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Physical Interpretation of SMEFT Effects in double boson productions at TeV-Scale Lepton Colliders

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Contents

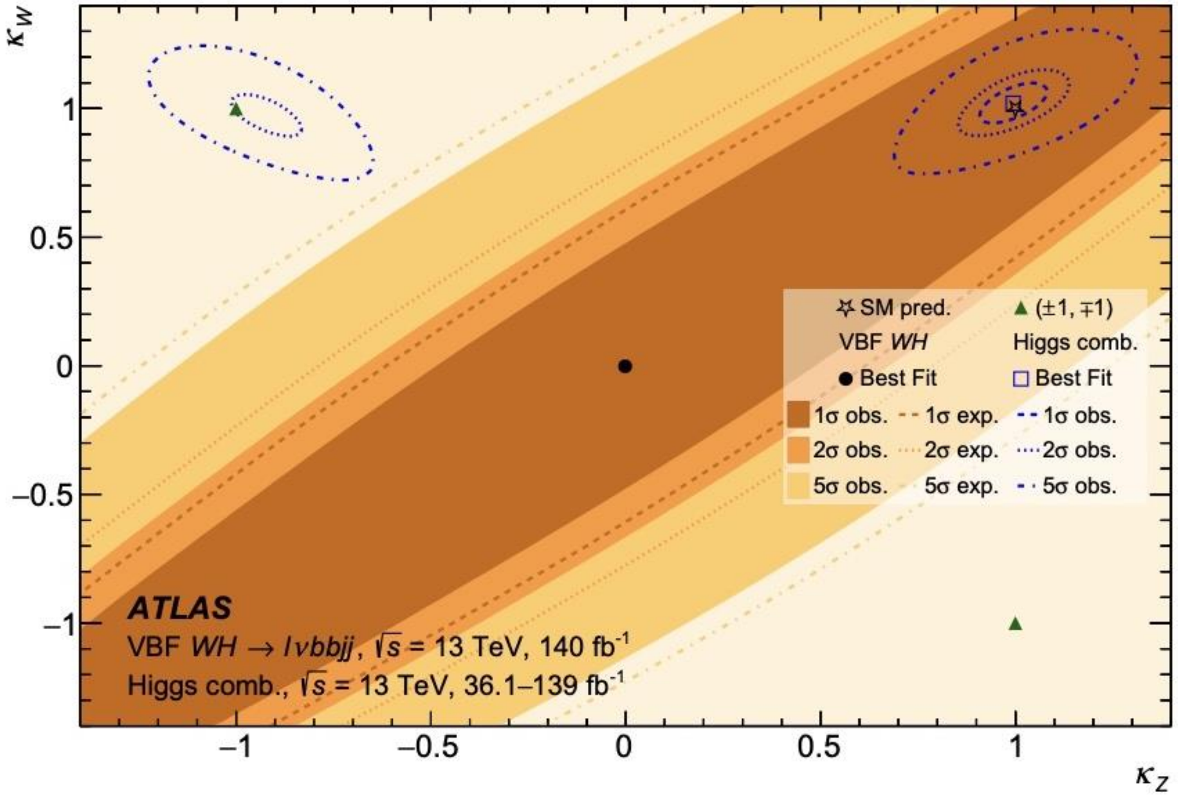
Contents

1. Introduction
2. $l^- l^+ \to \nu_l \bar{\nu}_l Z h$
3. FD gauge
4. SMEFT
5. Summary

1. Motivation

Determining the relative sign of the Higgs boson couplings to W and Z bosons using VBF WH production at ATLAS[1] and CMS[2]

[1] The ATLAS collaboration. "Determining the relative sign of the Higgs boson coupling to W and Z boson using VBF WH production with the ATLAS detector", Phys. Rev. Lett 133,141801, 2024.
[2] The CMS collaboration, "Study of WH production through vector boson scattering and extraction of the relative sign of the W and Z couplings to the Higgs boson in proton-proton collisions at $\sqrt{s} = 13$TeV", Phys. Lett. B (2025), 860, 139202.



The scenario with opposite-sign couplings of the W and Z to the Higgs
→This hypothesis is excluded at more than 5σ.

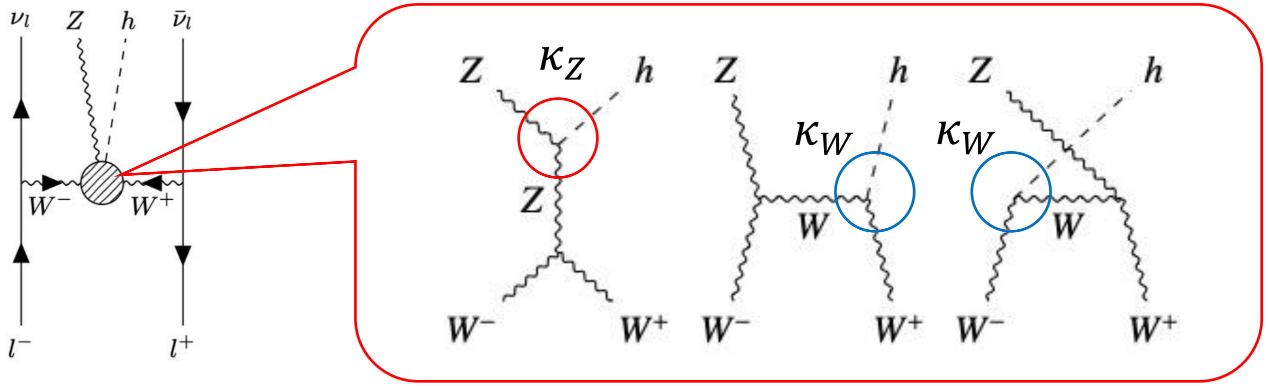
The cross section is highly dependent on the relative sign of the couplings.

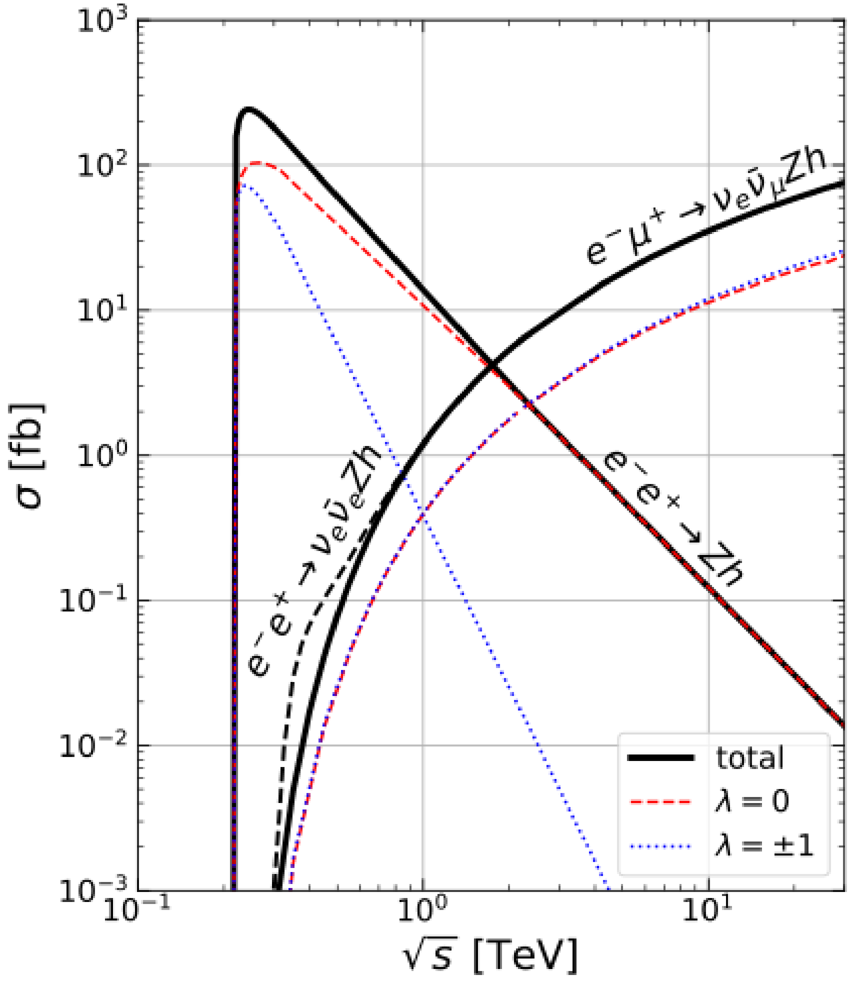
This process is sensitive to new physics contributions involving W, Z and Higgs.

1. Introduction

We focus on the $l^-l^+ \to \nu\bar{\nu}Zh$ process.

- At high energies, the cross section of $l^-l^+ \to \nu\bar{\nu}Zh$ is larger than the cross section of $l^-l^+ \to Zh$.
- The Vh production via vector boson scattering is included.





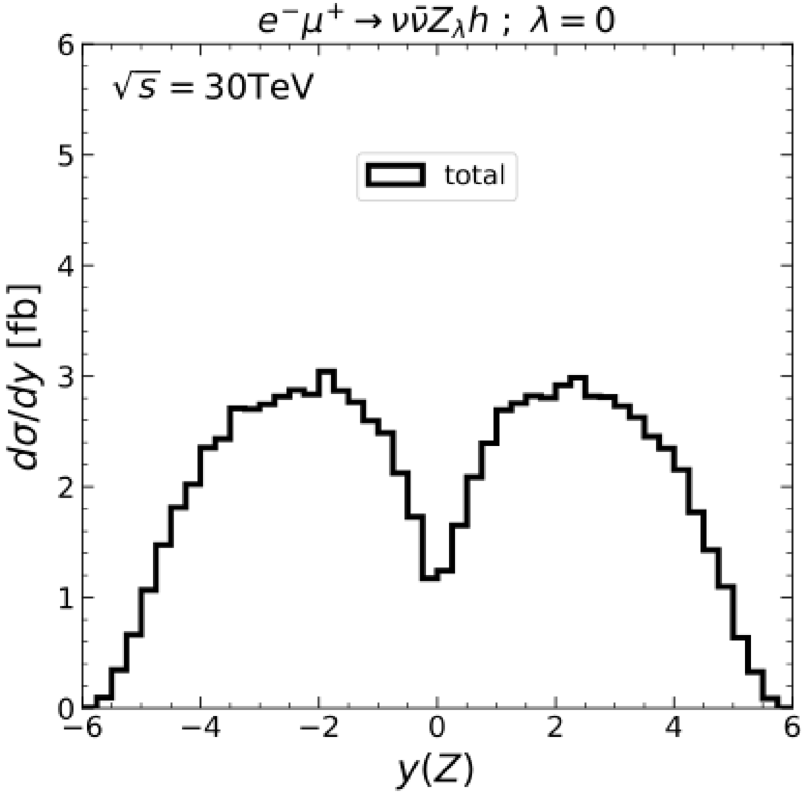
In the following, to make physics discussion simpler, we consider, $e^-\mu^+$ scattering in order to eliminate the s-channel contribution $Z \to \nu_e\bar{\nu}_e$ from $e^-e^+ \to ZZh$.

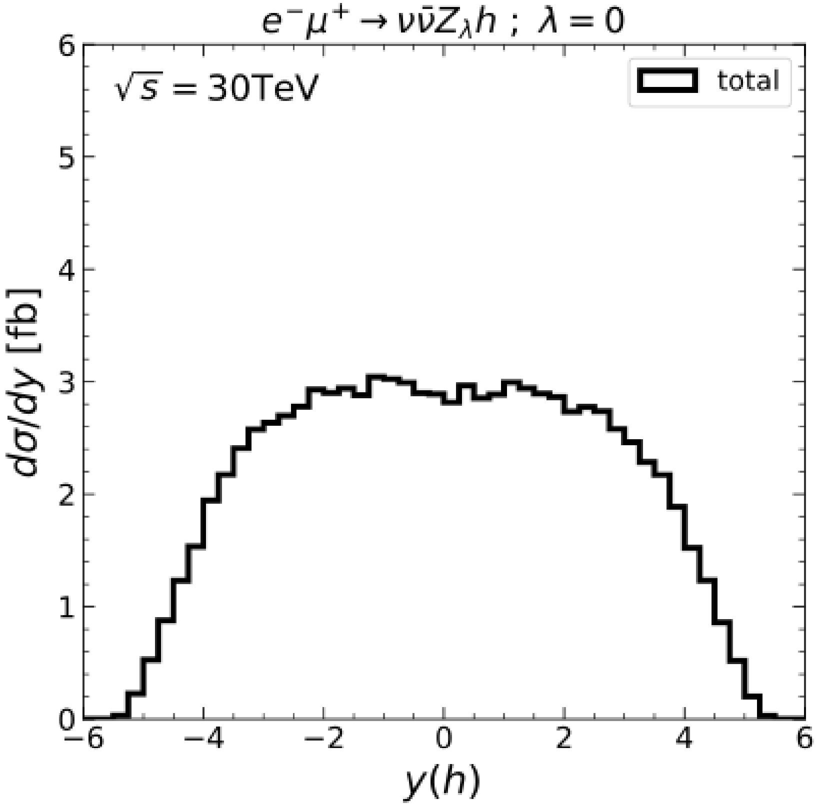
$l^-l^+ \to \nu\bar{\nu}Zh$ is important at high-energy lepton collider experiments to study Higgs coupling to the weak gauge bosons.

1. Introduction

Rapidity : $y = \frac{1}{2} \ln \frac{E+p_Z}{E-p_Z}$

・Describes motion along the beam (z) direction
・y=0: central, y>0: forward, y<0: backward





We naively expected that the distribution of the longitudinal Z boson and Higgs boson produced from $W^-W^+ \to Zh$ are similar.
However, this is not the case.

Why are y(Z) and y(h) different?

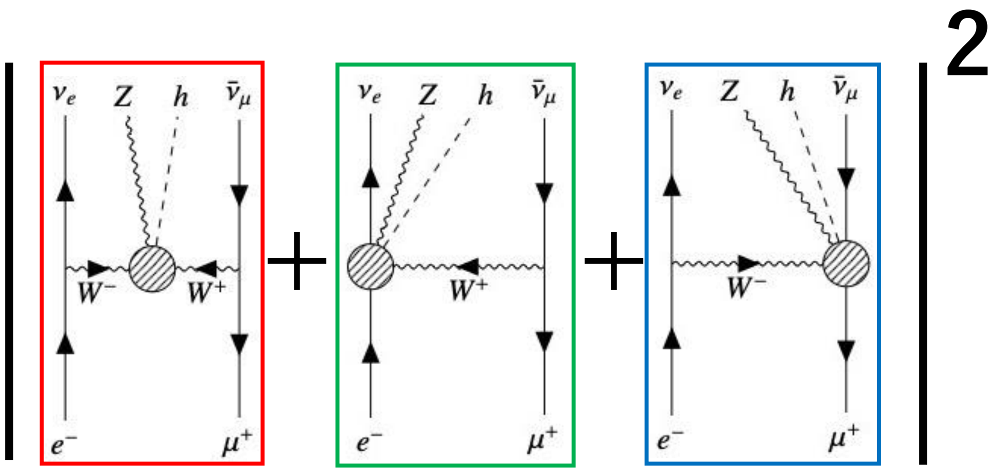
Contents

2. $l^-l^+ \to \nu_l\bar{\nu}_l Zh$

We divide $e^-\mu^+ \to \nu\bar{\nu}Zh$ into mainly 3 groups based on the topology of each Feynman diagram.

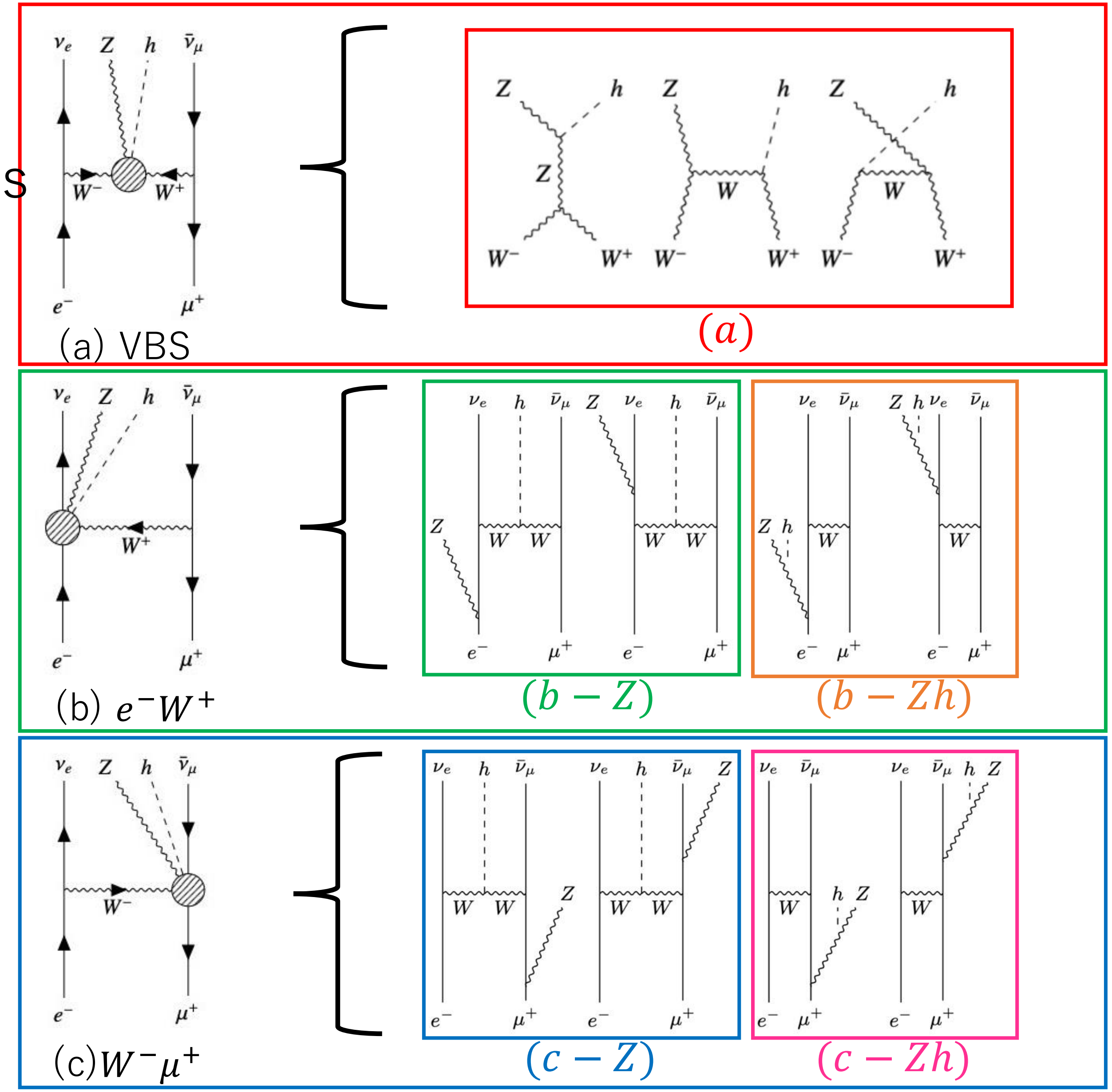
$\boldsymbol{\sigma} \propto$

Total



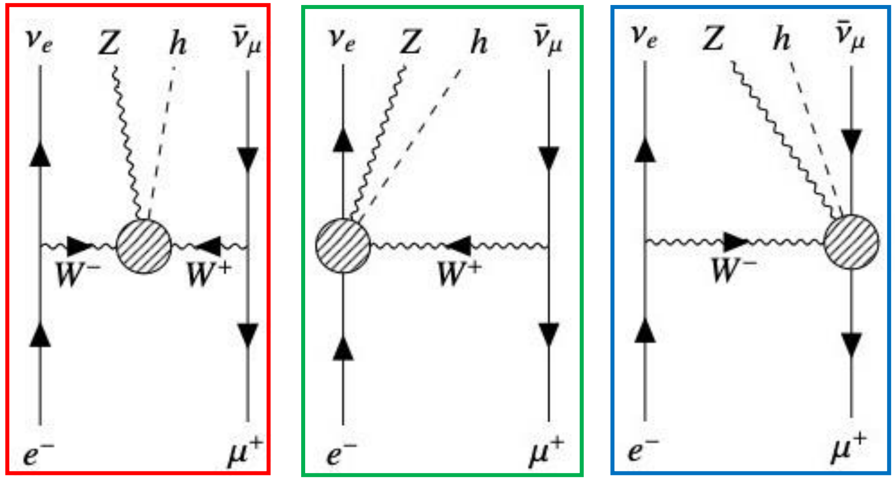
(a) (b) (c)

Needless to say, physical cross sections, obtained by squaring the sum of all relevant amplitudes, are gauge invariant, while contributions grouped according to the topology of each Feynman diagram are gauge dependent and unphysical.

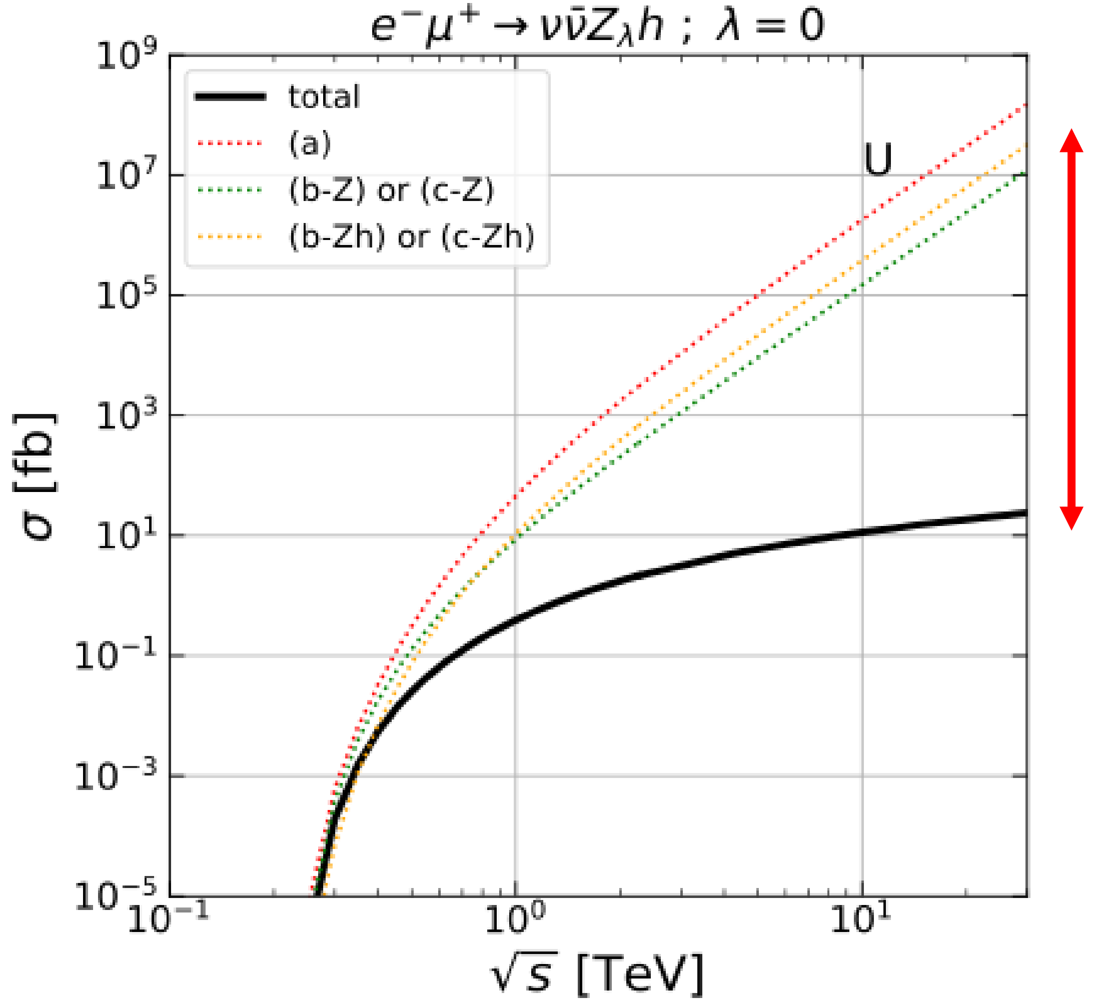


2. $l^- l^+ \to \nu_l \bar{\nu}_l Zh$

In such a process,
gauge cancellation occurs.
→This energy growth requires
significant cancellations.
Reducing the event rate, so unreliable...
This is a major obstacle of numerical simulation for future high-energy experiments.



(a) (b) (c)

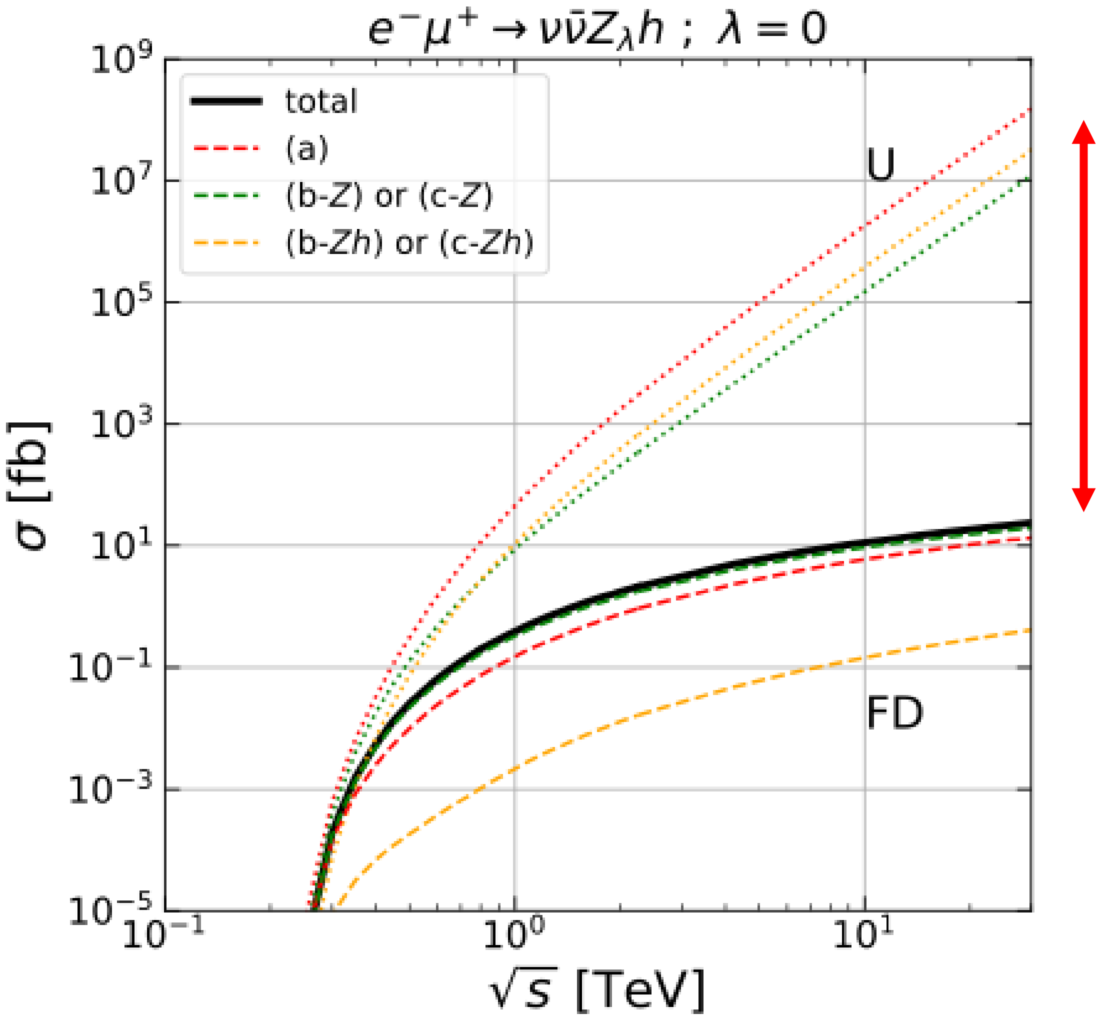


$\text{total} \propto |(a) + (b - Z) + (c - Z) + (b - Zh) + (c - Zh)|^2$,
$(a) \propto |M_{(a)}|^2, (b - Z) \propto |M_{(b-Z)}|^2, (c - Z) \propto |M_{(c-Z)}|^2, \ldots$

2. $l^- l^+ \to \nu_l \bar{\nu}_l Z h$

In such a process,
gauge cancellation occurs.
→This energy growth requires
significant cancellations.
This is a major obstacle of numerical simulation for future high-energy experiments.
A new gauge fixing was proposed and it indicates gauge cancellation among the interfering amplitudes can be avoided!!

Feynman-Diagram (FD) gauge



$$\text{total} \propto |(a) + (b-Z) + (c-Z) + (b-Zh) + (c-Zh)|^2,$$

$$(a) \propto |M_{(a)}|^2, (b-Z) \propto |M_{(b-Z)}|^2, (c-Z) \propto |M_{(c-Z)}|^2, \ldots$$

[3] J. Chen, K. Hagiwara, J. Kanzaki and K. Mawatari, "Helicity amplitudes without gauge cancellation for electroweak processes", Eur.Phys.J.C 83 (2023) 10, 922.
[4] J. Chen, K. Hagiwara, J. Kanzaki, K. Mawatari and Y. Zheng , "Helicity amplitudes in light-cone and Feynman-diagram gauges", Eur.Phys.J.Plus 139 332 (2024)
[5] K. Hagiwara, J. Kanzaki, O. Mattelaer, K. Mawatari and Y. Zheng, "Automatic generation of helicity amplitudes in Feynman-Diagram gauge", Phys. ReV. D 110, 056024 (2024)
[6] H.Furusato, K.Mawatari, Y.Suzuki, Y.Zheng, "W-boson pair production at lepton colliders in the Feynman-Diagram gauge", *Phys.Rev.D 110 (2024) 5, 053005*

Contents

U gauge vs FD gauge

U gauge (Covariant R_ξ gauge with $\xi \to \infty$)

Only weak gauge bosons (Nambu-Goldstone bosons are decoupled.)

$$\beta = \frac{q}{E} = \sqrt{1 - \frac{m^2}{E^2}}, \quad \gamma = \frac{E}{m} = \frac{1}{\sqrt{1-\beta^2}}.$$

・propagator $(\mu, \nu = 0, 1, 2, 3)$

$$P^{\mathrm{U}}_{\mu\nu} = \frac{i}{q^2 - m^2 + i\epsilon}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m^2}\right)$$

・polarization vector

$$\epsilon^\mu(q,0) = \frac{1}{m}(q,0,0,E) = \gamma(\beta,0,0,1)$$

FD gauge (Non-covariant light-cone gauge with $n^\mu = \left(\mathrm{sgn}(q^0), -\frac{\vec{q}}{|\vec{q}\,|}\right)$.

Weak gauge bosons and Nambu-Goldstone bosons are in Superposition state.

・5×5 propagator $(M, N = 0, 1, 2, 3, 4)$

$$P^{\mathrm{FD}}_{MN} = \frac{i}{q^2 - m^2 + i\epsilon}\begin{pmatrix} -\tilde{g}_{\mu\nu} & im\frac{n_\mu}{n\cdot q} \\ -im\frac{n_\nu}{n\cdot q} & 1 \end{pmatrix}$$

$$-\tilde{g}_{\mu\nu} = -g_{\mu\nu} + \frac{n_\mu q_\nu + q_\mu n_\nu}{n \cdot q}$$

・5 component polarization vector

$$\epsilon^M(q,0) = (\tilde{\epsilon}^\mu(q,0),\ i)$$

$$\tilde{\epsilon}^\mu(q,0) = \epsilon^\mu(q,0) - \frac{q^\mu}{Q} = \frac{1}{\gamma(1+\beta)}(-1,0,0,1)$$

3. $e^-\mu^+ \to \nu_e \bar{\nu}_\mu Zh$

We consider the case of $\lambda = 0$.

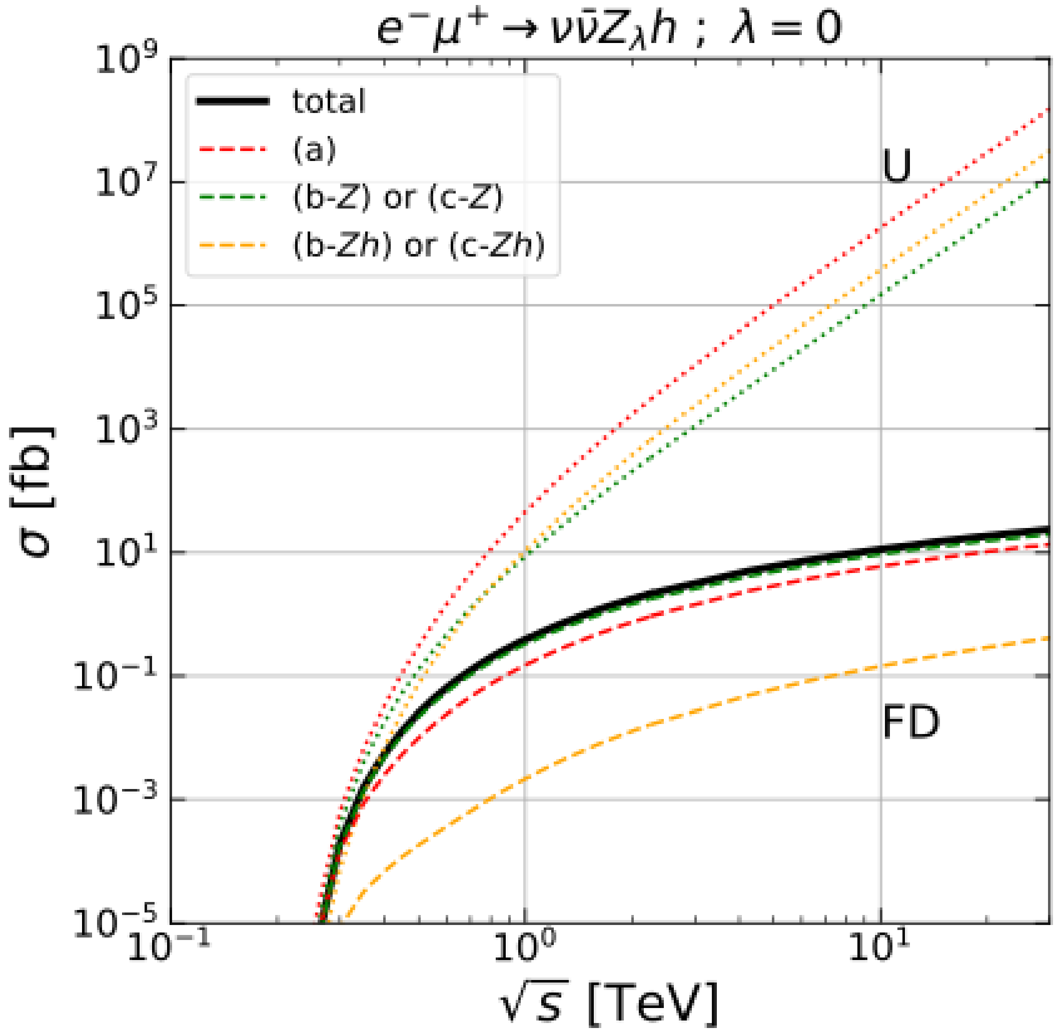
- **The solid black line** is the physical observable.
  total$\propto |(\mathrm{a}) + (b-Z) + (c-Z) + (b-Zh) + (c-Zh)|^2$
- Dotted lines (U gauge) and dashed lines (FD gauge) are unphysical quantities.

U gauge:

- Each category of the amplitudes has energy growth.
- Large cancellation among each group at high energies.

FD gauge :

- No such energy growth of each category at all.
- $(b-Z)$ and $(b-Zh)$ ($(c-Z)$ and $(c-Zh)$) differ by two orders of magnitude.
  $\to (b-Z) > (b-Zh)$ ($(c-Z) > (c-Zh)$).



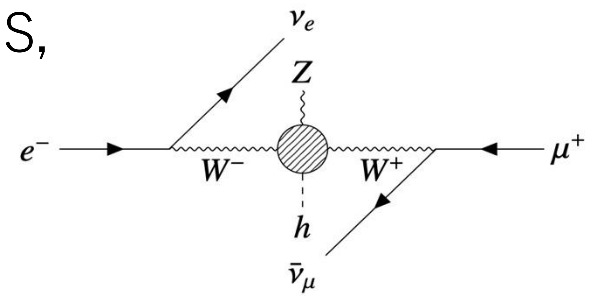
3. $e^- \mu^+ \to \nu_e \bar{\nu}_\mu Z h$

U gauge:

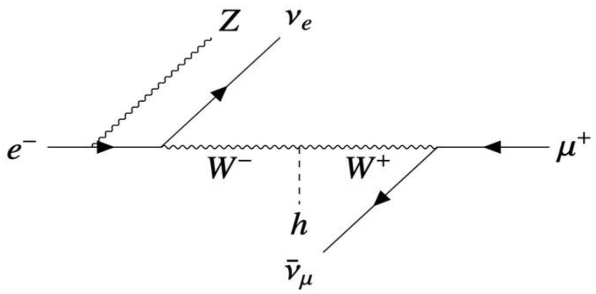
- No clue for the physical distribution from each contribution.

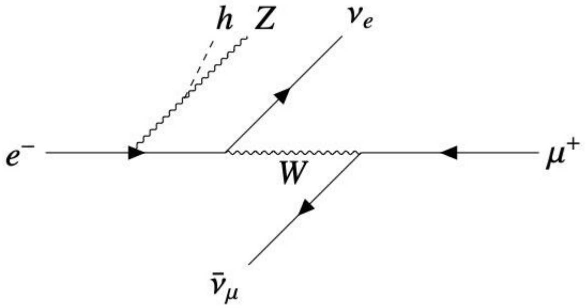
FD gauge :

- (a) : VBS,

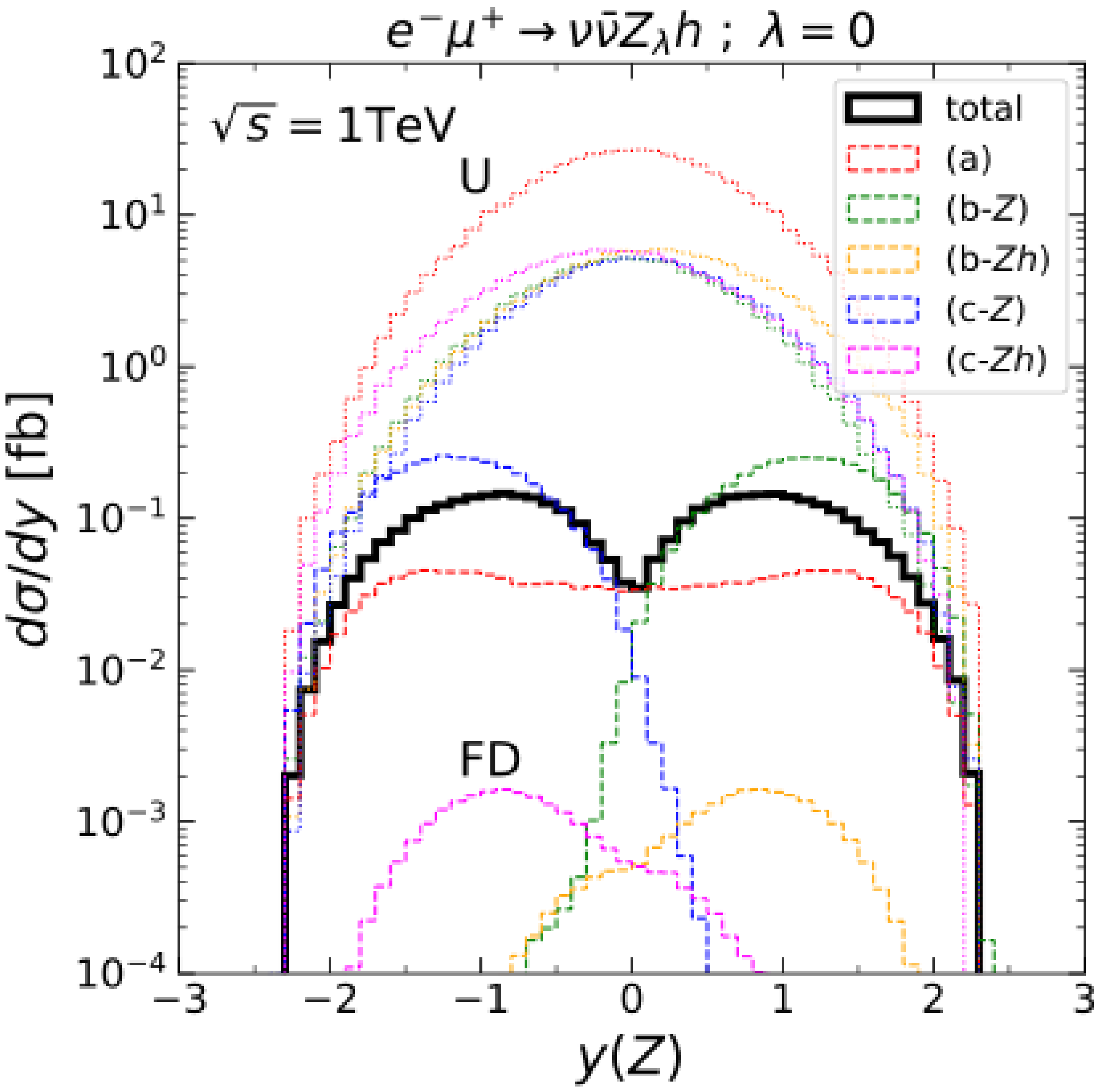


- $(b - Z)$, $(b - Zh)$: $e^- W^+$ scattering,



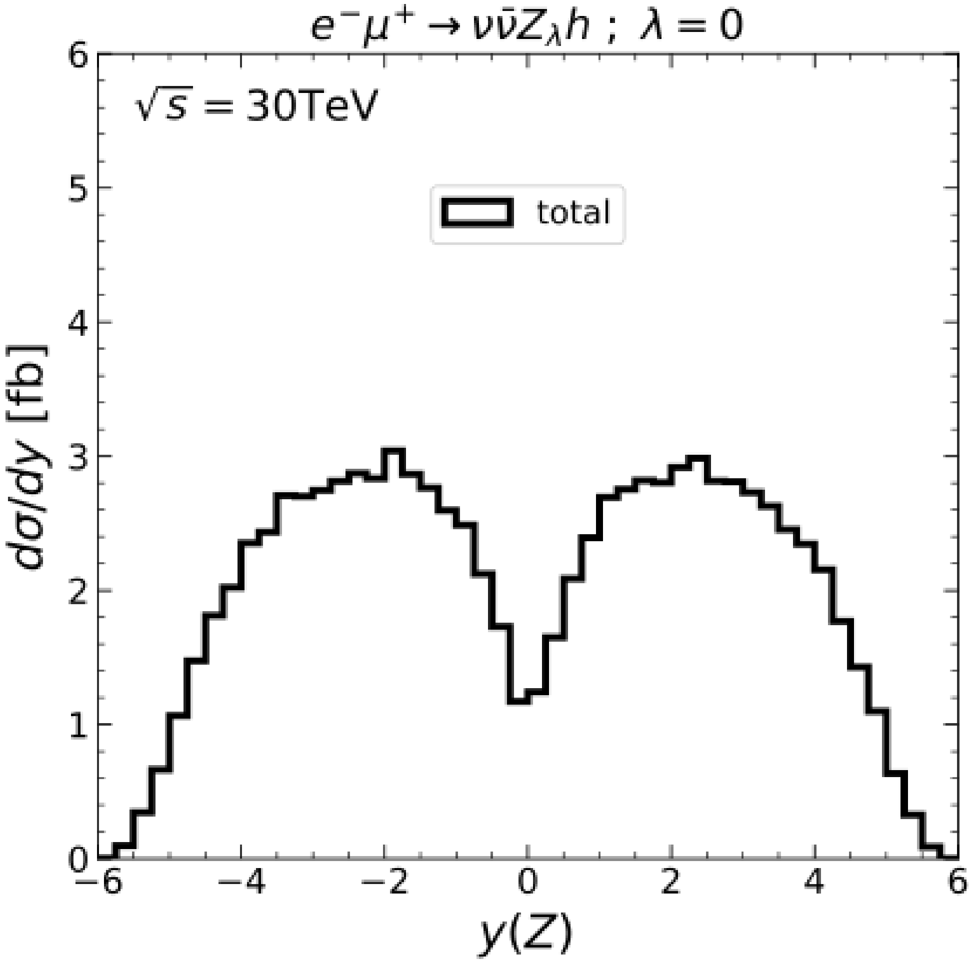


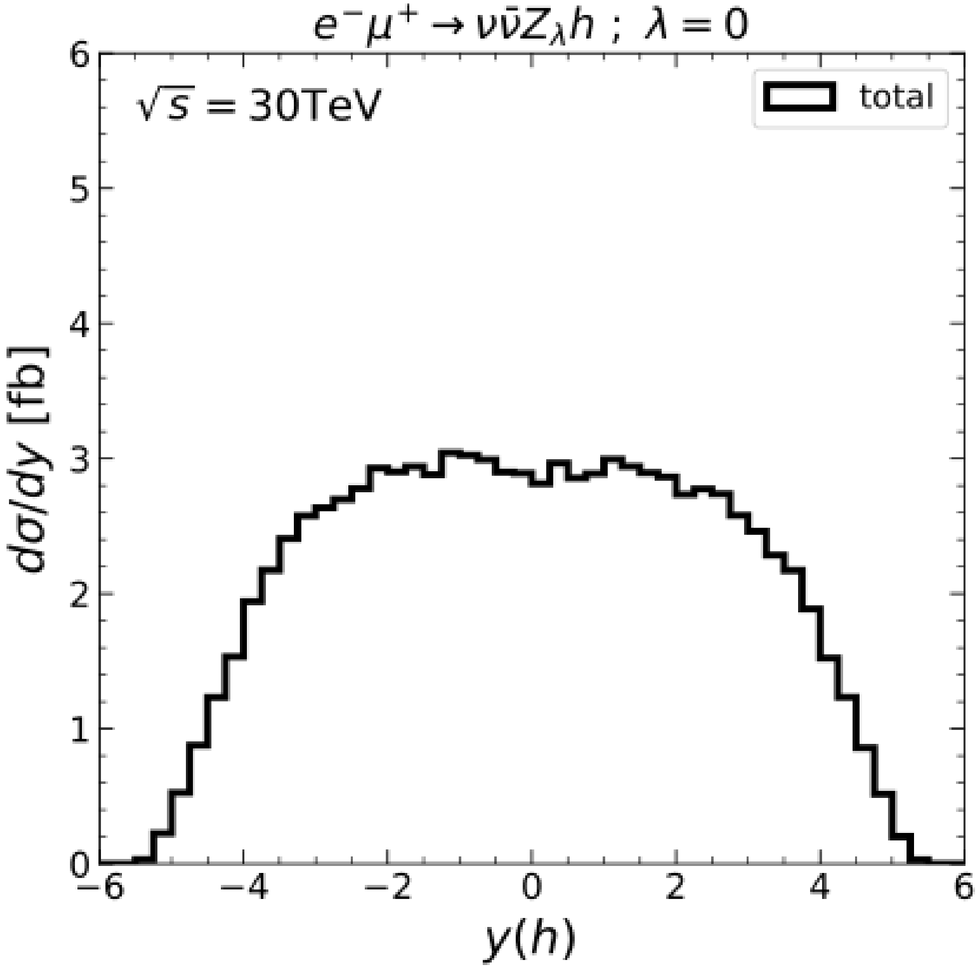
- $(c - Z)$, $(c - Zh)$: $W^- \mu^+$ scattering.



In the FD gauge,
the behavior of each group shows good agreement with our expectation.

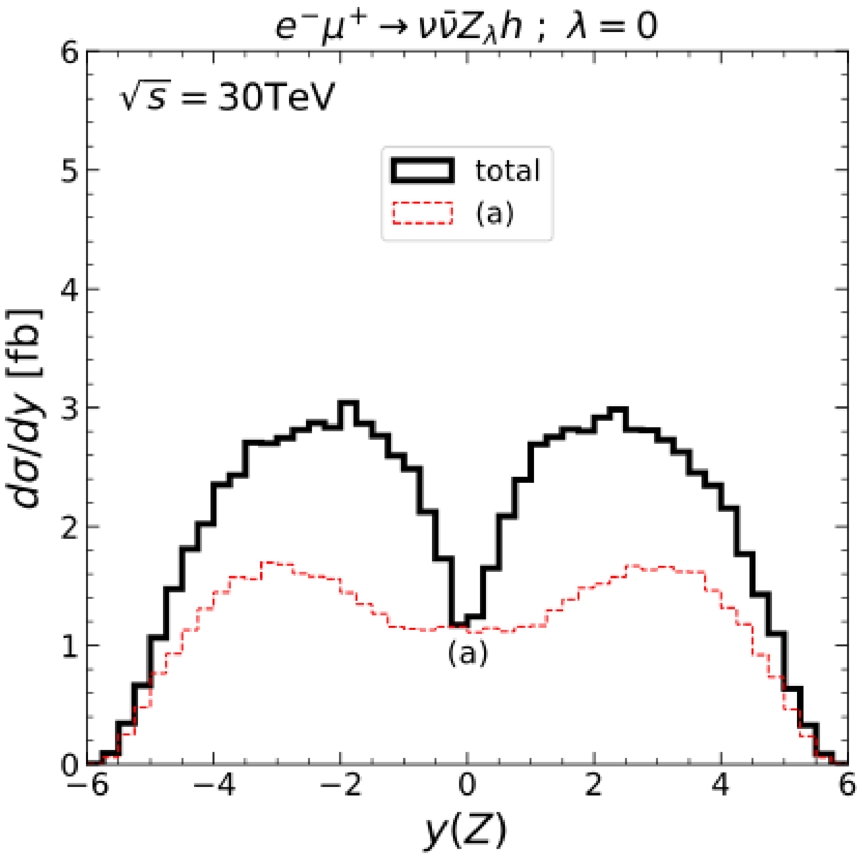
3. $e^-\mu^+ \to \nu_e \bar{\nu}_\mu Z h$

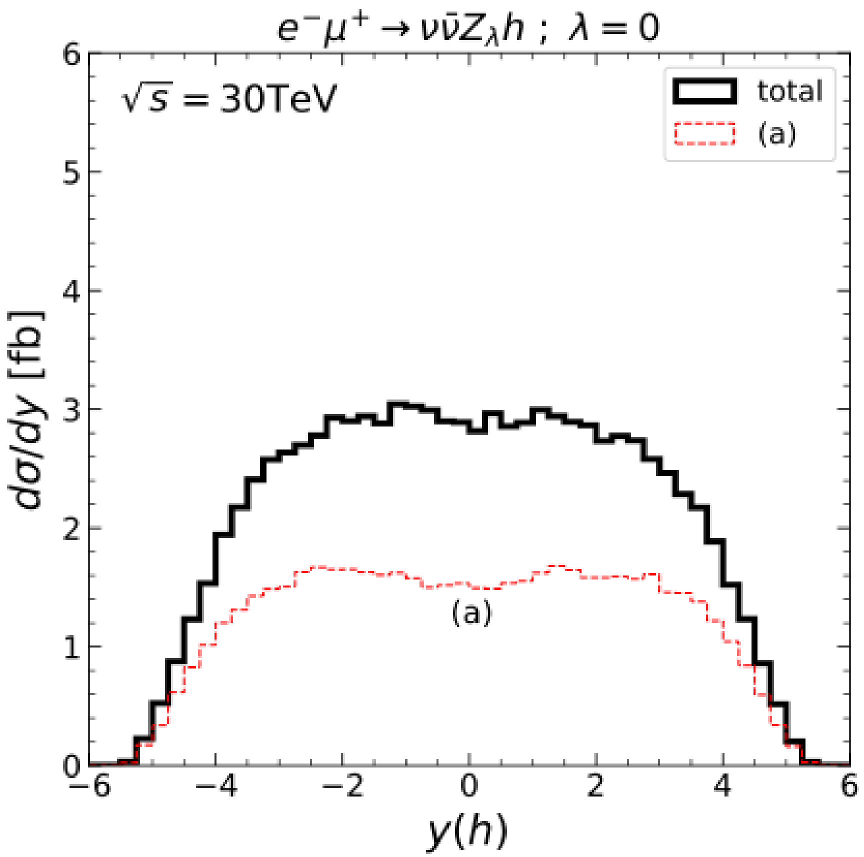




What causes the difference in the rapidity distributions of Z and Higgs?

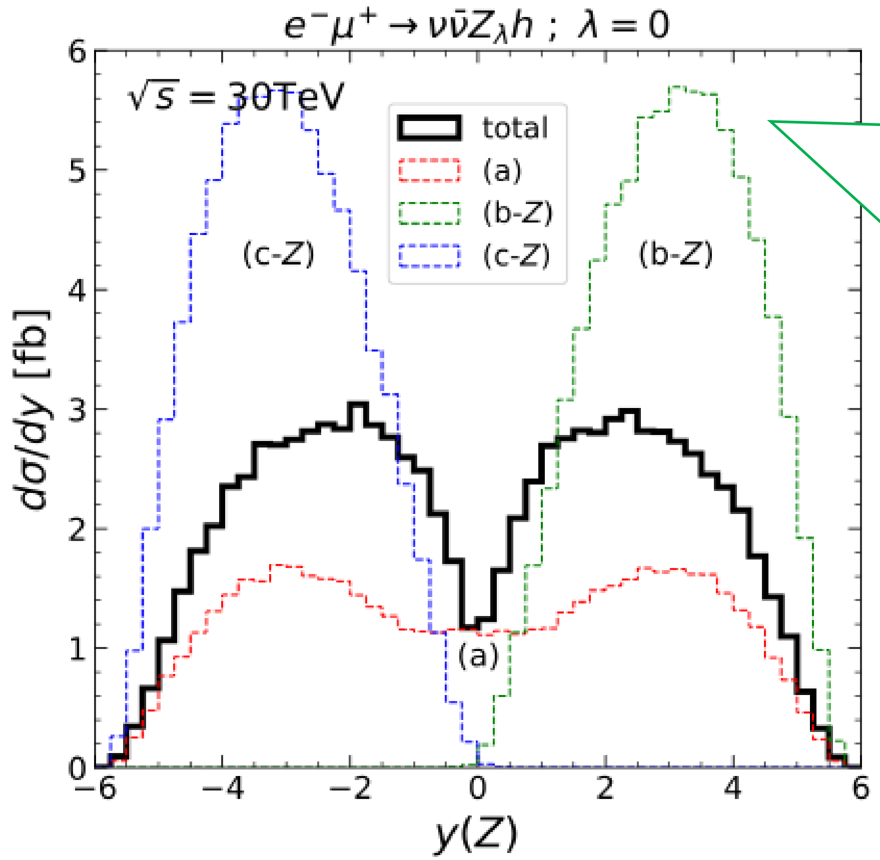
3. $e^-\mu^+ \to \nu_e\bar{\nu}_\mu Zh$

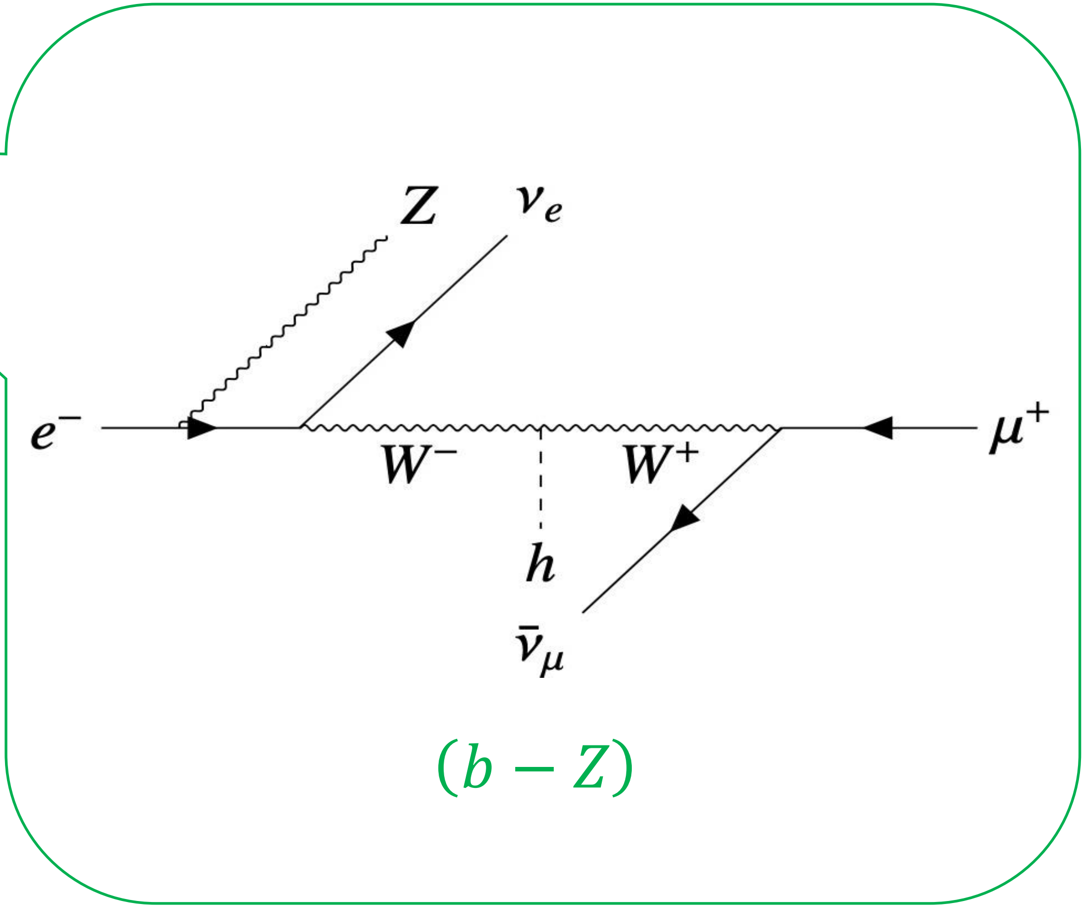




In the y(Z) and y(h), (a) are nearly identical and isotropic.
→As both particles are produced in the same $W^-W^+ \to Zh$ subprocess.
At y~0, (a) in y(Z) dominates, while, (a) in y(h) never dominates.

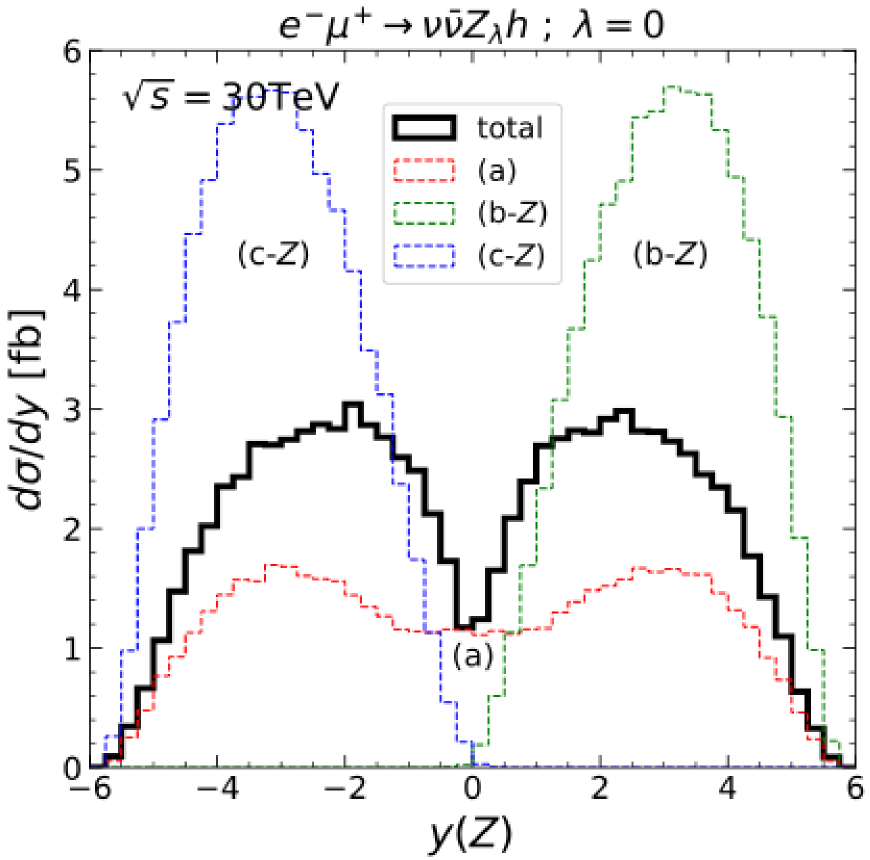
3. $e^- \mu^+ \to \nu_e \bar{\nu}_\mu Z h$

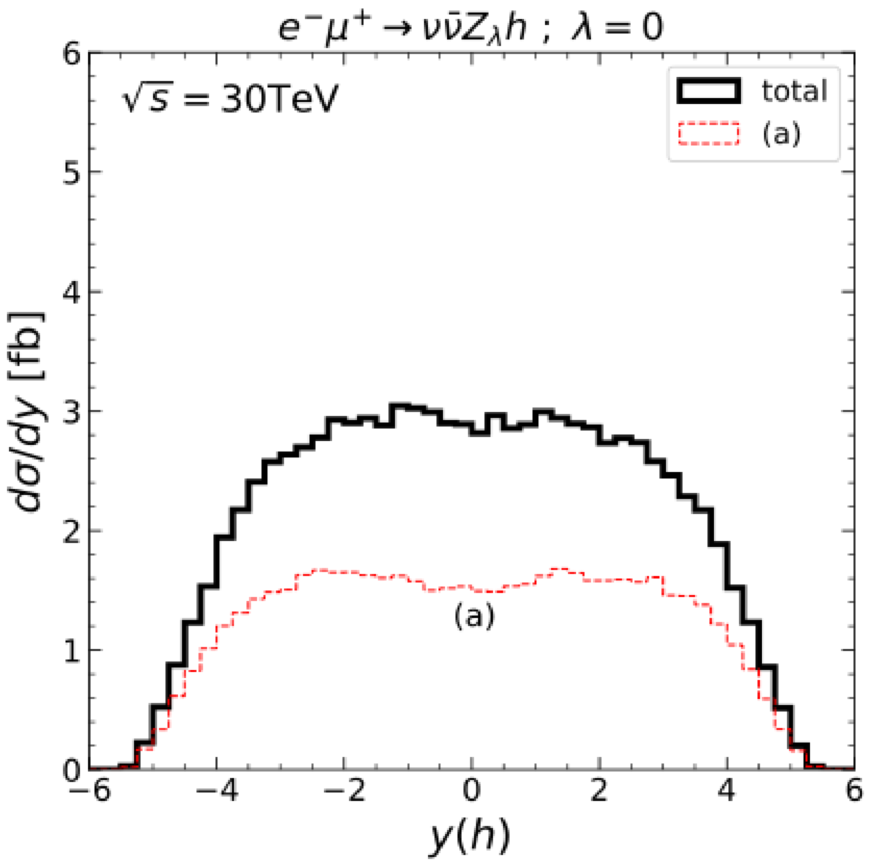




In y(Z),
Z is mainly emitted collinearly from the lepton lines, leading to a forward-peaked rapidity distribution.
→So $(b - Z)$ and $(c - Z)$ are distributed in forward and backward region, respectively.

3. $e^- \mu^+ \to \nu_e \bar{\nu}_\mu Z h$





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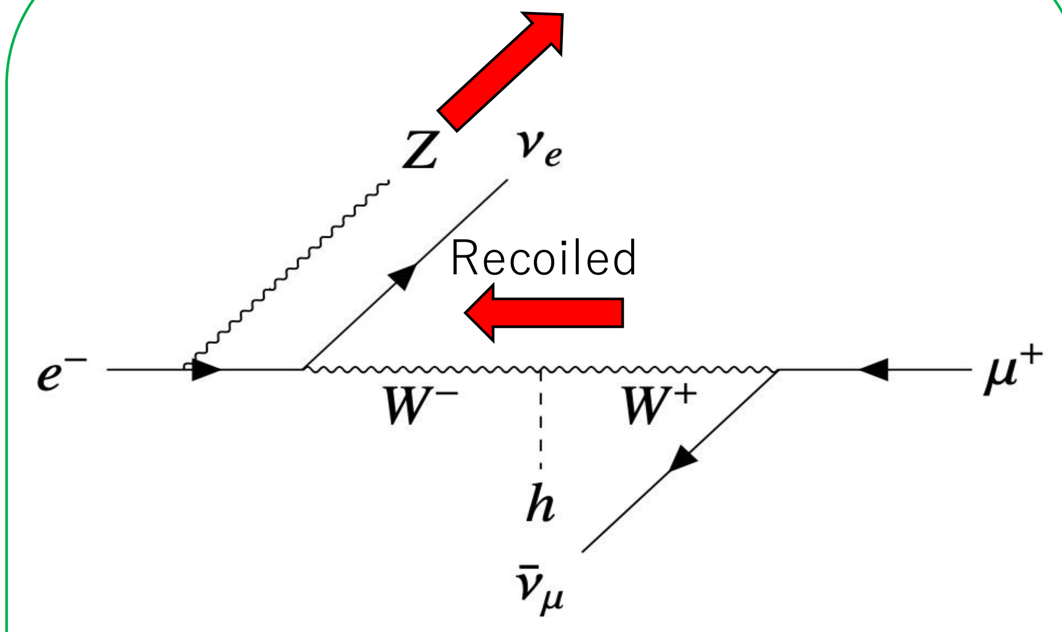
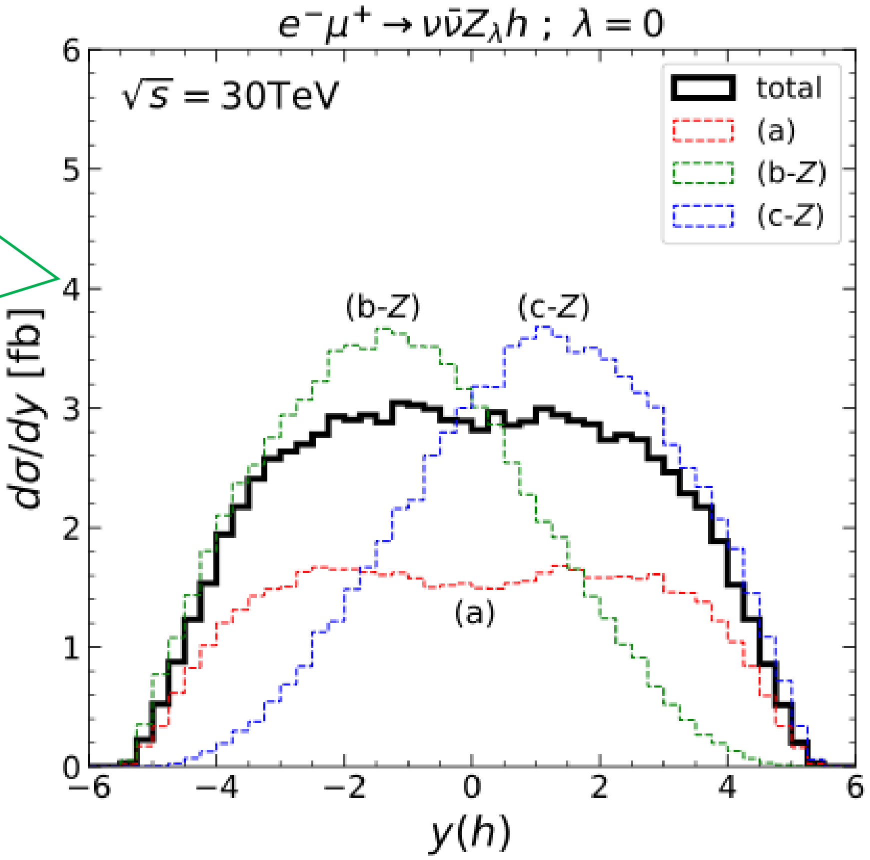
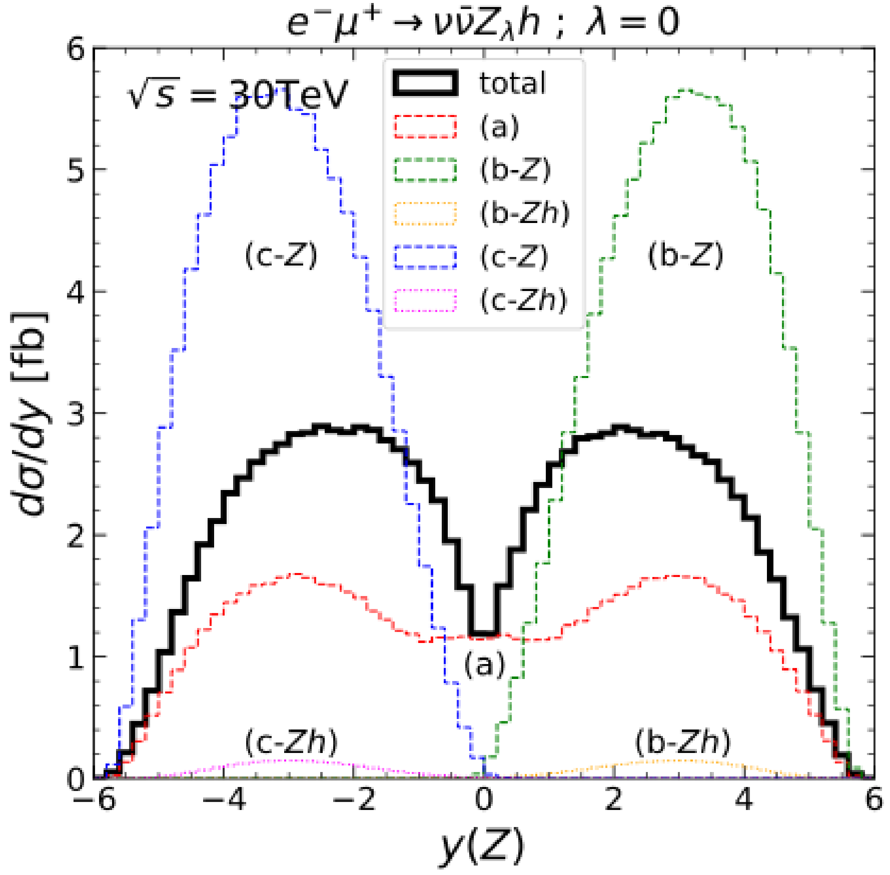


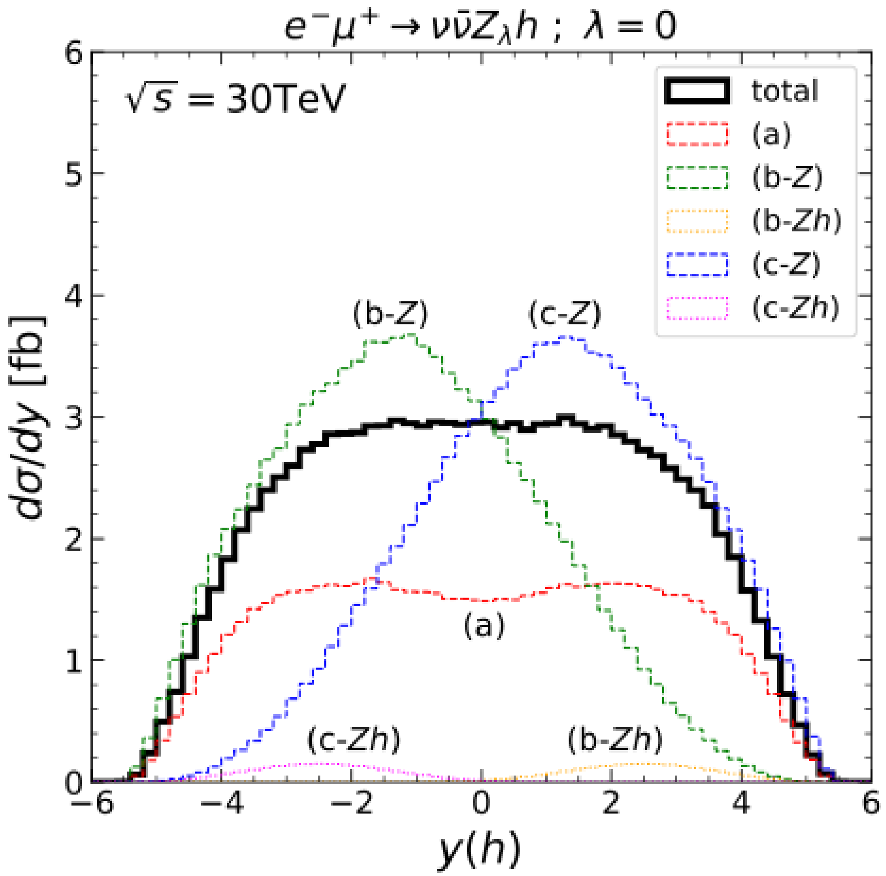
$(b-Z)$



In y(h),
$(b-Z)$ and $(c-Z)$ are swapped and they overlap significantly in the central region.
→Higgs via the VBF ($W^-W^+ \to h$) are isotropically distributed. The $W^-W^+ \to h$ system tends to be boosted into the opposite direction to Z, although the boost effect is not strong enough to completely separate the contributions from $(b-Z)$ and $(c-Z)$.

3. $e^-\mu^+ \to \nu_e\bar{\nu}_\mu Zh$





y(Z) :
The dominant amplitude contribution can be identified in each phase-space region.
Central : (a)
Forward : (a) and $(b-Z)$
Backward : (a) and $(c-Z)$

y(h) :
All amplitude groups, including their interference, contribute to the physical distribution.
→ All amplitude groups, including their interference, contribute to the physical distribution.

Contents

4. SMEFT

We study how the SMEFT contribution modifies each amplitude group in the FD gauge.
We introduce the dimension-6 operator O_{HW}.

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{c_{HW}}{\Lambda^2}\mathcal{O}_{HW}$$

$$\mathcal{O}_{HW} = H^\dagger H \; W^a_{\mu\nu} W^{a\;\mu\nu}$$

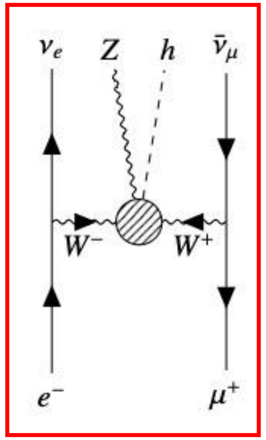
$$\supset vh\, W^a_{\mu\nu} W^{a\,\mu\nu}$$

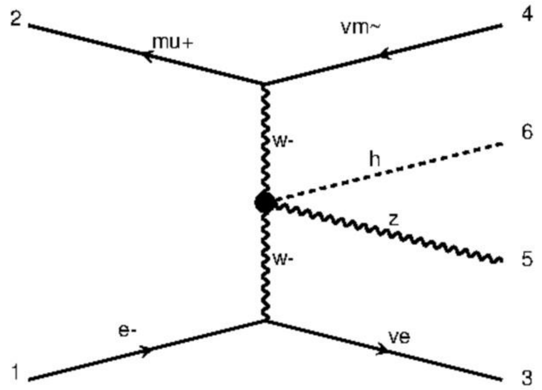
$$\tilde{c}_{HW} = \frac{c_{HW}}{\Lambda^2}$$

hWW vertex

$$\mathcal{V}^{\mu\nu}_{HW} \propto \tilde{c}_{HW}\left[(q_1\cdot q_2)g^{\mu\nu} - q_1^\nu q_2^\mu\right]$$

How does this modify in the FD gauge?





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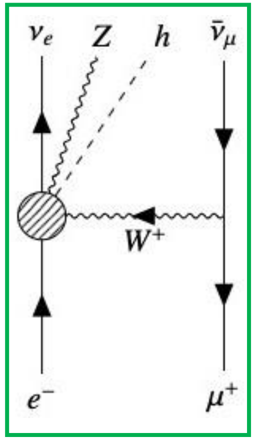

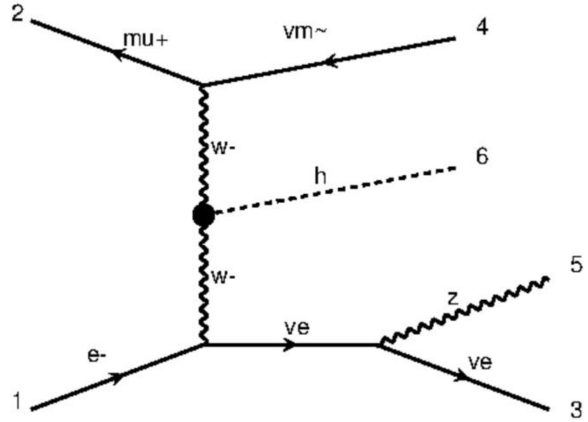

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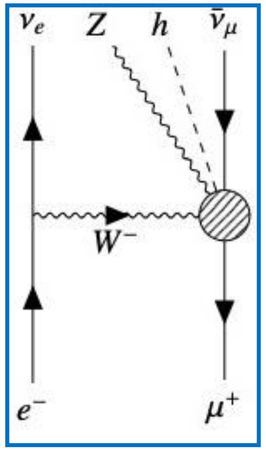

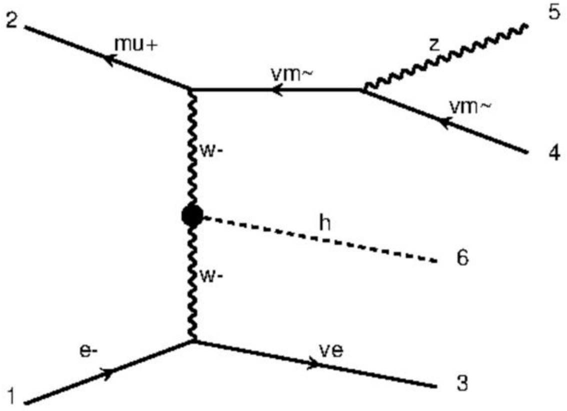

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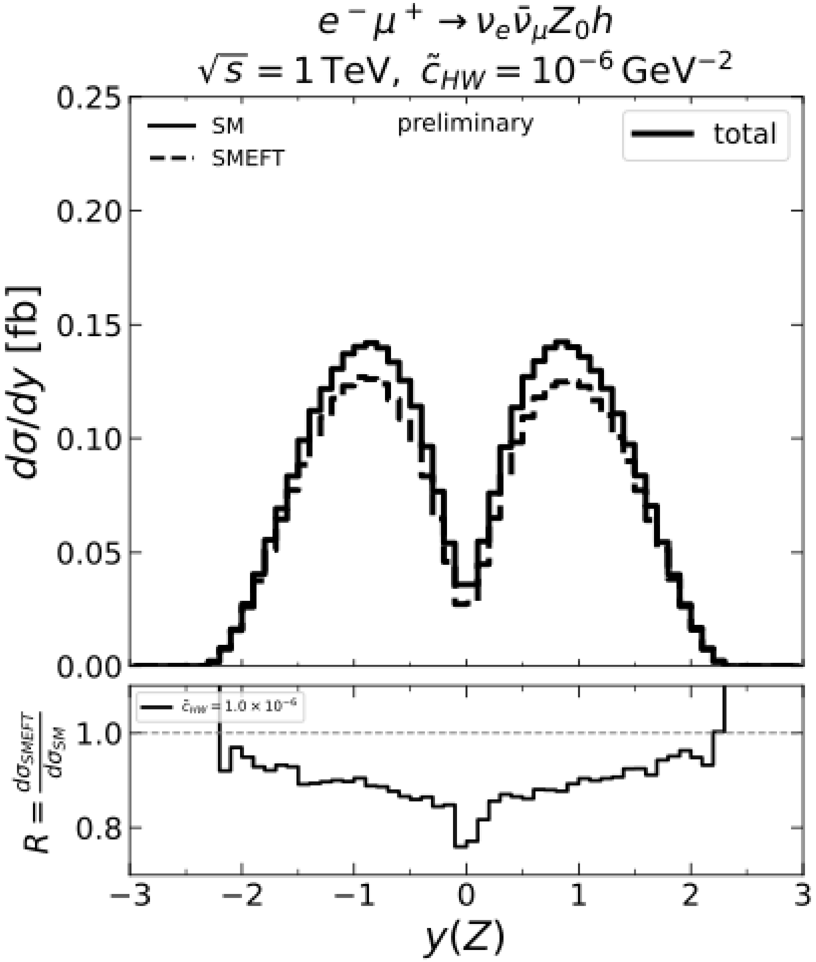
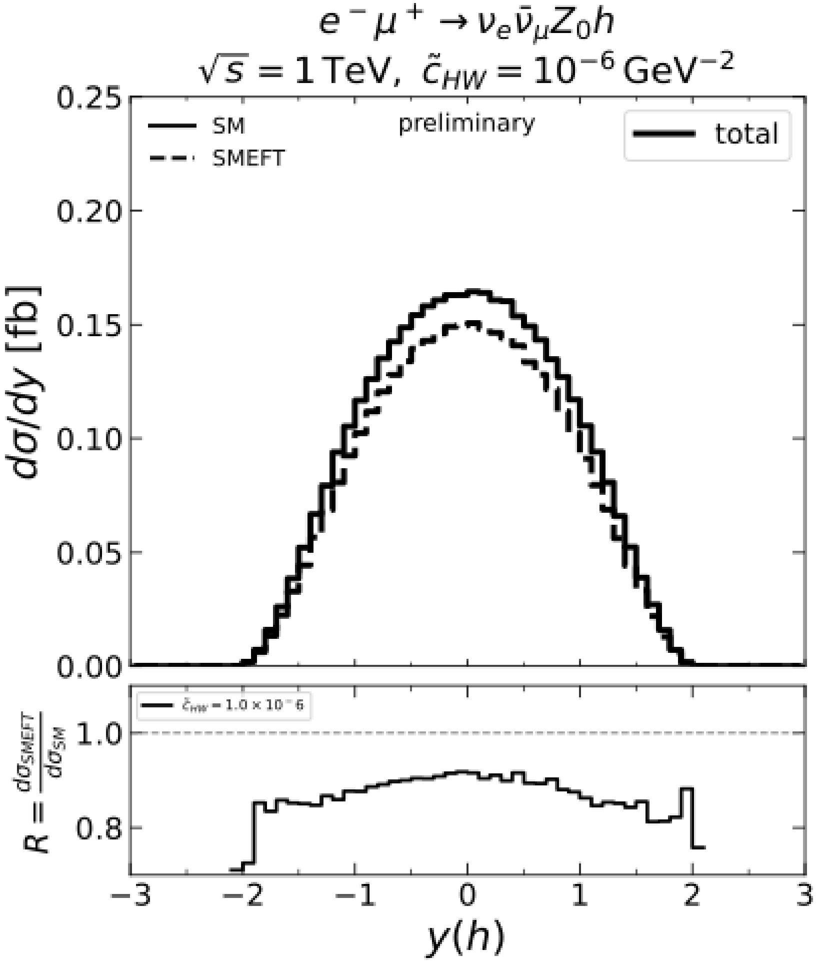
4. $e^-\mu^+ \to \nu_e\bar{\nu}_\mu Zh$ in SMEFT

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{c_{HW}}{\Lambda^2}\mathcal{O}_{HW}, \quad \tilde{c}_{HW} = \frac{c_{HW}}{\Lambda^2},$$

$$\mathcal{O}_{HW} = H^\dagger H \, W^a_{\mu\nu} W^{a\,\mu\nu}$$

For $\tilde{c}_{HW} > 0$, the SMEFT contribution suppresses the cross section.
The deviation is particularly large in the central y(Z) region.

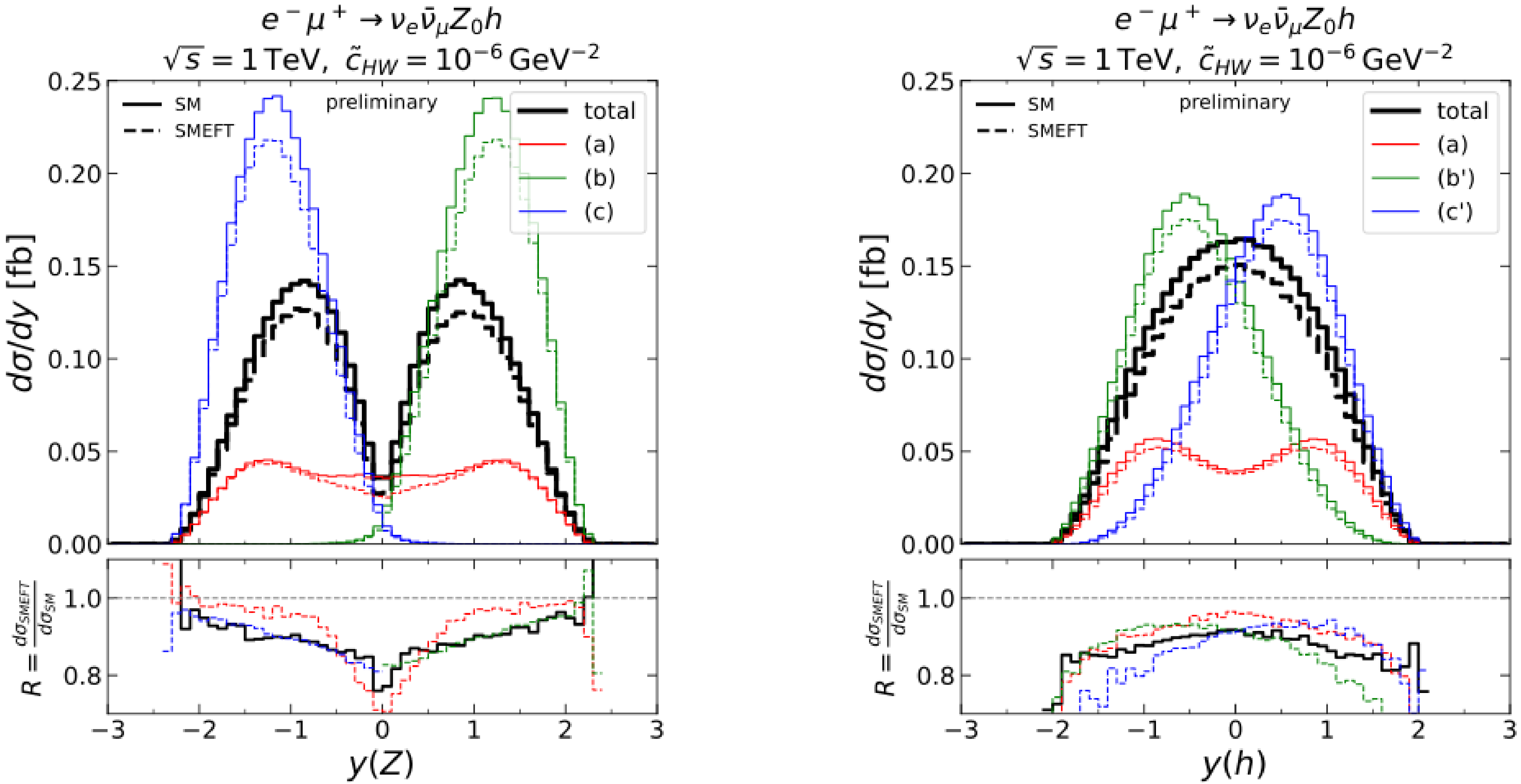




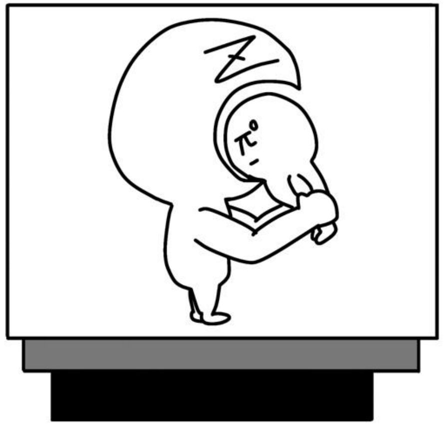
4. $e^- \mu^+ \to \nu_e \bar{\nu}_\mu Z h$ in SMEFT

$$\mathcal{V}^{\mu\nu}_{HW} \propto \tilde{c}_{HW} \left[(q_1 \cdot q_2) g^{\mu\nu} - q_1^\nu q_2^\mu \right]$$

The FD gauge reveals which amplitude is responsible for the SMEFT effect in each phase-space region.
The largest deviation from the SM is observed in the central y(Z) region, where contribution (a) is dominant.



Contents



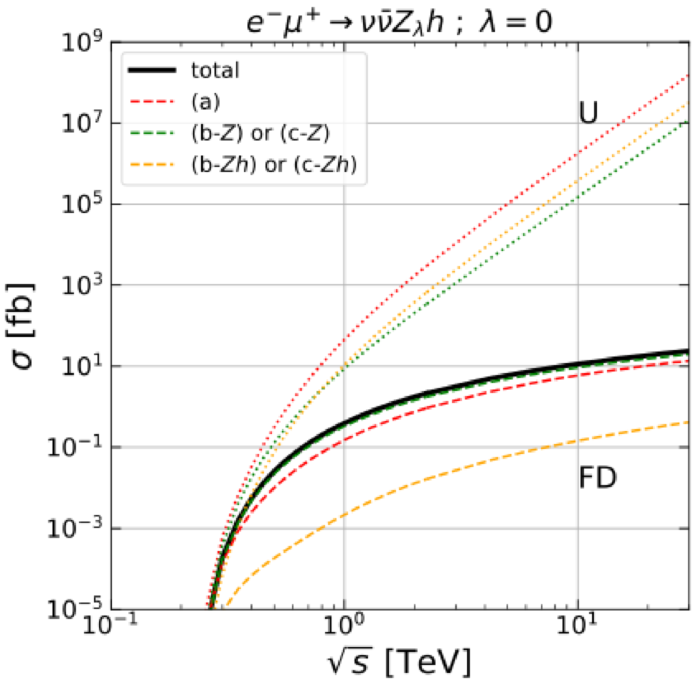
4. Summary

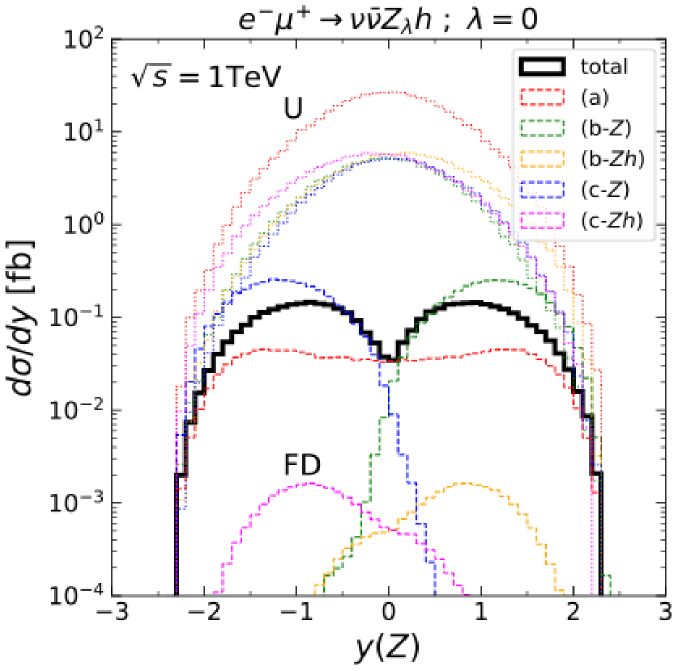
We studied $e^-\mu^+ \to \nu\bar{\nu}Zh$.
We found

U gauge:
- Each category of the amplitudes has energy growth.
  → Large cancellation among the three groups
  ((a),$(b-Z)$,$(c-Z)$, $(b-Zh)$, $(c-Zh)$) at high energies.
- No clue for the physical distribution from each contribution.

FD gauge :
- No such energy growth of each category at all.
- Naive expectations for the contribution of each group agree.
  In particular, it becomes clear which diagrams dominate in specific regions of phase space.

SMEFT :
- We introduced O_{HW} into this process.
- The FD gauge allows us to identify which individual amplitudes are responsible for the SMEFT effects observed in physical distributions.



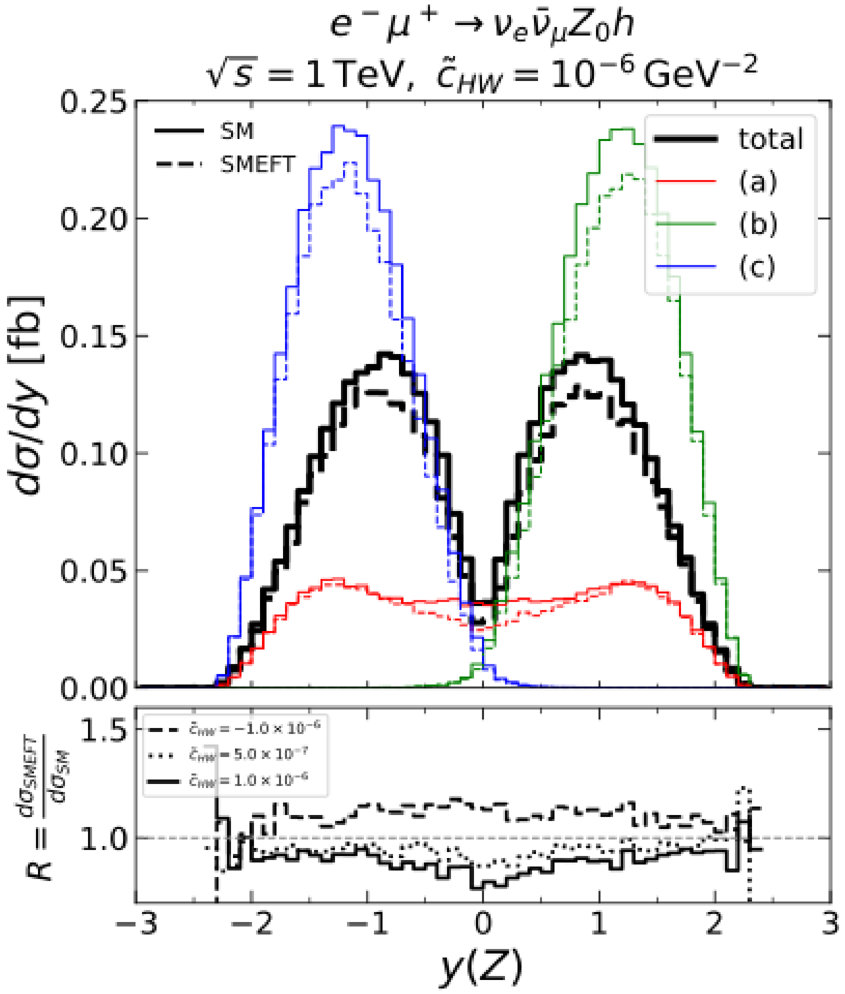


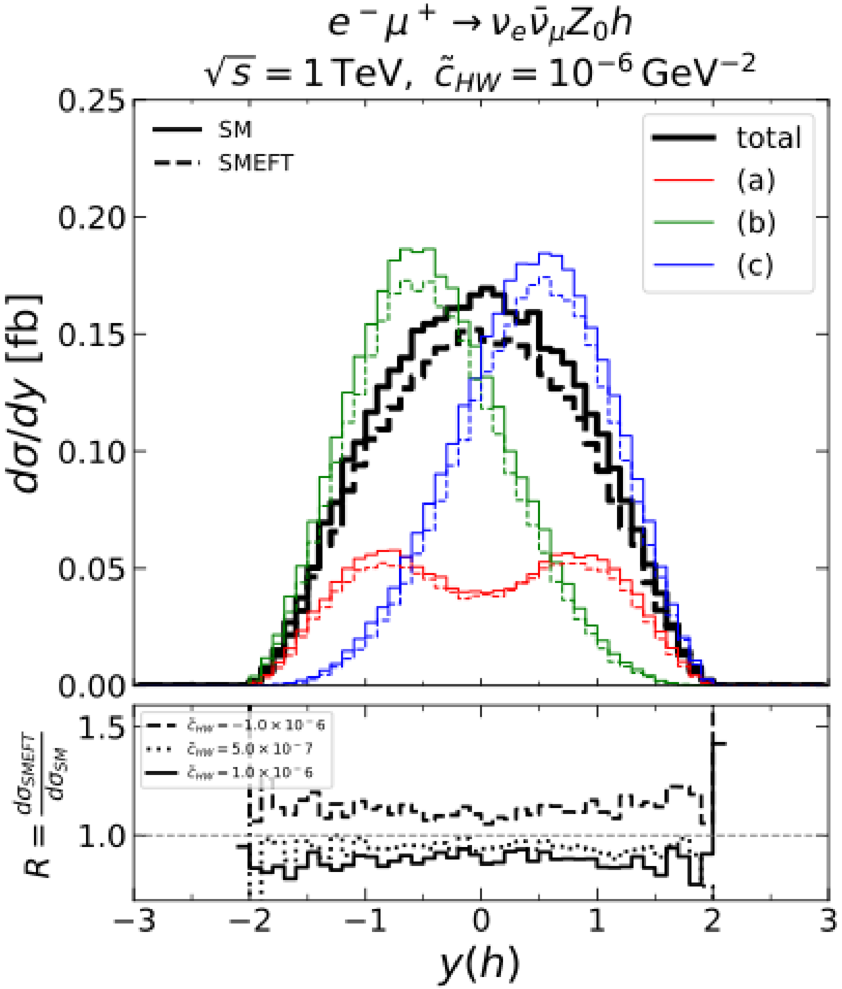
Back up

4. $e^-\mu^+ \to \nu_e\bar{\nu}_\mu Zh$ in SMEFT

$$d\sigma = d\sigma_{\rm SM} + \tilde{c}_{HW}\, d\sigma_{\rm int} + \mathcal{O}(\tilde{c}_{HW}^2)$$

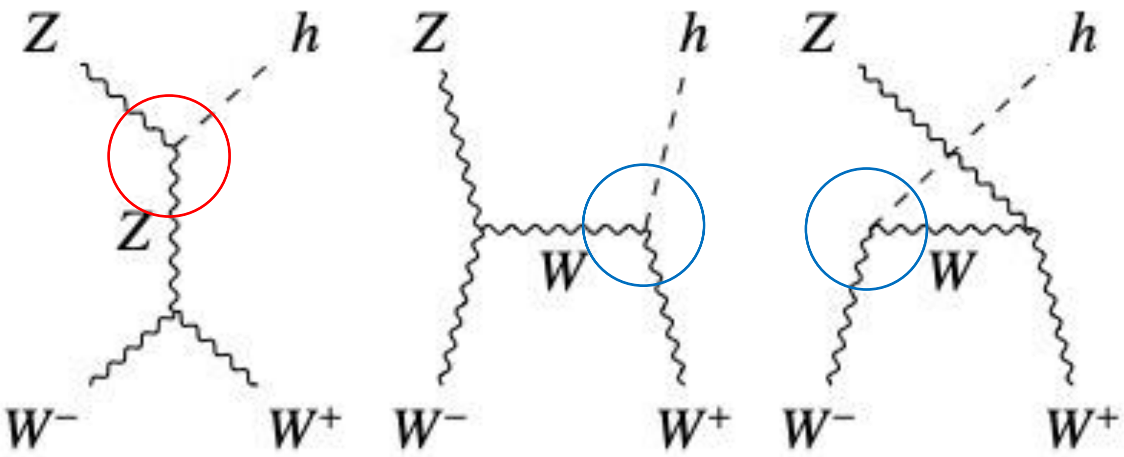
The sign and size of the deviation are controlled by $\tilde{c}_{HW}$.
The deviation changes sign for $\tilde{c}_{HW} \to -\tilde{c}_{HW}$ and decreases for smaller $|\tilde{c}_{HW}|$.





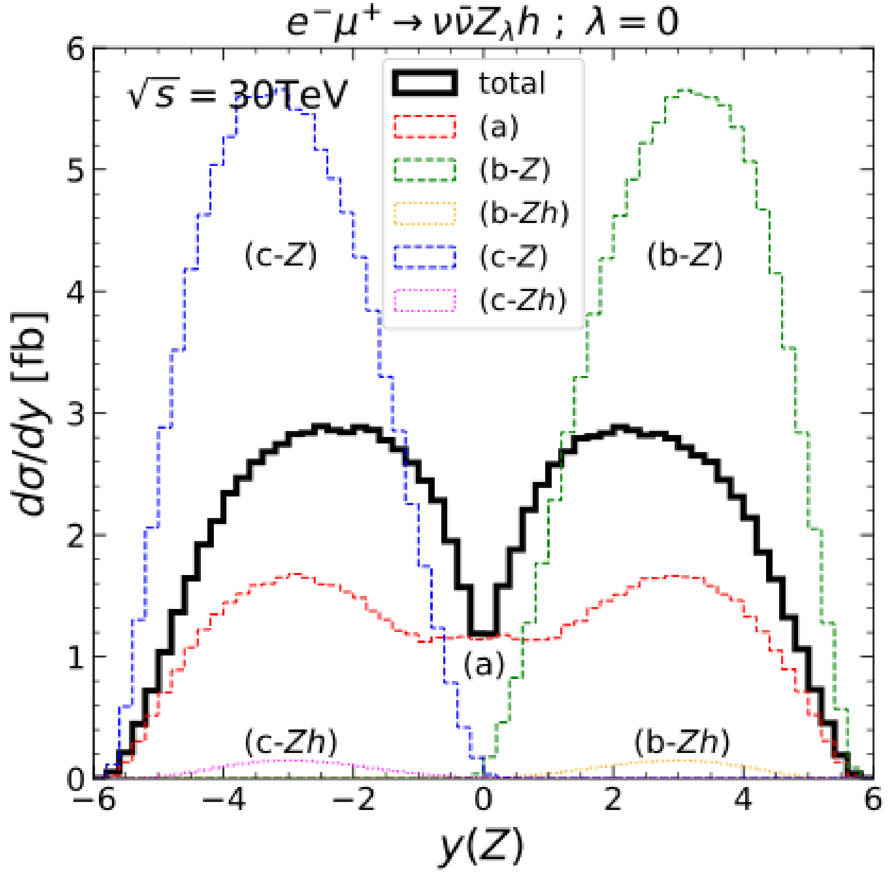
1. Introduction

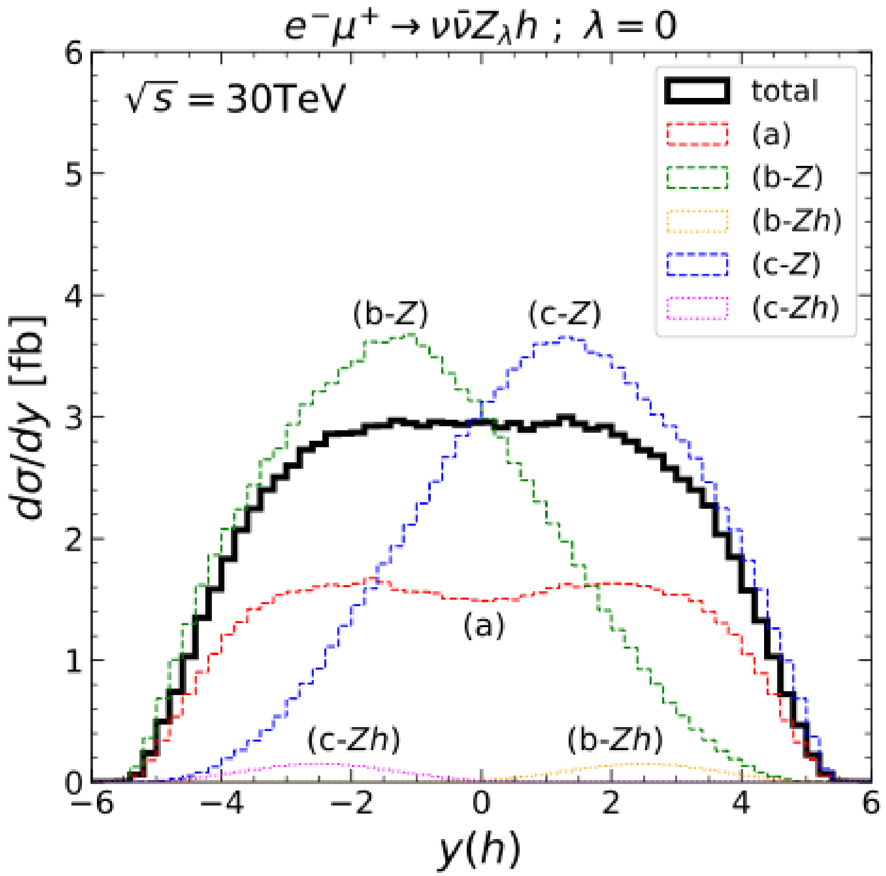
The discovery of the Higgs boson at the LHC completed the Standard Model.
However, there is much to understand about the Higgs boson.
→ **The origin of "Spontaneous Electroweak Symmetry Breaking"**
We focus on the
Vector-boson and the Higgs (Vh) production via Vector Boson Scattering (VBS).



This is an important process where the $\boxed{hZZ}$ and $\boxed{hWW}$ couplings can be determined simultaneously.

3. $e^- \mu^+ \to \nu_e \bar{\nu}_\mu Z h$



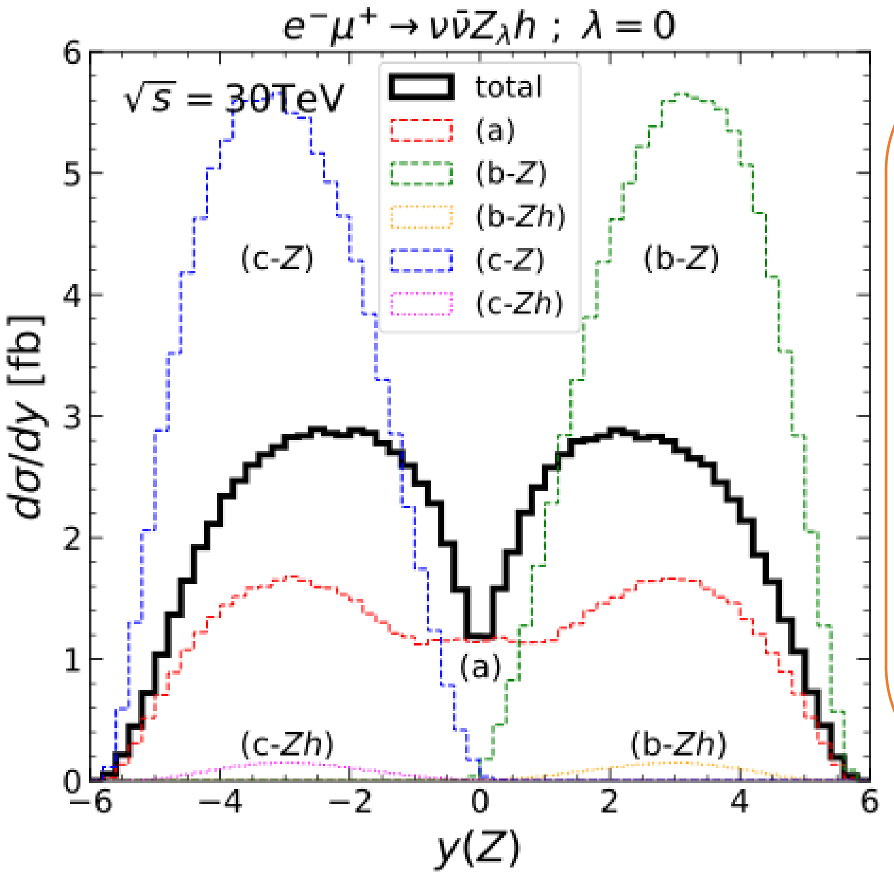


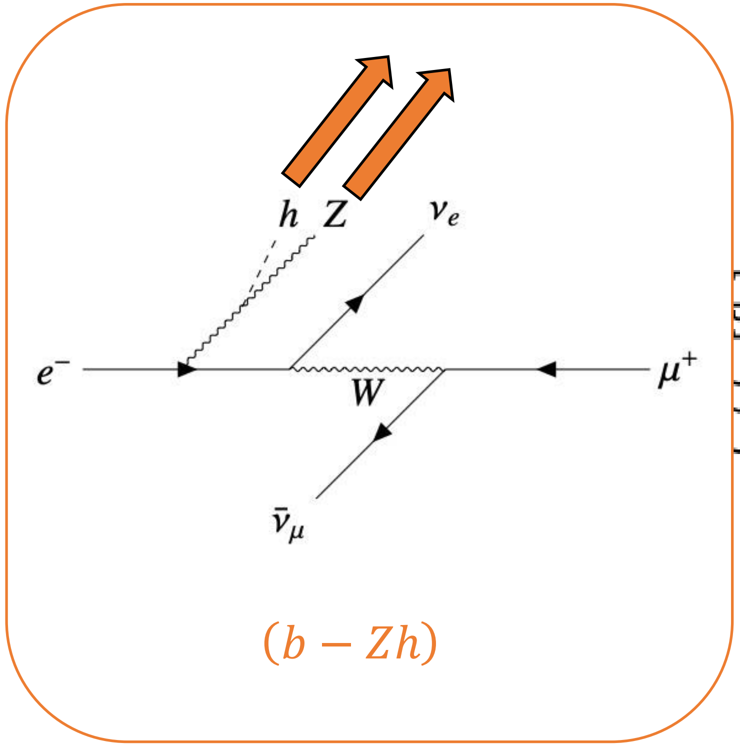
Rapidity distributions in the FD gauge

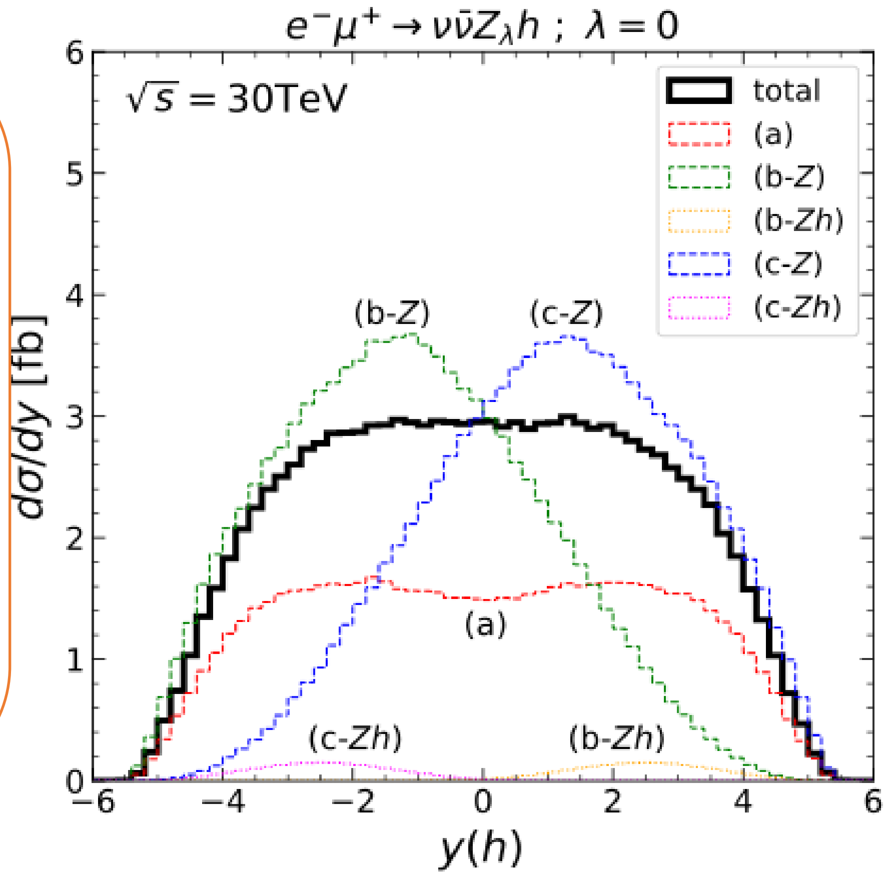
y(Z): The central , forward, and backward regions are dominated by (a), $(b - Z)$, and $(c - Z)$, respectively.
→ The dominant amplitude contribution can be identified in each phase-space region.

y(h): The contributions from (a), $(b - Z)$, and $(c - Z)$ largely overlap in the same phase-space region.
→ All amplitude groups, including their interference, contribute to the physical distribution.

3. $e^- \mu^+ \to \nu_e \bar{\nu}_\mu Z h$





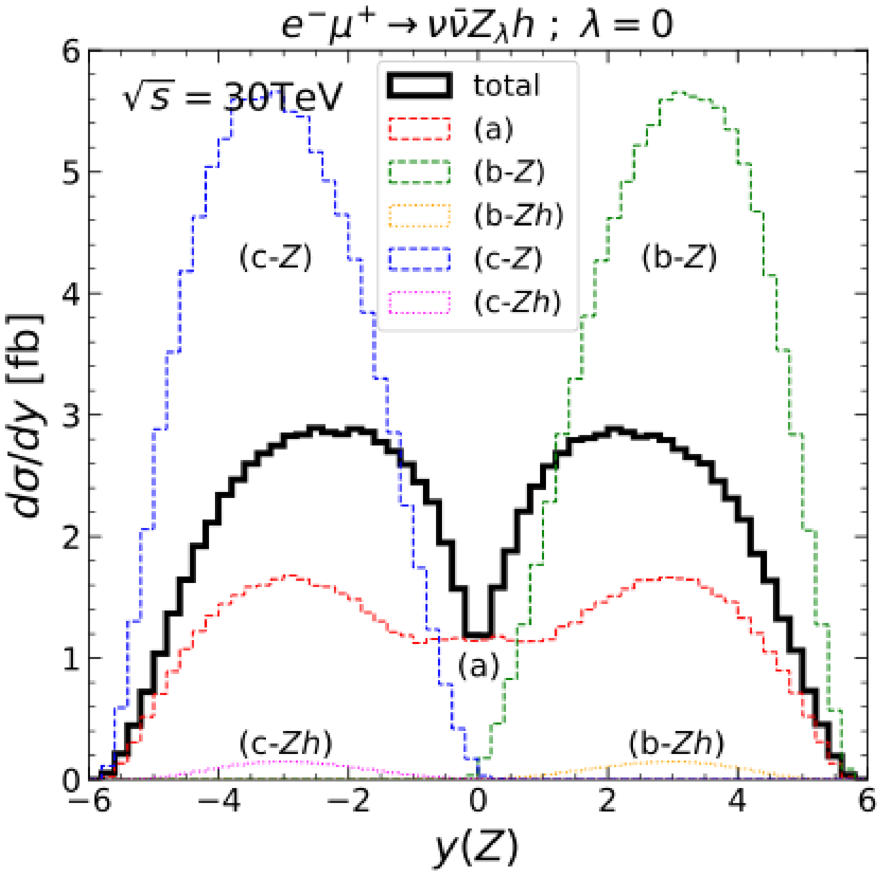


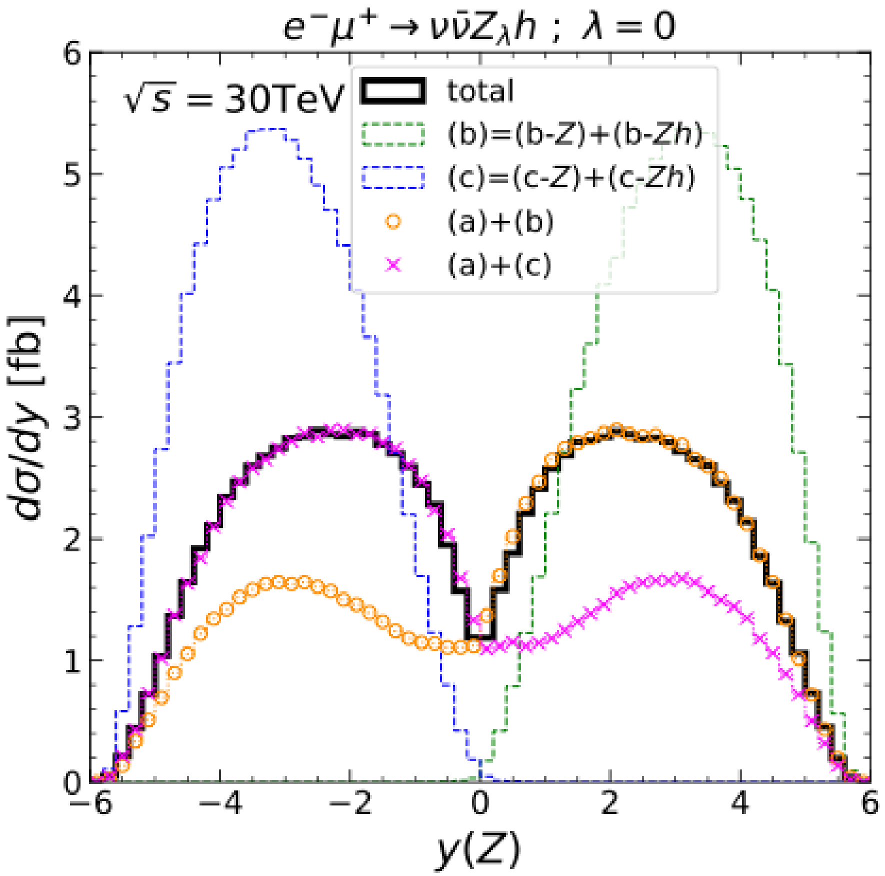
$(b - Zh)$ and $(c - Zh)$ are very similar in y(Z) and y(h).
→Z and Higgs from $(b - Zh)$ and $(c - Zh)$ are emitted together along the e^- line and the μ^+ line, respectively.

How can these contributions explain the physical observables?

3. $e^-\mu^+ \to \nu_e \bar{\nu}_\mu Zh$

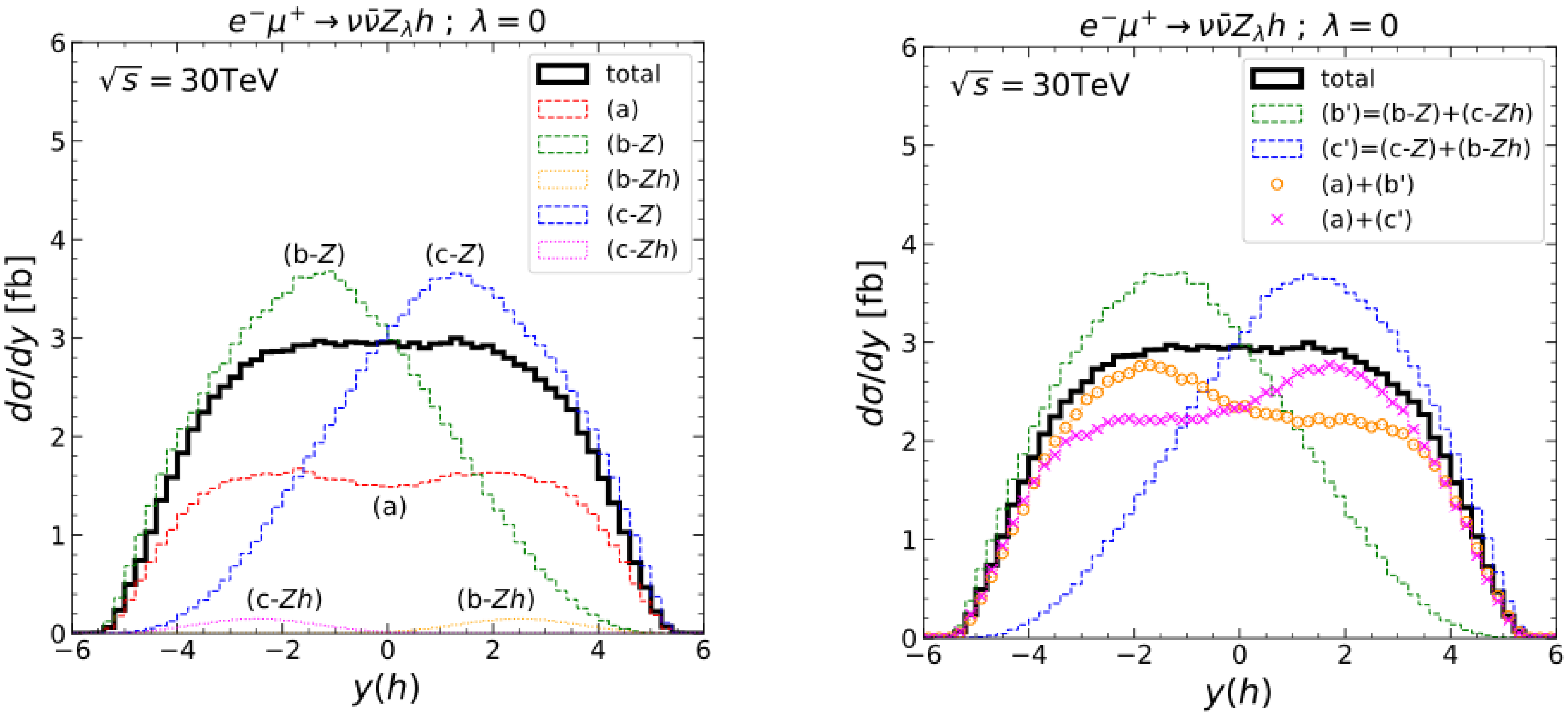
$$(a) + (b) = (a) + (b-Z) + (b-Zh)$$
$$\propto \left|M_{(a)} + M_{(b-Z)} + M_{(b-Zh)}\right|^2$$





Forward region : $(b) < (b-Z)$ →negative interference with $(b-Zh)$,
→Observable is described by the interference among
(a) and (b) ((a), $(b-Z)$ and $(b-Zh)$).
Backward region : Dominated by (a), $(c-Z)$ and $(c-Zh)$.

3. $e^-\mu^+ \to \nu_e \bar{\nu}_\mu Z h$

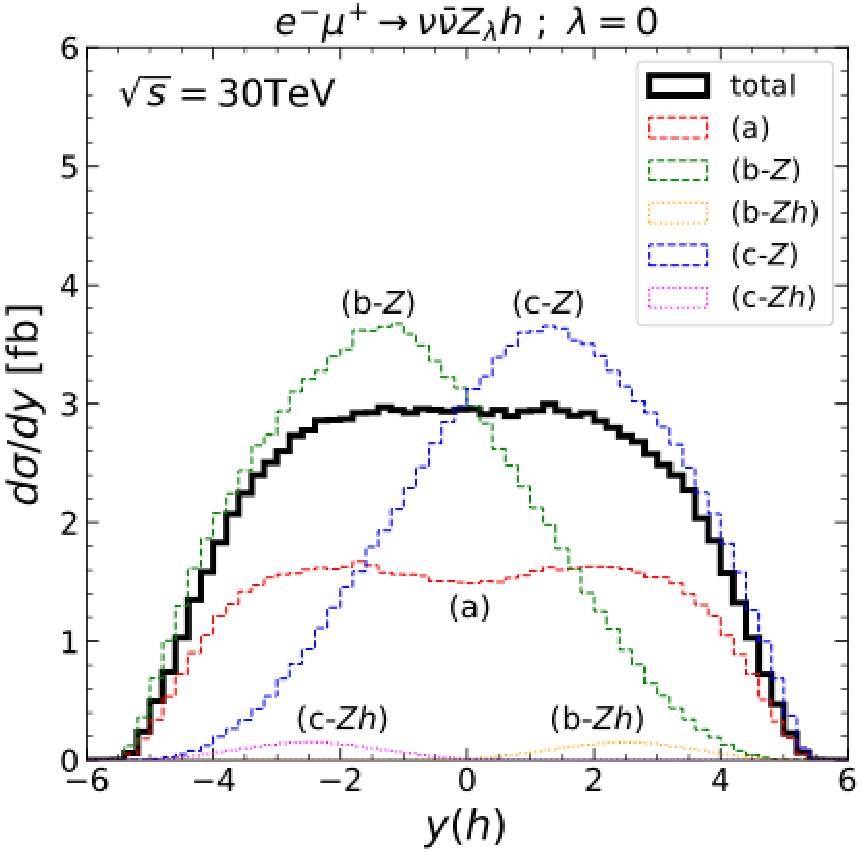


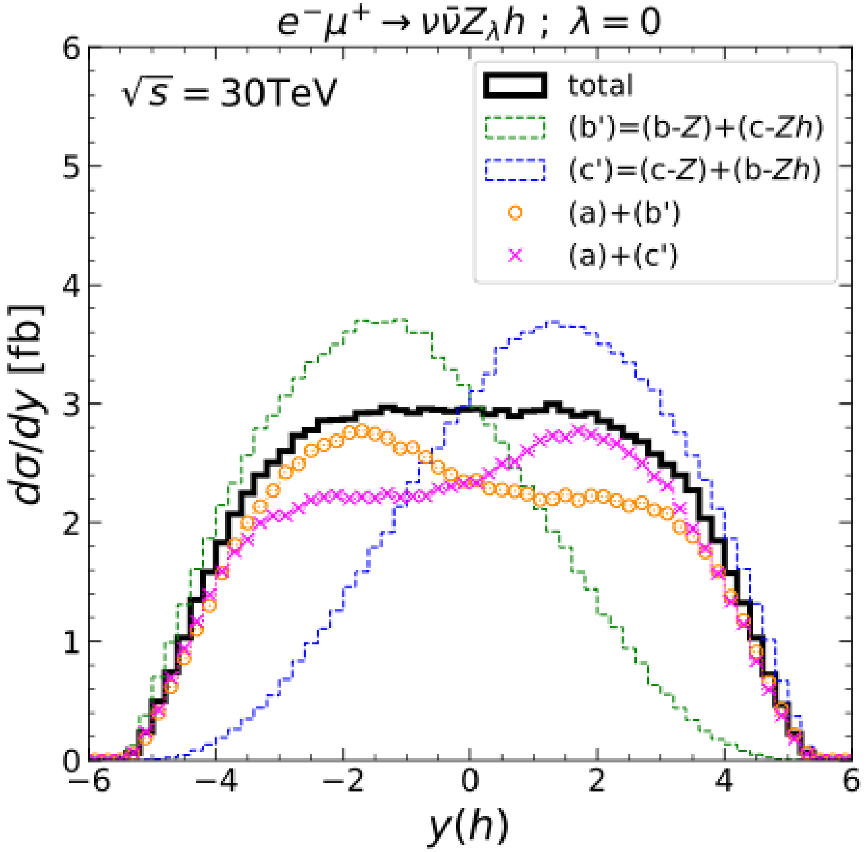
$(b') \sim (b-Z) \to$ small interference with $(c-Zh)$, interference between (a) and (b').

$$\sigma_{b'} \propto |\mathcal{M}_{b-Z} + \mathcal{M}_{c-Zh}|^2 \approx |\mathcal{M}_{b-Z}|^2 + |\mathcal{M}_{c-Zh}|^2$$

$(c') \sim (c-Z) \to$ small interference with $(b-Zh)$, interference between (a) and (c').

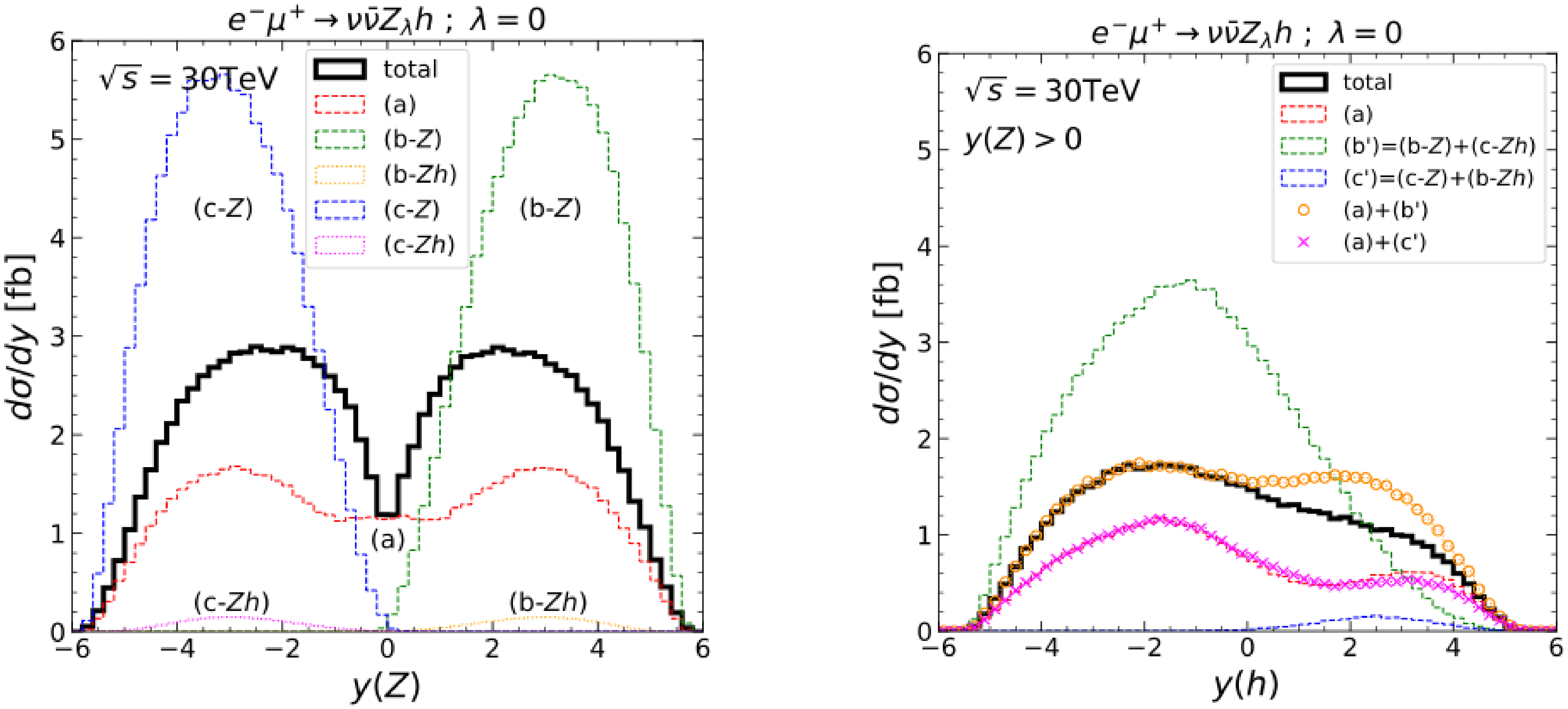
3. $e^- \mu^+ \to \nu_e \bar{\nu}_\mu Z h$





(a) and (b') cannot fully describe the observable distribution for y(h) $<$ 0.
Similarly, (a) and (c') for y(h)>0.
→Since (b') and (c') significantly overlap in y(h), unlike in y(Z).

3. $e^-\mu^+ \to \nu_e \bar{\nu}_\mu Z h$



・To remove the (c) contribution, we impose a kinematic cut $\boldsymbol{y(Z) > 0}$ based on y(Z).
→The backward region is completely dominated by $(a) + (b') \approx (a) + (b - Z)$.
・If we take the y(Z) < 0 cut, $(a) + (c')$ dominate in the forward region.

By using the FD gauge, appropriate kinematic cuts can be identified.

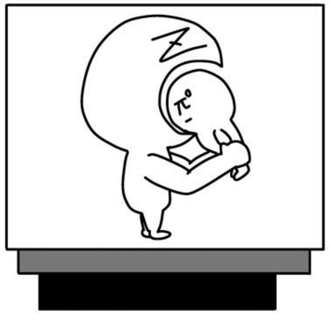
3. FD gauge

Electroweak
symmetry
breaking

NG boson

In the early universe,
gauge bosons run at the
speed of light (mass 0).

Unitary (U) gauge



Gauge bosons eat NG bosons and become fat
(run slower = get the mass).

Feynman-Diagram (FD) gauge

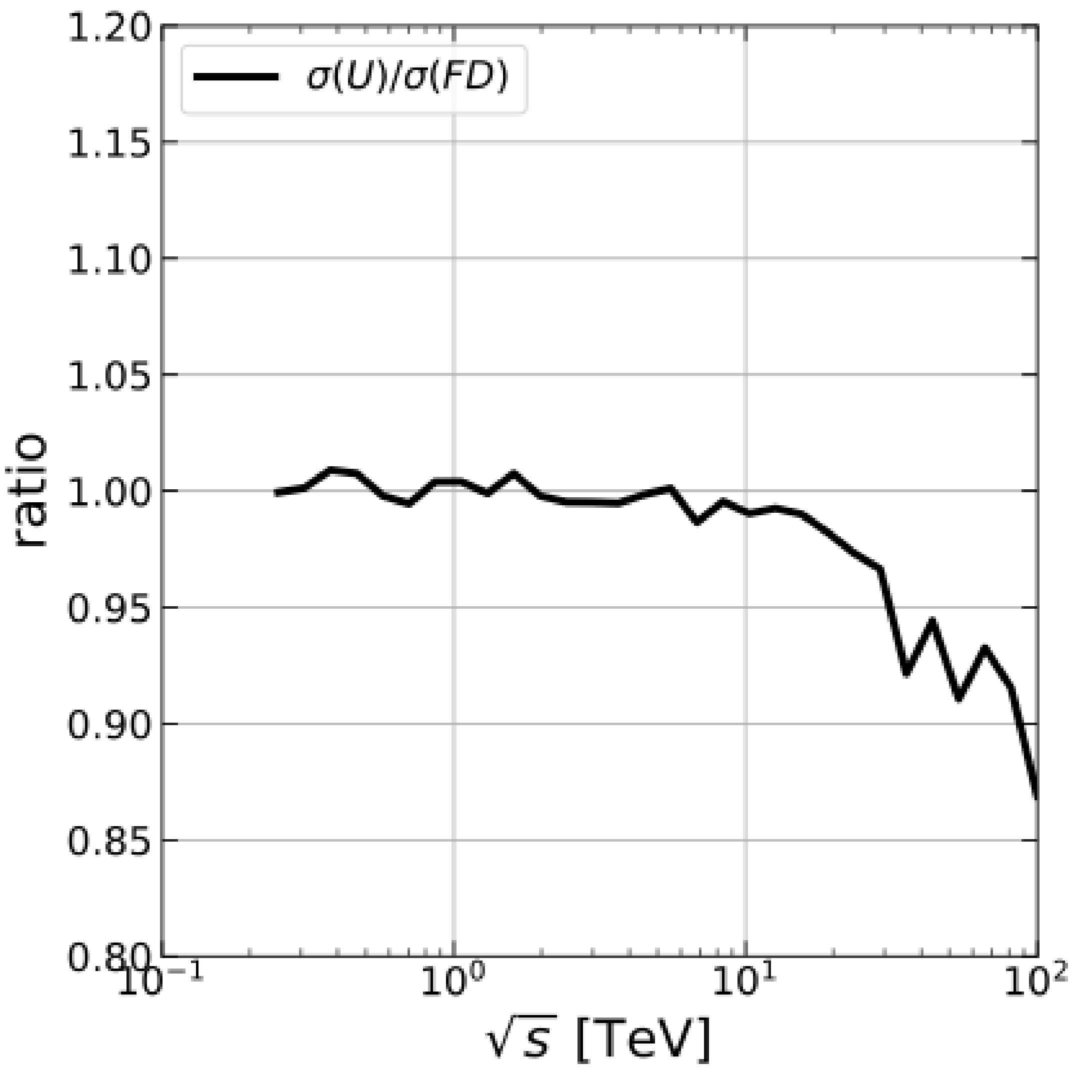
Three-legged race

Gauge bosons and NG bosons run together
(run slower = get the mass).

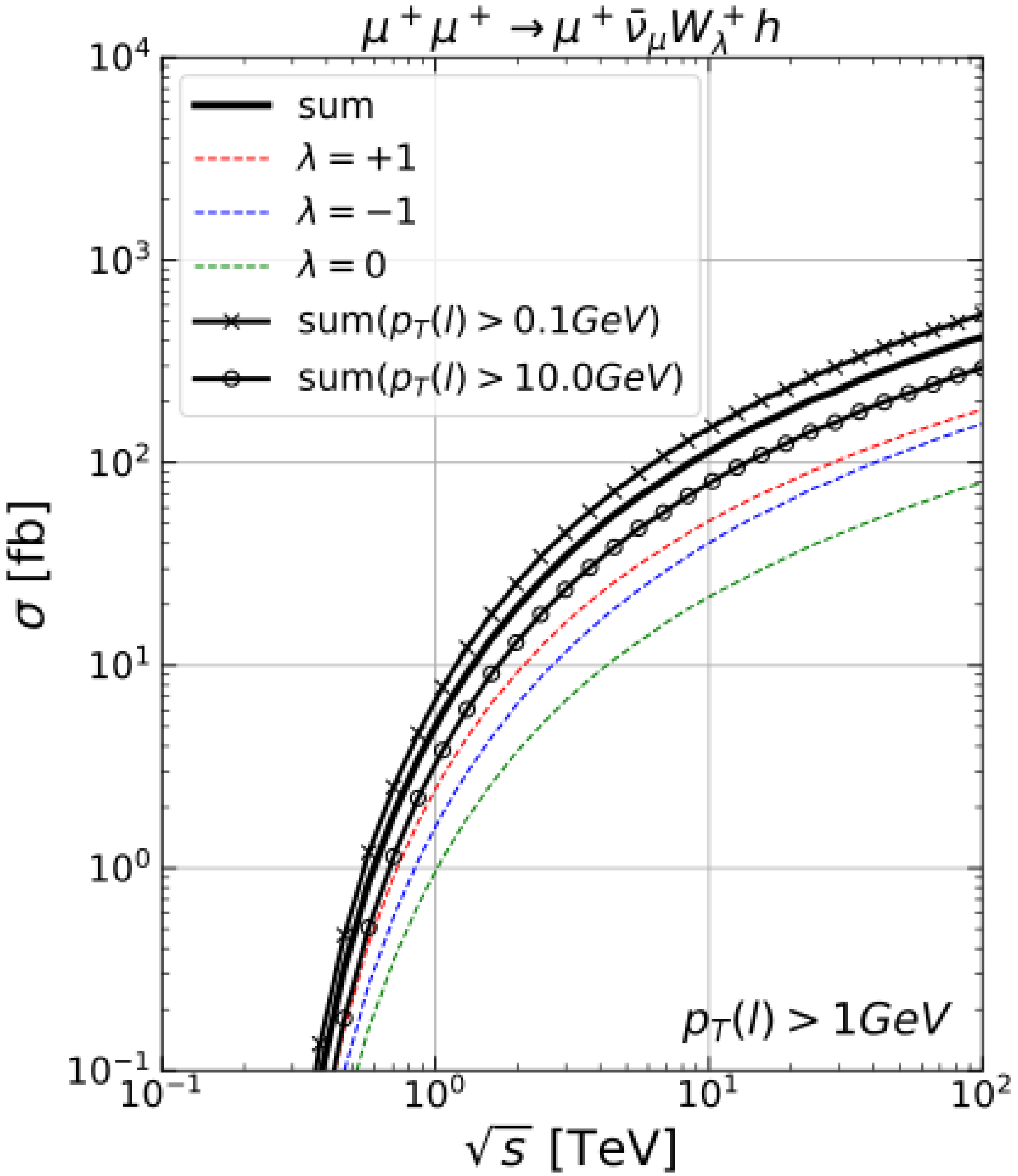
$\mu^+\mu^+ \to \mu^+\bar{\nu}_\mu W^+ h$

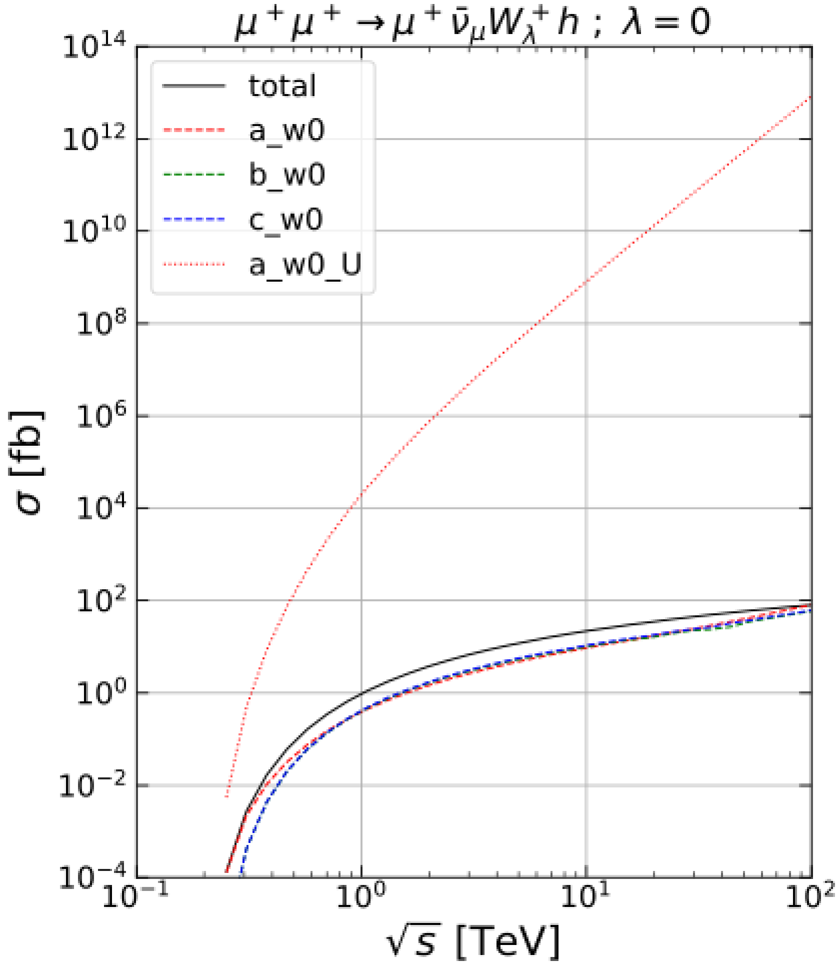
Gauge cancellations can lead to instabilities in Monte Carlo calculations, resulting in unreliable cross sections.

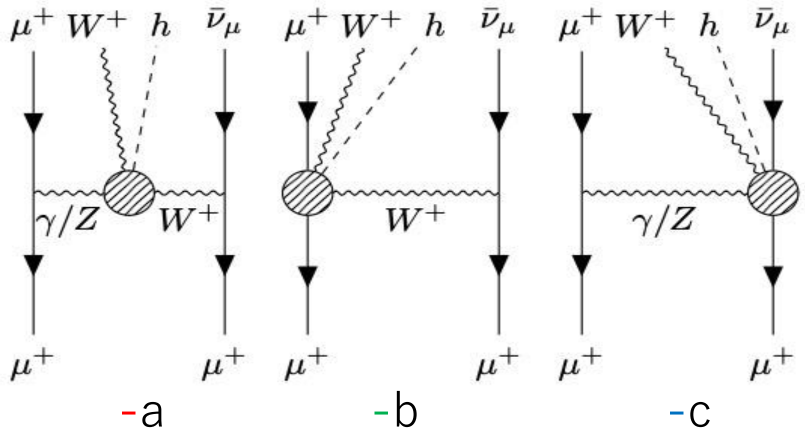
Run	Collider	Banner	Cross section (pb)	Events
full_w0_U_14	mu+ mu+ 1833.7946390713698 x 1833.7946390713698 GeV	tag_1	0.008521 ± 1.9e-05	10000
full_w0_U_15	mu+ mu+ 2254.638430806761 x 2254.638430806761 GeV	tag_1	0.0107 ± 2.7e-05	5762
full_w0_U_16	mu+ mu+ 2772.0631009396966 x 2772.0631009396966 GeV	tag_1	0.01309 ± 3.7e-05	597
full_w0_U_28	mu+ mu+ 33076.35093966178 x 33076.35093966178 GeV	tag_1	0.06202 ± 0.0012	21
full_w0_U_29	mu+ mu+ 40667.155629366185 x 40667.155629366185 GeV	tag_1	0.06725 ± 0.00055	178
full_w0_U_30	mu+ mu+ 50000.0 x 50000.0 GeV	tag_1	0.06911 ± 0.00047	180



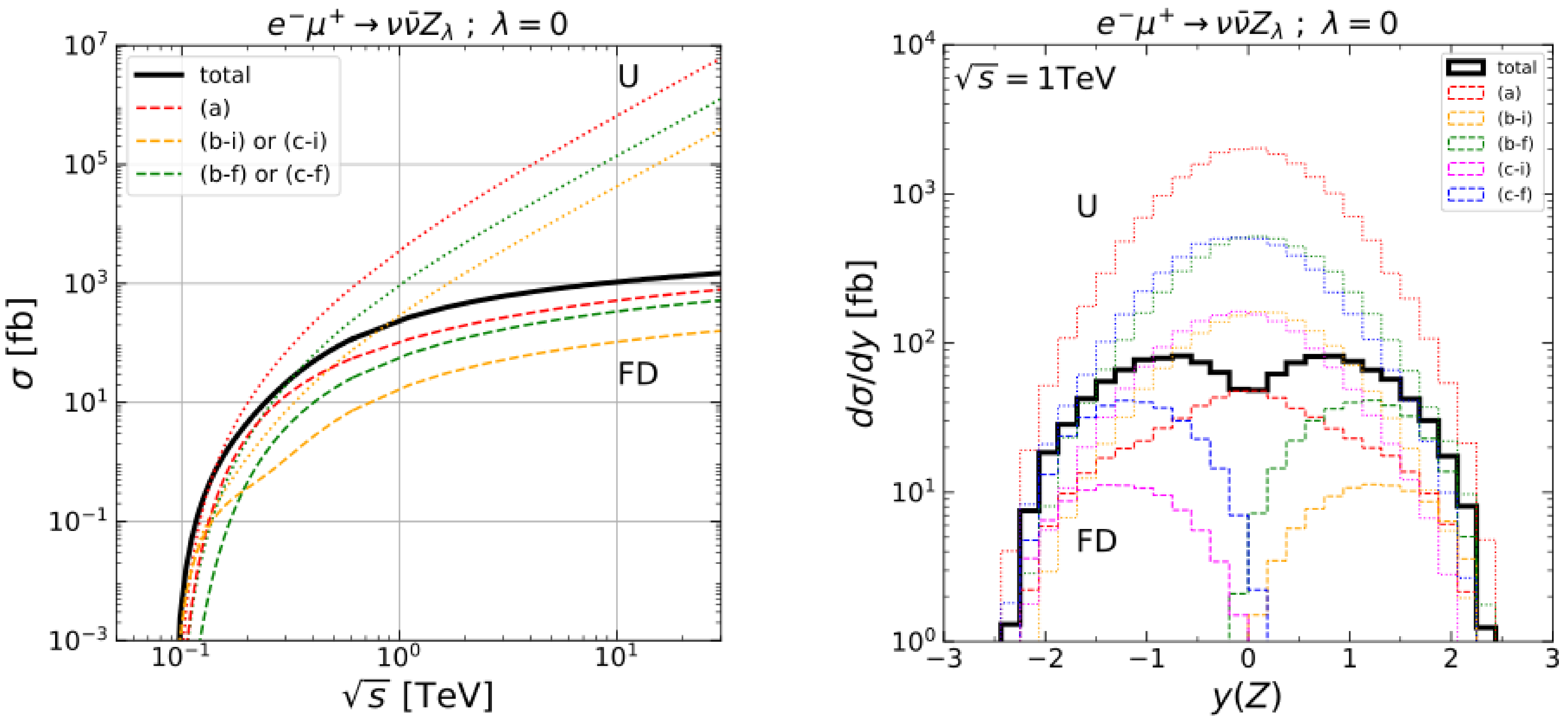
$\mu^+\mu^+ \to \mu^+\bar{\nu}_\mu W^+ h$



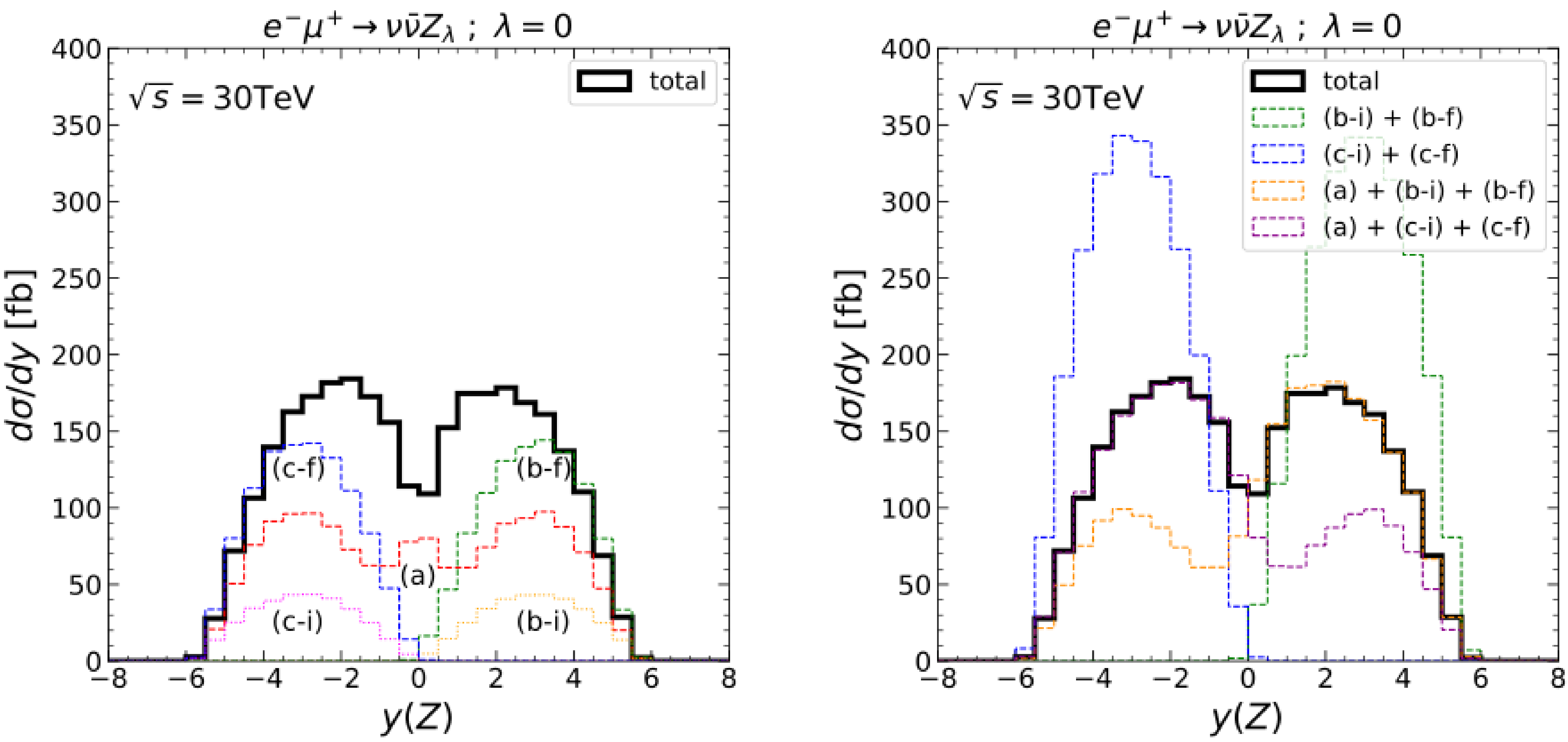




$$e^{-}\mu^{+} \to \nu_e \bar{\nu}_\mu Z$$



$e^- \mu^+ \to \nu\bar{\nu} Z_\lambda$; $\lambda = 0$
total
(a)
(b-i) or (c-i)
(b-f) or (c-f)
U
FD
σ [fb]
$\sqrt{s}$ [TeV]
$e^- \mu^+ \to \nu\bar{\nu} Z_\lambda$; $\lambda = 0$
$\sqrt{s} = 1\text{TeV}$
total
(a)
(b-i)
(b-f)
(c-i)
(c-f)
U
FD
$d\sigma/dy$ [fb]
$y(Z)$



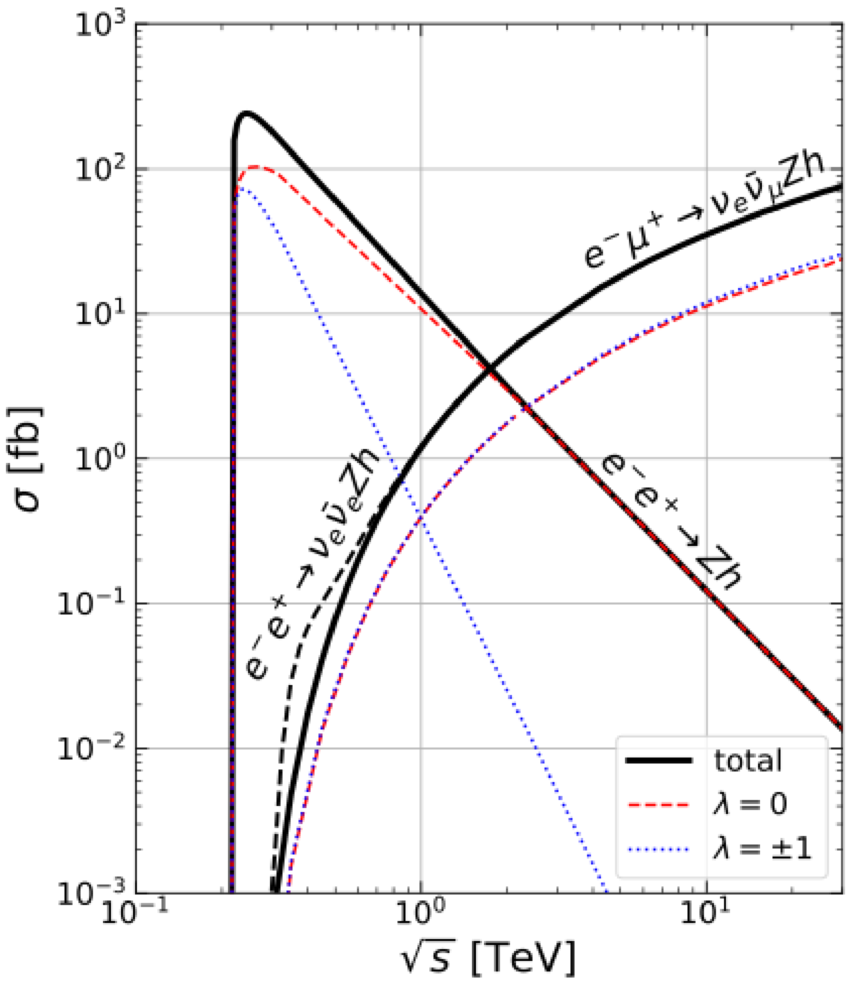
$e^-\mu^+ \to \nu\bar{\nu}Z_\lambda\ ;\ \lambda = 0$
$\sqrt{s} = 30\text{TeV}$
total
(c-f)
(b-f)
(a)
(c-i)
(b-i)
$d\sigma/dy$ [fb]
$y(Z)$
$e^-\mu^+ \to \nu\bar{\nu}Z_\lambda\ ;\ \lambda = 0$
$\sqrt{s} = 30\text{TeV}$
total
(b-i) + (b-f)
(c-i) + (c-f)
(a) + (b-i) + (b-f)
(a) + (c-i) + (c-f)
$d\sigma/dy$ [fb]
$y(Z)$

1. Introduction

Motivated by the Goldstone equivalence theorem, we focus on the longitudinal component; however, due to collinear enhancement in VBF, the transverse component gives a comparable contribution.

$$P_L \approx \frac{\alpha}{4\pi}\frac{1-x}{x}$$

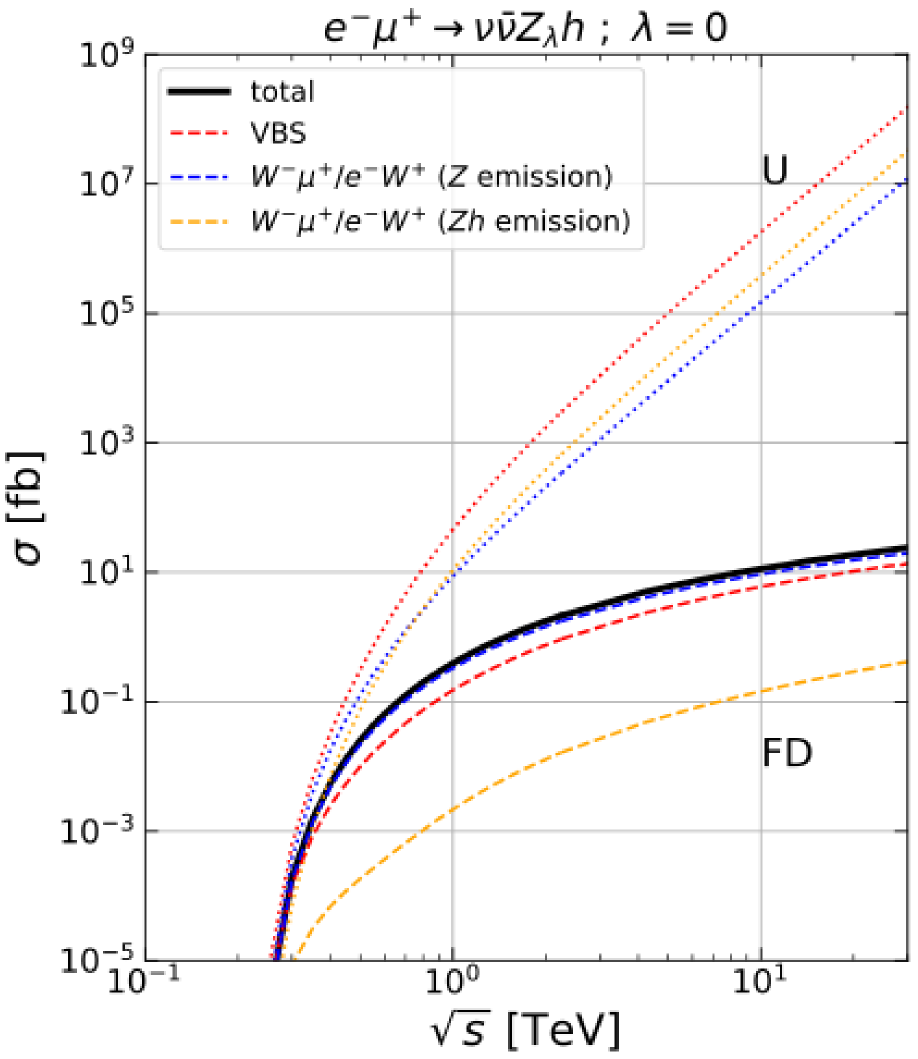
$$P_T \approx \frac{\alpha}{4\pi}\frac{1+(1-x)^2}{x}\log\left(\frac{s}{m_W^2}\right)$$

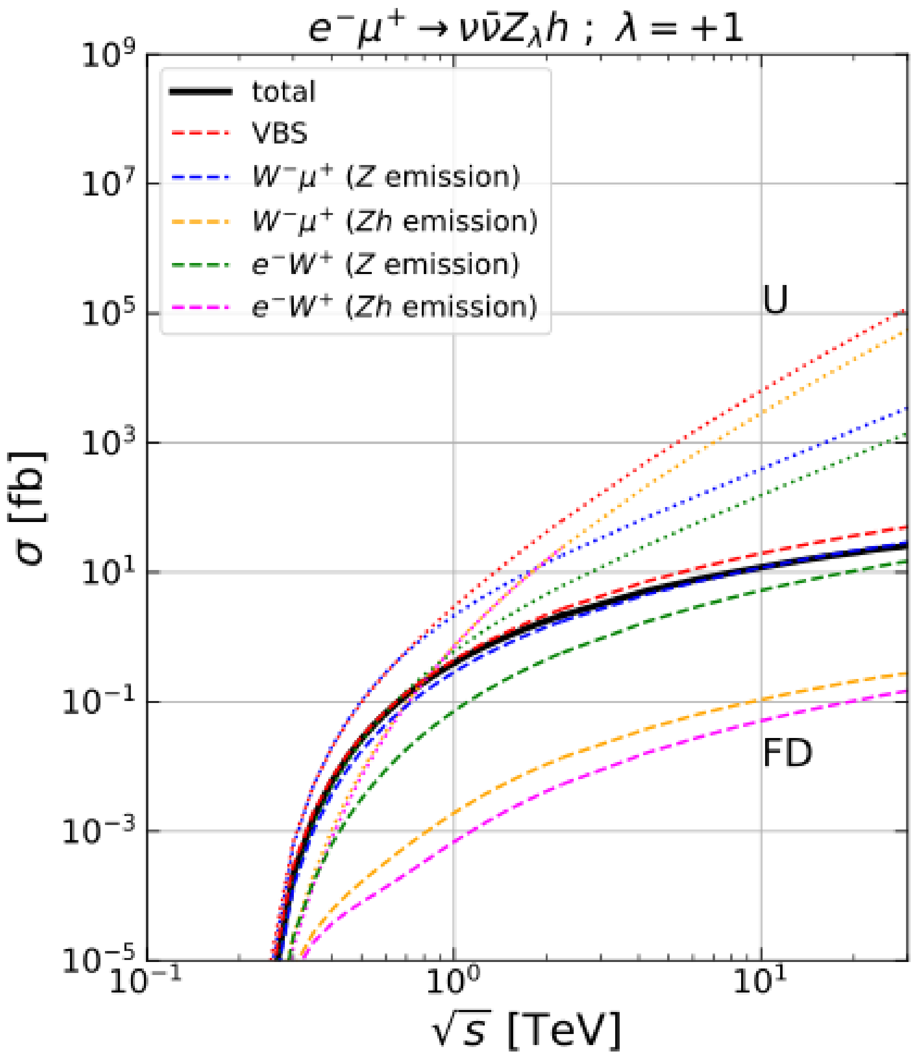


$l^-l^+ \to \nu\bar{\nu}Zh$ is important at high-energy lepton collider experiments to study Higgs coupling to the weak gauge bosons.

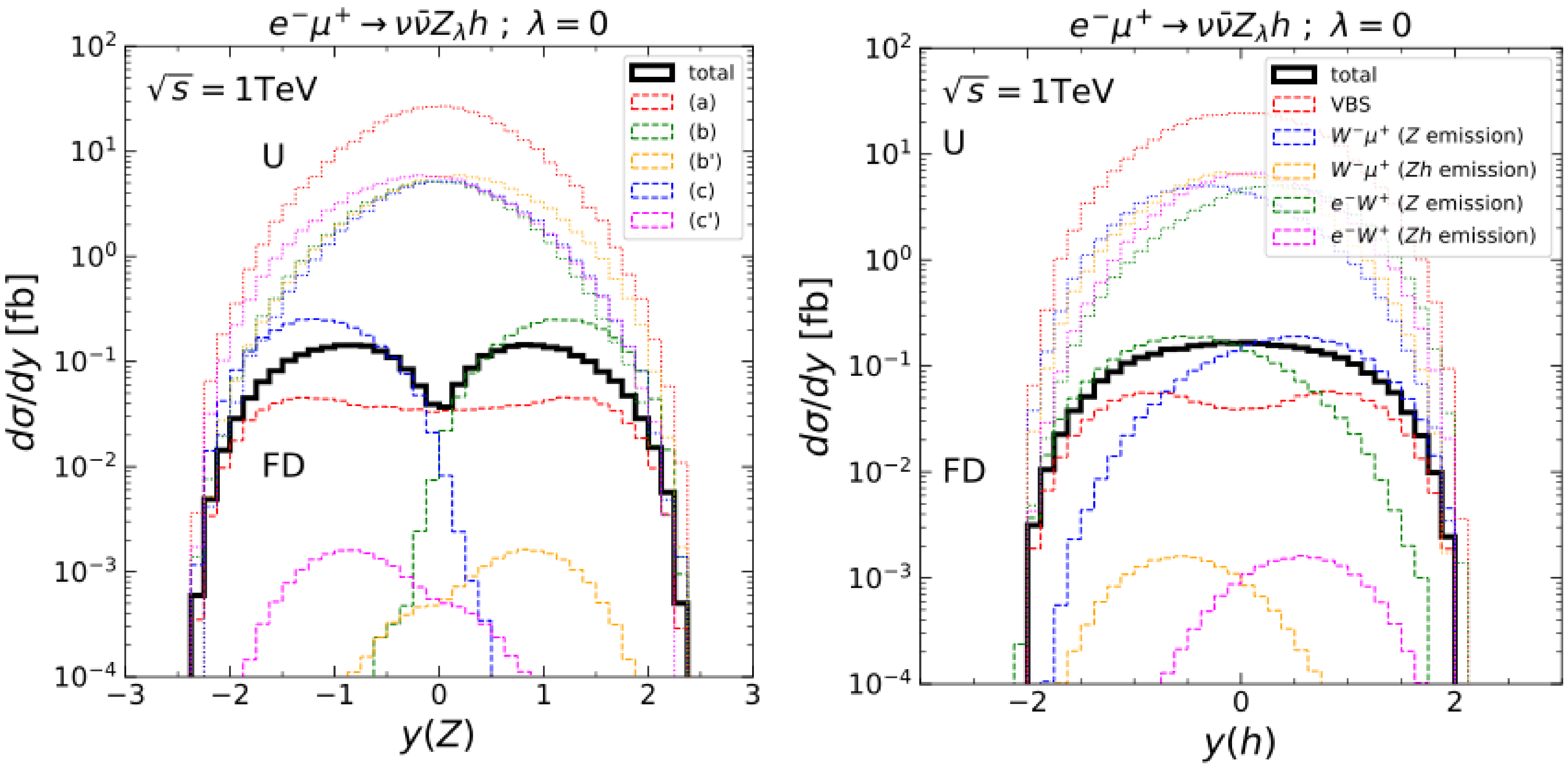
Show the contribution of all groups

Total cross section

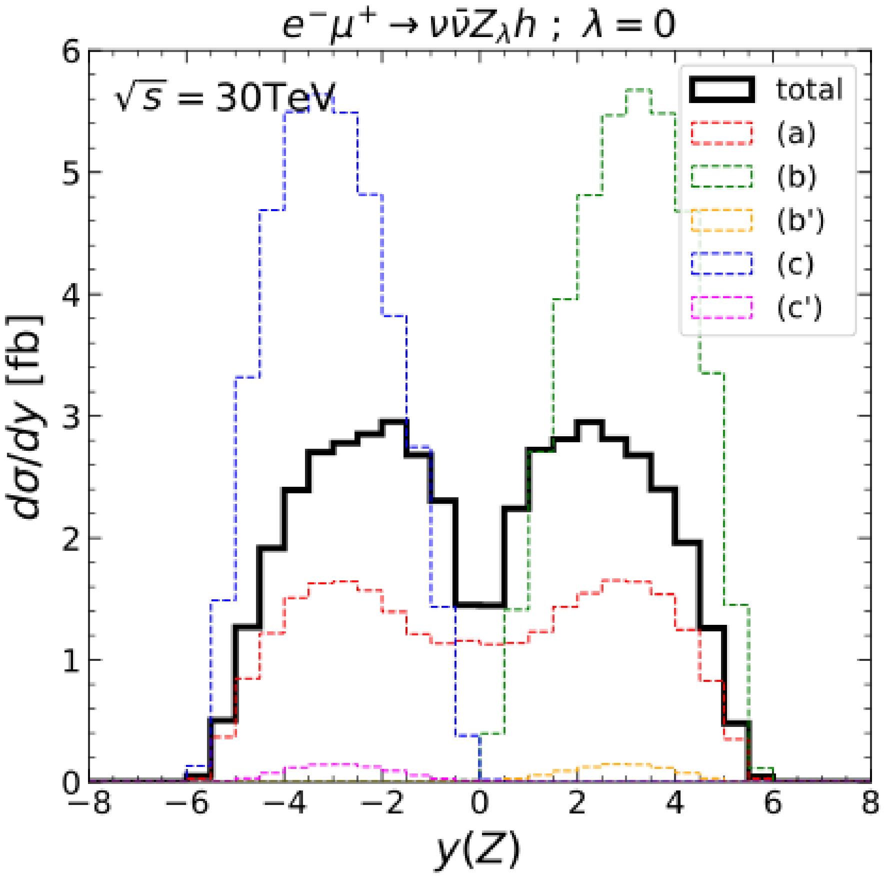


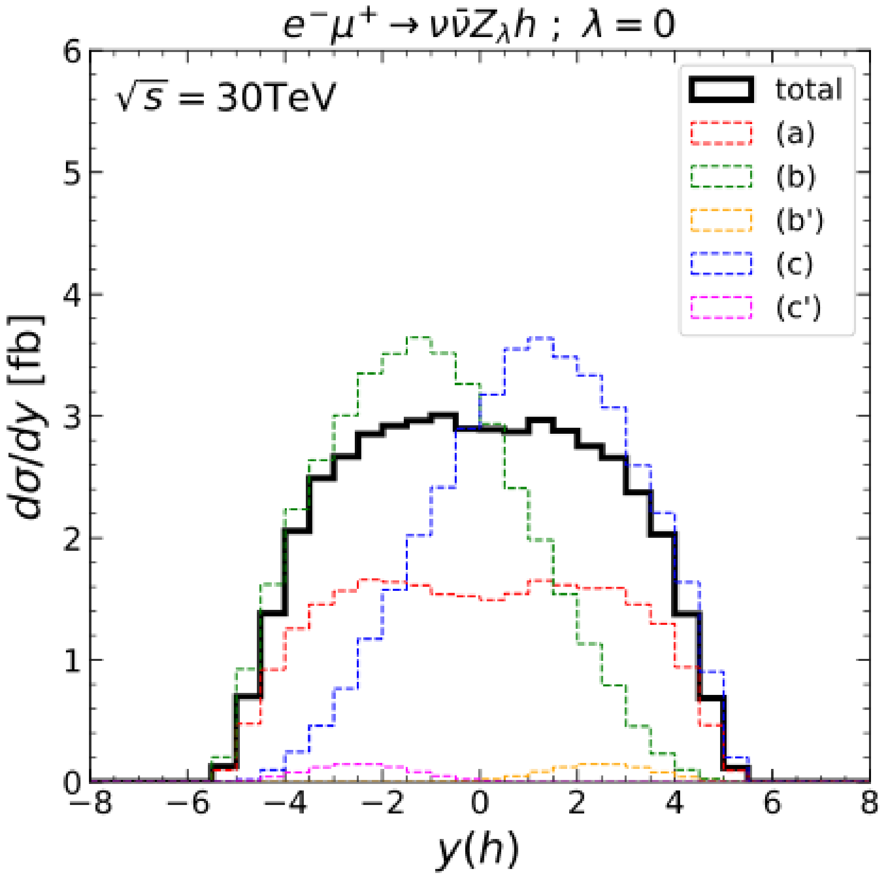


3. $e^-\mu^+ \to \nu_e\bar{\nu}_\mu Zh$



3. $e^-\mu^+ \to \nu_e \bar{\nu}_\mu Z h$

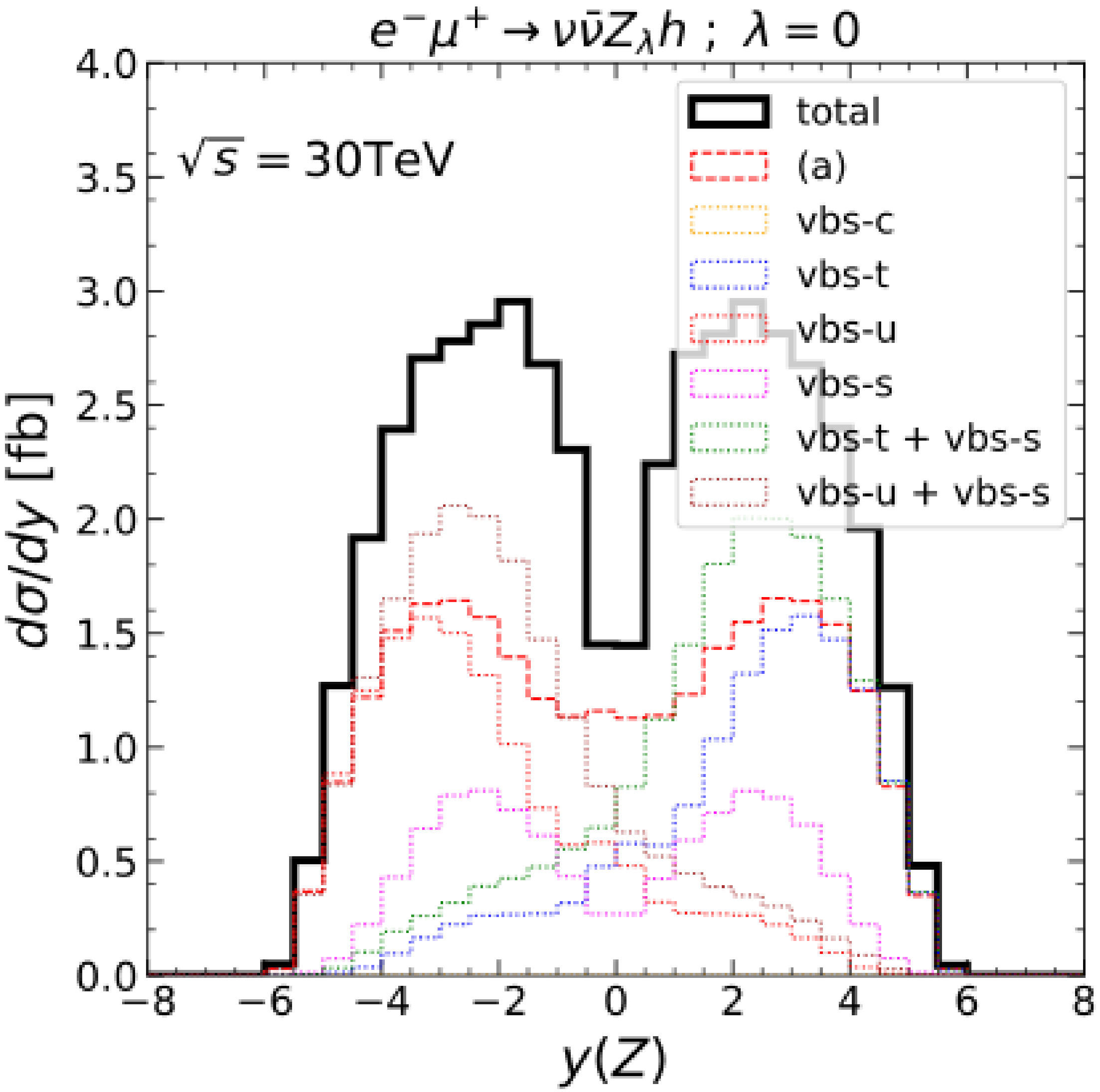


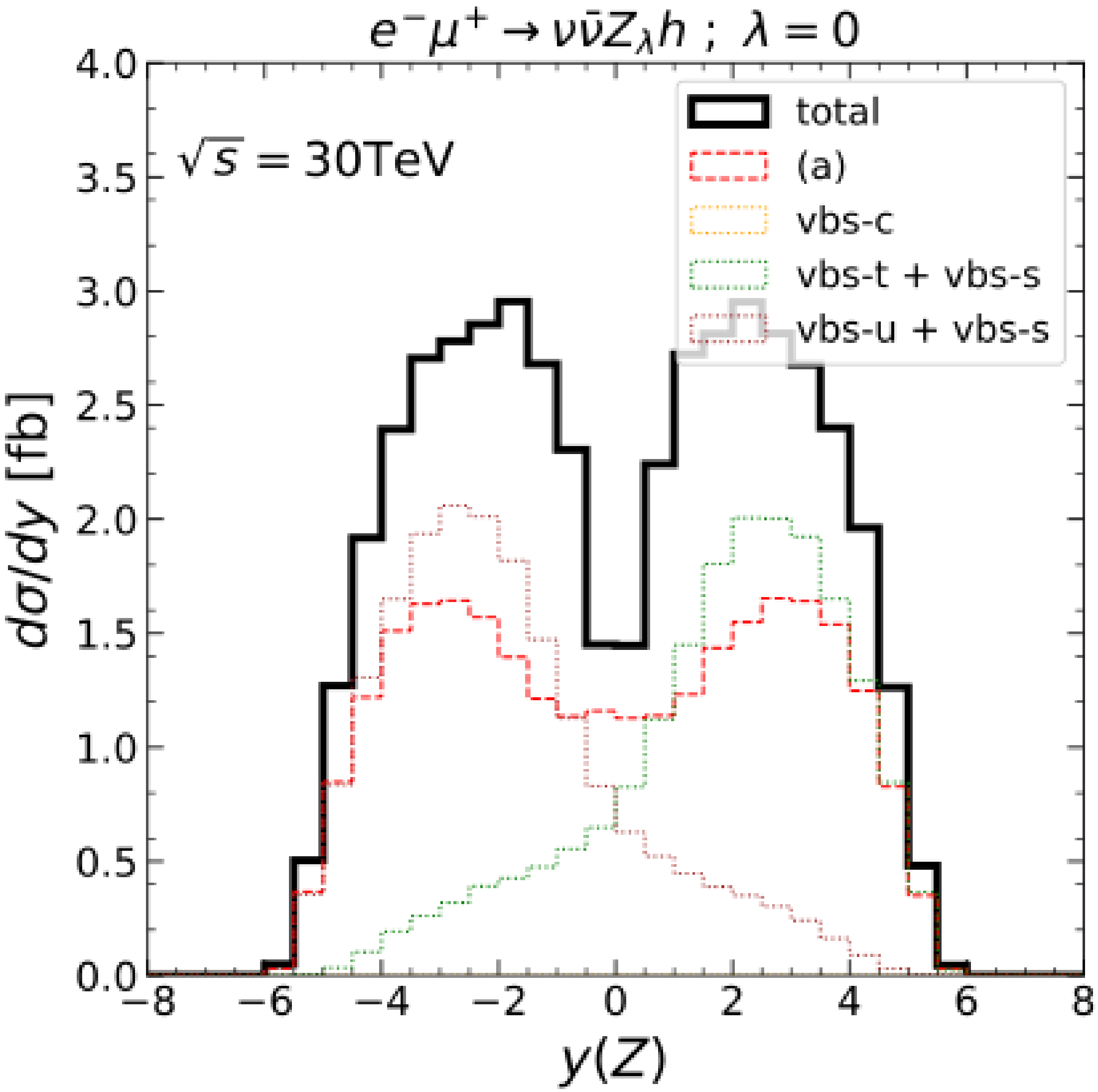


In $y(h)$,
- The $y(h)$ distributed in central region.
  →It does not receive collinear enhancement and thus tends to be emitted more isotropically, the Higgs is often produced as part of WW fusion or Zh system.
- (b) and (c) are swapped.
  → The boost effect.

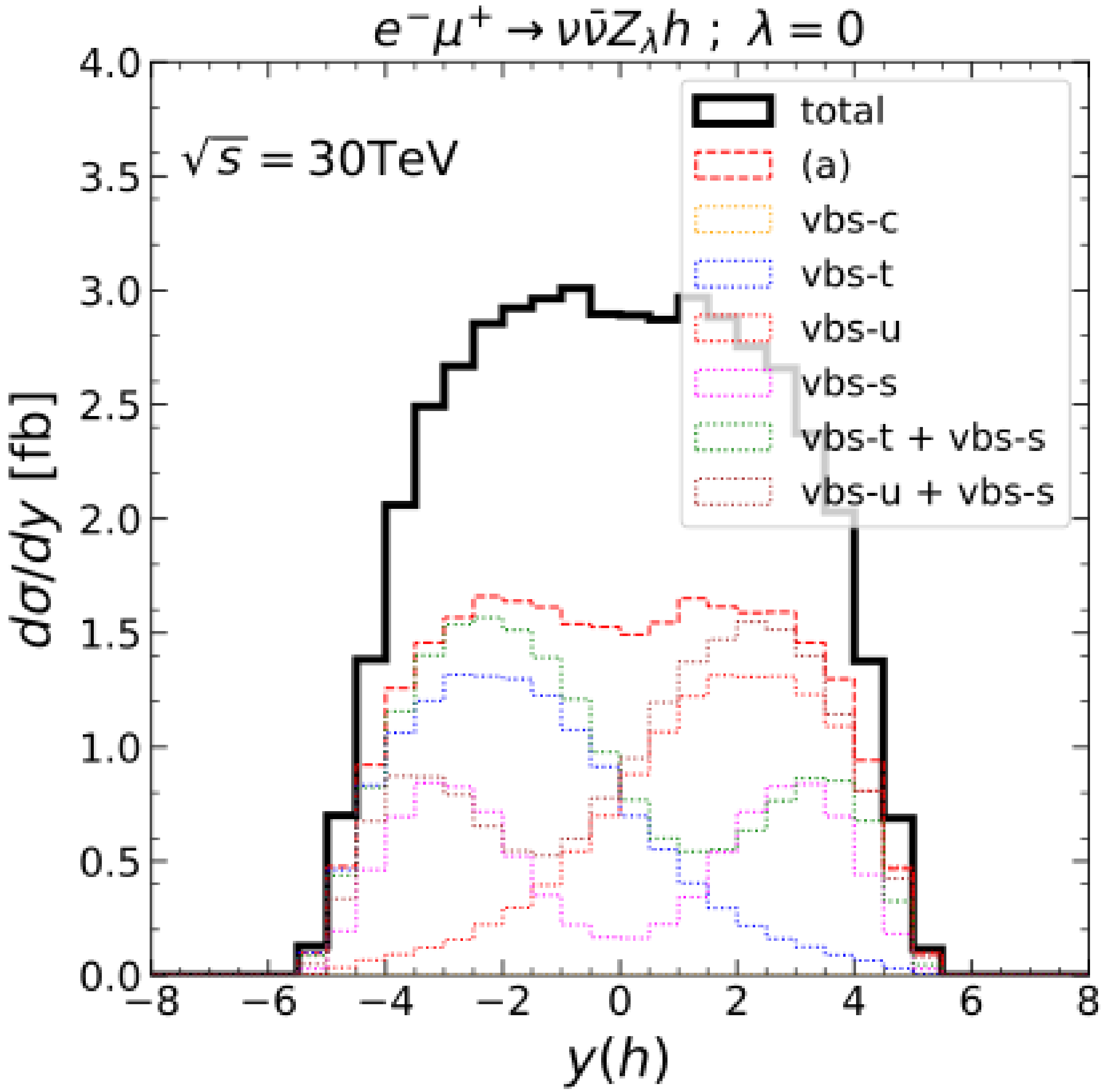
VBS

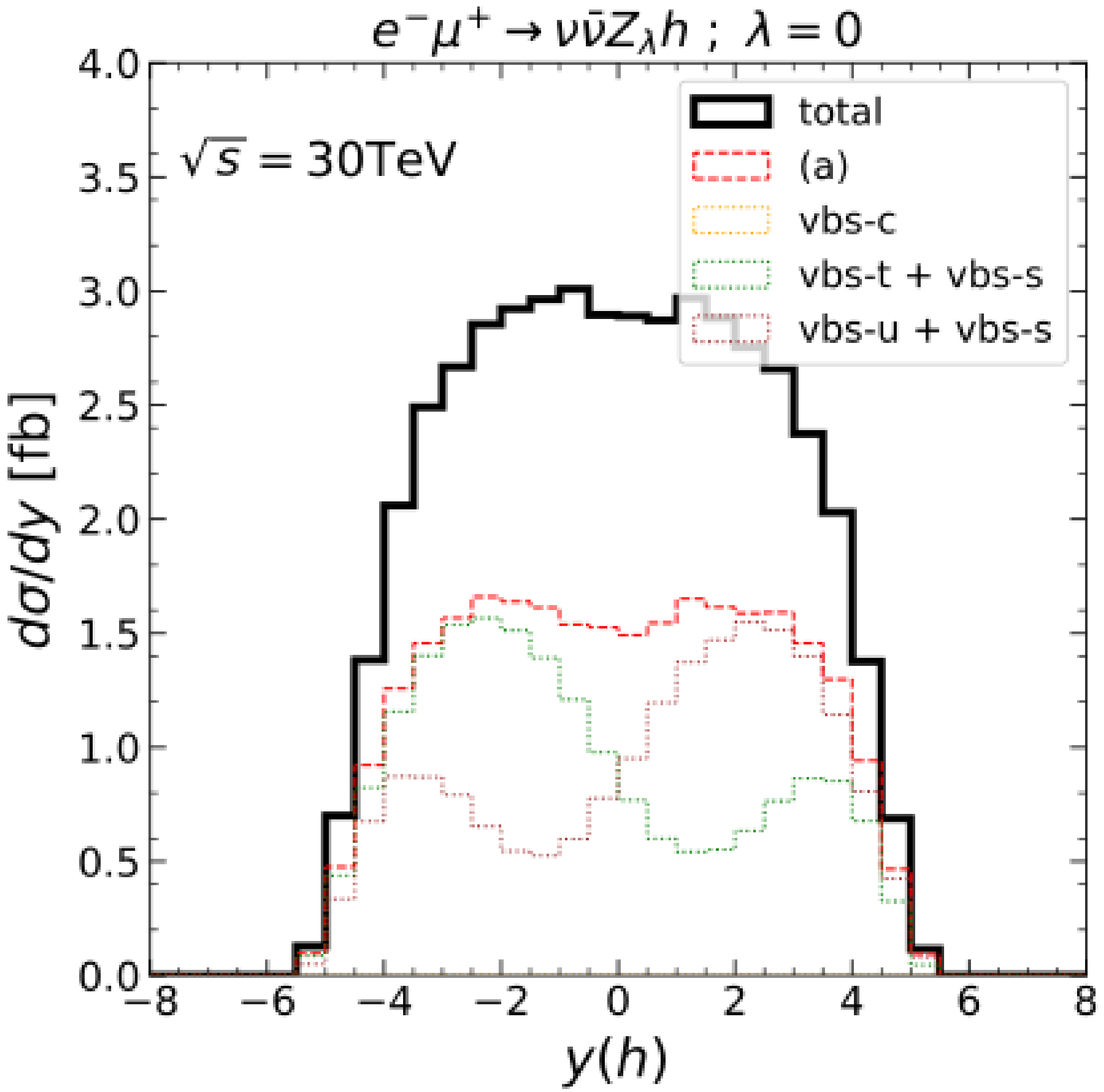
3. $e^-\mu^+ \to \nu_e\bar{\nu}_\mu Zh$

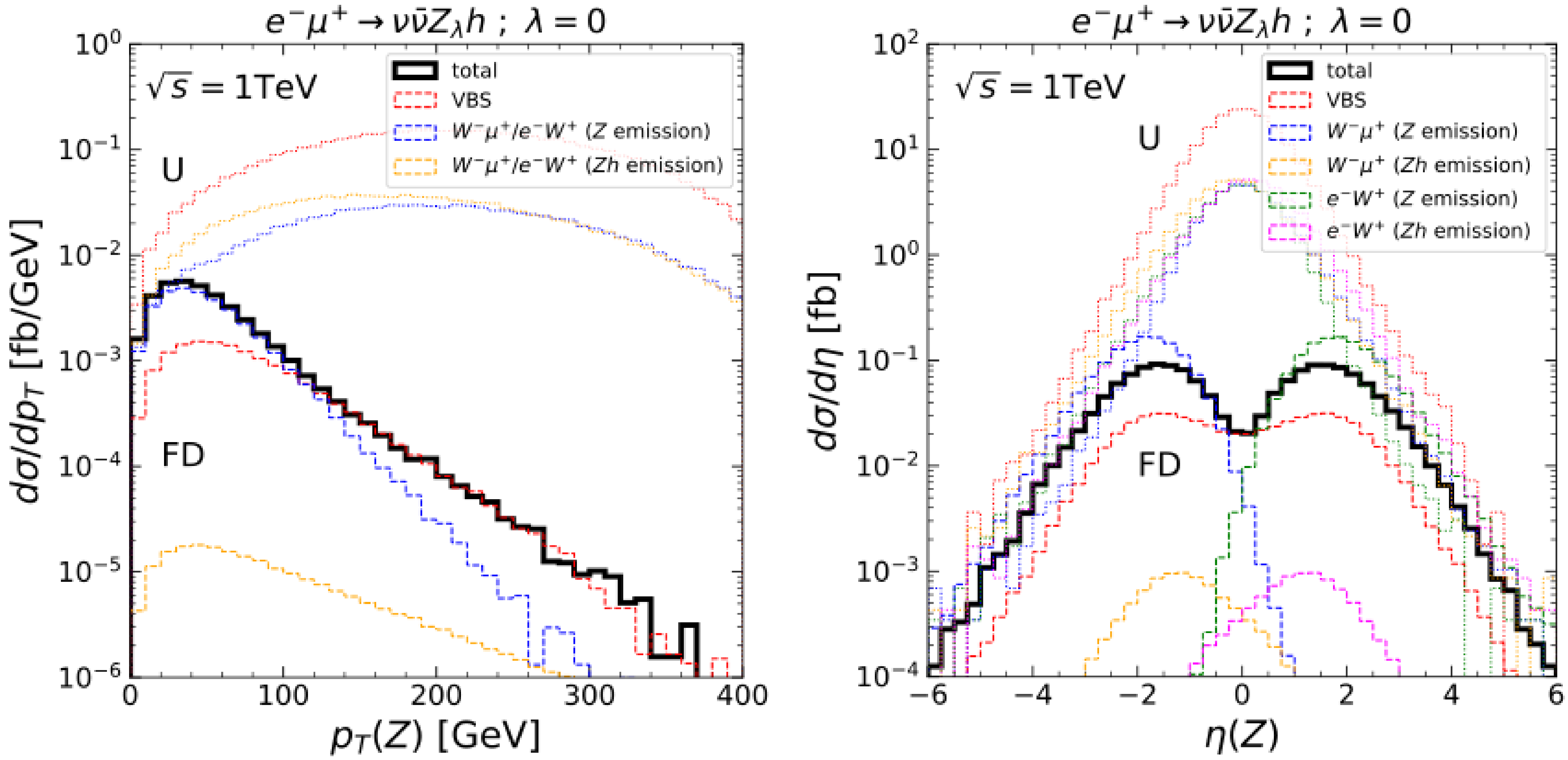




3. $e^{-}\mu^{+} \to \nu_e \bar{\nu}_\mu Z h$



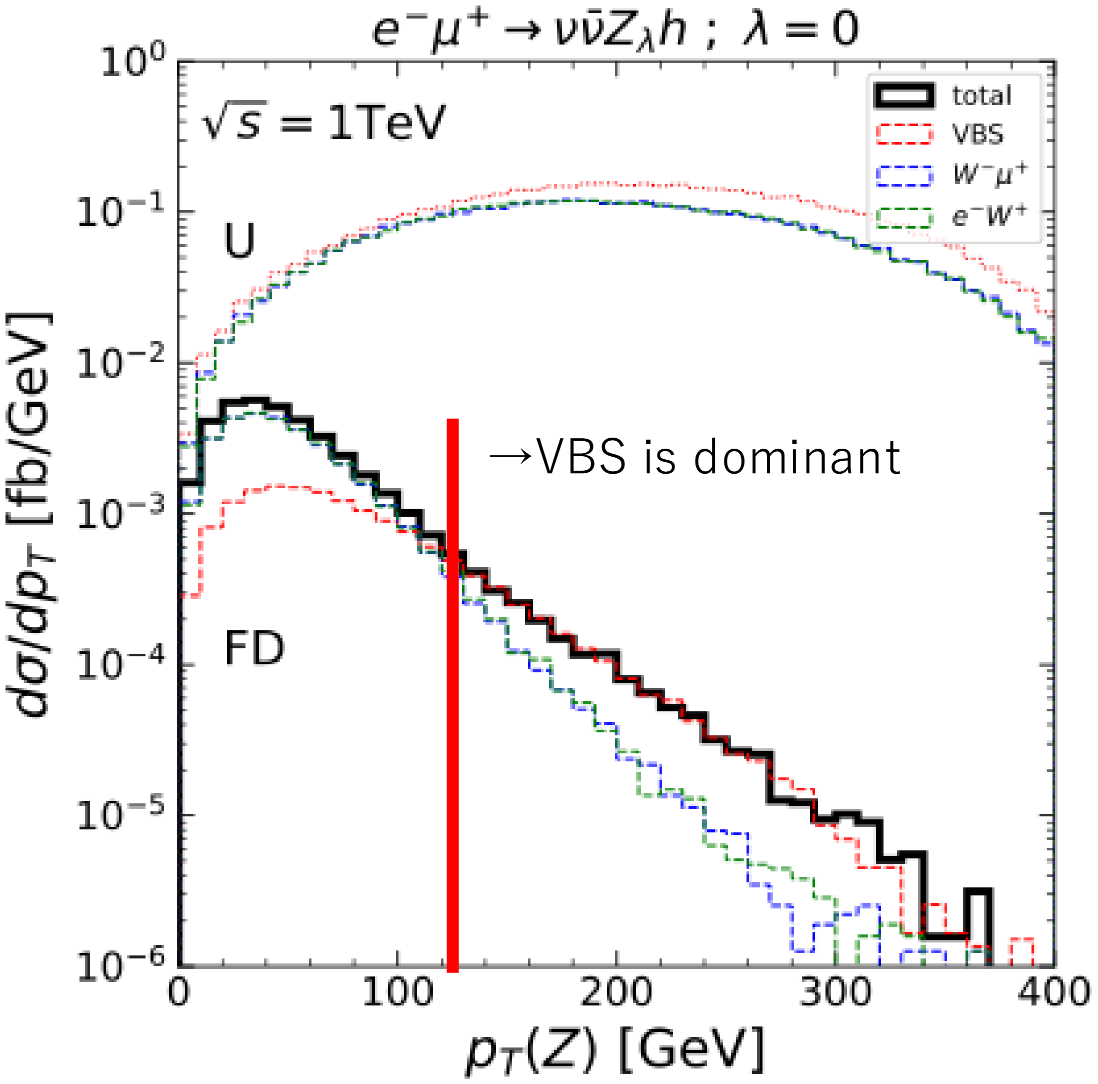


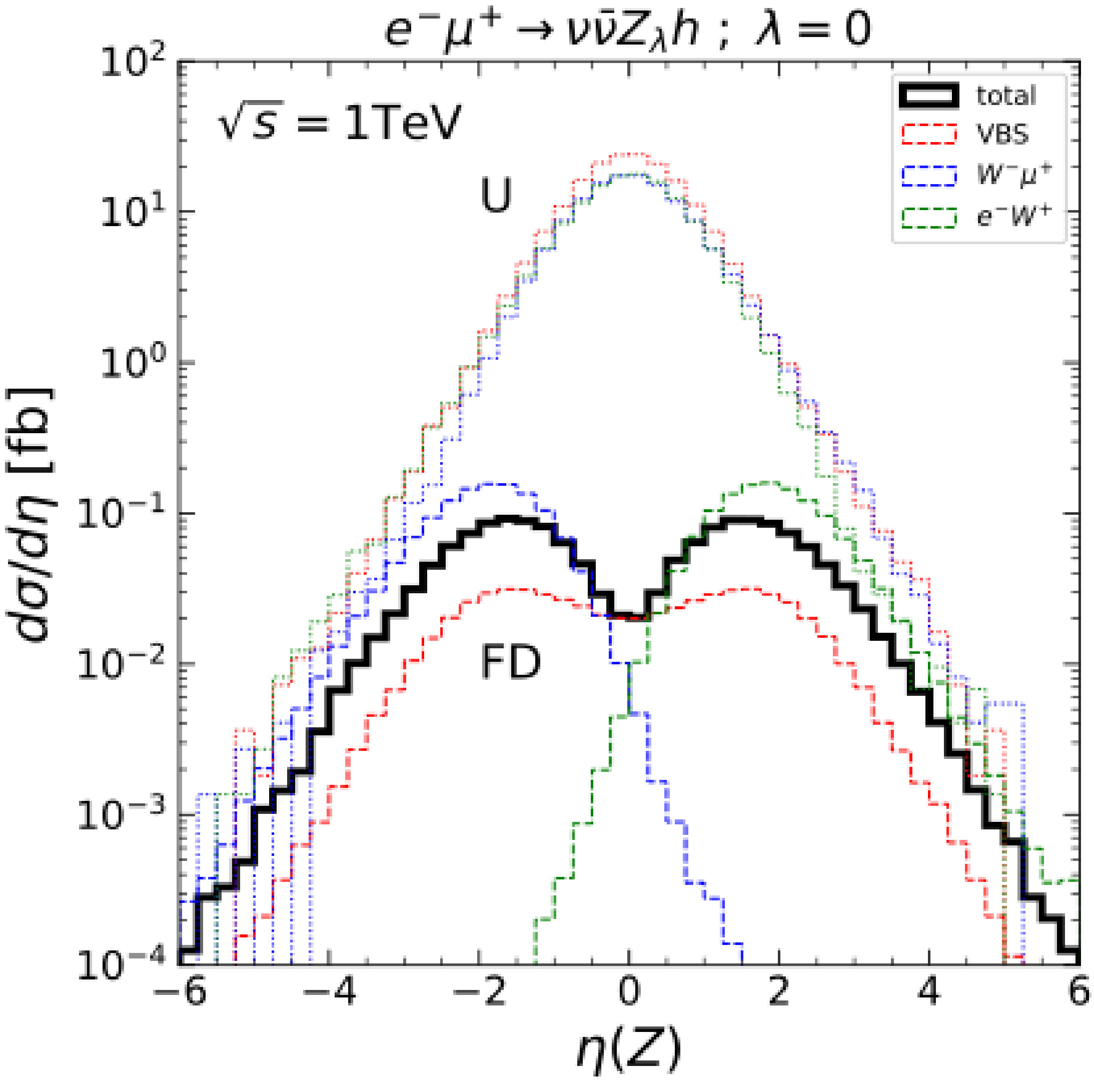


$e^-\mu^+ \to \nu\bar{\nu}Z_\lambda h\ ;\ \lambda = 0$
$\sqrt{s} = 1\text{TeV}$
total
VBS
$W^-\mu^+/e^-W^+$ (Z emission)
$W^-\mu^+/e^-W^+$ (Zh emission)
U
FD
$d\sigma/dp_T$ [fb/GeV]
$p_T(Z)$ [GeV]
$e^-\mu^+ \to \nu\bar{\nu}Z_\lambda h\ ;\ \lambda = 0$
$\sqrt{s} = 1\text{TeV}$
total
VBS
$W^-\mu^+$ (Z emission)
$W^-\mu^+$ (Zh emission)
e^-W^+ (Z emission)
e^-W^+ (Zh emission)
U
FD
$d\sigma/d\eta$ [fb]
$\eta(Z)$

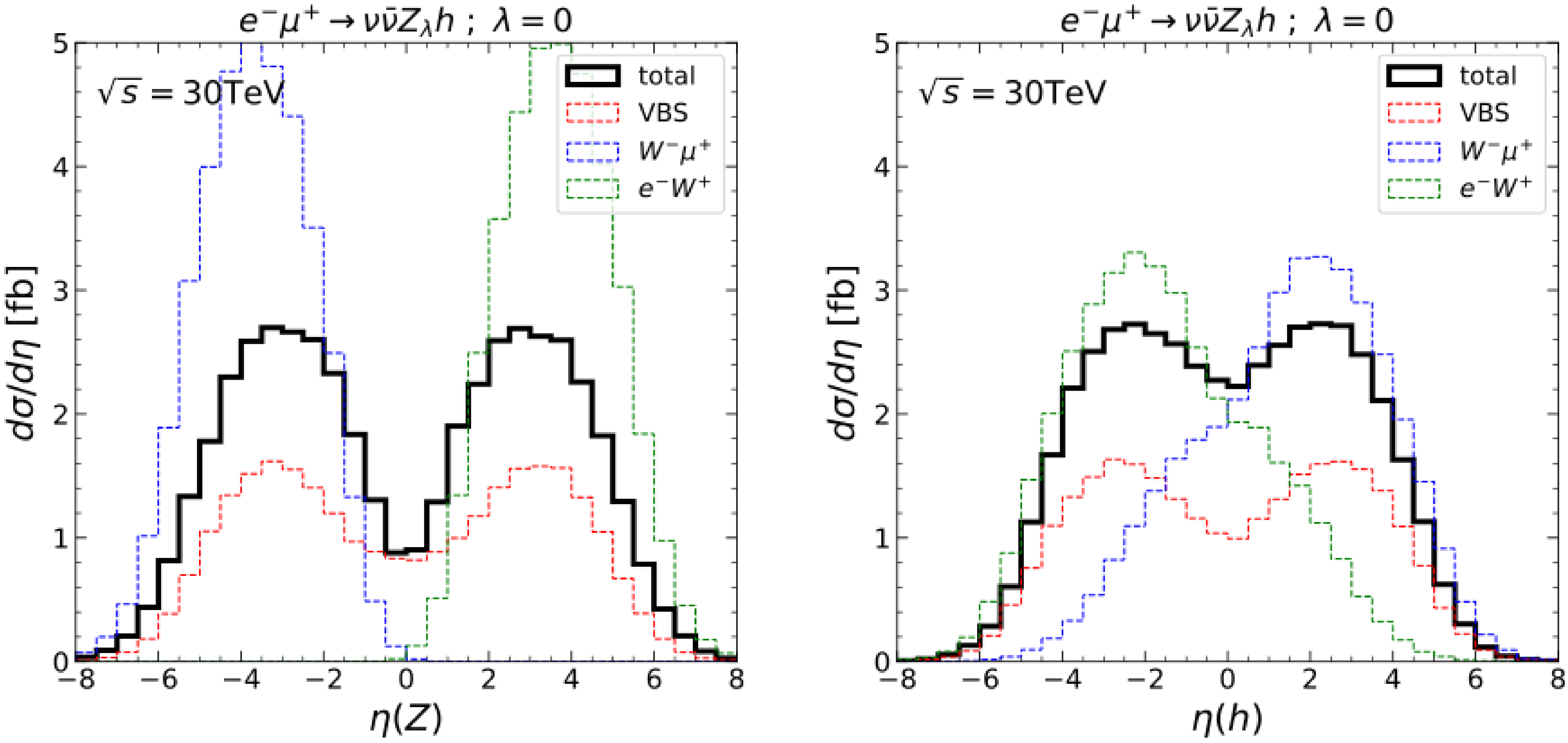
e- mu+ -> ve vm~ Z_{0} h
Kinematical cut pT(Z,h)>150GeV

$p_T(Z)$ distribution

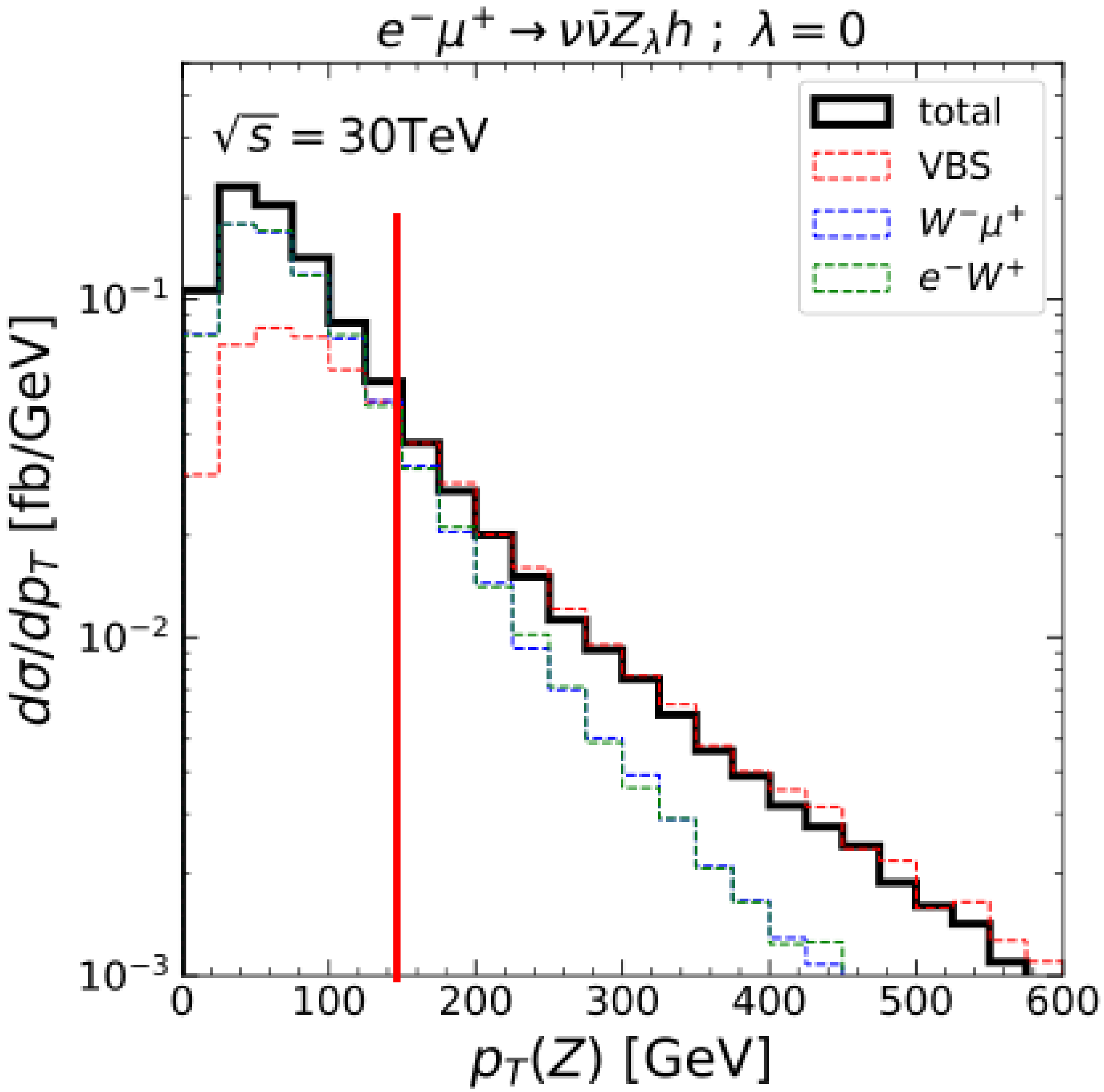


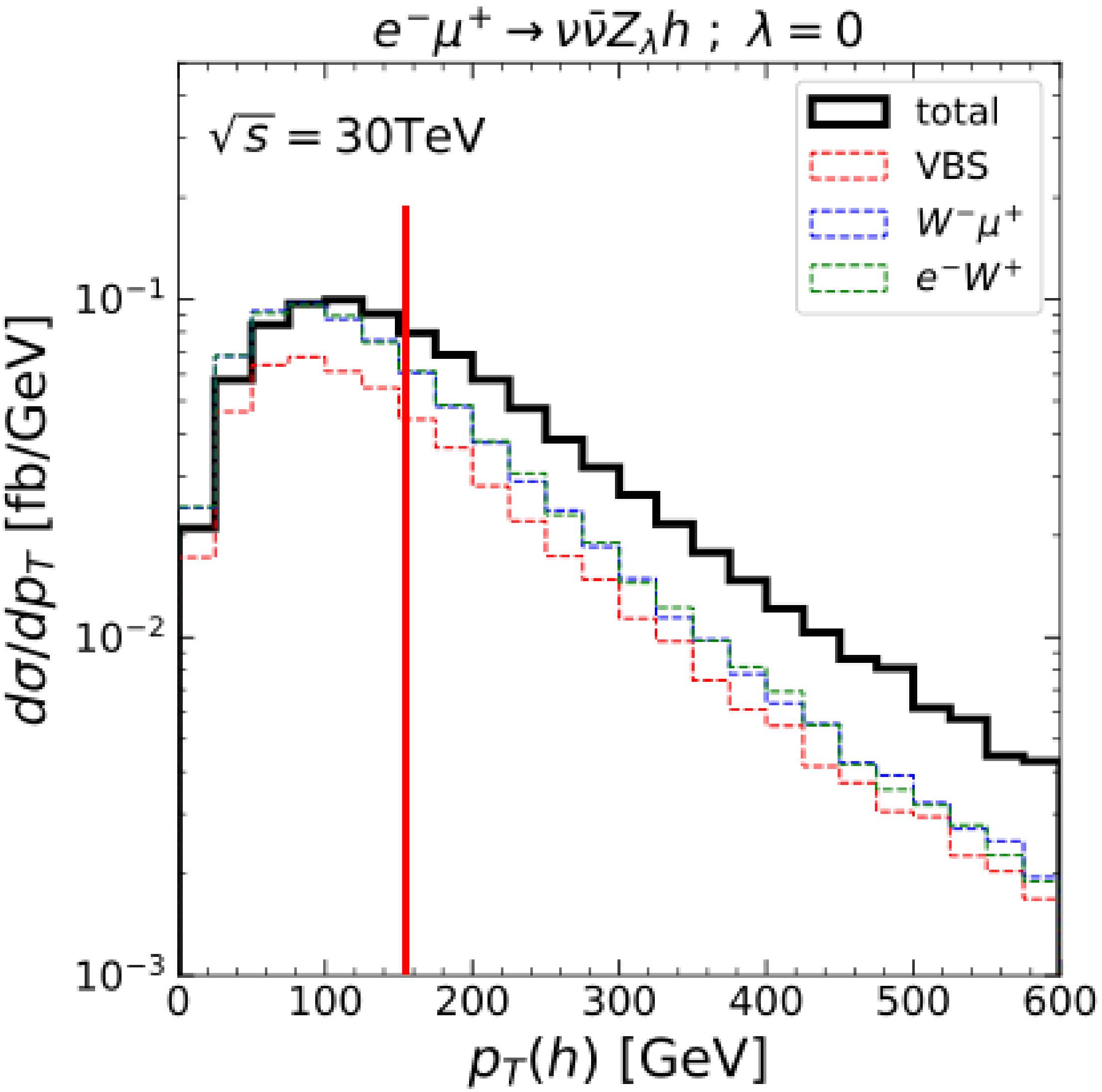


Enhancement of the VBS

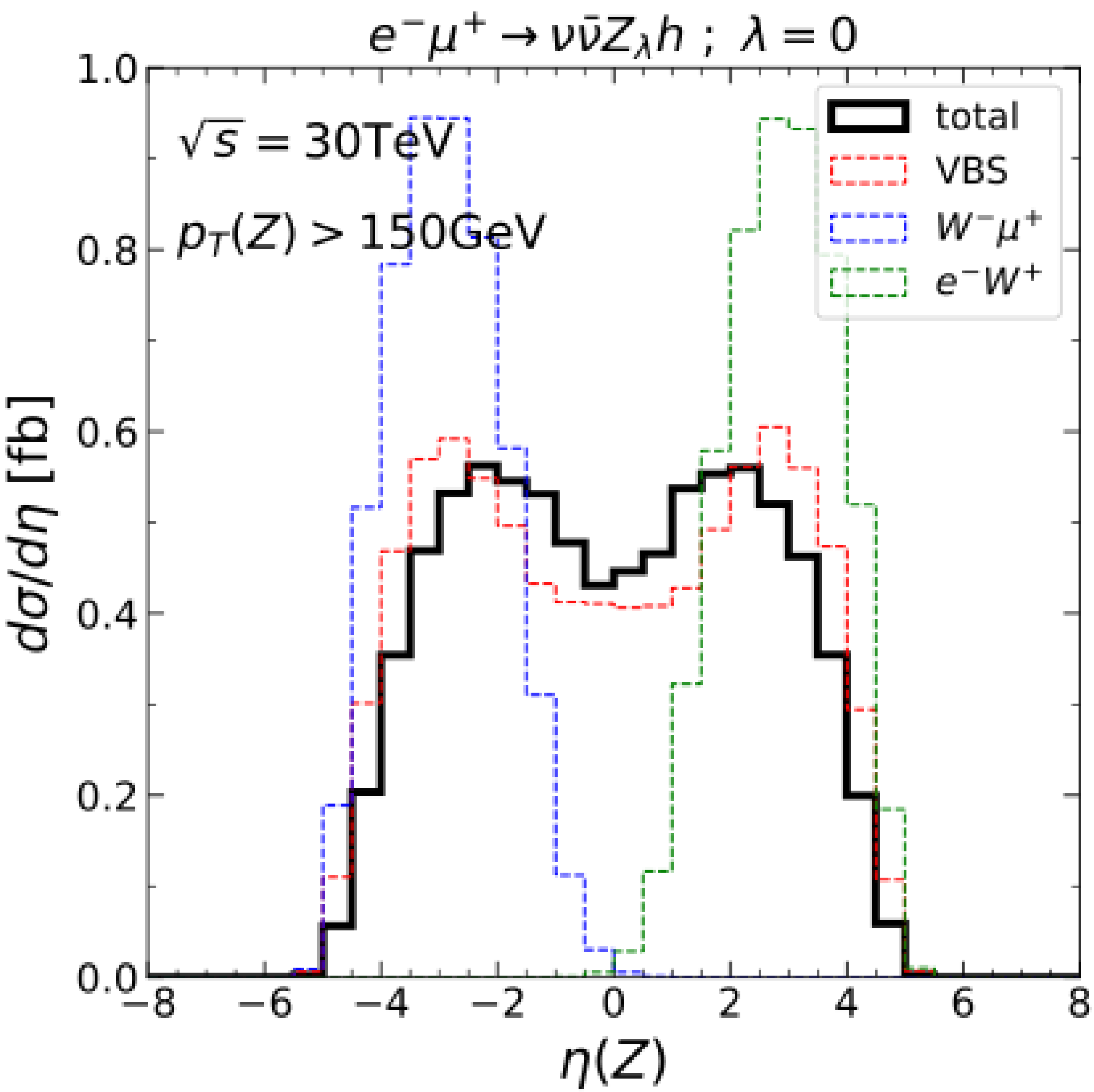


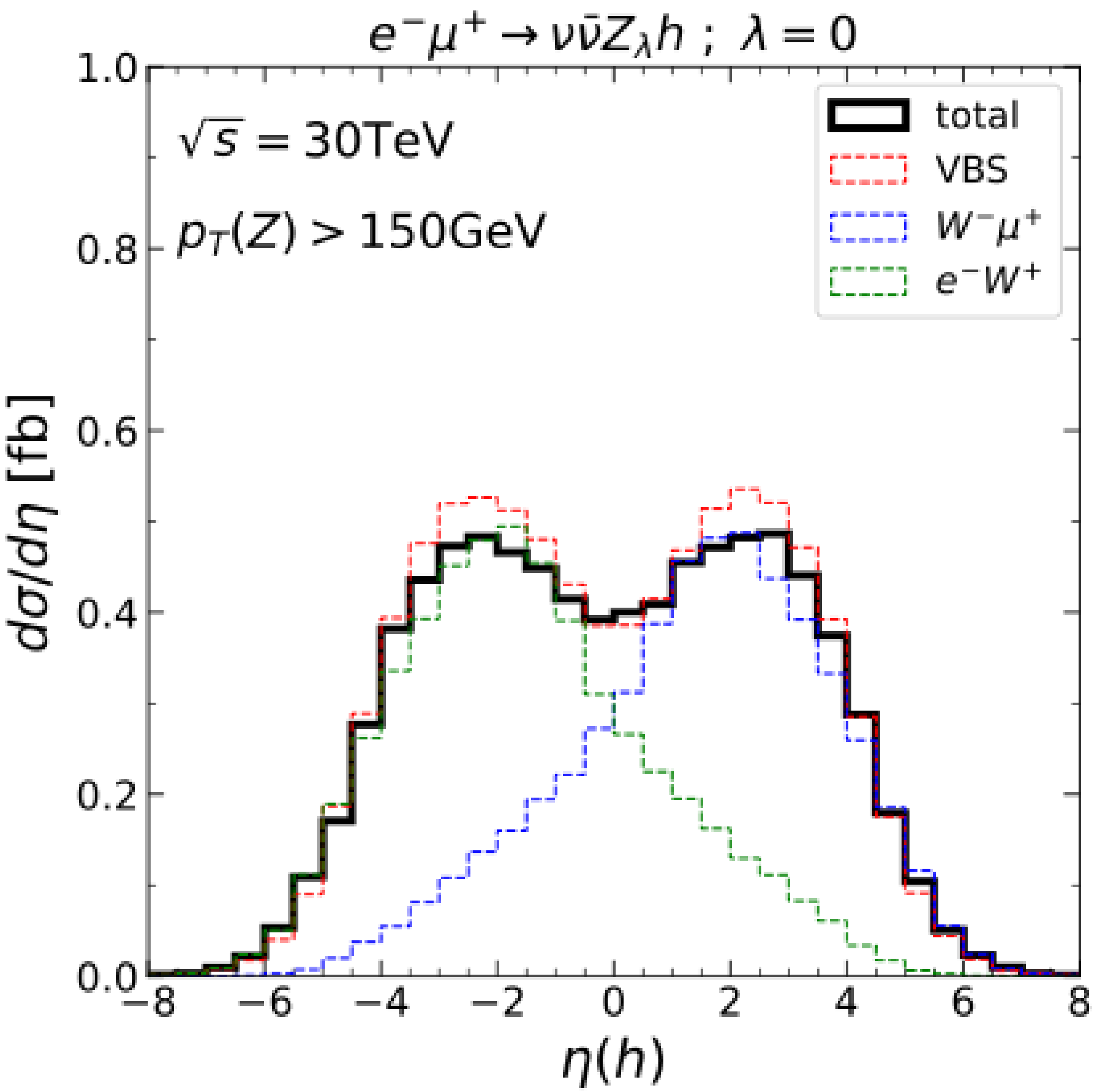
Enhancement of the VBS



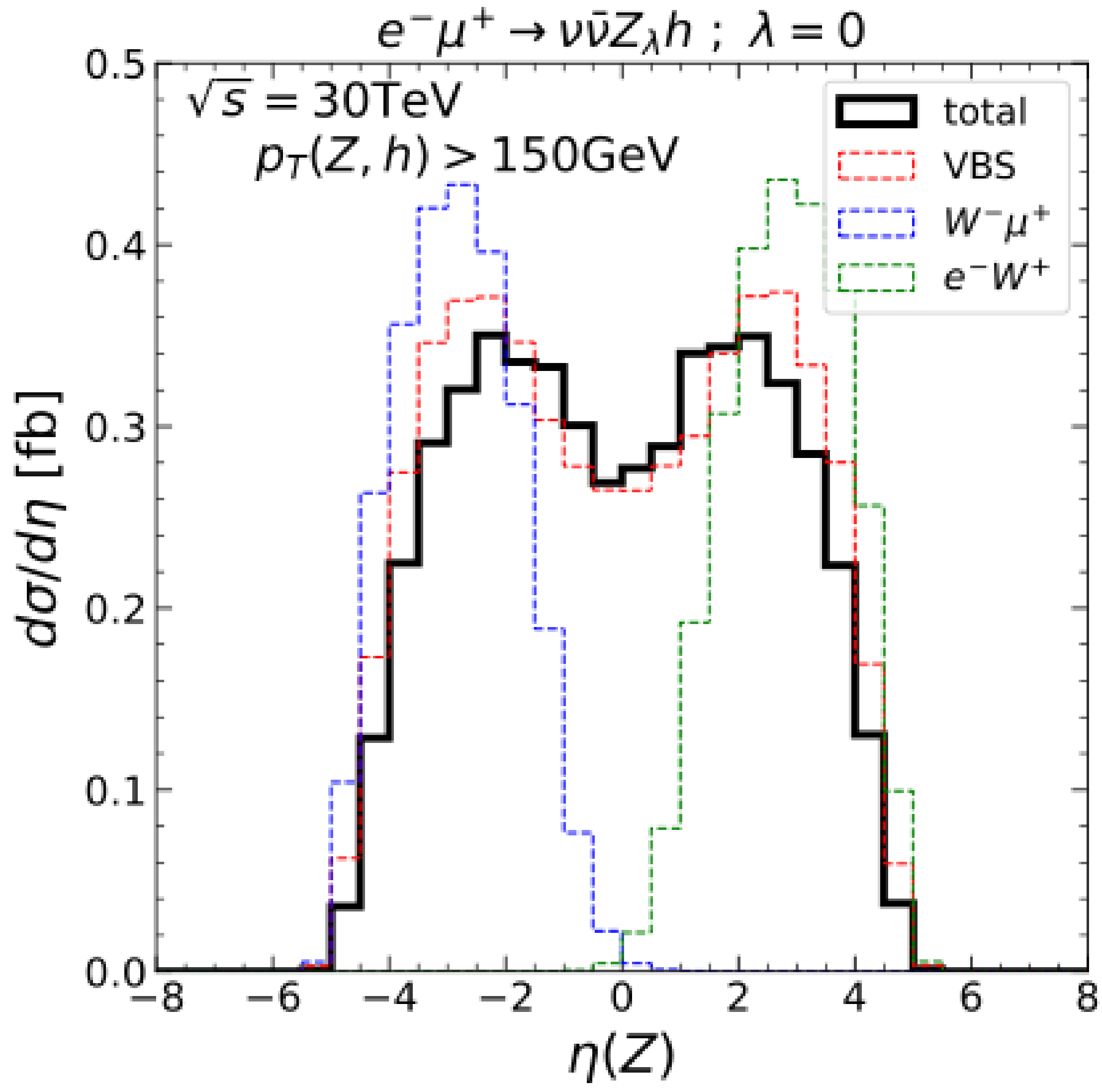


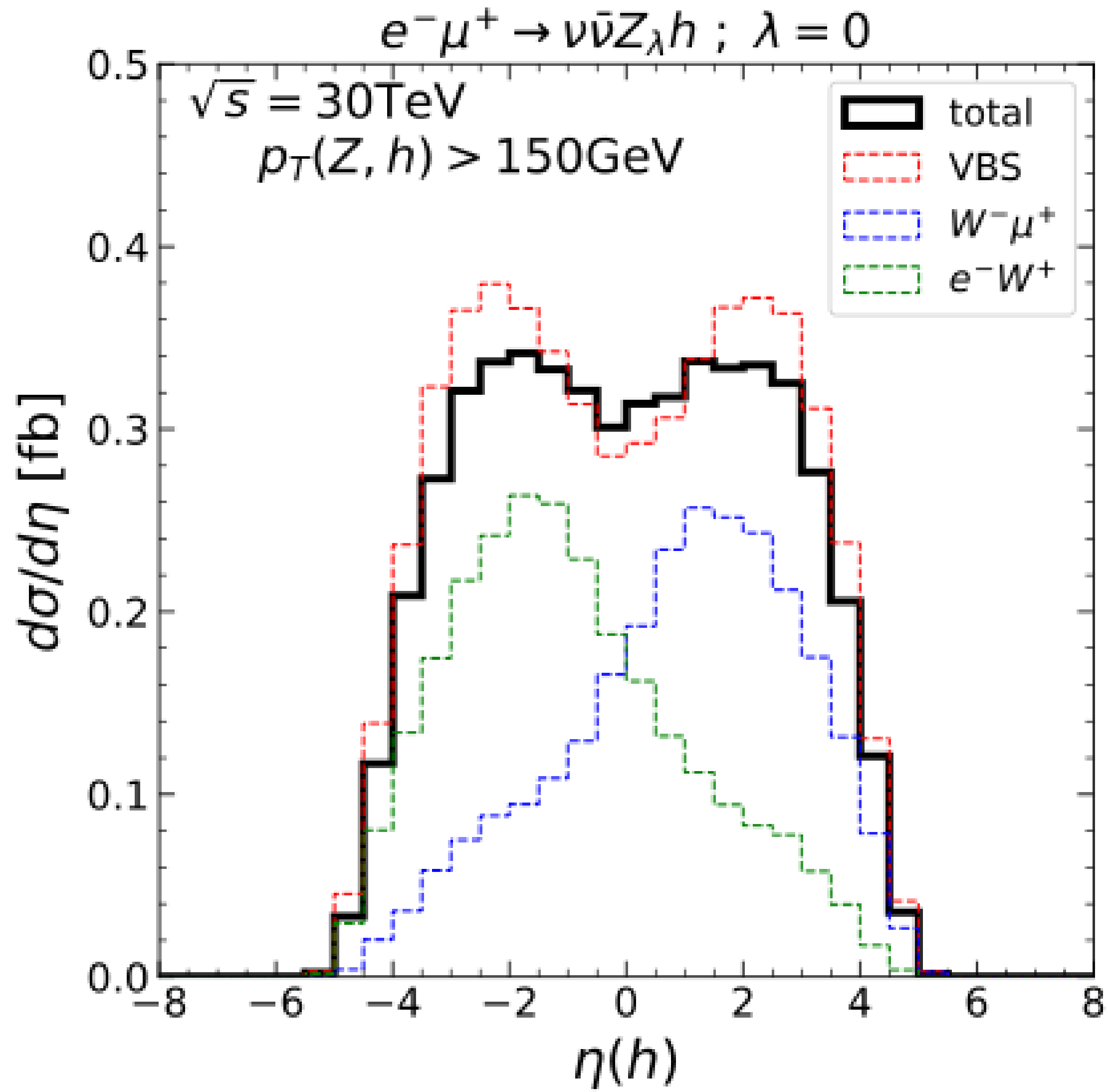
Enhancement of the VBS





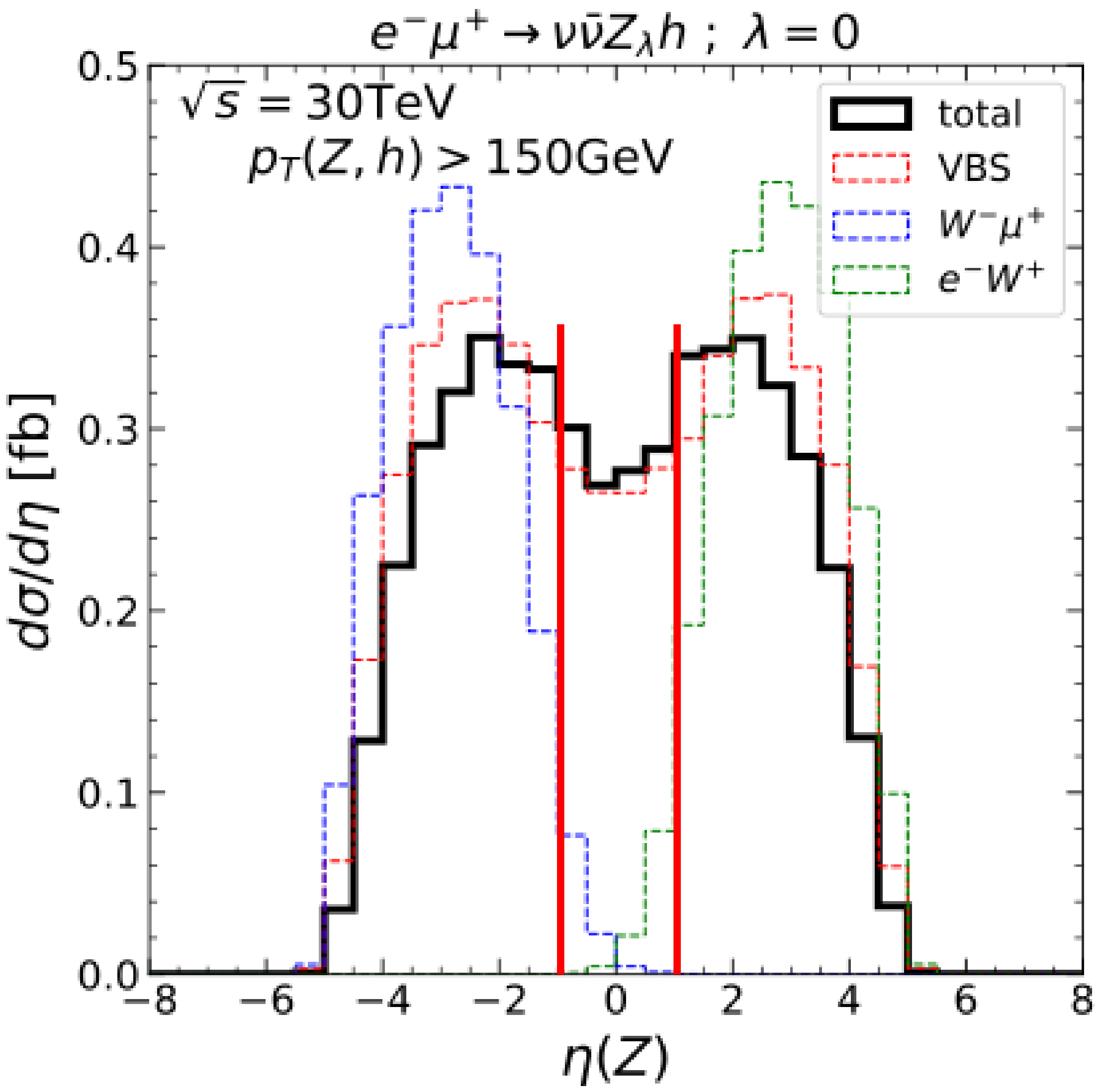
Enhancement of the VBS





Enhancement of the VBS

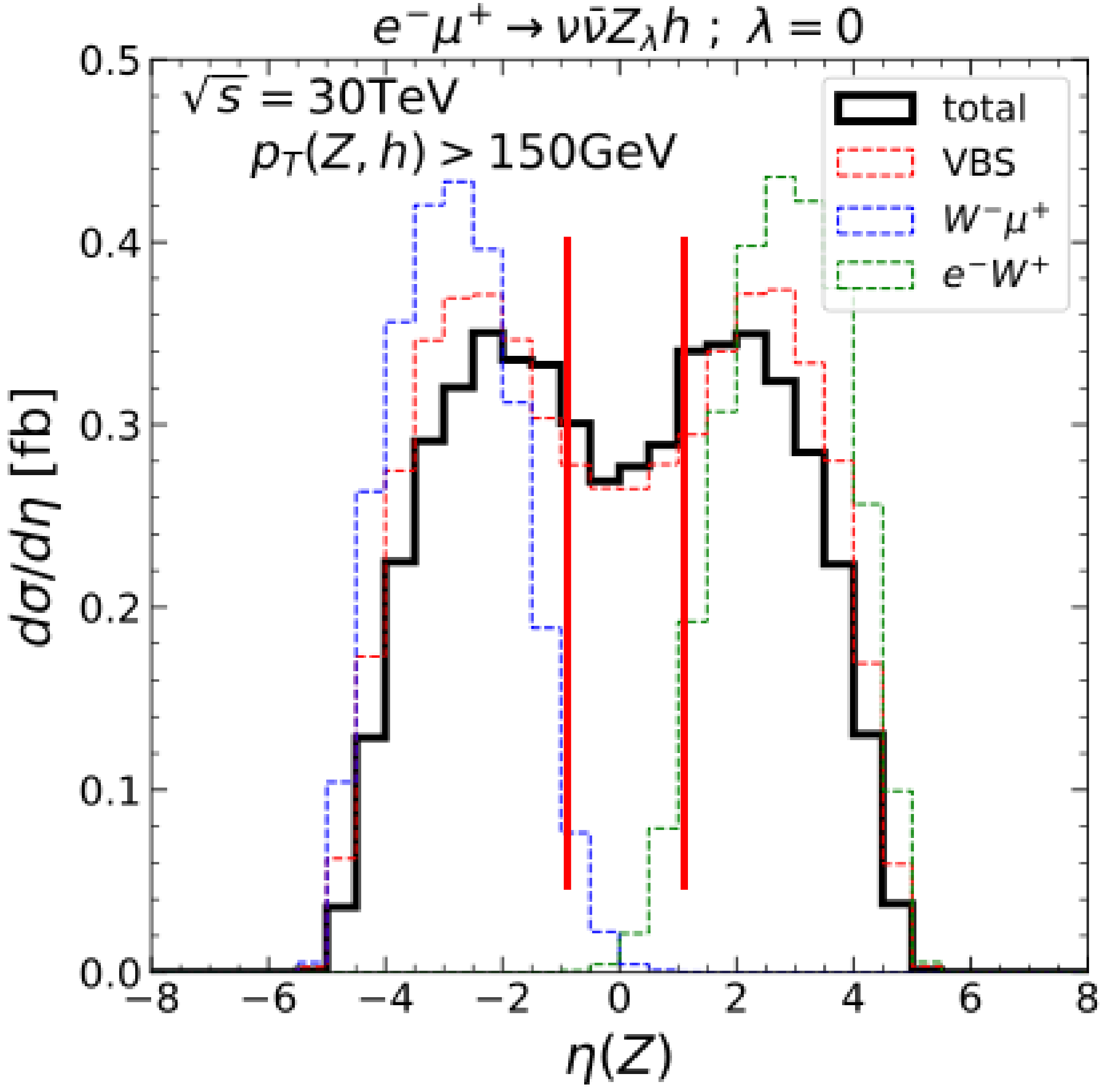
The VBS contribution can be enhanced by some kinematical cuts suggested by the distributions of the subgroups in the FD gauge.
At the same time, we also find that the dominance of the VBS contribution depends on the observables, and it is hard to isolate it from the $W-\mu+$ and $e-W+$ contribution, especially in the observables related to the Higgs boson in our naive study.



Enhancement of the VBS

The Weak boson PDFs provide a very good approximation under conditions where VBS is dominant. However, in this process the W－ μ＋ and e－W+ contributions are dominant, so the conditions under which VBS dominates are very limited, making the use of PDFs for the analysis rather inefficient.

Moreover, in the unitary gauge, the amplitude groups exhibit energy growth at high-energy region, which makes it extremely difficult to study the VBS contribution alone.



e- e+ -> W- W+

3. Helicity amplitudes

$$\beta = \frac{q}{E} = \sqrt{1 - \frac{m^2}{E^2}},$$
$$\gamma = \frac{E}{m} = \frac{1}{\sqrt{1-\beta^2}}.$$

U

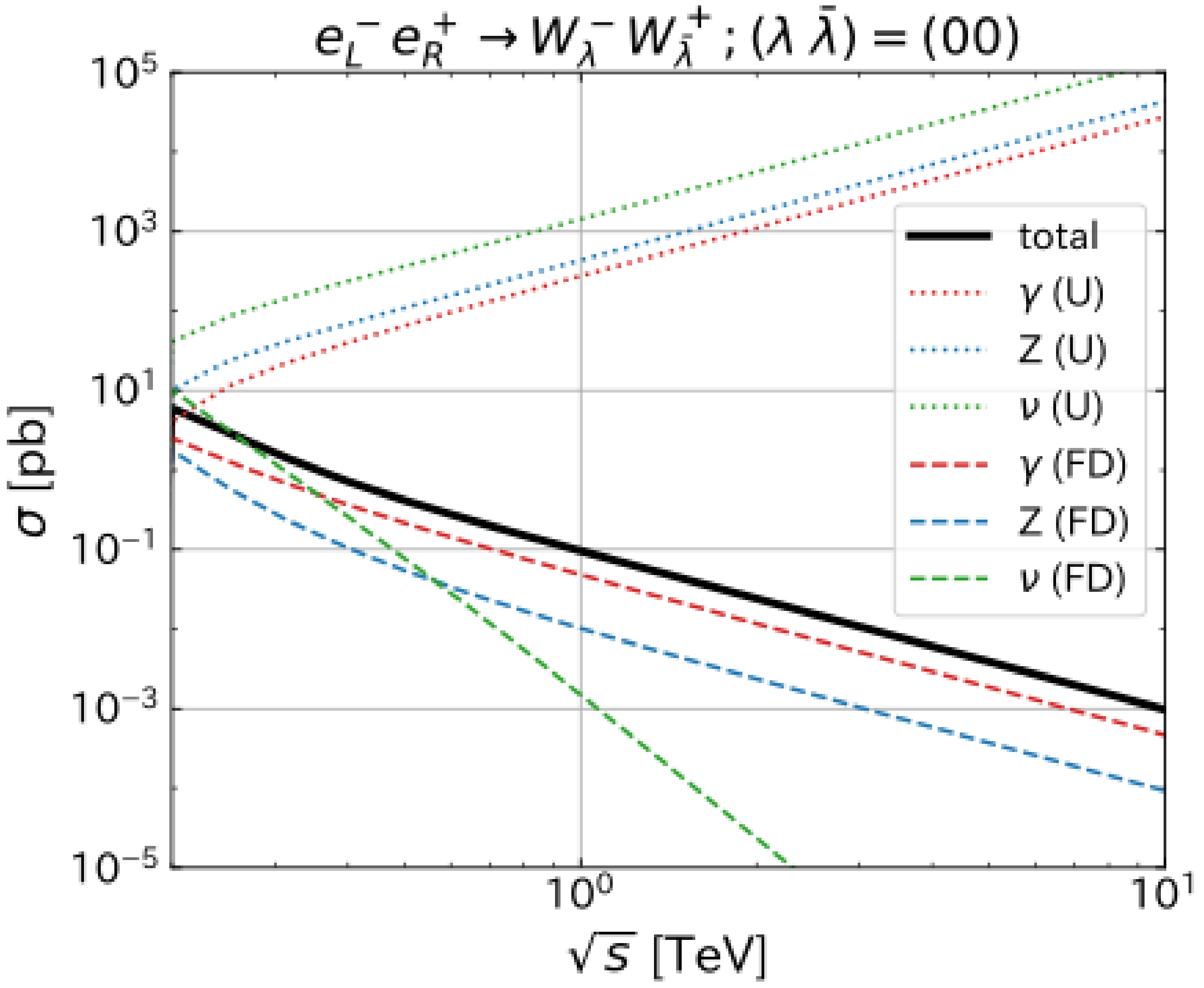
$\Delta\lambda$	$(\lambda, \bar{\lambda})$	$\tilde{\mathcal{M}}_\gamma{}^{\lambda\bar{\lambda}}(\theta)$	$\tilde{\mathcal{M}}_Z{}^{\lambda\bar{\lambda}}(\theta)$	$\tilde{\mathcal{M}}_\nu{}^{\lambda\bar{\lambda}}(\theta)$
0	$(0, 0)$	$-2\gamma^2\beta + \beta$	$2\gamma^2\beta - \beta$	$2\gamma^2(\beta - \cos\theta) - \beta$

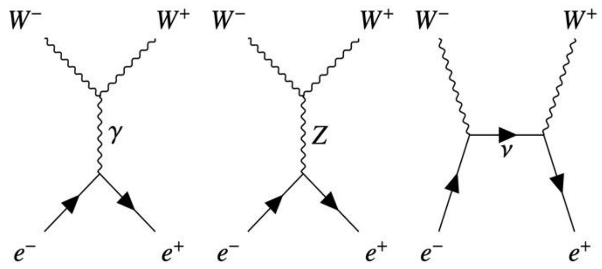
$$\tilde{\mathcal{M}} \propto E^2$$

FD

$\Delta\lambda$	$(\lambda, \bar{\lambda})$	$\tilde{\mathcal{M}}_\gamma{}^{\lambda\bar{\lambda}}(\theta)$	$\tilde{\mathcal{M}}_Z{}^{\lambda\bar{\lambda}}(\theta)$	$\tilde{\mathcal{M}}_\nu{}^{\lambda\bar{\lambda}}(\theta)$
0	$(0, 0)$	$\frac{1}{\gamma^2}\frac{3+\beta}{(1+\beta)^2} + 1$	$-\frac{1}{\gamma^2}\frac{3+\beta}{(1+\beta)^2} - \frac{s_W^2}{c_W^2}\left(\frac{\beta}{2s_W^2} - 1\right)$	$-\frac{1}{\gamma^2}\frac{2}{(1+\beta)^2}(1+\cos\theta)$

$$\tilde{\mathcal{M}} \cancel{\propto} E^2$$





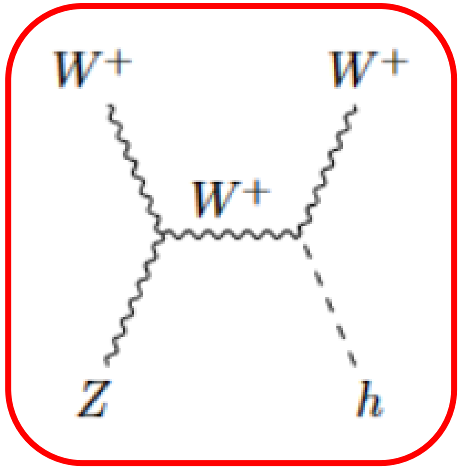
total$\propto |M_\gamma + M_Z + M_\nu|^2$,

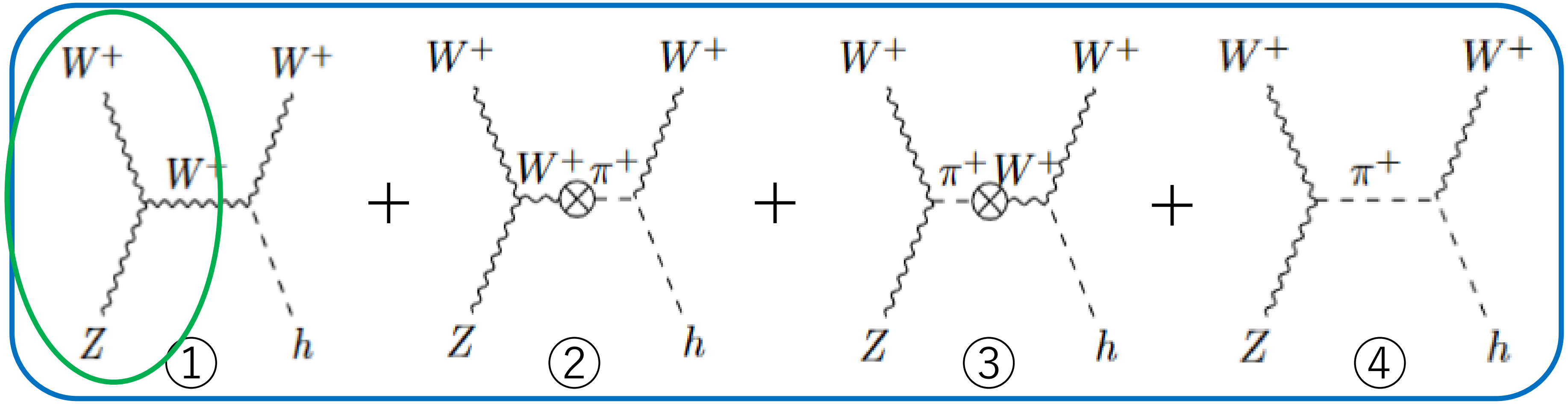
$\gamma \propto |M_\gamma|^2, Z \propto |M_Z|^2, \nu \propto |M_\nu|^2$

H.Furusato, K.Mawatari, Y.Suzuki, Y.Zheng, “W-boson pair production at lepton colliders in the Feynman-Diagram gauge”, *Phys.Rev.D 110 (2024) 5, 053005*

Subprocess VhVBS
W+ Z -> W+ h

Ex. the s-channel amplitude for $\lambda_1 = \lambda_2 = \lambda_3 = +1$ in the U and FD gauges

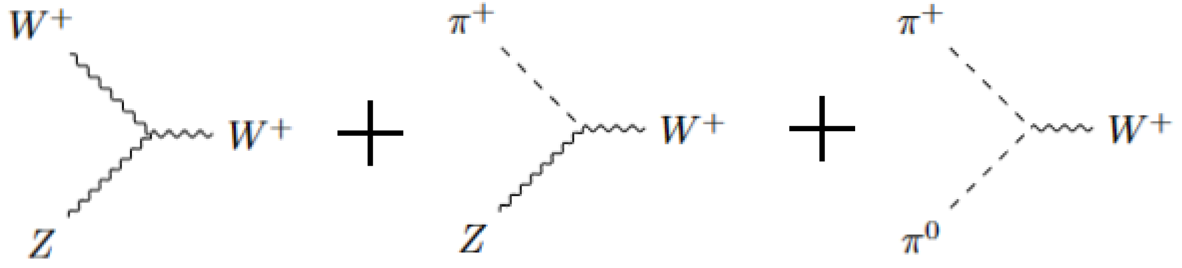




$$\mathcal{M}_s^{\mathrm{U}}(+,+,+) = J^{\mu}_{s\,++} P^{\mathrm{U}}_{s\,\mu\nu} J^{\nu}_{s\,+}$$

$$= J^{\mu}_{s\,++} \frac{i}{q^2 - m^2 + i\epsilon} \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m^2} \right) J^{\nu}_{s\,+}$$

for $\lambda_1 = \lambda_2 = \lambda_3 = 0$



$$\mathcal{M}_s^{\mathrm{FD}}(+,+,+) = J^{M}_{s\,++} P^{\mathrm{FD}}_{s\,MN} J^{N}_{s\,+}$$

$$= J^{\mu}_{s\,++} \frac{i}{q^2 - m^2 + i\epsilon} \left(-g_{\mu\nu} + \frac{n_\mu q_\nu + q_\mu n_\nu}{n \cdot q} \right) J^{\nu}_{s\,+} \quad \cdots ①$$

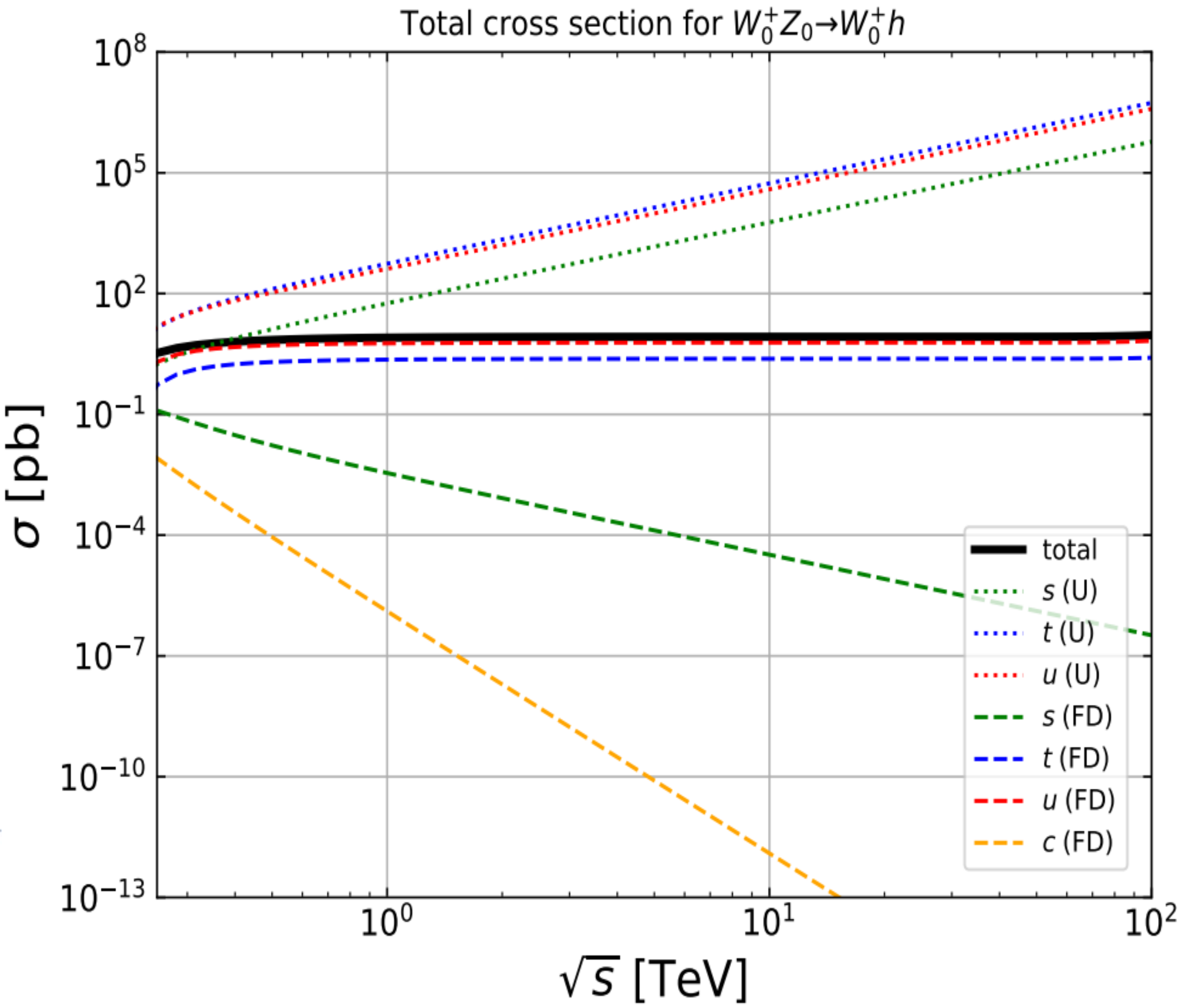
$$+ J^{\mu}_{s\,++} \frac{i}{q^2 - m^2 + i\epsilon} \left(im \frac{n_\mu}{n \cdot q} \right) J^{4}_{s\,+} \quad \cdots ②$$

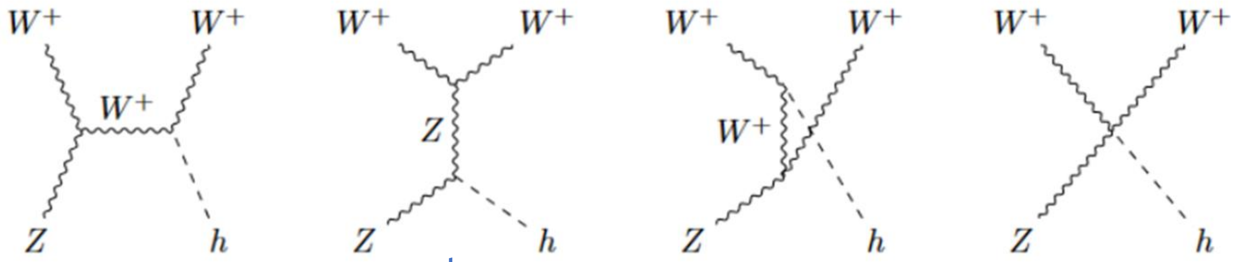
$$+ J^{4}_{s\,++} \frac{i}{q^2 - m^2 + i\epsilon} \left(-im \frac{n_\nu}{n \cdot q} \right) J^{\nu}_{s\,+} \quad \cdots ③$$

$$+ J^{4}_{s\,++} \frac{i}{q^2 - m^2 + i\epsilon} J^{4}_{s\,+} \quad \cdots ④$$

Comparison of cross sections in the U gauge and FD gauge

- **The solid black line** is the physical observable (obtained by summing the amplitudes of all channels (s, t, u, c) together and squaring them.)
- Dotted lines (U gauge) and dashed lines (FD gauge) are non-physical quantities (obtained by squaring the individual amplitudes s, t, u, c).
- Whichever one is chosen, the physical cross section (**solid black**) remains the same as expected.
  →That is, Gauge invariant.
- However, the contribution from each amplitude is quite different between the gauges!



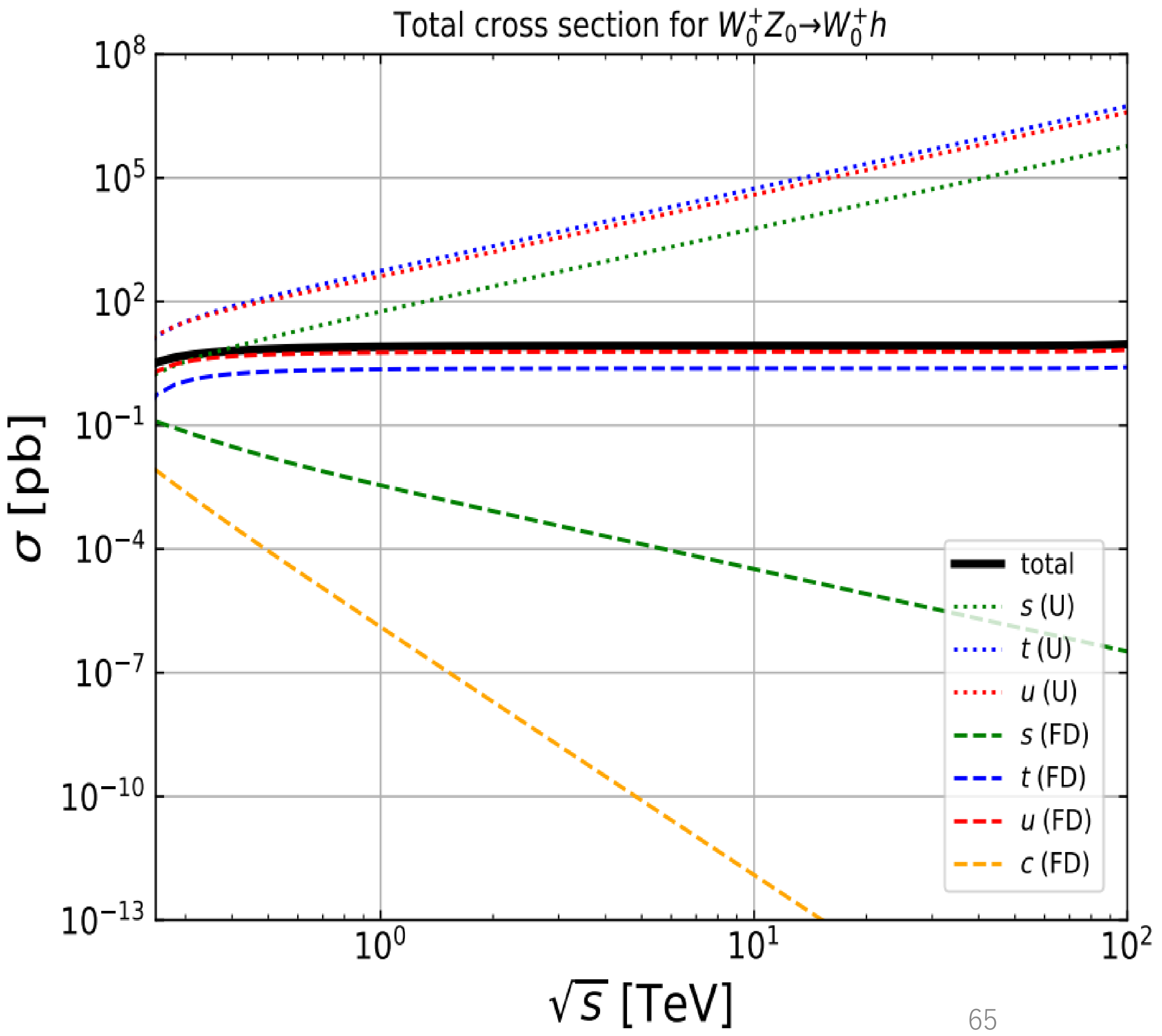


s t u c

Total cross section for $\lambda_1 = \lambda_2 = \lambda_3 = 0$

- U gauge :
  - Each amplitude has energy growth.
  - Large cancellation between the three amplitudes (s, t, u) at high energies.

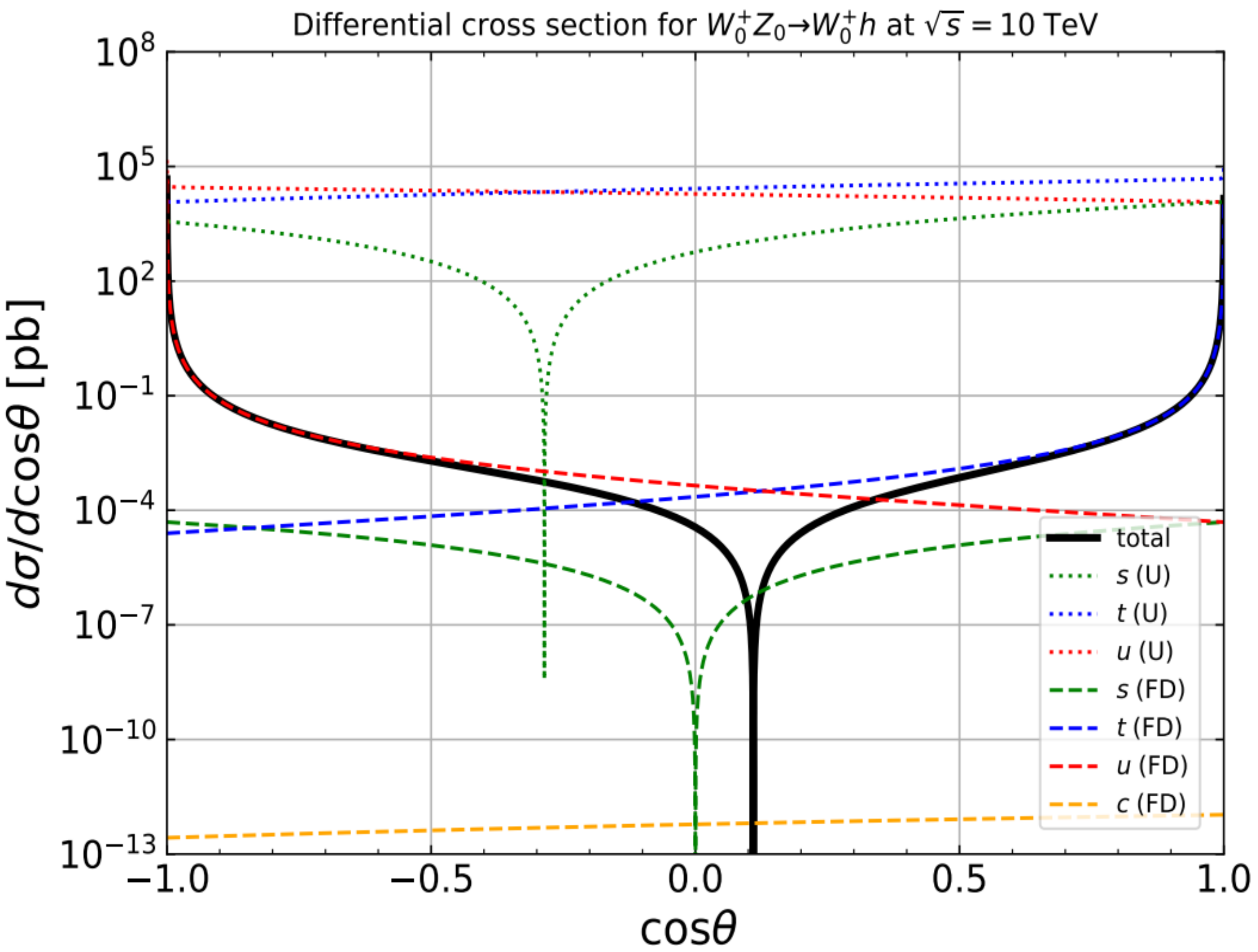
- FD gauge :
  - No such energy growth of each amplitude at all, i.e. no such gauge cancellation.
  - u and t are the main contributors.
  - At higher energies, s and c become smaller.



Angular Distribution for $\lambda_1 = \lambda_2 = \lambda_3 = 0$

- U gauge :
  - No clue for the physical distribution from each contribution.

- FD gauge :
  - Clear indication to the physical distribution from each contribution.
  - Forward scattering $(\cos\theta = 1)$ ： t is dominant.
  - Backward scattering $(\cos\theta = -1)$ ： u is dominant.



The FD gauge allows us to interpret the property of each Feynman amplitude and the interference among them.

We studied $pp \to Whjj$, especially the $W^+Z \to W^+h$ subprocess for longitudinally polarized gauge bosons. We found

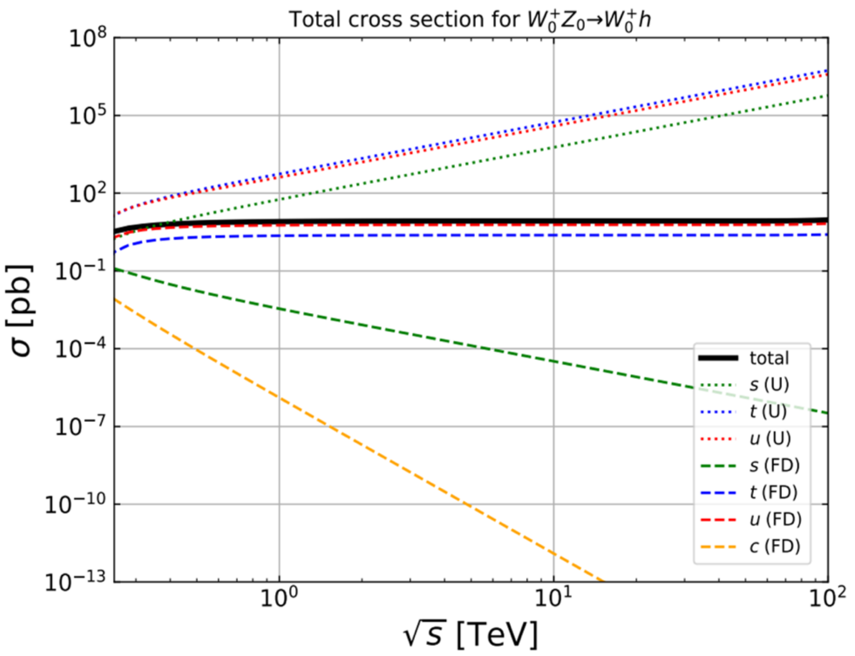
in the U gauge

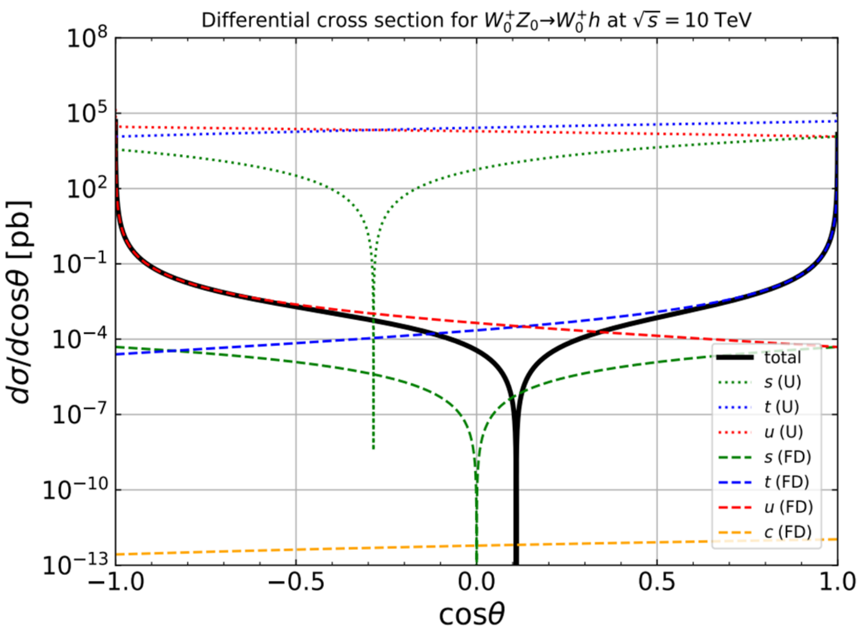
・huge gauge cancellation is needed at high energies.

in the FD

・no such cancellation.

・it is possible to see which are the dominant contributions and to interpret the property of the individual amplitudes.





B Goldstone boson couplings in the SM

In this appendix, we present all the Goldstone-boson couplings of the SM explicitly, because not only the magnitude but also the relative signs of all the gauge-boson and the corresponding Goldstone-boson couplings should be kept exact in order to keep the BRST invariance of the amplitudes. The Goldstone bosons appear in the four sector of the SM Lagrangian, the Higgs potential $\mathcal{V}_H$, the Higgs gauge interactions $\mathcal{K}_H$, the gauge-fixing term $\mathcal{L}_{GF}$, and the Yukawa term $\mathcal{L}_Y$. We show all of them explicitly below.

The minimal SM has just one $\mathrm{SU}(2)_L$ doublet Higgs field $\phi(x)$. The Lagrangian of the Higgs field is given by

$$\mathcal{L}_H = \mathcal{K}_H - \mathcal{V}_H \tag{B.1}$$

with

$$\mathcal{K}_H = (D^\mu\phi)^\dagger(D_\mu\phi), \tag{B.2}$$

$$\mathcal{V}_H = \frac{\lambda}{4}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2, \tag{B.3}$$

$$\mathcal{V}_H = \frac{m_H^2}{2}\left[H + \frac{H^2 + (\pi^1)^2 + (\pi^2)^2 + (\pi^3)^2}{2v}\right]^2, \tag{B.12}$$

$$\mathcal{K}_H = \frac{1}{2}(\partial H)^2 + (\partial\pi^+)(\partial\pi^-) + \frac{1}{2}(\partial\pi^0)^2 \tag{B.15a}$$

$$+ \left[\frac{g^2}{4}W^+W^- + \frac{g_Z^2}{4}\frac{Z^2}{2}\right]\left[(v+H)^2 + (\pi^0)^2\right] \tag{B.15b}$$

$$+ \frac{g_Z}{2}Z\left[(\partial\pi^0)(v+H) - (\partial H)\pi^0\right] \tag{B.15c}$$

$$+ \frac{g}{2}\left[W^+(\partial\pi^-) + W^-(\partial\pi^+)\right](v+H) \tag{B.15d}$$

$$- i\frac{g}{2}\left[W^+(\partial\pi^-) - W^-(\partial\pi^+)\right]\pi^0 \tag{B.15e}$$

$$+ i\frac{g}{2}(s_W^2 g_Z Z - eA)(W^+\pi^- - W^-\pi^+)(v+H) \tag{B.15f}$$

$$+ \frac{g}{2}(s_W^2 g_Z Z - eA)(W^+\pi^- + W^-\pi^+)\pi^0 \tag{B.15g}$$

$$- \frac{g}{2}\left[(W^+\pi^- + W^-\pi^+)(\partial H) + i(W^+\pi^- - W^-\pi^+)(\partial\pi^0)\right] \tag{B.15h}$$

$$- i\left[\left(\frac{1}{2} - s_W^2\right)g_Z Z + eA\right]\left[(\partial\pi^+)\pi^- - (\partial\pi^-)\pi^+\right] \tag{B.15i}$$

$$+ \left[\frac{g^2}{2}W^+W^- + \left(\left(\frac{1}{2} - s_W^2\right)g_Z Z + eA\right)^2\right]\pi^+\pi^-, \tag{B.15j}$$

J. Chen, K. Hagiwara, J. Kanzaki and K. Mawatari,
"Helicity amplitudes without gauge cancellation for electroweak processes", Eur.Phys.J.C 83 (2023) 10, 922.