Decay of scalar condensation: two different approaches

Ayuki Kamada (University of Warsaw)

UNIVERSITY OF WARSAW

Based on
AK and Kodai Sakurai, JHEP 2026 and work in preparation

Aug. 25, 2026 @ PPP2026

Motivation

Scalar condensation

- bosons are in the same quantum state
- described by macroscopic wavefunction
- follows classical equation of motion (Gross–Pitaevskii equation)
- appears not only in condensed matter physics, but also in cosmology
  - inflation, phase transition, axion…

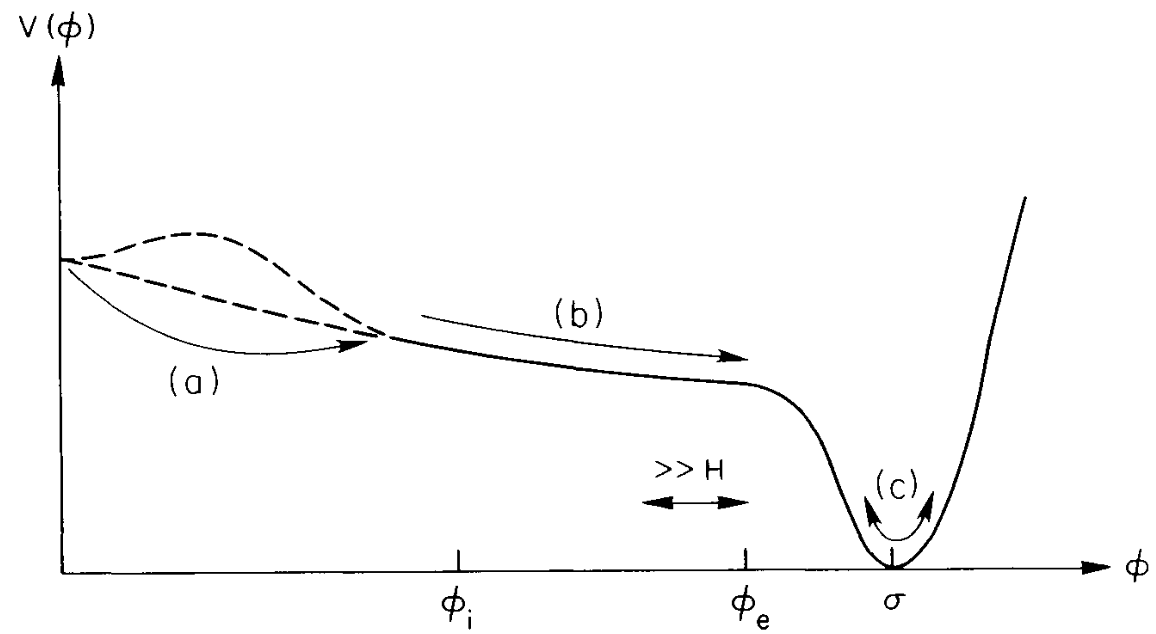


Fig. 8.1: Schematic illustration of an inflationary potential. Notice the "flatness of the potential—a feature common to all inflationary models. The three qualitativ phases of inflation are depicted: (a) barrier penetration (if necessary); (b) slow roll; an (c) coherent oscillations about the minimum of the potential.

Kolb and Turner, "The Early Universe"

Motivation

Decay of scalar condensation

- energy dissipation through interaction with other particles

$$\dot{\rho}_\varphi + 3H\dot{\varphi}^2 + m_\varphi \Gamma_\varphi(\varphi) = 0 \qquad \rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$$ - energy density

- how one can compute the rate (per volume)?

  - first glance: decay of each particle composing the condensation like $\phi \to \chi\chi$

  - second look: daughter particle is influenced by the condensation like time-dependent mass; not only decay but N≥2-body process is possible

Two approaches in the literature

- parametric resonance

- effective action

Are they identical with each other?

Contents

Review of two approaches

- parametric resonance
- effective action

Identical?

- yes for bosonic daughter particles
- ??? for fermionic daughter particles

Rabi model

- “identical" at perturbative level
- not identical at non-perturbative level

Toy model

Mass and interaction

$$\mathscr{L} \supset -\frac{1}{2}m_\varphi^2\varphi^2 - \frac{1}{2}m_\chi^2\chi^2 - \frac{1}{2}\mu\varphi\chi^2$$

- particle decay rate in vacuum

$$\gamma_{\varphi\to\chi\chi} = \frac{\mu^2}{32\pi m_\varphi}\beta_{N=1} \qquad \beta_{N=1} = \sqrt{1-\frac{4m_\chi^2}{m_\varphi^2}}$$

Coherent oscillation

$$\varphi = \frac{1}{2}A_\varphi\left(e^{-im_\varphi t} + e^{im_\varphi t}\right) \qquad \dot{\rho}_\varphi + m_\varphi\Gamma_\varphi(A_\phi) = 0 \qquad \rho_\varphi = \frac{1}{2}m_\varphi^2 A_\varphi^2$$

- we turn off cosmic expansion for simplicity

- extension to, say, quartic potential case will not be quite easy

    - mixture of different frequencies

Effective mass

$$m_{\chi,\mathrm{eff}}^2(t) = m_\chi^2 + \mu A_\varphi\cos(m_\varphi t)$$

- for large amplitude, it can be negative at some time

Parametric resonance

Kofman, Linde and Starobinsky, PRL, 1994

Yoshimura, PTP, 1995

Agenda

- equation of motion with time dependent mass

$$\ddot{\chi}_k + \left(k^2 + m^2_{\chi,\mathrm{eff}}(t)\right)\chi_k = 0$$

$$\rightarrow \quad \chi_k'' + (h_k - \theta_+ e^{2iz} - \theta\_e^{-2iz})\chi_k = 0$$

$$z = m_\varphi t/2 \qquad \theta_\pm = -\frac{2\mu A_\varphi}{m_\varphi^2}$$

$$h_k = \left(\frac{2E_{\chi,k}}{m_\varphi}\right)^2$$

- known as Mathieu equation, which admits Floquet solution

$$\chi_k = e^{\mu_k z} P_k(z)$$

- Floquet exponent (complex)

$$P_k(z+\pi) = P_k(z)$$

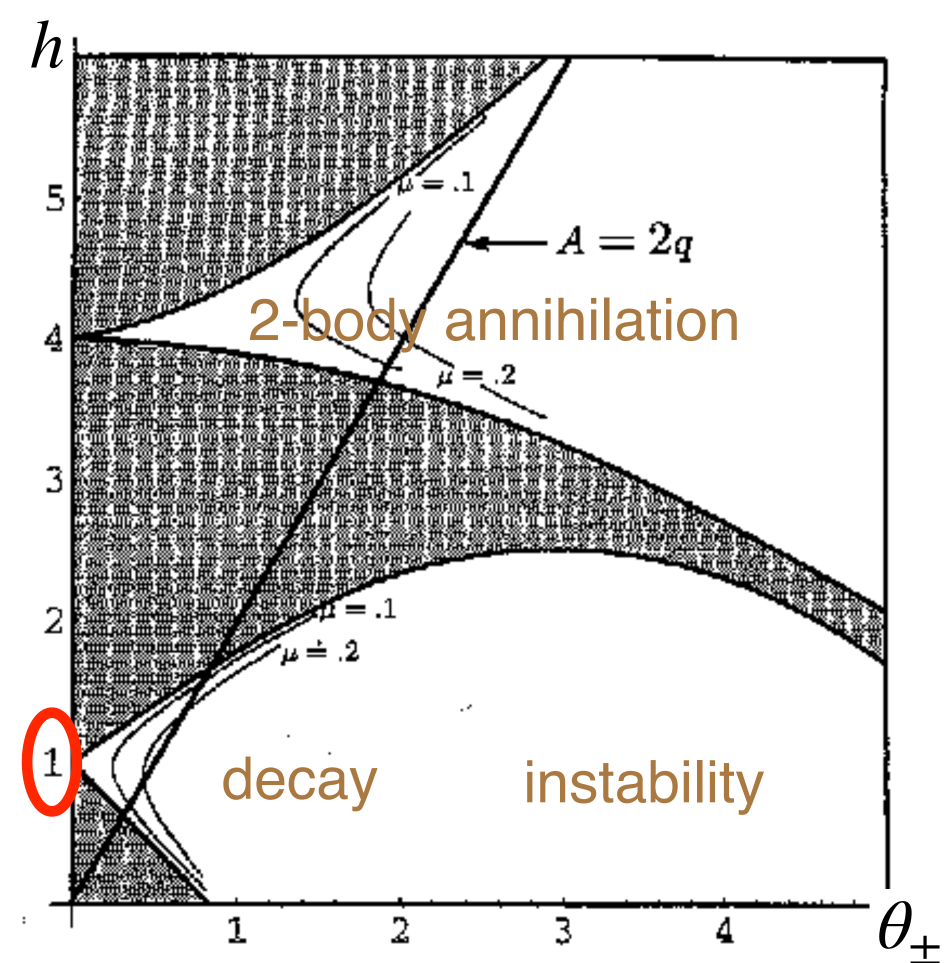
- Periodic function

- positive real of exponent ↔ instability

$$\mathrm{Re}(\mu_k) = \lambda_k > 0$$

- narrow resonance for small amplitude

- broad resonance for large amplitude



Parametric resonance

Instability

- regarded as daughter particle production from "vacuum"
(in the presence of background field)

- can be interpreted as decay of background field

Formulation

Yoshimura, PTP, 1995

- Schrödinger equation of "wave function" (2nd quantization)

$$i\frac{\partial}{\partial t}\Psi(\{q_k\},t) = H\Psi(\{q_k\},t) \qquad H(t) = \sum_k \frac{1}{2}\pi_k^2 + \frac{1}{2}E^2_{\chi,k,\mathrm{eff}}(t)q_k^2$$

- No mode mixing $\Psi(\{q_k\},t) = \prod_k \psi(q_k,t)$

$$i\frac{\partial}{\partial t}\psi(q_k,t) = -\frac{1}{2}\frac{\partial^2}{\partial q_k^2}\psi(q_k,t) + \frac{1}{2}E^2_{\chi,k,\mathrm{eff}}(t)\psi(q_k,t)$$

Parametric resonance

Formulation

- Gaussian solution

$$\psi(q_k, t) = \frac{1}{\sqrt{\chi_k}} \exp\left(\frac{i}{2} \frac{\dot{\chi}_k}{\chi_k} q_k^2 \right) \qquad \ddot{\chi}_k + E^2_{\chi,k,\mathrm{eff}}(t)\chi_k = 0$$

- initial condition: vacuum

$$\chi_k(t=0) = \left(\frac{\pi}{E_{\chi,k,\mathrm{eff}}(t=0)} \right)^{1/2} \qquad \frac{\dot{\chi}_k}{\chi_k}(t=0) = iE_{\chi,k,\mathrm{eff}}(t=0)$$

- one can find

$$\left| \langle \psi(q_k, t=0) \,|\, \psi(q_k, t) \rangle \right|^2 = \frac{1}{\sqrt{1 + N_k(t)}}$$

- produced number of particles

$$N_k(t) = \frac{1}{4\pi E_{\chi,k,\mathrm{eff}}(t=0)} \left(|\dot{\chi}_k|^2 + E^2_{\chi,k,\mathrm{eff}}(t=0) \, |\chi_k|^2 \right) - \frac{1}{2} \propto e^{\mathrm{Re}(\mu_k) m_\varphi t}$$ - asymptotically

Parametric resonance

Formulation

- probability to find initial vacuum for χ after long time

$$\left| \langle \Psi(\{q_k\}, t=0) \,|\, \Psi(\{q_k\}, t) \rangle \right|^2 \propto e^{-\Gamma_\varphi V t}$$

  - decay rate per volume $\Gamma_\varphi = \frac{1}{2} m_\varphi \int \frac{d^3k}{(2\pi)^3} \lambda_k$ $\qquad \dot{\rho}_\varphi + m_\varphi \Gamma_\varphi = 0$

Result for N=1 (LO)

$$\Gamma_\varphi^{N=1(\mathrm{LO})} = -\frac{1}{16\pi} \beta_{N=1} \left(\frac{\mu A_\varphi}{2} \right)^2 \sum_{p=1}^{\infty} \frac{(4p-7)!!}{p!(p-1)!} \left(\frac{\mu A_\varphi}{2 m_\varphi^2 \beta_{N=1}^2} \right)^{2(p-1)}$$

- there are corrections to decay in vacuum (p=1)

- expansion parameter is change in mass over momentum

  - above series is convergent only when the ratio is smaller than 1

Effective action

Matsumoto and Moroi, PRD, 2008

Agenda

- compute effective action for condensate

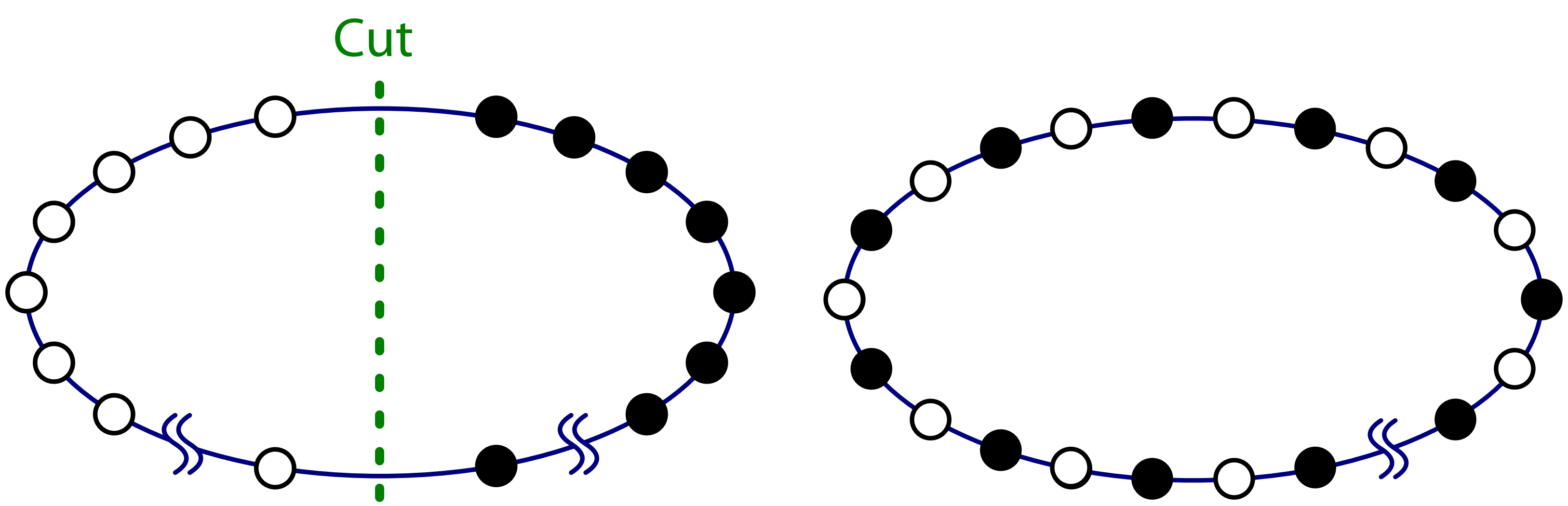
$$e^{iS_{\rm eff}[\phi]} = \int D\chi e^{iS[\phi,\chi]}$$

- imaginary part of it can be interpreted as decay rate

$$S_{\rm eff}[\phi] \supset i\Gamma_\varphi VT/2$$

Protocol

- effective action is given by sum of vacuum bubbles



- white: in-coming energy

$$\varphi_- = \frac{1}{2} A_\varphi e^{-im_\varphi t}$$

- black: out-going energy

$$\varphi_+ = \frac{1}{2} A_\varphi e^{im_\varphi t}$$

Effective action

Remark

- easy to resum all 1-loop diagrams for constant background

    - Coleman-Weinberg potential

- but now background is time-dependent, so evaluate diagram by diagram

Result for N=1 (LO)

- contribution from pair-wise insertion of in-coming energy and out-going energy

$$\Gamma_{\varphi}^{N=1(\mathrm{LO})} = -\frac{1}{16\pi}\beta_{N=1}\left(\frac{\mu A_{\varphi}}{2}\right)^{2}\sum_{p=1}^{\infty}\frac{(4p-7)!!}{p!(p-1)!}\left(\frac{\mu A_{\varphi}}{2m_{\varphi}^{2}\beta_{N=1}^{2}}\right)^{2(p-1)}$$

- p is number of pairs of white and black
- identical to parametric resonance

Contents

Review of two approaches

- parametric resonance
- effective action

Identical?

- yes for bosonic daughter particles
- ??? for fermionic daughter particles

Rabi model

- “identical” at perturbative level
- not identical at non-perturbative level

Important notion

AK and Sakurai, JHEP 2026

Decay rate of vacuum for χ in the presence of ϕ

- this is what we compute both in parametric resonance approach and effective action approach

$$\langle \Psi(\{q_k\}, t=0) \,|\, \Psi(\{q_k\}, t) \rangle \propto e^{iS_{\mathrm{eff}}[\varphi]}$$

They do not look identical

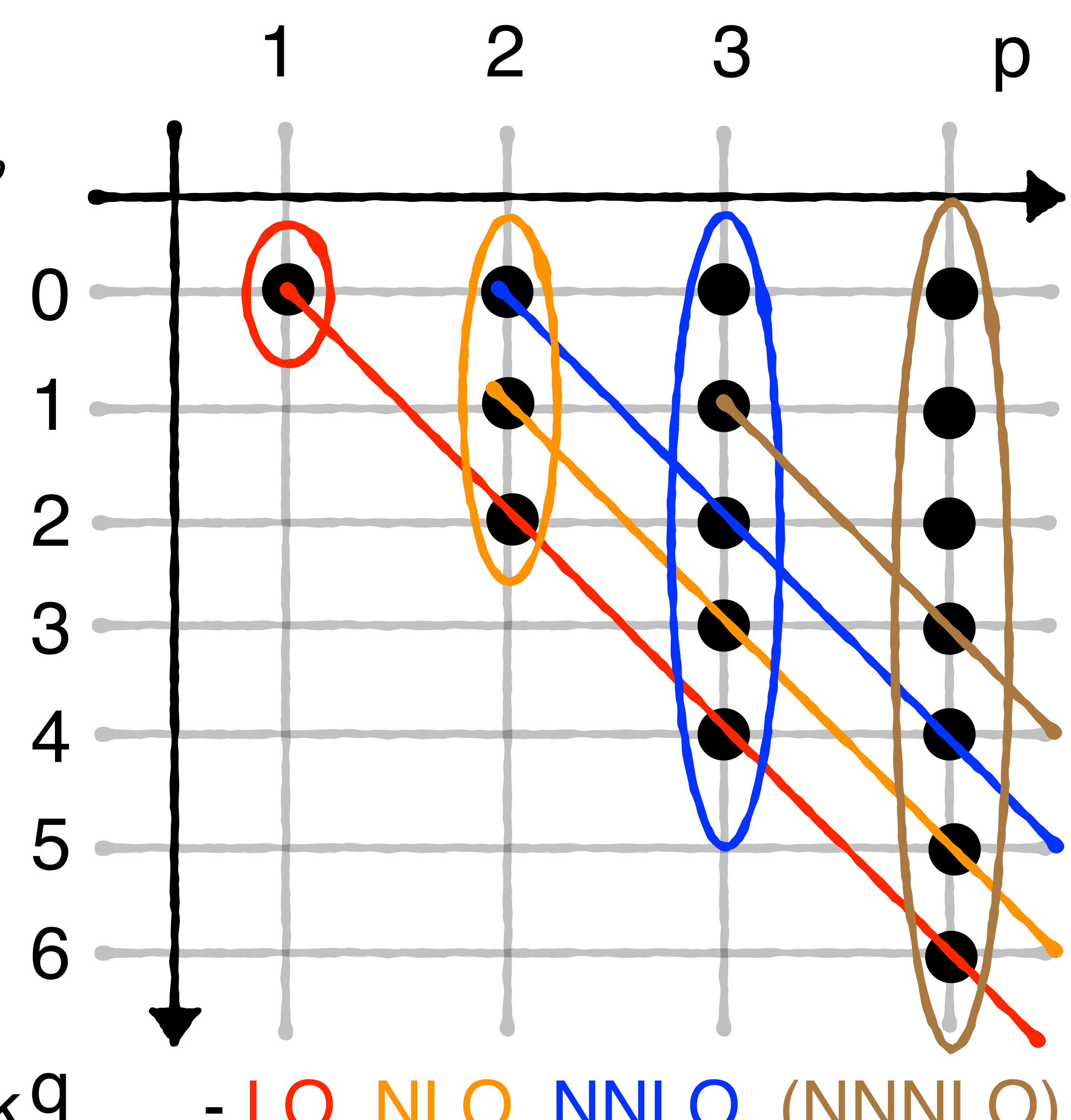
- double expansion of $\Gamma_{\varphi}^{N=1} \supset \beta_{N=1}^{1-2q}\theta^{2p}$

- grids along solid lines are obtained in each order of parametric resonance approach

$$\sqrt{\beta_{N=1}^2 + \epsilon}$$

- grids circled by lines are obtained in each of effective action approach

- p is number of pairs of white and black



- LO, NLO, NNLO, (NNNLO)

Low-order results

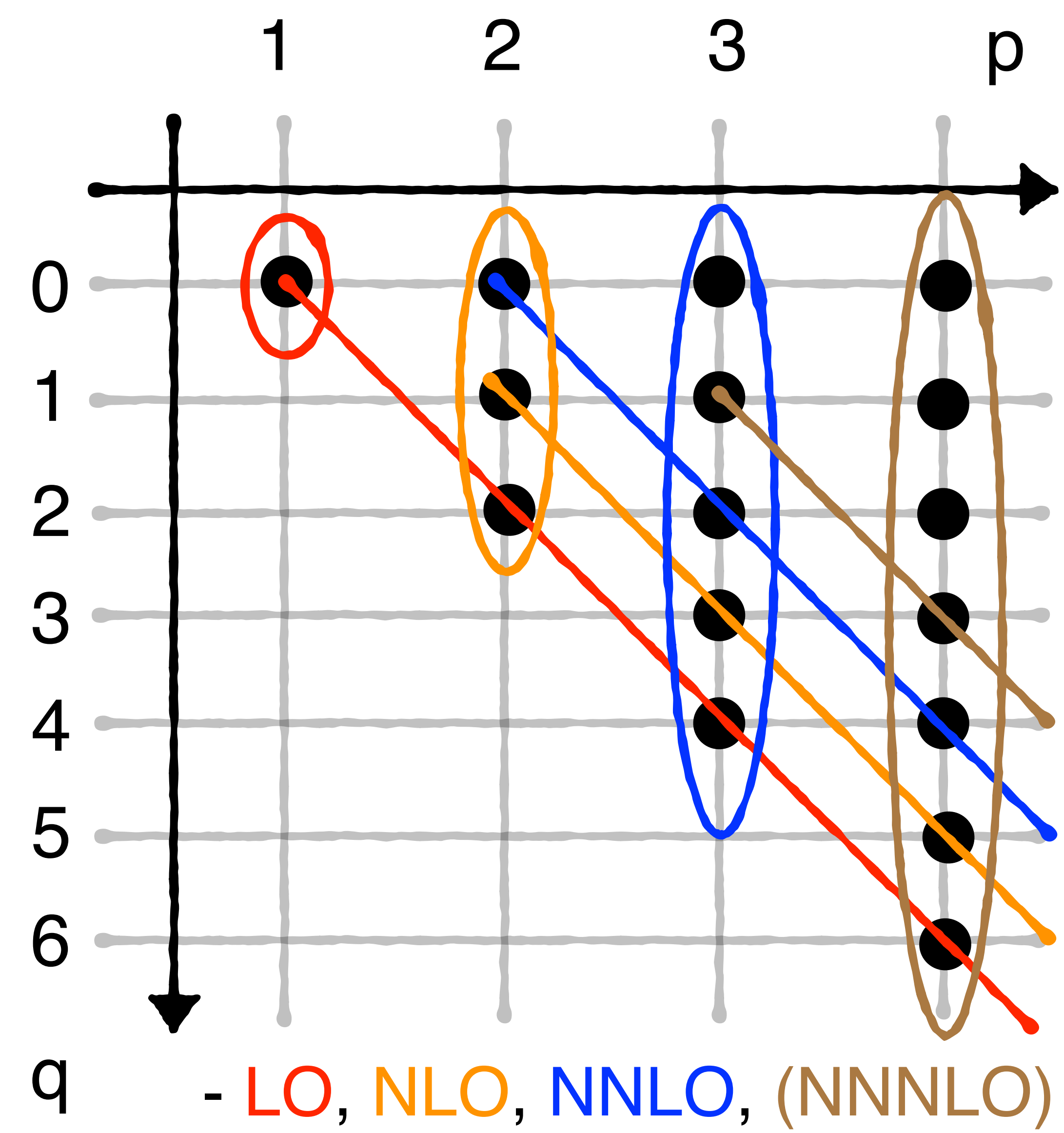
They are actually identical (N=1)

$$\Gamma_\varphi^{N=1,p=1} = \frac{1}{16\pi}\beta_{N=1}\left(\frac{\mu A_\varphi}{2}\right)^2$$

$$\Gamma_\varphi^{N=1,p=2} = \frac{1}{16\pi}\beta_{N=1}\left(\frac{\mu A_\varphi}{2}\right)^2 \times \frac{-1-3\beta_{N=1}^2-2\beta_{N=1}^4}{2}\left(\frac{\mu A_\varphi}{2m_\varphi^2\beta_{N=1}^2}\right)^2$$

$$\Gamma_\varphi^{N=1,p=3} = \frac{1}{16\pi}\beta_{N=1}\left(\frac{\mu A_\varphi}{2}\right)^2 \times \frac{-5-8\beta_{N=1}^2-2\beta_{N=1}^4}{4}\left(\frac{\mu A_\varphi}{2m_\varphi^2\beta_{N=1}^2}\right)^4$$

$$\Gamma_\varphi^{N=1} \supset \beta_{N=1}^{1-2q}\theta^{2p}$$



- LO, NLO, NNLO, (NNNLO)

Low-order results

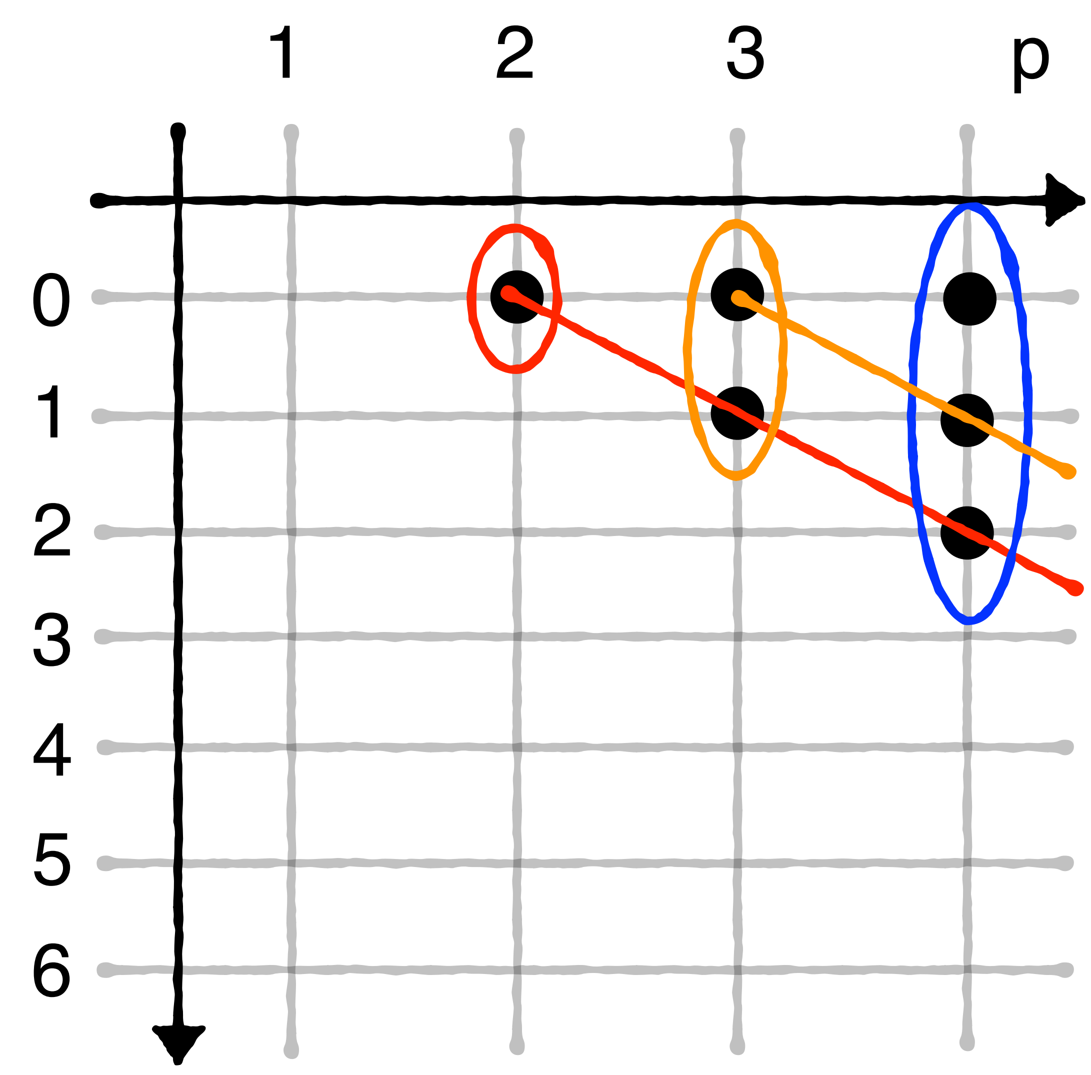
They are actually identical (N=2)

$$\Gamma_\varphi^{N=2,p=1} = \beta_{N=2}\frac{m_\varphi}{16\pi}\left(\frac{\mu A_\varphi}{2}\right)^2$$

$$\Gamma_\varphi^{N=2,p=2} = \frac{1}{16\pi}\beta_{N=2}\left(\frac{\mu A_\varphi}{2}\right)^2$$

$$\times\frac{1-8\beta_{N=2}^2}{3}\left(\frac{\mu A_\varphi}{2m_\varphi^2\beta_{N=2}^2}\right)^2$$

$$\Gamma_\varphi^{N=2} \supset \beta_{N=2}^{1-2q}\theta^{2p}$$



- LO, NLO, (NNLO)

Fermionic daughter particles

Parametric resonance

$$\left| \langle \psi(q_k, t=0) \,|\, \psi(q_k, t) \rangle \right|^2 = \frac{1}{\sqrt{1 + N_k(t)}} \to 1 - N_k(t)$$

- no growing mode because of Pauli blocking

- qualitatively different

Effective action

- each loop gets a minus sign corresponding to

$$\frac{1}{\sqrt{1 + N_k(t)}} \to 1 - N_k(t)$$

- no qualitative difference

Contents

Review of two approaches

- parametric resonance
- effective action

Identical?

- yes for bosonic daughter particles
- ??? for fermionic daughter particles

Rabi model

- "identical" at perturbative level
- not identical at non-perturbative level

Rabi model

Fermionic

$$\hat{H}(t) = \omega\left(\hat{a}^\dagger\hat{a} - \hat{c}\hat{c}^\dagger\right) + \gamma e^{i(mt-\phi)}\hat{c}\hat{a} + \gamma e^{-i(mt-\phi)}\hat{a}^\dagger\hat{c}^\dagger$$

- background field

$$\hat{J}_+ = \hat{a}^\dagger\hat{c}^\dagger \qquad \hat{J}_- = \hat{c}\hat{a} \qquad \hat{J}_3 = \frac{1}{2}\left(\hat{a}^\dagger\hat{a} - \hat{c}\hat{c}^\dagger\right)$$

$$[\hat{J}_+, \hat{J}_-] = 2\hat{J}_3 \qquad [\hat{J}_3, \hat{J}_\pm] = \pm\hat{J}_\pm$$ - SO(3) algebra

- can be found in standard textbooks, but in a matrix form (fundamental representation)

$$\hat{J}_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \hat{J}_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad \hat{J}_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- decay rate

$$\Gamma^P_\omega(T) = -\frac{1}{T}\ln\left(1 - \gamma^2\frac{\sin^2(\Omega T)}{\Omega^2}\right) \qquad \Omega^2 = \left(\omega - \frac{m}{2}\right) + \gamma^2$$

Rabi model

Bosonic

$$\hat{H}(t) = \frac{1}{2}\omega\left(\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger\right) + \frac{1}{2}\gamma e^{i(mt-\phi)}\hat{a}^2 + \frac{1}{2}\gamma e^{-i(mt-\phi)}\hat{a}^{\dagger 2}$$

- background field

$$\hat{J}_+ = -i\frac{1}{2}\hat{a}^2 \quad \hat{J}_- = -i\frac{1}{2}\hat{a}^{\dagger 2} \quad \hat{J}_3 = \frac{1}{4}\left(\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger\right)$$

$$[\hat{J}_+, \hat{J}_-] = 2\hat{J}_3 \qquad [\hat{J}_3, \hat{J}_\pm] = \pm\hat{J}_\pm$$

- SO(1,2) (complexified) algebra

- from Fermionic case: $\gamma \to i\gamma$; overall $-\frac{1}{2}$

- decay rate

$$\Gamma^P_\omega(T) = \frac{1}{2T}\ln\left(1 + \gamma^2\frac{\sinh^2(\Omega T)}{\Omega^2}\right) \qquad \Omega^2 = \left(\omega - \frac{m}{2}\right) - \gamma^2$$

Feynman-diagrammatic computation

Similarly to 1+3-dimension

AK and Sakurai, work in progress

- propagator

$$\frac{i}{p^2 - m^2 + i\epsilon} \to \frac{i}{\omega' - \omega + i\epsilon}$$

Fermionic

$$\Gamma^F_\omega(T) = \sum_{p=1}^{\infty} 2\pi \frac{(-1)^{p-1}}{p} \gamma^{2p} \frac{1}{[(p-1)!]^2} \lim_{\omega_1 \to \omega} \lim_{\omega_2 \to \omega} \frac{\partial^{p-1}}{\partial \omega_1^{p-1}} \frac{\partial^{p-1}}{\partial \omega_2^{p-1}} \delta(m - \omega_1 - \omega_2)$$

Bosonic

$$\Gamma^F_\omega(T) = \sum_{p=1}^{\infty} 2\pi \frac{1}{2p} \gamma^{2p} \frac{1}{[(p-1)!]^2} \lim_{\omega_1 \to \omega} \lim_{\omega_2 \to \omega} \frac{\partial^{p-1}}{\partial \omega_1^{p-1}} \frac{\partial^{p-1}}{\partial \omega_2^{p-1}} \delta(m - \omega_1 - \omega_2)$$

Identical?

“Yes” at perturbative level

- as far as standard “identity” in physics holds

$$\frac{\sin^2(\omega T)}{\omega^2 T} \to \pi\delta(\omega) \qquad T \to \infty$$

Non-perturbative level

- by integrating over ω, one finds the followings

- fermionic

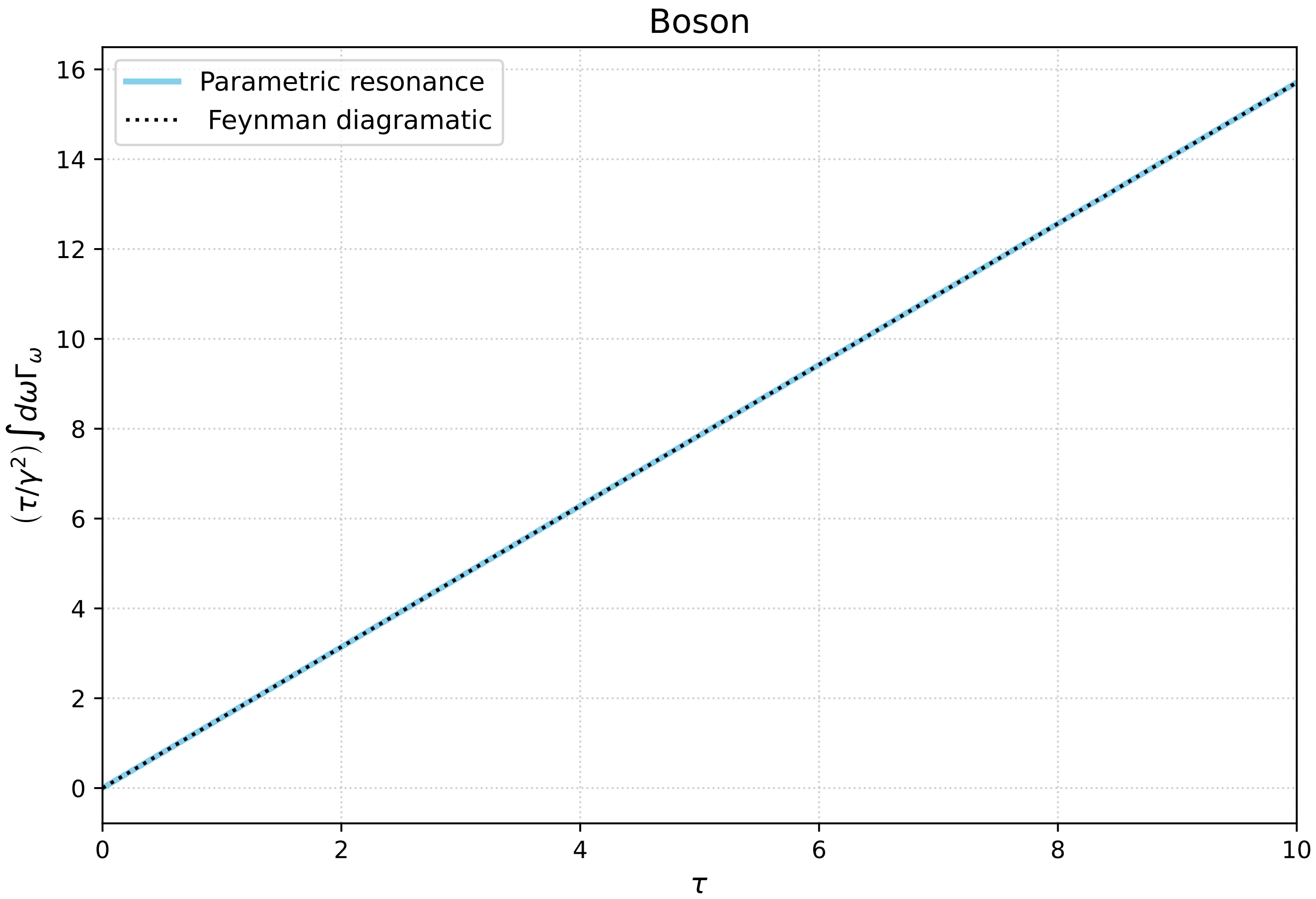
$$\int dx \ln\left(1 - \frac{\sin^2\sqrt{x^2+1}\,\tau}{x^2+1}\right) = -\pi\tau$$

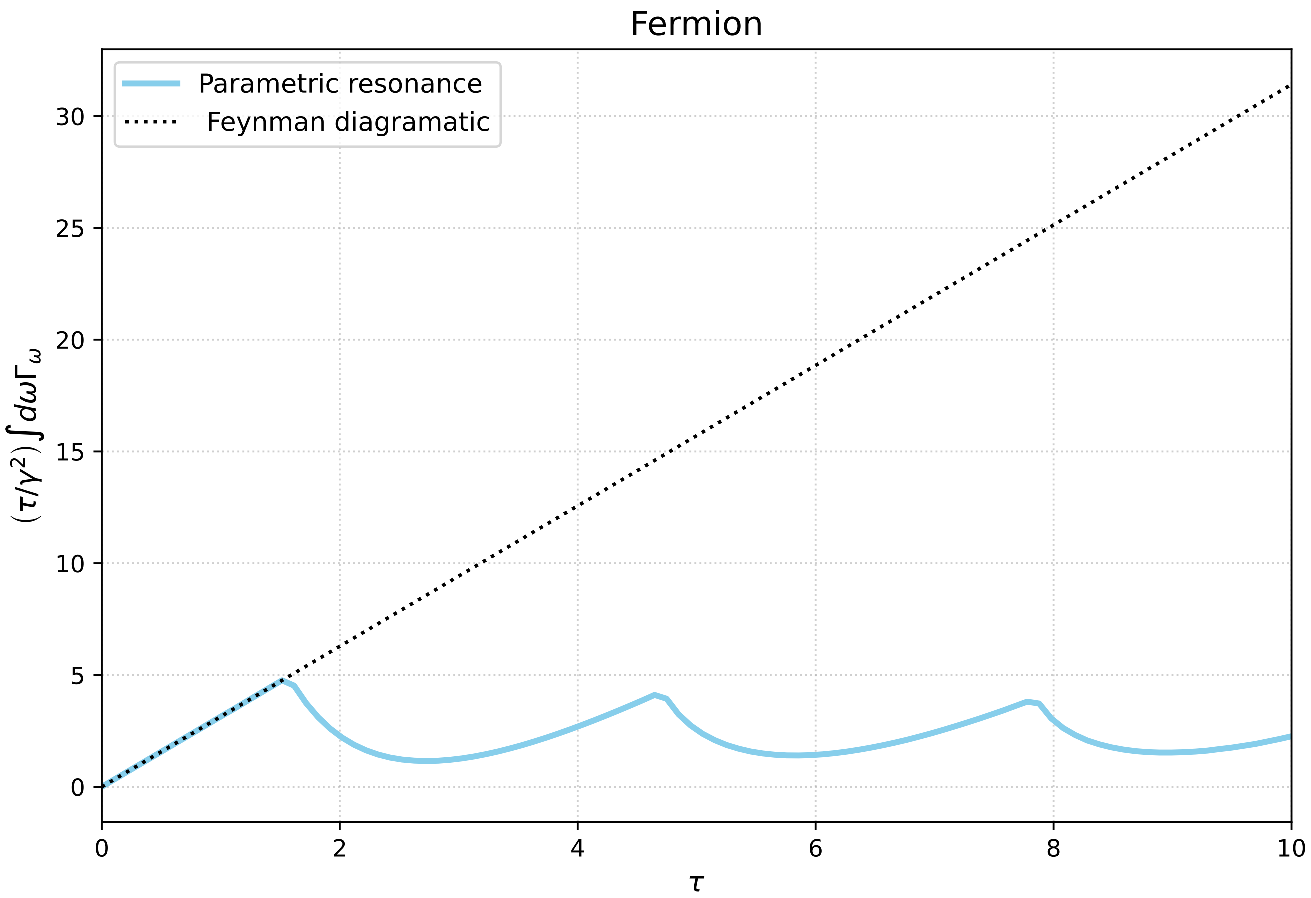
- bosonic

$$\int dx \ln\left(1 + \frac{\sin^2\sqrt{x^2-1}\,\tau}{x^2-1}\right) = \pi\tau$$

Numerical results

Bosonic identity holds but not fermionic one





Summary

Decay of scalar condensation

- plays an important role like reheating of the Universe

- two approaches can be found in the literature: parametric resonance; effective action

Identical?

- we confirm that what are obtained in both approaches are identical for bosonic daughter particles

- this would not be the case for fermionic daughter particles

- Rabi model tells us that they are identical at both perturbative and non-perturbative levels for bosonic case, but not identical at non-perturbative level for fermionic case

Thank you

Naive period average

Ahmed, Grzadkowski and Socha, PLB, 2022

Agenda

- decay is instantaneous compared to background evolution

Protocol

- compute decay rate for a given background but ignoring evolution

$$\Gamma_\varphi(t) = \frac{m_\varphi}{16\pi}\left(\frac{\mu A_\varphi}{2}\right)^2 \beta_{N=1,\mathrm{eff}}(t) \qquad \beta_{N=1,\mathrm{eff}}(t) = \sqrt{1 - \frac{4m^2_{\chi,\mathrm{eff}}(t)}{m^2_\varphi}}$$

- time dependent $\qquad m^2_{\chi,\mathrm{eff}}(t) = m^2_\chi + \mu A_\varphi \cos(m_\varphi t)$

- taking an oscillation average

$$\langle \Gamma_\varphi(t) \rangle_{\mathrm{osc}} = \frac{m_\varphi}{16\pi}\left(\frac{\mu A_\varphi}{2}\right)^2 \langle \beta_{N=1,\mathrm{eff}}(t) \rangle_{\mathrm{osc}}$$

$$= -\beta_{N=1}\frac{m_\varphi}{16\pi}\left(\frac{\mu A_\varphi}{2}\right)^2 \sum_{p=1}^{\infty} \frac{(4p-7)!!}{[(p-1)!]^2}\left(\frac{\mu A_\varphi}{m^2_\varphi \beta^2_{\varphi\to\chi\chi}}\right)^{2(p-1)}$$

Naive period average

Interpretation

$$\Gamma_\varphi = \langle \gamma_{\varphi\to\chi\chi,\mathrm{eff}}(t)\rangle_{\mathrm{osc}} \frac{\rho_\varphi}{m_\varphi} = -\beta_{N=1}\frac{m_\varphi}{16\pi}\left(\frac{\mu A_\varphi}{2}\right)^2 \sum_{p=1}^{\infty} \frac{(4p-7)!!}{[(p-1)!]^2}\left(\frac{\mu A_\varphi}{m_\varphi^2 \beta_{N=1}^2}\right)^{2(p-1)}$$

$$\dot{\rho}_\varphi + m_\varphi \Gamma_\varphi(A_\phi) = 0$$

- $p \geq 2$ contributions are corrections to decay in vacuum (N=1)
- expansion parameter is change in mass over momentum
    - above series is convergent only when the ratio is smaller than 1

Naive period average

Advantage of this approach

- straightforward extension to the case of $\mu A_\varphi \gg m_\varphi^2$
    - negative mass squared $m_{\chi,\mathrm{eff}}^2(t) = m_\chi^2 + \mu A_\varphi \cos(m_\varphi t)$
- daughter scalar develops non-zero field value around which mass squared is positive

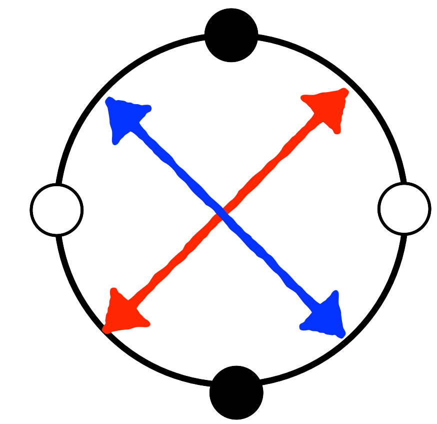
Doubt/concern

- "formation" time of daughter particles and "development" time of expectation value are comparable with time scale of background change
    - O(1) uncertainty is expected
- no N≥2-body process

Effective action

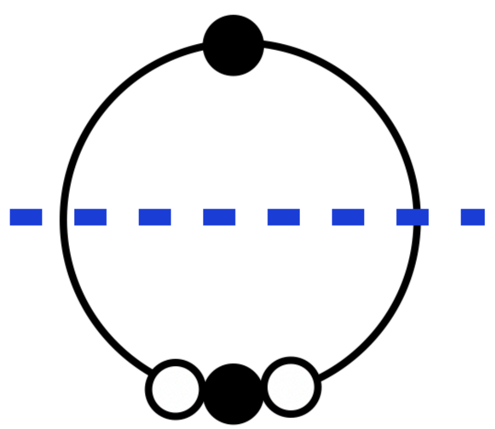
AK and Sakurai, JHEP 2026

Pair-wise insertion diagram (two pairs)



- two pairs of the same propagator

- the same propagator can be “pinched” by derivative with respect to mass



$$\frac{1}{(p^2 - m_\chi^2 + i\epsilon)^2} = \lim_{\xi \to m_\chi^2} \frac{\partial}{\partial \xi} \frac{1}{(p^2 - \xi + i\epsilon)}$$

- contribution is

$$\lim_{\xi_1, \xi_2 \to m_\chi^2} \frac{\partial}{\partial \xi_1} \frac{\partial}{\partial \xi_2} \frac{1}{16\pi} \left(\frac{\mu A_\varphi}{2} \right)^2 2 \frac{1}{4} \left(\frac{\mu A_\varphi}{2m_\varphi^2} \right)^2 m_\varphi^4 \beta_{N=1}(\xi_1, \xi_2)$$

- symmetric factor

$$\beta_{N=1}(\xi_1, \xi_2) = \sqrt{\left(1 - \frac{\xi_1}{m_\varphi^2} - \frac{\xi_2}{m_\varphi^2}\right)^2 - \frac{4\xi_1 \xi_2}{m_\varphi^4}}$$

$$\beta_{N=1}(\xi_1 = m_\chi^2, \xi_2 = m_\chi^2) = \beta_{N=1}$$

$$= -\frac{1}{16\pi} \beta_{N=1} \left(\frac{\mu A_\varphi}{2} \right)^2 \frac{1 + \beta_{N=1}^2}{2} \left(\frac{\mu A_\varphi}{2 m_\varphi^2 \beta_{N=1}^2} \right)^2$$