + Y_{10}\,\Sigma)\,\overline{\mathbf{10}} - \overline{\mathbf{5}}\,(M_5 + Y_5\,\Sigma)\,\mathbf{5} + \mathrm{h.c.}\,.$$



- After GUT breaking, the masses of GUT multiplets are split.
- Mass splittings affect the threshold corrections.

Result: Gauge Coupling Unification

Case with random $(M_{10,5})_{ij} = \mathcal{O}(1) \times M_0$, $(Y_{10,5})_{ij} = \mathcal{O}(1)$, $M_0 = 0.7 V_{\rm GUT}$



Origin of SM Fermions

Mass Term: $\mathcal{L}_M = -\mathbf{10}\,(M_{10} + Y_{10}\,\Sigma)\,\overline{\mathbf{10}} - \overline{\mathbf{5}}\,(M_5 + Y_5\,\Sigma)\,\mathbf{5} + \mathrm{h.c.}$

$\mathrm{rank}(M_{10,5} + Y_{10,5}\,\Sigma) = n \quad \Rightarrow$ Only 3 generations remain massless (SM fermions)

- SM fermions are linear combinations of multiple fermions.

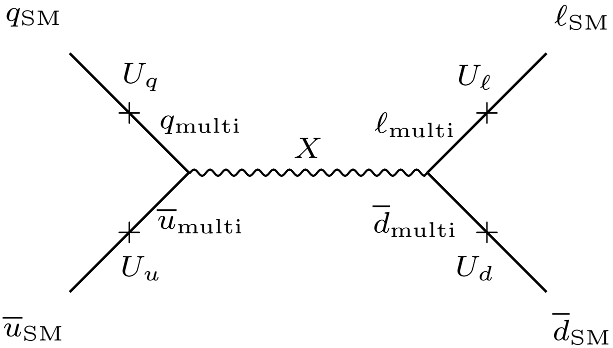
$$q_{\mathrm{SM}} = U_q\, q_{\mathrm{multi}}, \quad \overline{u}_{\mathrm{SM}} = U_u\, \overline{u}_{\mathrm{multi}}, \quad \overline{e}_{\mathrm{SM}} = U_e\, \overline{e}_{\mathrm{multi}}, \cdots$$

- After GUT breaking, the unitary matrices differ among SM fermions.

$$U_q \neq U_u \neq U_e, \quad U_d \neq U_\ell$$

Suppression of Proton Decay

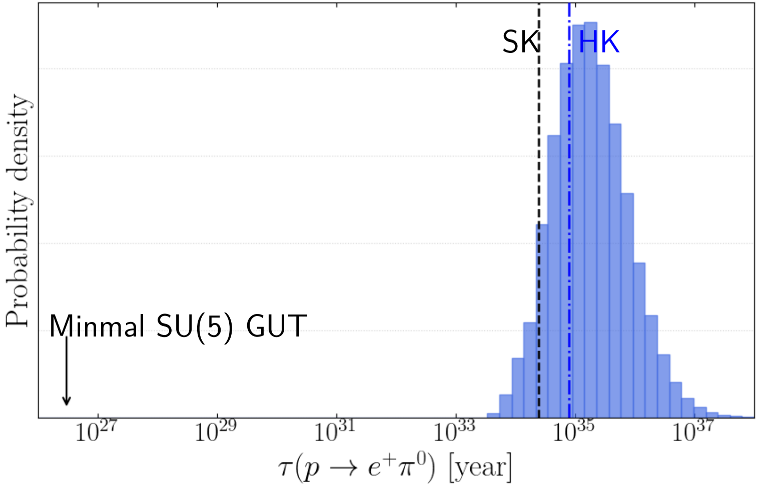
For large n,



$$\propto (U_q^\dagger U_u) \times (U_\ell^\dagger U_d) \sim \frac{1}{n}$$

For an $n \times n$ random unitary matrix, $U_{ij} = \mathcal{O}\left(\frac{1}{\sqrt{n}}\right)$, $(U^\dagger U')_{ij} = \mathcal{O}\left(\frac{1}{\sqrt{n}}\right)$

Result: Proton Lifetime Predictions



$$\text{SK}: \quad \tau(p \to \pi^0 + e^+) \gtrsim 2.4 \times 10^{34}\,\text{yr}\,, \quad \text{HK}: \quad \tau(p \to \pi^0 + e^+) \gtrsim 7.8 \times 10^{34}\,\text{yr}$$

Short Summary

- Minimality is not essential.
- Typical GUT can address the problems of minimal SU(5), including gauge unification and proton decay.

How about Yukawa?

- Similar sizes of down-type quark and lepton Yukawa couplings:

$$y_b \sim y_\tau\,, \qquad y_s \sim y_\mu\,, \qquad y_d \sim y_e$$

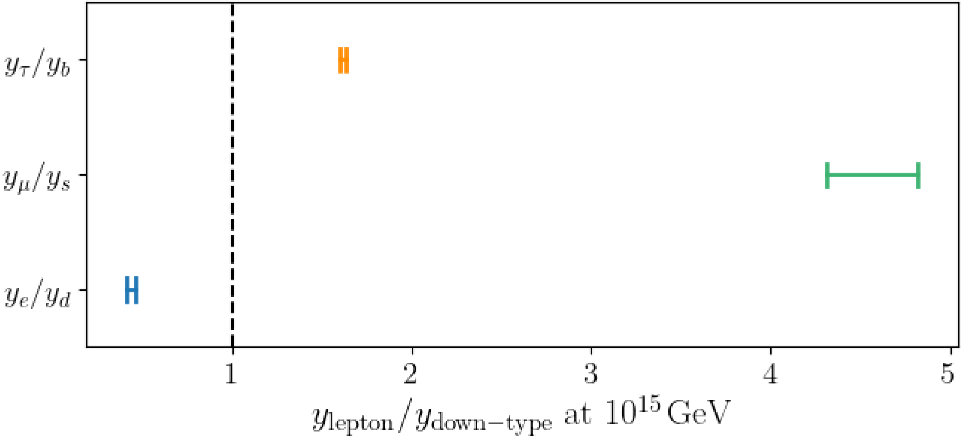
- In usual GUTs, they can originate from a common Yukawa interaction:

$$\mathcal{L}_Y = y_5\, H_5^\dagger \mathbf{10}\,\overline{\mathbf{5}} + \text{h.c.} \quad \Rightarrow \quad y_\text{down} = y_\text{lepton} = y_5$$

$$\Downarrow$$

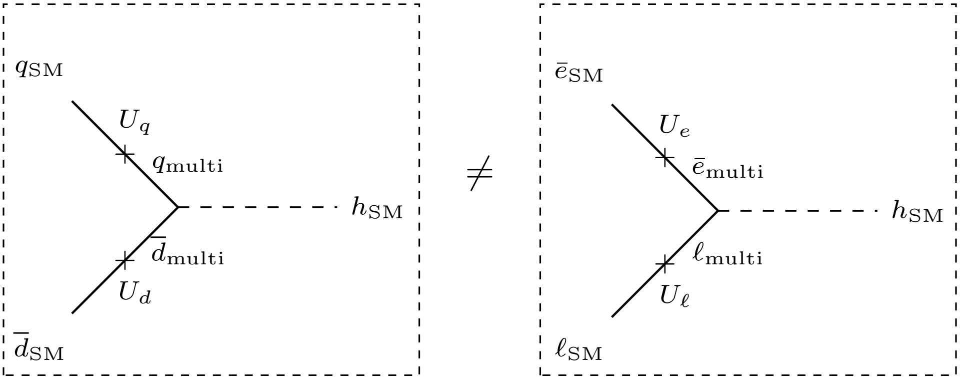
A hint of Grand Unification?

No Yukawa Unification in Minimal SU(5)

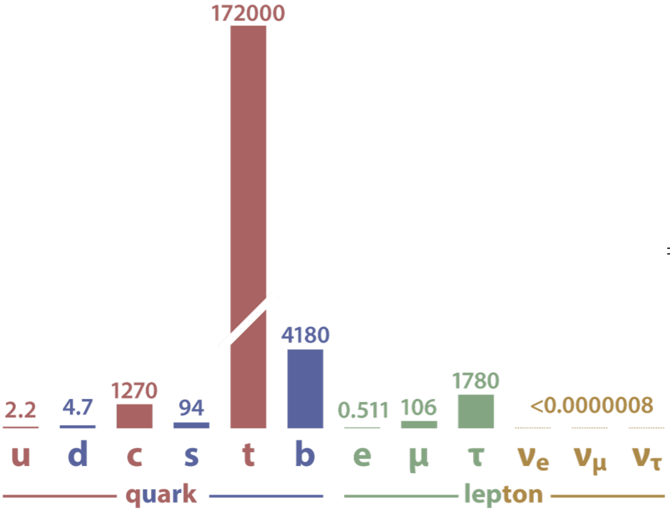


Typical GUT avoid Yukawa Unification

- SM Fermions as Linear Combinations of Multiple fermions
- Down-type quark and lepton Yukawa couplings need not be unified



Yukawa Hierarchy



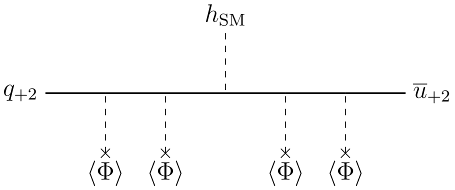
$\Rightarrow$ **Flavor Symmetry**

Froggatt-Nielsen Mechanism

Ref.: C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B **147** (1979) 277

- Global $\mathrm{U}(1)$ flavor symmetry
- SM fermions have generation-dependent $\mathrm{U}(1)$ charges (FN charges)
- $\mathrm{U}(1)_{\mathrm{FN}}$ broken by a complex scalar field Φ (FN charge $= -1$)
- Heavy fermions in the UV theory (often omitted...)
- Yukawa unification cannot be solved by FN.

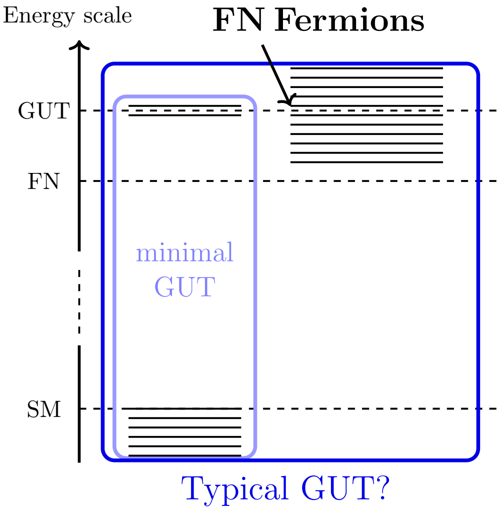
e.g. Yukawa interaction of q_{+2} and $\overline{u}_{+2}$



$$\longrightarrow \quad \mathcal{L} = \mathcal{O}(1) \times \left(\frac{\langle\Phi\rangle}{M}\right)^{(2+2)} q_{+2}\overline{u}_{+2}h_{\mathrm{SM}}$$

$$\epsilon \equiv \frac{\langle\Phi\rangle}{M} = \mathcal{O}(0.1)$$

Vector-like as FN fermions



- $\frac{\text{FN scale}}{\text{GUT scale}} \sim 0.1$
- FN fermions at GUT scale

- FN charges are assigned such that, after integrating out the heavy fermions,

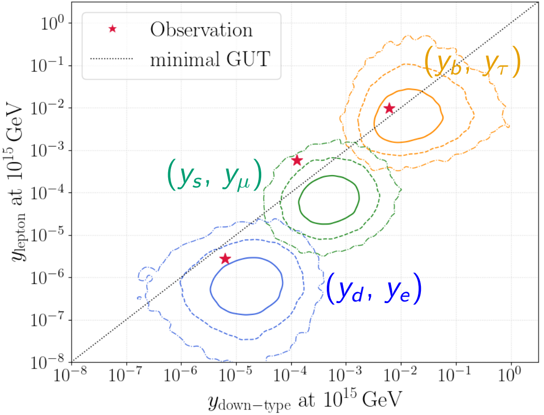
$$Q^{(\mathbf{10})} = (2, 1, 0), \qquad Q^{(\bar{\mathbf{5}})} = (2, 2, 2).$$

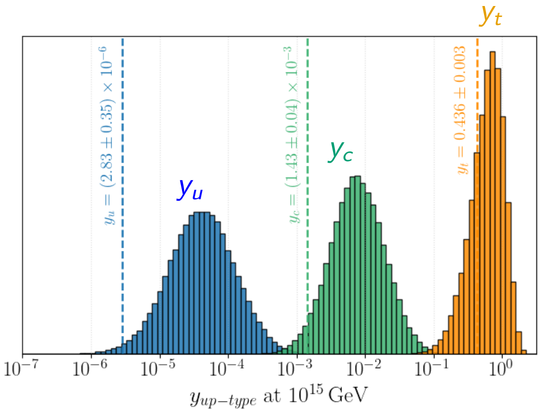
- This gives the hierarchical Yukawa textures

$$y_u \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon^1 \\ \epsilon^2 & \epsilon^1 & \epsilon^0 \end{pmatrix}, \qquad y_{d,\ell} \sim \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^4 \\ \epsilon^3 & \epsilon^3 & \epsilon^3 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \end{pmatrix}.$$

$$\epsilon = 0.06 \quad \Leftrightarrow \quad \text{Observation?}$$

Successful Yukawa Unification





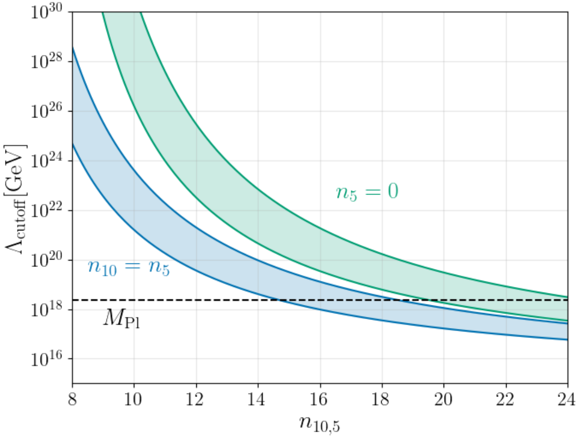
Summary

- Typical GUT model can achieve gauge coupling unification and sufficiently long proton lifetime.
- Yukawa unification (Bottom-tau problem) can be avoided.
- Heavy fermions can be interpreted as FN fermions.
- Typical GUT with the Froggatt-Nielsen mechanism can explain observed Yukawa couplings.

End

Upper Limit on the Number of Heavy Fermions

Good unification $\neq$ unlimited fermions: Landau poles set an upper bound.



- Many fermions accelerate the divergence of the coupling g_5.
- For $n_5 = n_{10} \lesssim 18$ or $n_5 = 0, n_{10} \lesssim 24$, no landau pole appears.

$g_5\left(\Lambda_{\rm cutoff}\right) = 4\pi$
$g_5(10^{15}\ \rm GeV) = 0.48\text{–}0.55$