Squashed Kaluza-Klein 帯電ブラックホール時空における光の赤方偏移

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Introduction

Higher-dimensional black hole spacetimes are actively discussed in the context of string theories and braneworld models.

- ✓ 5D squashed Kaluza-Klein black hole solutions asymptote to effective 4D spacetimes with twisted S^1 as extra dimension at infinity and represent fully 5D black holes near S^3 horizons.

- ➢ It could be regarded as candidates of realistic higher-dimensional models and would describe geometry around astrophysical compact objects.

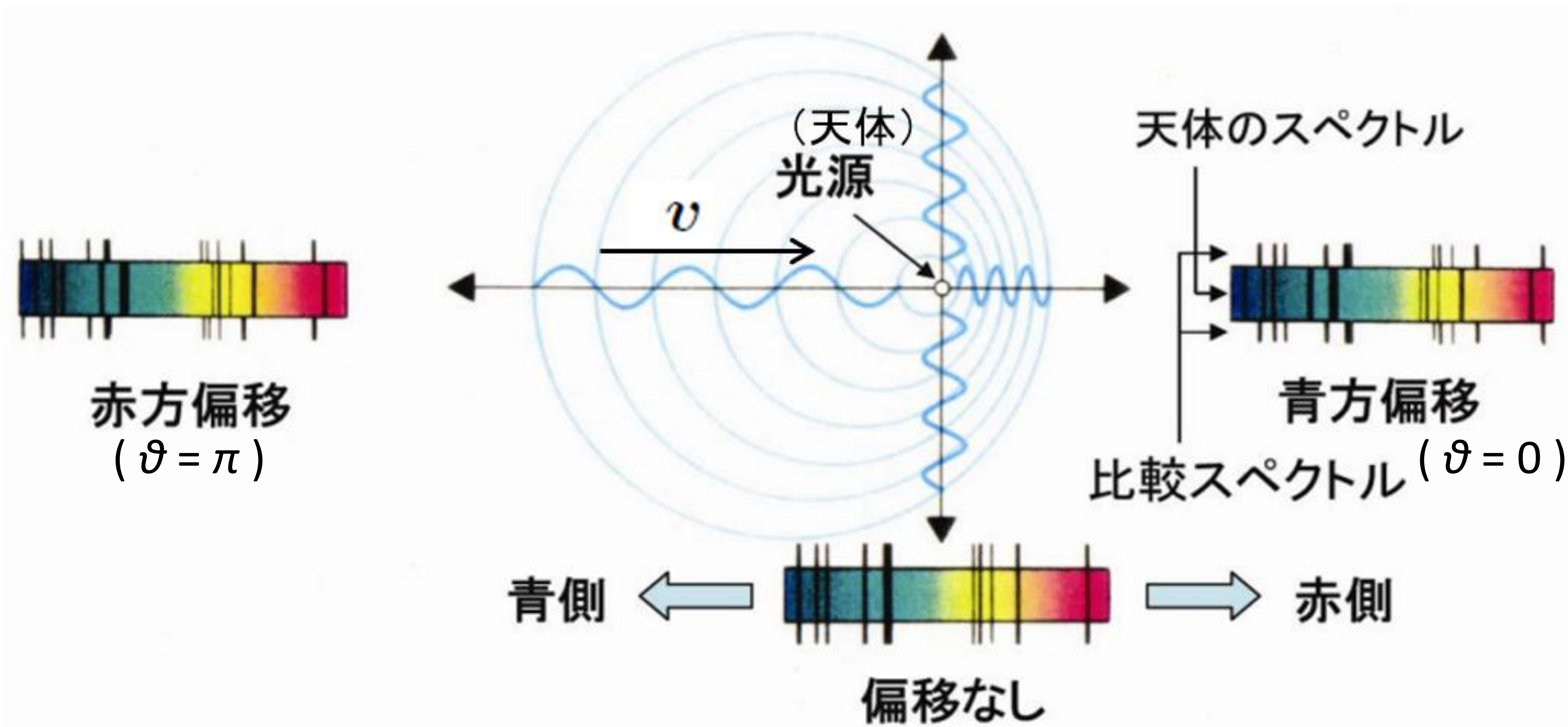
- ✓ We consider total frequency shift of photons emitted by massive test particles around charged squashed Kaluza-Klein black holes.

- ✓ We derive corrections of photon frequency shifts to general relativity, which are related to size of extra dimension and charges of objects.

Kinematical redshift

- 天体と観測者との相対運動に起因するドップラー効果による偏移

$$\omega_{\text{detector}} = \frac{\sqrt{1-v^2}}{1 - v\cos\theta}\,\omega_{\text{emitter}}$$



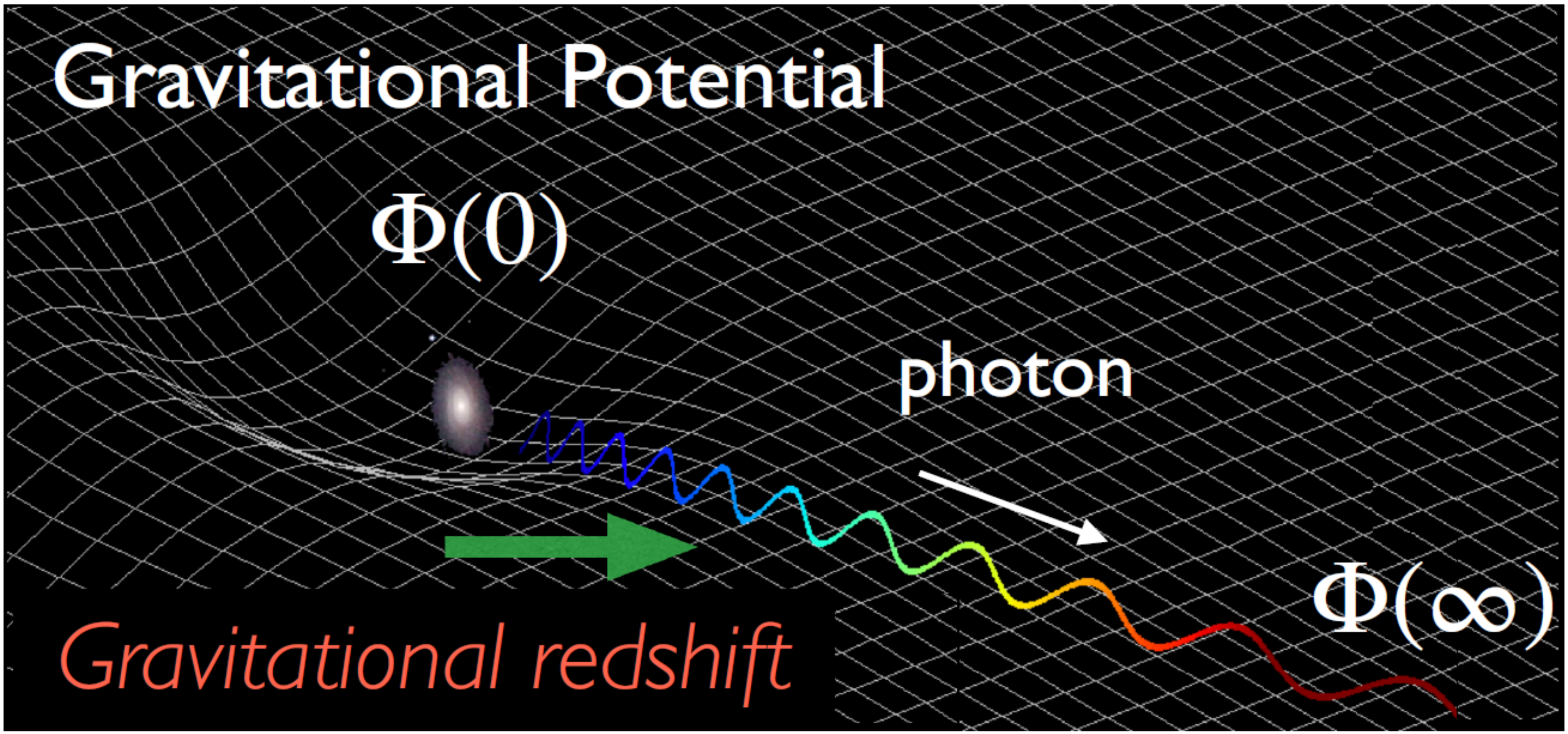
($\vartheta = \pi$)

($\vartheta = 0$)

Gravitational redshift

- 重力の強い領域から放出された光の波長が、放出された時点での波長より大きくなって観測される現象

✓ 4D Schwarzschild: $1+z=\frac{\omega_e}{\omega_d}=\frac{-k_\mu u^\mu|_e}{-k_\mu u^\mu|_d}=\sqrt{\frac{1-2M/r_d}{1-2M/r_e}}>1$

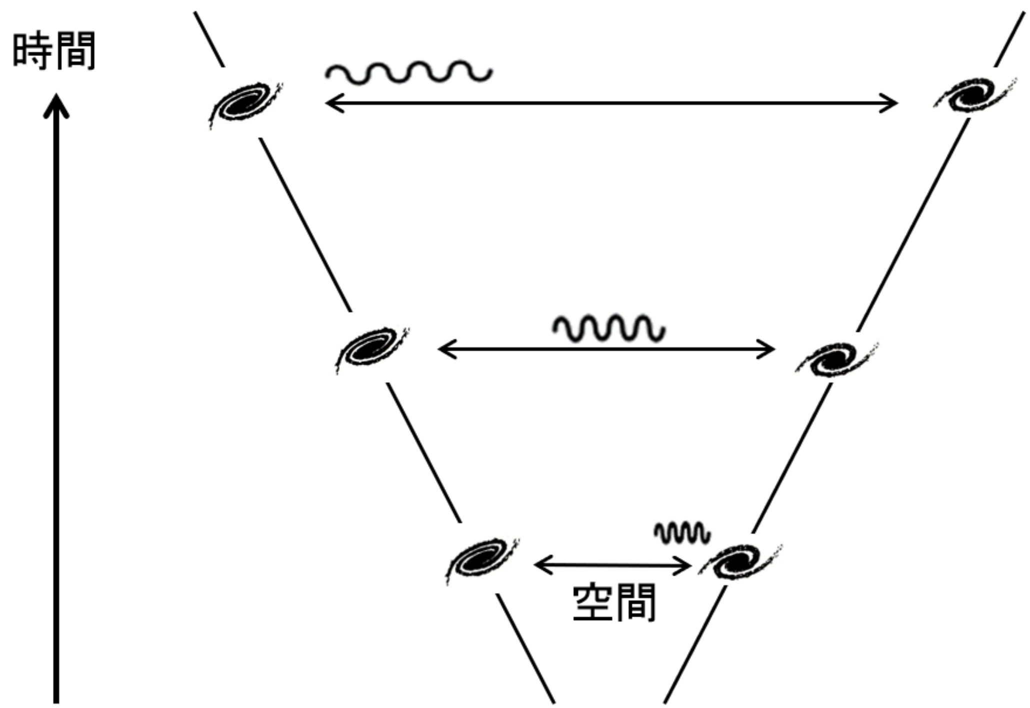


Cosmological redshift

- 十分遠方の天体を発した光が現在の観測者に届くまでの宇宙膨張によって、光の波長が大きくなる現象

✓ 4D FLRW: $ds^2 = a^2(\eta)\left[-d\eta^2 + \frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$

$$\Rightarrow \quad 1+z = \frac{\omega_e}{\omega_d} = \frac{-k_\mu u^\mu|_e}{-k_\mu u^\mu|_d} = \frac{a(\eta_0)}{a(\eta)} > 1 \quad (\eta_0 > \eta)$$



Kaluza-Klein black holes with squashed S^3 horizons

(H. Ishihara and K. Matsuno)

✓ Exact solution of five-dimensional minimal ungauged supergravity

$$
\begin{cases}
ds^2 = -F dt^2 + \dfrac{K^2}{F} d\rho^2 + \rho^2 K^2 (d\theta^2 + \sin^2\theta d\phi^2) + \dfrac{r_\infty^2}{4K^2} (d\psi + \cos\theta d\phi)^2 \\
A_\mu dx^\mu = -\dfrac{\sqrt{3}Q}{2\rho} dt, \qquad F = 1 - \dfrac{2M}{\rho} + \dfrac{Q^2}{\rho^2}, \qquad K^2 = 1 + \dfrac{\rho_0}{\rho} \\
\rho_0 = \sqrt{M^2 - Q^2 + r_\infty^2/4} - M, \qquad M \geq Q \geq 0, \qquad r_\infty > 0
\end{cases}
$$

(M : Komar mass, Q : black hole charge, r_∞ : extra dimension size at infinity)

- $\rho \to \infty$: twisted constant S^1 fiber over 4D Minkowski spacetime
- Parameter ρ_0: typical scale of transition from five dimensions to effective four dimensions
- $\rho_0 \to 0$ limit: four-dimensional Reissner-Nordström black hole with twisted constant S^1 fiber

Charged particle motions in charged Kaluza-Klein spacetime

$$\mathcal{L} = \frac{m}{2} g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu + q A_\mu \dot{x}^\mu$$: Lagrangian for charged massive test particle

- Constants of motion $\left(\hat{q} = \frac{\sqrt{3}q}{2m}\right)$:

$$\begin{cases} \hat{E} = \frac{E}{m} = F\dot{t} + \frac{\hat{q}Q}{\rho}, \quad p_\psi = \frac{m r_\infty^2}{4K^2}\left(\dot{\psi} + \cos\theta\dot{\phi}\right) \\ \hat{L} = \frac{L}{m} = \rho^2 K^2 \sin^2\theta\dot{\phi} + \frac{r_\infty^2 \cos\theta}{4K^2}\left(\dot{\psi} + \cos\theta\dot{\phi}\right) \end{cases}$$

✓ Assumption: $p_\psi = 0$ (no momentum in extra dimensional direction)

- Effective Lagrangian has same form in 4D spherically symmetric spacetime case:

$$\mathcal{L}_{\text{eff}} = \frac{m}{2}\left(-F\dot{t}^2 + \frac{K^2}{F}\dot{\rho}^2 + \rho^2 K^2 \dot{\theta}^2 + \rho^2 K^2 \sin^2\theta\dot{\phi}^2\right) - \frac{m\hat{q}Q}{\rho}\dot{t}$$

✓ We can concentrate on orbits with $\theta = \pi/2$ and $\dot{\theta} = 0$.

✓ Energy conservation equation:

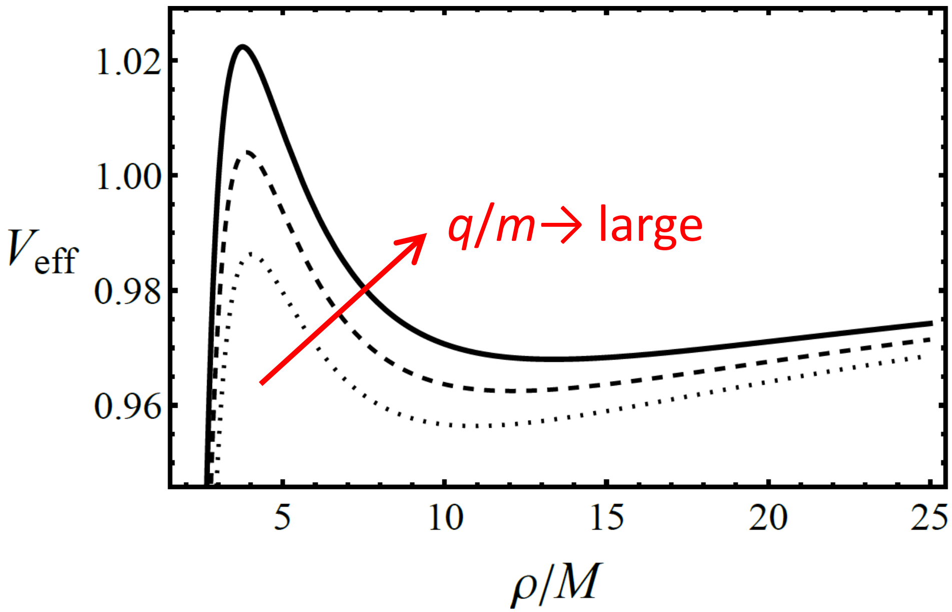
$$\begin{cases} \left(1 + \frac{\rho_0}{\rho}\right)\left(\frac{d\rho}{d\tau}\right)^2 = \left(\hat{E} - V_+\right)\left(\hat{E} - V_-\right) \\ V_\pm = \frac{\hat{q}Q}{\rho} \pm \sqrt{\left(1 - \frac{2M}{\rho} + \frac{Q^2}{\rho^2}\right)\left(1 + \frac{\hat{L}^2}{\rho(\rho+\rho_0)}\right)} \end{cases}$$

Charged particle motions in charged Kaluza-Klein spacetime

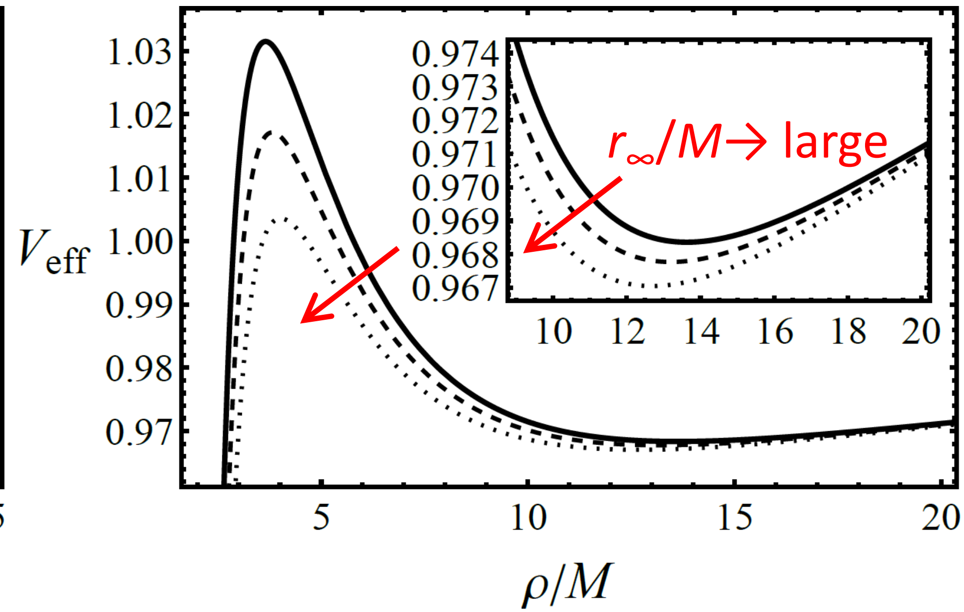
$$\left(1+\frac{\rho_0}{\rho}\right)\left(\frac{d\rho}{d\tau}\right)^2=\left(\hat{E}-V_{\text{eff}}\right)\left(\hat{E}-V_-\right),\quad V_{\text{eff}}=\frac{\hat{q}Q}{\rho}+\sqrt{\left(1-\frac{2M}{\rho}+\frac{Q^2}{\rho^2}\right)\left(1+\frac{\hat{L}^2}{\rho\left(\rho+\rho_0\right)}\right)}$$

✓ Charged particles behaves as ones in 4D spherically symmetric spacetime.

(L/M=4, Q/M=0.35, r_∞/M=1)



(L/M=4, Q/M=0.35, q/m=0.2)



✓ Radius of stable circular orbit satisfying $V_{\text{eff}}=0$, $V_{\text{eff}}'=0$, $V_{\text{eff}}''\leqq 0$

- increases with increasing particle charge q/m.
- decreases with increasing size of extra dimension r_∞/M.

Gravitational frequency shifts in charged Kaluza-Klein spacetime

- Emitter, observer and photon trajectories all lie in effective 4D spacetime.

$$\begin{cases} \text{Massless test particles:} \\ -F\left(k^t\right)^2+\frac{K^2}{F}\left(k^\rho\right)^2+\rho^2K^2\left(k^\theta\right)^2+\rho^2K^2\sin^2\theta\left(k^\phi\right)^2=0 \\ \text{Constants of motion:} \quad E_\gamma=Fk^t, \qquad L_\gamma=\rho^2K^2k^\phi \end{cases}$$

✓ Static emitter (e) and observer (d): $u_i^t=\left.\frac{1}{\sqrt{F}}\right|_i \quad (i=e,d)$ only.

➢ Photon frequency shifts ($\rho_d > \rho_e$):

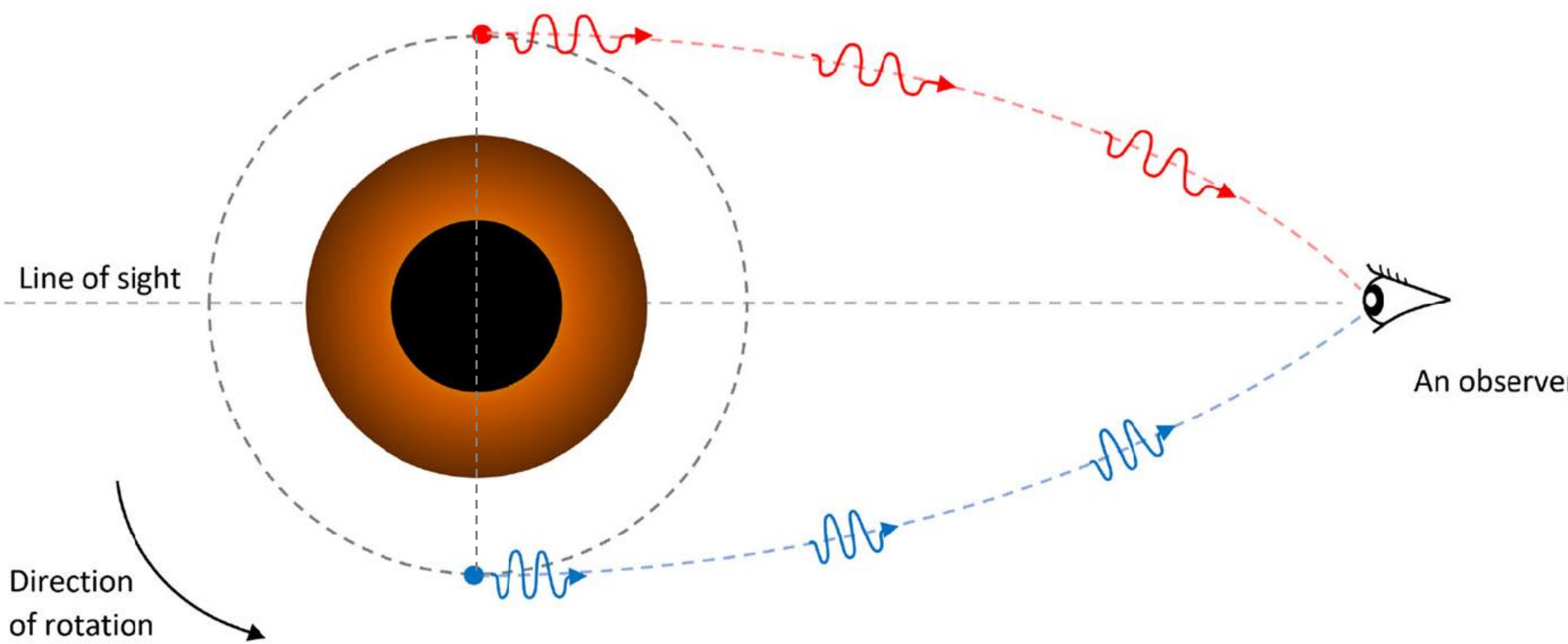
$$1+z=\frac{\omega_e}{\omega_d}=\frac{-k_\mu u^\mu|_e}{-k_\mu u^\mu|_d}=\sqrt{\frac{1-2M/\rho_d+Q^2/\rho_d^2}{1-2M/\rho_e+Q^2/\rho_e^2}}$$

✓ $\omega_e > \omega_d \Leftrightarrow \lambda_d > \lambda_e$: gravitational redshift

✓ No extra dimensional correction between static emitter and observer.

Photon frequency shifts in charged Kaluza-Klein spacetime

- Photon emitter moving around KK black hole in stable circular orbit



✓ Emitter, observer and photon trajectories with $k^\rho = 0$ all lie in $\vartheta = \pi/2$ in effective four-dimensional spacetime.

✓ When black hole is rotating, red and blue trajectories would not be symmetric due to frame dragging effect.

Photon frequency shifts in charged Kaluza-Klein spacetime

Massless test particles with $k^\rho = 0$ in $\vartheta = \pi/2$: $-F\left(k^t\right)^2 + \rho^2 K^2 \left(k^\phi\right)^2 = 0$

Constants of motion: $E_\gamma = F k^t, \quad L_\gamma = \rho^2 K^2 k^\phi$

- Impact parameters: $b_{\gamma 1,2} := \dfrac{L_\gamma}{E_\gamma} = \pm\sqrt{\dfrac{\rho^2 K^2}{F}}$

Emitter: uncharged massive test particle in stable circular orbit ρ

$$u_e^t = \sqrt{g'_{\phi\phi} \big/ \left(g_{\phi\phi} g'_{tt} - g_{tt} g'_{\phi\phi}\right)}, \quad u_e^\phi = \sqrt{g'_{tt} \big/ \left(g_{tt} g'_{\phi\phi} - g_{\phi\phi} g'_{tt}\right)}$$

Observer far away from black hole: $u_d^\mu = (1, 0, 0, 0)$

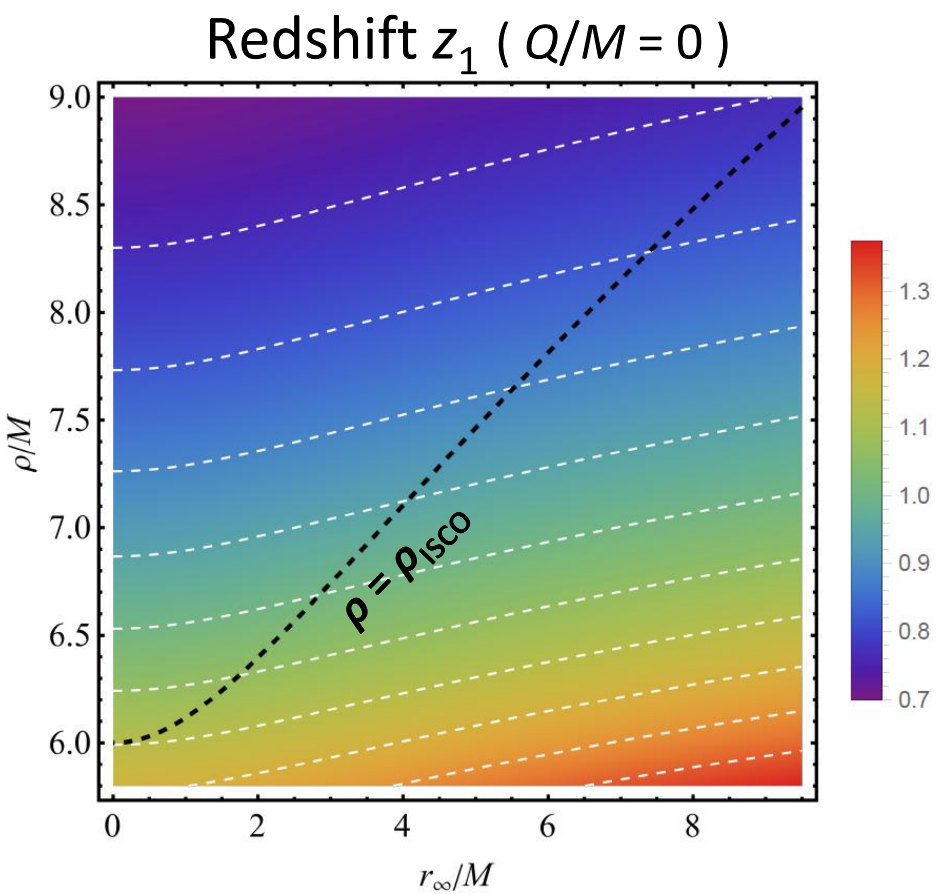
- Total photon frequency shifts (redshift z_1, blueshift z_2):

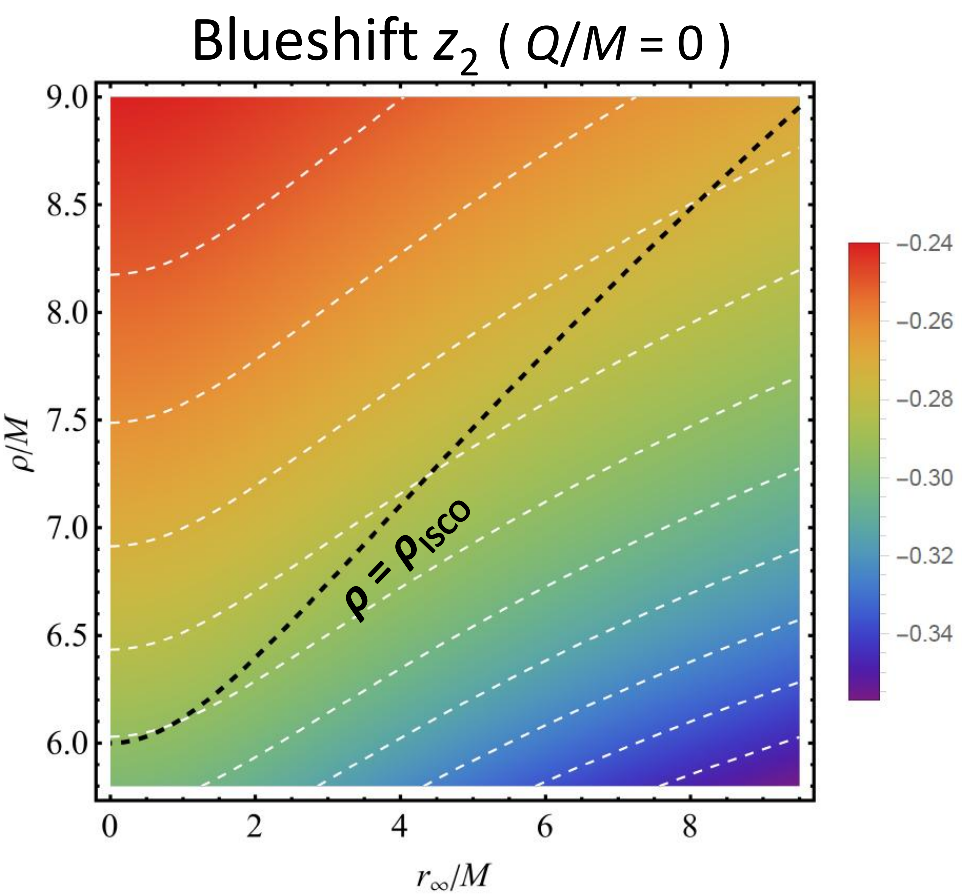
$$1 + z_{1,2} = 1 + z_{\text{grv}} \pm z_{\text{kin}} = \frac{\omega_e}{\omega_d} = \frac{-k_\mu u^\mu|_e}{-k_\mu u^\mu|_d} = u_e^t - b_{\gamma 1,2} u_e^\phi$$

$$= \left(\sqrt{2\rho + \rho_0} \pm \sqrt{\frac{2(\rho + \rho_0)\left(M\rho - Q^2\right)}{\rho^2 - 2M\rho + Q^2}}\right)\left[2\rho + \rho_0 - 2M\left(3 + \frac{2\rho_0}{\rho}\right) + \frac{(4\rho + 3\rho_0) Q^2}{\rho^2}\right]^{-1/2}$$

- $\rho_0 \to 0$ limit: total photon frequency shifts by 4D Reissner-Nordström black holes

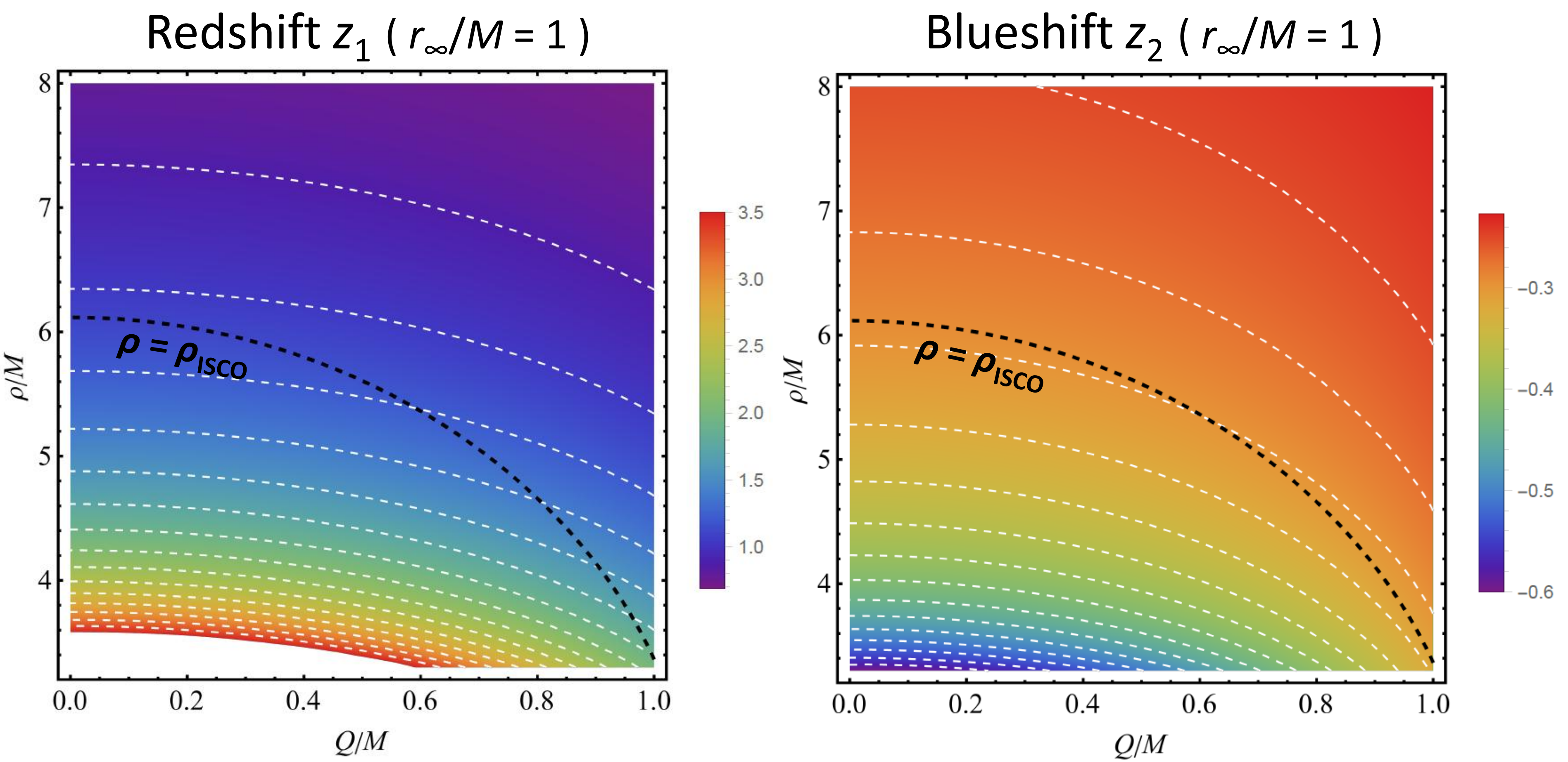
Photon frequency shifts in charged Kaluza-Klein spacetime





- ✓ For fixed emission radius ρ/M, z_1 and z_2 increase with increasing extra dimension size r_∞/M.
- ✓ $z_1(\rho_{ISCO})$ and $z_2(\rho_{ISCO})$ decrease with increasing r_∞/M.

Photon frequency shifts in charged Kaluza-Klein spacetime



- ✓ For fixed emission radius ρ/M, z_1 and z_2 decrease with increasing black hole charge Q/M.
- ✓ $z_1(\rho_{ISCO})$ and $z_2(\rho_{ISCO})$ increase with increasing Q/M.

Observational effects in photon frequency shifts

- Differences between total frequency shifts in squashed KK spacetime and in 4D Schwarzschild spacetime (i = 1, 2):

$$\frac{z_{i,\mathrm{KK}} - z_{i,\mathrm{Sch}}}{z_{i,\mathrm{Sch}}} \simeq \delta_1 - \delta_2, \quad \delta_1 := \frac{r_\infty^2}{32M\rho}, \quad \delta_2 := \frac{5Q^2}{8M\rho}$$

$$\left.\begin{array}{l} \delta_1 \simeq 10^{-35} \\ \delta_2 \lesssim 10^{-43} \end{array}\right\}$$: megamaser system at the center of NGC 4258 galaxy

$$\left(r_\infty \simeq 0.1 \text{ mm}, \; Q \lesssim 3\times 10^8 \text{ C}, \; M \simeq 8\times 10^{37} \text{ kg}, \; \rho \simeq 10^{15} \text{ m}\right)$$

➢ Corrections by extra dimension and charge for supermassive black holes would not appear to be relevant for present and near future observations.

✓ Uncharged black hole mass and extra dimension size in terms of observational quantities in squashed Kaluza-Klein spacetime

$X := (1+z_1)(1+z_2)$, $Y := (2+z_1+z_2)^2$:

$$M = \frac{\rho(X-1)}{2X}, \quad r_\infty = \frac{2\rho\sqrt{2(2X(X+1)-Y)(X(Y+8)-3Y)/X}}{|X(Y+4)-2Y|}$$

Summary

We study motions of massive and massless test particles around astrophysical compact objects in five-dimensional charged static squashed Kaluza-Klein black hole spacetime.

- We derive total frequency shifts of photons emitted by uncharged massive test particles around compact objects with corrections by extra dimension and Maxwell field.

✓ These corrections for supermassive black holes would not appear to be relevant for present and near future observations.

- If precise astrophysical observations of these phenomena by compact objects agree with expected values of general relativity, it would require rigorous upper limits of size of extra dimension, or it would exclude squashed Kaluza-Klein metric for describing geometry around such objects.

Appendix

Periapsis shift by squashed Kaluza-Klein black hole

- Energy conservation equation of charged particle ($u=1/\rho$):

$$\left(\frac{du}{d\phi}\right)^2 = \frac{\hat{E}^2-1}{\hat{L}^2} + \frac{2M+\left(\hat{E}^2-1\right)\rho_0 - 2\hat{q}\hat{E}Q}{\hat{L}^2}u + \left(2M - \frac{\left(1-\hat{q}^2\right)Q^2\rho_0}{\hat{L}^2}\right)u^3 - \left(1+\frac{\left(1-\hat{q}^2\right)Q^2 - 2M\rho_0 + 2\hat{q}\hat{E}Q\rho_0}{\hat{L}^2}\right)u^2 - Q^2u^4$$

- Bound orbit of charged particle (l: semilatus rectum, e: eccentricity):

$$\begin{cases} \dfrac{l}{\rho} = 1 + e\cos(\gamma\phi) + \dfrac{\sigma e^2 M \sin^2(\gamma\phi)}{l} + O\left(\dfrac{M^2}{l^2}\right) \\ \gamma = 1 - \dfrac{\alpha M}{l} - \dfrac{\beta M^2}{l^2}, \quad \alpha,\beta,\sigma = \text{const.} \end{cases}$$

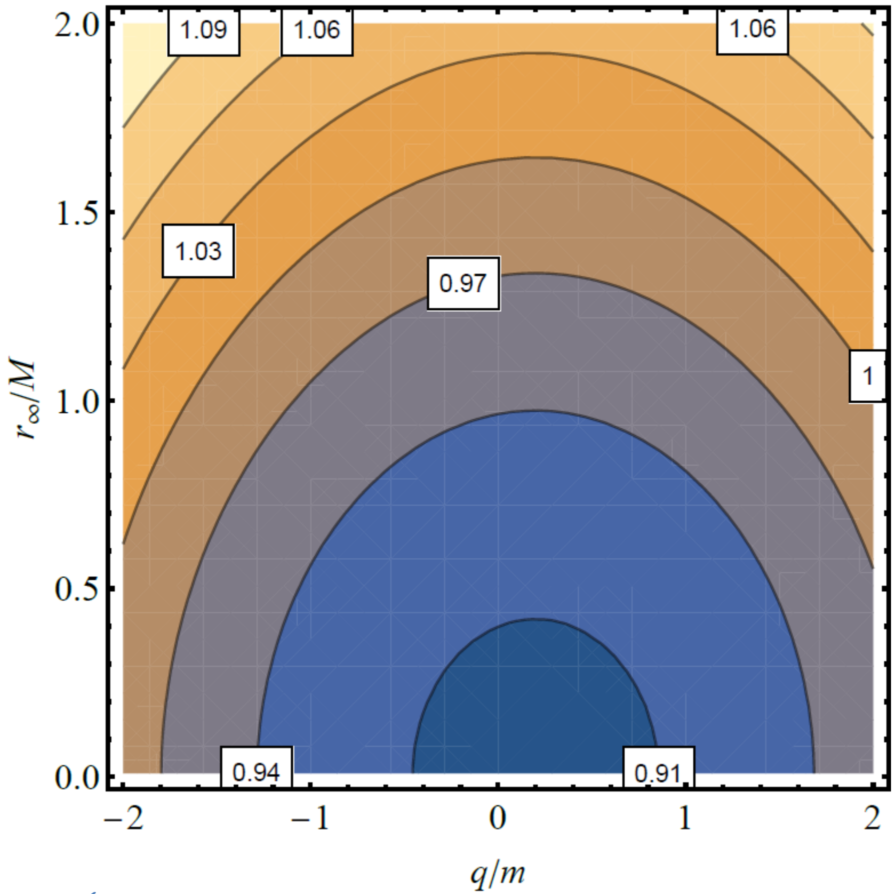
✓ Periapsis shift angle of charged test particle:

$$\delta\phi_{\mathrm{KK}} = \frac{\pi M}{l}\left[6 + \frac{2\rho_0}{M} - \frac{Q^2}{M^2} - \frac{qQ^3}{mM^3} + \frac{q^2Q^2}{m^2M^2}\left(1-\frac{Q^2}{M^2}\right)\right] + O\left(\frac{M^2}{l^2}, \frac{q^3}{m^3}\right)$$

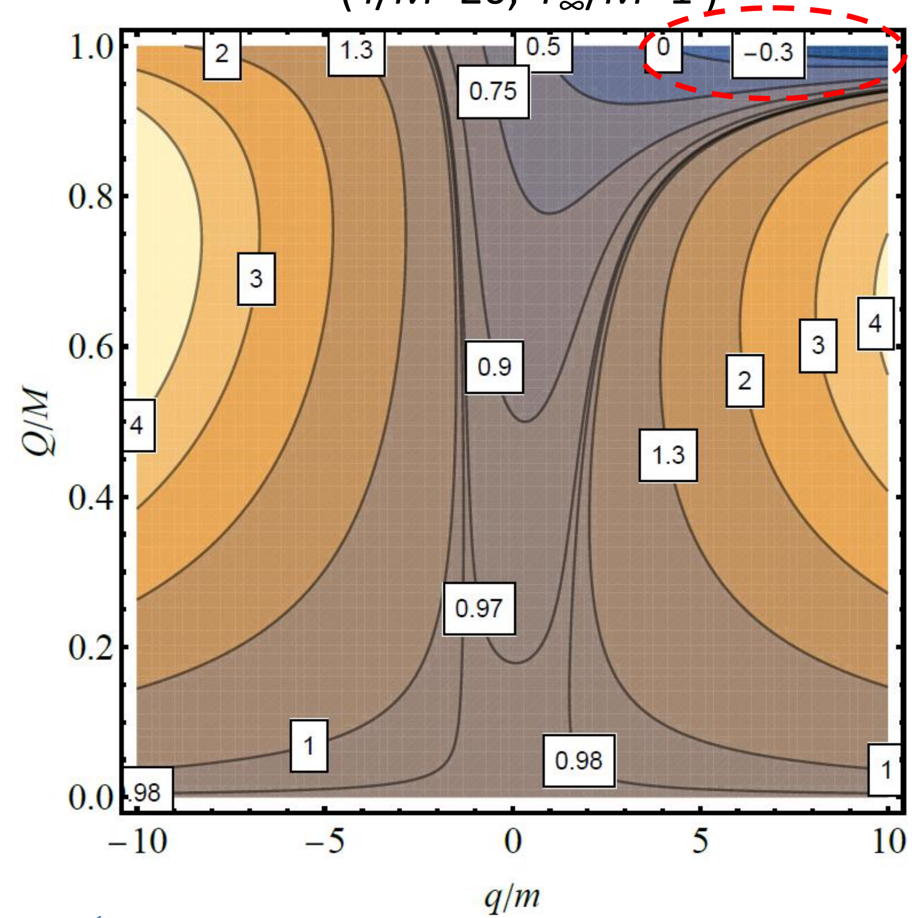
Periapsis shift angle in charged Kaluza-Klein spacetime

$$\delta\phi_{\rm KK} = \frac{\pi M}{l}\left[6 + \frac{2\rho_0}{M} - \frac{Q^2}{M^2} - \frac{qQ^3}{mM^3} + \frac{q^2Q^2}{m^2M^2}\left(1 - \frac{Q^2}{M^2}\right)\right] + O\left(\frac{M^2}{l^2}, \frac{q^3}{m^3}\right)$$

(l/M=20, Q/M=0.35)



(l/M=20, r_∞/M=1)



✓ Periapsis shift angle increases with increasing extra dimension size r_∞/M.

✓ Negative periapsis shift angle exists in the case of large Q/M and q/m.

Observational effects in periapsis shift

- Difference between shift angles in sq. KK spacetime and in 4D Schwarzschild spacetime:

$$\frac{\delta\phi_{\mathrm{KK}} - \delta\phi_{\mathrm{Sch}}}{\delta\phi_{\mathrm{Sch}}} \simeq -\tilde{\delta}_1 + \tilde{\delta}_2, \quad \tilde{\delta}_1 := \frac{Q^2}{3M^2}, \quad \tilde{\delta}_2 := \frac{r_\infty^2}{24M^2}$$

$\tilde{\delta}_1 \lesssim 10^{-6},\ \tilde{\delta}_2 \simeq 10^{-16}$: Mercury around Sun
$\left(Q \lesssim 4 \times 10^{17}\ \mathrm{C},\ r_\infty \simeq 0.1\ \mathrm{mm}\right)$

$\tilde{\delta}_1 \lesssim 10^{-5},\ \tilde{\delta}_2 \simeq 10^{-5}$: LAGEOS II satellite around Earth
$\left(Q \lesssim 3 \times 10^{12}\ \mathrm{C},\ r_\infty \simeq 0.1\ \mathrm{mm}\right)$

$\tilde{\delta}_1 \lesssim 10^{-37},\ \tilde{\delta}_2 \simeq 10^{-29}$: S2 around Sgr A*
$\left(Q \lesssim 3 \times 10^{8}\ \mathrm{C},\ r_\infty \simeq 0.1\ \mathrm{mm}\right)$

✓ Correction by extra dimension might be detected in future observations around Earth.