Forward Scattering Probability and Final-State Dependence in Quantum Field Theory with Gaussian Wave Packets

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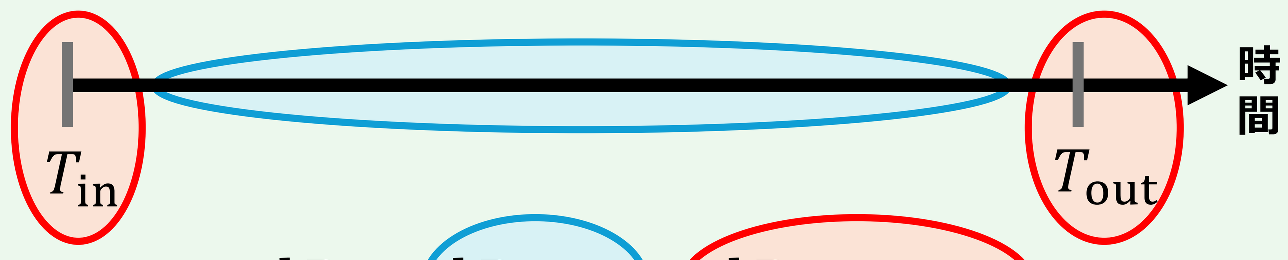
Motivation

有限時間の散乱

ミクロ粒子の散乱 → 場の理論による記述

通常の平面波計算：$|T_{\mathrm{out}} - T_{\mathrm{in}}| \to \infty$



$$\mathrm{d}P = \mathrm{d}P_{\mathrm{bulk}} + \mathrm{d}P_{\mathrm{in-bdry}}$$

有限散乱時間では 時間境界からの寄与 も存在

ガウス波束基底を用いて計算→有限値の振幅・確率

振幅の干渉項による forward contribution

散乱確率：$\mathrm{d}P \propto \left|\mathcal{S}^{(0)} + \mathcal{S}^{(1)} + \cdots\right|^2$

$$\boxed{\mathrm{d}P^{(1)} \propto 2\mathrm{Re}\left[\mathcal{S}^{(0)*}\mathcal{S}^{(1)}\right]}$$

平面波：寄与は完全前方に局在

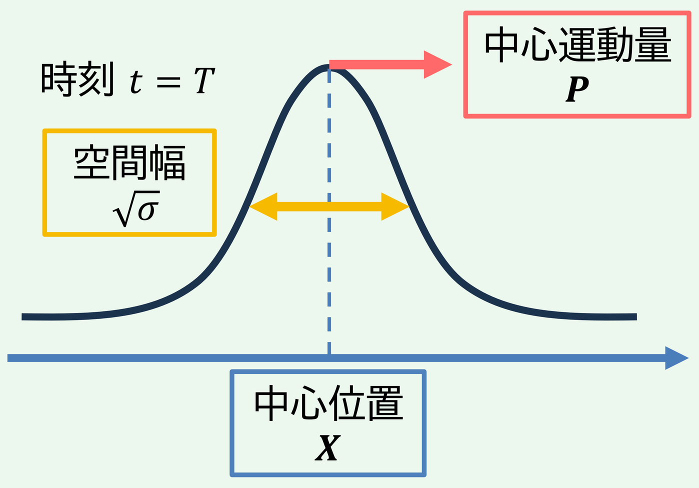
波　束：有限のforward probability

→ この干渉項の bulk / boundary 成分と
終状態への依存性を解析する

Method

ϕ^4相互作用で2体散乱確率を計算

ガウス波束基底: $|\sigma, T, \boldsymbol{X}, \boldsymbol{P}\rangle = |\sigma, \Pi\rangle = \hat{A}^\dagger_\sigma(\Pi)|0\rangle$



交換関係: $\left[\hat{A}_{\sigma_i}(\Pi_i), \hat{A}^\dagger_{\sigma_j}(\Pi_j)\right] = \langle\sigma_i, \Pi_i|\sigma_j, \Pi_j\rangle$

For large-σ,

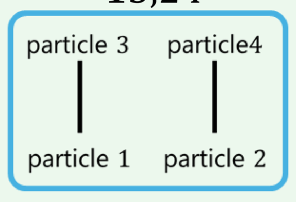
$$\langle\sigma_i, \Pi_i|\sigma_j, \Pi_j\rangle \propto e^{-\frac{\sigma_{ij}}{2}(\boldsymbol{P}_i - \boldsymbol{P}_j)^2} e^{-\frac{1}{2(\sigma_i+\sigma_j)}\left((\boldsymbol{X}_i-\boldsymbol{X}_j)-\boldsymbol{V}(\tilde{\boldsymbol{P}}_{ij})(T_i-T_j)\right)^2}$$

$$\mathrm{d}^6\Pi = \frac{\mathrm{d}^3X\mathrm{d}^3P}{(2\pi)^3}, E(\boldsymbol{P}) \equiv \sqrt{m^2 + \boldsymbol{P}^2}, \boldsymbol{V}(\boldsymbol{P}) \equiv \frac{\boldsymbol{P}}{E(\boldsymbol{P})}, \sigma_{ij} \equiv \frac{\sigma_i\sigma_j}{\sigma_i+\sigma_j}, \boldsymbol{V}(\boldsymbol{P}) \equiv \frac{\boldsymbol{P}}{E(\boldsymbol{P})}, \tilde{\boldsymbol{P}}_{ij} \equiv \frac{\sigma_i\boldsymbol{P}_i + \sigma_j\boldsymbol{P}_j}{\sigma_i + \sigma_j}$$

$\mathcal{S}$-matrix:

$$\mathcal{S} = \langle\mathrm{out}; \sigma_3, \Pi_3; \sigma_4, \Pi_4|\sigma_1, \Pi_1; \sigma_2, \Pi_2; \mathrm{in}\rangle = \mathcal{S}^{(0)} + \mathcal{S}^{(1)} + \cdots$$

$$\mathcal{S}^{(0)} = \langle\sigma_3, \Pi_3; \sigma_4, \Pi_4|\sigma_1, \Pi_1; \sigma_2, \Pi_2\rangle = \mathcal{S}^{(0)}_{13,24} + \mathcal{S}^{(0)}_{14,23}$$



$$\mathcal{S}^{(1)} \equiv \mathcal{S}^{(1)}_{\mathrm{bulk}} + \mathcal{S}^{(1)}_{\mathrm{in-bdry}} + \mathcal{S}^{(1)}_{\mathrm{out-bdry}}$$

$$\mathcal{S}^{(1)}_{\mathrm{bulk}} \propto i\lambda e^{i(\mathfrak{T}(\delta\omega))} \cdot e^{-\frac{\sigma_s}{2}(\delta\boldsymbol{P})^2} \cdot e^{-\frac{\sigma_t}{2}(\delta\omega)^2}$$

$$\mathcal{S}^{(1)}_{\mathrm{in-bdry}} \propto i\lambda e^{i(T_{\mathrm{in}}(\delta\omega))} \cdot e^{-\frac{\sigma_s}{2}(\delta\boldsymbol{P})^2} \cdot e^{-\frac{(\mathfrak{T}-T_{\mathrm{in}})^2}{2\sigma_t}} \frac{1}{\mathfrak{T} - T_{\mathrm{in}} + i\sigma_t(\delta\omega)}$$

散乱確率: $\mathrm{d}P = \mathrm{d}^6\Pi_3\mathrm{d}^6\Pi_4|\mathcal{S}|^2 = \mathrm{d}P^{(0)} + \mathrm{d}P^{(1)} + \cdots$

$$\mathrm{d}P^{(1)} \equiv \mathrm{d}^6\Pi_3\mathrm{d}^6\Pi_4\left(2\mathrm{Re}\left[\mathcal{S}^{*(0)}\mathcal{S}^{(1)}\right]\right)$$
$$= \mathrm{d}P^{(1)}_{\mathrm{bulk}} + \mathrm{d}P^{(1)}_{\mathrm{in-bdry}} + \mathrm{d}P^{(1)}_{\mathrm{out-bdry}}$$

References
K. Ishikawa and T. Shimomura, *PTP* **114**, 1201 (2006)
K. Ishikawa and Y. Tobita, *PTEP* **2013**, 073B02 (2013)
K. Ishikawa and K.-y. Oda, *PTEP* **2018**, 123B01 (2018)
K. Ishikawa, K. Nishiwaki and K.-y. Oda, *PTEP* **2020**, 103B04 (2020)
K. Ishikawa, *Ann. Phys.* **460**, 169571 (2024)
K. Ishikawa and M. Takesada, arXiv:2605.28847 (2026)
K. Ishikawa and Y. N, in preparation.

Result

$$\mathrm{d}P^{(1)}_{13,24} = \mathrm{d}^6\Pi_3\mathrm{d}^6\Pi_4\left(2\mathrm{Re}\left[\mathcal{S}^{*(0)}_{13,24}\mathcal{S}^{(1)}\right]\right) = \mathrm{d}P^{(1)}_{\mathrm{bulk}} + \mathrm{d}P^{(1)}_{\mathrm{in-bdry}} + \mathrm{d}P^{(1)}_{\mathrm{out-bdry}}$$

$$\mathrm{d}P^{(1)}_{\mathrm{bulk}} \propto \lambda Q e^{-\frac{\sigma_t}{2}(\delta\omega)^2}\sin(\Theta + \mathfrak{T}\delta\omega)$$

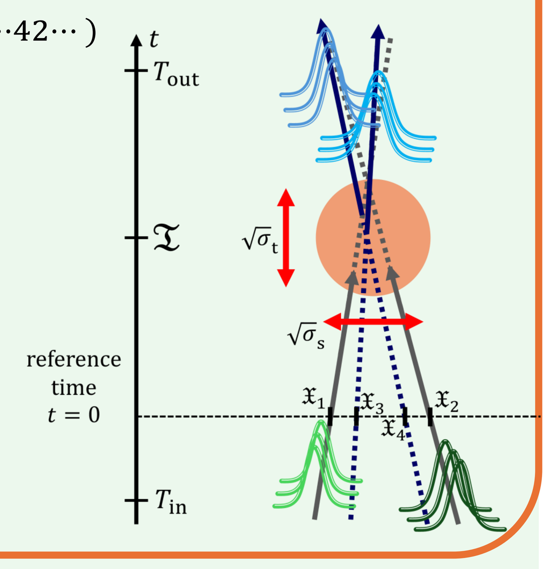
$$\mathrm{d}P^{(1)}_{\mathrm{in-bdry}} \propto \lambda Q e^{-\frac{(\mathfrak{T}-T_{\mathrm{in}})^2}{2\sigma_t}} \frac{-(\mathfrak{T} - T_{\mathrm{in}})\sin(\Theta + T_{\mathrm{in}}\delta\omega) + \sigma_t(\delta\omega)\cos(\Theta + T_{\mathrm{in}}\delta\omega)}{(\mathfrak{T} - T_{\mathrm{in}})^2 + \sigma_t^2(\delta\omega)^2}$$

$$Q = e^{-\frac{\sigma_s}{2}(\delta\boldsymbol{P})^2 - \mathcal{R} - \frac{\sigma_{13}}{2}(\boldsymbol{P}_1-\boldsymbol{P}_3)^2 - \frac{\left((\boldsymbol{X}_3-\boldsymbol{X}_1)-\boldsymbol{V}(\tilde{\boldsymbol{P}}_{31})(T_3-T_1)\right)^2}{2(\sigma_3+\sigma_1)} - (\cdots 42\cdots)}$$

$$\Theta \equiv -E(\tilde{\boldsymbol{P}}_{31})(T_3 - T_1) - E_3T_3 + E_1T_1$$
$$+(\boldsymbol{P}_3 - \boldsymbol{P}_1) \cdot \left(\frac{\sigma_3\boldsymbol{X}_3 + \sigma_1\boldsymbol{X}_1}{\sigma_3+\sigma_1}\right) + (\cdots 42\cdots)$$

$\mathfrak{T}$：Intersection time,

$\mathcal{R}$：Overlap exponent



重心系での散乱確率

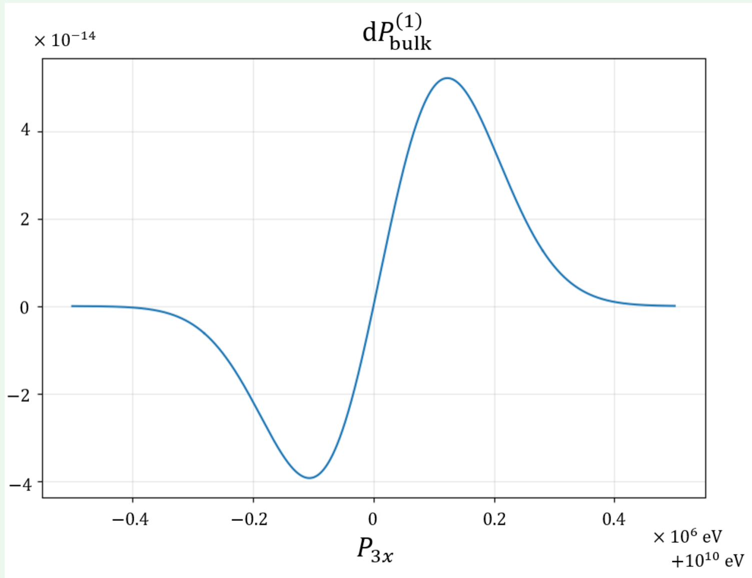
$\sigma_{1,2,3,4} = \sigma$,　$\boldsymbol{P}_1 = -\boldsymbol{P}_2, \boldsymbol{P}_3 = -\boldsymbol{P}_4$,　$\boldsymbol{X}_1 = -\boldsymbol{X}_2, \boldsymbol{X}_3 = -\boldsymbol{X}_4$,

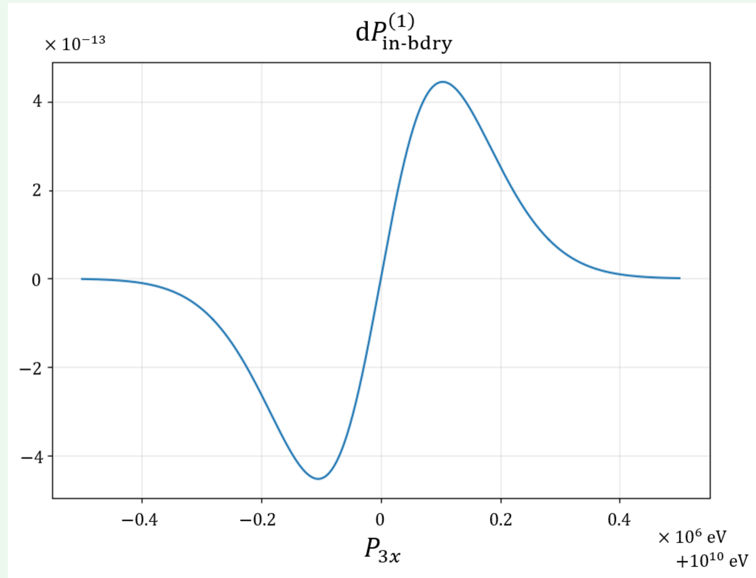
$T_1 = T_2$,　$T_3 = T_4 = 50\ \mathrm{eV}^{-1}$,　$T_{\mathrm{in}} = -10\ \mathrm{eV}^{-1}, T_{\mathrm{out}} = 50\ \mathrm{eV}^{-1}$,

$\lambda = 0.1$,　$X_{1x,3x} = V_{1,3x}(T_{1,3} - T_{\mathrm{in}}) - 0.1\sqrt{\sigma/2}$

終状態運動量 P_{3x} についてのプロット

$m = 10^8$ eV,　$P_{1x} = 10^{10}$ eV,　$\sqrt{\sigma} = 10^{-12}$ m



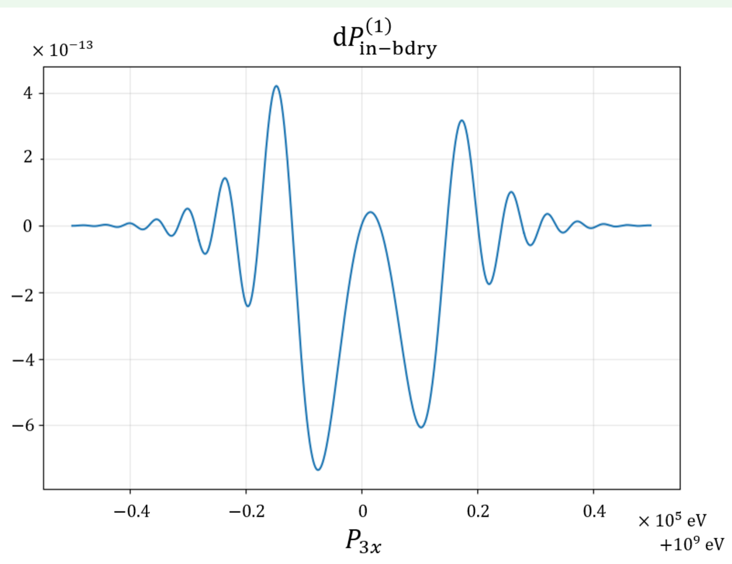


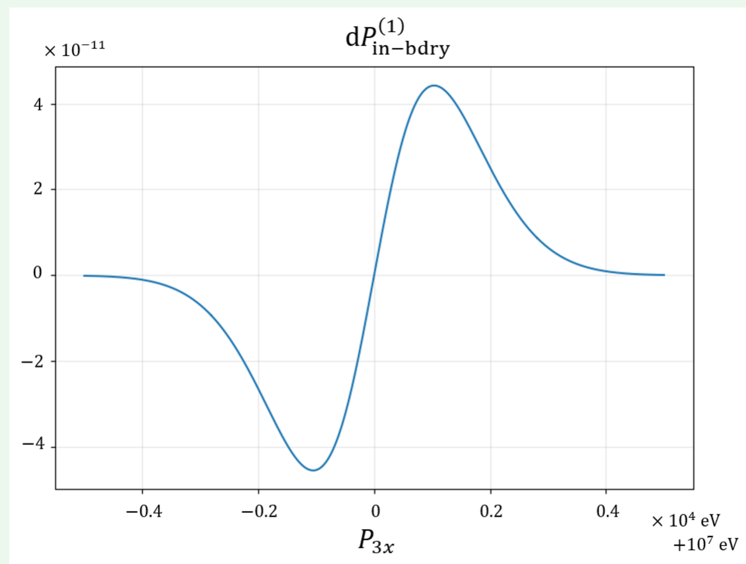
$\mathrm{d}P^{(1)}_{\mathrm{in-bdry}}$ のプロット

$m = 10^9$ eV,　$P_{1x} = 10^9$ eV,
$\sqrt{\sigma} = 10^{-11}$ m

$m = 5 \times 10^5$ eV, $P_{1x} = 10^7$ eV,
$\sqrt{\sigma} = 10^{-10}$ m

終状態運動量 P_{3x} について

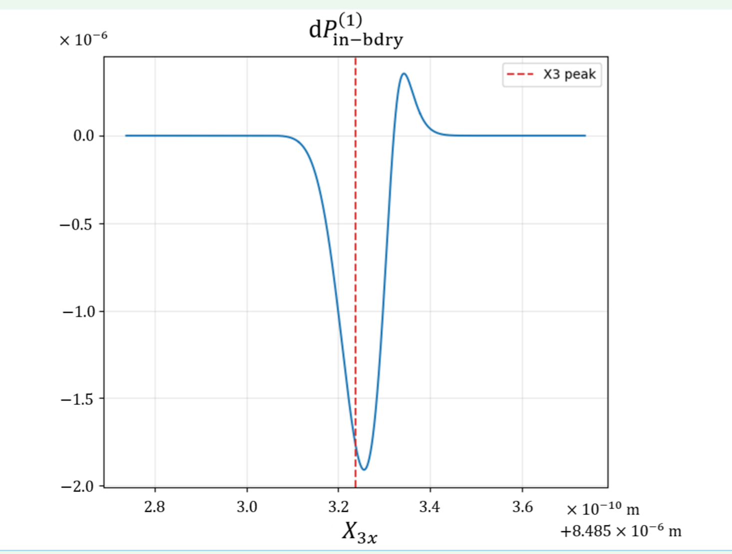


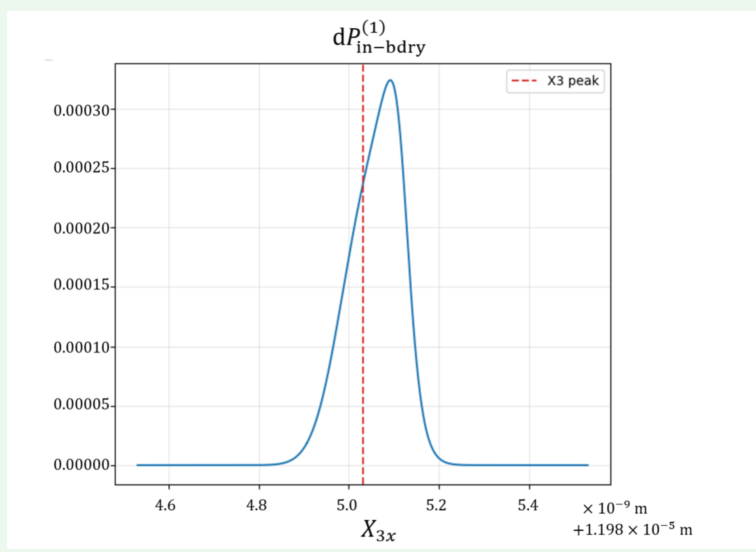


終状態位置 X_{3x} について

$P_{3x} - P_{1x} = 10^4$ eV

$P_{3x} - P_{1x} = 10^3$ eV





Summary

- ガウス波束基底により、振幅の干渉項

$$\mathrm{d}P^{(1)} \propto 2\mathrm{Re}\left[\mathcal{S}^{(0)*}\mathcal{S}^{(1)}\right]$$

を有限な確率として記述し、終状態依存性を解析した

- この確率には有限散乱時間による 2種類の寄与 があり、終状態波束に対して異なる依存性を示す
- 有限時間境界の寄与が bulk 寄与を上回る場合がある