格子ゲージ理論と量子計算の進展

Arata Yamamoto (RIKEN)

Self-introduction

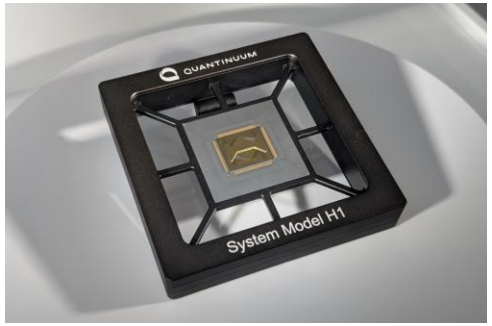
name: **Arata Yamamoto**

research: **QCD & hadron physics, lattice gauge theory**

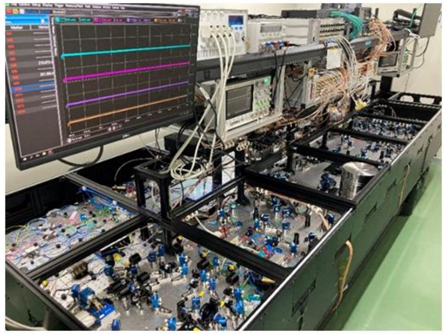
affiliation: **RIKEN iTHEMS**

A & A-II

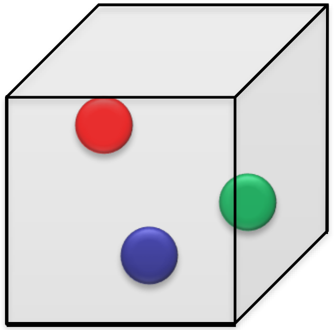
ibm_kobe



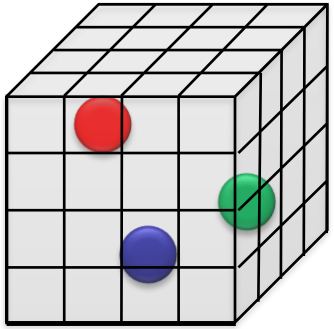
Reimei



optical QC



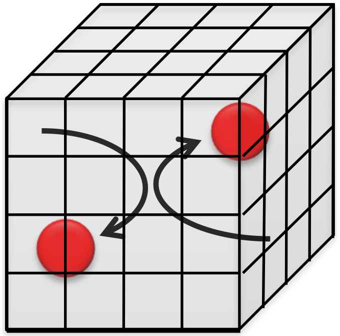
gauge theory



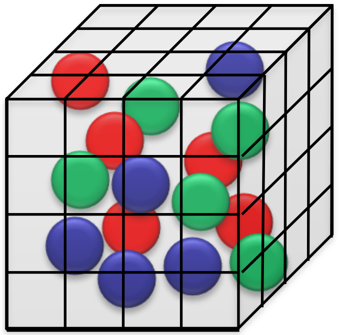
lattice gauge theory

supercomputer

open problems in lattice gauge theory



real-time dynamics



dense matter

quantum computer

open problems in lattice gauge theory

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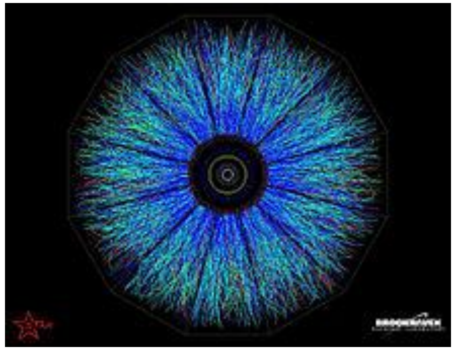
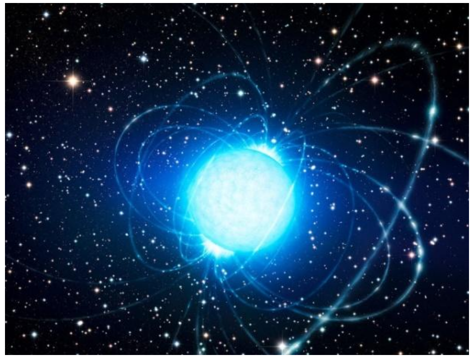


real-time dynamics

https://en.wikipedia.org/wiki/Magnetar



dense matter

$\longrightarrow$

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quantum computer

Contents

2. Quantum computing

Quantum computing

classical computer

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quantum computer

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$$\text{c-bit} = 0 \text{ or } 1$$

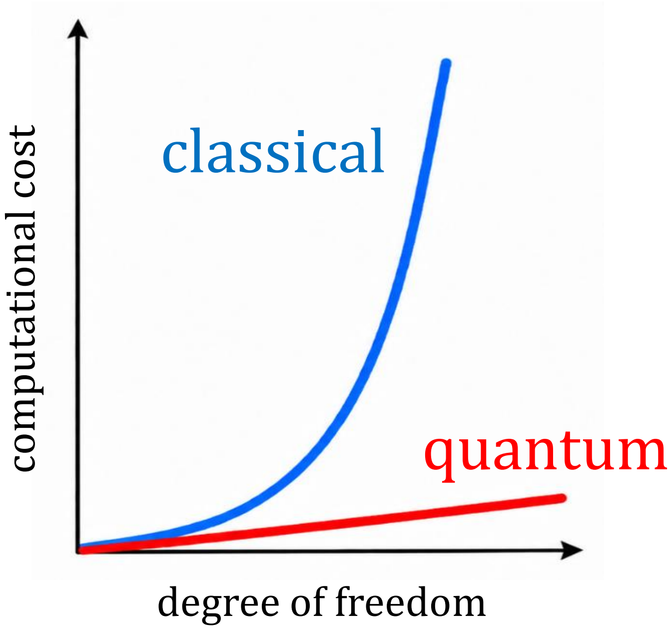
$$\text{q-bit} = c_0|0\rangle + c_1|1\rangle$$

$$N \text{ c-bits} = \underbrace{0110010\cdots}_{N \text{ digits}}$$

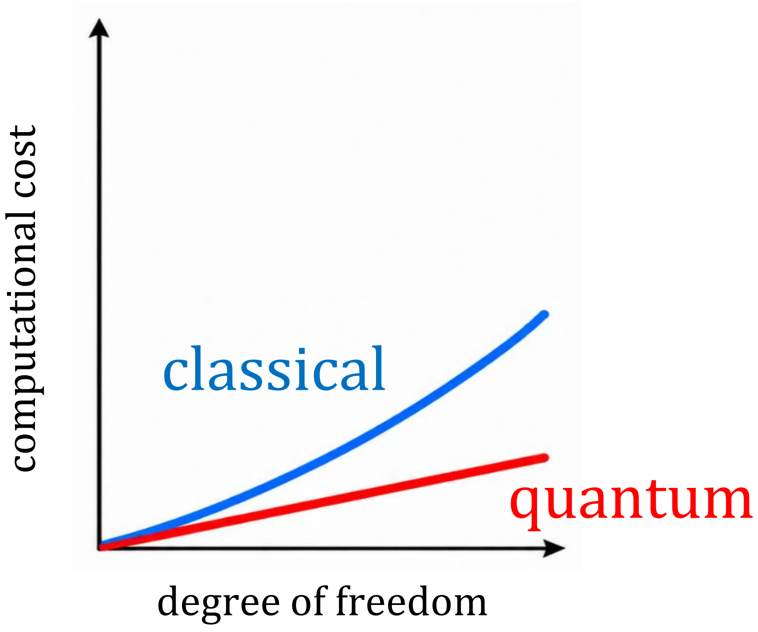
$$N \text{ q-bits} = \underbrace{c_0|000\cdots\rangle + c_1|100\cdots\rangle + c_2|010\cdots\rangle\cdots}_{2^N \text{ states}}$$

Quantum computing

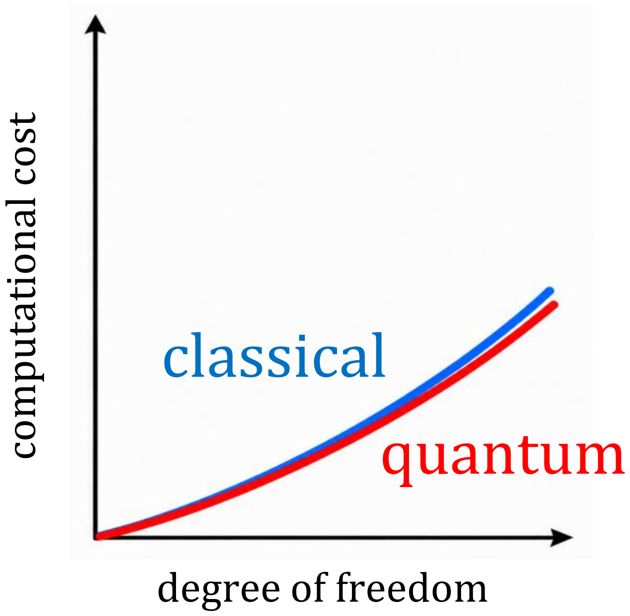
exponential speedup



polynomial speedup

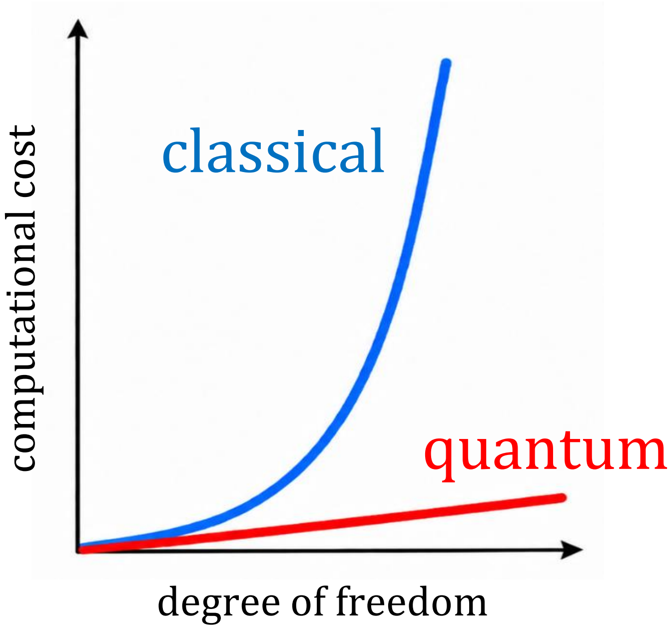


no speedup

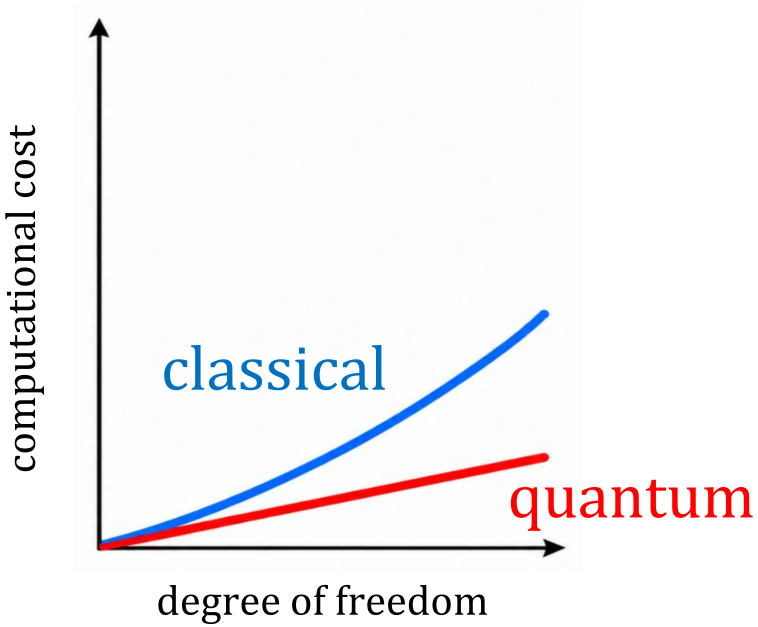


Quantum computing

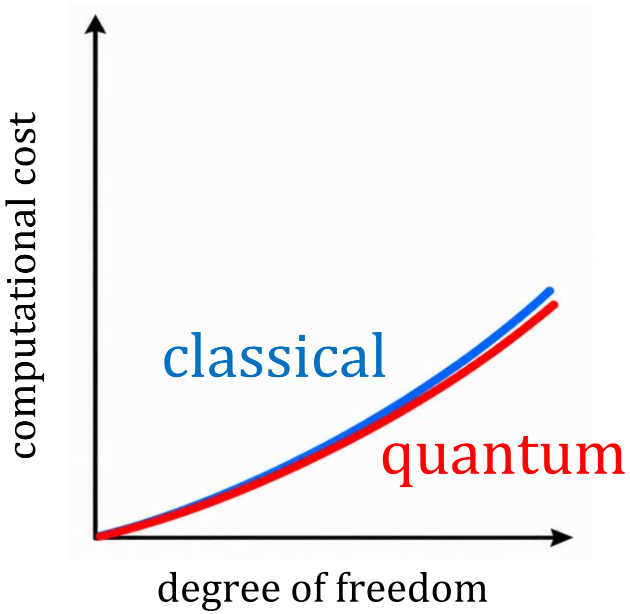
exponential speedup



polynomial speedup



no speedup



quantum simulation

noisy intermediate-scale quantum (NISQ)

Quantum computing

noisy intermediate-scale quantum (NISQ)

small number of q-bits

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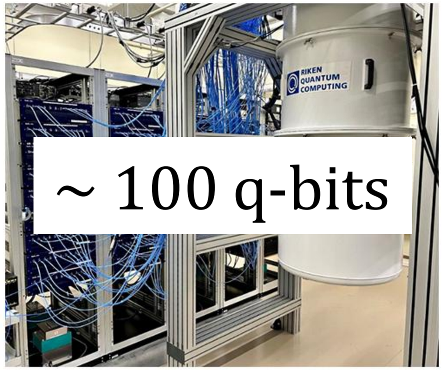

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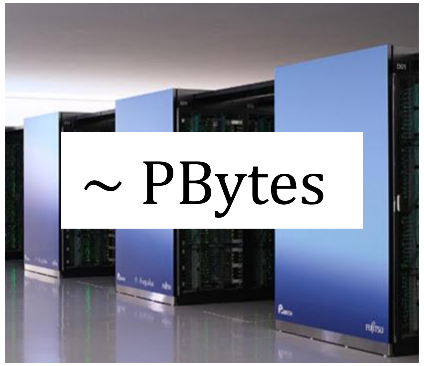


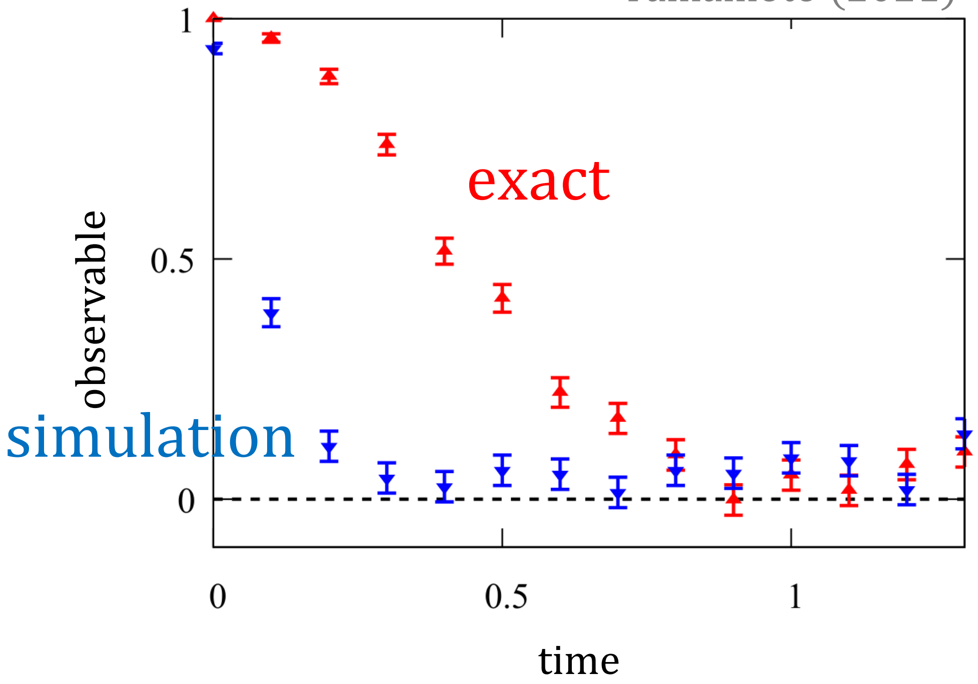
100 q-bits $\sim$ 10^{16} PBytes

noisy intermediate-scale quantum (NISQ)

device error

small number of q-bits

Yamamoto (2021)



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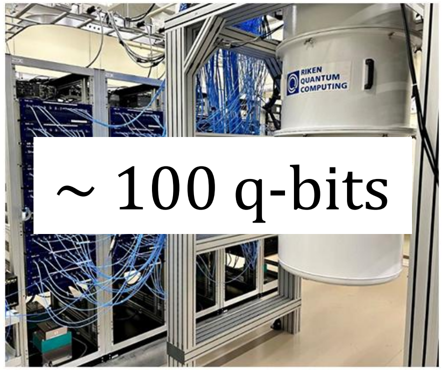

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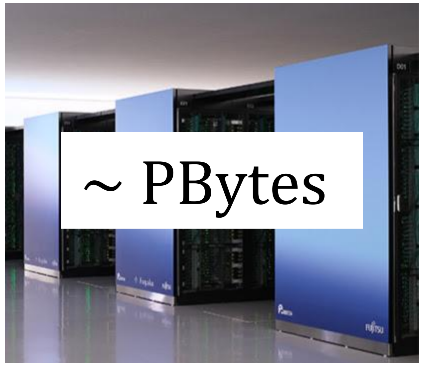


100 q-bits $\sim$ 10^{16} PBytes

quantum device roadmap

NISQ

(early) fault tolerant

quantum simulation roadmap

2026	2029 ~	20XX ~
✓ NISQ ✓ benchmark test ✓ usage & algorithm	✓ small-scale QC ✓ toy model	✓ large-scale QC ✓ QCD, SUSY, etc.

3. Hamiltonian lattice gauge theory

Hamiltonian lattice gauge theory

path integral formalism

$$Z = \int d\Psi \, e^{-S}$$

c-number classical action

Hamiltonian formalism

$$E = \langle \Psi | H | \Psi \rangle$$

Hamiltonian operator quantum state

Hamiltonian lattice gauge theory

path integral formalism

Wilson (1974)

classical computing (1980s)

high performance computing (now)

Hamiltonian formalism

Kogut, Susskind (1975)

quantum computing (now)

fermion

$$|\psi\rangle = a|\mathrm{empty}\rangle + b|\mathrm{occupied}\rangle \qquad \leftrightarrow \qquad \text{1 qubit}$$

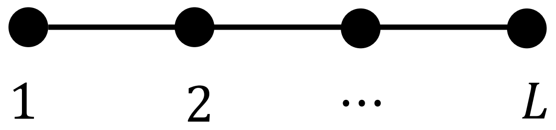
Hamiltonian lattice gauge theory

fermion

$$|\psi\rangle = a|\text{empty}\rangle + b|\text{occupied}\rangle \quad \leftrightarrow \quad 1 \text{ qubit}$$

total state vector $$|\Psi\rangle = \prod_{x=1}^{L} |\psi(x)\rangle \quad \leftrightarrow \quad L \text{ qubits}$$

total dimension $$D = 2^L$$



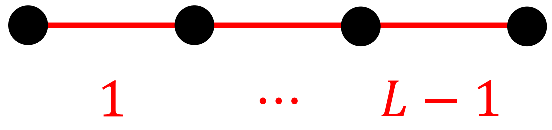
Hamiltonian lattice gauge theory

gauge field

Z_2 gauge $\quad |g\rangle = c_0|+1\rangle + c_1|-1\rangle \quad \leftrightarrow \quad$ 1 qubit

total state vector $\quad |\Psi\rangle = \prod_{x=1}^{L-1} |g(x)\rangle \quad \leftrightarrow \quad L-1$ qubits

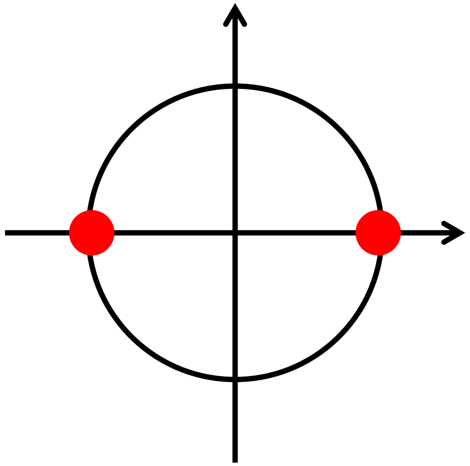
total dimension $\quad D = 2^{L-1}$



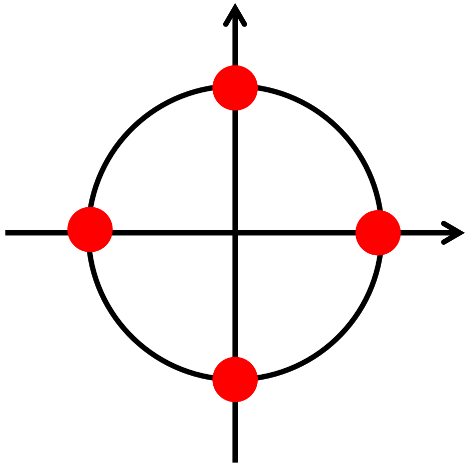
Hamiltonian lattice gauge theory

how to formulate continuous gauge group?

$Z_2 = \{+1, -1\}$

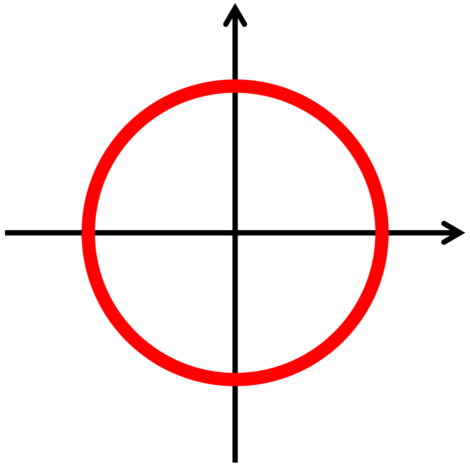


$Z_4 = \left\{e^{i0}, e^{i\pi/2}, e^{i\pi}, e^{i3\pi/2}\right\}$



$\xrightarrow[N \to \infty]{}$

$U(1) = \left\{e^{i\theta}\right\}$



how to formulate continuous gauge group?

$Z_2 \qquad |g\rangle = c_0|+1\rangle + c_1|-1\rangle \qquad \leftrightarrow \qquad$ 1 qubit

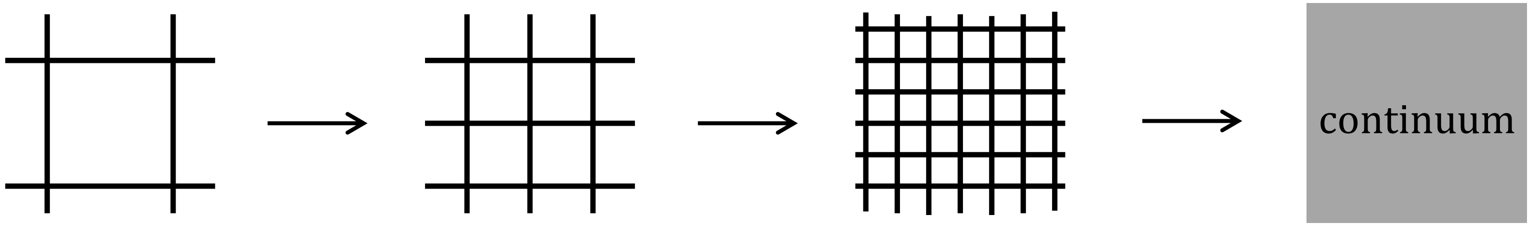
$Z_4 \qquad |g\rangle = c_0\left|e^{i0}\right\rangle + c_1\left|e^{i\pi/2}\right\rangle + c_2\left|e^{i\pi}\right\rangle + c_3\left|e^{i3\pi/2}\right\rangle \qquad \leftrightarrow \qquad$ 2 qubits

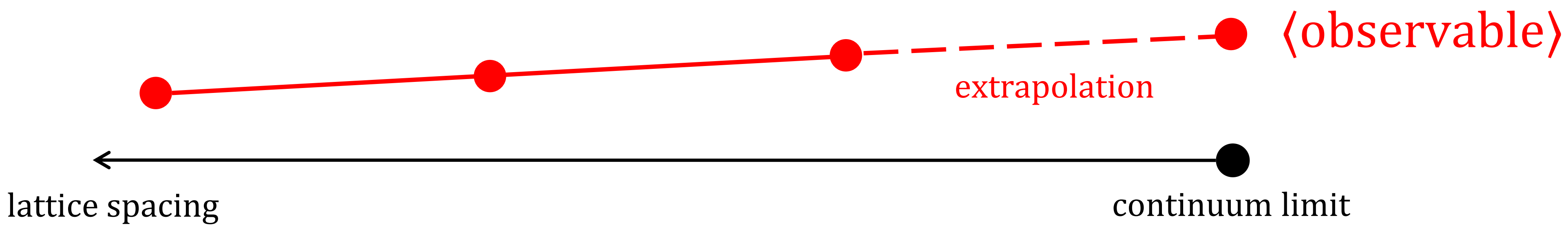
$\vdots$

$Z_N \qquad |g\rangle = c_0\left|e^{i0}\right\rangle + c_1\left|e^{i2\pi/N}\right\rangle + \cdots + c_{N-1}\left|e^{i2\pi(N-1)/N}\right\rangle \qquad \leftrightarrow \qquad$ $\log_2 N$ qubits

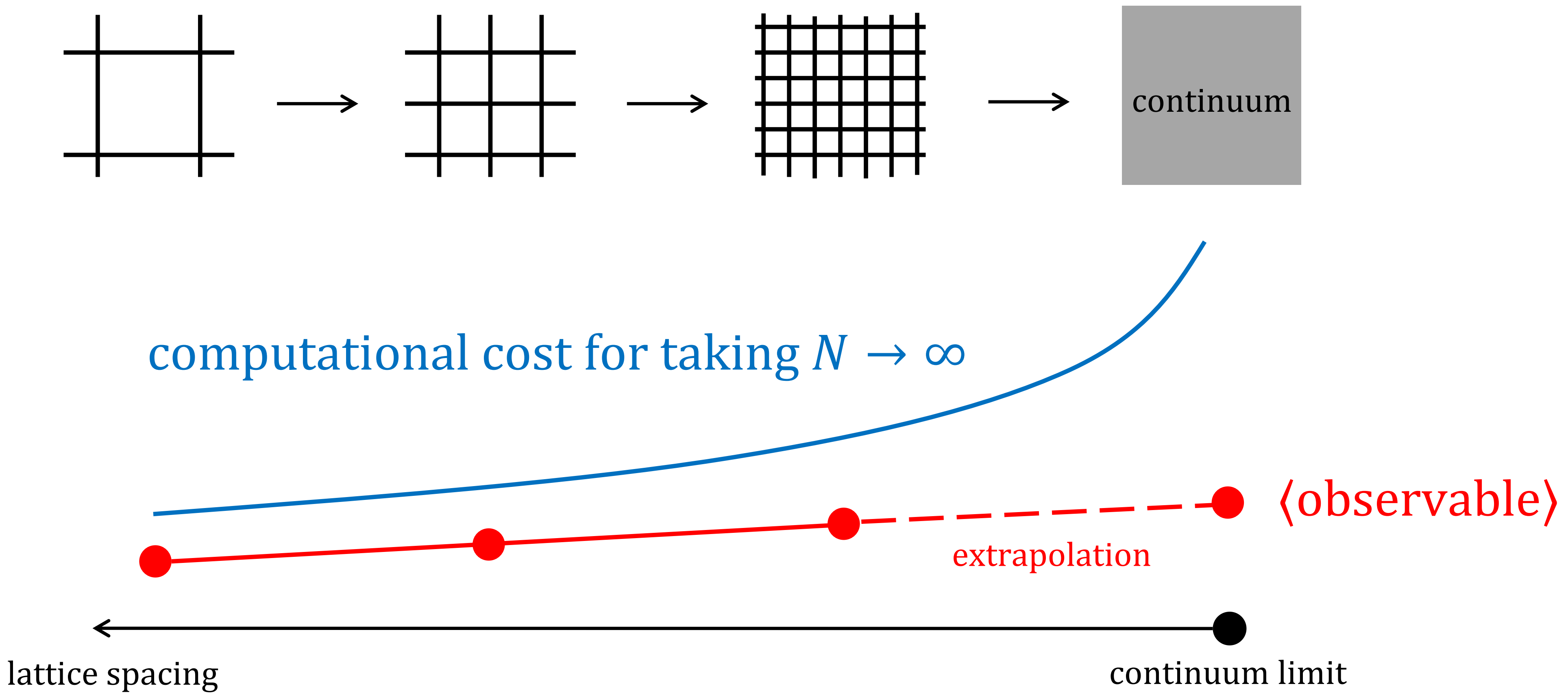
$\xrightarrow[N \to \infty]{} \qquad U(1) \qquad \leftrightarrow \qquad$ ∞ qubits

how to formulate continuous gauge group?





how to formulate continuous gauge group?



how to formulate chiral lattice fermion?

Nielsen-Ninomiya no go theorem Nielsen, Ninomiya (1981)

"chiral symmetry (or one of basic properties) is violated on lattices"

how to formulate chiral lattice fermion?

Nielsen-Ninomiya no go theorem   Nielsen, Ninomiya (1981)

"chiral symmetry (or one of basic properties) is violated on lattices"

path integral formalism

$$\gamma^5 D + D\gamma^5 = D\gamma^5 D$$

Dirac operator

Hamiltonian formalism

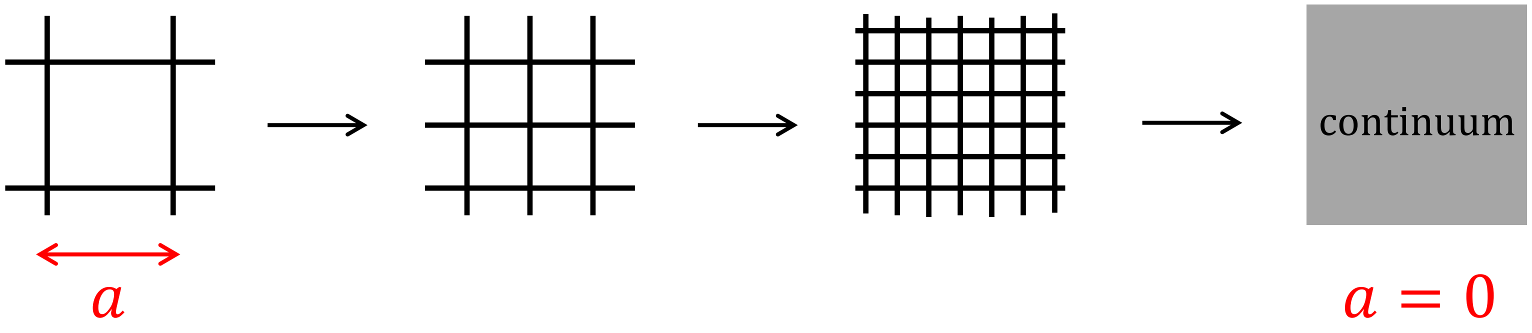
$$[Q, H] = 0$$

chiral charge

how to formulate chiral lattice fermion?

nonconserved chiral charge $Q = \int d^D x\, \psi^+ \gamma^5 \psi$ $[Q, H] \neq 0$

conserved chiral charge $Q = \int d^D x\, \psi^+ \gamma^5 \psi + O(a)$ $[Q, H] = 0$



how to formulate chiral lattice fermion?

chiral anomaly in 1+1 dim. Hidaka, Yamamoto (2025)

nonconserved chiral charge $\frac{d}{dt}\langle Q\rangle = O(a) \to 0$ no anomaly???

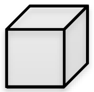
conserved chiral charge $\frac{d}{dt}\langle Q\rangle = \frac{1}{\pi}\int dx\, eE_{\mathrm{ex}}$

perturbation reproduces anomaly

4. Application

Application

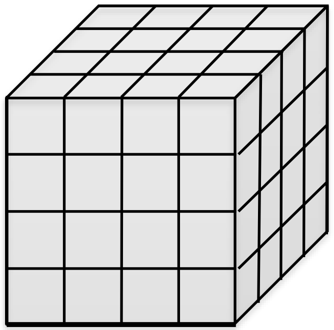
how to verify quantum simulation?



small lattice

↑

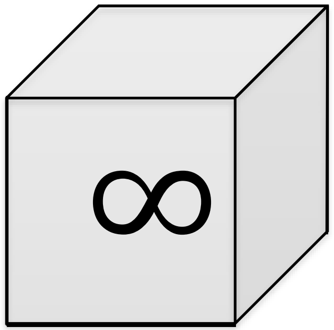
exact diagonalization



large lattice

↑

tensor networks etc.

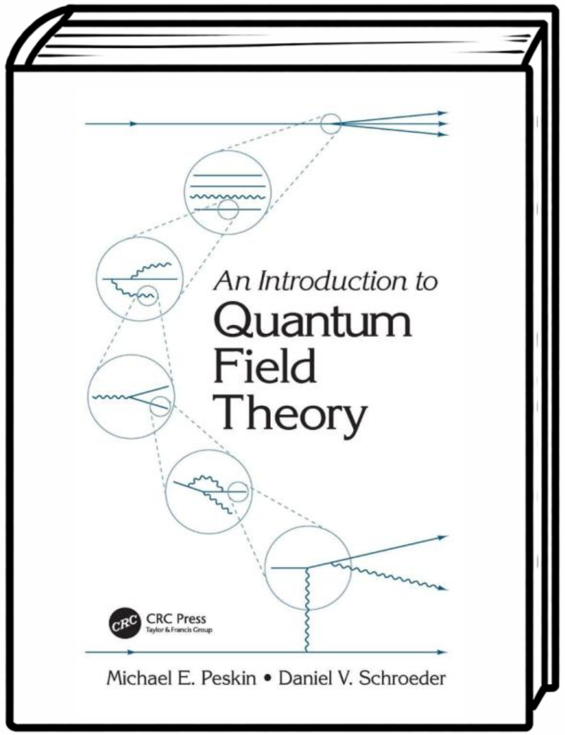


quantum field theory

↑

???

Application



chiral anomaly in 1+1 dim.

Fourier transform of the field equation

$$\partial_\mu j^{\mu 5} = \frac{e}{2\pi}\epsilon^{\mu\nu}F_{\mu\nu}.$$

the axial vector current is not conserved in the pres
fields, as the result of an anomalous behavior of its
iagram.

chiral anomaly in 1+3 dim.

e symmetric limit $\epsilon \to 0$ in four dimensions. We find

$$\partial_\mu j^{\mu 5} = -\frac{e^2}{16\pi^2}\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}.$$

n, which expresses the anomalous nonconservation of th
axial current, is known as the Adler-Bell-Jackiw anomaly

Application

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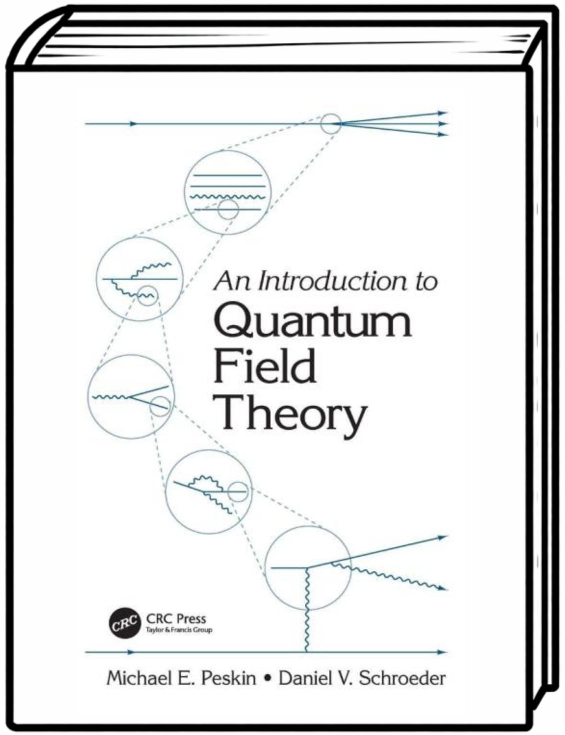

“exact” in all orders

chiral anomaly in 1+1 dim.

Fourier transform of the field equation

$$\partial_\mu j^{\mu 5} = \frac{e}{2\pi}\epsilon^{\mu\nu}F_{\mu\nu}.$$

the axial vector current is not conserved in the pres
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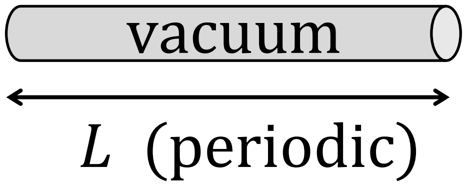
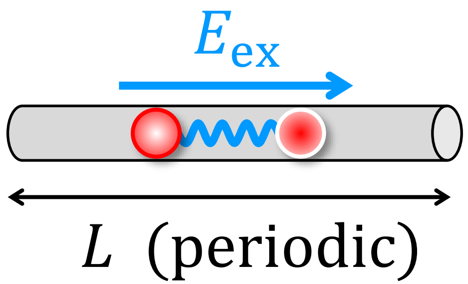
Application



external electric field

$$\frac{d}{dt}\langle Q\rangle = \frac{1}{\pi} L e E_{\mathrm{ex}}$$

exact number

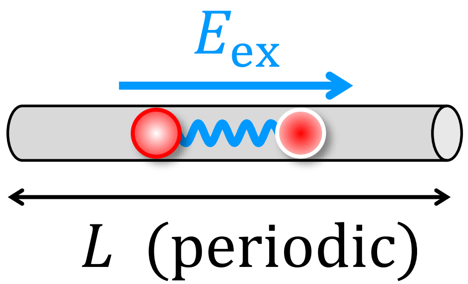


$\langle Q\rangle$ = # of right-handed electrons − # of left-handed electrons input parameter

$$\frac{d}{dt}\langle Q\rangle = C L e E_{\mathrm{ex}}$$

output data

fitting result



if $C = \frac{1}{\pi} \pm$ (statistical error) $\Longrightarrow$ simulation is correct

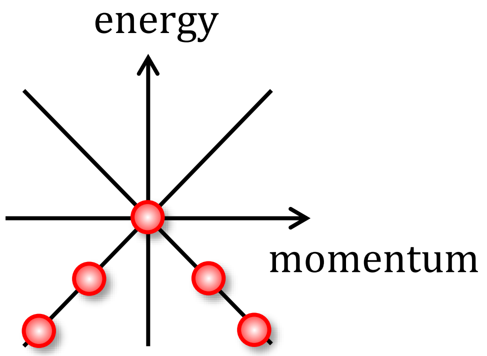
Application

“Reimei”

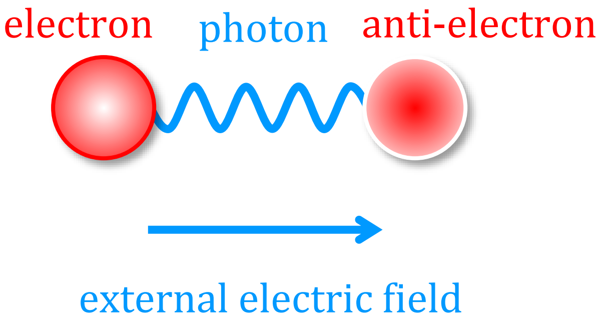
- ✓ deployed at RIKEN
- ✓ Quantinuum System Model H1
- ✓ all-to-all connectivity
- ✓ 20 qubits

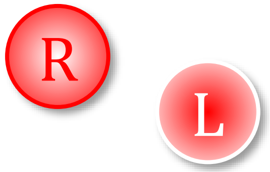
Application

1. initial state preparation	2. time evolution	3. measurement



free electron vacuum





$$\langle Q \rangle_{t=0} = 0$$

$$e^{-iH\delta t}$$

$$\langle Q \rangle_{t=\delta t} > 0$$

Z_N gauge theory on a finite lattice

1. U(1) gauge $Z_4, Z_8, Z_{16} \to Z_\infty$
2. infinitesimal time $\delta t \to 0$
3. infinite volume $L = 3, 4, 5 \to \infty$
4. continuum limit $e \to 0$

U(1) gauge theory in infinite continuous space

Application

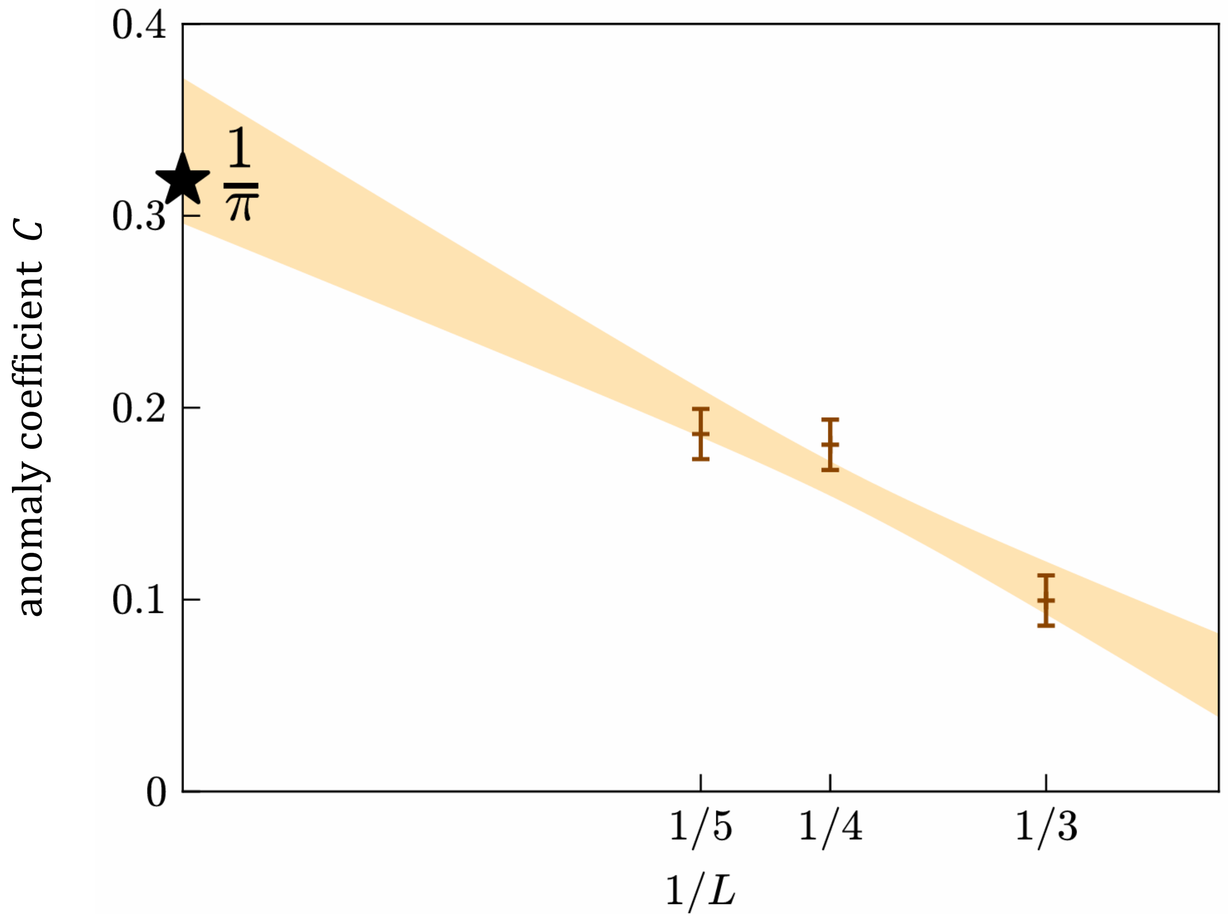
Z_N gauge theory on a finite lattice

1. U(1) gauge $Z_4, Z_8, Z_{16} \to Z_\infty$
2. infinitesimal time $e^2 \delta t = 0.3, 0.2, 0.1 \to 0$
3. infinite volume $L = 3, 4, 5 \to \infty$
4. ~~continuum limit~~ $\cancel{e \to 0}$ for a single Trotter step

U(1) gauge theory in infinite continuous space

Hayata, Yamamoto (2026)

simulation on Reimei (no error mitigation)



good agreement !!

Summary

recent trends on quantum computation for high-energy physics

- ✓ device: NISQ → early fault tolerant
- ✓ theory: continuous gauge group, chiral lattice fermion
- ✓ application: chiral anomaly in 1+1 dim.