レプトジェネシスに基づく再加熱温度の自然性評価

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When Did the Hot Universe Begin?

$$T_{\mathrm{RH}} = ?$$

- Reheating temperature determines our thermal history and subsequent matter abundances.
- However, high T_{RH} may overproduce unwanted relics.

T_{RH} : the most important parameter of the early Universe

- Can particle physics determine the range of T_{RH}?

The Mystery of the BAU

$$\text{Why is } \eta_B^{\text{obs}} \simeq 6.0 \times 10^{-10}?$$

$$\underbrace{\text{Inflation} \to T_{\text{RH}} \to \text{EW} \to \text{QCD} \to \text{BBN}}_{\text{BAU generated?}} \to \text{CMB}$$

- The SM cannot generate the observed BAU.
  (1) CP violation is too small.
  (2) Insufficient departure from equilibrium.

New Physics Is Required!

Seesaw Mechanism and Leptogenesis

Adding right-handed neutrinos can solve both problems!

$$\mathcal{L} \supset -Y_{\alpha i}\overline{L_\alpha}\widetilde{H}N_i - \frac{1}{2}M_i\overline{N_i^c}N_i + \text{h.c.}$$

(1) Explain tiny neutrino masses (Seesaw mechanism)

$$m_\nu \sim \frac{Y^2 v_{\text{EW}}^2}{M_N}$$

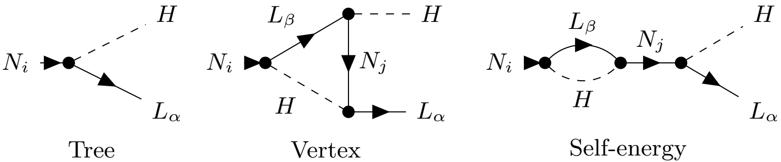
A large M_N naturally explains the tiny neutrino-mass scale.

(2) Explain the baryon asymmetry of the universe

$\Rightarrow$ CP-violating decays are the key.

Origin of CP Violation

- Quantum interference of tree and one-loop diagrams.



$$\varepsilon_1 = \frac{\Gamma(N_1 \to LH) - \Gamma(N_1 \to \overline{LH})}{\Gamma(N_1 \to \text{All})} \neq 0$$

$$\underbrace{\Delta L}_{\text{Lepton number violation}} \propto \varepsilon_1 = \mathcal{O}\left(Y^2\right)$$

Leptogenesis Scenario

Step1 $T > M_N$: N_1 neutrinos are in the thermal bath.

Step2 $T \sim M_N$: N_1 decays (CP-violation and out-of-eq)

$\Rightarrow$ Generate a lepton asymmetry

Step3 Lepton asymmetry $\Rightarrow$ Baryon asymmetry

$$\Delta L \neq 0 \Rightarrow \Delta B \neq 0$$

(Automatically converted by sphaleron processes)

Conventional Analyses Give a Lower Bound on T_{RH}

- For fixed neutrino masses, a low M_N implies small Y and ϵ.

$$T_{\mathrm{RH}} \gtrsim 10^{10}\,\mathrm{GeV}$$

is required for successful leptogenesis.

[Davidson , Ibarra 02]

However, this bound relies on simplifying assumptions.

- Strong hierarchy: $M_1 \ll M_2, M_3$
- Flavor effects are neglected.

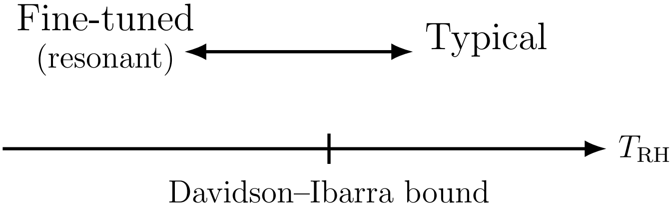
Mass Degeneracy Can Lower the Bound on T_{RH}

- When $\Delta M_N \simeq \Gamma_N/2 \ll M_{1,2}$, ε is resonantly enhanced.

  [Pilaftsis 97; Pilaftsis, Underwood 03]

- Heavy neutrino mass scale can be as low as the EW scale.

  [Pilaftsis, Underwood 05; Deppisch, Pilaftsis 10]

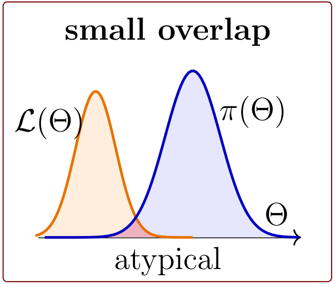


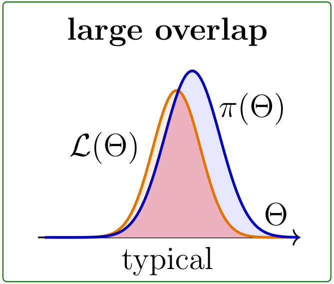
What is the natural T_{RH} range for leptogenesis?

How do we quantify naturalness?

Naturalness has no unique definition.
Here, we quantify it by the prior-likelihood overlap.

- prior($\pi(\Theta)$) : Naive theoretical expectation
- likelihood($\mathcal{L}(\Theta)$) : Observables

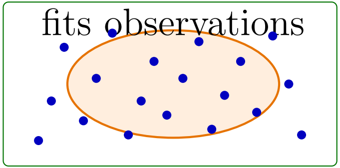




A Successful Point Is Not Necessarily Typical

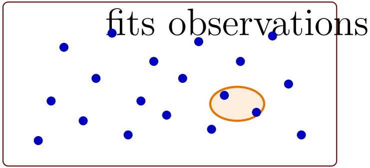
We treat the Yukawa and mass matrices as random variables.

Typical / natural



Many samples from the prior reproduce the observations.

Fine-tuned



A good fit exists, but only for rare prior samples.

A successful point can reproduce the observations.

Naturalness asks whether they are typical under the prior.

Prior: flavor anarchy

Y and M are unknown, so we treat them as random.

$$\mathcal{L} \supset -Y_{\alpha b}\overline{L_\alpha}\widetilde{H}N_{Rb} - \frac{1}{2}M_{ab}\overline{N^c_{Ra}}N_{Rb} + \text{h.c.}$$

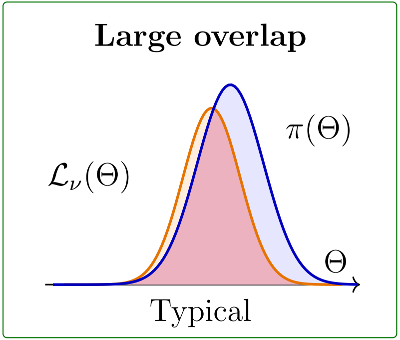
Anarchy: no preferred flavor structure at high energies.

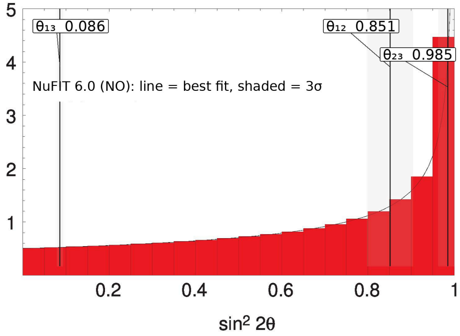
[Hall, Murayama, Weiner 00; Haba, Murayama 01; Lu,Murayama 14]

$$\Downarrow$$

$$Y_{\alpha b} \sim \mathcal{O}(1) \times \underbrace{Y_0}_{\text{Yukawa scale}}, \quad M_{ab} \sim \mathcal{O}(1) \times \underbrace{M_0}_{\text{Majorana mass scale}}$$

Why Anarchy? – A Benchmark for Typicality





[Haba, Murayama 01; NuFIT 6.1]

Oscillation data are typical in Anarchy.

$\Rightarrow$ **Anarchy is a benchmark for typicality.**

What about leptogenesis?

Likelihood: neutrino data + baryon asymmetry

The likelihood measures agreement with observations.

$$\mathcal{L}(\mathbf{x}) \propto \prod_i \exp\left[-\frac{(x_i - x_i^{\mathrm{obs}})^2}{2\sigma_i^2}\right]$$

Neutrino oscillation data	Baryon asymmetry
$\sin^2\theta_{12}, \quad \sin^2\theta_{13}, \quad \sin^2\theta_{23}$ $\delta_{\mathrm{CP}}, \quad \Delta m_{21}^2, \quad \Delta m_{31}^2$	$\eta_B^{\mathrm{obs}} \simeq 6.0 \times 10^{-10}$

Bayes Factors

- Marginal likelihood Z :

$$Z = \int \mathcal{L}(\Theta)\, \pi(\Theta)\, d\Theta$$

- The ratio of marginal likelihoods is called the Bayes factor:

$$\mathrm{BF}_{12} = \frac{Z_1}{Z_2}$$

- If $\mathrm{BF}_{12} > 1$, Model 1 is favored over Model 2.
- Jeffreys' scale: $\mathrm{BF}_{12} < 1/100$ strongly favors Model 2.

What's New?

- Neutrino Anarchy [Hall, Murayama, Weiner 00; Haba, Murayama 01]
- Anarchy + Leptogenesis [Jeong, Takahashi 12; Lu, Murayama 14]
- **Our Work**

(1) Quantify naturalness using Bayesian statistics.

(2) Include flavor effects in the Boltzmann equations.

Lepton asymmetry evolve differently for each flavor.

From Model Parameters to $Z(Y_0, M_0, T_{\mathbf{RH}})$

Step 1: Fix the Yukawa scale Y_0 and Majorana mass scale M_0.

Step 2: Randomly generate $Y \sim \mathcal{O}(1)Y_0$ and $M \sim \mathcal{O}(1)M_0$.

Step 3: Compute the observables.

Step 4: Evaluate the integral by Monte Carlo:

$$Z(Y_0, M_0, T_{\mathrm{RH}}) = \int (\text{likelihood}) \times (\text{prior})\, dY\, dM$$

From $Z(Y_0, M_0, T_{\mathbf{RH}})$ to $Z(M_0, T_{\mathbf{RH}})$

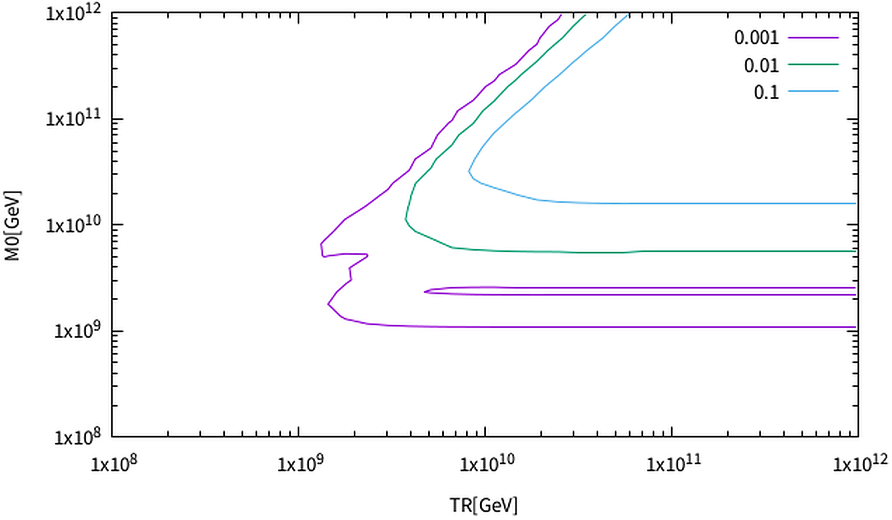
Step 5: Marginalize over Y_0 with a log-flat measure:

$$Z(M_0, T_{\rm RH}) = \int \frac{dY_0}{Y_0} \, Z(Y_0, M_0, T_{\rm RH})$$

- $Z(M_0, T_{\rm RH})$: How natural is a given $(M_0, T_{\rm RH})$?

Step 6: Plot the Bayes Factor in the $(T_{\rm RH}, M_0)$ plane.

Results I : Contour plot of Bayes Factor (preliminary)



From $Z(M_0, T_{\mathbf{RH}})$ to $Z(T_{\mathbf{RH}})$

- Marginalize over M_0 with a log-flat measure:

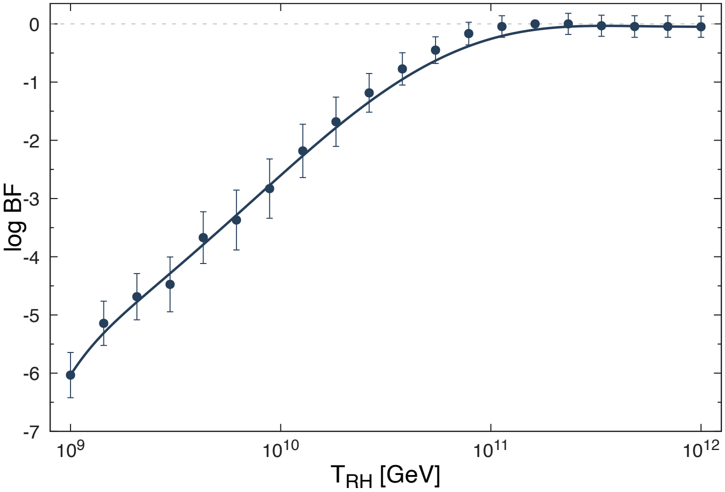
$$Z(T_{\rm RH}) = \int \frac{dM_0}{M_0} \, Z(M_0, T_{\rm RH})$$

- $Z(T_{\rm RH})$: How natural is a given reheating temperature?

Q. Why log-flat measure?

A. Assuming no preferred scale below $M_{\rm pl}$, we adopt a log-flat prior.

Results II : Plot of log Bayes Factor (preliminary)



Low Reheating Temperatures Are Disfavored

- According to Jeffreys' scale, $T_{\mathrm{RH}} \lesssim 10^{10}\,\mathrm{GeV}$ is strongly disfavored.
- For $T_{\mathrm{RH}} \gtrsim 10^{11}\,\mathrm{GeV}$, the Bayes Factor reaches a plateau.
- Can flavor effects lower T_{RH}? [Blanchet, Di Bari '09]
  $\Rightarrow$ No such tendency is observed in our analysis.
  $\Rightarrow$ Are sizable flavor effects atypical?

Low T_{RH} is atypical for leptogenesis.

Backups

Thermal leptogenesis : Part I

- Add heavy right-handed neutrinos in the SM.
- In the early universe, Hot Standard Model Plasma can make right-handed neutrinos.

Thermal leptogenesis : Part II

- When the temperature of the universe become $T \sim M$, the abundance of the right-handed neutrinos are exponentially surpressed.
- Out-of equilibrium decay can left non-zero lepton asymmetry.
- Lepton asymmetry change to Baryon asyymetry by using the spharelon process.

$$\eta_{\mathrm{B}} \approx 6.0 \times 10^{-10}$$

Time-evolution of B-L Charge : rough sketch

- Assume 10^{12} GeV $\ll M_1 \ll M_2, M_3$ and Higgs asymmetry can neglect.
- In this case, time evolution of lepton asymmetry can be approximated by integrated Boltzmann equation

$$\frac{dN_{N_1}}{dz} = -D(z)(N_{N_1} - N_{N_1}^{\mathrm{eq}})$$
$$\frac{dN_{B-L}}{dz} = -\varepsilon_1 D(z)(N_{N_1} - N_{N_1}^{\mathrm{eq}}) - W_{\mathrm{ID}}(z) N_{B-L}$$

- where

$$D(z) = Kz\frac{K_1(z)}{K_2(z)}, W_{\mathrm{ID}}(z) = \frac{1}{4}Kz^3K_1(z), N_{N_1}^{\mathrm{eq}} = \frac{3}{8}z^2K_2(z)$$

$K_n(z)$: modified Bessel functions.

- What is ε_1 and K ?

Very important parameters : ε_1 and K

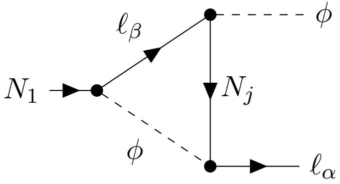
- $\mathcal{L} \supset -Y_{\alpha i}\overline{\ell}_\alpha \widetilde{\phi} N_i + \mathrm{h.c.}$
- The definition of ε_1 is

$$\varepsilon_1 := \frac{\Gamma(N_1 \to \ell\phi) - \Gamma(N_1 \to \overline{\ell\phi})}{\Gamma(N_1 \to \mathrm{ALL})} = \frac{3}{16\pi}\frac{1}{(Y^\dagger Y)_{11}}\sum_{j=2,3}\mathrm{Im}[(Y^\dagger Y)^2_{1j}]\frac{M_1}{M_j}$$

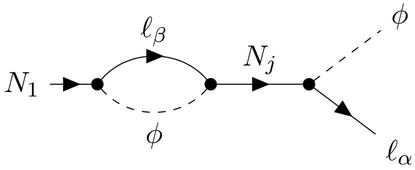
- $\varepsilon_1 \neq 0 \to$ CP is violating !
- The definition of K is

$$K := \frac{\Gamma(N_1 \to \mathrm{ALL})}{H(T = M_1)} = \frac{(Y^\dagger Y)_{11} M_1}{8\pi}\frac{M_{\mathrm{pl}}}{1.66\sqrt{g_*}M_1^2}$$

Feynman graph of this phenomena



Vertex



Self-energy

Theoretical estimation of $\eta_{\rm B}$

- In this setup, $\eta_{\rm B}$ is approximated by

$$\eta_{\rm B}^{\rm th} \approx 0.96 \times 10^{-2} \varepsilon_1 \kappa_{\rm f}$$

where

$$\kappa_{\rm f} = -\frac{4}{3} \int_{z_{\rm i}}^{\infty} dz' \frac{dN_{N_1}}{dz'} \exp\left(- \int_{z'}^{\infty} dz'' W_{\rm ID}(z'') \right)$$

- Very important bound here

$$|\varepsilon_1| \leq \frac{3}{16\pi} \frac{M_1}{v^2} (m_3 - m_1), \quad \kappa_{\rm f} \leq 1$$

- Therefore high $\eta_{\rm B}^{\rm th}$ need high M_1 and high $T_{\rm RH}$!