TOHOKU UNIVERSITY

GPPU GRADUATE PROGRAM ON PHYSICS FOR THE UNIVERSE
宇宙創成物理学
国際共同大学院

# 軽いミニ荷電粒子模型に現れる整数論的構造

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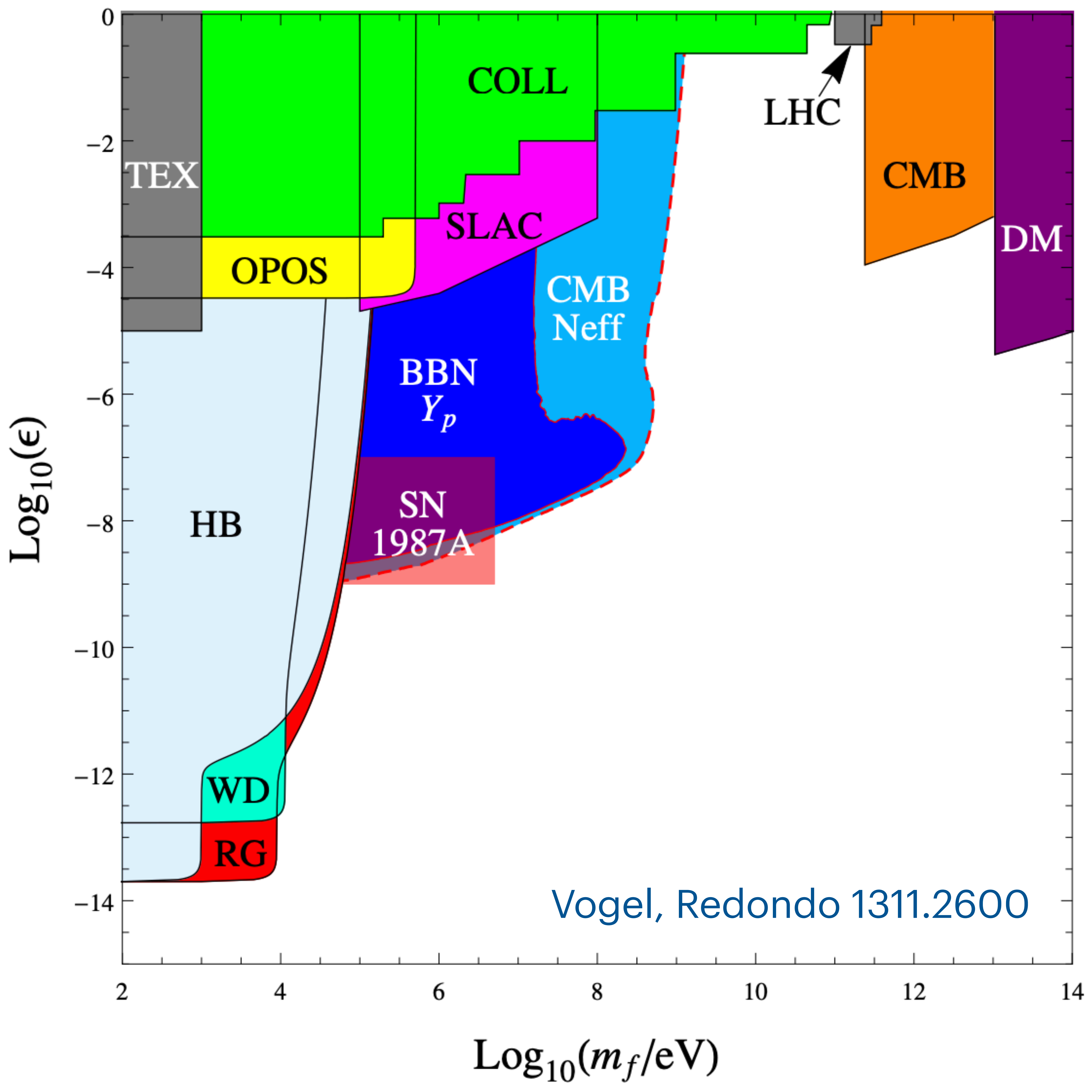
# Minicharged particle

Minicharged particle (mCP) is a particle $\Psi$ with **electric charge** $Q_{\rm eff} \ll e$ and mass $m_\Psi$.

Small charge makes them weakly interacting but not completely invisible.

mCPs are testable by stellar cooling, laboratory searches, cosmological observations, …



# Extra $U(1)$

Abelian gauge symmetries appear naturally in GUT, string, and dark-sector constructions. Wen, Witten `85, Holdom `86, …

They can remain hidden if the new matter is Standard-Model neutral.

The simplest connection to us is a dimension-four portal.

Standard Model — $\chi B_{\mu\nu} F'^{\mu\nu}$ — Hidden $U(1)_{\mathrm{H}}$

fermion $\Psi$

# Hidden charge becomes minicharge

For an unbroken hidden $U(1)_{\rm H}$ , one can change the basis to remove kinetic mixing term:

$$A'_\mu \to A'_\mu - \chi B_\mu \,.$$

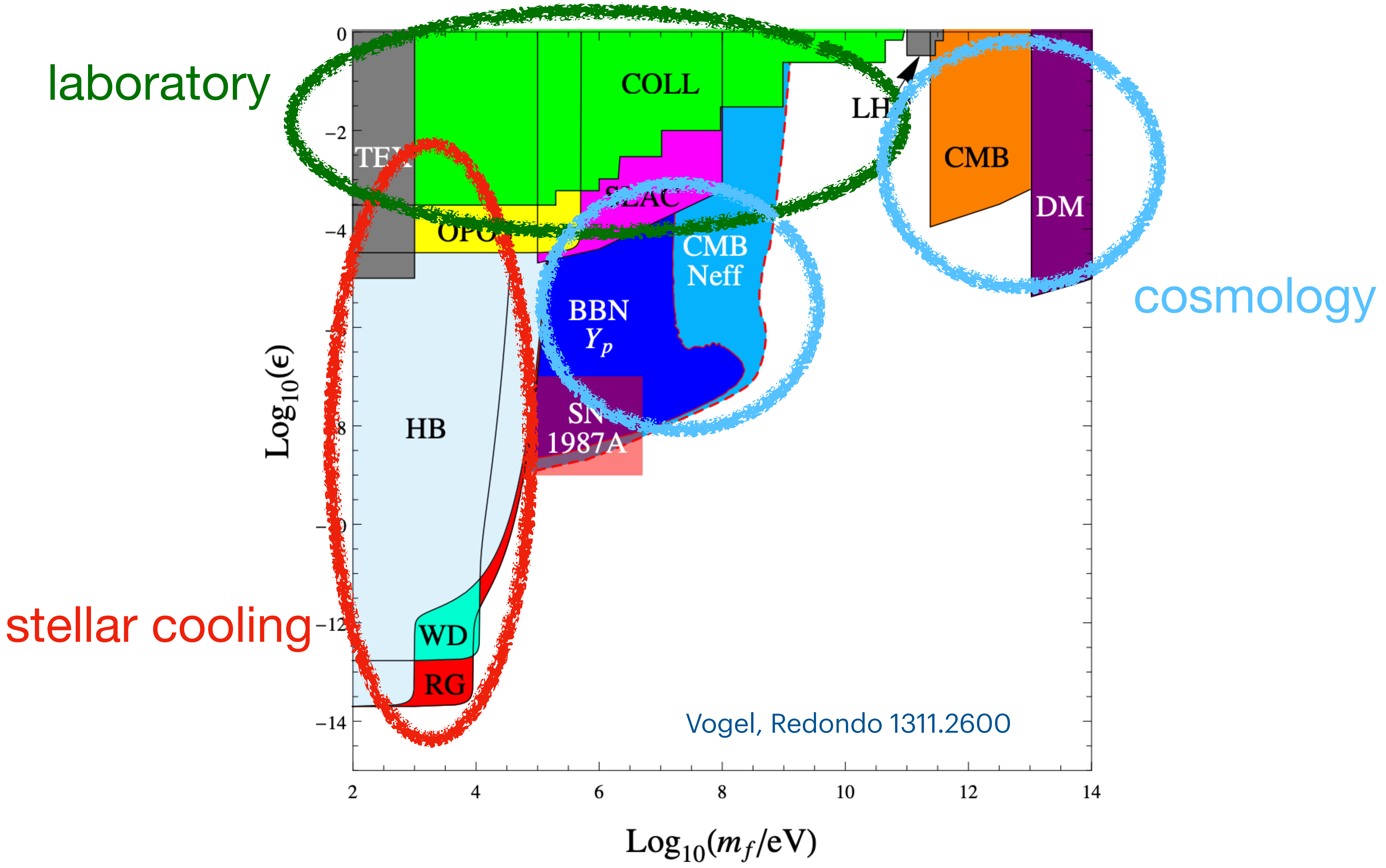
A hidden fermion $\Psi$ with

$$\mathscr{L}_{\rm int} \supset -g_{\rm H} q_{\rm H} A'_\mu \bar{\Psi}\gamma^\mu \Psi$$

obtains an effective electromagnetic charge,

$$Q_{\rm eff} \simeq -\chi g_{\rm H} q_{\rm H} \cos\theta_W \,.$$

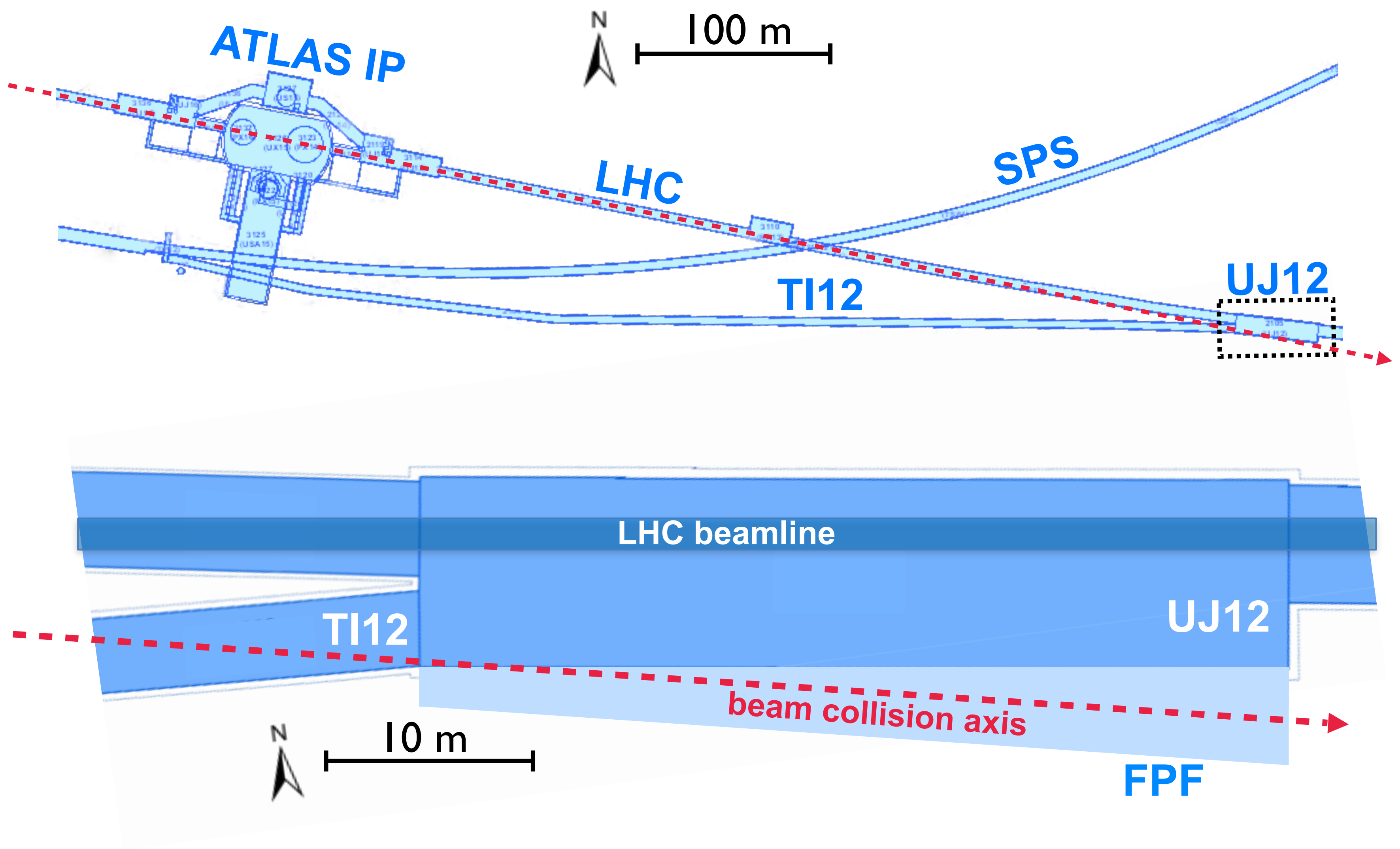
Minicharged Particle (mCP)



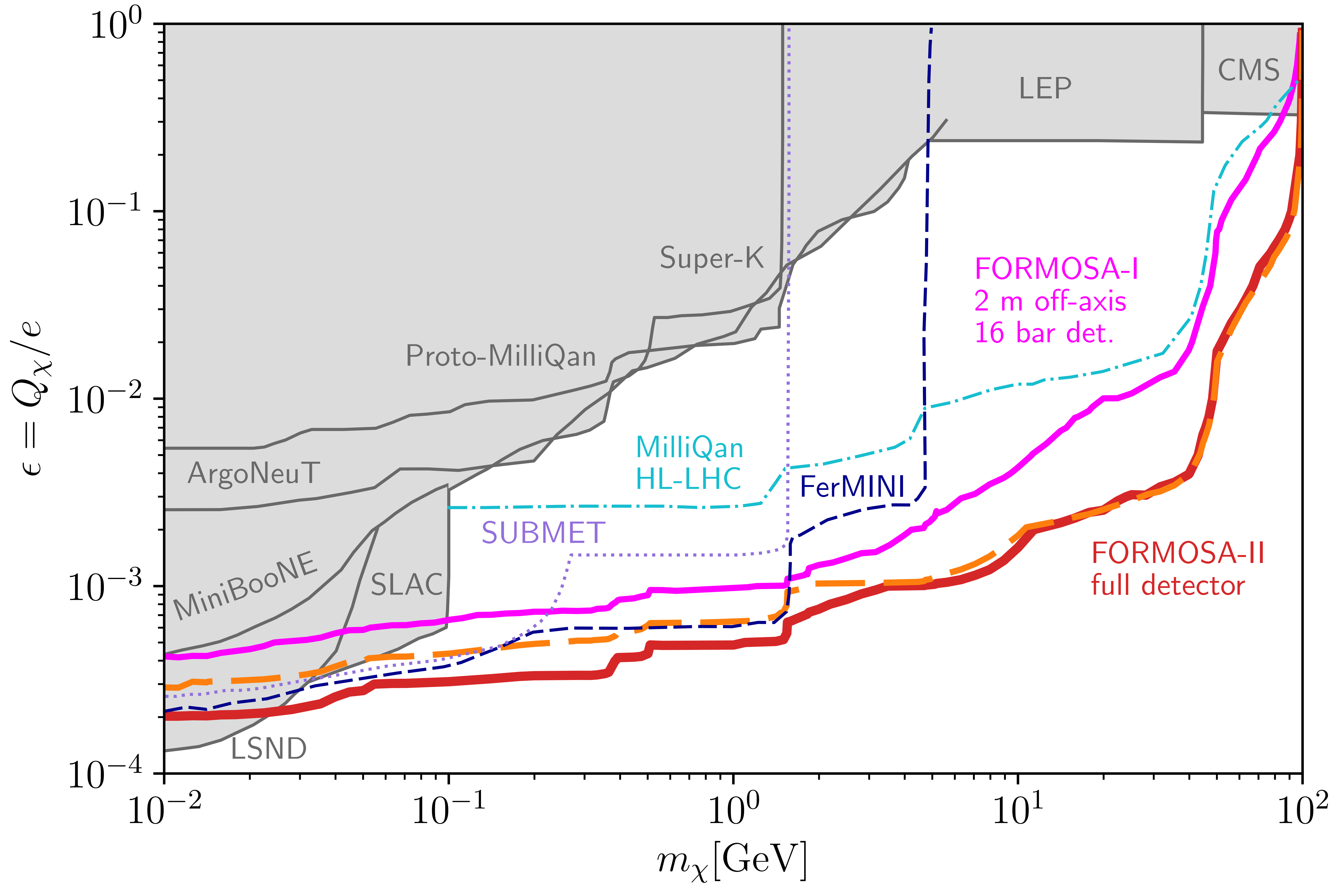
laboratory
stellar cooling
cosmology
COLL
LH
CMB
DM
SLAC
OPO
CMB
Neff
BBN
$Y_p$
HB
SN
1987A
WD
RG
0
−2
−4
−8
−12
−14
2
4
6
8
10
12
14
$\mathrm{Log}_{10}(\epsilon)$
$\mathrm{Log}_{10}(m_f/\mathrm{eV})$
Vogel, Redondo 1311.2600

# FORMOSA as an example

Future scintillator-based experiments to be located in the far-forward region at the LHC.




Foroughi-Abari, Kling, Tsai 2010.07941



# Why is mCP light?

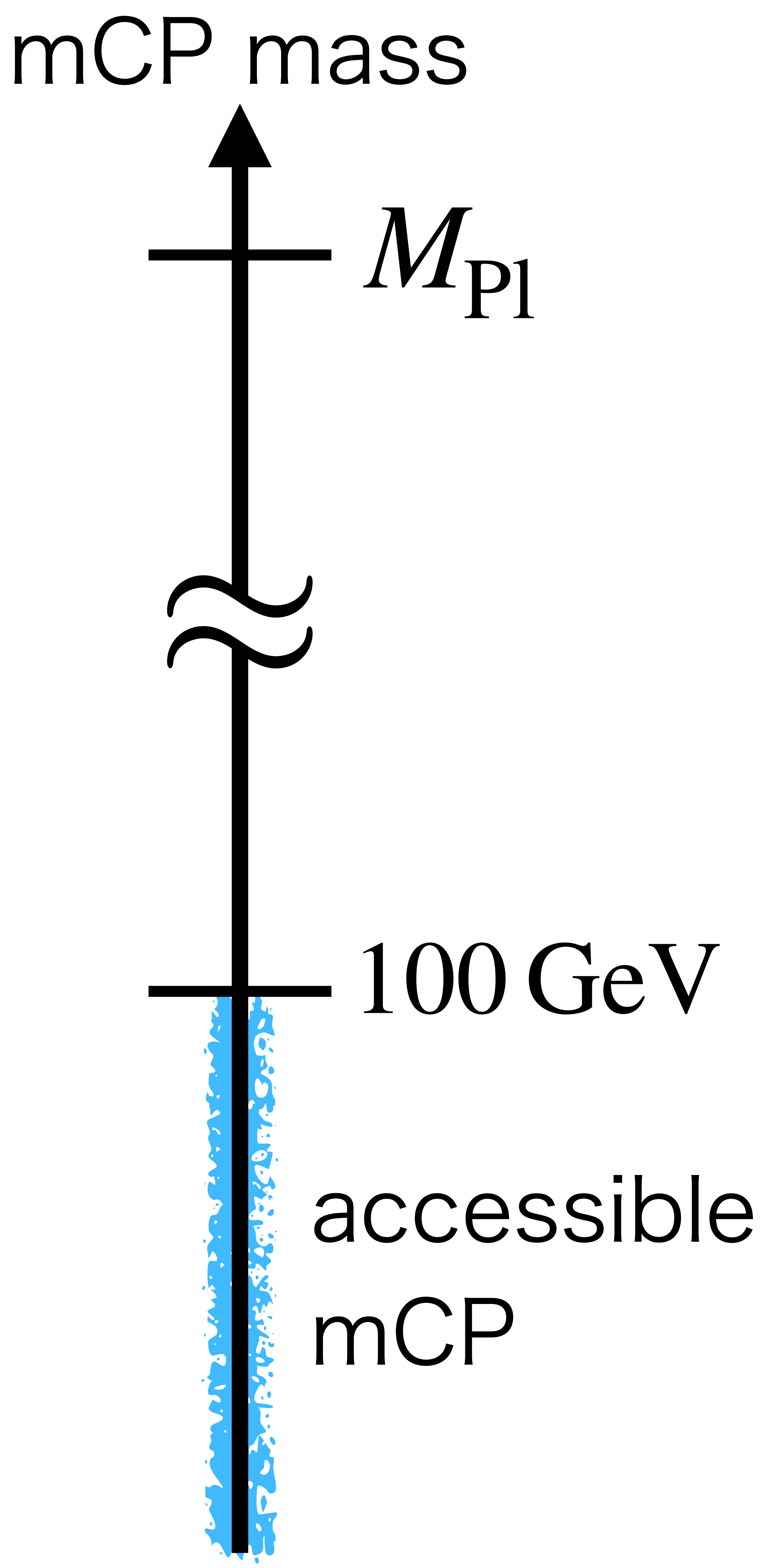
In the minimal setup, a Dirac mass is allowed:

$$-m_\Psi \bar{\Psi}\Psi\,.$$

If the cutoff is $M_c$, effective field theory suggests

$$m_\Psi \sim M_c\,,$$

unless the small mass is technically natural.



# Strategy: symmetry protects small mass

Introduce a chiral gauge symmetry $U(1)_X$ , and forbid a mass term.

$$U(1)_H \times U(1)_X$$

$$\cancel{\psi_1\psi_2 + \mathrm{h.c.}}$$

| | | $\psi_1$ | $\psi_2$ |
|---|---|---|---|
| Vector-like | $U(1)_H$ | $+1$ | $-1$ |
| Chiral | $U(1)_X$ | $a$ | $-b$ |

$$a \neq b$$

# Strategy: symmetry protects small mass

Introduce a chiral gauge symmetry $U(1)_\mathrm{X}$ , and forbid a mass term.

When $U(1)_\mathrm{X}$ is broken by $\phi$, a small fermion mass is allowed.

$$U(1)_\mathrm{H} \times U(1)_\mathrm{X}$$

$$\cancel{\psi_1 \psi_2 + \mathrm{h.c.}}$$

$$y M_c \left( \frac{\phi^{(*)}}{M_c} \right)^{|a-b|} \psi_1 \psi_2 + \mathrm{h.c.}$$

( $y = \mathcal{O}(1)$ complex coefficient )

| | | $\psi_1$ | $\psi_2$ | $\phi$ (Dark Higgs) |
|---|---|---|---|---|
| Vector-like | $U(1)_\mathrm{H}$ | $+1$ | $-1$ | $0$ |
| Chiral | $U(1)_\mathrm{X}$ | $a$ | $-b$ | $1$ |

$$a \neq b$$

# Strategy: symmetry protects small mass

| $U(1)_{\mathrm{H}}$ | $U(1)_{\mathrm{X}}$ |
|---|---|
| • unbroken | • broken by Higgs $\phi$ |
| • vector-like | • chiral |
| • gives the minicharge through kinetic mixing | • protects the light mass |

# Strategy: symmetry protects small mass

| $U(1)_{\mathrm{H}}$ | $U(1)_{\mathrm{X}}$ |
| --- | --- |
| • unbroken | • broken by Higgs $\phi$ |
| • vector-like | • chiral |
| • gives the minicharge through kinetic mixing | • protects the light mass |

We must cancel chiral gauge anomalies.

# Anomaly cancellation

In general, we may introduce $2n$ Weyl fermions $\psi_i$ .

| | $\psi_i$ | $\psi_j$ | $\phi$ |
|---|---|---|---|
| $U(1)_{\mathrm{H}}$ | $+1$ | $-1$ | $0$ |
| $U(1)_{\mathrm{X}}$ | $a_i$ | $-b_j$ | $1$ |

$i = 1, 2, \cdots, n$

$j = n+1, n+2, \cdots, 2n$

$a_i \neq b_j$

# Anomaly cancellation

In general, we may introduce $2n$ Weyl fermions $\psi_i$ .

$U(1)_\mathrm{X} - U(1)_\mathrm{X} - U(1)_\mathrm{X}$ anomaly

$$\sum_i a_i^3 = \sum_j b_j^3$$

$U(1)_\mathrm{X} - U(1)_\mathrm{X} - U(1)_\mathrm{H}$ anomaly

$$\sum_i a_i^2 = \sum_j b_j^2$$

$U(1)_\mathrm{X} - \mathrm{graviton} - \mathrm{graviton}$ anomaly

$$\sum_i a_i = \sum_j b_j$$

| | $\psi_i$ | $\psi_j$ | $\phi$ |
|---|---|---|---|
| $U(1)_\mathrm{H}$ | $+1$ | $-1$ | $0$ |
| $U(1)_\mathrm{X}$ | $a_i$ | $-b_j$ | $1$ |

$$i = 1, 2, \cdots, n$$
$$j = n+1, n+2, \cdots, 2n$$

$$a_i \neq b_j$$

Euler, Goldbach (1750), Prouhet (1851), Escott (1910), Tarry (1895),
Review: Borwein (Springer, `02)

# Prouhet-Tarry-Escott problem

degree $k$ and size $n$ PTE problem:

Given a positive integer $k$, find two distinct sets of integer solutions $A = \{a_1, a_2, \cdots, a_n\}$ and $B = \{b_1, b_2, \cdots, b_n\}$ that satisfy

$$\sum_{i=1}^{n} a_i^\ell = \sum_{i=1}^{n} b_i^\ell \qquad \text{for } \ell = 1, 2, \cdots, k\,.$$

The smallest possible size is $n = k + 1$.

This can be proved using Newton's identities.

Such solutions are called **ideal solutions**.

# Prediction I: at least four mCPs

| | $\psi_i$ | $\psi_j$ | $\phi$ |
|---|---|---|---|
| $U(1)_{\mathrm{H}}$ | $+1$ | $-1$ | $0$ |
| $U(1)_{\mathrm{X}}$ | $a_i$ | $-b_j$ | $1$ |

$i = 1, 2, 3, 4$

$j = 5, 6, 7, 8$

$a_i \neq b_j$

$\epsilon = \dfrac{\langle \phi \rangle}{M_c}$

$$\mathcal{L} \supset y_{ij} M_c \left( \frac{\phi^{(*)}}{M_c} \right)^{|a_i - b_j|} \psi_i \psi_j + \mathrm{h.c.}$$

( $y_{ij} = \mathcal{O}(1)$ complex coefficients )

$\Rightarrow$

$4 \times 4$ mass matrix

$$\boxed{M_{ij} \simeq M_c \epsilon^{|a_i - b_j|}}$$

In this talk, we focus on the <u>ideal case</u>.

Four mCPs

# More about PTE problem

There are solutions with good properties: **symmetric solutions**.

The solution is called symmetric when it remains unchanged under the sign flip, $\{a_i\} = \{-a_i\}$ and $\{b_i\} = \{-b_i\}$.

e.g.) $$A = \{-8, -1, 1, 8\}, \quad B = \{-7, -4, 4, 7\}$$

Up to a common shift, **many ideal solutions are symmetric**.

# Nota Bene: affine transformation

Mathematically,

$$A \to A' = \{ca_i + d\}, \qquad B \to B' = \{cb_i + d\}$$

$$(c, d : \text{rational numbers}, \quad c \neq 0)$$

maps a PTE solution to another solution, generating an equivalence class.

$\ast$ We extend the definition of a symmetric solution, allowing a shift.
e.g.) $A = \{0, 7, 9, 16\}$, $B = \{1, 4, 12, 15\}$ is a symmetric solution.

Affine-related PTE solutions are physically distinct.

# Why a symmetric solution is good

For $a_i \in \{-a, a\}$ and $b_i \in \{-b, b\}$, the eigenvalues of the matrix $\epsilon^{|a_i - b_j|}$ are degenerate.

$$\boxed{M_{ij} \simeq M_c \epsilon^{|a_i - b_j|}}$$

| | $-a$ | $a$ |
|---|---|---|
| $-b$ | $\lvert a-b \rvert$ | $\lvert a+b \rvert$ |
| $b$ | $\lvert a+b \rvert$ | $\lvert a-b \rvert$ |

$$a \neq b \text{ and } a, b \geq 0$$

# Symmetric solutions give paired exponents

For $a_i \in \{-a_1, a_1, -a_2, a_2\}$ and $b_i \in \{-b_1, b_1, -b_2, b_2\}$, the exponent in the mass eigenvalues appear in pairs.

$$M_{ij} \simeq M_c \epsilon^{|a_i - b_j|}$$

| | $-a_1$ | $a_1$ | $-a_2$ | $a_2$ |
|---|---|---|---|---|
| $-b_1$ | $\lvert a_1 - b_1 \rvert$ | $\lvert a_1 + b_1 \rvert$ | higher power | |
| $b_1$ | $\lvert a_1 + b_1 \rvert$ | $\lvert a_1 - b_1 \rvert$ | | |
| $-b_2$ | higher power | | $\lvert a_2 - b_2 \rvert$ | $\lvert a_2 + b_2 \rvert$ |
| $b_2$ | | | $\lvert a_2 + b_2 \rvert$ | $\lvert a_2 - b_2 \rvert$ |

Mathematical proof can be found in JL, Takahashi, Tsai 2603.12320

# Prediction II: doublet structure


Degeneracy is in power of $\epsilon$. $\mathcal{O}(1)$ coefficients split the pair.

# Symmetric solutions dominates ideal solutions

For the symmetric cases, the linear and cubic sum conditions are trivial.

Write

$$A = \{-a_1, a_1, -a_2, a_2\}, \qquad B = \{-b_1, b_1, -b_2, b_2\},$$

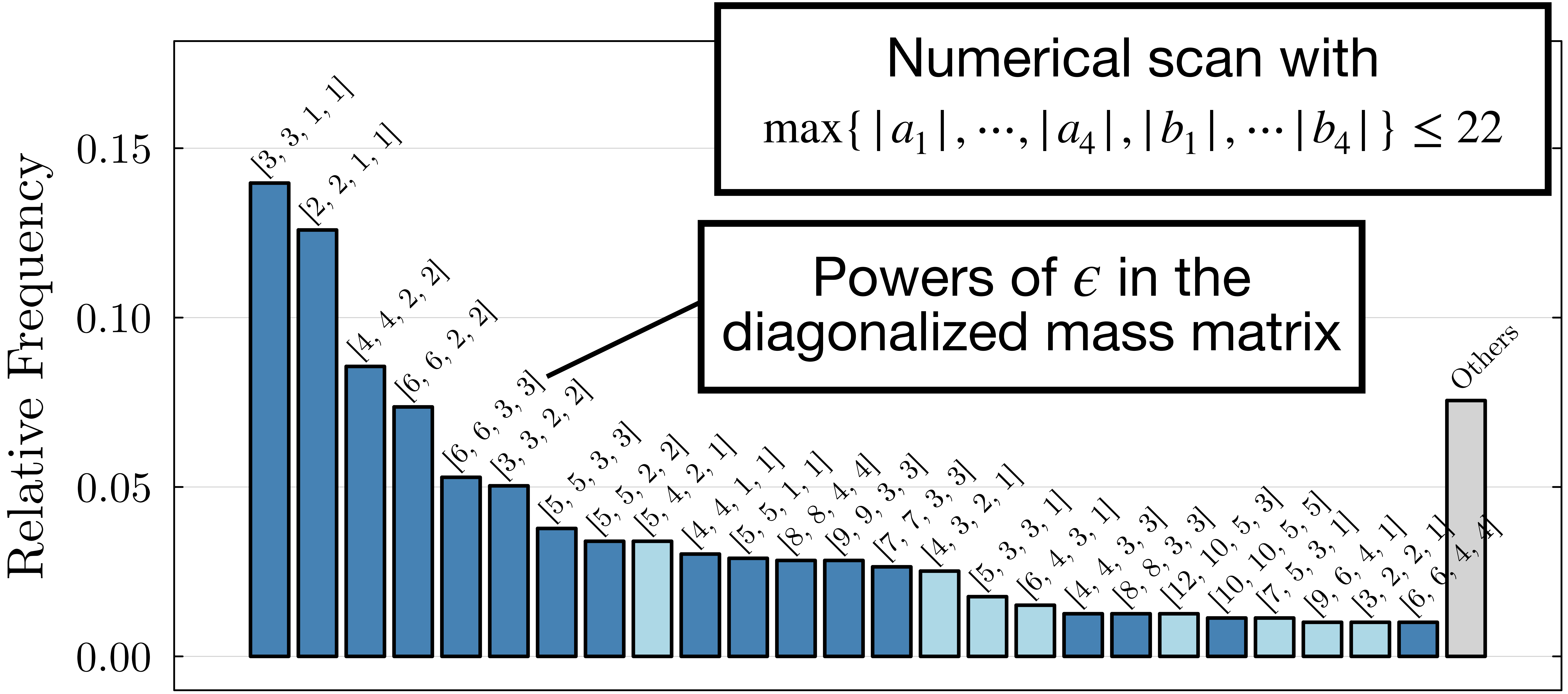
the anomaly cancellation conditions reduce to

$$a_1^2 + a_2^2 = b_1^2 + b_2^2 \, .$$

At least for sets of integers that are not too large, symmetric solutions appear numerically dominant among all possible solutions.

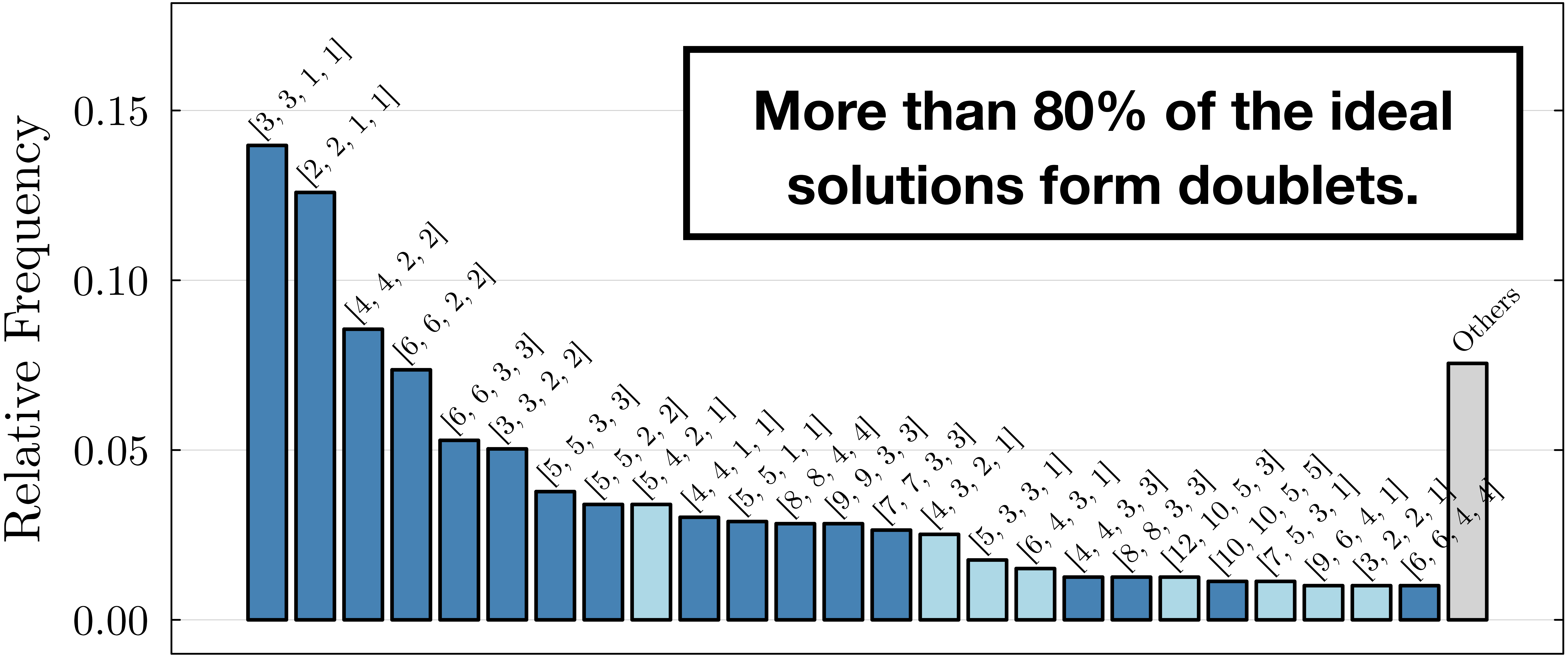
JL, Takahashi, Tsai 2606.07735

# mCPs likely form doublets



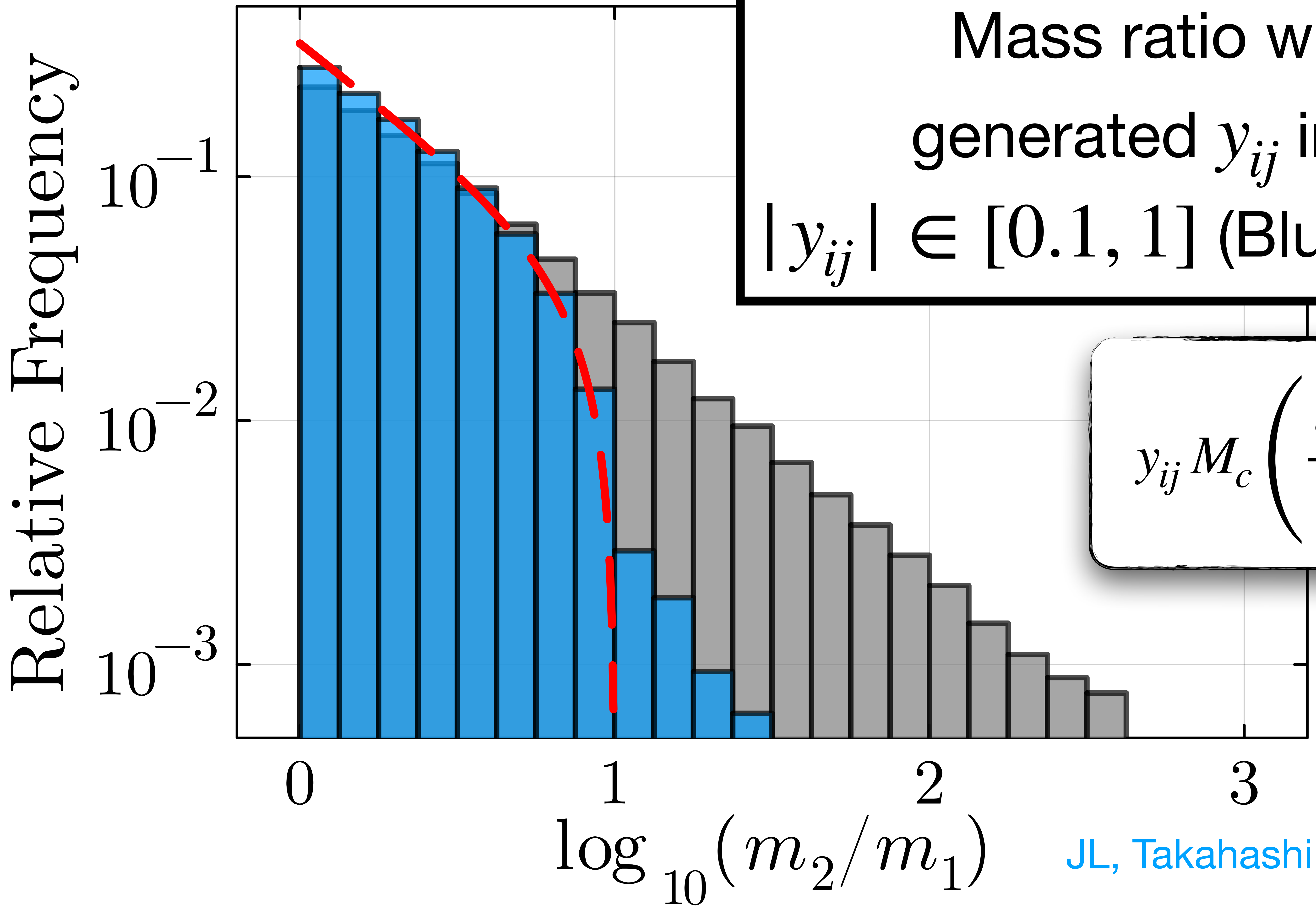

JL, Takahashi, Tsai 2603.12320

# mCPs likely form doublets




JL, Takahashi, Tsai 2603.12320

# mCP mass ratio of the lightest doublet




Mass ratio with randomly generated $y_{ij}$ in the range of $|y_{ij}| \in [0.1, 1]$ (Blue) and $[0, 1]$ (Gray)

$$y_{ij} M_c \left(\frac{\phi^{(*)}}{M_c}\right)^{|a_i - b_j|} \psi_i \psi_j + \mathrm{h.c.}$$

JL, Takahashi, Tsai 2603.12320

The lightest two states usually remain relatively close in mass.

# Summary

- Light mCP masses can be protected by chiral $U(1)_{\mathrm{X}}$ .
- Anomaly cancellation is equivalent to the degree-3 Prouhet-Tarry-Escott Problem.
- Minimal consistency predicts four mCPs and generically doublet spectra.

If one mCP is found, do not immediately fit it as a one-species model.
Look for a partner with the same minicharge and a nearby mass.

# Back Up

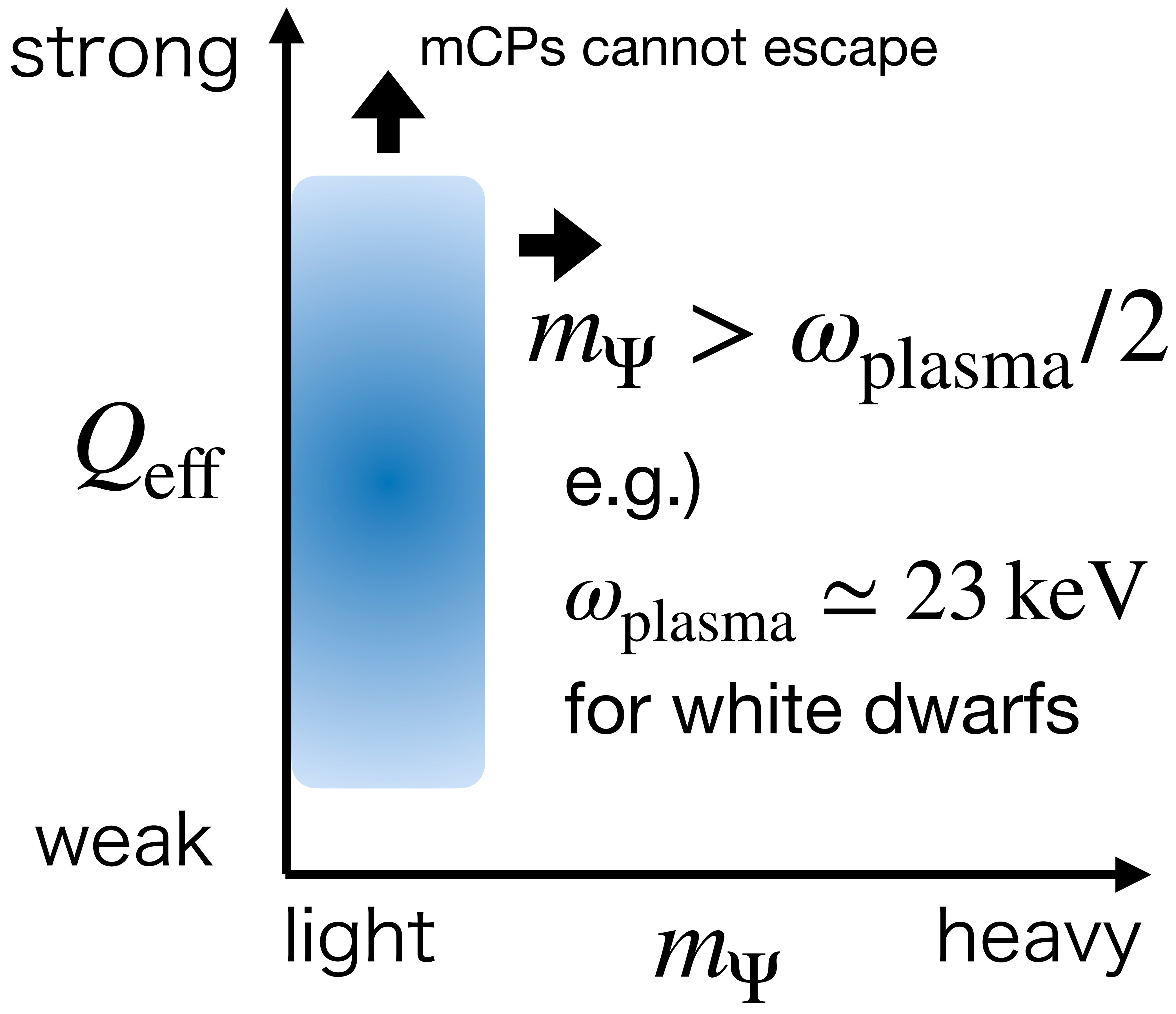
# Stellar cooling

Davidson, Campbell, Bailey `91, Raffelt `96

In a stellar plasma, a photon can produce light mCP pairs.

$$\gamma^* \to \Psi\bar{\Psi}$$

Escaping mCPs carry away invisible energy and constrain light masses.



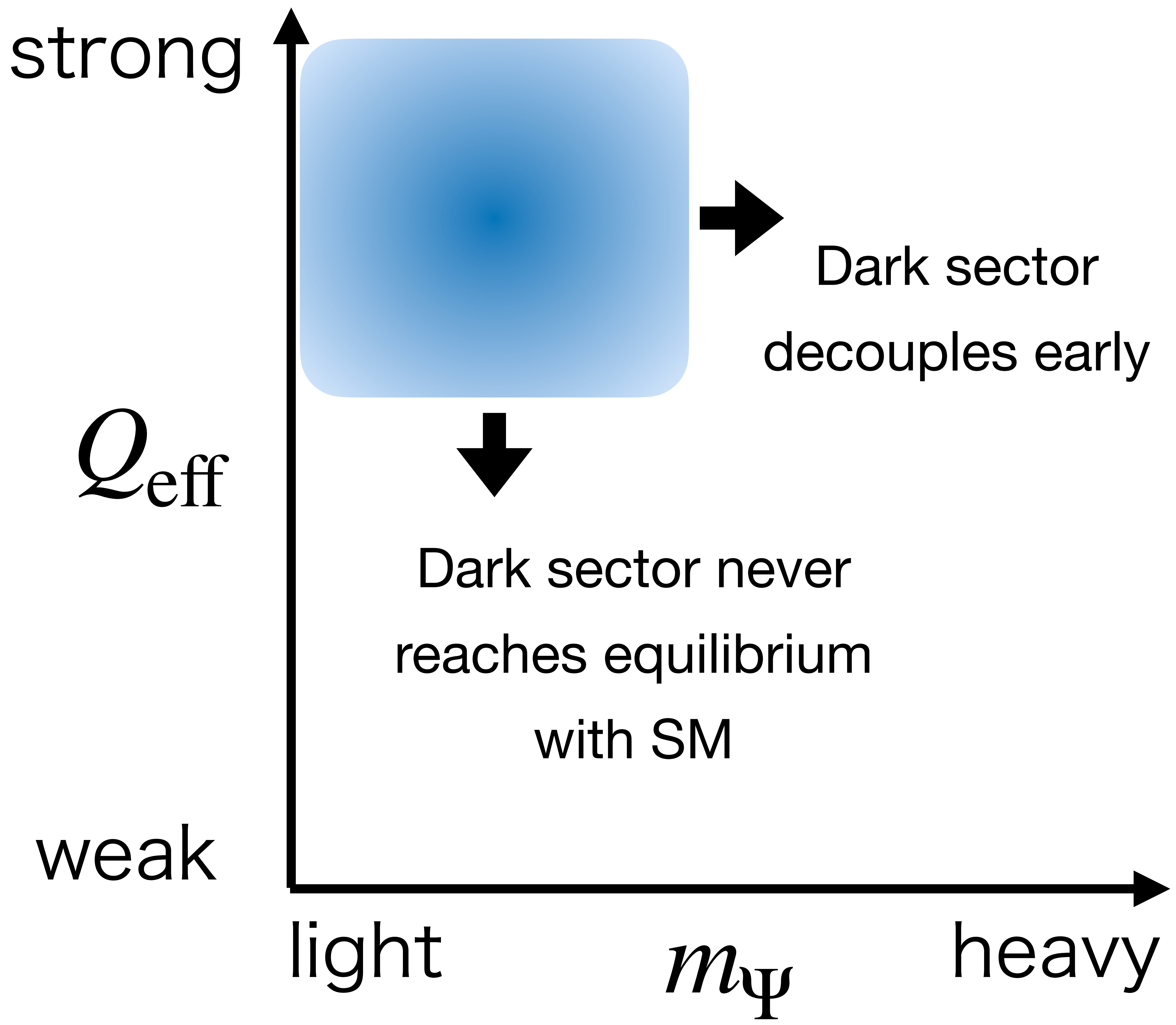
# Cosmology

Davidson, Campbell, Bailey `91, Vogel, Redondo 1311.2600

Hidden sector particles are efficiently produced by

$$e^+e^- \to \Psi\bar{\Psi}, \quad \gamma^* \to \Psi\bar{\Psi}$$

in the early time, and may contribute to $N_{\rm eff}$ at the BBN and CMB epoch.



# Laboratory searches

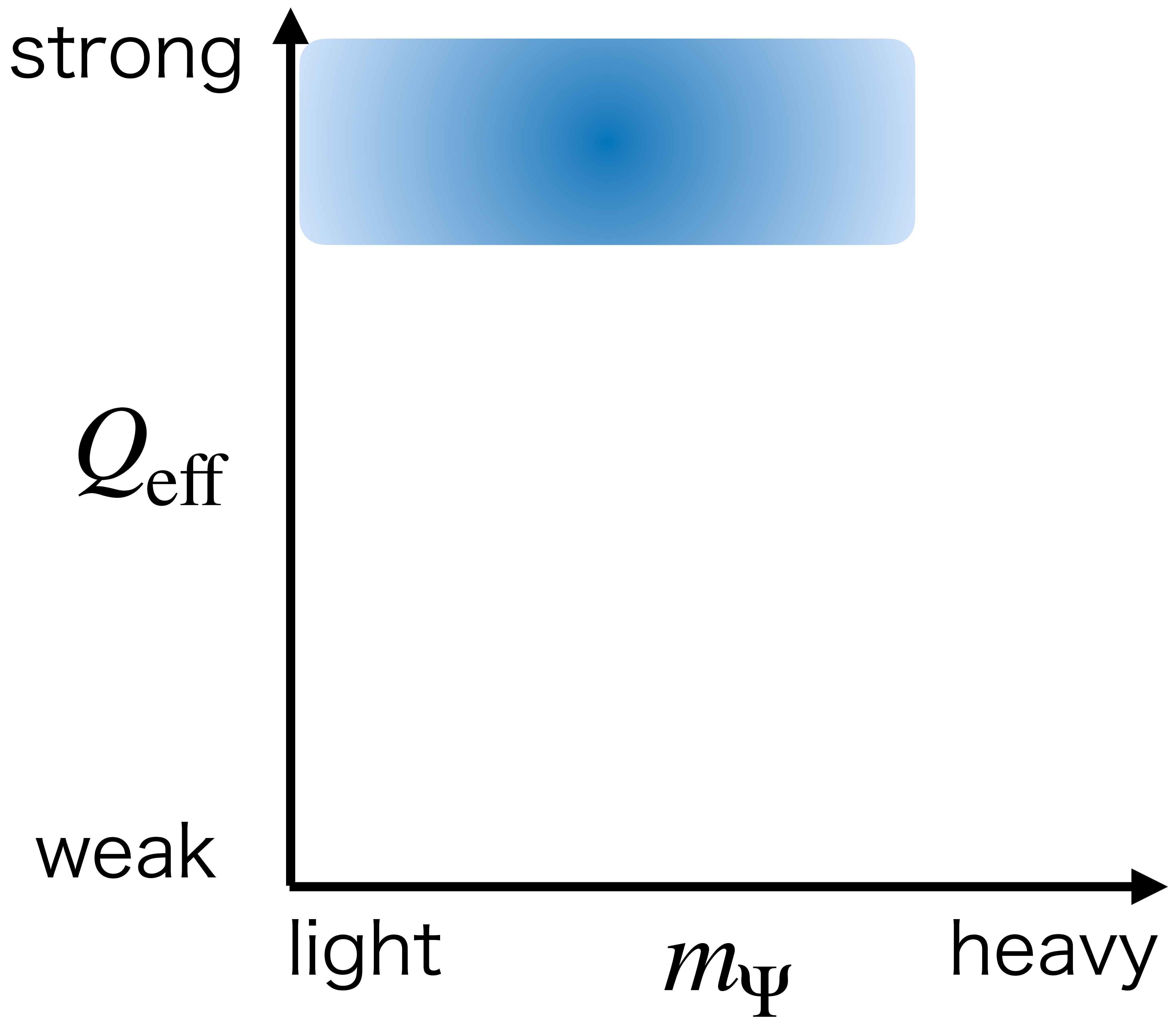
Various experiments constrain a large mixing regime.

Beam-dump

Collider

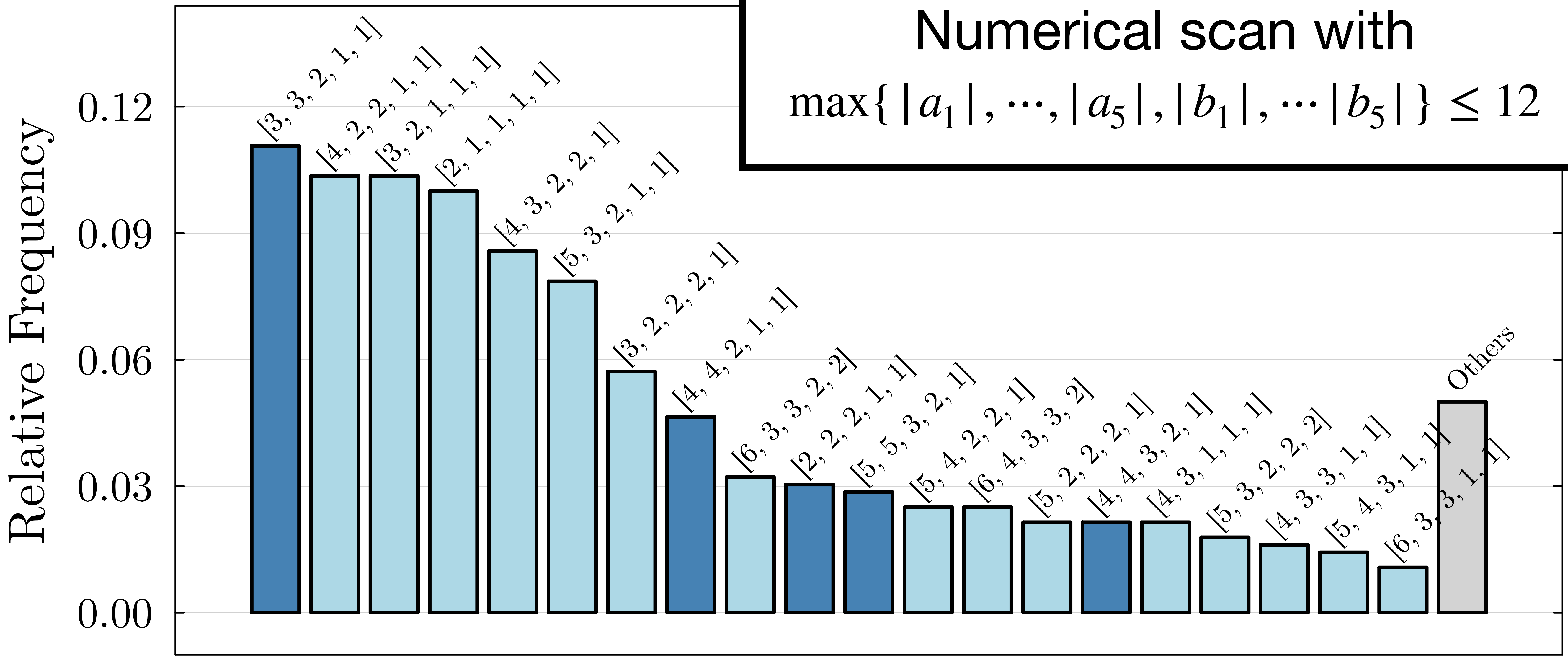
Positronium decay

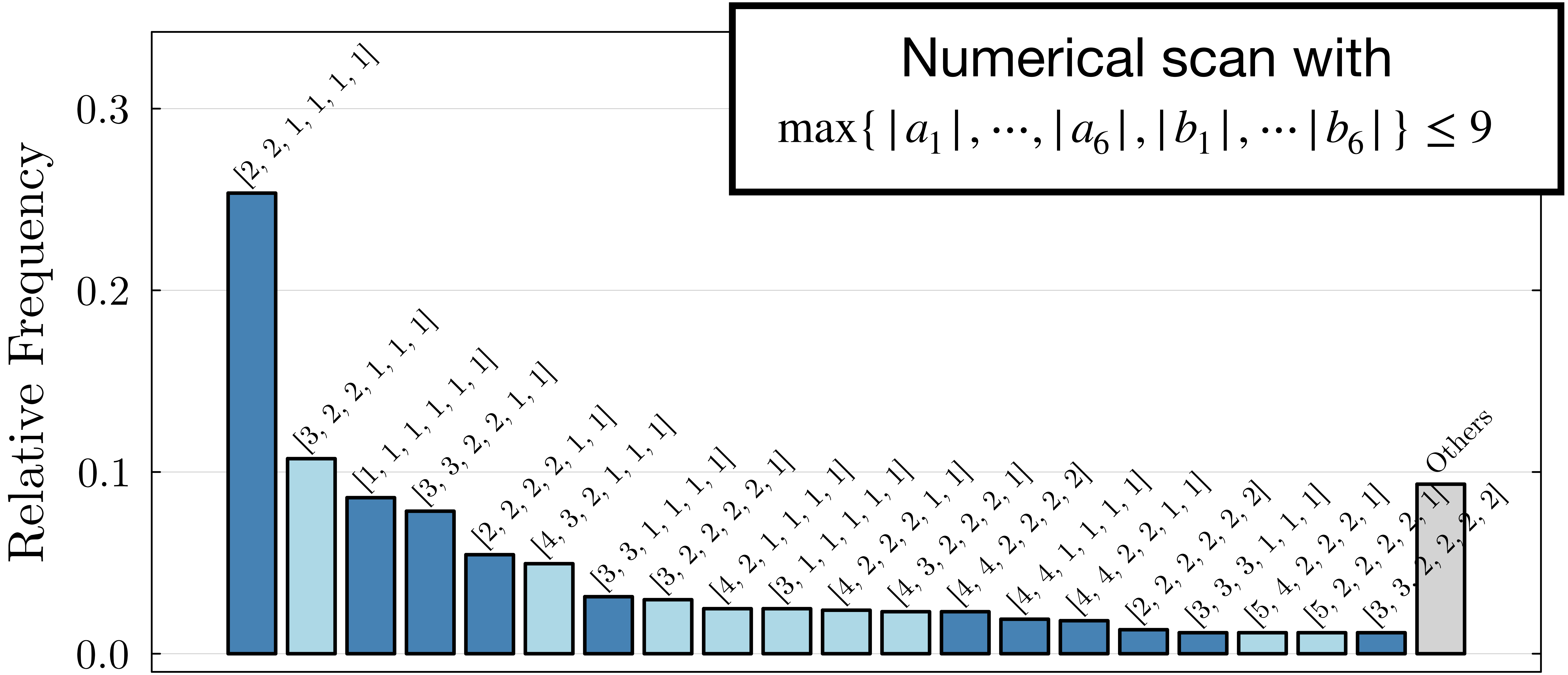
...



Numerical scan with

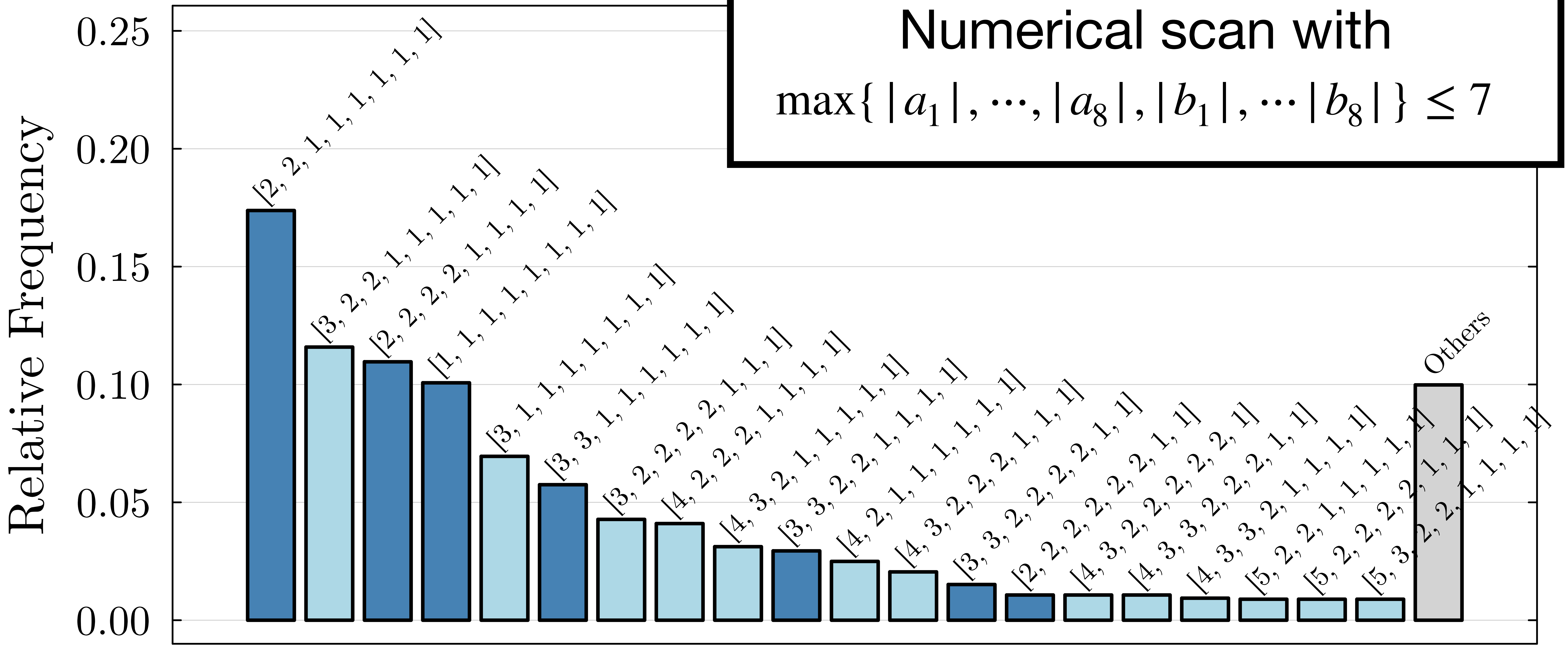
$$\max\{\,|a_1|,\cdots,|a_5|,|b_1|,\cdots|b_5|\,\}\leq 12$$

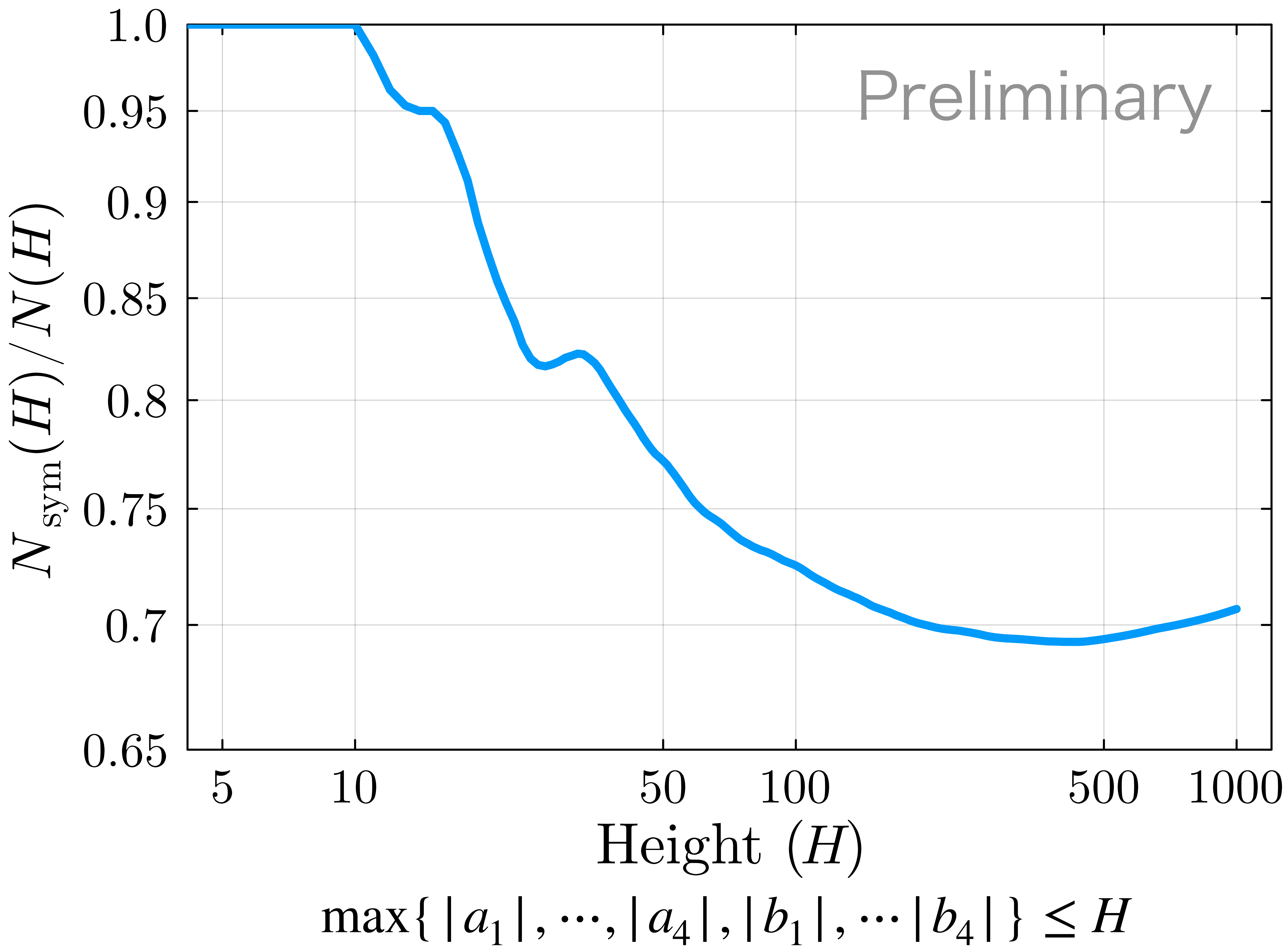



Numerical scan with
$\max\{\,|a_1|,\cdots,|a_6|,|b_1|,\cdots|b_6|\,\} \leq 9$
Relative Frequency
0.3
0.2
0.1
0.0
[2, 2, 1, 1, 1, 1]
[3, 2, 2, 1, 1, 1]
[1, 1, 1, 1, 1, 1, 1]
[3, 3, 2, 2, 1, 1]
[2, 2, 2, 2, 1, 1]
[4, 3, 2, 1, 1, 1]
[3, 3, 1, 1, 1, 1]
[3, 2, 2, 2, 2, 1]
[4, 2, 1, 1, 1, 1]
[3, 1, 1, 1, 1, 1]
[4, 2, 2, 2, 1, 1]
[4, 3, 2, 2, 2, 1]
[4, 4, 2, 2, 2, 2]
[4, 4, 1, 1, 1, 1]
[4, 4, 2, 2, 1, 1]
[2, 2, 2, 2, 2, 2]
[3, 3, 3, 1, 1, 1]
[5, 4, 2, 2, 2, 1]
[5, 2, 2, 2, 2, 1]
[3, 3, 2, 2, 2, 2]
Others

**Numerical scan with**

$\max\{|a_1|, \cdots, |a_8|, |b_1|, \cdots |b_8|\} \leq 7$





Preliminary
1.0
0.95
0.9
0.85
0.8
0.75
0.7
0.65
$N_{\mathrm{sym}}(H)/N(H)$
5
10
50
100
500
1000
Height $(H)$
$\max\{|a_1|, \cdots, |a_4|, |b_1|, \cdots |b_4|\} \leq H$